$\mathcal{T}^{\rm jet}$ resummation in Higgs production at NLL$'$ + NLO

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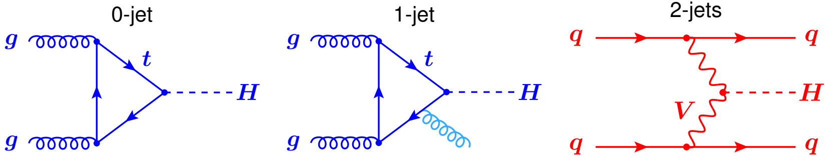
Outline

Introduction

- Jet binning is important :
  Distinguish different processes based on number of jets in the final state.

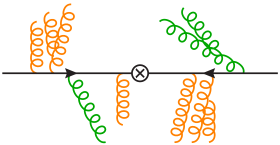


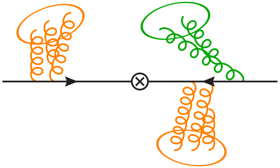
  Separate different backgrounds : eg in H $\rightarrow$ W W* any hard jets with $p_T^{\text{jet}} > p_T^{\text{cut}} \sim 20 - 30\,\text{GeV}$ are vetoed.
- Any type of such exclusive measurements or vetoes induce large Sudakov double logs of the veto scale p_T^{cut} : $\alpha_s^n \log^m [p_T^{\text{cut}}/m_H]$.
- Eg: gg $\rightarrow H + 0$ jet

$$\sigma_0(p_T^{\text{cut}}) \propto \sigma_B(1 - 2\frac{\alpha_s C_A}{\pi} \log^2 \frac{p_T^{\text{cut}}}{m_H} + ...)$$

- These large logarithms must be resummed to all orders to obtain reliable precision predictions.

Jet veto Observables





Global Veto
restricts $\sum$ of all emissions

"beam broadening"

$$E_T = \sum_i |\vec{p}_{Ti}|$$

"beam thrust"

$$\mathcal{T} = \sum_i |\vec{p}_{T\,i}| e^{-|y_i - Y|}$$

(y_i and Y are the jet and Higgs rapidity respectively.)

Local Veto
restrict (local chunks of) individual emissions

"jet p_T"

$$\vec{p}_T^{\,jet} = \max_{j \in \text{jets}} |\vec{p}_{Tj}|$$

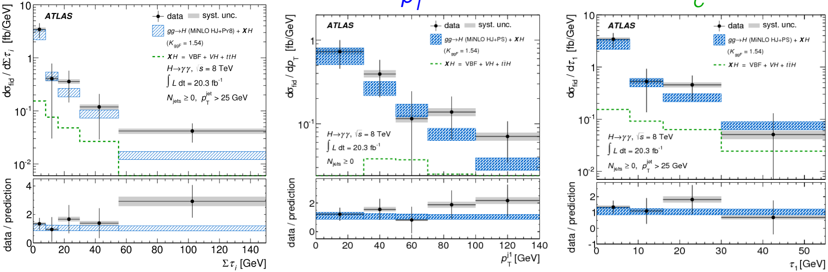
"Rapidity-Dependent"

$$\mathcal{T}_f^{\text{jet}} = \max_{j \in \text{jets}} |p_{Tj}| f(y_j)$$

Jet veto Observables

- Fiducial differential cross section measurements exist for different jet veto observables.

$\sum \mathcal{T}$ $\quad$ p_T^{jet} $\quad$ $\mathcal{T}_C^{\mathrm{jet}}$



- Resummation of various jet veto observables has been done :
  1. Beam Thrust resummation: Berger, Marcantonini, Stewart, Tackmann, Waalewijn
  2. p_T^{jet} resummation :
     H + 0-jet: Banfi, Monni, Salam, Zanderighi [1203.5773, 1206.4998]
     H + 0-jet: Becher, Neubert, Rothen [1205.3806, 1307.0025]
     H + 0-jet: Stewart, Tackmann, Walsh, Zuberi [1307.1808]
  3. In this talk : $\mathcal{T}_{B(C)}^{\mathrm{jet}}$ resummation

Rapidity dependent Jet Vetoes $\mathcal{T}_{B,C}$

$$p_T^{\text{jet}} = \max_{j\in\text{jets}}\{|\vec{p}_{Tj}|\theta(|y_j| < y^{\text{cut}})\}\,, \quad \mathcal{T}_{fj} = |p_{Tj}| f(y_j) \;\rightarrow\; \mathcal{T}_f^{\text{jet}} = \max_{j\in\text{jets}} \mathcal{T}_{fj} < \mathcal{T}^{\text{cut}}$$

$$\mathcal{T}_{Bcm}^{\text{jet}} : f(y_j) = e^{-|y_j|}$$

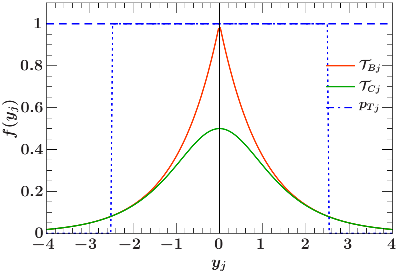
$$\mathcal{T}_{B}^{\text{jet}} : f(y_j) = e^{-|y_j - Y|}$$

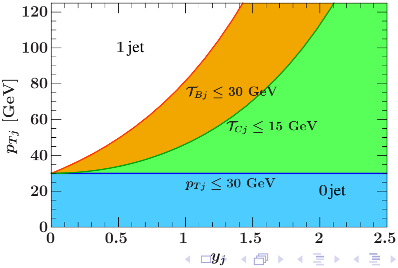
$$\mathcal{T}_{Ccm}^{\text{jet}} : f(y_j) = \frac{1}{2\cosh(y_j)}$$

$$\mathcal{T}_{C}^{\text{jet}} : f(y_j) = \frac{1}{2\cosh(y_j - Y)}$$

0-jet bin: $\mathcal{T}_f^{\text{jet}} < \mathcal{T}^{\text{cut}}$

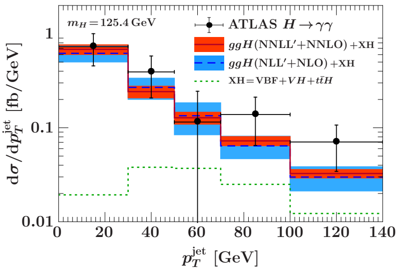
$\geq$ 1-jet bin: $\mathcal{T}_f^{\text{jet}} > \mathcal{T}^{\text{cut}}$

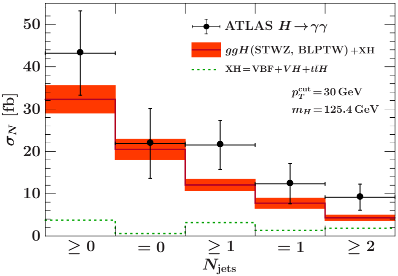




Motivation

- Jets can only be reconstructed down to some minimum p_T : limit on the jet veto cut. Low p_T jets hard to measure at forward rapidities.
- Rapidity dependent observables : A tight veto on central jets and veto constraint getting looser for forward jets.
- $\mathcal{T}_B^{\text{jet}}$ and $\mathcal{T}_C^{\text{jet}}$ are rapidity weighted p_T^{jet}, resummable to same level as p_T^{jet}.
- Provide complementary information in the exclusive jet bins.





[Stewart, Tackmann, Walsh, Zuberi], [Boughezal, Petriello, Liu, Walsh, Tackmann]

Soft-collinear Factorization and Resummation

The cross section in SCET for $gg \to H$:

$$d\sigma_0 = |C_{ggH}|^2 \langle p_a p_b | O^\dagger_{ggH} M^{veto} O_{ggH} | p_a p_b \rangle$$

$$O_{ggH} \sim H O_a O_s O_b \sim H B^\mu_{n_a} T[Y^\dagger_{n_a} Y_{n_b}] B_{n_b \mu}$$

Measurement function M^{veto}

- Implements phase space constraints due to jet veto.
- Soft-collinear factorization implies

$$M^{veto} = M^{veto}_a \times M^{veto}_b \times M^{veto}_s + \mathrm{O}(R^2)$$

- Matrix elements factorize into independent soft and collinear matrix elements

$$B_{a,b}(\mu) = \langle p_a | O^\dagger_{a,b} M_{a,b} O_{a,b} | p_b \rangle$$

$$S_{a,b}(\mu) = \langle 0 | O^\dagger_s M_s O_s | 0 \rangle$$

Jet algorithm effects in local vetoes

- Measurement function for $\mathcal{T}_f^{\rm jet} < \mathcal{T}^{\rm cut}$: product of measurement functions on individual jets.

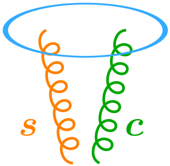
$$M_f^{\rm jet}(\mathcal{T}^{\rm cut}) = \theta(\mathcal{T}_f^{\rm jet} < \mathcal{T}^{\rm cut}) = \prod_{j \in J(R)} \theta(\mathcal{T}_{fj} < \mathcal{T}^{\rm cut})$$

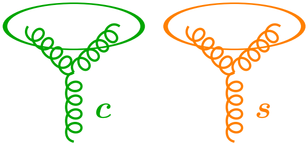
- Measurement factorized into soft and collinear components:

$$M_f^{\rm jet} = (M_{fa}^{\rm jet} + \Delta M_{fa}^{\rm jet})(M_{fb}^{\rm jet} + \Delta M_{fa}^{\rm jet})(M_{fs}^{\rm jet} + \Delta M_{fs}^{\rm jet}) + \delta M_f^{\rm jet}$$

$\delta M_f^{\rm jet}$: $O(R^n)$ clustering of soft and collinear components into the same jet. Relevant for large R $\sim$ 1.

$\Delta M_f^{\rm jet}$: $O(\ln^n R)$ clustering within each sector. Relevant for small $R \ll 1$





- Typical jet radius used in experiments ATLAS: 0.4, CMS: 0.5

Soft collinear Factorization

The full H+0-jet cross section differential in the Higgs rapidity Y and with $\mathcal{T}^{jet}_{B,C} < \mathcal{T}^{cut}$

$$\frac{d\sigma_0}{dY} = \sigma_B H_{gg}(m_t, m_H^2, \mu)\, B_g(m_H \mathcal{T}^{\rm cut}, x_a, R, \mu) B_g(m_H \mathcal{T}^{\rm cut}, x_b, R, \mu) \times S_{gg}^{B,C}(\mathcal{T}^{\rm cut}, R, \mu) + \frac{d\sigma_0^{\rm Rsub}}{dY}(\mathcal{T}^{\rm jet}_{B,C} < \mathcal{T}^{\rm cut}, R) + \frac{d\sigma_0^{\rm nons}}{dY}(\mathcal{T}^{\rm jet}_{B,C} < \mathcal{T}^{\rm cut})$$

For $\mathcal{T}^{jet}_{Bcm,Ccm} < \mathcal{T}^{cut}$

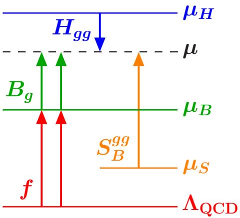
$$\frac{d\sigma_0}{dY} = \sigma_B H_{gg}(m_t, m_H^2, \mu)\, B_g(m_H \mathcal{T}^{\rm cut} e^{Y}, x_a, R, \mu) B_g(m_H \mathcal{T}^{\rm cut} e^{-Y}, x_b, R, \mu) \times S_{gg}^{B,C}(\mathcal{T}^{\rm cut}, R, \mu) + \frac{d\sigma_0^{\rm Rsub}}{dY}(\mathcal{T}^{\rm jet}_{Bcm,Ccm} < \mathcal{T}^{\rm cut}, R) + \frac{d\sigma_0^{\rm nons}}{dY}(\mathcal{T}^{\rm jet}_{Bcm,Ccm} < \mathcal{T}^{\rm cut})$$

Resummation

- Only difference between $\mathcal{T}_B^{\text{jet}}$ and $\mathcal{T}_C^{\text{jet}}$ to all orders: different soft functions.
- Beam functions same: Describe collinear ISR i.e. emissions with forward rapidities where $\mathcal{T}_B^{\text{jet}}$ and $\mathcal{T}_C^{\text{jet}}$ measurements are equal.
- Logarithms are split apart and resummed using RGE

$$\ln^2 \frac{\mathcal{T}^{cut}}{m_H} = 2\ln^2 \frac{m_H}{\mu} - \ln^2 \frac{\mathcal{T}^{cut} m_H}{\mu^2} + 2\ln^2 \frac{\mathcal{T}^{cut}}{\mu}$$

- Natural scales for Hard, beam and soft functions:
  $\mu_H \simeq m_H$, $\mu_B \simeq \sqrt{\mathcal{T}^{\text{cut}} m_H}$, $\mu_S \simeq \mathcal{T}^{\text{cut}}$
- Hard function contains: $\ln^2\left[-m_H^2/\mu_H^2\right]$
- Beam functions: $\ln^2\left[m_H \mathcal{T}^{cut}/\mu_B^2\right]$
- Soft function: $\ln^2\left[\mathcal{T}^{cut}/\mu_S\right]$



Resummation

- Resummation Structure:

$$\ln \sigma_0(\mathcal{T}^{\rm cut}) \sim \sum_n \alpha_s^n \ln\left[\frac{\mathcal{T}^{\rm cut}}{m_H}\right]^{n+1} (1+\alpha_s+\alpha_s^2+\cdots) \sim \text{LL}+\text{NLL}+\text{NNLL}+\cdots$$

- Including the RGE running the resummed cross section for $\mathcal{T}^{jet}$

$$\sigma_0(\mathcal{T}^{cut}) = H_{ggH}(m_H,\mu_H)\, U_H(m_H,\mu_H,\mu) \times B_g(x_a,R,\mathcal{T}^{cut},\mu_B)$$
$$B_g(x_b,R,\mathcal{T}^{cut},\mu_B)\, U_B(m_H,\mu_B,\mu)^2 \times S_{gg}^{B,C}(R,\mathcal{T}^{cut},\mu_S)\, U_S(\mu_S,\mu)$$

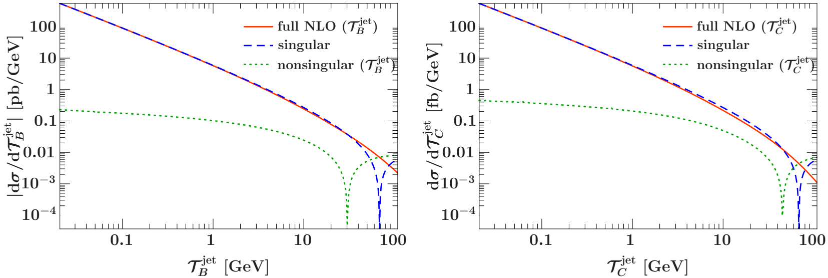
Log counting:	Fixed-order corrections matching	nonsingular	Resummation input $\gamma^{\mu}_{H,B,S}$	$\Gamma_{\rm cusp}$	β
NLL	1	-	1-loop	2-loop	2-loop
NLL′+ NLO	NLO	NLO	1-loop	2-loop	2-loop
NNLL′+NNLO	NNLO	NNLO	2-loop	3-loop	3-loop

Nonsingular Contribution

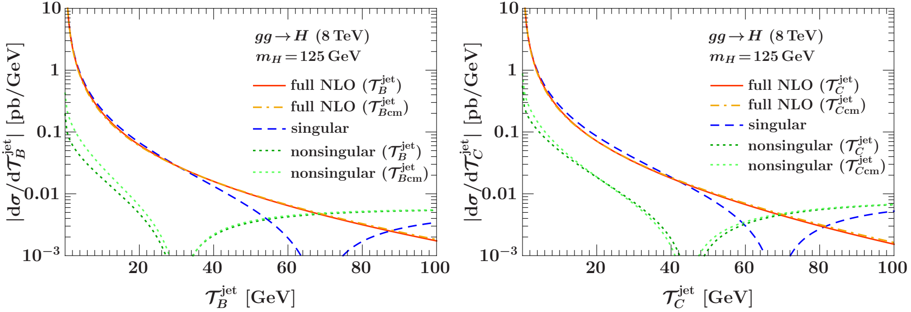
- To compute nonsingular corrections for different jet veto observables:

$$\frac{\mathrm{d}\sigma_0^{\mathrm{nonsing}}}{\mathrm{d}\mathcal{T}_f^{\mathrm{jet}}\mathrm{d}Y} = \frac{\mathrm{d}\sigma_0^{\mathrm{FO}}}{\mathrm{d}\mathcal{T}_f^{\mathrm{jet}}\mathrm{d}Y} - \frac{\mathrm{d}\sigma_0^{\mathrm{resum}}}{\mathrm{d}\mathcal{T}_f^{\mathrm{jet}}\mathrm{d}Y}(\mu_H = \mu_B = \mu_S = \mu_{\mathrm{FO}})$$

- $\mathrm{d}\sigma_0^{\mathrm{FO}}$: Computed full FO cross section differential in Y and $\mathcal{T}_f^{\mathrm{jet}}$. Implemented non-trivial measurement functions for $\mathcal{T}_f^{\mathrm{jet}}$.
- Validation plots:



Resummation scales



- Resummation region: Logs are resummed using canonical scaling

$$|\mu_H| \sim m_H \qquad \mu_S \sim \mathcal{T}^{\rm cut} \qquad \mu_B \sim \sqrt{m_H \mathcal{T}^{\rm cut}}$$

- FO region: Resummation is turned off to get the right FO cross section at large $\mathcal{T}^{\rm cut}$

$$\mu_B, \mu_S \to \mu_{\rm FO} \sim m_H$$

- Transition region: Profiles for μ_B, μ_S provide smooth transition from resummation to fixed-order region.

Perturbative Uncertainties in Jet Binning

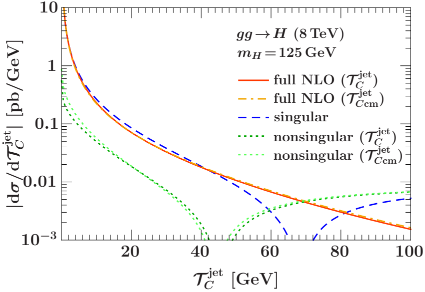
$$\sigma_{\text{total}} = \int_0^{\mathcal{T}^{\text{cut}}} d\mathcal{T}^{\text{jet}} \frac{d\sigma}{d\mathcal{T}^{\text{jet}}} + \int_{\mathcal{T}^{\text{cut}}}^{\infty} d\mathcal{T}^{\text{jet}} \frac{d\sigma}{d\mathcal{T}^{\text{jet}}}$$
$$\equiv \sigma_0(\mathcal{T}^{\text{cut}}) + \sigma_{\geq 1}(\mathcal{T}^{\text{cut}})$$

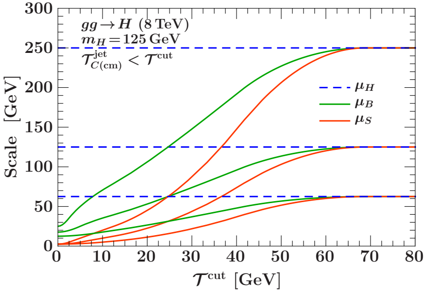
- Uncertainties in the jet binning can be described by fully correlated (yield) and fully anticorrelated (migration) components of a covariance matrix $\{\sigma_0, \sigma_{\geq 1}\}$ [Stewart,Tackmann: 1107.2117, SG,Tackmann: 1302.4437]

$$C = \begin{pmatrix} (\Delta_0^{\text{y}})^2 & \Delta_0^{\text{y}}\,\Delta_{\geq 1}^{\text{y}} \\ \Delta_0^{\text{y}}\,\Delta_{\geq 1}^{\text{y}} & (\Delta_{\geq 1}^{\text{y}})^2 \end{pmatrix} + \begin{pmatrix} \Delta_{\text{cut}}^2 & -\Delta_{\text{cut}}^2 \\ -\Delta_{\text{cut}}^2 & \Delta_{\text{cut}}^2 \end{pmatrix}$$

- yield uncertainty $\Delta^y \equiv \Delta_{FO}$: Fixed-order scale uncertainty in σ_{total}
- migration uncertainty $\Delta_{\text{cut}} \equiv \Delta_{\text{resum}}$: Resummation uncertainty induced by the binning cut and drops out in the sum $\sigma_0 + \sigma_{\geq 1}$.
  $\Delta_0^2(\mathcal{T}^{\text{cut}}) = \Delta_{\text{FO}}^2(\mathcal{T}^{\text{cut}})$ + $\Delta_{\text{resum}}^2(\mathcal{T}^{\text{cut}})$

Resummation Scales and Fixed-Order Uncertainty





- Central profiles:

$$\mu_H = -i\mu_{\rm FO}\,, \quad \mu_{\rm ns} = \mu_{\rm FO}\,,$$
$$\mu_S(\mathcal{T}^{\rm cut}) = \mu_{\rm FO} f_{\rm run}(\mathcal{T}^{\rm cut}/m_H)\,,$$
$$\mu_B(\mathcal{T}^{\rm cut}) = \sqrt{\mu_S(\mathcal{T}^{\rm cut})\mu_{\rm FO}} = \mu_{\rm FO}\sqrt{f_{\rm run}(\mathcal{T}^{\rm cut}/m_H)}$$

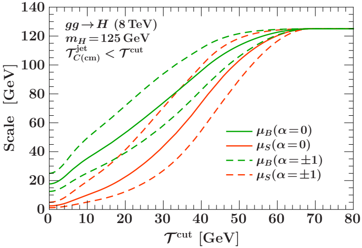
- Fixed Order Uncertainty Δ_{FO}:
  - Maximum of collective variation of all scales by a factor of 2 keeping scale ratios fixed.
  - Reproduces the inclusive cross section uncertainty for large $\mathcal{T}^{\rm cut}$.

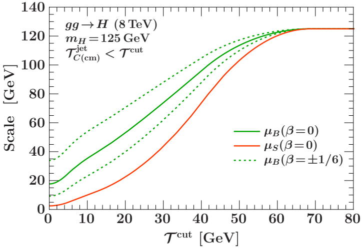
Resummation Uncertainty

- Varying argument of logs estimates their size and missing higher log terms.
- These variations are smoothly turned off at large $\mathcal{T}^{cut}$ where the resummation turns off.

$$\mu_S^{\rm vary}(x,\alpha) = f_{\rm vary}^{\alpha}(x)\,\mu_S(x) = \mu_{\rm FO}\, f_{\rm vary}^{\alpha}(x)\, f_{\rm run}(x)$$

$$\mu_B^{\rm vary}(x,\alpha,\beta) = \mu_S^{\rm vary}(x,\alpha)^{1/2-\beta}\,\mu_{\rm FO}^{1/2+\beta} = \mu_{\rm FO}\left[f_{\rm vary}^{\alpha}(x)\, f_{\rm run}(x)\right]^{1/2-\beta}$$





Resummation Uncertainties

Choosing α and β for μ_S and μ_B variation (for SCET-I type observables)

- Retain natural scale hierarchy: $\mu_{\rm FO} \sim \mu_H \gg \mu_B = \sqrt{\mu_S \mu_{\rm FO}} \gg \mu_S$.
- A systematic way to vary μ_B and μ_S without double counting.

$$\mu_S^{\rm vary}(x,\alpha) = \mu_{\rm FO}\, f_{\rm vary}^{\alpha}(x)\, f_{\rm run}(x)\,, \quad \mu_B^{\rm vary}(x,\alpha,\beta) = \mu_{\rm FO}\left[f_{\rm vary}^{\alpha}(x)\, f_{\rm run}(x)\right]^{1/2-\beta}$$

- Independent scale ratios entering the resummed logs are

$$\frac{\mu_B^2}{\mu_H^2} \sim \frac{\mathcal{T}^{\rm cut}}{m_H}\,, \qquad \frac{\mu_S^2}{\mu_B^2} \sim \frac{\mathcal{T}^{\rm cut}}{m_H}$$

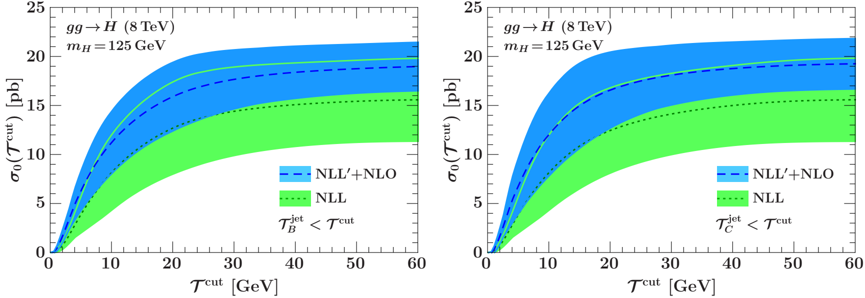
- α variation for both μ_S and μ_B: Equal changes in the log ratios.

$$\ln\frac{\mu_B^2}{\mu_H^2} \to \ln f_{\rm vary}^{\alpha} + \ln\frac{\mu_B^2}{\mu_H^2}\,, \quad \ln\frac{\mu_S^2}{\mu_B^2} \to \ln f_{\rm vary}^{\alpha} + \ln\frac{\mu_S^2}{\mu_B^2}\,.$$

- β variation: Equal magnitude opposite sign changes in the log ratios.

$$\ln\frac{\mu_B^2}{\mu_H^2} \to (1-2\beta)\ln\frac{\mu_B^2}{\mu_H^2}\,, \quad \ln\frac{\mu_S^2}{\mu_B^2} \to (1+2\beta)\ln\frac{\mu_S^2}{\mu_B^2}\,.$$

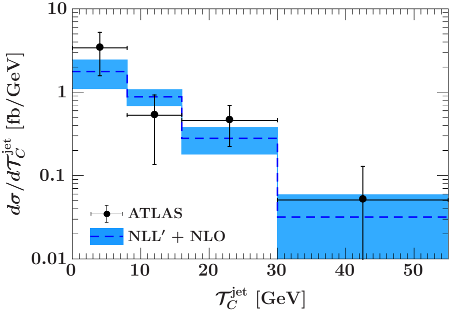
NLL′+NLO results for H+0-jet



- Good convergence and reduced uncertainties going from NLL to NLL′+NLO.
- At $\mathcal{T}^{\rm cut} \sim 25\,\mathrm{GeV}$, 20% uncertainty for *NLL*′+NLO,comparable to the precision for $p_T^{\rm jet}$.
- Expect significant reduction in uncertainties going to NNLL′+NNLO.

Comparison to ATLAS data

- The resummed results for different jet veto observables can be directly compared to the ATLAS measurements.
- Comparison of our NLL′+NLO predictions in bins of $\mathcal{T}_C^{\rm jet}$ with the recent ATLAS measurement in $H \to \gamma\gamma$ channel.
- Several correction factors implemented given in ATLAS pub:1407.4222 for $H \to \gamma\gamma$ branching ratio, photon isolation efficiency, diphoton kinematic acceptance.



Summary

- Resummation of jet veto logs: important for accurate cross section predictions.
- Rapidity dependent jet vetoes:
  1. A natural and efficient central jet veto, Resummable to same level of precision as $p_T^{\rm jet}$.
  2. Provide complementary way to divide phase space into exclusive jet bins. $\longrightarrow$ Motivation to measure rapidity-dependent jet-vetoes in other processes.
- Determined NLL$'$+NLO resummed results for $\mathcal{T}_{Bcm}^{\rm jet}$, $\mathcal{T}_{B}^{\rm jet}$, $\mathcal{T}_{Ccm}^{\rm jet}$ and $\mathcal{T}_{C}^{\rm jet}$ with robust uncertainty estimates. Comparison with ATLAS data!
- Next step is resummation to NNLL$'$+NNLO order :
  1. Beam and soft functions at 2 loops + ln R^2 clustering corrections (W.I.P)
  2. Extension to full NNLL$'$+NNLO.

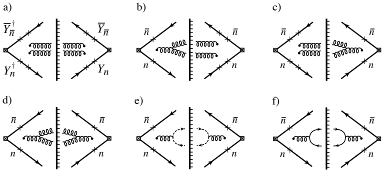
Outlook: Clustering correction to soft function

- Dijet hemisphere soft function

$$\Delta S^{(2)}(k_1, k_2) = \int \frac{d^d k_1}{(2\pi)^d} \frac{d^d k_2}{(2\pi)^d} A_j(k_1, k_2) \Delta M^{jet(2)}(k_1, k_2) C(k_1) C(k_2)$$

$$\int \frac{d^d k_1}{(2\pi)^d} \frac{d^d k_2}{(2\pi)^d} = \int_0^\infty dk_1^+ dk_2^+ dk_1^- dk_2^- (k_1^+ k_1^- k_2^+ k_2^-)^{-\epsilon} \int_0^\pi d\Delta\phi \sin^{-2\epsilon} \Delta\phi$$

- For the clustering corrections only non-abelian matrix elements considered. [Hornig, Lee, Stewart, Walsh, Zuberi '11]



- For a $\mathcal{T}_B$ veto, the full measurement function can be written as [following Tackmann, Walsh, Zuberi '12]

$$M_i^{\text{jet}}(\mathcal{T}^{\text{cut}}) = M_i(\mathcal{T}^{\text{cut}}) + \Delta M_i^{\text{jet}}(\mathcal{T}^{\text{cut}})$$

Outlook: Clustering correction to soft function

Inclusive $\mathcal{T}_B$ measurement

$$M_i(\mathcal{T}^{\rm cut}) = \theta(\sum_j \mathcal{T}_j < \mathcal{T}^{\rm cut})$$

Measurement function for the clustering correction relative to M_i

$$\begin{aligned}\Delta M^{\rm jet}(\mathcal{T}^{\rm cut}) &= 2[\theta(\Delta R_{12} < \Delta R)\theta(\mathcal{T}_B^{\rm jet} < \mathcal{T}^{\rm cut}) + \theta(\Delta R_{12} > R)\theta(\mathcal{T}_{B1} < \mathcal{T}^{\rm cut})\theta(\mathcal{T}_{B2} < \mathcal{T}^{\rm cut}) \\ &\quad -\theta(\mathcal{T}_{B1} + \mathcal{T}_{B2} < \mathcal{T}^{\rm cut})] \\ &= 2\theta(\Delta R_{12} > R)[\theta(\mathcal{T}_{B1} < \mathcal{T}^{\rm cut})\theta(\mathcal{T}_{B2} < \mathcal{T}^{\rm cut}) - \theta(\mathcal{T}_{B1} + \mathcal{T}_{B2} < \mathcal{T}^{\rm cut})]\end{aligned}$$

The variables we use are

$$y_t = \tfrac{1}{2}(y_1 + y_2) \quad \Delta y = y_1 - y_2 \quad z = \frac{\mathcal{T}_{B1}}{\mathcal{T}_{B1}+\mathcal{T}_{B2}} = \frac{k_1^+}{k_1^+ + k_2^+}$$

$$\mathcal{T}_T = \mathcal{T}_{B1} + \mathcal{T}_{B2} \quad \cos\Delta\phi = \frac{k_1^\perp . k_2^\perp}{|k_1^\perp||k_2^\perp|} = \frac{1/2(k_1^+ k_2^- + k_2^+ k_1^-) - k_1 . k_2}{\sqrt{k_1^+ k_1^- k_2^+ k_2^-}}$$

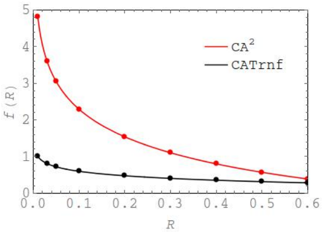
$$\Delta S^{(2)}(\mathcal{T}^{\rm cut}) = \frac{8}{(16\pi^2)^2}\int \frac{\mathcal{T}_T^{-1-4\epsilon}}{z(1-z)} \mathcal{T}_T^4 z^2(1-z)^2 (z^2(1-z)^2)^{-\epsilon} e^{-4\epsilon y_t} A_j \Delta M^{jet(2)} e^{4y_t} d\mathcal{T}_t dz dy_t d\Delta y$$

$$\begin{aligned}\Delta S^{(2)}(\mathcal{T}^{\rm cut}) &= \frac{-8}{(16\pi^2)^2}\frac{1}{4\epsilon}\left(\frac{\mu}{\mathcal{T}^{cut}}\right)^{4\epsilon}\int_0^1 dz \int_{-\infty}^{\infty} d\Delta y \int_0^\pi d\Delta\phi \sin^{-2\epsilon}\Delta\phi \frac{\log[\max(z,1-z)]}{z(1-z)}(z(1-z))^{-2\epsilon} \\ &\quad \theta(\Delta R_{12} > \Delta R)\mathcal{T}_T^4 z^2(1-z)^2 A_j\end{aligned}$$

Outlook: Clustering correction to soft function

- Remaining integrals finite and result of the form: $a \log R + b + O(R)$.
- To extract log R, express A_i in the small $\Delta R = \sqrt{\Delta y^2 + \Delta\phi^2}$ limit as A_i^R. Write $A = (A - A^R) + A^R$
- To determine a : Analytic integration of A^R. Extract b numerically by integrating $A - A^R$.

$$\Delta S^{(2)}(\mathcal{T}^{\rm cut}) = \left(\frac{\alpha_s}{\pi}\right)^2 \left(\frac{\mu}{\mathcal{T}^{cut}}\right)^{4\epsilon} \Bigg[\frac{1}{4\epsilon}\Bigg[C_A^2\Big\{\frac{1}{18}(131 - 12\pi^2 - 132\log 2)\log R - 0.937 + 0.652R^2 + ...)\Big\}$$
$$+C_A T_R n_f\Big\{\frac{-1}{9}(23 - 24\log 2)\log R + 0.747 + 0.019R^2 + ...)\Big\}\Bigg] + C_A^2\{-4.254\log R$$
$$+1.096\log^2 R + 1.713 + O(R^2)\} + C_A T_R n_f\{-0.451\log R + 0.177\log^2 R + 0.184 + O(R^2)\}\Bigg]$$

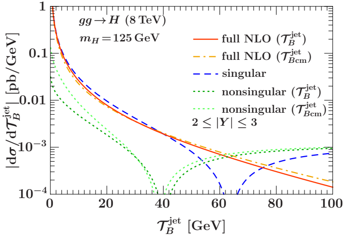


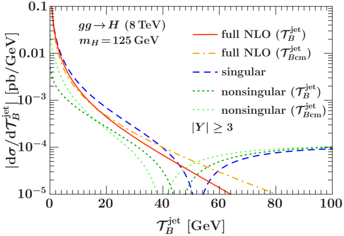
Analogous but more tedius calculation for $\mathcal{T}_C^{\rm jet}$ gives the same anom. dim. as $\mathcal{T}_B^{\rm jet}$.

BackUp slides

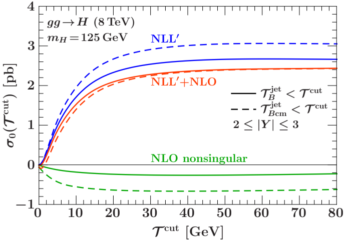
NLL′+NLO results for H+0-jet

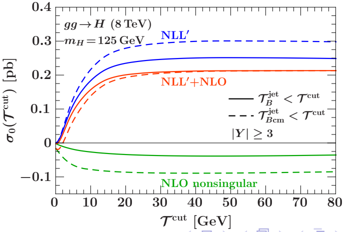
- Differential singular, nonsingular and full fixed order cross sections for $\mathcal{T}_B^{\text{jet}}$ in different Y bins.





- Cumulant NLL′ and nonsingular for $\mathcal{T}_B^{\text{jet}}$ in different Y bins.





Soft function and Resummation uncertainty

- Soft function for $\mathcal{T}_B^{\rm jet}$ and $\mathcal{T}_C^{\rm jet}$:

$$S_{gg}^C(\mathcal{T}^{\rm cut},\mu) = 1 + \frac{\alpha_s(\mu)C_A}{\pi}\left(-2\ln^2\frac{\mathcal{T}^{\rm cut}}{\mu} + \frac{\pi^2}{4}\right)$$

$$S_{gg}^B(\mathcal{T}^{\rm cut},\mu) = 1 + \frac{\alpha_s(\mu)C_A}{\pi}\left(-2\ln^2\frac{\mathcal{T}^{\rm cut}}{\mu} + \frac{\pi^2}{12}\right),$$

$$f_{\rm run}(x) = \begin{cases} x_0\left(1+\frac{x}{4x_0}\right) & x \le 2x_0 \\ x & 2x_0 \le x \le x_1 \\ x + \frac{(2-x_2-x_3)(x-x_1)^2}{2(x_2-x_1)(x_3-x_1)} & x_1 \le x \le x_2 \\ 1 - \frac{(2-x_1-x_2)(x-x_3)^2}{2(x_3-x_1)(x_3-x_2)} & x_2 \le x \le x_3 \\ 1 & x_3 \le x \end{cases}$$

$$f_{\rm vary}(x) = \begin{cases} 2(1-x^2/x_3^2) & 0 \le x \le x_3/2 \\ 1+2(1-x/x_3)^2 & x_3/2 \le x \le x_3 \\ 1 & x_3 \le x \end{cases},$$

Clustering Correction

- The variables we use are

$$y_1 = \frac{1}{2}\log\frac{k_1^-}{k_1^+} \qquad y_2 = \frac{1}{2}\log\frac{k_2^-}{k_2^+}$$

$$y_t = \tfrac{1}{2}(y_1 + y_2) \quad \Delta y = y_1 - y_2 \quad z = \frac{\mathcal{T}_{B1}}{\mathcal{T}_{B1}+\mathcal{T}_{B2}} = \frac{k_1^+}{k_1^+ + k_2^+}$$

$$\mathcal{T}_T = \mathcal{T}_{B1} + \mathcal{T}_{B2} \quad \cos\Delta\phi = \frac{k_1^\perp . k_2^\perp}{|k_1^\perp||k_2^\perp|} = \frac{1/2(k_1^+ k_2^- + k_2^+ k_1^-) - k_1 . k_2}{\sqrt{k_1^+ k_1^- k_2^+ k_2^-}}$$

$$A_f^R(\mathcal{T}^{cut}) = \frac{4g^4 C_A T_R n_f \mu^{4\epsilon} e^{-4y_t}}{\mathcal{T}_T^4 z^2(1-z)^2}\frac{4}{\Delta R^4}\left[\frac{z(1-z)}{2}\left(\Delta R^2 - 4z(1-z)\Delta y^2\right)\right]$$

$$A_A^R(\mathcal{T}^{cut}) = \frac{4g^4 C_A^2 \mu^{4\epsilon} e^{-4y_t}}{\mathcal{T}_T^4 z^2(1-z)^2}\frac{2}{\Delta R^2}\left[(1-z+z^2) - z(1-z) + \frac{2z^2(1-z)^2\Delta y^2(1-\epsilon)}{\Delta R^2}\right]$$

$$A_f(\mathcal{T}^{\text{cut}}) = \frac{4g^4 C_A T_R n_f \mu^{4\epsilon}}{\mathcal{T}_T^4 z^2(1-z)^2}\left[\frac{1}{(\cosh\Delta y - \cos\Delta\phi)(\cosh\Delta y - (1-2z)\sinh\Delta y)}\right]^2 \frac{z(1-z)}{2}\Big[1 - 2\cos\Delta\phi\cosh\Delta y +$$
$$2(1-2z)\cos\Delta\phi\sinh\Delta y + (\cosh\Delta y - (1-2z)\sinh\Delta y)^2\Big]$$

$$A_A(\mathcal{T}^{cut}) = \frac{4g^4 C_A^2 \mu^{4\epsilon}}{\mathcal{T}_T^4 z^2(1-z)^2}\frac{1}{(\cosh\Delta y - \cos\Delta\phi)(\cosh\Delta y - (1-2z)\sinh\Delta y)}\Big[(1-z+z^2)\cos\Delta\phi\cosh\Delta y -$$
$$(1-2z)\cos\Delta y\sinh\Delta\phi - z(1-z) + \frac{(1-\epsilon)z^2(1-z)^2\sinh\Delta y^2}{(\cosh\Delta y - \cos\Delta\phi)(\cosh\Delta y - (1-2z)\sinh\Delta y)}\Big]$$