Resumming Non-global Logs

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Motivation

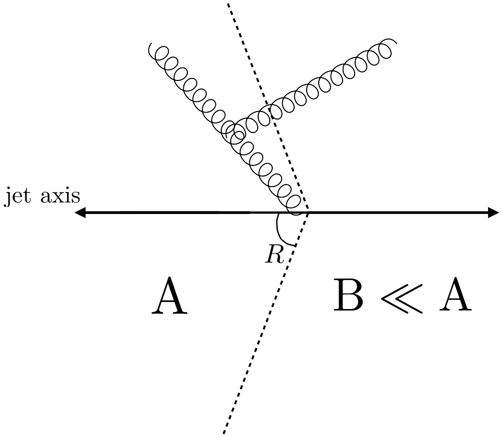
- Collider observables often do not constrain all of phase-space.
- Such observables have so-called **non-global logarithms (NGLs)**, corresponding to correlated splittings in different phase-space regions [Dasgupta, Salam].
- Resummation has resisted traditional factorization theorem techniques, relying on evolution equations [Banfi, Marchesini, Smye; Weigert; Caron-Huot] or Monte Carlos [Dasgupta, Salam].
- The pattern of these NGLs at fixed order do not exhibit any straightforward exponentiation, but still have rich structure [Schwartz, Zhu].

Outline

- What is an NGL?
- Isolating phase-space regions that dominate NGLs.
- Exhibit factorization theorems for such phase-space regions.
- From the RG structure of such factorization theorems, resum NGLs.
- Realizing Jets as the quasi-particles of pQCD: integrating jets not gluons to resum more inclusive quantities.

NGL: Correlated soft emmission.

At fixed order, soft configuration giving rise to NGLs:



Beyond Global Factorization.

The hard, collinear, and soft factorization of the cross-section:

$$\frac{d\sigma}{dAdB} = H_{n\bar{n}} J_n(A) \otimes J_{\bar{n}}(B) \otimes S_{n\bar{n}}(A, B)$$

- Simply integrates over these soft splittings in $S_{n\bar{n}}$.
- Details of the dynamics leading to the NGL lost.

Isolating Soft Jets

To study the NGLs,

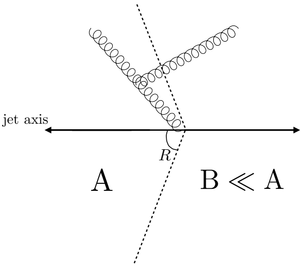
- Need a detailed understanding of the history of soft radiation
- Move from the inclusive cross-section to a more exclusive cross-section.

$$\frac{d\sigma}{dA\,dB} \to \frac{d\sigma}{de_2^{(\alpha)}\,de_2^{(\beta)}\,de_3^{(\beta)}\,dB}$$

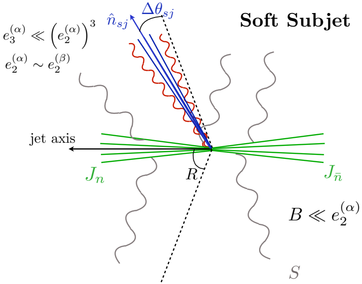
Study its factorization properties.

From Gluons to Jets

Inclusive to exclusive cross-section: Study soft jets, not gluons.



Versus



Isolating Soft Jets

To control the soft jet spectrum: energy-energy correlators:

$$e_2^{(\alpha)} = \frac{1}{E_J^2} \sum_{i<j} E_i E_j \left(\frac{p_i \cdot p_j}{E_i E_j} \right)^{\alpha/2}$$

$$e_3^{(\beta)} = \frac{1}{E_J^3} \sum_{i<j<k} E_i E_j E_k \left(\frac{p_i \cdot p_j}{E_i E_j} \frac{p_j \cdot p_k}{E_j E_k} \frac{p_k \cdot p_i}{E_k E_i} \right)^{\beta/2}$$

[Larkoski, Thaler, Salam]

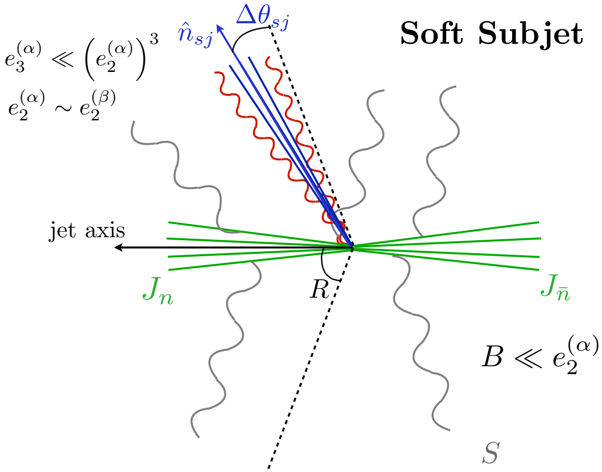
$d\sigma$... what?? Why so many observables?

We simultaneously impose constraints from $e_2^{(\alpha)}, e_2^{(\beta)}$, and $e_3^{(\beta)}$:

- Inclusive and IRC safe definition of the splitting angle and energy of the subjet.
- $e_2^{(\alpha)}, e_2^{(\beta)} \to z, \theta$.
- $e_3^{(\beta)}$ demarcates resolved partons (jets) from unresolved partons (also called partons).
- Relative scalings of $e_2^{(\alpha)}, e_2^{(\beta)}$ and $e_3^{(\beta)}$ map out 1v2-prong structure: Ian Moult's talk.

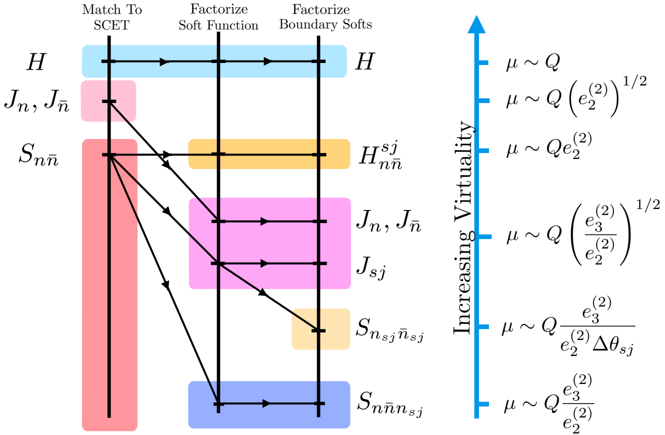
Where Are We?

In the soft subjet region, $e_2^{(\alpha)} \sim e_2^{(\beta)}$ and $e_3^{(\beta)} \ll \left(e_2^{(\beta)}\right)^3$.



Soft Subjet Factorization

$e^+e^- \to 2_j + 1_{sj}$:



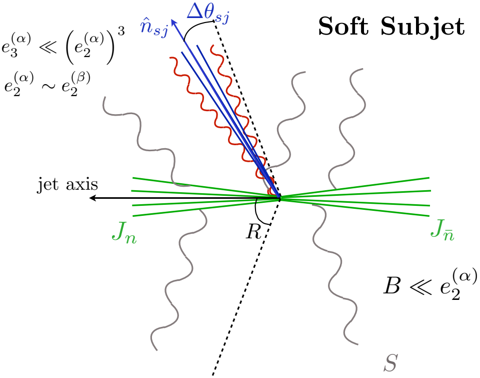
Soft Subjet Factorization

$e^+e^- \to 2_j + 1_{sj}$:

$$\frac{d\sigma}{de_2^{(\alpha)} de_2^{(\beta)} de_3^{(\beta)} dB} = H_{n\bar{n}} H_{n\bar{n}}^{sj}(e_2^{(\alpha)}, e_2^{(\beta)}) J_{n_{sj}}(e_3^{(\beta)}) \otimes S_{n_{sj}\bar{n}_{sj}}(e_3^{(\beta)}) \otimes S_{n\bar{n}n_{sj}}(e_3^{(\beta)}; B) \otimes J_n(e_3^{(\beta)}) \otimes J_{\bar{n}}(B)$$

- $H_{n\bar{n}}$ creation of initial dijets.
- $H_{n\bar{n}}^{sj}$ creation of soft subjet off $n\bar{n}$ dipole.
- $J_{n_{sj}}$ soft jet collinear modes.
- $S_{n_{sj}\bar{n}_{sj}}$ *boundary soft* modes.
- $S_{n\bar{n}n_{sj}}$ soft modes.

The Role of Boundary Softs



Boundary soft modes sensitive to the soft jet's angular distance to fat-jet boundary $\Delta\theta_{sj}$.

The Role of Boundary Softs

The boundary softs are subtracted from the $S_{n\bar{n}n_{sj}}$ function.

- Following the zero-bin procedure of [Manohar, Stewart].
- Removes $\Delta\theta_{sj}$ from the "in-jet" region of $S_{n\bar{n}n_{sj}}$.
- Does *not* remove $\Delta\theta_{sj}$ from the "out-jet" region (scaleless).
- Anom. Dim. dependence on $\Delta\theta_{sj}$ cancels between "out-jet" region and boundary soft modes.
- Thus resums a hierarchy of *soft* energy scales, with support in differing angular regions of phase-space.

Connecting to Inclusive Cross-Section

Having resummed an NGL in exclusive cross-section.

- Return to inclusive cross-section by laplace transforms:

$$\frac{d\sigma}{dz d\theta d\tilde{e}_3^{(\beta)} dB} = \int_0^\infty de_3^{(\beta)}\, e^{-\tilde{e}_3^{(\beta)} e_3^{(\beta)}} \frac{d\sigma}{dz d\theta de_3^{(\beta)} dB}$$

- And marginalizing:

$$\frac{d\sigma}{dAdB} = \int dz d\theta\, \delta\Big(A - F(z,\theta)\Big) \lim_{\tilde{e}_3^{(\beta)} \to 0} \frac{d\sigma}{dz d\theta d\tilde{e}_3^{(\beta)} dB}$$

This is the one soft sub-jet contribution to $\frac{d\sigma}{dAdB}$.

Dressing the Gluon as a Soft Jet.

Re-associate elements of the factorization theorem using their RG equations [Hornig, Lee, Walsh, Zuberi]:

$$W_{n\bar{n}}(z,\theta) = \lim_{\tilde{e}_3^{(\beta)}\to 0} H_{n\bar{n}}^{sj}(z,\theta) J_{n_{sj}}(\tilde{e}_3^{(\beta)}) S_{n_{sj}\bar{n}_{sj}}(\tilde{e}_3^{(\beta)}) \frac{S_{n\bar{n}n_{sj}}(\tilde{e}_3^{(\beta)})}{S_{n\bar{n}}(\tilde{e}_3^{(\beta)})}\Big|_{in}$$

$$G_{n\bar{n}n_{sj}}(B) = \lim_{\tilde{e}_3^{(\beta)}\to 0} \frac{S_{n\bar{n}n_{sj}}(\tilde{e}_3^{(\beta)};B)}{S_{n\bar{n}}(\tilde{e}_3^{(\beta)};B)}\Big|_{out+NG}$$

So:

$$\lim_{\tilde{e}_3^{(\beta)}\to 0} \frac{d\sigma}{dz\,d\theta\,d\tilde{e}_3^{(\beta)}\,dB} = \lim_{\tilde{e}_3^{(\beta)}\to 0} H_{n\bar{n}} W_{n\bar{n}}(z,\theta) G_{n\bar{n}n_{sj}}(B) J_n(\tilde{e}_3^{(\beta)}) J_{\bar{n}}(B) S_{n\bar{n}}(\tilde{e}_3^{(\beta)};B)$$

$W_{n\bar{n}}(z,\theta)G_{n\bar{n}n_{sj}}(B)$ *is an RG invariant!*

Resumming the NGLs

Evolving $G_{n\bar{n}n_{sj}}(B)$ resums the NGLs from out-of-jet emmissions off of the soft sub-jet:

$$W_{n\bar{n}}(z,\theta)G_{n\bar{n}n_{sj}}(B) = \frac{\alpha_s C_F}{4\pi^2}\frac{1}{z\sin^2\theta}\left(1-\tan^2\frac{\theta}{2}\right)^{\frac{\alpha_s C_A}{\pi}\ln\left(\frac{\mu}{B}\right)}$$

Connecting to Inclusive Cross-Section: Thrust

$$S^{(1)}(t_H, t_L) = \int [d^d p]_+ \frac{n \cdot \bar{n}}{n \cdot p \, p \cdot \bar{n}} \theta(t_H - n \cdot p) \theta(\bar{n} \cdot p - n \cdot p) \left\{ \theta(t_L - n \cdot p) + \theta(n \cdot p - t_L) \left(1 - \frac{n \cdot p}{\bar{n} \cdot p}\right)^{\frac{\alpha_s C_A}{\pi} \ln\left(\frac{n \cdot p}{t_L}\right)} \right\}$$

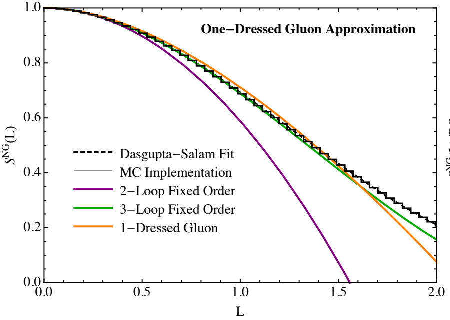
- $A = t_H$ and $B = t_L$.
- Use soft jet description when the factorization is valid: $z \sim t_H > t_L$, and $0 \ll \theta < \frac{\pi}{2}$.

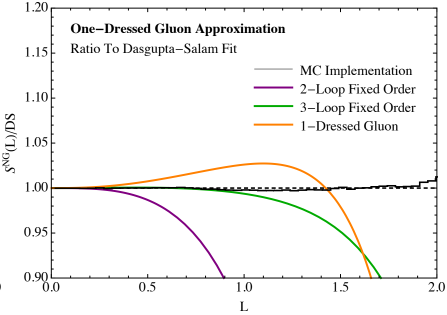
Connecting to Inclusive Cross-Section: Thrust

$$S^{(1)}(t_H, t_L) = \int [d^d p]_+ \frac{n \cdot \bar{n}}{n \cdot p \, p \cdot \bar{n}} \theta(t_H - n \cdot p)\theta(\bar{n} \cdot p - n \cdot p)$$
$$\left\{ 1 + \theta(n \cdot p - t_L) \left(\left(1 - \frac{n \cdot p}{\bar{n} \cdot p} \right)^{\frac{\alpha_s C_A}{\pi} \ln\left(\frac{n \cdot p}{t_L}\right)} - 1 \right) \right\}$$

- "1" is global contribution.
- Remove region where soft subjet is not a soft subjet but an unresolved parton.
- When unresolved, linked to the virtual corrections via KLN.

Single Soft Jet Contribution To Full LL NGLs





$$L = \frac{\alpha_s C_A}{\pi} \ln \frac{t_H}{t_L}$$

Single Soft Jet NGL contribution

Begins to break down at $L \sim 1$.

- A single soft jet factorization resums only the NGLs originating off of the jet.
- Does not resum NGLs of $\frac{e_3^{(\beta)}}{B}$.
- The inclusive hemi-sphere thrust distribution includes contributions from potentially arbitrary number soft subjets.

Including arbitrary soft strongly-ordered subjets.

Need a general factorization formula for $N \to N + 1_{sj}$ process.**Conjecture:**

$$\frac{d\sigma(B_N)}{dz_{sj}d\Omega_{sj}de_{res}} = \mathbf{tr}\Big[\mathbf{H}_N \sum_{\{i,...,k\}\subset\{1,...,N\}} \mathbf{H}^{sj}_{i...k}\frac{\mathbf{S}_{i...k\,n_{sj}}}{\mathbf{S}_{i...k}}\mathbf{S}_N\Big]\tilde{J}_{sj}\prod_{i=1}^{N} J_i$$

- **Given original hard and soft factors $\mathbf{H}_N \cdot \mathbf{S}_N$ of N-jet factorization, add soft jet.**
- **$\mathbf{H}^{sj}_{i...k}$ hard function encoding creation of soft jet off of eikonal lines $i...k$.**
- **$\mathbf{S}_{i...k\,n_{sj}}\mathbf{S}^{-1}_{i...k}$ ratio of soft eikonal wilson lines describing soft interations of soft subjet.**

Including arbitrary soft strongly-ordered subjets.

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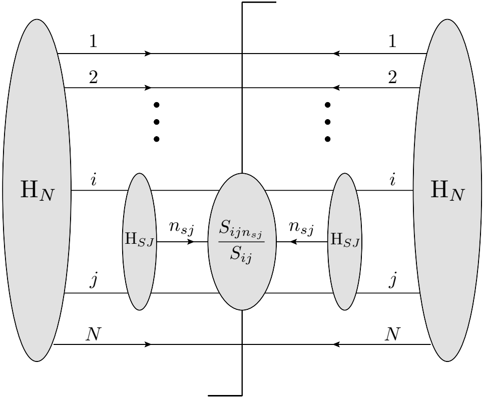
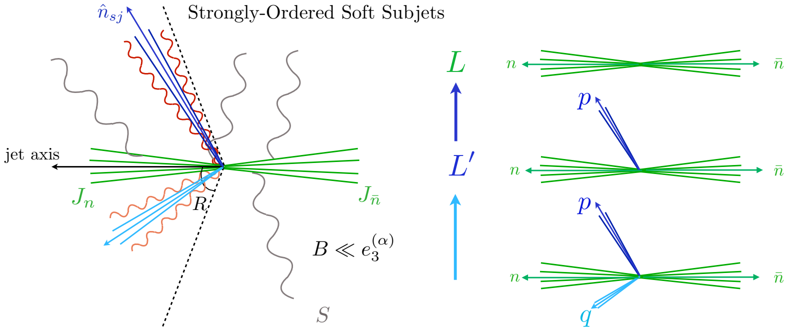


Iterating Soft jet factorization.



Sequence of EFTs [Bauer,Schwartz; Baumgart, Marcantonini, Stewart] to improve NGL resummation:

$$0_{sj} \to 1_{sj} \to 2_{sj}$$

Iterating Soft jet factorization.

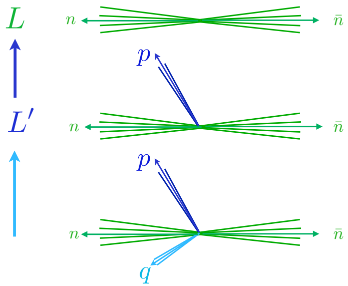
- For each hard and soft factor pairs of the previous factorization, apply $N \to N + 1sj$.
- Form appropriate **W** and **G** functions from the factorization.
- Resum NGLs thru RG of factorizations.
- Collinear limits subtracted to remove overlap with collinear subjet factorizations.

$$\frac{d\sigma(B_N)}{dz_{sj}d\Omega_{sj}de_{res}} = \mathrm{tr}\Big[\mathbf{H}_N \sum_{\{i,...,k\}\subset\{1,...,N\}} \mathbf{H}^{sj}_{i...k}\frac{\mathbf{S}_{i...k\, n_{sj}}}{\mathbf{S}_{i...k}}\mathbf{S}_N\Big]\tilde{J}_{sj}\prod_{i=1}^{N} J_i$$

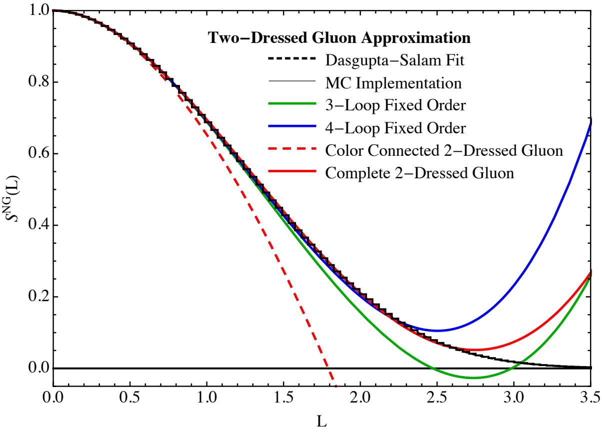
Jets as the quasi-particles of pQCD

In order to resum the NGLs of an inclusive cross-section:

- Reorganize perturbative series as an expansion in **jets (quasi-particles)**, not partons.
- Define jets through physical observables: z, θ, e_{res}
- Factorizations in corners of phase-space resum NGLs.
- Keep quasi-particle description to regions where it is valid.
- Sum over the possible jet histories.



Two Soft Jet Contribution



$$L = \frac{\alpha_s C_A}{\pi} \ln \frac{t_H}{t_L}$$

Soft Jet Expansion Versus Evolution equations.

The soft jet expansion can be derived from evolution equation resummation of NGLs [Banfi, Marchesini, Smye; Caron-Huot]:

$$\partial_L g_{n\bar{n}}(L) = \int_{in} \frac{d\Omega_j}{4\pi} W_{n\bar{n}}(j) \Big\{ U_{n\bar{n}j}(L) g_{nj}(L) g_{j\bar{n}}(L) - g_{n\bar{n}}(L) \Big\}$$

$$U_{n\bar{n}j}(L) = \text{Exp}\left[L \int_{out} \frac{d\Omega_\ell}{4\pi} W_{n\bar{n}}(\ell) - W_{nj}(\ell) - W_{j\bar{n}}(\ell) \right]$$

$$g_{n\bar{n}} = \text{Exp}\left[\sum_{k=1}^{\infty} g_{n\bar{n}}^{(k)} \right]$$

Each $g_{n\bar{n}}^{(k)}$ corresponds to a marginalization over a resummed factorization theorem with k-soft jets with all collinear limits subtracted.

Conclusions

- Resummation of NGLs realize jets as the quasi-particles of perturbative QCD.
- Expansion for NGLs can be mapped to physically observable processes.
- Full NGL resummation must sum over all possible physically realizable histories.
- The diluteness of weak coupling jets and collinear subtractions implies only few such physical processes needed to provide an accurate description for phenomological NGLs.

Directions

- Calculate to subleading orders in factorization theorems for subleading NGL contributions.
- Collinear Splittings at the Jet Boundary.
- Size of $L \leftrightarrow$ average (sub)jet multiplicity.
- What does truncation of soft jet expansion truncate in?
- Buffer region of k soft jets $\rightarrow$ asymptotic behavoir of $g_{n\bar{n}}^{(k)}$ as $L \rightarrow \infty$.