Factorization and resummation for generic jet hierarchies

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based on work with
Frank Tackmann, Wouter Waalewijn

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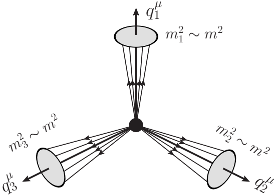
Collider processes with fixed jet multiplicity

- typically considered in SCET: energetic and well-separated jets

$e^+e^- \to 3$ jets:



$q_i^\mu = Q_i \frac{n_i^\mu}{2}$

energetic: $Q_i \sim Q$

well-separated: $n_i \cdot n_j \sim 1$

$\to m$ fixed by arbitrary jet resolution variable

$\to$ here for definiteness: 3-jettiness (SCET I) $\quad \mathcal{T}_3 = \sum_k \min_i \left\{ \frac{2 q_i \cdot p_k}{Q_i} \right\}$

[Stewart, Tackmann, Waalewijn (2010)]

$\to \ln(m^2/Q^2) \sim \ln(\mathcal{T}_3/Q)$ resummed by standard SCET framework

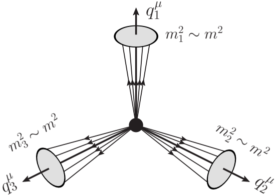
- common experimental cuts: hierarchies between the jets (e.g. $p_{T,1} \gg p_{T,2}$)

  not addressed in this talk:
  - hierarchies between two resolution measurements on the same jet $\to$ Lisa's talk
  - hierarchies between resolution measurements on different jets $\to$ Duff's talk

Jets with hierarchies

- typically considered in SCET: energetic and well-separated jets

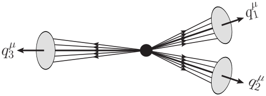
$e^+e^- \to 3$ jets:



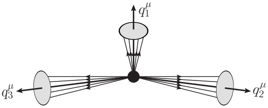
$$q_i^\mu = Q_i \frac{n_i^\mu}{2}$$

$$Q_i \sim Q,\; n_i \cdot n_j \sim 1$$

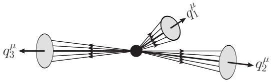
- generic additional hierarchies for $e^+e^- \to 3$ jets:



$n_1 \cdot n_2 \ll 1$



$Q_1 \ll Q$



$n_1 \cdot n_2 \ll 1$, $Q_1 \ll Q$

$\to$ aim: set up factorization theorems for all hierarchies and recombine them

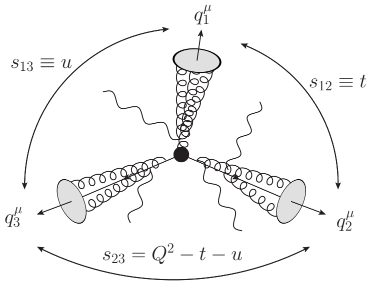
$\Rightarrow$ SCET$_+$

Outline

1. Hard, well-separated jets
2. Two jets close to each other
3. One soft jet
4. One soft jet close to a hard jet
5. Combining all EFTs
6. Generalizations
7. Summary

Outline

Kinematic setup & modes



$$q_i^\mu = Q_i \frac{n_i^\mu}{2},\ s_{ij} \equiv 2 q_i \cdot q_j \gg m^2$$

labelling of jets:

$$t \equiv s_{12} < u \equiv s_{13} < s_{23} \sim Q^2$$

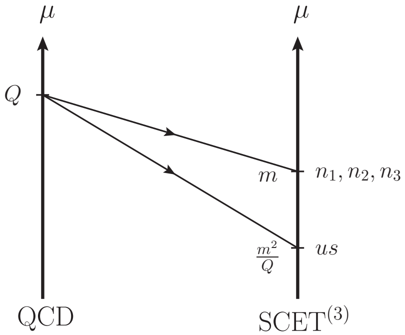
- Hard, well-separated jets: $Q_i \sim Q, \quad n_i \cdot n_j \sim 1 \longleftrightarrow \quad t \sim u \sim Q^2$
- modes ($m^2 \sim Q\mathcal{T}_3$):

mode	$p^\mu = (+, -, \perp)$	$\sqrt{p^2}$
collinear (n_1, n_2, n_3)	$\left(\frac{m^2}{Q}, Q, m\right)$	m
usoft	$\left(\frac{m^2}{Q}, \frac{m^2}{Q}, \frac{m^2}{Q}\right)$	$\frac{m^2}{Q}$

Factorization

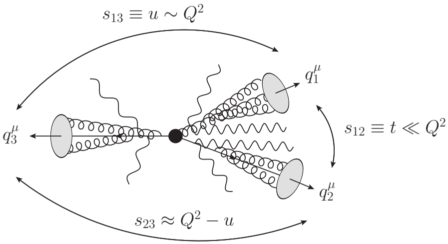
Factorization theorem for $t \sim u \sim Q^2$:

$$\frac{d\sigma_3}{dt\, du\, d\mathcal{T}_3} = H_3(t, u, Q^2, \mu) \prod_{i=1}^{3} J_i(Q_i \mathcal{T}_3, \mu) \otimes S_3\left(\frac{\mathcal{T}_3}{\sqrt{n_i \cdot n_j}}, \mu\right) + \mathcal{O}\left(\frac{m^2}{Q^2}\right)$$



Outline

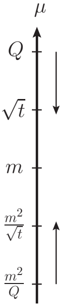
Kinematic setup & modes



jets 1 and 2 close to each other:

$Q_i \sim Q$, $n_1 \cdot n_2 \ll 1$

$\longleftrightarrow m^2 \ll t \ll u \sim Q^2$



unresummed logs in SCET$^{(3)}$:

$\ln(t/Q^2)$ in H_3 and $\ln(n_1 \cdot n_2) \sim \ln(t/Q^2)$ in S_3

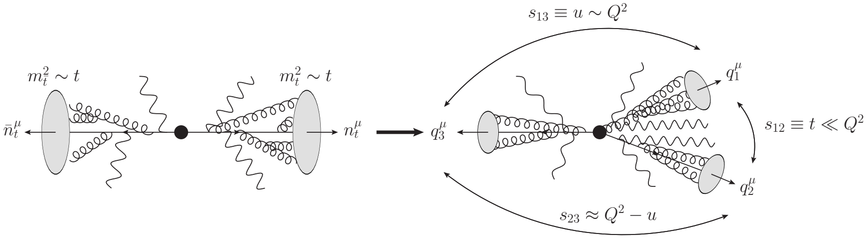
$\Rightarrow$ extension: SCET$_{c+}$ [Bauer, Tackmann, Walsh, Zuberi (2011)]

mode	$p^\mu = (+, -, \perp)$	$\sqrt{p^2}$
collinear (n_1, n_2, n_3)	$\left(\frac{m^2}{Q}, Q, m\right)$	m
csoft$_{n_1,n_2}$	$\left(\frac{m^2}{Q}, \frac{Qm^2}{t}, \frac{m^2}{\sqrt{t}}\right)$	$\frac{m^2}{\sqrt{t}}$
usoft	$\left(\frac{m^2}{Q}, \frac{m^2}{Q}, \frac{m^2}{Q}\right)$	$\frac{m^2}{Q}$

Factorization in SCET$_{c+}$

[Bauer, Tackmann, Walsh, Zuberi (2011)]

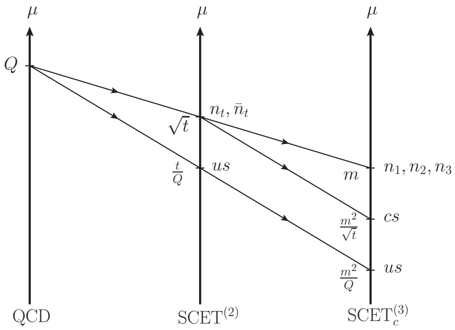
construction of SCET$_{c+}$: conveniently via two-step matching procedure



$\rightarrow$ jets 1 and 2 originate from n_t-collinear sector in SCET$^{(2)}$ with $p^\mu_{n_t} \sim (\frac{t}{Q}, Q, \sqrt{t})$

Factorization in SCET$_{c+}$

- construction of SCET$_{c+}$: conveniently via two-step matching procedure



- decoupled Lagrangian after field redefinitions

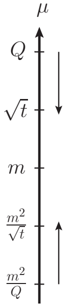
$$\mathcal{L}^{(c)} = \sum_{i=1}^{3} \mathcal{L}_{n_i} + \mathcal{L}_{cs} + \mathcal{L}_{us} \,.$$

$\mathcal{L}_{cs}$: csoft Lagrangian = collinear Lagrangian with
$\rightarrow$ label direction $n_t \sim n_1 \sim n_2$
$\rightarrow$ label momentum $\sim Q \times m^2/t \ll Q$

Factorization in SCET$_{c+}$

Factorization theorem for $t \ll u \sim Q^2$:

$$\frac{\mathrm{d}\sigma_c}{\mathrm{d}t\,\mathrm{d}u\,\mathrm{d}\mathcal{T}_3} = H_2(Q^2,\mu)\, H_c\left(t, \frac{u}{Q^2}, \mu\right) \prod_{i=1}^{3} J_i(Q_i \mathcal{T}_3, \mu) \otimes S_c\left(\frac{\mathcal{T}_3}{\sqrt{n_1 \cdot n_2}}, \mu\right) \otimes S_2(\mathcal{T}_3, \mu) + \mathcal{O}\left(\frac{m^2}{t}, \frac{t}{Q^2}\right)$$



Ingredients:

- $H_2(Q^2,\mu)$: dijet hard function
- $H_c(t, \frac{u}{Q^2}, \mu)$: $\sim$ collinear splitting amplitude
- $J_i(Q_i \mathcal{T}_3, \mu)$: jet functions (same as in SCET$^{(3)}$)
- $S_c(\frac{\mathcal{T}_3}{\sqrt{n_1 \cdot n_2}}, \mu)$: csoft function resolving the jets 1 and 2
- $S_2(\mathcal{T}_3, \mu)$: soft function resolving the well-separated jets (12) and 3

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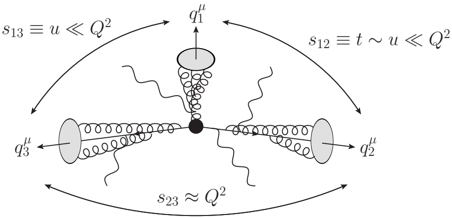
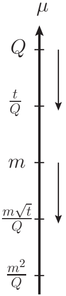
Kinematic setup & modes



jet 1 is soft:

$Q_1 \ll Q$, $n_i \cdot n_j \sim 1$

$\longleftrightarrow m^2 \ll t \sim u \ll Q^2$



unresummed logs in SCET$^{(3)}$: $\ln(t/Q^2)$ in H_3

$\Rightarrow$ extension: SCET$_{s+}$ [P.P., Tackmann, Waalewijn (in preparation)]

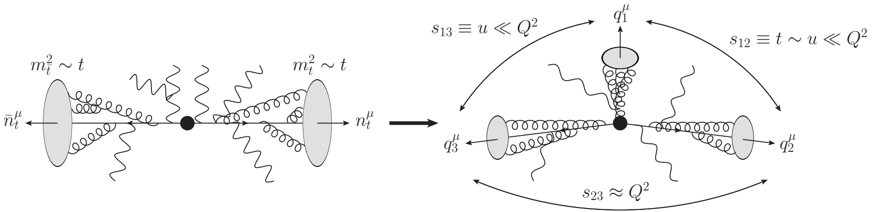
[see also: Larkoski, Moult, Neill (2015)]

mode	$p^\mu = (+, -, \perp)$	$\sqrt{p^2}$
collinear (n_2, n_3)	$\left(\frac{m^2}{Q}, Q, m\right)$	m
soft-collinear (n_1)	$\left(\frac{m^2}{Q}, \frac{t}{Q}, \frac{m\sqrt{t}}{Q}\right)$	$\frac{m\sqrt{t}}{Q}$
usoft	$\left(\frac{m^2}{Q}, \frac{m^2}{Q}, \frac{m^2}{Q}\right)$	$\frac{m^2}{Q}$

Factorization in SCET$_{s+}$

[P.P., Tackmann, Waalewijn (in preparation)]
[Larkoski, Moult, Neill (2015)]

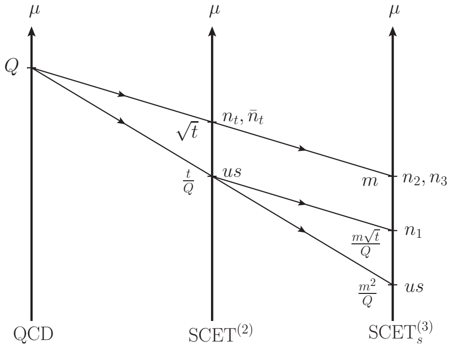
construction of SCET$_{s+}$: use again two-step matching procedure



$\rightarrow$ jet 1 originates from usoft sector in SCET$^{(2)}$ with $p_{us}^\mu \sim (\frac{t}{Q}, \frac{t}{Q}, \frac{t}{Q})$

Factorization in SCET$_{s+}$

- construction of SCET$_{s+}$: use again two-step matching procedure



- decoupled Lagrangian after field redefinitions

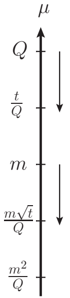
$$\mathcal{L}^{(s)} = \sum_{i=2}^{3} \mathcal{L}_{n_i} + \mathcal{L}_{n_1} + \mathcal{L}_{us} \,.$$

$\mathcal{L}_{n_1}$: soft-collinear Lagrangian = collinear Lagrangian with
$\rightarrow$ label momentum $\sim t/Q \ll Q$

Factorization in SCET_{s+}

Factorization theorem for $t \sim u \ll Q^2$:

$$\frac{d\sigma_s}{dt\,du\,d\mathcal{T}_3} = H_2(Q^2,\mu)\, H_s\left(\frac{t\,u}{Q^2},\mu\right) \prod_{i=2}^{3} J_i(Q_i\mathcal{T}_3,\mu) \otimes J_1(Q_1\mathcal{T}_3,\mu) \otimes S_3\left(\frac{\mathcal{T}_3}{\sqrt{n_i \cdot n_j}},\mu\right) + \mathcal{O}\left(\frac{m^2}{t},\frac{u}{Q^2}\right)$$

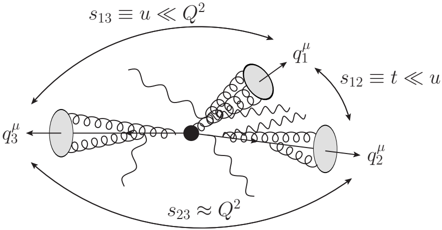


Ingredients:

- $H_2(Q^2,\mu)$: dijet hard function
- $H_s(\frac{t\,u}{Q^2},\mu)$: $\sim$ soft splitting amplitude
  up to two loops extracted from [Li, Zhu (2013); Duhr (2013)]
- $J_{2/3}(Q_{2/3}\mathcal{T}_3,\mu) = J_{q/\bar{q}}$: (anti-)quark jet functions
- $J_1(Q_1\mathcal{T}_3,\mu) = J_g$: gluon jet function
- $S_3(\frac{\mathcal{T}_3}{\sqrt{n_i \cdot n_j}}\mu)$: soft function as in $\mathrm{SCET}^{(3)}$

Outline

Kinematic setup & modes

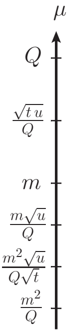


jet 1 is soft and close to jet 2:

$n_1 \cdot n_2 \ll 1$, $Q_1 \ll Q$

$\longleftrightarrow t \ll u$, $u \ll Q^2$

(with $t \gg m^2$)



unresummed logs in SCET$_{c+}$: $\ln(u/Q^2)$ in H_c

unresummed logs in SCET$_{s+}$: $\ln(t/u)$ in S_3

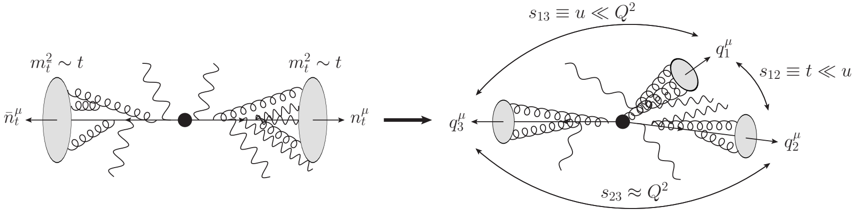
$\Rightarrow$ extension: SCET$_{cs+}$ [P.P., Tackmann, Waalewijn (in preparation)]

mode	$p^\mu = (+, -, \perp)$	$\sqrt{p^2}$
collinear (n_2, n_3)	$\left(\frac{m^2}{Q}, Q, m\right)$	m
soft-collinear (n_1)	$\left(\frac{m^2}{Q}, \frac{u}{Q}, \frac{m\sqrt{u}}{Q}\right)$	$\frac{m\sqrt{u}}{Q}$
csoft$_{n_1,n_2}$	$\left(\frac{m^2}{Q}, \frac{m^2 u}{Q\,t}, \frac{m^2\sqrt{u}}{Q\sqrt{t}}\right)$	$\frac{m^2\sqrt{u}}{Q\sqrt{t}}$
usoft	$\left(\frac{m^2}{Q}, \frac{m^2}{Q}, \frac{m^2}{Q}\right)$	$\frac{m^4}{Q^2}$

Factorization in SCET$_{cs+}$

[P.P., Tackmann, Waalewijn (in preparation)]

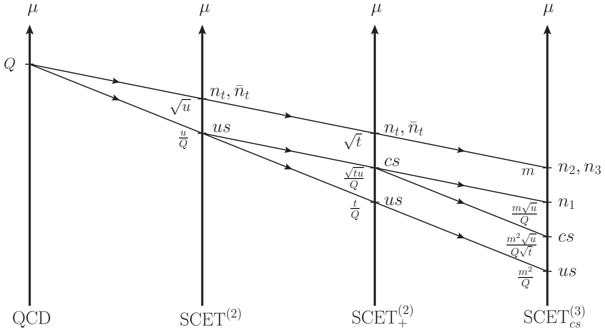
construction of SCET$_{cs+}$: again via multi-step matching procedure



$\rightarrow$ jet 1 originates from additional csoft sector in SCET$^{(2)}$ with $p_{cs}^\mu \sim (\frac{t}{Q}, \frac{u}{Q}, \frac{\sqrt{tu}}{Q})$

Factorization in SCET$_{cs+}$

- construction of SCET$_{cs+}$: again via multi-step matching procedure



- decoupled Lagrangian after field redefinitions

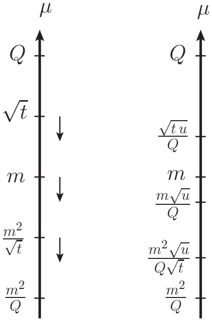
$$\mathcal{L}^{(cs)} = \sum_{i=2}^{3} \mathcal{L}_{n_i} + \mathcal{L}_{n_1} + \mathcal{L}_{cs} + \mathcal{L}_{us}\,.$$

$\mathcal{L}_{n_1}$, $\mathcal{L}_{cs}$ = collinear Lagrangians with
$\to$ label momenta $\sim Q \times u/Q \ll Q$ (soft-coll.), $\sim u/Q \times m^2/t \ll u/Q$ (csoft)
$\to$ label direction for csoft modes: $n_t \sim n_1 \sim n_2$

Factorization in SCET_{cs+}

Factorization theorem for $t \ll u$ & $u \ll Q^2$:

$$\frac{\mathrm{d}\sigma_{cs}}{\mathrm{d}t\,\mathrm{d}u\,\mathrm{d}\mathcal{T}_3} = H_2(Q^2,\mu)\, H_{cs}\left(\frac{t\,u}{Q^2},\mu\right) \prod_{i=2}^{3} J_i(Q_i\mathcal{T}_3,\mu) \otimes J_1(Q_1\mathcal{T}_3,\mu)$$
$$\otimes\, S_c\left(\frac{\mathcal{T}_3}{\sqrt{n_1\cdot n_2}},\mu\right) \otimes S_2(\mathcal{T}_3,\mu) + \mathcal{O}\left(\frac{m^2}{t},\frac{t}{u},\frac{u}{Q^2}\right)$$



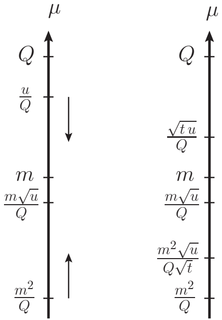
compared to SCET_{c+} ($t \ll u \sim Q^2$):

- $H_c(t, \frac{u}{Q^2},\mu) \xrightarrow{u \ll Q^2} H_{cs}(\frac{t\,u}{Q^2},\mu)$
- virtuality of $J_1 = J_g$ and S_c lowered

Factorization in SCET_{cs+}

Factorization theorem for $t \ll u$ & $u \ll Q^2$:

$$\frac{d\sigma_{cs}}{dt\,du\,d\mathcal{T}_3} = H_2(Q^2,\mu)\,H_{cs}\left(\frac{t\,u}{Q^2},\mu\right)\prod_{i=2}^{3} J_i(Q_i\mathcal{T}_3,\mu)\otimes J_1(Q_1\mathcal{T}_3,\mu)$$
$$\otimes\, S_c\left(\frac{\mathcal{T}_3}{\sqrt{n_1\cdot n_2}},\mu\right)\otimes S_2(\mathcal{T}_3,\mu)+\mathcal{O}\left(\frac{m^2}{t},\frac{t}{u},\frac{u}{Q^2}\right)$$



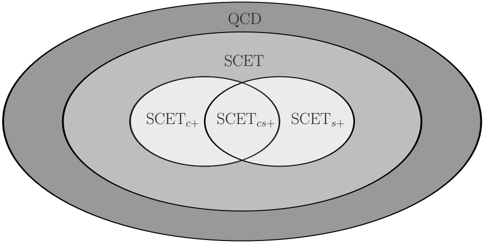
compared to SCET_{s+} ($t \sim u \ll Q^2$):

- $H_s(\frac{t\,u}{Q^2},\mu) \xrightarrow{t\ll u} H_{cs}(\frac{t\,u}{Q^2},\mu)$
  $\rightarrow$ here (trivial color): $H_{cs}(\frac{t\,u}{Q^2},\mu) = H_s(\frac{t\,u}{Q^2},\mu)$
- $S_3(\frac{\mathcal{T}_3}{\sqrt{n_i\cdot n_j}},\mu) \xrightarrow{t\ll u} S_c(\frac{\mathcal{T}_3}{\sqrt{n_1\cdot n_2}},\mu)\otimes S_2(\mathcal{T}_3,\mu)$

Outline

Result with full fixed-order content & log-resummation

- fixed order content of the theories:



- diff. cross section resumming all kinematic logs with full fixed-order content:

$$\mathrm{d}\sigma = \mathrm{d}\sigma_{cs} + \mathrm{d}\sigma_c^{ns} + \mathrm{d}\sigma_s^{ns} + \mathrm{d}\sigma_3^{ns} + \mathrm{d}\sigma_{\mathrm{QCD}}^{ns}$$

$\rightarrow \mathrm{d}\sigma_{cs}$: resums all logs $\ln(m^2/t)$, $\ln(t/u)$, $\ln(u/Q^2)$

$\rightarrow \mathrm{d}\sigma_c^{ns} = \mathrm{d}\sigma_c - \mathrm{d}\sigma_{cs}|_{u\sim Q^2}$: resums logs $\ln(m^2/t)$, $\ln(t/u)$

$\rightarrow \mathrm{d}\sigma_s^{ns} = \mathrm{d}\sigma_s - \mathrm{d}\sigma_{cs}|_{t\sim u}$: resums logs $\ln(m^2/t)$, $\ln(u/Q^2)$

$\rightarrow \mathrm{d}\sigma_3^{ns} = \mathrm{d}\sigma_3 - \mathrm{d}\sigma_c|_{t\sim Q^2} - \mathrm{d}\sigma_s|_{u\sim Q^2} + \mathrm{d}\sigma_{cs}|_{t\sim u\sim Q^2}$: resums logs $\ln(m^2/Q^2)$

$\rightarrow \mathrm{d}\sigma_{\mathrm{QCD}}^{ns} = \mathrm{d}\sigma_{\mathrm{QCD}} - \mathrm{d}\sigma_3|_{m^2\sim Q^2}$: only fixed-order

Outline

$pp \to N$ jets: One nonstandard hierarchy

- additional complications compared to $e^+e^- \to 3$ jets:
  $\to$ initial state radiation, more involved kinematics, color correlations
- well-separated energetic jets:

$$\mathrm{d}\sigma_N \sim \mathrm{Tr}\left[\hat{H}_N \times \mathcal{B}_a \otimes \mathcal{B}_b \otimes \prod_{i=1}^{N} J_i \otimes \hat{S}_N\right]$$

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$$\mathrm{d}\sigma_N \sim \mathrm{Tr}\left[\hat{H}_N \times \mathcal{B}_a \otimes \mathcal{B}_b \otimes \prod_{i=1}^{N} J_i \otimes \hat{S}_N\right]$$

- two close-by jets or jet close to beam: [Bauer, Tackmann, Walsh, Zuberi (2011)]

$$\mathrm{d}\sigma_c \sim \mathrm{Tr}\left[\hat{H}_{N-1} \times H_c \times \mathcal{B}_a \otimes \mathcal{B}_b \otimes \prod_{i=1}^{N} J_i \otimes S_c \otimes \hat{S}_{N-1}\right]$$

$pp \to N$ jets: One nonstandard hierarchy

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$$\mathrm{d}\sigma_c \sim \mathrm{Tr}\left[\hat{H}_{N-1} \times H_c \times \mathcal{B}_a \otimes \mathcal{B}_b \otimes \prod_{i=1}^{N} J_i \otimes S_c \otimes \hat{S}_{N-1}\right]$$

- one soft jet:

$$\mathrm{d}\sigma_s \sim \mathrm{Tr}\left[\hat{C}_{N-1,s} \times \hat{H}_{N-1} \times \hat{C}^\dagger_{N-1,s} \times \mathcal{B}_a \otimes \mathcal{B}_b \otimes \prod_{i=2}^{N} J_i \otimes J_1 \otimes \hat{S}_N\right]$$

$pp \to N$ jets: One nonstandard hierarchy

- additional complications compared to $e^+e^- \to 3$ jets:
  $\to$ initial state radiation, more involved kinematics, color correlations
- well-separated energetic jets:

$$\mathrm{d}\sigma_N \sim \mathrm{Tr}\left[\hat{H}_N \times \mathcal{B}_a \otimes \mathcal{B}_b \otimes \prod_{i=1}^{N} J_i \otimes \hat{S}_N\right]$$

- two close-by jets or jet close to beam: [Bauer, Tackmann, Walsh, Zuberi (2011)]

$$\mathrm{d}\sigma_c \sim \mathrm{Tr}\left[\hat{H}_{N-1} \times H_c \times \mathcal{B}_a \otimes \mathcal{B}_b \otimes \prod_{i=1}^{N} J_i \otimes S_c \otimes \hat{S}_{N-1}\right]$$

- one soft jet:

$$\mathrm{d}\sigma_s \sim \mathrm{Tr}\left[\hat{C}_{N-1,s} \times \hat{H}_{N-1} \times \hat{C}^{\dagger}_{N-1,s} \times \mathcal{B}_a \otimes \mathcal{B}_b \otimes \prod_{i=2}^{N} J_i \otimes J_1 \otimes \hat{S}_N\right]$$

- one soft jet close to a jet or beam:

$$\mathrm{d}\sigma_{cs} \sim \mathrm{Tr}\left[\hat{H}_{N-1} \times H_{cs} \times \mathcal{B}_a \otimes \mathcal{B}_b \otimes \prod_{i=2}^{N} J_i \otimes J_1 \otimes S_c \otimes \hat{S}_{N-1}\right]$$

$pp \to N$ jets: More hierarchies

- strongly ordered limit: iterations of single hard splittings, e.g. for M soft jets with $Q_1 \ll Q_2 \ll \cdots \ll Q_M \ll Q$:

$$\mathrm{d}\sigma \sim \mathrm{Tr}\left[\hat{C}_{N-1,s} \ldots \hat{C}_{N-M,s} \times \hat{H}_{N-M} \times \hat{C}^{\dagger}_{N-M,s} \ldots \hat{C}^{\dagger}_{N-1,s}\, \mathcal{B}_a \otimes \mathcal{B}_b \otimes \prod_{i=1}^{N} J_i \otimes \hat{S}_N\right]$$

$pp \to N$ jets: More hierarchies

- strongly ordered limit: iterations of single hard splittings, e.g. for M soft jets with $Q_1 \ll Q_2 \ll \cdots \ll Q_M \ll Q$:

$$\mathrm{d}\sigma \sim \mathrm{Tr}\left[\hat{C}_{N-1,s} \ldots \hat{C}_{N-M,s} \times \hat{H}_{N-M} \times \hat{C}^{\dagger}_{N-M,s} \ldots \hat{C}^{\dagger}_{N-1,s}\, \mathcal{B}_a \otimes \mathcal{B}_b \otimes \prod_{i=1}^{N} J_i \otimes \hat{S}_N\right]$$

- not strongly ordered: multiple splittings at a single scale, e.g. for M soft jets with $Q_1 \sim Q_2 \sim \cdots \sim Q_M \ll Q$:

$$\mathrm{d}\sigma \sim \mathrm{Tr}\left[\hat{C}_{N-M,\underbrace{ss\ldots s}_{M}} \times \hat{H}_{N-M} \times \hat{C}^{\dagger}_{N-M,\underbrace{ss\ldots s}_{M}} \times \mathcal{B}_a \otimes \mathcal{B}_b \otimes \prod_{i=1}^{N} J_i \otimes \hat{S}_N\right]$$

$\hat{C}_{N-M,\underbrace{ss\ldots s}_{M}}$: $1 \to M+1$ soft splitting amplitudes

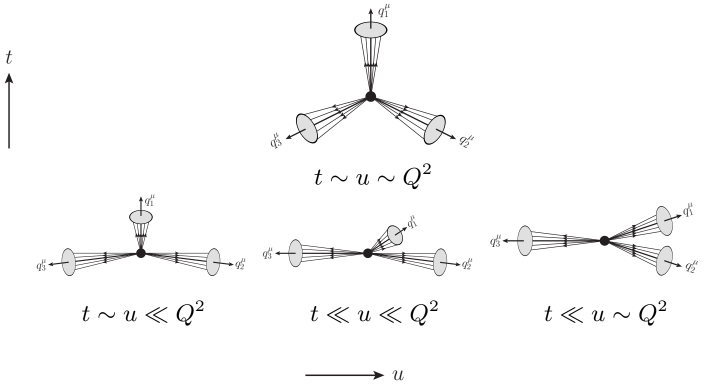
(for M jets close to each other: $1 \to M$ collinear splitting amplitudes)

$\Rightarrow$ results for all hierarchies can be obtained from these cases

Outline

Summary

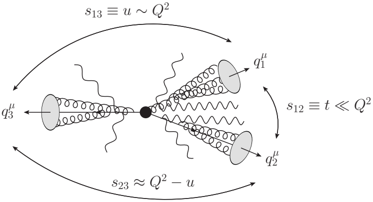
- EFT framework for generic hierarchies between jets: SCET_+



- all scales are disentangled in collinear+soft limit
  $\rightarrow$ resummation of all potentially large kinematic logarithms possible
- systematic incorporation of power corrections via sequence of nonsingular terms

Outline

Scaling of the modes: SCET$_{c+}$ ($t \ll u \sim Q^2$)



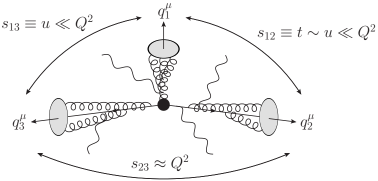
scaling of the csoft mode

- csoft mode has to be collinear to resolve the jets 1 and 2
  $\to p_{cs}^\mu \sim Q_{cs}(\lambda_{cs}^2, 1, \lambda_{cs})$ with $\lambda_{cs}^2 \sim t/Q^2$
- csoft mode has to contribute to measurement
  $\to p_{cs}^+ \sim \mathcal{T}_3$

$$\Rightarrow p_{cs}^\mu \sim \frac{Q^2}{t}\,\mathcal{T}_3\left(\frac{t}{Q^2}, 1, \frac{\sqrt{t}}{Q}\right)$$

mode	$p^\mu = (+, -, \perp)$	p^2
collinear (n_1, n_2, n_3)	$Q\left(\frac{\mathcal{T}_3}{Q}, 1, \sqrt{\frac{\mathcal{T}_3}{Q}}\right)$	$Q\mathcal{T}_3$
csoft$_{n_1,n_2}$	$\frac{Q^2}{t}\mathcal{T}_3\left(\frac{t}{Q^2}, 1, \frac{\sqrt{t}}{Q}\right)$	$\frac{Q^2}{t}\mathcal{T}_3^2$
usoft	$\mathcal{T}_3\left(1, 1, 1\right)$	$\mathcal{T}_3^2$

Scaling of the modes: SCET$_{s+}$ ($t \sim u \ll Q^2$)



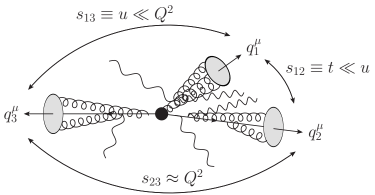
scaling of the soft-collinear mode $Q_{n_1}(\lambda_{n_1}^2, 1, \lambda_{n_1})$

- soft jet leads to dijet invariant masses $\sim t$: $p_{n_1} \cdot p_{n_i} \sim t$ $\to p_{n_1}^- \sim t/Q$
- soft-collinear mode has to contribute to measurement $\to p_{n_1}^+ \sim \mathcal{T}_3$

$$\Rightarrow p_{n_1}^\mu \sim \left(\mathcal{T}_3, \frac{t}{Q}, \sqrt{\frac{t}{Q}\mathcal{T}_3}\right)$$

mode	$p^\mu = (+, -, \perp)$	p^2
collinear (n_2, n_3)	$Q\left(\frac{\mathcal{T}_3}{Q}, 1, \sqrt{\frac{\mathcal{T}_3}{Q}}\right)$	$Q\mathcal{T}_3$
soft-collinear (n_1)	$\frac{t}{Q}\left(\frac{Q\mathcal{T}_3}{t}, 1, \sqrt{\frac{Q\mathcal{T}_3}{t}}\right)$	$\frac{t}{Q^2}Q\mathcal{T}_3$
usoft	$\mathcal{T}_3\Big(1, 1, 1\Big)$	$\mathcal{T}_3^2$

Scaling of the modes: SCET$_{cs+}$ ($t \sim u \ll Q^2$)



scaling of the soft-collinear mode $Q_{n_1}(\lambda_{n_1}^2, 1, \lambda_{n_1})$

- soft jet leads to the large dijet invariant mass u: $p_{n_1} \cdot p_{n_3} \sim u \to p_{n_1}^- \sim u/Q$
- soft-collinear mode has to contribute to measurement $\to p_{n_1}^+ \sim \mathcal{T}_3$

$$\Rightarrow p_{n_1}^\mu \sim \left(\mathcal{T}_3, \tfrac{u}{Q}, \sqrt{\tfrac{u}{Q}\mathcal{T}_3}\right)$$

scaling of the csoft mode

- csoft mode has to be collinear to resolve the jets 1 and 2 $\to p_{cs}^\mu \sim Q_{cs}(\lambda_{cs}^2, 1, \lambda_{cs})$ with $\lambda_{cs}^2 \sim t/u$
- csoft mode has to contribute to measurement $\to p_{cs}^+ \sim \mathcal{T}_3$

$$\Rightarrow p_{cs}^\mu \sim \mathcal{T}_3 \tfrac{u}{t}\left(\tfrac{t}{u}, 1, \sqrt{\tfrac{t}{u}}\right)$$

Current operators

- current in SCET$^{(2)}$

$$\mathcal{J}_2 = \bar{\chi}_{n_t} Y_{n_t}^\dagger \, \Gamma \, Y_{\bar{n}_t} \chi_{\bar{n}_t} \ ,$$

- current in SCET$^{(3)}$ and in SCET$_{s+}$ (difference only in the label momenta)

$$\mathcal{J}_3^\mu = \bar{\chi}_{n_q} Y_{n_q}^\dagger Y_{n_g} \, \Gamma \, \mathcal{B}^\mu_{n_g \perp,} Y_{n_g}^\dagger Y_{n_{\bar{q}}} \chi_{n_{\bar{q}}} \ .$$

- current in SCET$_{c+}$ and in SCET$_{cs+}$ (difference only in the label momenta)

$$\mathcal{J}_{3,c}^\mu = \bar{\chi}_{n_q} \mathcal{B}^{\mu,A}_{n_g \perp} \, \Gamma \, \left[X_{n_q}^\dagger X_{n_g} T^A X_{n_g}^\dagger V_{n_t} \right] \left[Y_{n_t}^\dagger Y_{n_{\bar{q}}} \right] \chi_{n_{\bar{q}}} \ ,$$

- current in SCET$_+^{(2)}$

$$\mathcal{J}_{2+} = \bar{\chi}_{n_t} X_{n_t}^\dagger V_{n_t} Y_{n_t}^\dagger \, \Gamma \, Y_{\bar{n}_t} \chi_{\bar{n}_t} \ ,$$

Matrix elements in SCET$_{c+}$

- csoft function

$$S_c^{\{g,q,\bar{q}\}}(k_1,k_2,\mu) = \frac{1}{N_C\,C_F}\,\langle 0|\bar{T}\left[V_{n_t}^\dagger X_{n_1} T^A X_{n_1}^\dagger X_{n_2}\right]\delta\left(k_1\sqrt{\hat{s}_t} - n_1\cdot\hat{P}_1\right)$$
$$\times\,\delta\left(k_2\sqrt{\hat{s}_t} - n_2\cdot\hat{P}_2\right) T\left[X_{n_2}^\dagger X_{n_1} T^A X_{n_1}^\dagger V_{n_t}\right]|0\rangle\,,$$

- Wilson lines ($V_n \leftrightarrow W_n$, $X_n \leftrightarrow Y_n$)

$$V_n = \mathrm{P}\exp\left[-ig\int_{-\infty}^{0}\mathrm{d}s\,\bar{n}_t\cdot A_n^{cs}(s\bar{n}^\mu + x^\mu)\right]$$
$$X_n = \overline{\mathrm{P}}\exp\left[ig\int_{0}^{\infty}\mathrm{d}s\,n\cdot A_n^{cs}(sn^\mu + x^\mu)\right]$$

- soft function

$$S_2(\ell_1,\ell_2,\ell_3,\mu) = \frac{1}{N_C}\,\langle 0|\bar{T}\left[Y_{n_3}^\dagger Y_{n_t}\right]\delta\left(\ell_1 - n_1\cdot\hat{P}_1\right)\delta\left(\ell_2 - \bar{n}\cdot\hat{P}_2\right)$$
$$\times\,\delta\left(\ell_3 - \bar{n}\cdot\hat{P}_3\right) T\left[Y_{n_t}^\dagger Y_{n_3}\right]|0\rangle\,,$$

Overview: factorization theorems

Factorization theorem in SCET$_{c+}$ (for $t \ll u \sim Q^2$):

$$\frac{d\sigma_c}{dt\,du\,d\mathcal{T}_3} = H_2(Q^2,\mu)\,H_c\left(t,\frac{u}{Q^2},\mu\right)\prod_{i=1}^{3} J_i(Q_i\mathcal{T}_3,\mu)\otimes S_c\left(\frac{\mathcal{T}_3}{\sqrt{n_1\cdot n_2}},\mu\right)\otimes S_2(\mathcal{T}_3,\mu)$$

Factorization theorem in SCET$_{s+}$ (for $t \sim u \ll Q^2$):

$$\frac{d\sigma_s}{dt\,du\,d\mathcal{T}_3} = H_2(Q^2,\mu)\,H_s\left(\frac{t\,u}{Q^2},\mu\right)\prod_{i=1}^{3} J_i(Q_i\mathcal{T}_3,\mu)\otimes S_3(\mathcal{T}_3,n_i\cdot n_j,\mu)$$

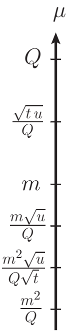
Factorization theorem in SCET$_{cs+}$ (for $t \ll u \ll Q^2$):

$$\frac{d\sigma_{cs}}{dt\,du\,d\mathcal{T}_3} = H_2(Q^2,\mu)\,H_s\left(\frac{t\,u}{Q^2},\mu\right)\prod_{i=1}^{3} J_i(Q_i\mathcal{T}_3,\mu)\otimes S_c\left(\frac{\mathcal{T}_3}{\sqrt{n_1\cdot n_2}},\mu\right)\otimes S_2(\mathcal{T}_3,\mu)$$

Factorization theorem in SCET$_{cs+}$

Factorization theorem for $t \ll u \ll Q^2$:

$$\frac{\mathrm{d}\sigma_{cs}}{\mathrm{d}t\,\mathrm{d}u\,\mathrm{d}\mathcal{T}_3} = H_2(Q^2,\mu)\, H_{cs}\left(\frac{t\,u}{Q^2},\mu\right) \prod_{i=1}^{3} J_i(Q_i\mathcal{T}_3,\mu) \otimes S_c\left(\frac{\mathcal{T}_3}{\sqrt{n_1\cdot n_2}},\mu\right) \otimes S_2(\mathcal{T}_3,\mu)$$



Ingredients:

- $H_2(Q^2,\mu)$: dijet hard function
- $H_{cs}(\frac{tu}{Q^2},\mu)$: $\sim$ csoft (=soft) splitting amplitude
- $J_{2/3}(Q_{2/3}\mathcal{T}_3,\mu) = J_q$: (anti-)quark jet functions
- $J_1(Q_1\mathcal{T}_3,\mu) = J_g$: gluon jet function
- $S_c(\frac{\mathcal{T}_3}{\sqrt{n_1\cdot n_2}},\mu)$: csoft function resolving the closeby jets with directions n_1 and n_2
- $S_2(\mathcal{T}_3,\mu)$: soft function resolving the well-separated jets with directions n_t and n_3

Tree level expressions for hard splitting functions

$$H_3^{\{g,q,\bar{q}\}}(t,u,Q^2,\mu) = \frac{\alpha_s C_F}{2\pi} \frac{(Q^2-t)^2+(Q^2-u)^2}{Q^4\, t\, u}\,,$$

$$H_c^{\{g,q,\bar{q}\}}(t,z,\mu) = \frac{\alpha_s C_F}{2\pi} \frac{1+(1-z)^2}{z\, Q^2 t}$$

$$H_s\left(\frac{t\,u}{Q^2},\mu\right) = \frac{\alpha_s C_F}{\pi} \frac{1}{t\,u} = H_{cs}\left(\frac{t\,u}{Q^2},\mu\right)$$