Toward soft-gluon resummation for the associated production of a Higgs boson and a top-quark pair

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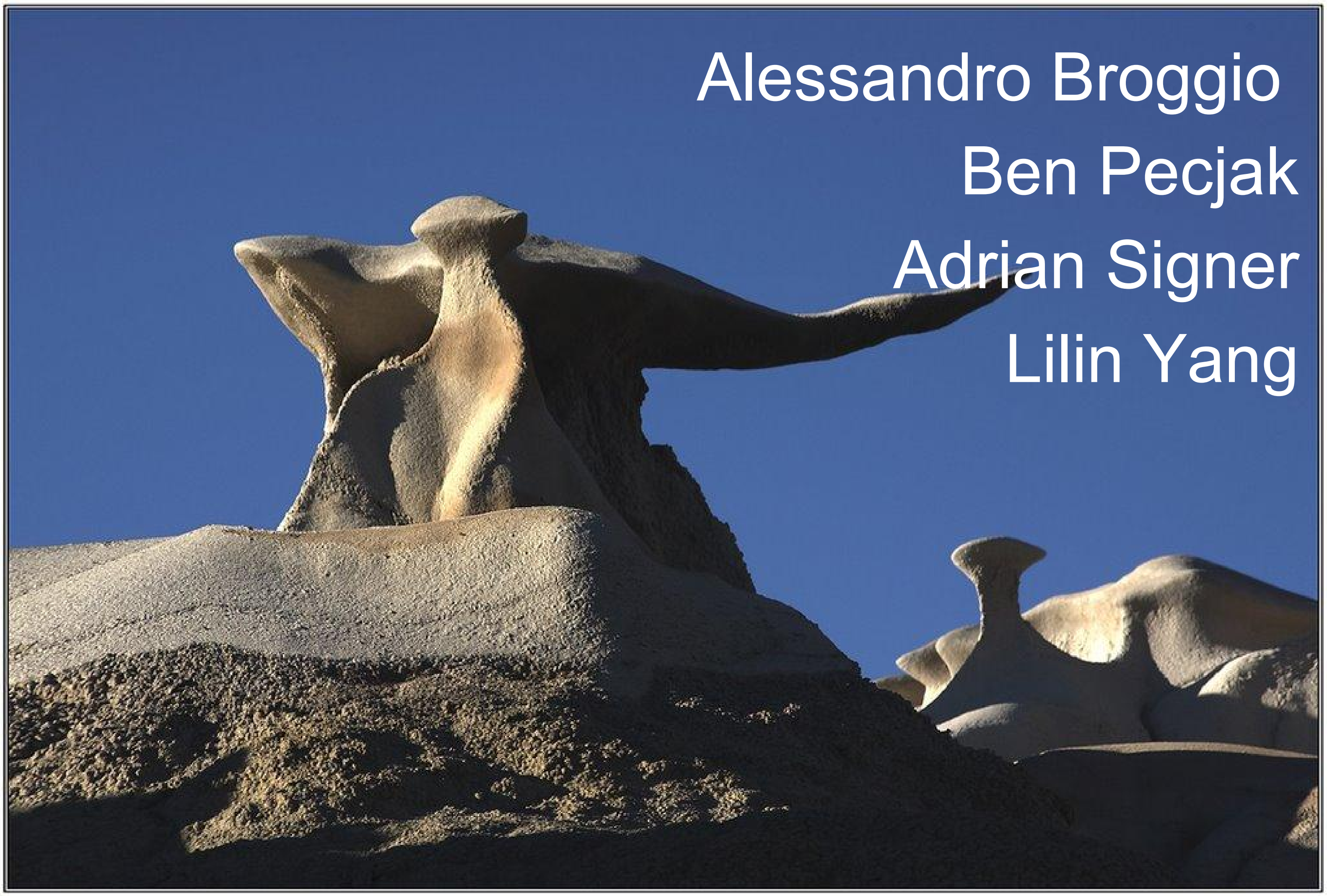
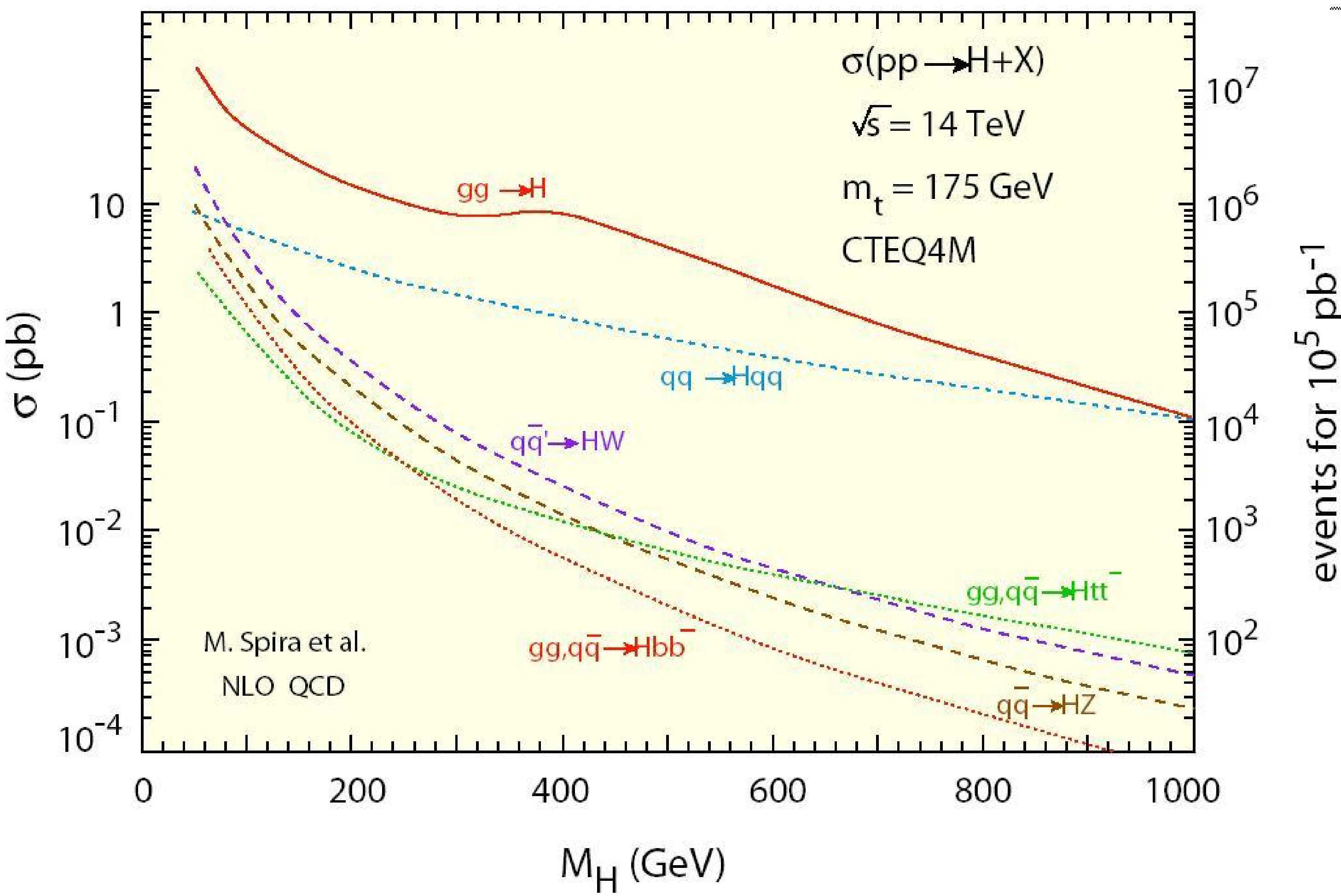
Ongoing work in collaboration with



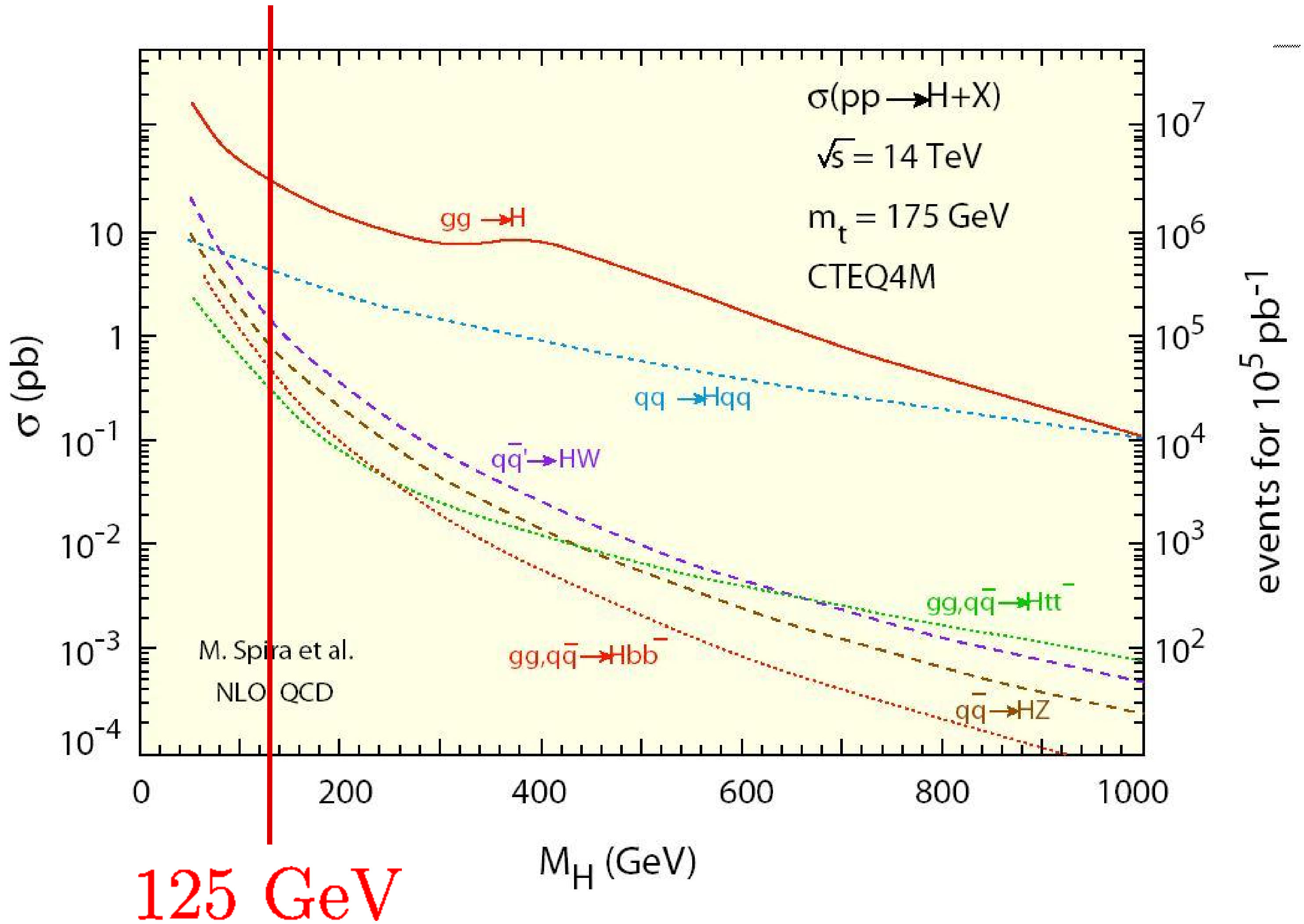
Outline

- Introduction / Motivation
- Factorization of the partonic cross section
- Implementation & numerical calculations

Associated production of top-pair+Higgs boson at the LHC



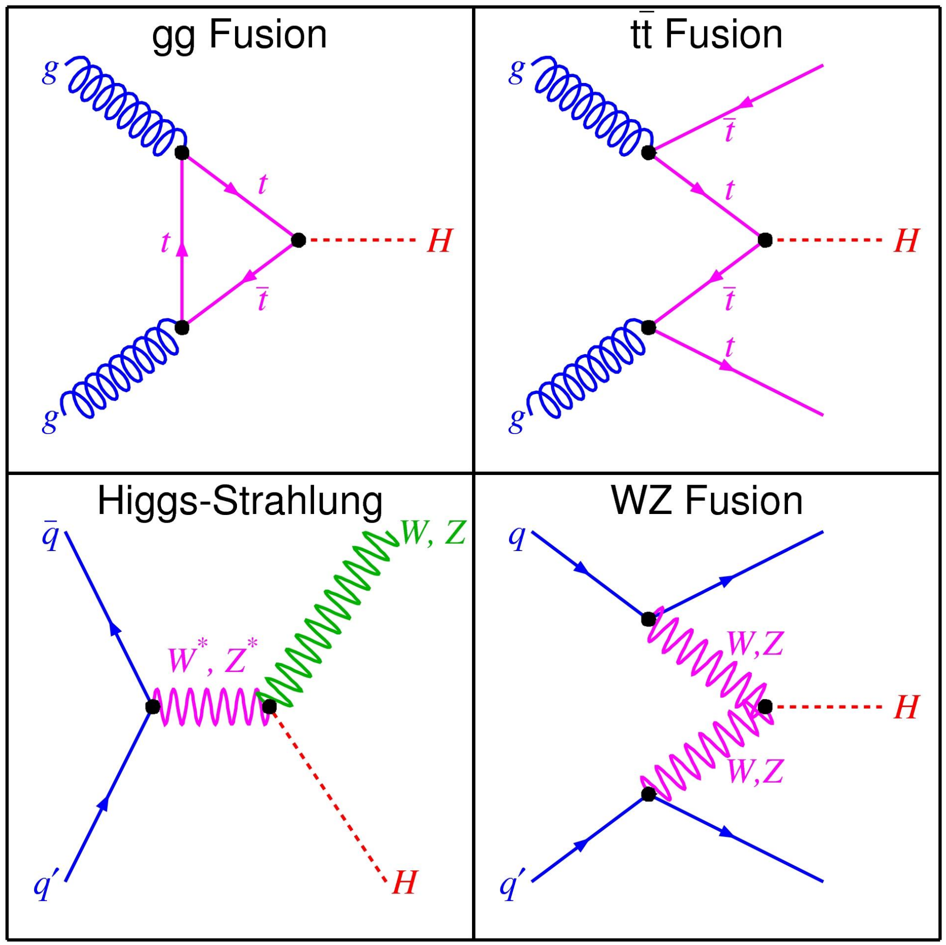
Associated production of top-pair+Higgs boson at the LHC



Associated production of top-pair+Higgs boson at the LHC

$\sigma \sim 49$ pb

$\sigma \sim 0.6$ pb



$\sigma_W \sim 1.5$ pb

$\sigma_Z \sim 0.97$ pb

$\sigma \sim 4.2$ pb

LHC @ 14 TeV

Associated production of top-pair+Higgs boson at the LHC

- Direct access top Yukawa coupling at the LHC

- Small cross section (and large background):

$$\sim 0.6 \text{ pb} \quad \text{for} \quad \sqrt{s} = 14 \text{ TeV}$$

- Cross section and some distributions known up to NLO, scale uncertainty $\sim^{+6\%}_{-9\%}$

Beenakker, Dittmaier, Kraemer, Pluember, Spira, Zerwas ('01-'02)
Dawson, Reina, Wackeroth, Orr, Jackson ('01,'03)

Associated production of top-pair+Higgs boson at the LHC

In a 2 → 3 process ("multileg process"), analytic NLO calculations become cumbersome: This was one of the first processes to be used to test modern automated one loop calculational tools

Friedrix et al. and Hirshi et al. ('11)
Garzelli, at al. Bevilacqua et al. ('11)

EW corrections to the parton level cross sections are known

Frixione, Hirshi, Pagani, Shao, Zaro ('14)
Zhang, Ma, Zhang, Chen, Guo ('14)

NLO QCD corrections were interfaced with SHERPA and with POWHEG BOX

Gleisberg, Hoeche, Krauss, Schonherr,Schaumann('09)
Hartanto, Jaeger, Reina, Wackeroth ('15)

Soft-gluon emission, heavy particles and SCET

- Top-quark pair production (total cross section and some differential distributions) was studied in SCET up to NNLL / approximate NNLO

Beneke, Falgari, Schwinn ('10-'12)
Ahrens, AF, Neubert, Pecjak, Yang ('09-'11)

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Ahrens, AF, Neubert, Pecjak, Yang (‘09–’11)

Top-quark decays added to SCET based approximate formulas in a fully differential parton level MC

Broggio, Papanastsiou, Signer (’14)

Complete NNLO calculations for the top quark pair production total cross section and top quark FB asymmetry at the Tevatron are available. The NNLO calculations of other differential distributions are in progress.

Czakon, Mitov, Fiedler (‘13–’14)

Soft-gluon emission, heavy particles and SCET

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- The formalism can be easily generalized to the case of the productions of a top-quark pair + color neutral particles

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- Top-quark pair production (total cross section and some differential distributions) was studied in SCET up to NNLL / approximate NNLO

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- The first application of this kind was the calculation of tT+W boson

Li, Li and Li ('14)

Goal

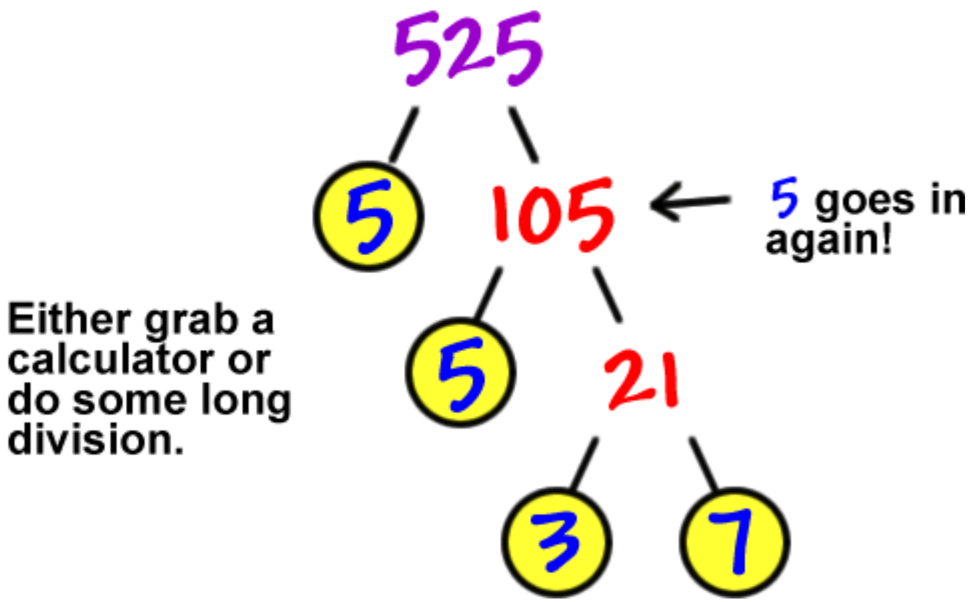
Analyze the factorization of

$$p + p \longrightarrow t + \bar{t} + H + X$$

in the soft-gluon emission limit

- Obtain approximate NNLO formulas / implement NNLL resummation
- Evaluate the total cross section and differential distributions depending on the 4-momenta of the final state particles

Factorization



Kinematics

The partonic processes that survive in the soft emission limit are the same ones present at tree level:

$$\text{quark ann.} \quad q(p_1) + \bar{q}(p_2) \to t(p_3) + \bar{t}(p_4) + H(p_5)$$

$$\text{gluon fusion} \quad g(p_1) + g(p_2) \to t(p_3) + \bar{t}(p_4) + H(p_5)$$

Hard scales: top and Higgs mass and Mandelstam invariants

$$\hat{s} = (p_1 + p_2)^2 \,, \qquad s_{t\bar{t}} = (p_t + p_{\bar{t}})^2$$

$$\tilde{s}_{ij} = 2 p_i \cdot p_j \,, \qquad (i = 1, 2; j = 3, 4)$$

Factorization in PIM kinematics

We define the partonic threshold in PIM kinematics:

$$M^2 \equiv (p_t + p_{\bar{t}} + p_H)^2 \qquad z = \frac{M^2}{\hat{s}}$$

In the soft limit $z \to 1$ the CS factors as

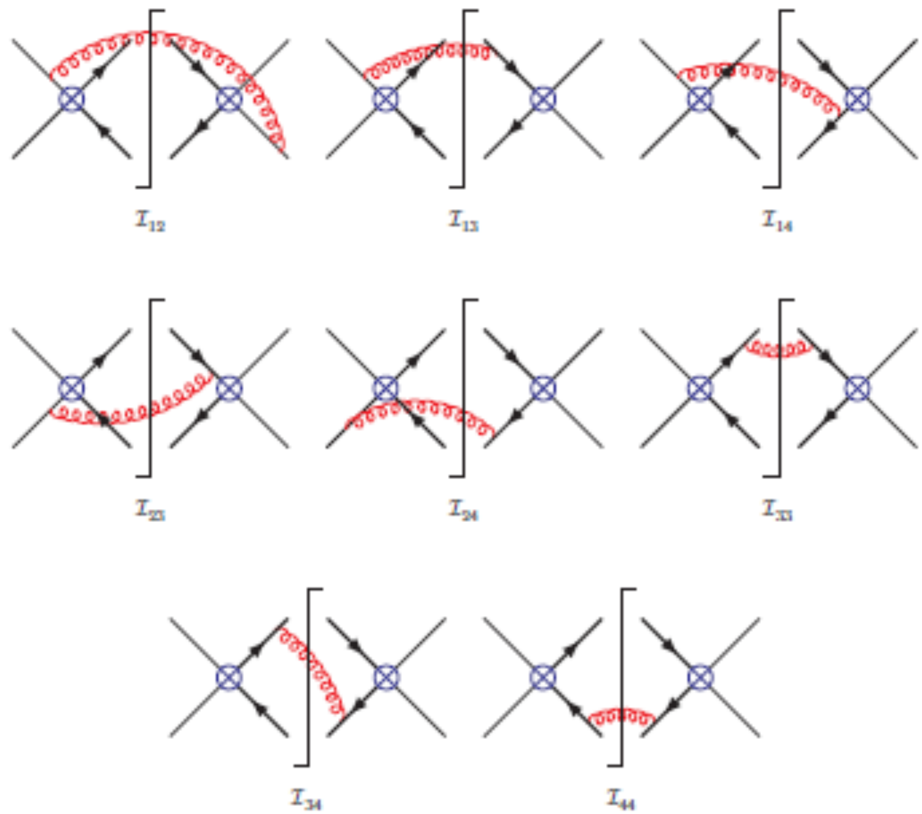
$$d\sigma \propto \underbrace{ff}_{\text{parton lum.}} \otimes \underbrace{\mathbf{H}}_{\text{hard function}} \otimes \underbrace{\mathbf{S}}_{\text{soft function}}$$

NLO soft function

The soft function receives contributions from the same soft emission diagrams as in the top quark pair case and it has precisely the same color structure. It is identical to the soft function for the tTW production case

$$W^{(1)}_{\text{bare}}(\epsilon, x_0, \mu) = \sum_{ij} w_{ij} \mathcal{I}_{ij}(\epsilon, x_0, \mu)$$

The renormalized soft function can then be obtained by subtracting the poles from the bare function



NLO soft function

Momentum-space soft function

$$\mathbf{S}\left(\sqrt{\hat{s}}(1-z),\cdots,\mu\right)=\sqrt{\hat{s}}\int\frac{dx_0}{4\pi}e^{i\sqrt{\hat{s}}(1-z)x_0/2}\mathbf{W}\left(x_0,\vec{x}=0,\mu\right)$$

Laplace-space soft function

$$\begin{aligned}\tilde{\mathbf{s}}\left(L,\cdots,\mu\right)&=\frac{1}{\sqrt{\hat{s}}}\int_0^\infty d\omega\exp\left(-\frac{\omega}{e^{\gamma_E}\mu e^{L/2}}\right)\mathbf{S}\left(\omega,\cdots,\mu\right)\\&=\mathbf{W}\left(x_0=\frac{-2i}{e^{\gamma_E}\mu e^{L/2}},\mu\right)\end{aligned}$$

NLO soft function

$$\tilde{\mathbf{s}} = \tilde{\mathbf{s}}^{(0)} + \left(\frac{\alpha_s}{4\pi}\right)\tilde{\mathbf{s}}^{(1)} + \left(\frac{\alpha_s}{4\pi}\right)^2\tilde{\mathbf{s}}^{(2)} + \cdots$$

Fixed order expansions are power series in the Laplace parameter we used

$$\tilde{\mathbf{s}}^{(1)} = c_{12}L^2 + c_{11}L + c_{10}$$
$$\tilde{\mathbf{s}}^{(2)} = c_{24}L^4 + c_{23}L^3 + c_{22}L^2 + c_{21}L + c_{20}$$

NLO soft function

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Approximate formulas can be obtained by translating Ls in $P_n(z) \equiv \left[\frac{\ln^n(1-z)}{1-z}\right]_+$

$$1 \to \delta(1-z)$$
$$L \to 2P_0(z) + \delta(1-z)L_M$$
$$L^2 \to 8P_1(z) + 4P_0(z)L_M + \delta(1-z)\left(L_M^2 - \frac{2}{3}\pi^2\right) - 4\frac{\ln z}{1-z}$$
$$\cdots$$

NLO hard function

$$\mathbf{H} = \frac{\alpha_s^2}{d_R^2}\left(\mathbf{H}^{(0)} + \frac{\alpha_s}{4\pi}\mathbf{H}^{(1)} + \cdots\right)$$

$$d_R = N_c \quad \text{for} \quad q\bar{q}\,, \qquad d_R = N_c^2 - 1 \quad \text{for} \quad gg$$

As for the soft functions, we deal with matrices in color space:

$$H_{IJ}^{(1)} = \frac{1}{4}\left[\langle c_I|\mathcal{M}^{(0)}\rangle\langle\mathcal{M}^{(1)}|c_J\rangle + \langle c_I|\mathcal{M}^{(1)}\rangle\langle\mathcal{M}^{(0)}|c_J\rangle\right]$$

As usual, we need to implement a subtraction of the IR poles

NLO hard function

$$\mathbf{H} = \frac{\alpha_s^2}{d_R^2}\left(\mathbf{H}^{(0)} + \frac{\alpha_s}{4\pi}\mathbf{H}^{(1)} + \cdots\right)$$

$d_R =$ ... gg

As for th...

$H_{IJ}^{(1)} =$ ... $|c_J\rangle\Big]$

As usua...

We need to evaluate one-loop corrections to 2 → 3 processes:
It is convenient to take advantage the automated tools available on the market. However, to date all require some level of customization in order to provide the information needed for the calculation of the hard function.

Modified version of **GoSam** (see later)

NNLL resummation / approximate NNLO

Once NLO soft and hard functions are available, one can proceed as usual

- Use Renormalization Group Equations in order to determine the coefficients of <u>all</u> of the plus distributions appearing in the partonic cross section at NNLO

→ **Approximate NNLO**

and / or

- Evaluate hard functions and soft functions at their characteristic scales and use RGE to resum large logs in the scale ratio

→ **NNLL Resummation**

Implementation & numerical calculations

Modified version of GoSam

- GoSam is a One Loop Provider which works for generic QCD (and EW) processes. It was tested on a number of multileg processes

Cullen, Greiner, Heinrich, Luisoni, Mastrolia, Ossola, Reiter, Tramontano ('11-'14)

- Out of the box GoSam provides squared amplitudes summed over colors
- In order to build the hard function we need to combine color decomposed (complex) amplitudes

→ New function implemented in GoSam

Snippets from the modified code

```
+--------------------------------------------------------------------+
|   __   __   ___   __   __ __                  GoSam                 |
|  / _) /  \ / __) (  ) (  \/  )        An Automated One-Loop        |
| ( (/\(  () )\__ \ /__\  )    (        Matrix Element Generator     |
|  \__/ \__/ (___/(_)(_)(_/\/\_)        Version 2.0.1 Rev: 735       |
|                                                                     |
|                           (c) The GoSam Collaboration 2011-2014    |
```

(code modified with the help of N. Greiner and G. Ossola)

Fix the phase space point:

```
Phase space point
vec_1   250.00000000000000        0.0000000000000000        0.0000000000000000        250.00000000000000
vec_2   250.00000000000000        0.0000000000000000        0.0000000000000000       -250.00000000000000
vec_3   187.73850389979739      -58.983229676638324       -41.627869607566652        -11.808257084878599
vec_4   184.32670746268576       39.113315116582932        50.299577830700244        -4.6932008990400051
vec_5   127.93478863751679       19.869914560055363        -8.6717082231335887        16.501457983918591
```

Output LO hard function and (UV renormalized) NLO hard function in the quark annihilation channel, after rotation to the desired color basis:

```
Print Tree Level Hard Function:
(-1.1117307432712692OE-021,  0.0000000000000000      ) (-3.8116482626443515E-021,  0.0000000000000000      )
(-3.3881317890172013GE-021,  0.0000000000000000      ) ( 9.1403180646541744IE-005,  0.0000000000000000      )

Print One Loop Hard Function:
( 1.4772254600114996OE-018, 3.3881317890172013GE-021) (-2.3089383098535146GE-004, 1.4722285402959297OE-004)
(-2.3089383098535146GE-004,-1.4722285402959297OE-004) (-2.4868451361348187OE-004,  0.0000000000000000      )
```

Snippets from the modified code

```
+----------------------------------------------------------------------+
|                                                     GoSam            |
```

(code modified with the ... einer and G.

Fix the ph...

```
Phase space p
vec_1    250.0                                   0000000000000
vec_2    250.0                                   0000000000000
vec_3    187.7                                   3257084878599
vec_4    184.3                                   2008990400051
vec_5    127.9                                   1457983918591
```

Output L... n the quark an...

```
Print Tree Le
(-1.111730743                                    0000000          )
(-3.388131789                                    0000000          )

Print One Loop hard function:
( 1.4772254600114996OE-018, 3.38813178901720136E-021) (-2.30893830985351466E-004, 1.47222854029592970E-004)
(-2.30893830985351466E-004,-1.47222854029592970E-004) (-2.48684513613481870E-004,  0.00000000000000000      )
```

The IR poles of the HF are can be subtracted by using the Becher-Neubert formula for the IR poles in QCD amplitudes

$$\mathbf{H}^{(1)} = \mathbf{H}^{(1)\mathrm{IR}} - \left(\mathbf{Z}^{(1)}\mathbf{H}^{(0)} + \mathbf{H}^{(0)}\mathbf{Z}^{(1)\dagger}\right)$$

The calculation of the hard function was also implemented by modifying MadLoop. The GoSam and MadLoop implementations are in agreement.

GoSam takes about 100ms to calculate the HF in a phase space point

Implementation of the differential cross section

Approximate formulas require shorter running times than resummation, therefore we start by implementing those

The integration formula for the cross section can be written as

$$\int d\sigma = \frac{1}{2s} \int_{\tau_{\min}}^{1} \frac{d\tau}{\tau} \int_{\tau}^{1} \frac{dz}{z^{\alpha}} ff\left(\frac{\tau}{z}\right) \int d\Phi_{t\bar{t}H} \mathrm{Tr}\left[\mathbf{H}(M, \cdots, \mu)\mathbf{S}\left(\sqrt{\hat{s}}(1-z), \cdots, \mu\right)\right]$$

$$\tau_{\min} = \frac{(2m_t + m_H)^2}{s} \qquad ff(y) = \int_y^1 \frac{dx}{x} f_{i/N_1}(x) f_{j/N_2}\left(\frac{y}{x}\right)$$

where we kept all integrations over the final state momenta explicit, since ultimately we will want to be able to obtain arbitrary distributions depending on the momenta of the top-quarks and Higgs boson

$$d\Phi_{t\bar{t}H} = \frac{d^3\vec{p}_t}{(2\pi)^2 2E_t} \frac{d^3\vec{p}_{\bar{t}}}{(2\pi)^2 2E_{\bar{t}}} \frac{d^3\vec{p}_H}{(2\pi)^2 2E_H} (2\pi)^4 \delta^{(4)}\left(q - p_t - p_{\bar{t}} - p_H\right)$$

Implementation of the differential cross section

In the rest frame of a particle pair one can write

$$\int \frac{d\vec{p}_1}{(2\pi)^3 2E_1} \frac{d\vec{p}_2}{(2\pi)^3 2E_2} (2\pi)^4 \delta^{(4)} \left(P - p_1 - p_2\right) = \int \frac{d\Omega}{16\pi^2} \frac{1}{2\hat{s}} K\left(\hat{s}, m_1, m_2\right)$$

$$\hat{s} \equiv P^2 \qquad K = \text{Källen function}$$

The final state phase space can then be factored as follows

$$\int d\Phi_{t\bar{t}H} = \int \frac{ds_{t\bar{t}}}{2\pi} \frac{1}{2\hat{s}} \frac{d\Omega}{16\pi^2} K\left(\hat{s}, s_{t\bar{t}}, m_H^2\right) \frac{d\Omega^*}{16\pi^2} K\left(s_{t\bar{t}}, m_t^2, m_t^2\right)$$

Implementation of the differential cross section

The exponent α changes according to the way we treat prefactors in deriving the formula for the integration of the cross section

$$\text{if } M^2 \simeq \hat{s} \qquad \longrightarrow \alpha = 1$$
$$\text{if } M^2/\hat{s} = z \longrightarrow \alpha = 1/2$$

The two implementation are equivalent up to subleading terms of $\mathcal{O}(1-z)$

However, the choice of α has some numerical impact on the results

The option α = 1 was chosen in previous work on top-pair production (and other processes)

The choice α = ½ provides results which are numerically closer to the ones obtained by "direct QCD" methods

α = ½ and vs "direct QCD"

The choice α =½ is equivalent to modify the soft function in the SCET resummation formula:

$$S(z, M^2, \mu_s) = \tilde{s}\left(\frac{M^2}{\mu_s^2} + \partial_\eta, \mu_s\right) \frac{\sqrt{z}}{1-z} \left(\frac{1-z}{\sqrt{z}}\right)^{2\eta} \frac{e^{-2\gamma_E \eta}}{\Gamma(2\eta)}$$

One can also chose to reconstruct a SCET like soft function from the "direct QCD" resummation formula

Becher, Neubert, Xu ('07)
Bonvini, Forte, Ridolfi, Rottoli ('14)

$$S(z, M^2, \mu_s) = \tilde{s}\left(\frac{M^2}{\mu_s^2} + \partial_\eta, \mu_s\right) (-\ln z)^{-1+2\eta} \frac{e^{-2\gamma_E \eta}}{\Gamma(2\eta)}$$

With α =½ , the two implementation differ by quadratic, rather than linear terms:

$$(-\ln z)^{-1+2\eta} = \frac{\sqrt{z}}{1-z} \left(\frac{1-z}{\sqrt{z}}\right)^{2\eta} \left[1 + \mathcal{O}\left((1-z)^2\right)\right]$$

Integration

The integration of the cross section can be implemented in a "parton level Monte Carlo" . For our purposes that means

- Randomly generate

$$\tau \in [\tau_{\min}, 1] \,, \qquad z \in [\tau, 1] \,, \qquad x \in \left[\frac{\tau}{z}, 1\right]$$

and the momenta of the top-quark, antitop and Higgs boson

- Have a subroutine that, given the full kinematics for a phase space point, evaluates the integrand (PDFs, hard function and soft function) at the desired order in the strong coupling constant
- Integrate over the integrand with a MC integrator, such as Vegas implemented in the Cuba library

T. Hahn ('04-'14)

- By retaining and binning appropriately the information on the final-state momenta, one can obtain predictions for differential distributions

Benchmark values and preliminary tests

Total cross section and
scale variation

LHC 7 TeV (Spira et al.) MSTW2008PDF	σ [fb]
LO	87.7+34.9-23.0
NLO	89.8+3.0-8.4

$$(m_H = 125 \text{ GeV}\,, \quad \mu_f = (2m_t + m_H)/2 = 235 \text{ GeV})$$

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Our implementation numerical results (preliminary)

Approx. NLO PIM (1/z)	85.1+0.0-5.1
Approx. NLO PIM (1/z^1/2)	87.9+0.0-6.7

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PRELIMINARY

Approx. NNLO PIM (1/z^1/2) *Not matched*	82.2 [fb]

Work in Progress and Conclusions

Urgent issues:

- Control and minimize running times for the evaluation of the approximate NNLO formulas (we need roughly 3 million points for a reasonably small numerical error on the total cross section)
- Matching to complete NLO results using aMC@NLO which allows for the NLO calculation of the distributions we are interested in

To do

- Switch to GoSam as One Loop Provider in aMC@NLO (same OLP for the calculation of the hard function and NLO for matching)
- Implement NNLL resummation and study the dependence on hard and soft scales
- Implement distributions in the code
- Phenomenological study