Renormalization of Subleading Dijet Operators in Soft-Collinear Effective Theory

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Work completed with Simon Freedman:
arXiv:1408.6240

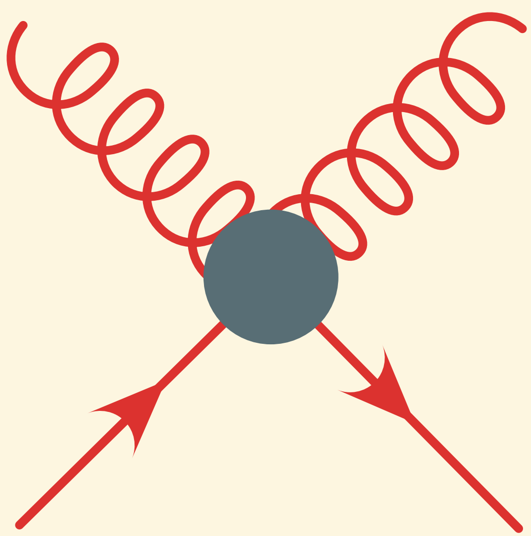
Physics
UNIVERSITY OF TORONTO

SCET 2015
Santa Fe, NM

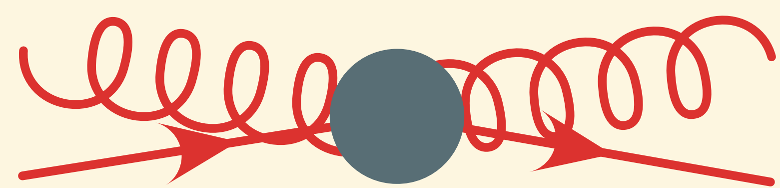
Thrust

$$\tau = 1 - \frac{1}{Q} \min_{\vec{t}} \sum_i |\vec{t} \cdot \vec{p_i}|$$

Q : Centre of mass energy

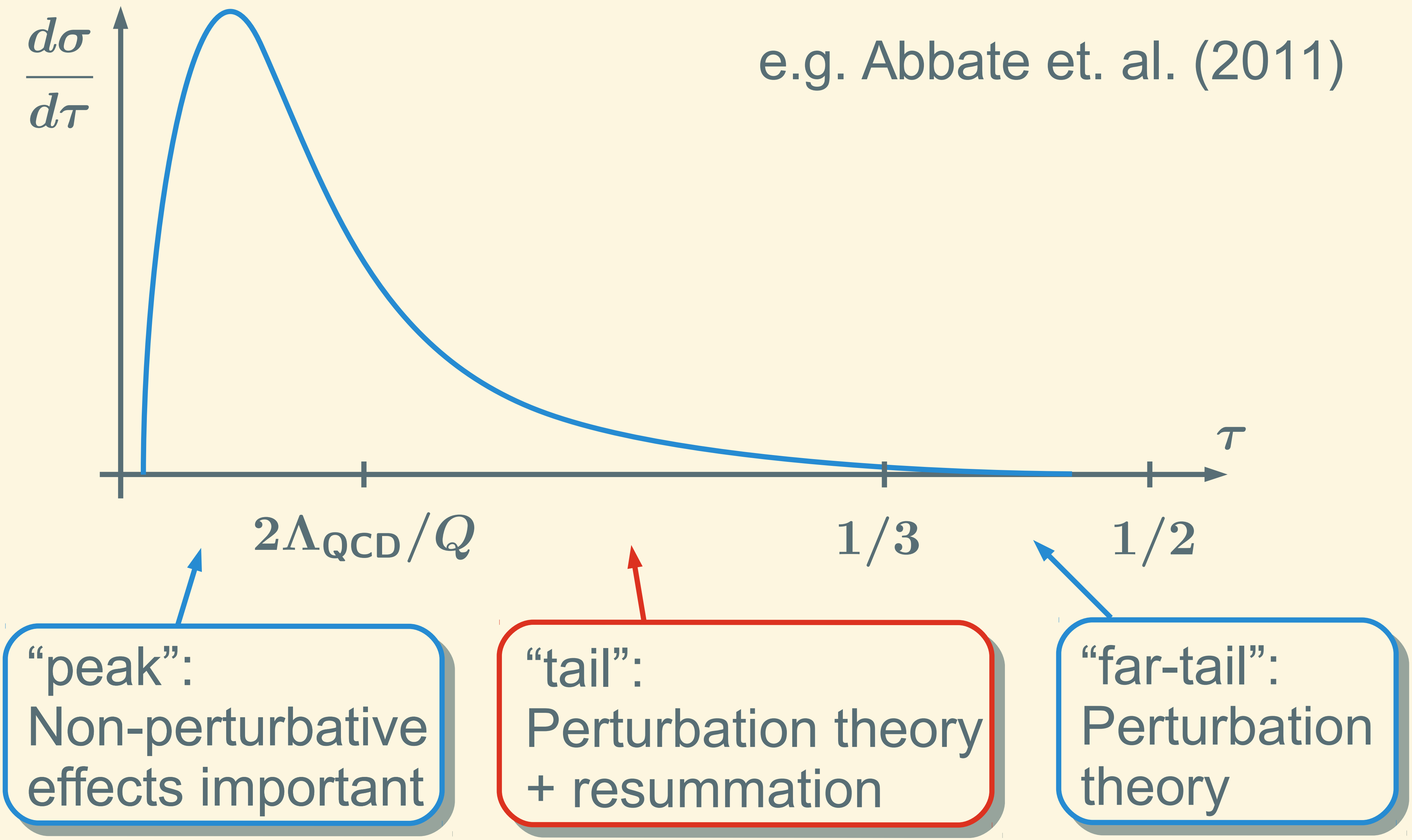


$\tau \sim 1/2$



$\tau \sim 0$

Thrust



$\frac{d\sigma}{d\tau}$
e.g. Abbate et. al. (2011)
τ
$2\Lambda_{\text{QCD}}/Q$
$1/3$
$1/2$
"peak": Non-perturbative effects important
"tail": Perturbation theory + resummation
"far-tail": Perturbation theory

Thrust

$$\frac{d\sigma}{d\tau} = \frac{d\sigma}{d\tau}^{(0)} + D(\tau)$$

$$\lim_{\tau \to 0} \int_0^\tau d\tau' D(\tau') = 0$$

$\mathbf{N^3LL}$ → $\frac{d\sigma}{d\tau}^{(0)}$

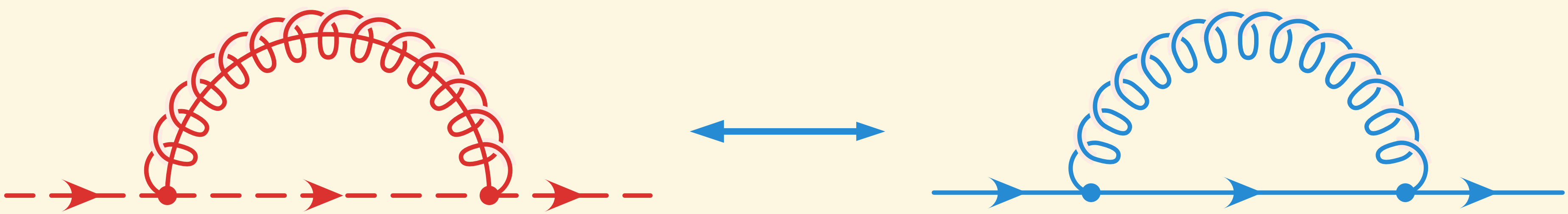
$\mathbf{NLO}$ → $D(\tau)$

Becher & Schwartz (2008)

Eventually, resummation in $D(\tau)$ will become important, requiring subleading SCET.

Alternate Formalism for SCET

Freedman & Luke (2012)



$\mathcal{L}_\xi = \mathcal{L}_{\xi\xi}^{(0)} + \cdots$

$\mathcal{L}_{\mathsf{QCD}}$

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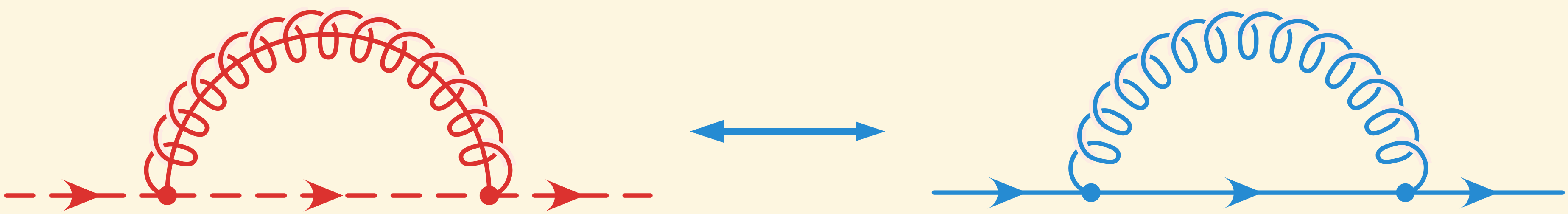

$$\mathcal{L}_\xi = \mathcal{L}_{\xi\xi}^{(0)} + \cdots \qquad \qquad \mathcal{L}_{\text{QCD}}$$

Idea:

$$\mathcal{L} = \sum_{i \in \text{sectors}} \mathcal{L}^i_{\text{QCD}}$$

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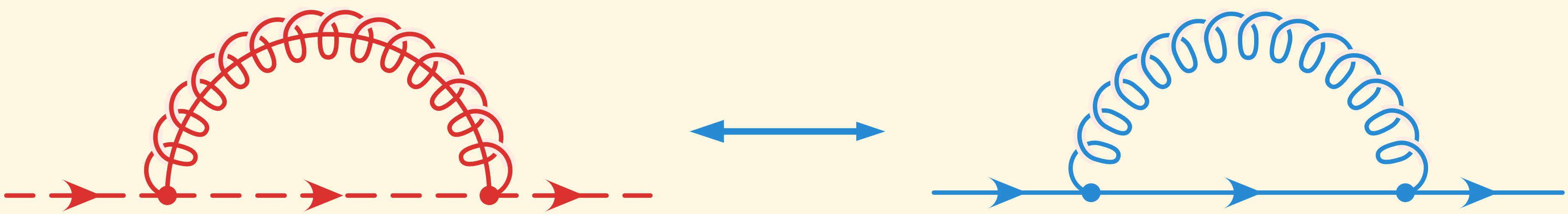

$\mathcal{L}_\xi = \mathcal{L}_{\xi\xi}^{(0)} + \cdots$

$\mathcal{L}_{\text{QCD}}$

Idea:

$$\mathcal{L} = \sum_{i \in \text{sectors}} \mathcal{L}_{\text{QCD}}^{i}$$

$$O_{\text{QCD}} \to C^{(0)} O^{(0)} + \frac{1}{Q} \sum_i C^{(i)} O^{(i)} + \mathcal{O}\left(\lambda^2\right)$$

Dijet Operators

Leading order	Subleading orders
$\mathcal{L}^{(0)}_{\text{SCET}}$	$\mathcal{L}^{(1)}_{\text{SCET}} + \cdots$
$O_2^{(0)} = \bar{\xi}_n W_n Y_n^\dagger \Gamma Y_{\bar{n}} W_{\bar{n}}^\dagger \xi_{\bar{n}}$	$O_2^{(1)} + \cdots$
$\sum_{i \in \text{sectors}} \mathcal{L}^i_{\text{QCD}}$	
$O_2^{(0)} = \bar{\psi}_n W_n Y_n^\dagger P_{\bar{n}} \Gamma P_{\bar{n}} Y_{\bar{n}} W_{\bar{n}}^\dagger \psi_{\bar{n}}$	$O_2^{(1)} + \cdots$

Alternate Formalism for SCET

$$\langle p_n, p_{\bar{n}}, p_s | \, O_2^{(0)} \, |0\rangle$$

$$= \langle p_n, p_{\bar{n}}, p_s | \, \bar{\psi}_n W_n \Gamma^{(0)} Y_n^\dagger Y_{\bar{n}} W_{\bar{n}}^\dagger \psi_{\bar{n}} \, |0\rangle$$

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$$= \langle p_n | \, \bar{\psi}_n W_n \, |0\rangle \, \Gamma^{(0)} \, \langle p_s | \, Y_n^\dagger Y_{\bar{n}} \, |0\rangle \, \langle p_{\bar{n}} | \, W_{\bar{n}}^\dagger \psi_{\bar{n}} \, |0\rangle$$

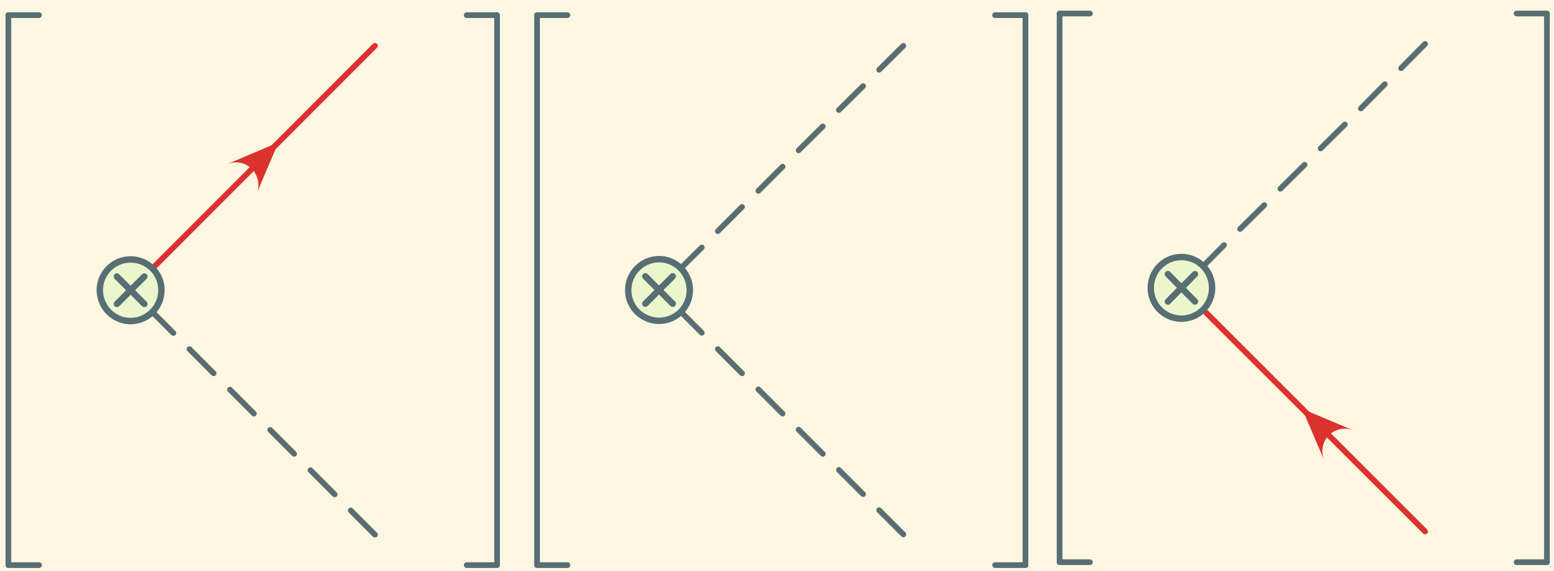
Alternate Formalism for SCET

$$\langle p_n, p_{\bar{n}}, p_s | O_2^{(0)} |0\rangle$$

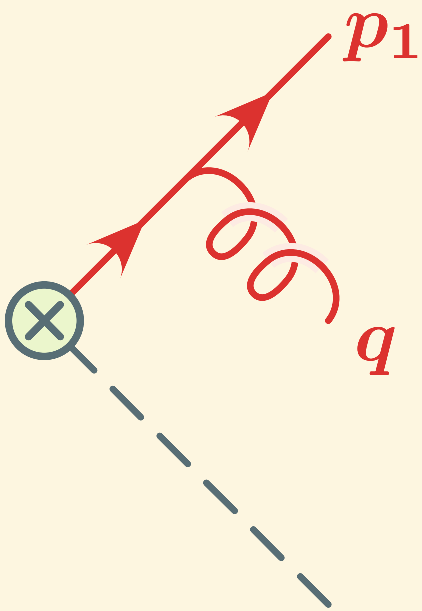
$$= \langle p_n, p_{\bar{n}}, p_s | \bar{\psi}_n W_n \Gamma^{(0)} Y_n^\dagger Y_{\bar{n}} W_{\bar{n}}^\dagger \psi_{\bar{n}} |0\rangle$$

$$= \langle p_n | \bar{\psi}_n W_n |0\rangle \, \Gamma^{(0)} \, \langle p_s | Y_n^\dagger Y_{\bar{n}} |0\rangle \, \langle p_{\bar{n}} | W_{\bar{n}}^\dagger \psi_{\bar{n}} |0\rangle$$

$$\langle p_1, p_2 | O_2^{(0)} |0\rangle =$$



Alternate Formalism for SCET



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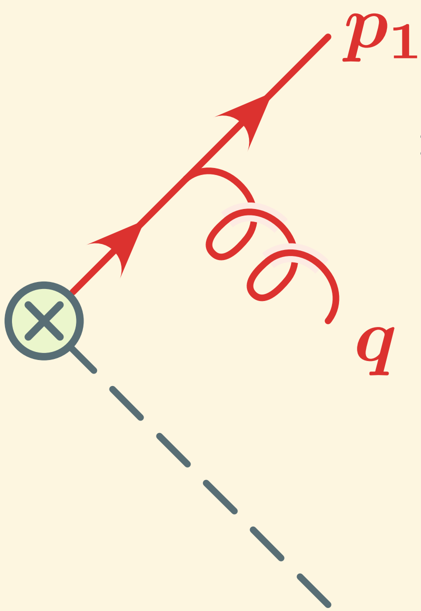
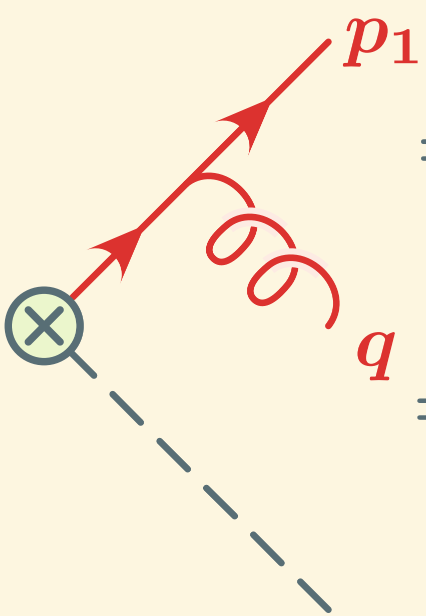


$$= -igT^a \bar{u}(p_1)\gamma^\alpha \frac{\not{p}_1 + \not{q}}{2p_1 \cdot q} P_{\bar{n}} \Gamma P_{\bar{n}} v(p_2) \epsilon^*_\alpha(q)$$

Alternate Formalism for SCET



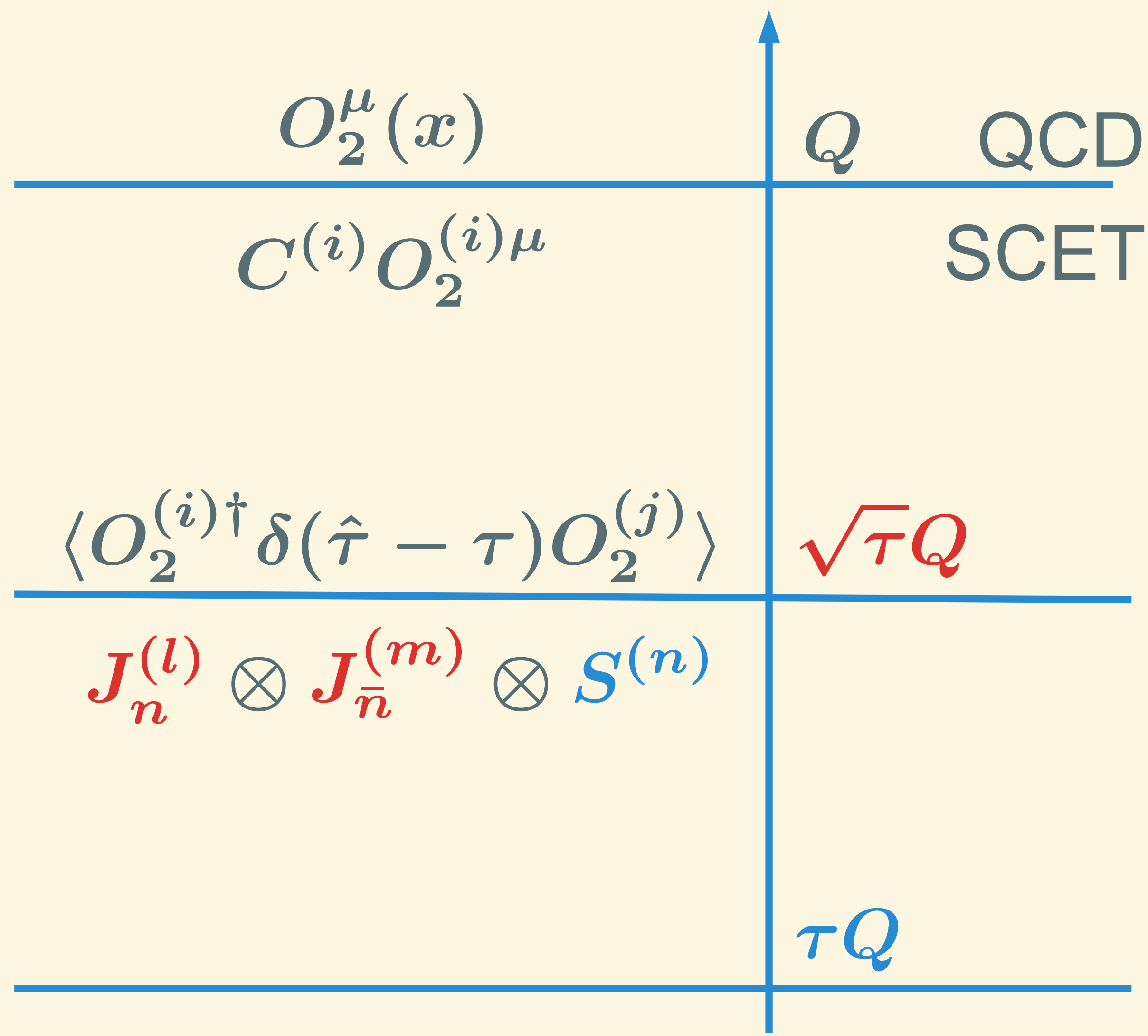
$$= -igT^a \bar{u}(p_1)\gamma^\alpha \frac{\not{p}_1 + \not{q}}{2p_1\cdot q} P_{\bar{n}}\Gamma P_{\bar{n}} v(p_2)\epsilon^*_\alpha(q)$$

$$= -igT^a \bar{\xi}_{n,\tilde{p}_1}(k_1)\Bigg(n^\alpha + \frac{\gamma_\perp^\alpha \not{p}_{1\perp}}{\bar{n}\cdot p_1} + \frac{(\not{p}_{1\perp}+\not{q}_\perp)\gamma_\perp^\alpha}{\bar{n}\cdot(p_1+q)} + \frac{(\not{p}_{1\perp}+\not{q}_\perp)\not{p}_{1\perp}}{\bar{n}\cdot p_1 \bar{n}\cdot(p_1+q)}\bar{n}^\alpha\Bigg)\frac{\not{\bar{n}}}{2}$$

$$\left(\frac{\not{n}}{2}\frac{\bar{n}\cdot(p_1+q)}{n\cdot(k_1+k_3)\bar{n}\cdot(p_1+q) + (p_{1\perp}+q_\perp)^2}\right)$$

$$\times\, \Gamma \xi_{\bar{n},\tilde{p}_2}(k_2)\epsilon^*_{n,\tilde{q},\alpha}(k_3)$$

To sum subleading logs in Thrust



To sum subleading logs in Thrust

$$\tau \sim \lambda^2$$

	Scale	
$O_2^{\mu}(x)$	Q	QCD
$C^{(i)} O_2^{(i)\mu}$		SCET
$\langle O_2^{(i)\dagger} \delta(\hat{\tau} - \tau) O_2^{(j)} \rangle$	$\sqrt{\tau} Q$	
$J_n^{(l)} \otimes J_{\bar{n}}^{(m)} \otimes S^{(n)}$		
	τQ	

To sum subleading logs in Thrust

$\tau \sim \lambda^2$

$O_2^{\mu}(x)$ — Q — QCD

$C^{(i)} O_2^{(i)\mu}$ — SCET

$\langle O_2^{(i)\dagger} \delta(\hat{\tau} - \tau) O_2^{(j)} \rangle$ — $\sqrt{\tau} Q$

$J_n^{(l)} \otimes J_{\bar{n}}^{(m)} \otimes S^{(n)}$

Freedman (2013)

τQ

To sum subleading logs in Thrust

$\tau \sim \lambda^2$

Need anomalous dimensions:

$O^{(1i)}, O^{(2i)}$

and

$J_n^{(l)} \otimes J_{\bar{n}}^{(m)} \otimes S^{(n)}$

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RG & Freedman (2014)

Subleading Operators: Add Derivatives

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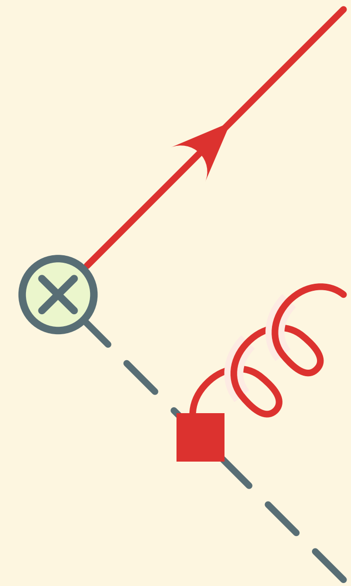
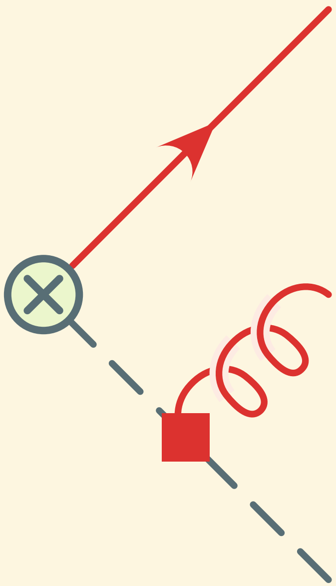
$$\langle \boldsymbol{p}, \boldsymbol{q}| \, \bar{\psi}_n \boldsymbol{W}_n(\boldsymbol{x}, \boldsymbol{y}) \boldsymbol{D}^{\boldsymbol{\mu}}(\boldsymbol{y}) \boldsymbol{W}_n(\boldsymbol{y}, \boldsymbol{\infty}) \, |0\rangle =$$



Subleading Operators: Add Derivatives

$$\langle p, q | \, \bar{\psi}_n W_n(x,y) D^\mu(y) W_n(y,\infty) \, |0\rangle =$$



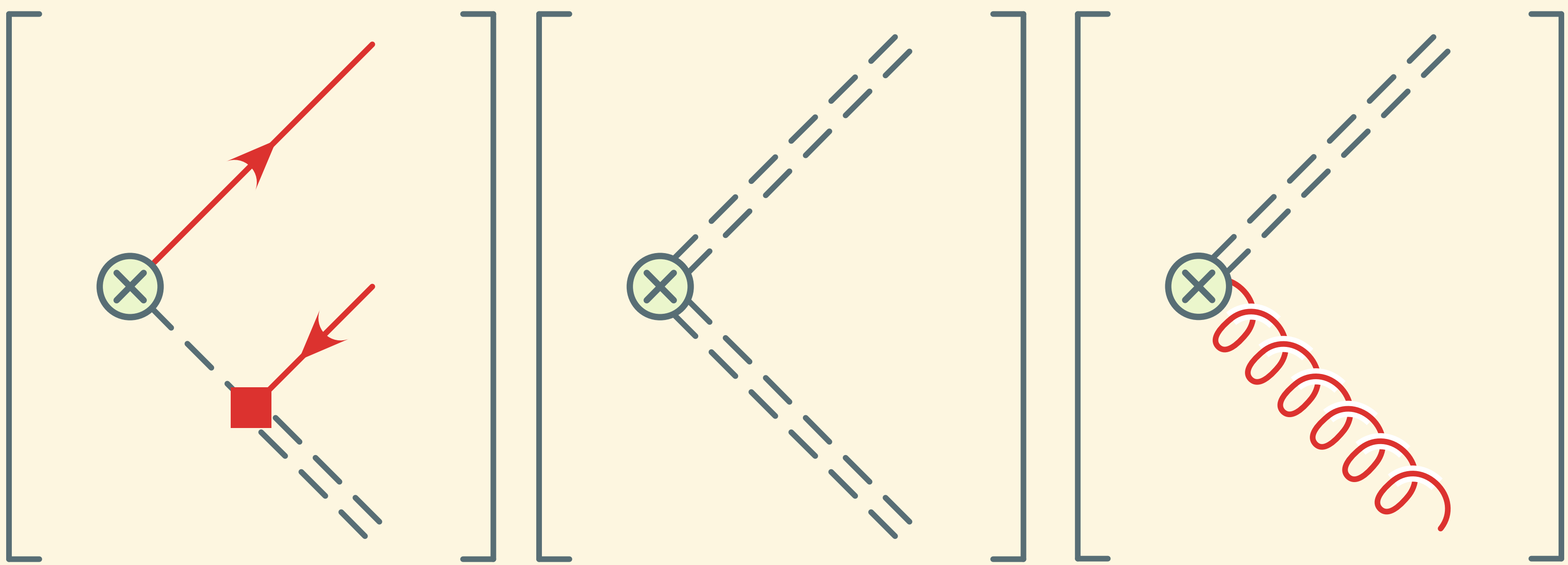
Note: $\langle p | \, \bar{\psi}_n W_n D^\mu W_n \, |0\rangle = 0$

Subleading Operators: Colour Structure

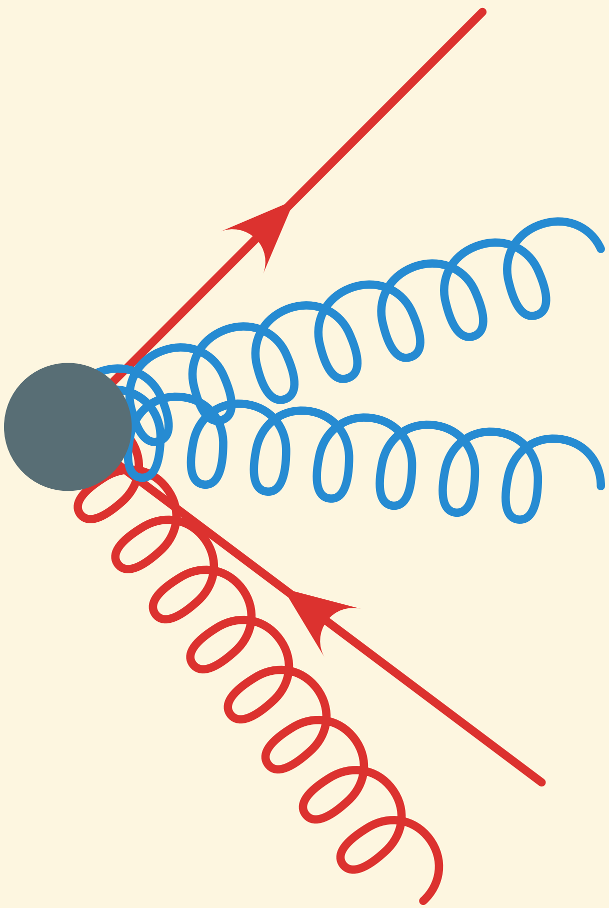
$$\langle p_1, p_2, q| O_2^{1e} |0\rangle =$$

$$\langle p_1, p_2| \bar{\psi}_n(x) W_n(x,y) T^a \psi_n(y) W_n^{ab}(y,\infty) |0\rangle$$

$$\langle 0| Y_n^{\dagger bc} Y_{\bar{n}}^{cd} |0\rangle \, \Gamma_e^{\mu} \, \langle q| W_{\bar{n}}^{\dagger de} i g n_\nu G_{\bar{n}}^{e\mu\nu} |0\rangle$$



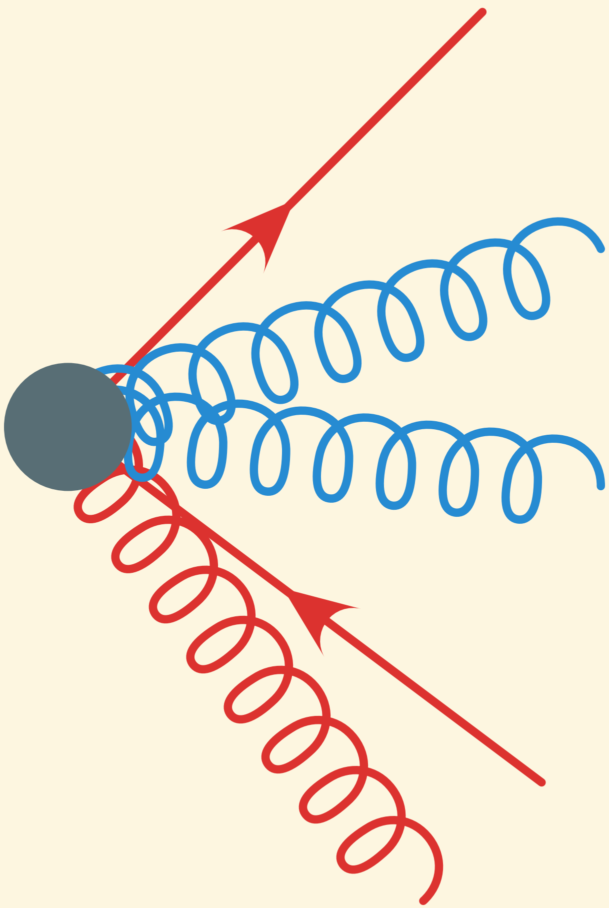
Multipole Expansion



$$\delta\left(Q^\mu - p^\mu\right) =$$

$$2\delta\left(Q - \bar{n}\cdot p\right)\delta\left(Q - n\cdot p\right)\delta\left(\vec{p}_\perp\right)$$

Multipole Expansion



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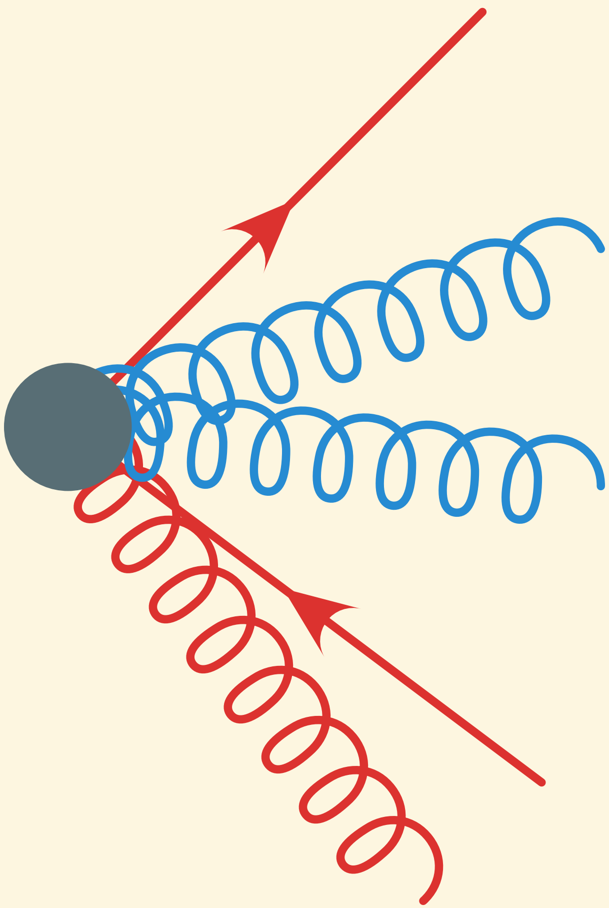
$$p^\mu = p_n^\mu + p_{\bar{n}}^\mu + p_s^\mu$$

Multipole Expansion



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↓

$$p^{\mu}=p_{n}^{\mu}+p_{\bar{n}}^{\mu}+p_{s}^{\mu}$$

$$\delta_{\mathsf{SCET}}(Q;p)=$$

$$2\delta\left(Q-\bar{n}\cdot p_{n}\right)\delta\left(Q-n\cdot p_{\bar{n}}\right)$$

$$\delta\left(\vec{p}_{n\perp}+\vec{p}_{\bar{n}\perp}\right)+\mathcal{O}\left(\lambda\right)$$

Anomalous Dimensions

After applying various constraints:

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Total of 4 independent anomalous dimensions

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Two options: Delta-regulator

Gluon mass

Delta regulator

Chui et. al. (2009)

$$\frac{1}{(p_i+k)^2} \to \frac{1}{(p_i+k)^2 - \Delta_i}\,; \qquad \frac{\bar{n}_i^\alpha}{k\cdot\bar{n}_i} \to \frac{\bar{n}_i^\alpha}{k\cdot\bar{n}_i - \delta_{p_j}},$$

Delta regulator

Chui et. al. (2009)

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$$\left\langle p_1^n, q^n, p_2^{\bar{n}} \right| O_2^{(0)} \left|0\right\rangle$$

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Chui et. al. (2009)

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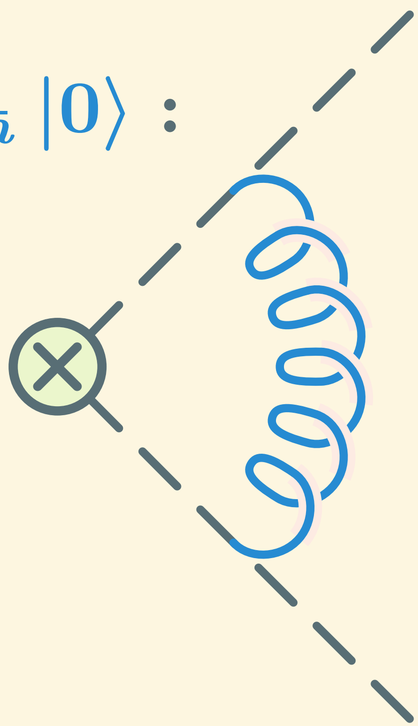
$$\langle p_1^n, q^n, p_2^{\bar{n}} |\, O_2^{(0)}\, |0\rangle$$

$$\langle 0|\, Y_n^\dagger Y_{\bar{n}}\, |0\rangle \ :$$



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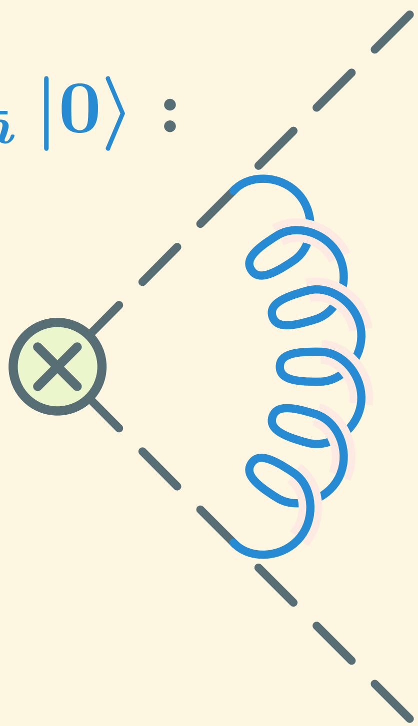
$\langle 0|\, Y_n^\dagger Y_{\bar{n}}\, |0\rangle$:

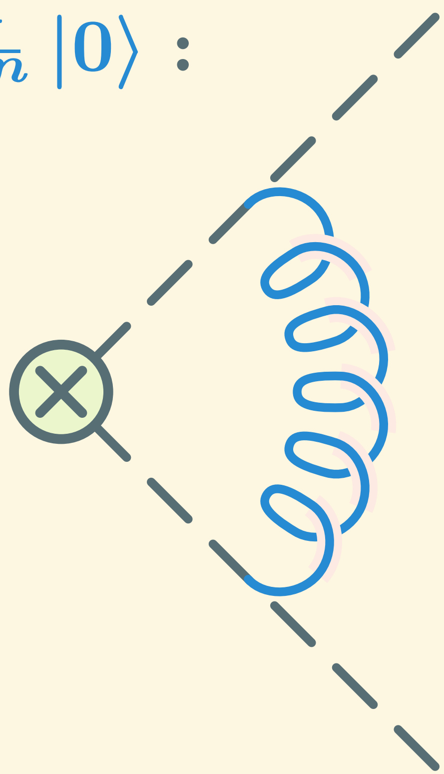


$$\propto \int d^dk \frac{1}{(k^2-\Delta_q)(\bar{n}\cdot k + \delta_{p_2})} \times \left(\frac{C_F + C_A}{n\cdot k - \delta_{p_1}} - \frac{C_A}{n\cdot k - \delta_q} \right)$$

Gluon Mass

$$\frac{1}{k^2} \to \frac{1}{(k^2 - M^2)}$$

$\langle 0| \, Y_n^\dagger Y_{\bar{n}} \, |0\rangle$:



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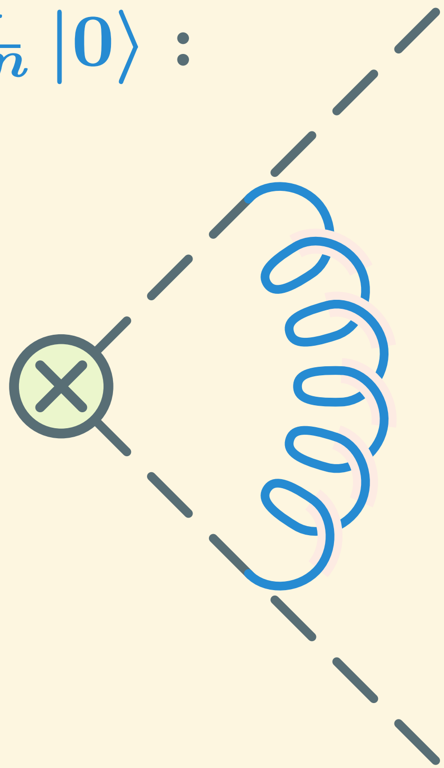

$$\propto C_F M^{-2\epsilon} \int_0^\infty \frac{dk^-}{k^-}$$

Gluon Mass

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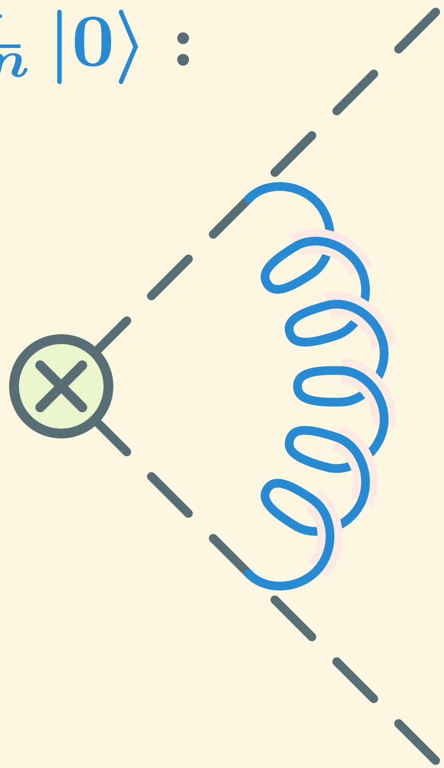

Divergent integral

$$\propto C_F M^{-2\epsilon} \int_0^\infty \frac{dk^-}{k^-}$$

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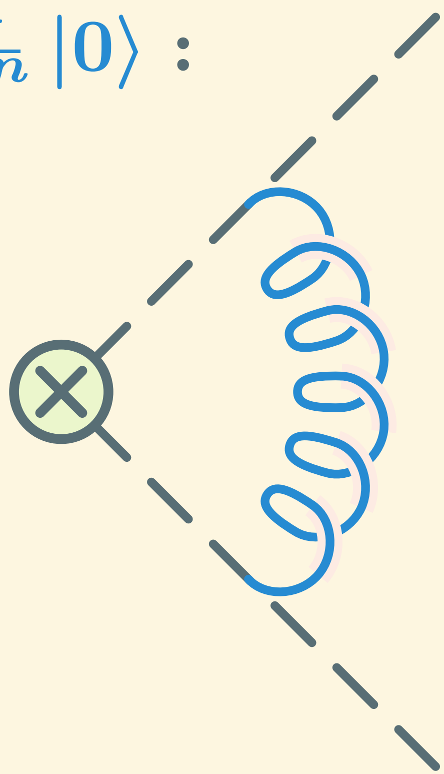


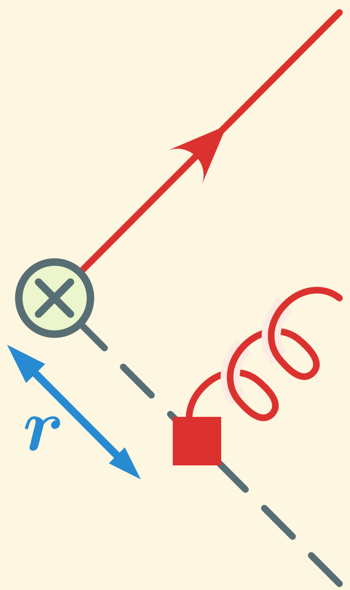
Divergent integral

$$\propto C_F M^{-2\epsilon} \int_0^\infty \frac{dk^-}{k^-}$$

Add collinear integrands before integrating – finite.

Chiu et. al. (2009)

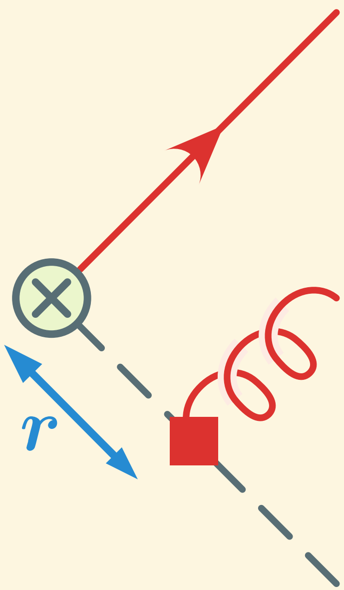
Anomalous Dimensions



$$O_2^{(i)\mathbf{bare}}(u) = \int dv\, Z^{(i)}(u,v)\, O_2^{(i)\mathbf{ren}}(v)$$

($u \leftrightarrow r$, momentum space)

Anomalous Dimensions



$$O_2^{(i)\mathbf{bare}}(u) = \int dv\, Z^{(i)}(u,v)\, O_2^{(i)\mathbf{ren}}(v)$$

($u \leftrightarrow r$, momentum space)

$$\gamma_{2(i)}(u,v) = -\int Z^{-1}_{(i)}(u,w)\, \frac{d}{d\log\mu} Z_{(i)}(w,v)$$

Results

$$\gamma_{(1a)}(u,v) = \frac{\alpha_s \delta(u-v)\theta(\bar{v})}{\pi}\left(C_F\left(-\frac{3}{2}+\log\frac{-Q^2}{\mu}+\log\bar{v}\right)+\frac{C_A}{2}\right)$$

$$+\frac{\alpha_s}{\pi}\left(C_F-\frac{C_A}{2}\right)\bar{u}\left(\theta(1-u-v)\frac{uv}{\bar{u}\bar{v}}+\theta(\bar{u})\theta(\bar{v})\theta(u+v-1)\frac{uv+u+v-1}{uv}\right)$$

$$-\frac{\alpha_s C_A}{2\pi}\bar{u}\left(\theta(\bar{u})\theta(u-v)\frac{\bar{v}-uv}{u\bar{v}}+\theta(\bar{v})\theta(v-u)\frac{\bar{u}-uv}{v\bar{u}}+\frac{1}{\bar{u}\bar{v}}\frac{\bar{u}\theta(\bar{u})\theta(u-v)}{u-v}+\frac{\bar{v}\theta(\bar{v})\theta(v-u)}{v-u}\right)$$

$$\gamma_{(1b)}(u,v) = \gamma_{(1a_n)}(u,v)$$

$$\gamma_{(1B)} = \gamma_{(1\delta)} = \gamma_{(0)} = \frac{\alpha_s C_F}{\pi}\left(-\frac{3}{2}+\log\frac{-Q^2}{\mu}\right)$$

$$\gamma_{(1c)}(u,v) = \frac{\alpha_s\delta(u-v)}{\pi}\left(C_F\left(-\frac{3}{2}+\log\frac{-Q^2}{\mu}\right)+\frac{C_A}{2}\log v\right)$$

$$-\frac{\alpha_s C_A}{\pi}\left(\left[\frac{\theta(v-u)\theta(u)}{v-u}+\frac{\theta(u-v)\theta(v)}{u-v}\right]_+-\frac{\theta(u-v)}{u}-\frac{\theta(v-u)}{v}\right)$$

$$\gamma_{(1d)}(u,v) = \frac{\alpha_s\delta(u-v)}{\pi}\left(-\frac{C_F}{2}+C_A\left(\log\frac{-Q^2}{\mu}+\log(v)-\frac{1}{2}\right)\right)$$

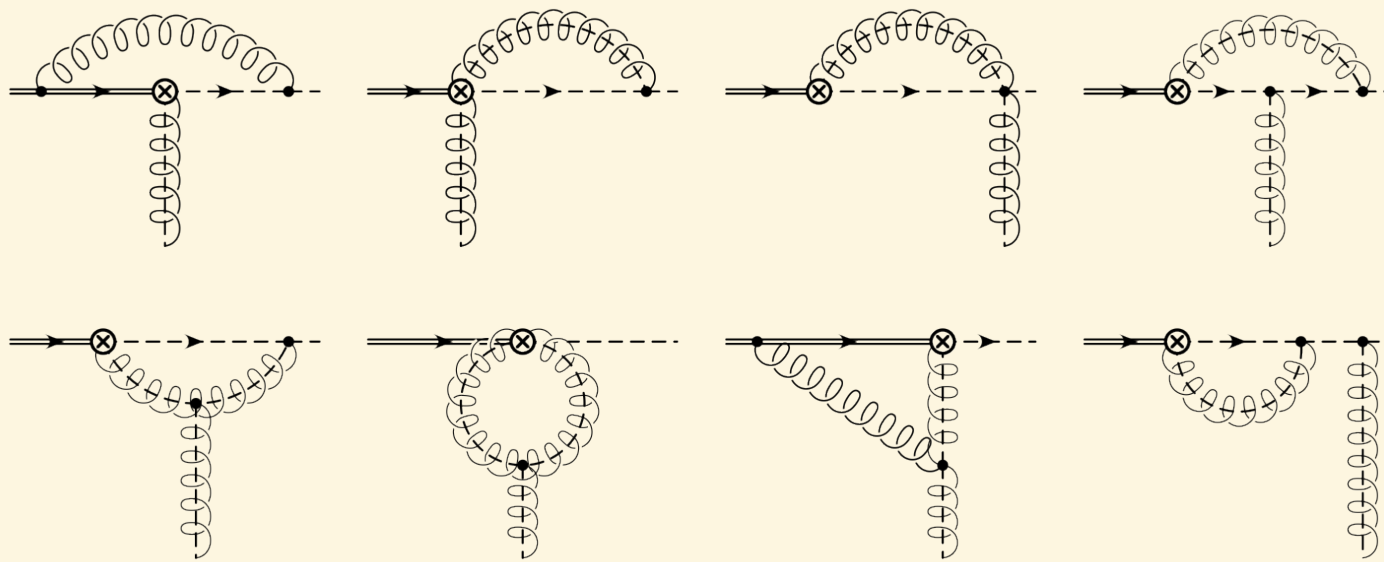
$$-\frac{\alpha_s}{\pi}\left(C_F-\frac{C_A}{2}\right)\frac{1}{v}\left[\frac{v\theta(u-v)\theta(v)}{u-v}+\frac{u\theta(v-u)\theta(u)}{v-u}\right]_+$$

$$\gamma_{(1e)}(u,v) = \frac{\alpha_s\delta(u-v)\theta(\bar{v})}{\pi}\left(\frac{C_F}{2}+C_A\left(\log\frac{-Q^2}{\mu}+\log(v)-1\right)\right)-\frac{\alpha_s}{\pi}\left(C_F-\frac{C_A}{2}\right)\frac{1}{v\bar{v}}\Big(\theta(\bar{v})\theta(v-u)u\bar{v}$$

$$+\theta(\bar{u})\theta(u-v)v\bar{u}+\left[\frac{\bar{u}v\theta(\bar{u})\theta(u-v)}{u-v}+\frac{u\bar{v}\theta(\bar{v})\theta(v-u)}{v-u}\right]_+\Big)$$

Checks on the Results

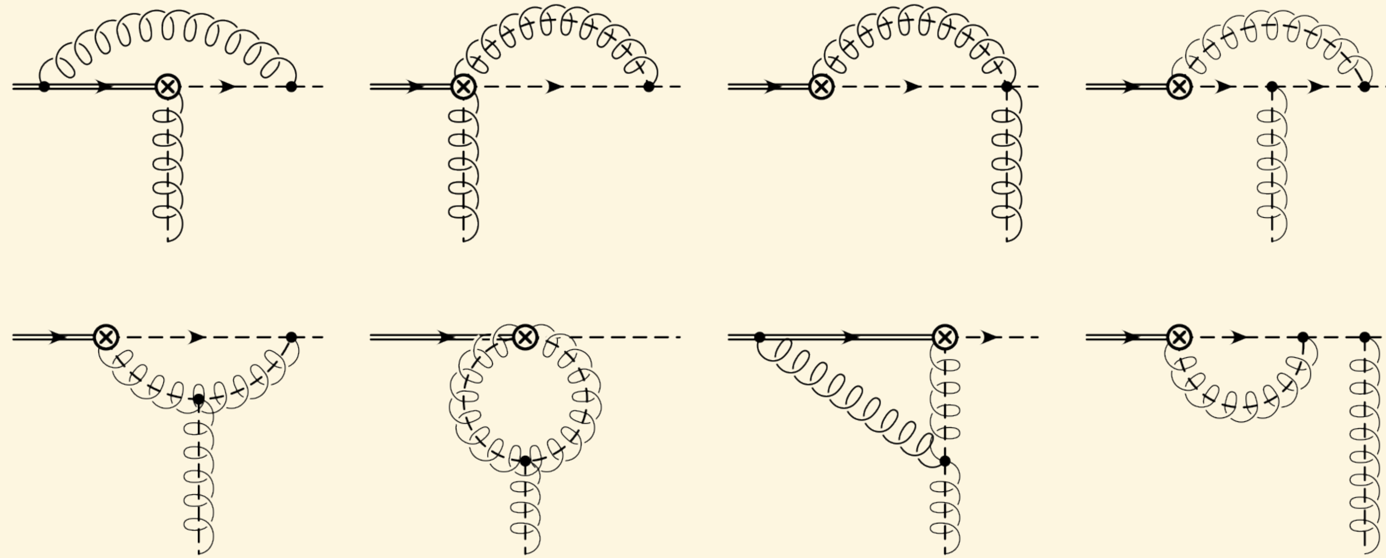
Heavy-to-light vector currents: Off-diagonal terms agree



Hill et.al. (2004)

Checks on the Results

Heavy-to-light vector currents: Off-diagonal terms agree



Hill et.al. (2004)

Cusp anomalous dimension agrees:

$$\gamma_{(i)}(u,v) = \delta(u-v)\Gamma_{\mathbf{cusp}} \log\left(\frac{-Q^2}{\mu^2}\right) + \gamma^{NC}_{(i)}(u,v)$$

Summary

Towards summing subleading logs in Thrust:

Computed anomalous dimension of $\mathcal{O}(\lambda)$ dijet operators in SCET

Used a formalism of SCET as multiple copies of QCD coupled to Wilson Lines.

To sum logs, still need anomalous dimensions of $\mathcal{O}(\lambda^2)$ and soft functions

Summary

Towards summing subleading logs in Thrust:

- Computed anomalous dimension of $\mathcal{O}(\lambda)$ dijet operators in SCET
- Used a formalism of SCET as multiple copies of QCD coupled to Wilson Lines.
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(coming soon!)