Factorization and RG-improved predictions on the $t\bar{t}+0j$ process at hadron colliders

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University of Bern

SCET2015 Santa Fe NM

- In collaboration with C.S. Li & H.T. Li (in progress)
- Related work done by Li, Li, DYS, Yang & Zhu (PRL(2013),PRD(2013))

Motivation

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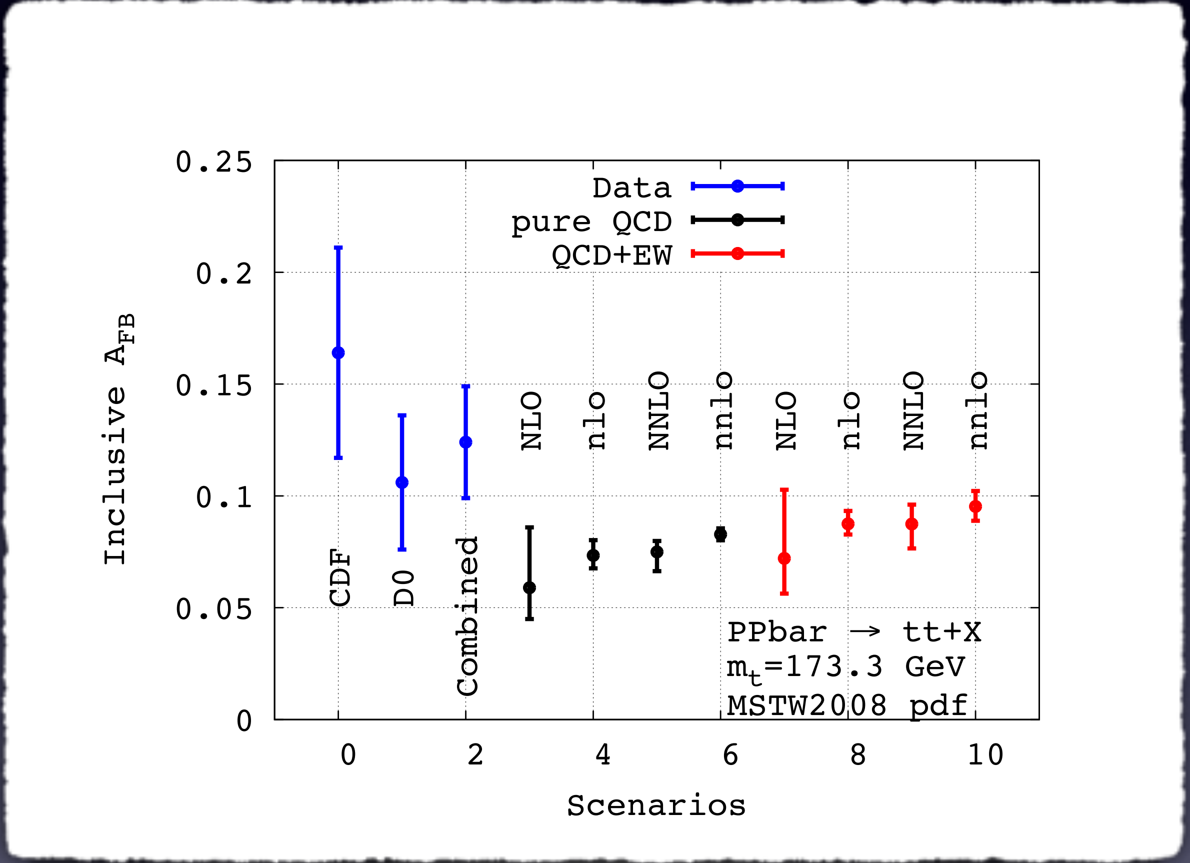
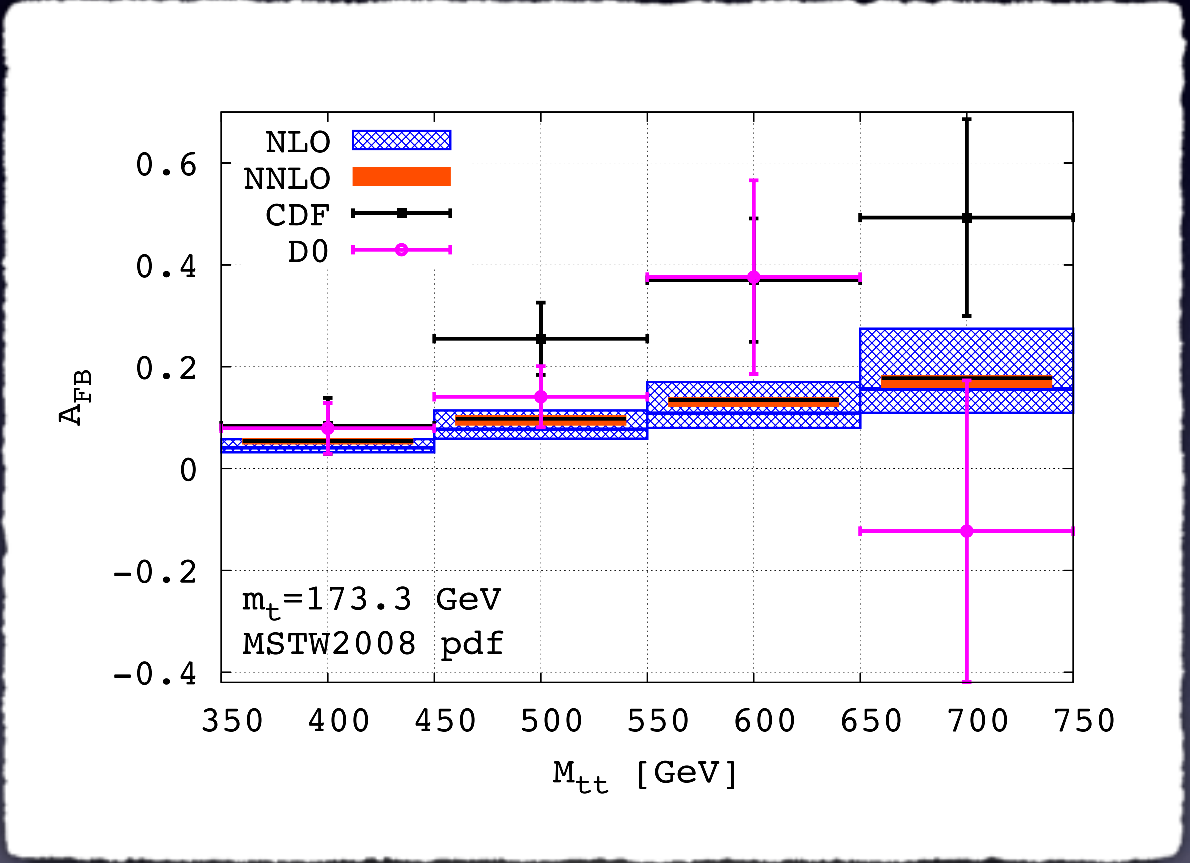
- Differential cross section in QCD NNLO (M. Czakon, P. Fiedler & A. Mitov 1411.3007)





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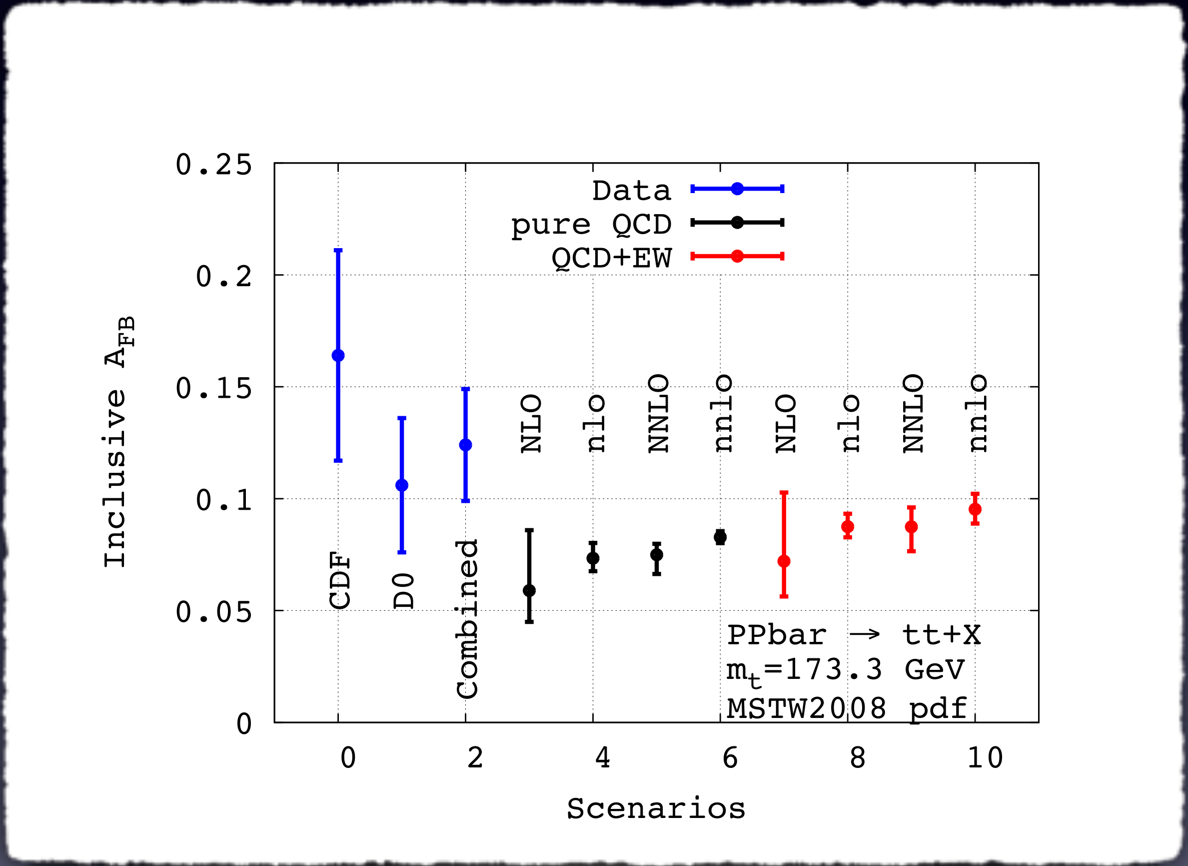

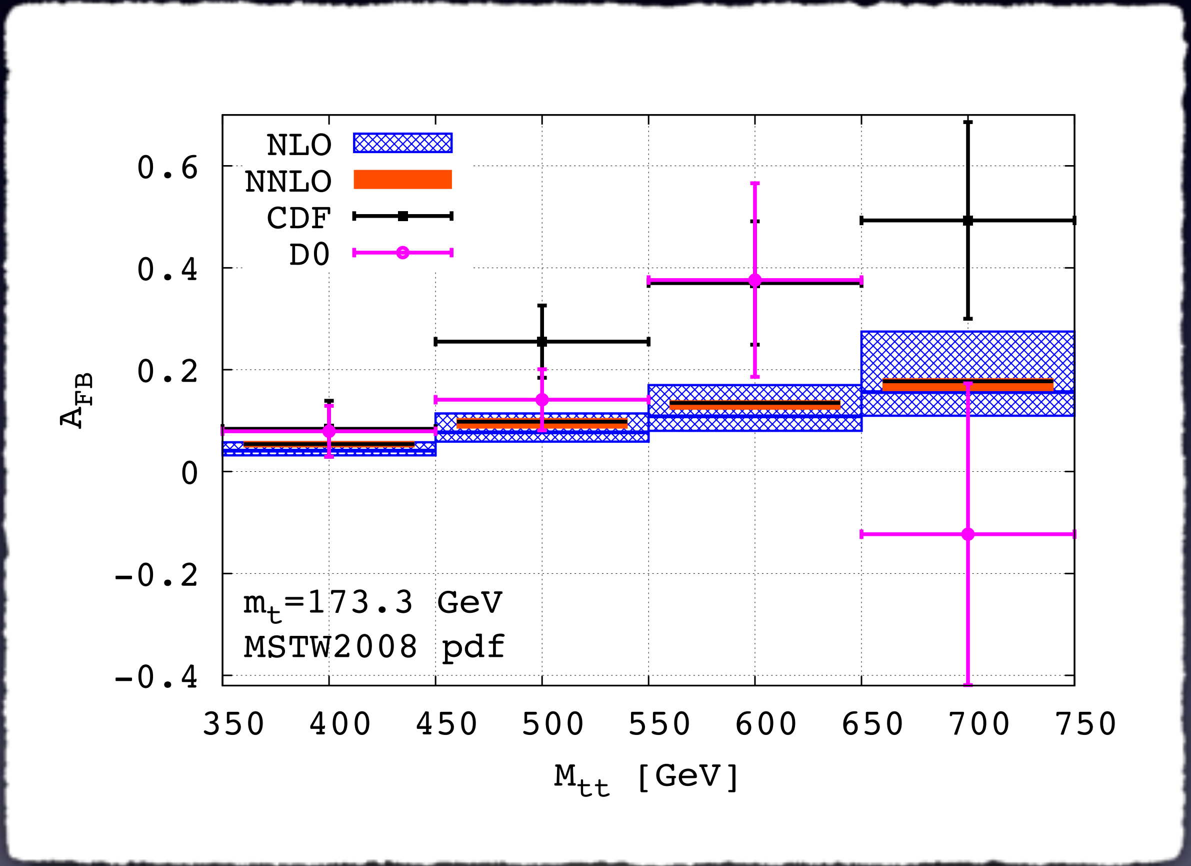

- Jet multiplicities in ttbar production: Fixed Order+Parton Shower

Eg. ttbar+0j & ttbar+1j @ QCD NLO: A_{FB} (Hoche, Huang, Lusioni, Winter and Schonherr 2013)

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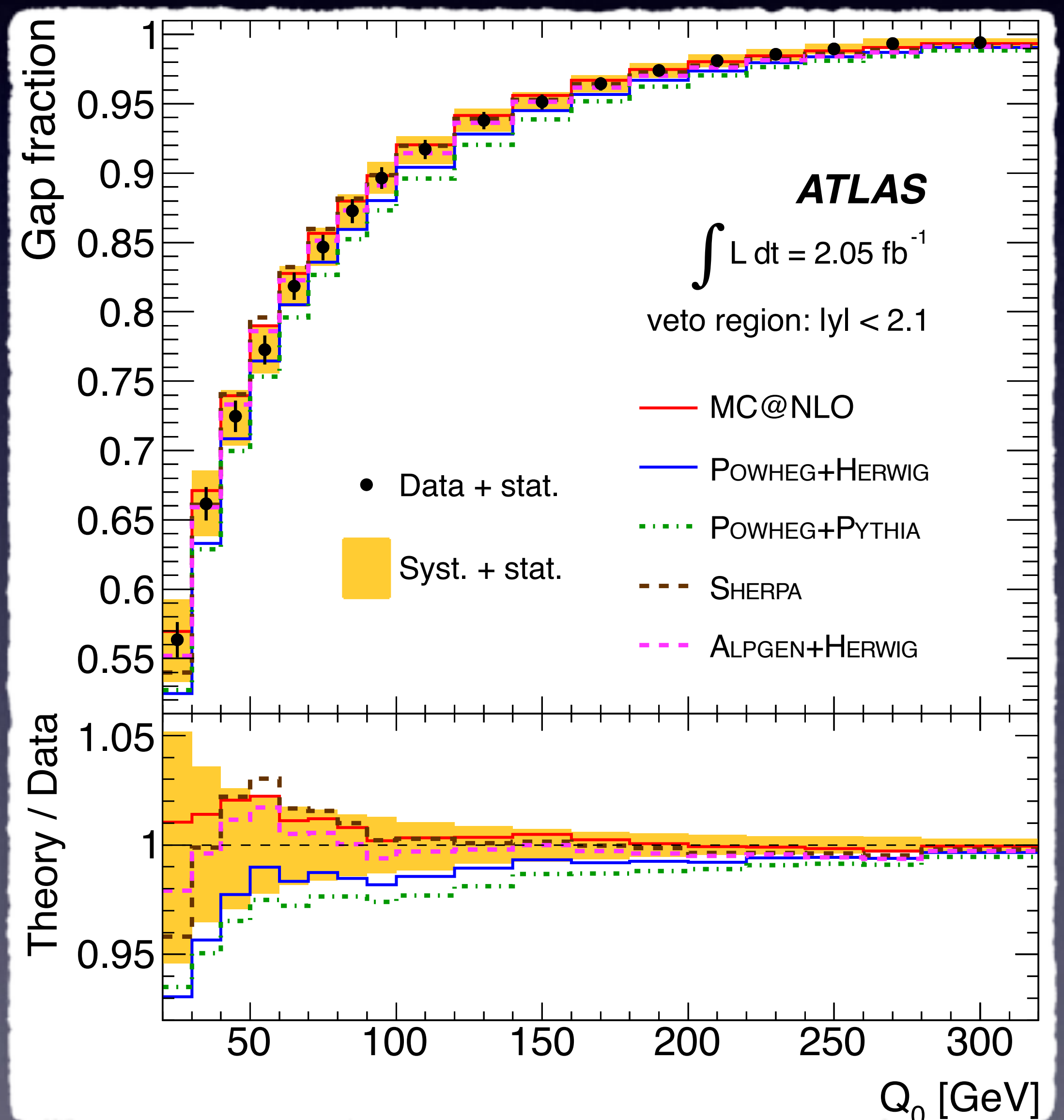
$\sigma(t\bar{t} + 0j)/\sigma(t\bar{t}X)$



EPJC72(2012)2043

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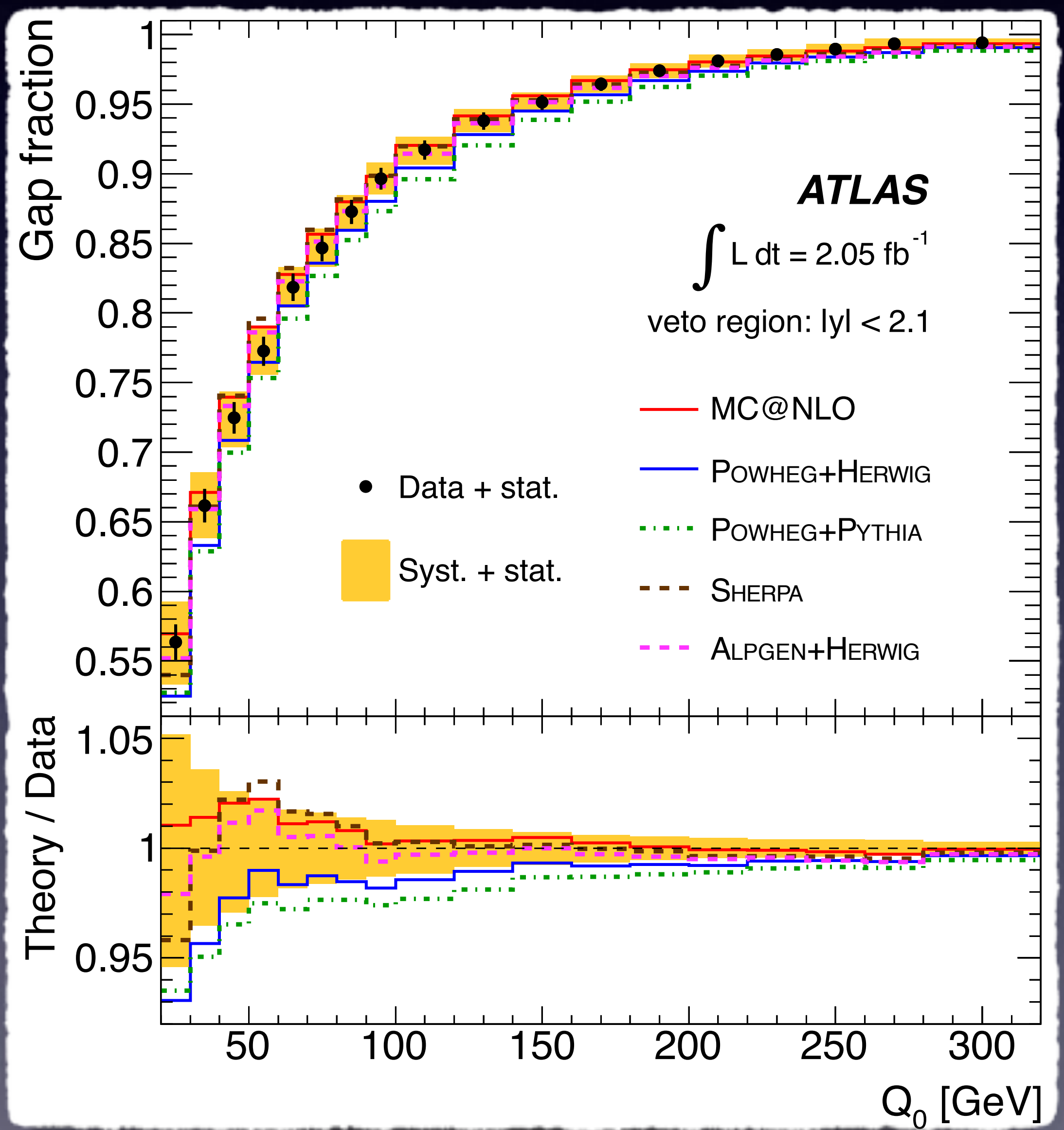

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1. Test pQCD at top quark scale

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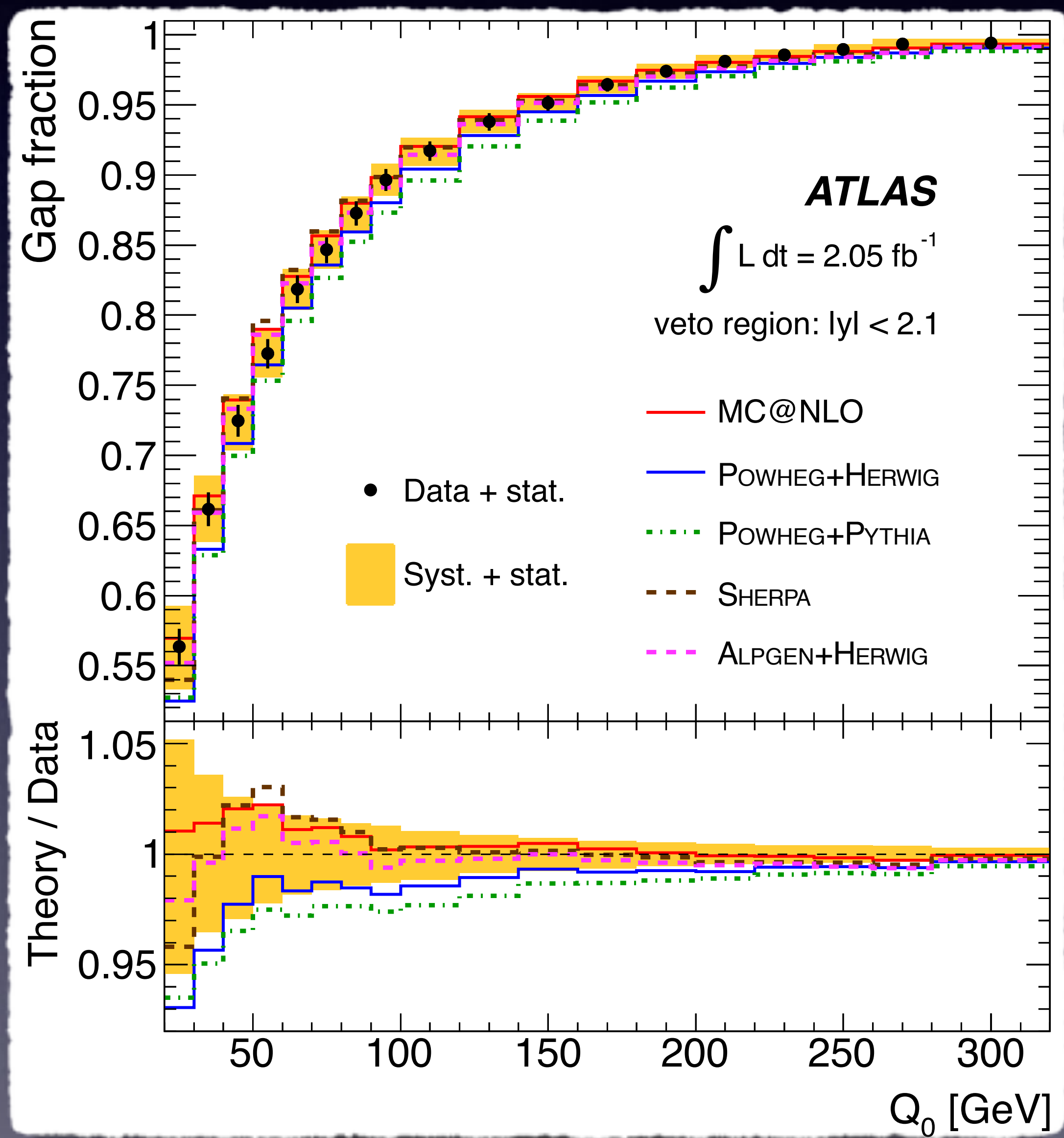

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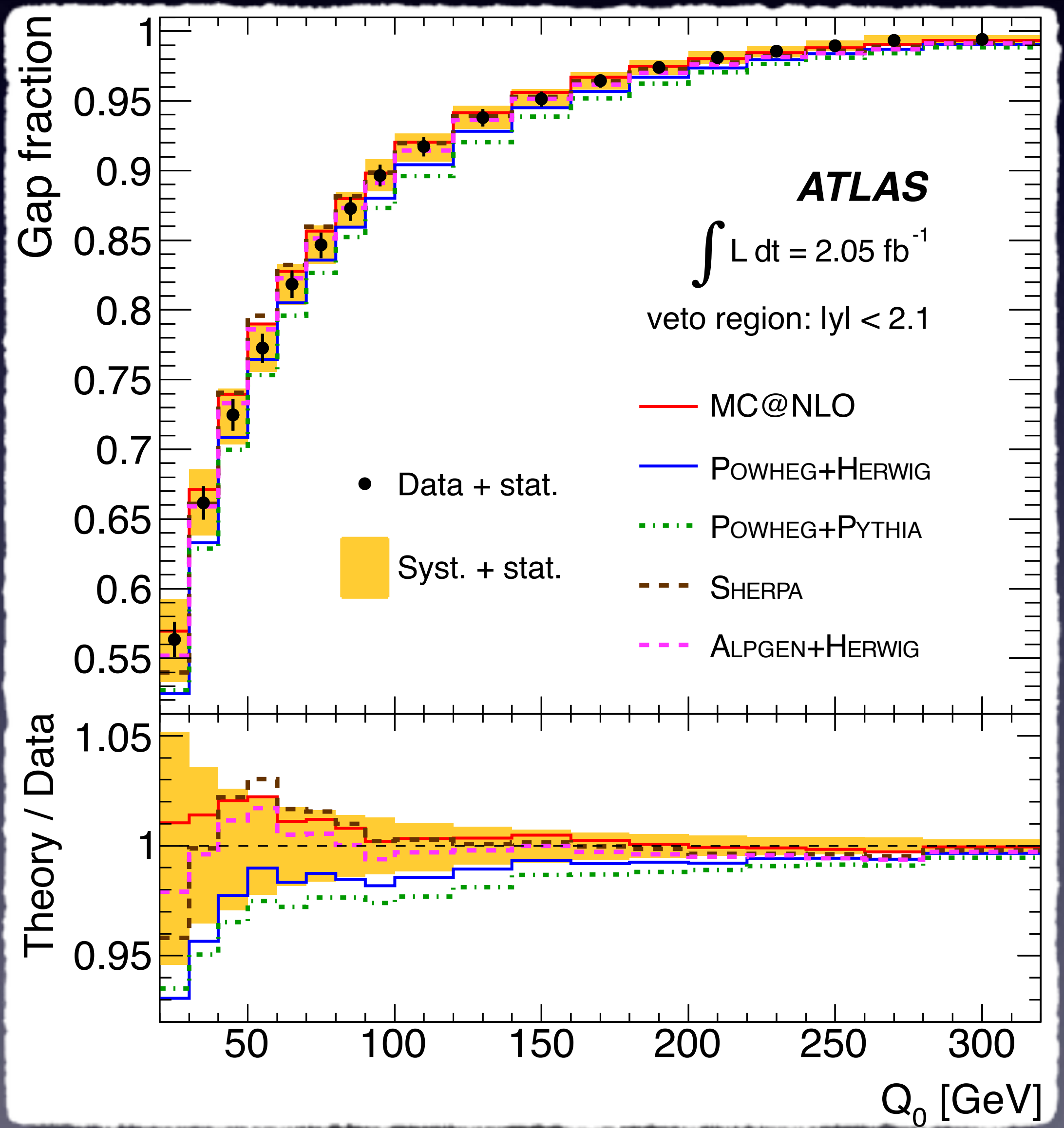

EPJC72(2012)2043

1. Test pQCD at top quark scale
2. Constrain model uncertainties in Monte-Carlo generator
3. Enhanced new physics signals

Sun, PRD80(2012)094020

Ask et.al. JHEP1201(2012)018

……

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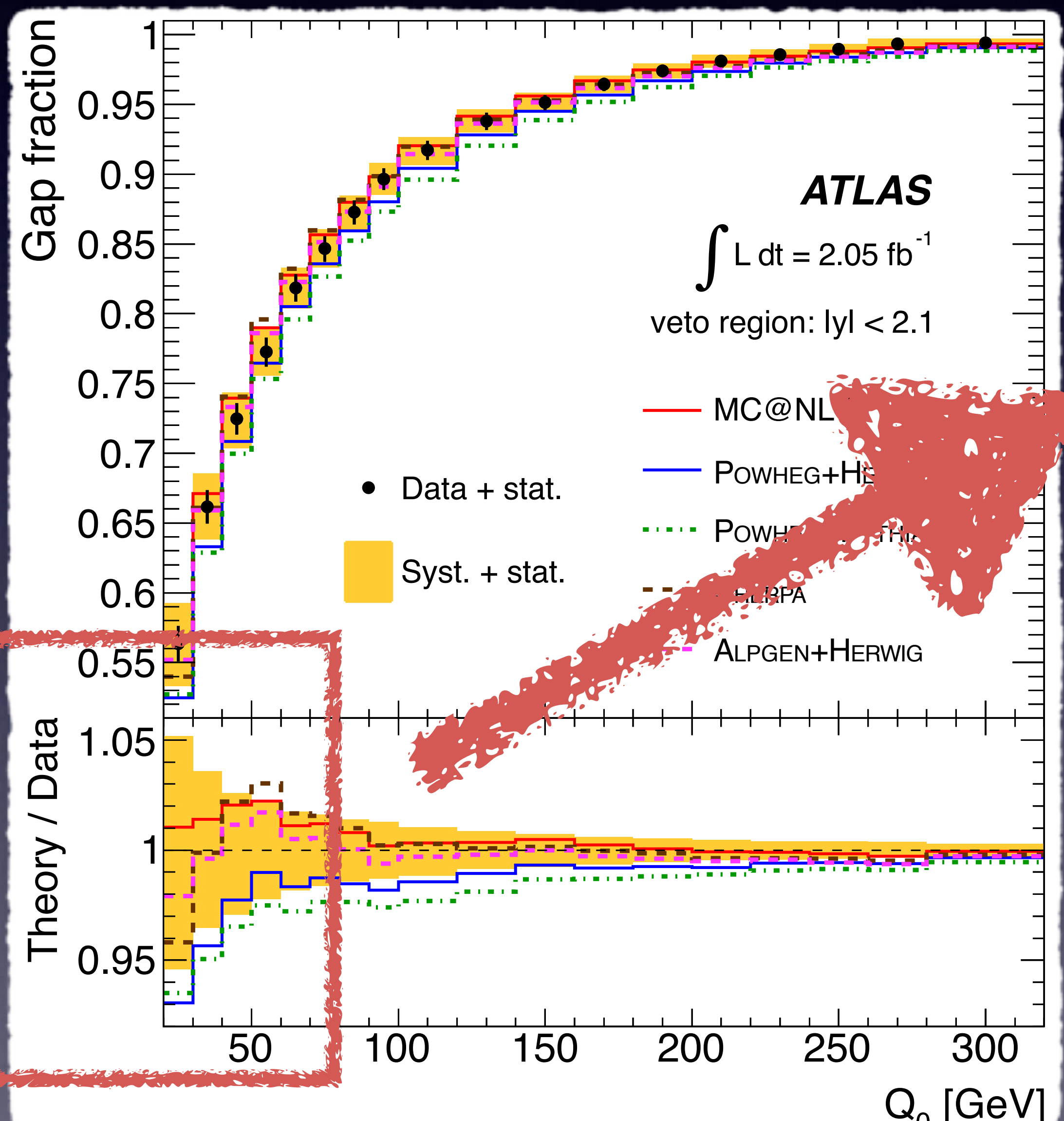
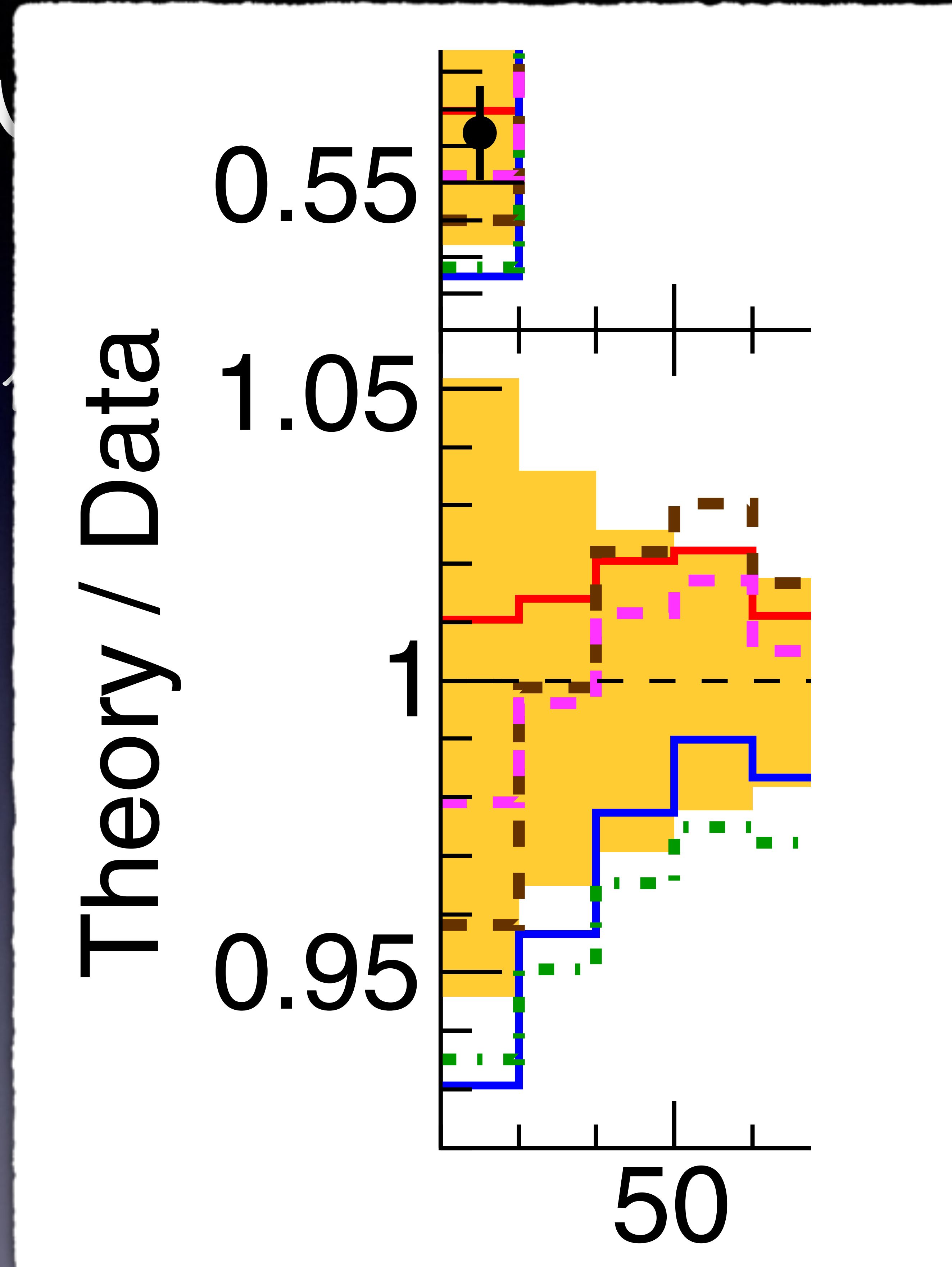
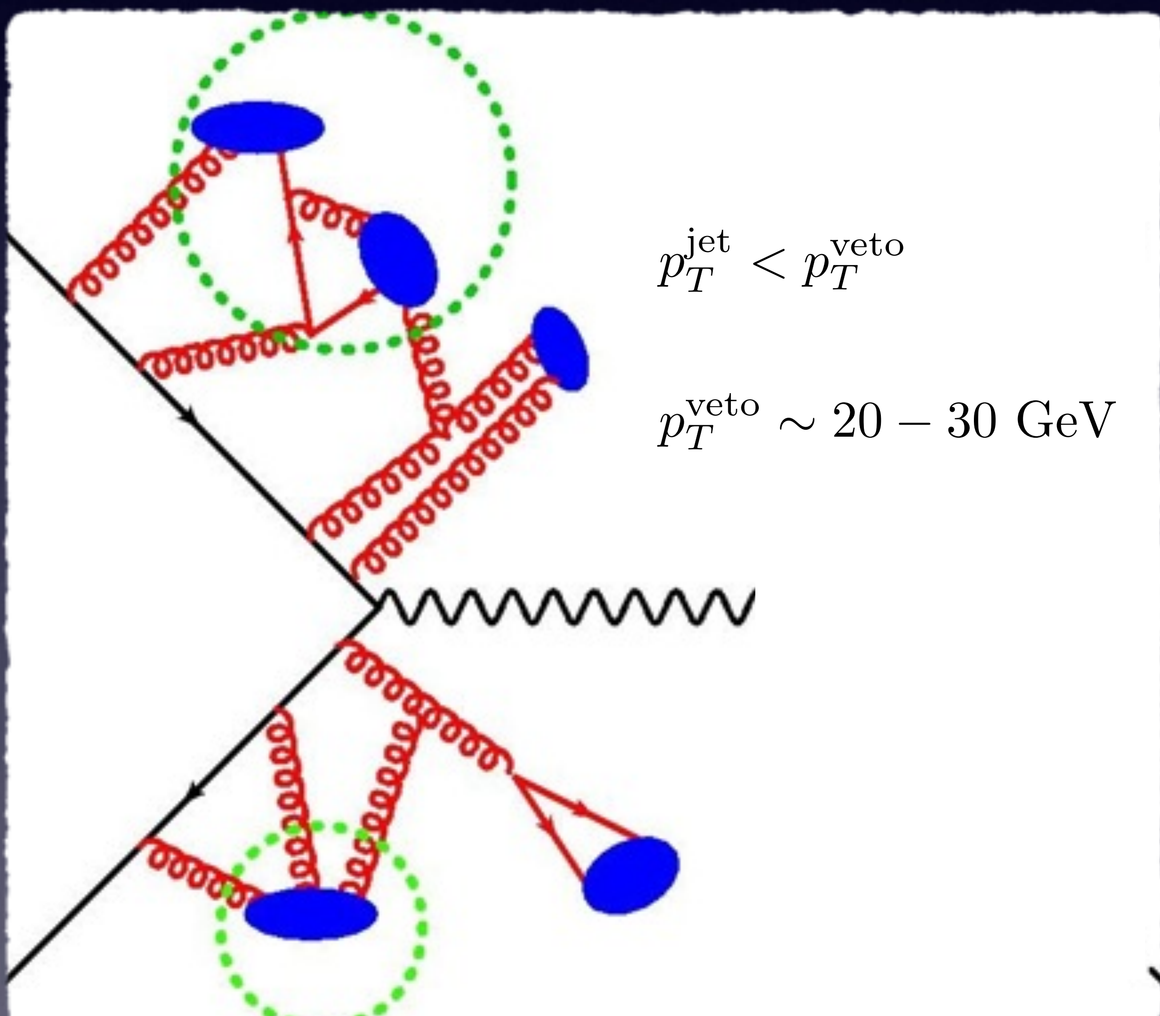




EPJC72(2012)2043

Difference between Drell-Yan like process



h(V)+0j

Salam, et.al.,2012;……

Becher, et.al. 2013;……

Stewart, et.al.,2013;……

hV+0j

DYS, et.al. 2014;

Li, et.al.2014;……

VV+0j

Jaiswal, et.al. 2014;

Becher, et.al.2014;……

……

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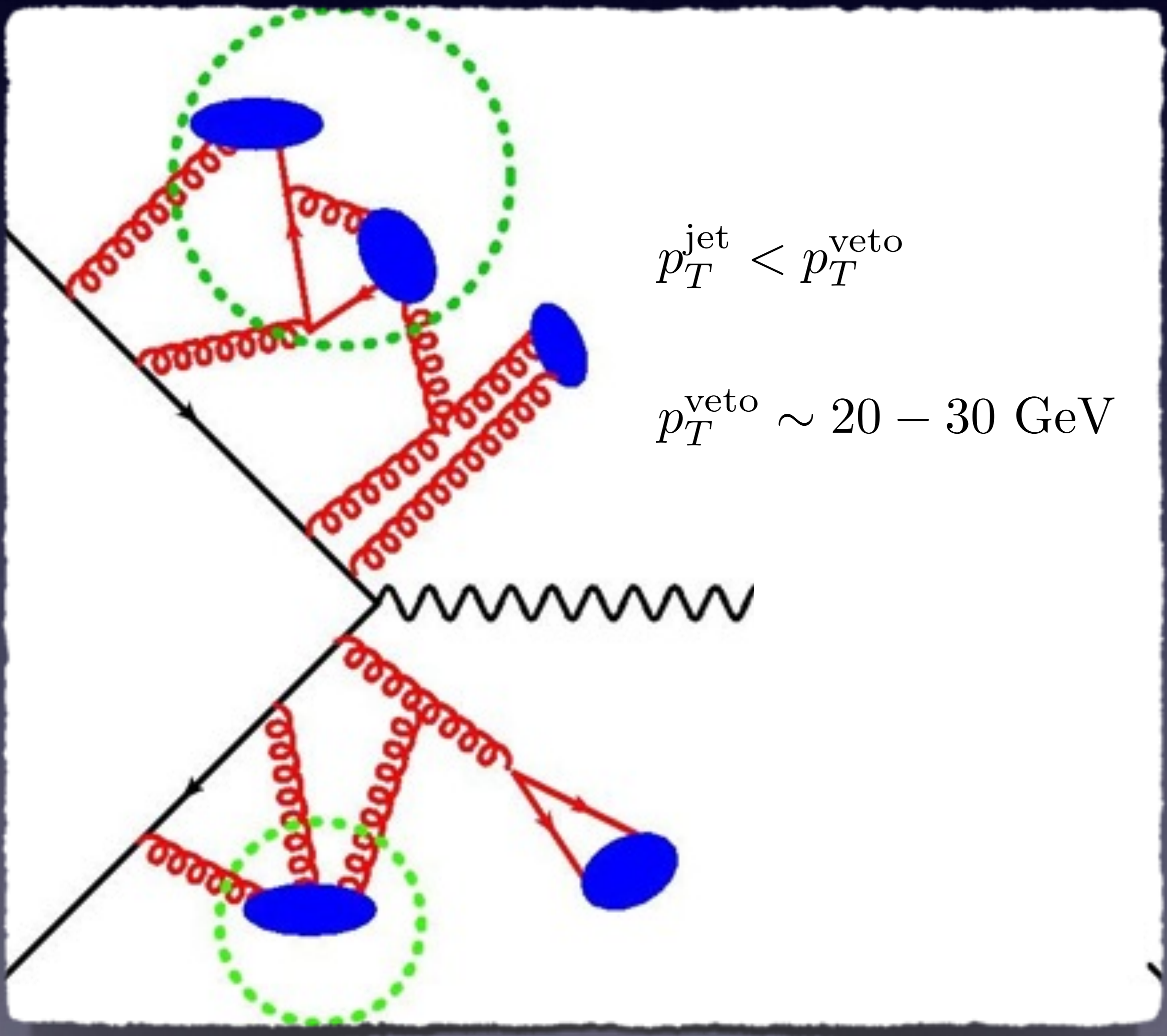

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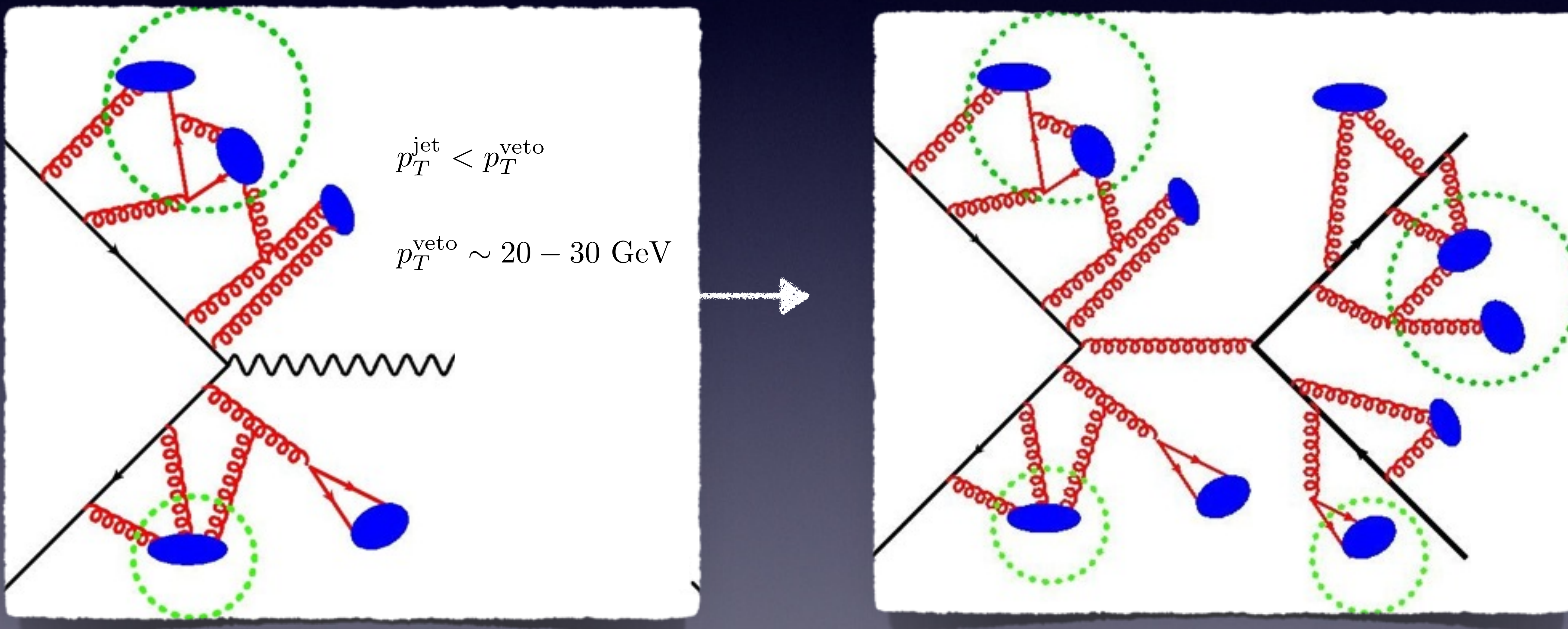

Wilson lines in HQET

C.Bauer, D.Pirjol & I. Stewart PRD65(2002)054022

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The leading HQET Lagrangian and heavy-to-heavy current

$$\mathcal{L}_{\rm HQET} = \sum_v \bar{h}_v iv \cdot D h_v \qquad \mathcal{J}_{v_t, v_{\bar{t}}} = \bar{h}_{v_t} \Gamma h_{v_{\bar{t}}}$$

where $h_v(x) = \frac{1 + \not{v}}{2} e^{-im_t v \cdot x} t(x) \qquad iv \cdot D = iv \cdot \partial^\mu + gv \cdot A_s$

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The Wilson line along v directions: (Korchemsky & Radyushkin, PLB279(1992)359)

$$S_v(x) = \mathbf{P} \exp \left[ig \int ds v \cdot A(vs) \right]$$

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After field redefinition,

$$h_v(x) = S_v(x) h_v^{(0)}(x)$$

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The new field $h_v^{(0)}$ annihilate and create top quark, but do not interact with soft gluon. All soft gluon interactions are absorbed into the Wilson lines.

Factorization formalism

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Consider the process $t\bar{t} + 0j$

$$N_1(P_1) + N_2(P_2) \to t(p_3) + \bar{t}(p_4) + X'(P_X)$$

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In the Born approximation

$$q(p_1) + \bar{q}(p_2) \to t(p_3) + \bar{t}(p_4) \qquad g(p_1) + g(p_2) \to t(p_3) + \bar{t}(p_4)$$

define the kinematic invariants

$$s = (P_1 + P_2)^2, \quad \hat{s} = (p_1 + p_2)^2, \quad M^2 = (p_3 + p_4)^2,$$
$$t_1 = (p_1 - p_3)^2 - m_t^2, \quad u_1 = (p_1 - p_4)^2 - m_t^2.$$

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the kinematic region we are interested in

$$\hat{s}, M^2, |t_1|, |u_1|, m_t^2 \gg (p_T^{\text{veto}})^2 \gg \Lambda_{\text{QCD}}^2$$

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Expanding parameter: $\lambda \sim p_T^{\mathrm{veto}}/M$

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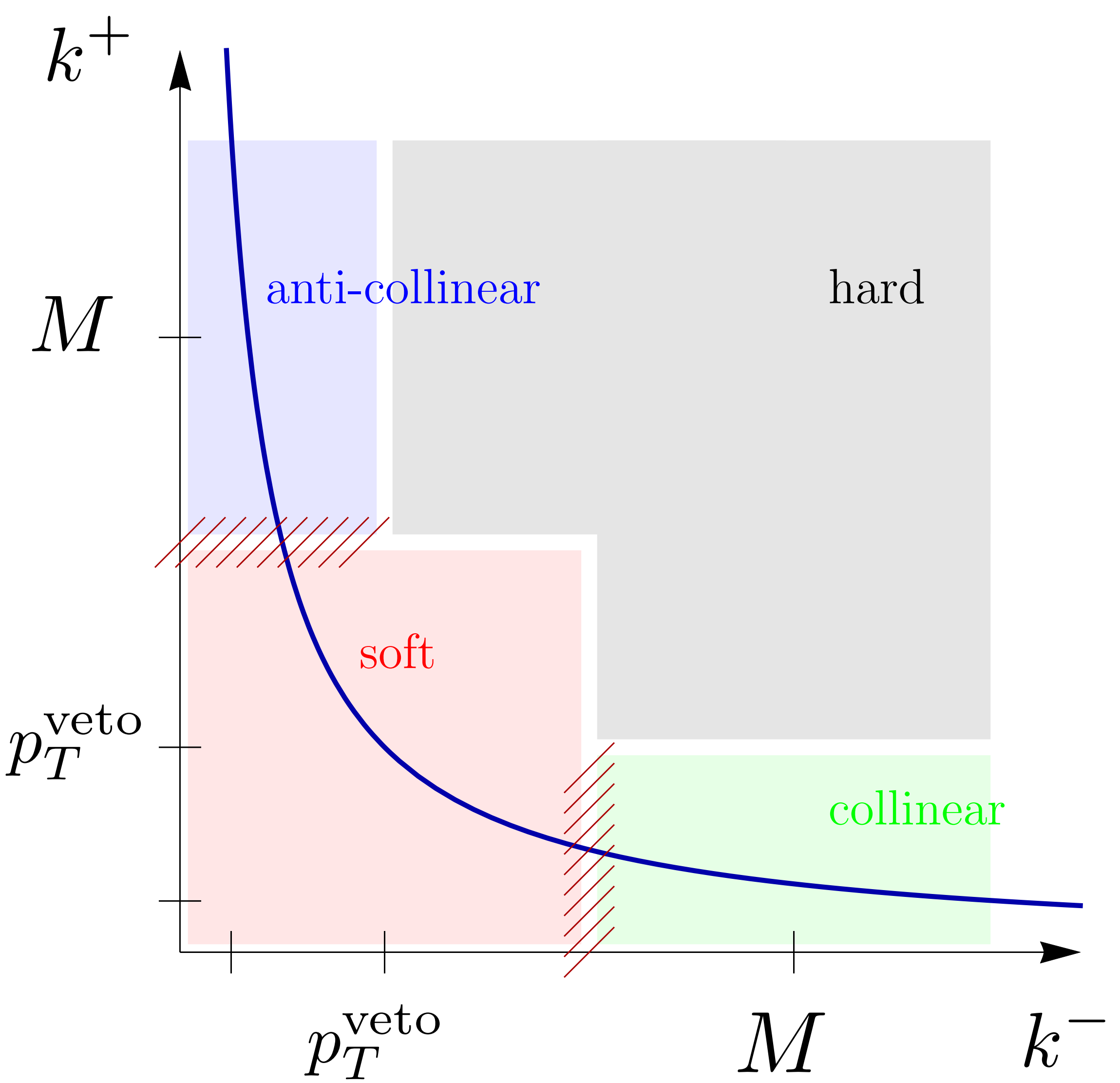
Mom. Scale	$(k^+, k^-, k_\perp)$	Virtuality
Hard	$(1,1,1)\,M$	M
Collinear	$(1,\lambda^2,\lambda)\,M$	$\lambda M \sim p_T^{\text{veto}}$
Anti-Col.	$(\lambda^2,1,\lambda)\,M$	$\lambda M \sim p_T^{\text{veto}}$
Soft	$(\lambda,\lambda,\lambda)\,M$	$\lambda M \sim p_T^{\text{veto}}$

$$k^2 = k^+k^- + k_\perp^2$$



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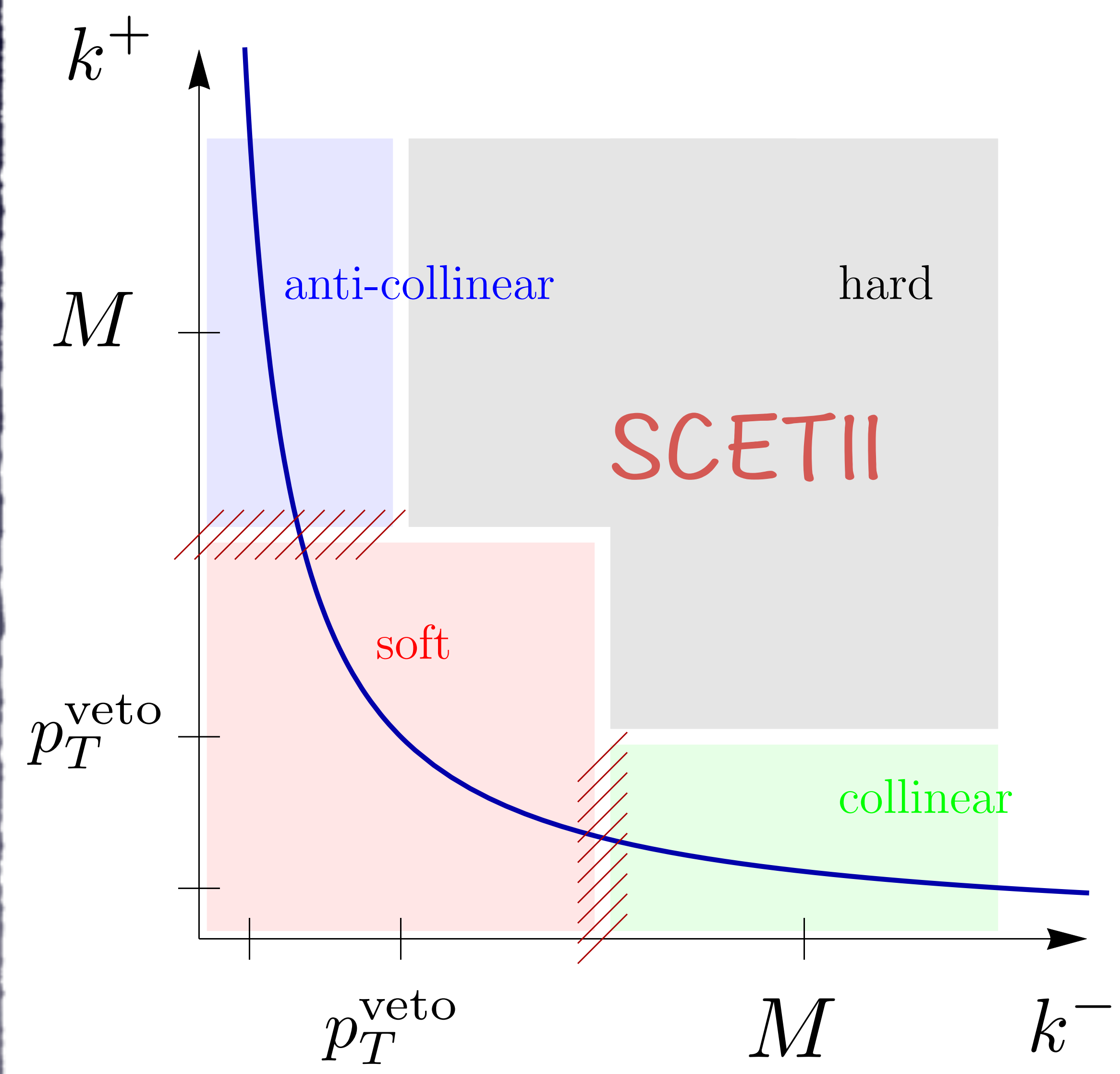

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The effective Hamiltonian for this process:

$$\mathcal{H}_{\text{eff}}(x) = \sum_{Im} \int dt_1 dt_2 e^{im_t(v_t+v_{\bar{t}})\cdot x} \left[\widetilde{C}^{q\bar{q}}_{Im}(t_1,t_2) O^{q\bar{q}}_{Im}(x,t_1,t_2) + \widetilde{C}^{gg}_{Im}(t_1,t_2) O^{gg}_{Im}(x,t_1,t_2) \right],$$

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Eg. for the gluon-gluon fusion channel

$$O^{gg}_{Im}(x,t_1,t_2) = \sum_{\{a\},\{b\}} (c^{gg}_I)_{\{a\}} \left[O^h_m(x) \right]^{\mu\nu}_{b_3,b_4} \left[O^c(x,t_1,t_2) \right]^{b_1,b_2}_{\mu\nu} \left[O^s(x) \right]^{\{a\},\{b\}}$$

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$$\left[O^h_m(x)\right]^{\mu\nu}_{b_3,b_4} = \bar{h}^{b_3}_{v_t}(x)\Gamma^{\mu\nu}_m h^{b_4}_{v_{\bar{t}}}(x) \qquad \left[O^c(x,t_1,t_2)\right]^{b_1b_2}_{\mu\nu} = \mathcal{A}^{b_1}_{n\mu\perp}(x+t_1\bar{n})\mathcal{A}^{b_2}_{\bar{n}\nu\perp}(x+t_2 n)$$

Factorization formalism

The effective Hamiltonian for this process:

$$\mathcal{H}_{\rm eff}(x) = \sum_{Im} \int dt_1 dt_2 e^{im_t(v_t+v_{\bar{t}})\cdot x} \left[\widetilde{C}^{q\bar{q}}_{Im}(t_1,t_2) O^{q\bar{q}}_{Im}(x,t_1,t_2) + \widetilde{C}^{gg}_{Im}(t_1,t_2) O^{gg}_{Im}(x,t_1,t_2) \right],$$

Eg. for the gluon-gluon fusion channel

$$O^{gg}_{Im}(x,t_1,t_2) = \sum_{\{a\},\{b\}} (c^{gg}_I)_{\{a\}} \left[O^h_m(x)\right]^{\mu\nu}_{b_3,b_4} \left[O^c(x,t_1,t_2)\right]^{b_1,b_2}_{\mu\nu} \left[O^s(x)\right]^{\{a\},\{b\}}$$

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$$\left[O^s(x)\right]^{\{a\},\{b\}} = \left[S^\dagger_{v_t}(x)\right]^{b_3a_3}\left[S_{v_{\bar{t}}}(x)\right]^{a_4b_4}\left[S^\dagger_{\bar{n}}(x)\right]^{b_2a_2}\left[S_n(x)\right]^{a_1b_1}$$

Factorization formalism

Factorization formalism

The differential cross section:

$$d\sigma = \frac{1}{2s}\frac{d^3\vec{p}_3}{(2\pi)^3 2E_3}\frac{d^3\vec{p}_4}{(2\pi)^3 2E_4}\sum_X{}' \int d^4x \langle \mathcal{M}(x)|\mathcal{M}(0)\rangle$$

Factorization formalism

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$$\sum_X{}' \langle \mathcal{M}(x)|\mathcal{M}(0)\rangle = \sum_{mm'}\int dt_1 dt_2 dt_1' dt_2' e^{-i(p_3+p_4)\cdot x}\langle 0|\left[\mathcal{O}_m'^h(0)\right]^{\rho\sigma}|t\bar{t}\rangle\langle t\bar{t}|\left[\mathcal{O}_m^h(0)\right]^{\mu\nu}|0\rangle$$
$$\times \sum\!\!\!\!\!\!\!\int_{X_n,\mathrm{reg}} \mathcal{M}_{\mathrm{veto}}(p_T^{\mathrm{veto}}, R, \{\underline{p_n}\})\langle N_1|\mathcal{A}_{n\rho\perp}(x^+ + x_\perp + t_1'\bar{n})|X_c\rangle\langle X_c|\mathcal{A}_{n\mu\perp}(t_1\bar{n})|N_1\rangle$$
$$\times \sum\!\!\!\!\!\!\!\int_{X_{\bar{n}},\mathrm{reg}} \mathcal{M}_{\mathrm{veto}}(p_T^{\mathrm{veto}}, R, \{\underline{p_{\bar{n}}}\})\langle N_2|\mathcal{A}_{\bar{n}\sigma\perp}(x^+ + x_\perp + t_2'\bar{n})|X_{\bar{c}}\rangle\langle X_{\bar{c}}|\mathcal{A}_{n\nu\perp}(t_2\bar{n})|N_2\rangle$$
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Factorization formalism

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Hard

$$\times \sum\!\!\!\!\!\!\int_{X_n,\mathrm{reg}} \mathcal{M}_{\mathrm{veto}}(p_T^{\mathrm{veto}}, R, \{\underline{p_n}\})\langle N_1|\mathcal{A}_{n\rho\perp}(x^+ + x_\perp + t_1'\bar{n})|X_c\rangle\langle X_c|\mathcal{A}_{n\mu\perp}(t_1\bar{n})|N_1\rangle$$

$$\times \sum\!\!\!\!\!\!\int_{X_{\bar{n}},\mathrm{reg}} \mathcal{M}_{\mathrm{veto}}(p_T^{\mathrm{veto}}, R, \{\underline{p_{\bar{n}}}\})\langle N_2|\mathcal{A}_{\bar{n}\sigma\perp}(x^+ + x_\perp + t_2'\bar{n})|X_{\bar{c}}\rangle\langle X_{\bar{c}}|\mathcal{A}_{n\nu\perp}(t_2\bar{n})|N_2\rangle$$

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Factorization formalism

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$$\sum_X{}'\langle\mathcal{M}(x)|\mathcal{M}(0)\rangle = \sum_{mm'}\int dt_1 dt_2 dt_1' dt_2' e^{-i(p_3+p_4)\cdot x}\langle 0|\left[\mathcal{O}_m'^h(0)\right]^{\rho\sigma}|t\bar{t}\rangle\langle t\bar{t}|\left[\mathcal{O}_m^h(0)\right]^{\mu\nu}|0\rangle$$

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Hard

Collinear

Factorization formalism

The differential cross section:

$$d\sigma = \frac{1}{2s}\frac{d^3\vec{p}_3}{(2\pi)^3 2E_3}\frac{d^3\vec{p}_4}{(2\pi)^3 2E_4}\sum_X{}'\int d^4x\langle\mathcal{M}(x)|\mathcal{M}(0)\rangle$$

$$\sum_X{}'\langle\mathcal{M}(x)|\mathcal{M}(0)\rangle = \sum_{mm'}\int dt_1 dt_2 dt_1' dt_2' e^{-i(p_3+p_4)\cdot x}\langle 0|\left[\mathcal{O}_m'^h(0)\right]^{\rho\sigma}|t\bar{t}\rangle\langle t\bar{t}|\left[\mathcal{O}_m^h(0)\right]^{\mu\nu}|0\rangle$$

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Hard

Collinear

Anti-Collinear

Factorization formalism

The differential cross section:

$$d\sigma = \frac{1}{2s}\frac{d^3\vec{p}_3}{(2\pi)^3 2E_3}\frac{d^3\vec{p}_4}{(2\pi)^3 2E_4}\sum_X{}' \int d^4x \langle \mathcal{M}(x)|\mathcal{M}(0)\rangle$$

$$\sum_X{}' \langle \mathcal{M}(x)|\mathcal{M}(0)\rangle = \sum_{mm'}\int dt_1 dt_2 dt_1' dt_2' e^{-i(p_3+p_4)\cdot x} \langle 0| \left[\mathcal{O}_m'^h(0)\right]^{\rho\sigma} |t\bar{t}\rangle\langle t\bar{t}| \left[\mathcal{O}_m^h(0)\right]^{\mu\nu} |0\rangle$$

$$\times \sum\!\!\!\!\!\!\!\int_{X_n,\mathrm{reg}} \mathcal{M}_{\mathrm{veto}}(p_T^{\mathrm{veto}}, R, \{\underline{p_n}\})\langle N_1|\mathcal{A}_{n\rho\perp}(x^+ + x_\perp + t_1'\bar{n})|X_c\rangle\langle X_c|\mathcal{A}_{n\mu\perp}(t_1\bar{n})|N_1\rangle$$

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Hard

Collinear

Anti-Collinear

Soft

Factorization formalism

Factorization formalism

The collinear matrix element

$$\mathcal{B}^{\mu\nu,\,n}_{g/N}(z, x_\perp, p_T^{\text{veto}}, \mu) =$$

$$-\frac{z\bar{n}\cdot p}{2\pi}\int\frac{dt}{2\pi}e^{-izt\bar{n}\cdot p}\sum\!\!\!\!\!\!\!\int_{X_n,\text{reg}}\mathcal{M}_{\text{veto}}(p_T^{\text{veto}}, R, \{\underline{p_n}\})\langle N(p)|\bar{\chi}_n(t\bar{n})|X_n\rangle\langle X_n|\chi_n(0)|N(p)\rangle$$

Factorization formalism

The collinear matrix element

$$\mathcal{B}^{\mu\nu,\,n}_{g/N}(z, x_\perp, p_T^{\rm veto}, \mu) =$$

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The hard function

$$\mathcal{H}^{\mu\nu\rho\sigma}_{gg}(M, m_t, v_3, \mu) = \frac{3}{8}\frac{1}{(4\pi)^2}\frac{1}{d_g}\sum_{mm'} |C_m\rangle\langle C'_m| \langle 0| \left[O^{h\dagger}_{m'}(0)\right]^{\rho\sigma} |t(p_3)\bar{t}(p_4)\rangle$$

$$\times \langle t(p_3)\bar{t}(p_4)| \left[O^{h}_{m}(0)\right]^{\mu\nu} |0\rangle,$$

Factorization formalism

The collinear matrix element

$$\mathcal{B}^{\mu\nu,\,n}_{g/N}(z, x_\perp, p_T^{\rm veto}, \mu) = -\frac{z\bar{n}\cdot p}{2\pi}\int\frac{dt}{2\pi}e^{-izt\bar{n}\cdot p}\sum\!\!\!\!\!\!\!\int_{X_n,\rm reg}\mathcal{M}_{\rm veto}(p_T^{\rm veto}, R, \{\underline{p_n}\})\langle N(p)|\bar{\chi}_n(t\bar{n})|X_n\rangle\langle X_n|\chi_n(0)|N(p)\rangle$$

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The soft function

$$\mathcal{W}(x_\perp, p_T^{\rm veto}, \mu) = \frac{1}{d_R}\sum\!\!\!\!\!\!\!\int_{X_s,\rm reg}\mathcal{M}_{\rm veto}(p_T^{\rm veto}, R, \{\underline{p_s}\})\langle 0|\overline{\mathbf{T}}\left[\mathcal{O}^{s\dagger}(x_\perp)\right]|X_s\rangle\langle X_s|\overline{\mathbf{T}}\left[\mathcal{O}^s(0)\right]|0\rangle$$

Factorization formalism

The collinear matrix element

$$\mathcal{B}^{\mu\nu,\,n}_{g/N}(z,x_\perp,p_T^{\rm veto},\mu)=$$
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$$\mathcal{H}^{\mu\nu\rho\sigma}_{gg}(M,m_t,v_3,\mu)=\frac{3}{8}\frac{1}{(4\pi)^2}\frac{1}{d_g}\sum_{mm'}|C_m\rangle\langle C'_m|\langle 0|\left[O^{h\dagger}_{m'}(0)\right]^{\rho\sigma}|t(p_3)\bar{t}(p_4)\rangle$$
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The factorized differential cross section

$$\frac{d^3\sigma}{dy\,dM\,d\cos\theta}=\frac{8\pi\beta_t}{3sM}\mathcal{B}_{i/N_1}(\zeta_1,p_T^{\rm veto},\mu)\mathcal{B}_{\bar{i}/N_2}(\zeta_2,p_T^{\rm veto},\mu)\mathbf{Tr}\left[\mathcal{H}_{i\bar{i}}(M,m_t,\cos\theta,\mu)\mathcal{S}_{i\bar{i}}(p_T^{\rm veto},M,m_t,\cos\theta,\mu)\right].$$

where $\mathcal{S}_{i\bar{i}}(p_T^{\rm veto},M,m_t,\cos\theta,\mu)=\int_0^{2\pi}\frac{d\phi_t}{2\pi}\mathcal{W}_{i\bar{i}}(0,p_T^{\rm veto},\mu)$

Factorization formalism

The collinear matrix element

$$\mathcal{B}^{\mu\nu,\,n}_{g/N}(z, x_\perp, p_T^{\rm veto}, \mu) = - \frac{z\bar{n}\cdot p}{2\pi} \int \frac{dt}{2\pi} e^{-izt\bar{n}\cdot p} \sum\!\!\!\!\!\!\!\int_{X_n,{\rm reg}} \mathcal{M}_{\rm veto}(p_T^{\rm veto}, R, \{\underline{p_n}\}) \langle N(p)|\bar{\chi}_n(t\bar{n})|X_n\rangle\langle X_n|\chi_n(0)|N(p)\rangle$$

The hard function

$$\mathcal{H}^{\mu\nu\rho\sigma}_{gg}(M, m_t, v_3, \mu) = \frac{3}{8}\frac{1}{(4\pi)^2}\frac{1}{d_g}\sum_{mm'} |C_m\rangle\langle C'_m|\langle 0|\left[O^{h\dagger}_{m'}(0)\right]^{\rho\sigma}|t(p_3)\bar{t}(p_4)\rangle \times \langle t(p_3)\bar{t}(p_4)|\left[O^h_m(0)\right]^{\mu\nu}|0\rangle,$$

The soft function

$$\mathcal{W}(x_\perp, p_T^{\rm veto}, \mu) = \frac{1}{d_R} \sum\!\!\!\!\!\!\!\int_{X_s,{\rm reg}} \mathcal{M}_{\rm veto}(p_T^{\rm veto}, R, \{\underline{p_s}\}) \langle 0|\overline{\mathbf{T}}\left[\mathcal{O}^{s\dagger}(x_\perp)\right]|X_s\rangle\langle X_s|\overline{\mathbf{T}}\left[\mathcal{O}^s(0)\right]|0\rangle$$

The factorized differential cross section

Ahrens, Ferroglia, Neubert, Pecjak and Yang, JHEP(2010), JHEP(2011)······

$$\frac{d^3\sigma}{dy\,dM\,d\cos\theta} = \frac{8\pi\beta_t}{3sM}\mathcal{B}_{i/N_1}(\zeta_1, p_T^{\rm veto}, \mu)\mathcal{B}_{\bar{i}/N_2}(\zeta_2, p_T^{\rm veto}, \mu)\,\mathbf{Tr}\left[\mathcal{H}_{i\bar{i}}(M, m_t, \cos\theta, \mu)\mathcal{S}_{i\bar{i}}(p_T^{\rm veto}, M, m_t, \cos\theta, \mu)\right].$$

where $\mathcal{S}_{i\bar{i}}(p_T^{\rm veto}, M, m_t, \cos\theta, \mu) = \int_0^{2\pi} \frac{d\phi_t}{2\pi}\mathcal{W}_{i\bar{i}}(0, p_T^{\rm veto}, \mu)$

Beam function

$$\frac{d^3\sigma}{dydM\,d\cos\theta} = \frac{8\pi\beta_t}{3sM}\mathcal{B}_{i/N_1}(\zeta_1, p_T^{\text{veto}}, \mu)\mathcal{B}_{\bar{i}/N_2}(\zeta_2, p_T^{\text{veto}}, \mu)\mathbf{Tr}\left[\,\mathcal{H}_{i\bar{t}}(M, m_t, \cos\theta, \mu)\mathcal{S}_{i\bar{i}}(p_T^{\text{veto}}, M, m_t, \cos\theta, \mu)\,\right].$$

Beam function

$$\frac{d^3\sigma}{dydMd\cos\theta} = \frac{8\pi\beta_t}{3sM}\mathcal{B}_{i/N_1}(\zeta_1, p_T^{\text{veto}}, \mu)\mathcal{B}_{\bar{i}/N_2}(\zeta_2, p_T^{\text{veto}}, \mu)\mathbf{Tr}\left[\mathcal{H}_{i\bar{t}}(M, m_t, \cos\theta, \mu)\mathcal{S}_{i\bar{i}}(p_T^{\text{veto}}, M, m_t, \cos\theta, \mu)\right].$$

$$\mathcal{B}^n_{q/N}(\zeta, p_T^{\text{veto}}, \mu) = \sum_{i=g,q,\bar{q}}\int_\zeta^1 \frac{dz}{z}\mathcal{I}_{q\leftarrow i}(z, p_T^{\text{veto}}, \mu)f_{i/N}(\zeta/z, \mu)$$

Beam function

$$\frac{d^3\sigma}{dy dM d\cos\theta} = \frac{8\pi\beta_t}{3sM} \mathcal{B}_{i/N_1}(\zeta_1, p_T^{\text{veto}}, \mu) \mathcal{B}_{\bar{i}/N_2}(\zeta_2, p_T^{\text{veto}}, \mu) \textbf{Tr}\left[\mathcal{H}_{i\bar{t}}(M, m_t, \cos\theta, \mu) \mathcal{S}_{i\bar{i}}(p_T^{\text{veto}}, M, m_t, \cos\theta, \mu)\right].$$

$$\mathcal{B}^n_{q/N}(\zeta, p_T^{\text{veto}}, \mu) = \sum_{i=g,q,\bar{q}} \int_\zeta^1 \frac{dz}{z} \mathcal{I}_{q\leftarrow i}(z, p_T^{\text{veto}}, \mu) f_{i/N}(\zeta/z, \mu)$$

Matching Coefficient

Beam function

$$\frac{d^3\sigma}{dydMd\cos\theta} = \frac{8\pi\beta_t}{3sM}\mathcal{B}_{i/N_1}(\zeta_1, p_T^{\text{veto}}, \mu)\mathcal{B}_{\bar{i}/N_2}(\zeta_2, p_T^{\text{veto}}, \mu)\,\mathbf{Tr}\left[\mathcal{H}_{i\bar{i}}(M, m_t, \cos\theta, \mu)\mathcal{S}_{i\bar{i}}(p_T^{\text{veto}}, M, m_t, \cos\theta, \mu)\right].$$

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Matching Coefficient

"The short distance Wilson coefficient of any operators has to be independent of the long distance physics in the process, and in particular it has to be independent of the external states chosen for the matching calculation."

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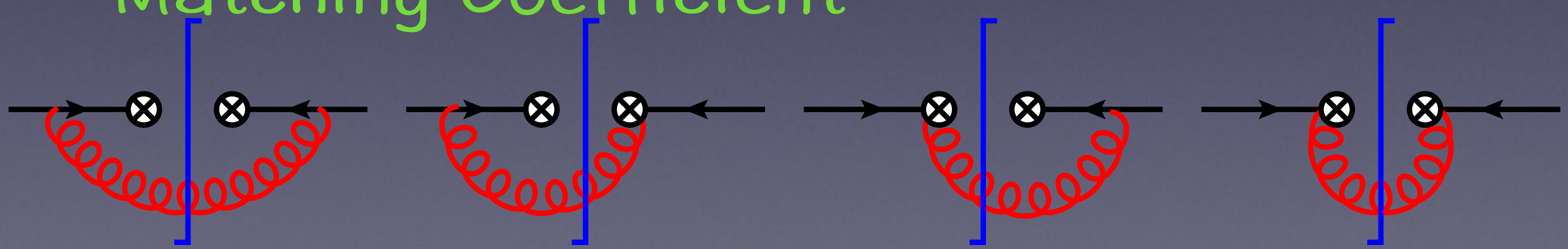

Soft Function

Soft Function

Top Quark Pair Production:

Threshold (M, S_4):

Ahrens, Ferroglia, Neubert, Pecjak and Yang, JHEP(2010), JHEP(2011)⋯⋯

Small transverse momentum:

Li, Li, DYS, Yang and Zhu, PRL(2013), PRD(2013)

Jet Veto:

this talk

Soft Function

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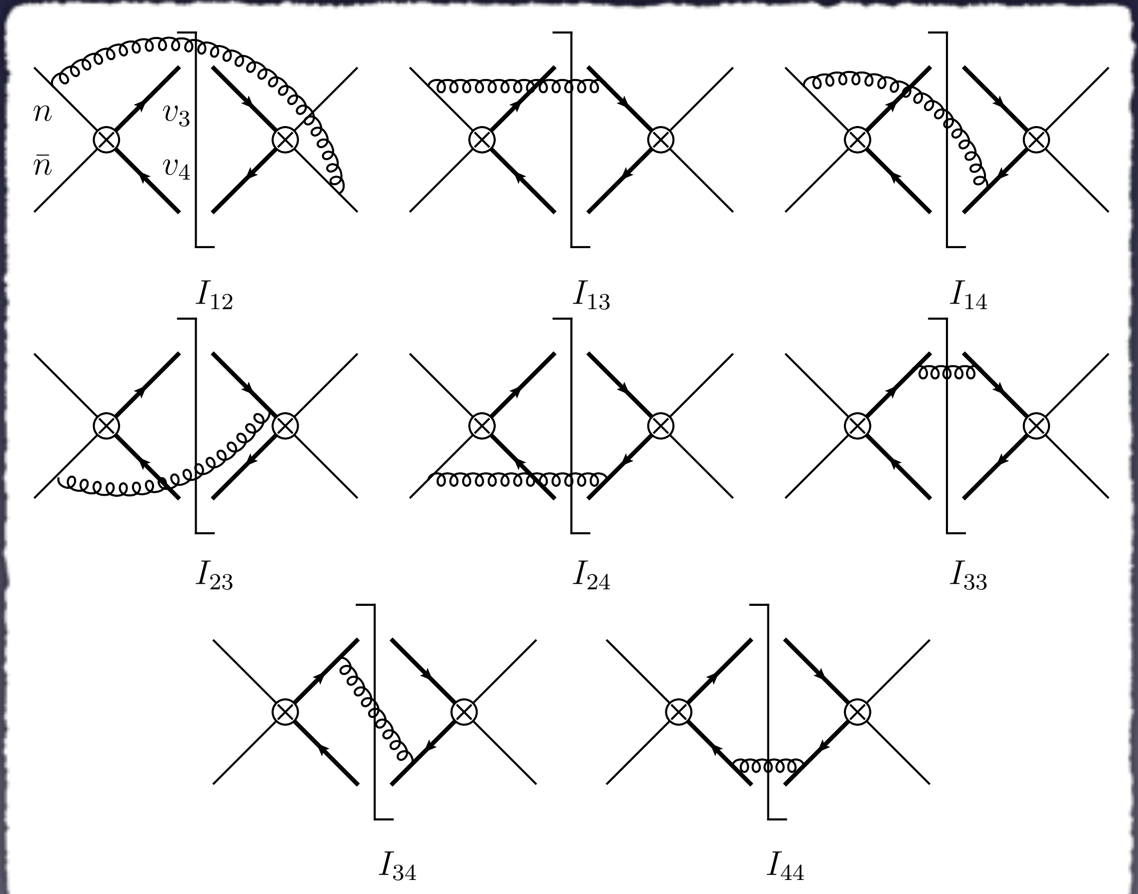
Up to NLO, the soft function

$$\mathcal{S}(p_T^{\text{veto}}) \sim \int d^D k \delta^+(k^2)\theta(p_T^{\text{veto}} - k_T)\frac{n_i \cdot n_j}{n_i \cdot k \; n_j \cdot k}$$



Regularization of light-cone singularity

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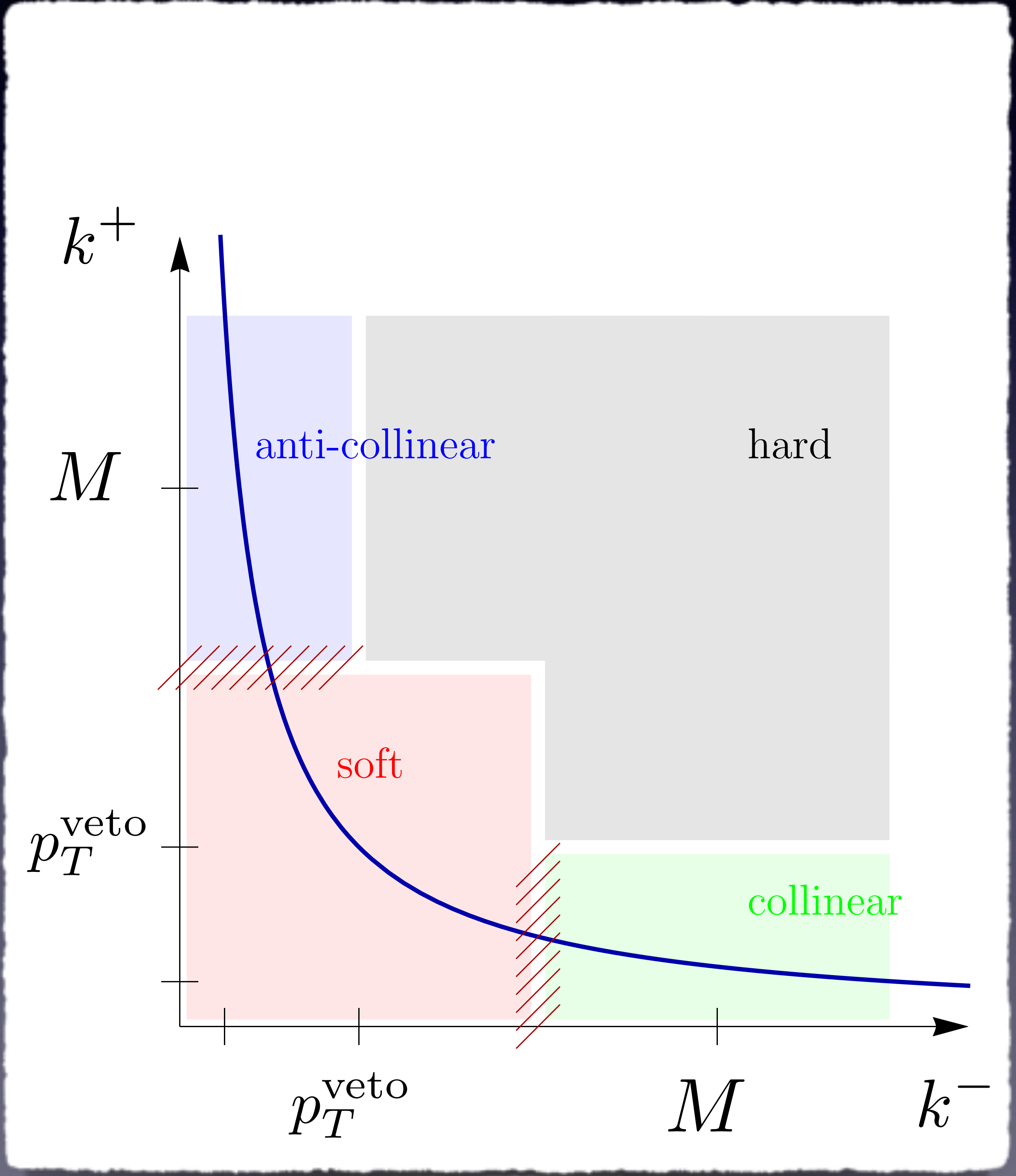

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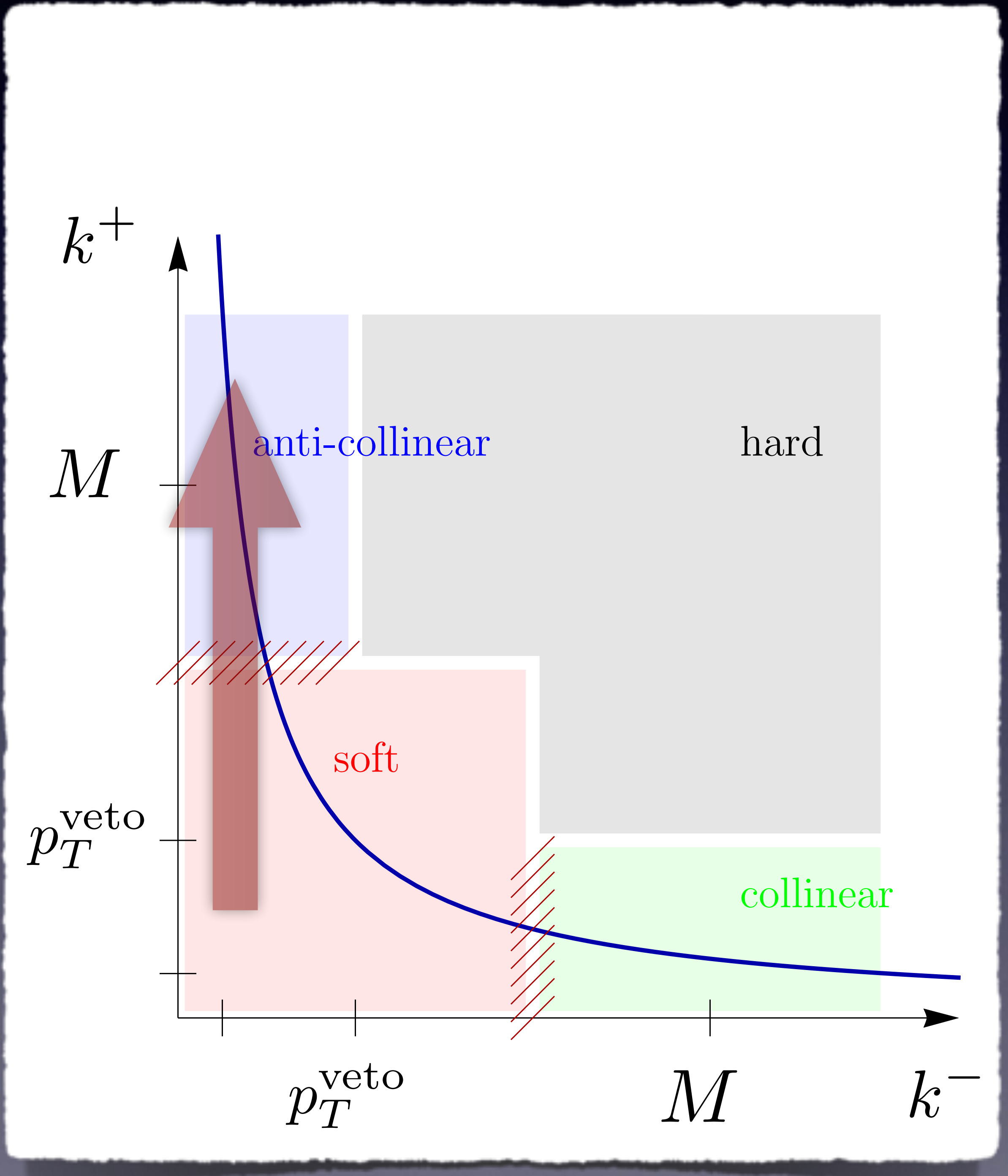

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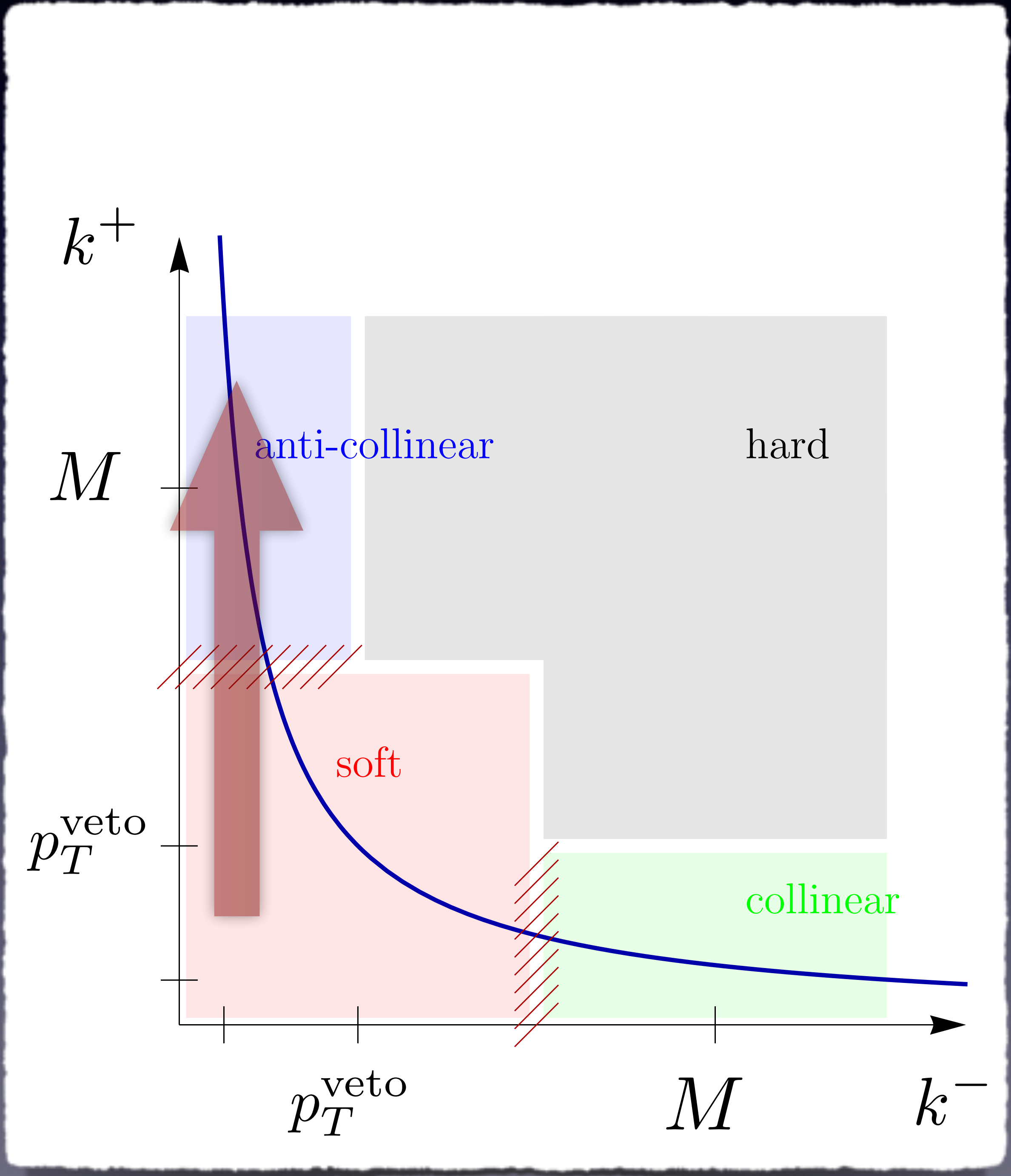
Sector Decomposition

$$I = \int_{p_T^{\text{veto}}}^{\Lambda} \frac{dk^+}{k^+} + \int_{\Lambda}^{M} \frac{dk^+}{k^+}$$



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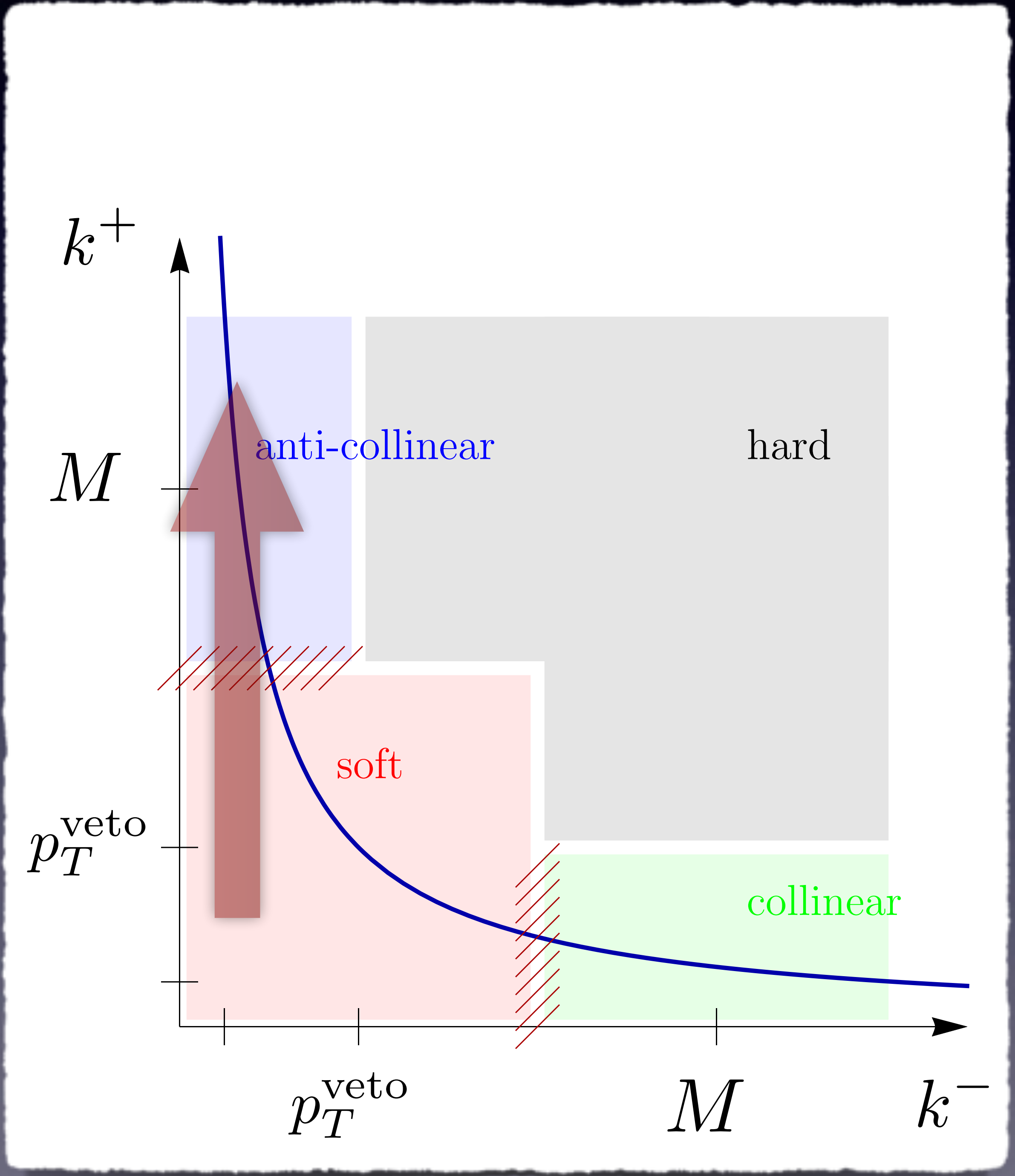
$$I = \int_{p_T^{\text{veto}}}^{\infty} \frac{dk^+}{k^+} + \int_{0}^{M} \frac{dk^+}{k^+}$$

(Collins, hep-ph:0304122)



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- Delta regulator(Chui, et.al., 2009)
- rapidity renormalization group (Chui, et.al., 2011)
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"our choice"

Soft function

Soft function

At Leading Order

$$\mathcal{S}_{q\bar{q}}^{(0)} = \begin{pmatrix} N_C & 0 \\ 0 & C_F/2 \end{pmatrix}, \qquad \mathcal{S}_{gg}^{(0)} = \begin{pmatrix} N_C & 0 & 0 \\ 0 & N_C/2 & 0 \\ 0 & 0 & (N_C^2-4)/2N_C \end{pmatrix}$$

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Up to Next-Leading Order

$$\mathcal{S}^{(1)}_{q\bar{q}(gg)} = \sum_{jk} w^{q\bar{q}(gg)}_{jk} I_{jk}$$

where

$$\overline{I}(p_T^{\text{veto}}, a_{ij}) = \int d^D k \left(\frac{\nu}{k^+}\right)^{\alpha} \delta^+(k^2)\theta(p_T^{\text{veto}} - k_T) a_{ij} \ , \quad \text{with} \quad a_{ij} = \frac{n_i \cdot n_j}{n_i \cdot k \ n_j \cdot k}$$

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After evaluating the phase space integration measure

$$\overline{I}(p_T^{\rm veto}, a_{ij}) = -\frac{\mu^{-2\epsilon}\Omega_{-2\epsilon}}{2(\alpha+2\epsilon)} \left(\frac{\mu}{\nu}\right)^{-\alpha} \left(\frac{\mu}{p_T^{\rm veto}}\right)^{\alpha+2\epsilon} \int_0^\pi d\theta_1 \frac{\sin^{1+\alpha}\theta_1}{(1-\cos\theta_1)^\alpha} \int_0^\pi d\theta_2 \sin^{-2\epsilon}\theta_2 \left[|\vec{k}|^2 a_{ij}\right]$$

Soft function

Soft function

After parametrizing the momentum,

$$k=|\vec{k}|(1,\cdots,\sin\theta_1\sin\theta_2,\sin\theta_1\cos\theta_2,\cos\theta_1),\quad n_1=(1,0,\cdots,0,1),\quad n_2=(1,0,\cdots,0,-1),$$
$$n_3=\frac{1}{\sqrt{1-\beta_t^2}}(1,0,\cdots,0,\beta_t\sin\theta,\beta_t\cos\theta),\quad n_4=\frac{1}{\sqrt{1-\beta_t^2}}(1,0,\cdots,0,-\beta_t\sin\theta,-\beta_t\cos\theta)$$

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The angular integral in the soft function can be expressed as

$$\Sigma^{(k,\,l,\,m)} = \int_0^\pi d\theta_1 \frac{\sin^{1+\alpha}\theta_1}{(1-\cos\theta_1)^\alpha} \int_0^\pi d\theta_2 \sin^{-2\epsilon}\theta_2$$
$$\times \frac{(a + b\cos\theta_1)^{-k}}{(A_1 + B_1 \sin\theta_1 \cos\theta_2 + C_1 \cos\theta_1)^l (A_2 + B_2 \sin\theta_1 \cos\theta_2 + C_2 \cos\theta_1)^m}$$

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Where the angular integral $\Sigma^{(1,1,0)}$, $\Sigma^{(0,2,0)}$, and $\Sigma^{(0,1,1)}$ is obtained firstly in (Li,Li,DYS,Yang&Zhu 2013)

RG revolution and resummation

RG revolution and resummation

The product of initial state beam functions can be factorized as

(Becher, Neubert & Rothen 2012,2013,2014)

$$\left[\mathcal{I}_{i\leftarrow a}(z_1, p_T^{\rm veto}, \mu_f)\mathcal{I}_{\bar{i}\leftarrow b}(z_2, p_T^{\rm veto}, \mu_f)\right]_{q^2=M^2} =$$

$$\left(\frac{M}{p_T^{\rm veto}}\right)^{-2F_{i\bar{i}}(p_T^{\rm veto}, \mu_f)} e^{2h_i(p_T^{\rm veto}, \mu_f)}\bar{I}_{i\leftarrow a}(z_1, p_T^{\rm veto}, \mu_f)\bar{I}_{\bar{i}\leftarrow a}(z_2, p_T^{\rm veto}, \mu_f)$$

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Solving the RG equation for the hard function (Ahrens, Ferroglia, Neubert, Pecjak and Yang, 2010, 2011 ……)

$$\mathcal{H}_{i\bar{i}}(M, m_t, \cos\theta, \mu_f) = \left|\exp\left[4S_i(\mu_h, \mu_f) - 4a_{\gamma^i}(\mu_h, \mu_f)\right]\right| \left|\left(\frac{-M^2}{\mu_h^2}\right)^{-2a_{\Gamma_i}(\mu_h,\mu_f)}\right| \times u_{i\bar{i}}^h(M, m_t, \cos\theta, \mu_h, \mu_f)\mathcal{H}_{i\bar{i}}(M, m_t, \cos\theta, \mu_h)u_{i\bar{i}}^{h\dagger}(M, m_t, \cos\theta, \mu_h, \mu_f)$$

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Exponentiating the logarithmic terms in the soft function

$$\mathcal{S}_{i\bar{i}}(p_T^{\text{veto}},M,m_t,\cos\theta,\mu) = u^{s,\dagger}_{i\bar{i}}(M,m_t,\cos\theta,\mu)\overline{\mathcal{S}}_{i\bar{i}}(p_T^{\text{veto}},M,m_t,\cos\theta)u^s_{i\bar{i}}(M,m_t,\cos\theta,\mu)$$

RG improved cross section

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Integrate out the rapidity, the differential cross section

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$$\times\int_\tau^1 \overline{I\!I}_{i\bar{i}\leftarrow ab}(z,p_T^{\rm veto},\mu_f)\,f\!f_{ab}(\tau/z,\mu_f)$$

where define RG-invariant hard function

$$\overline{\mathcal{H}}_{i\bar{i}}(M,m_t,\cos\theta) = \left(\frac{M}{p_T^{\rm veto}}\right)^{-2F_{i\bar{i}}(p_T^{\rm veto},\mu_f)} e^{2h_i(p_T^{\rm veto},\mu_f)}$$
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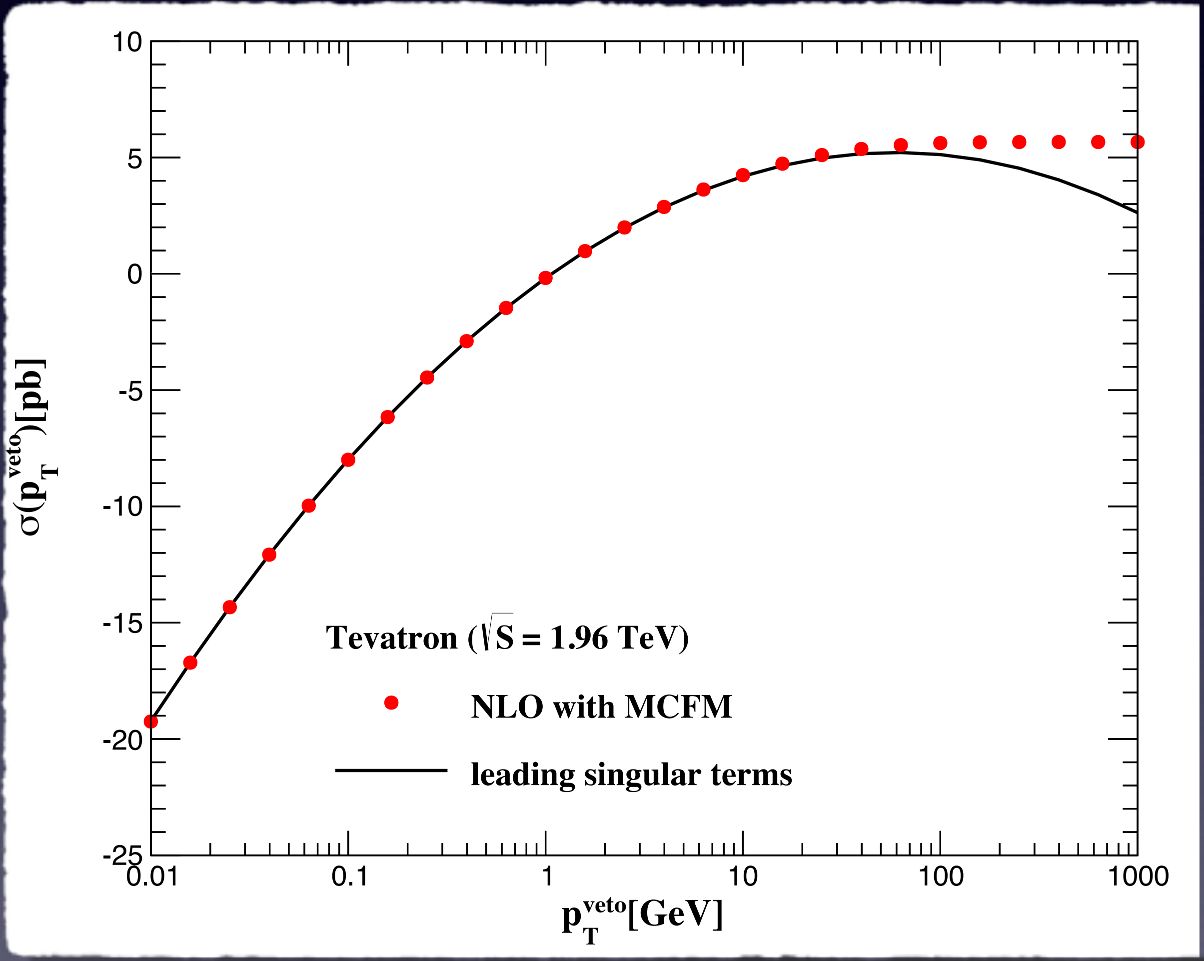
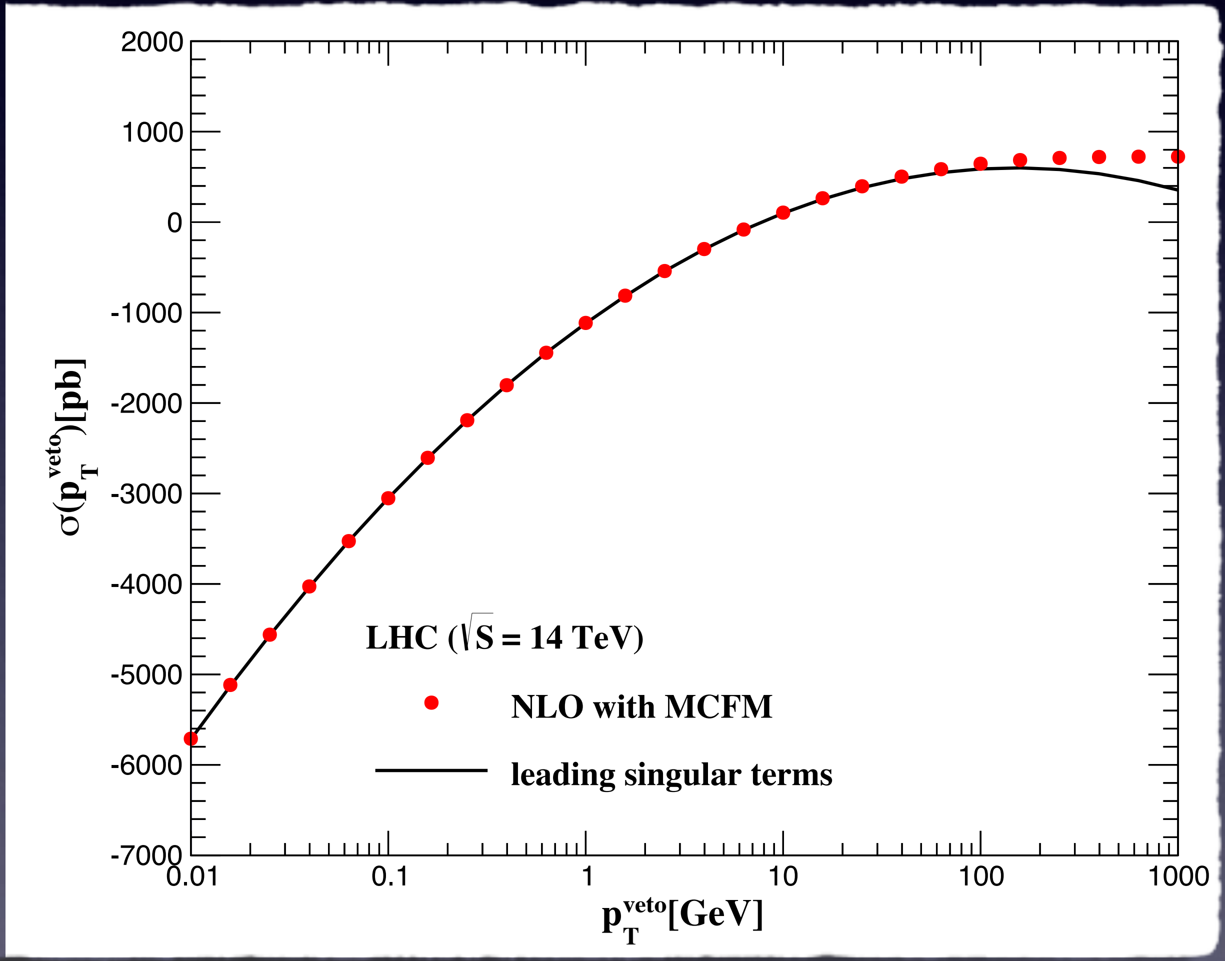
Finally, the RG improved cross section

$$\frac{d\sigma^{\rm NLO+NNLL}(p_T^{\rm veto})}{dM\,d\cos\theta} = \frac{d\sigma^{\rm NNLL}(p_T^{\rm veto})}{dM\,d\cos\theta} + \left[\frac{d\sigma^{\rm NLO}}{dM\,d\cos\theta} - \frac{d\sigma^{\rm NNLL}(p_T^{\rm veto})}{dM\,d\cos\theta}\right]_{\rm expand\ to\ NLO}$$

Leading Singular Terms





Resummation results

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Scale choice: $\mu_f \sim p_T^{\text{veto}}, \quad \mu_h \sim iM_{t\bar{t}}$

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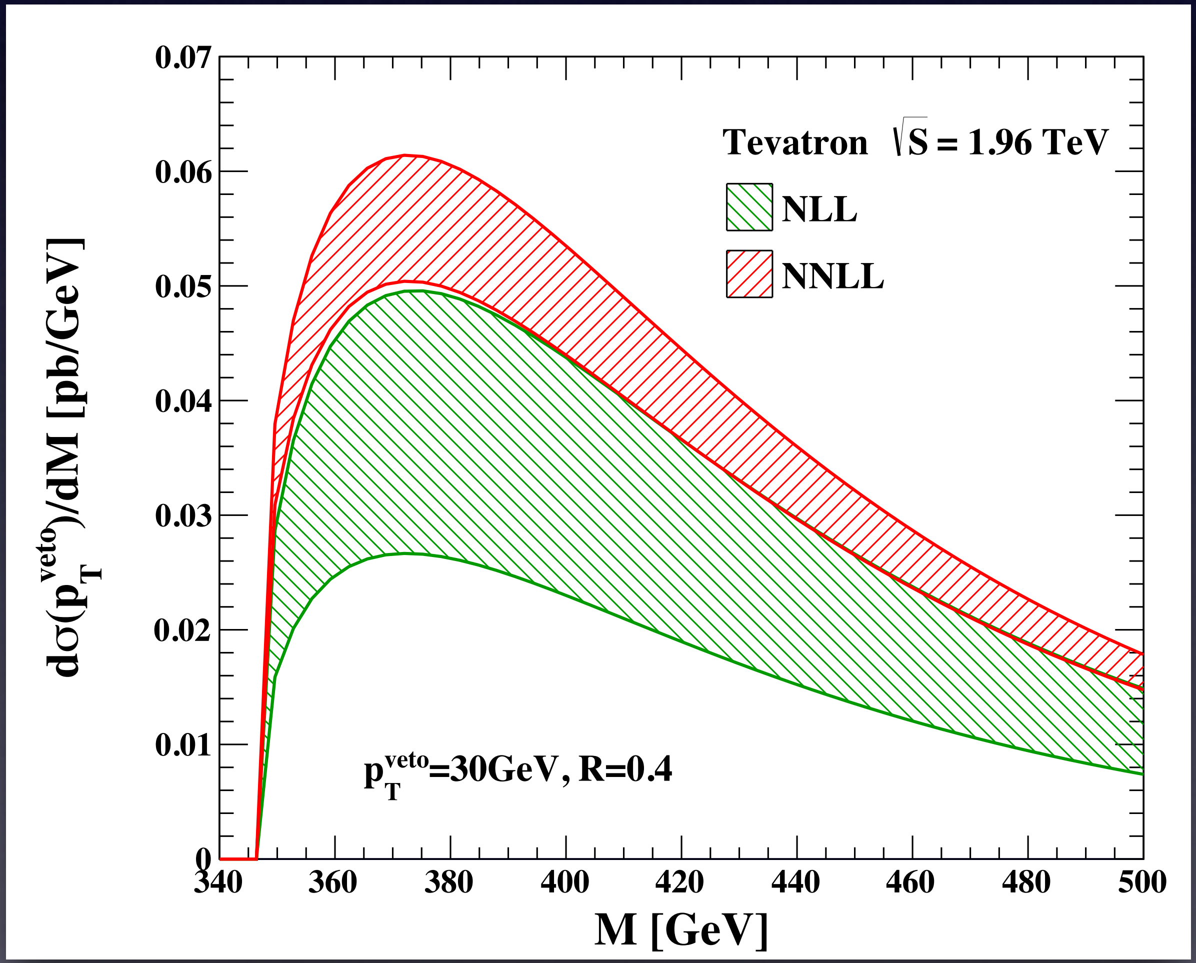
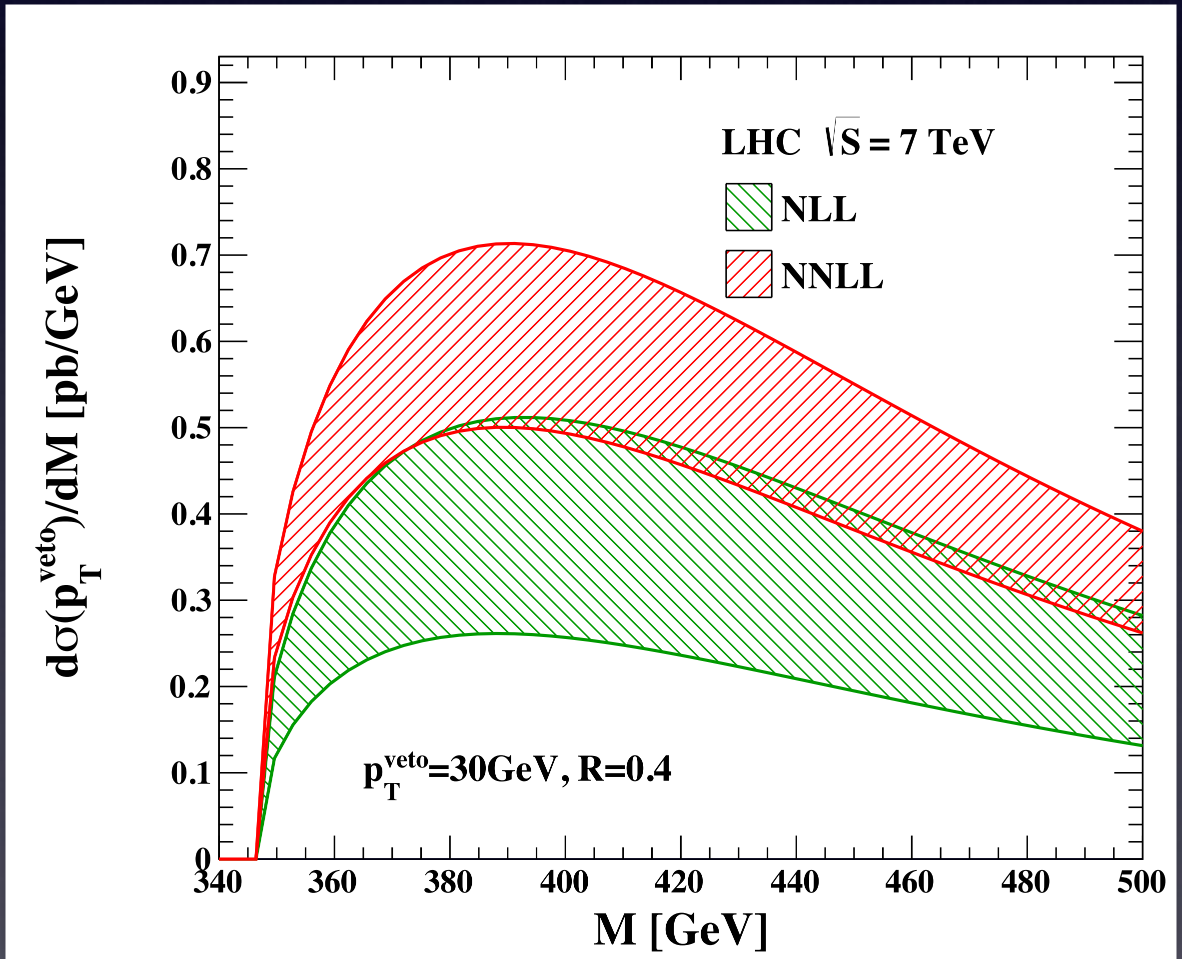
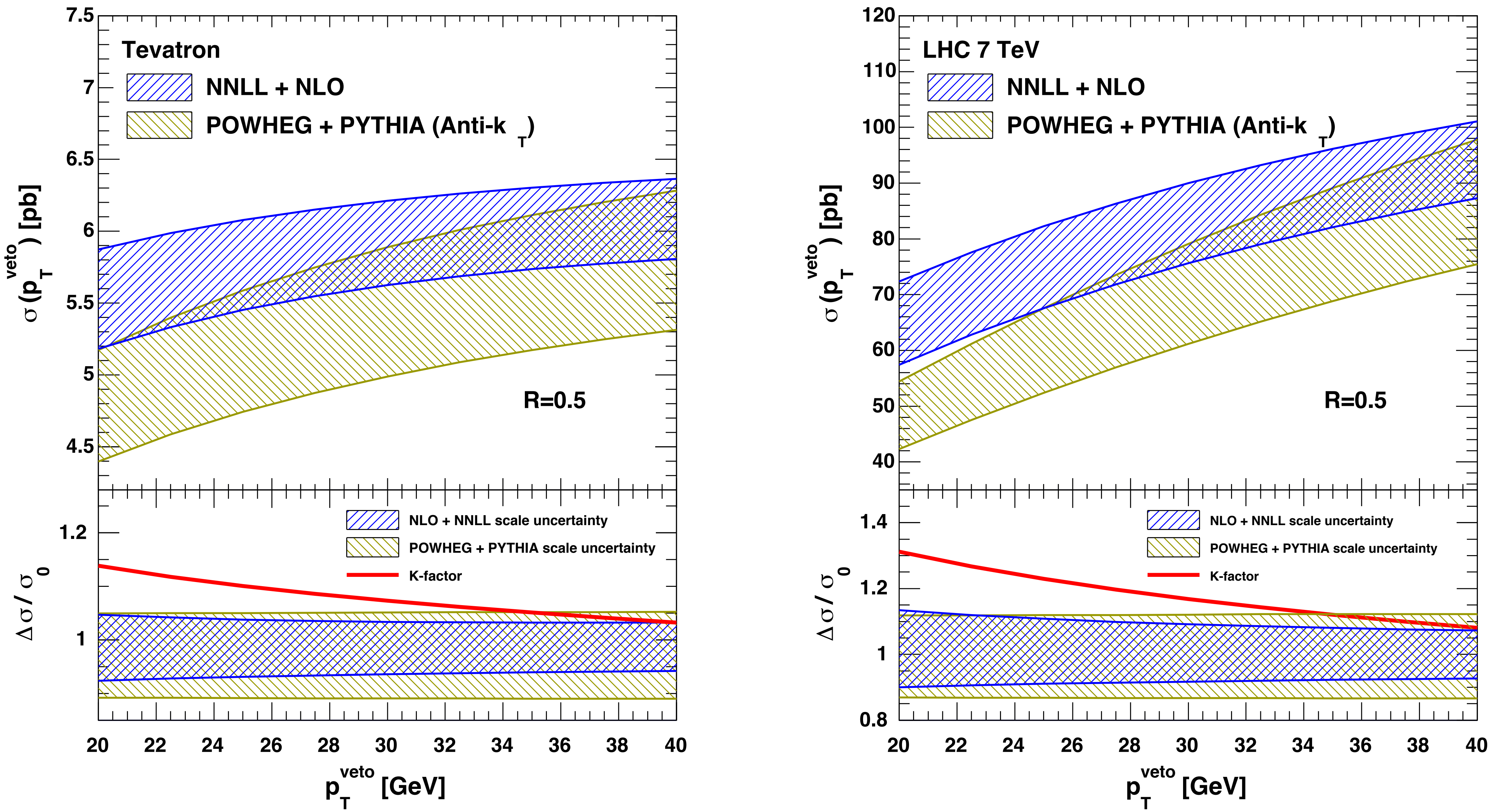




NLO+NNLL v.s. MC Tools



Conclusion and Outlook

Based on SCET and HQET, we have studied the factorization formalism for "$t\bar{t}+0j$" process at hadron colliders, and propose a jet vetoed soft function to describe the soft gluon radiation from massive colour final states;

Our formalism can be extended to other process with massive colour final states, e.g. tW, squark pair, gluino pair production etc.

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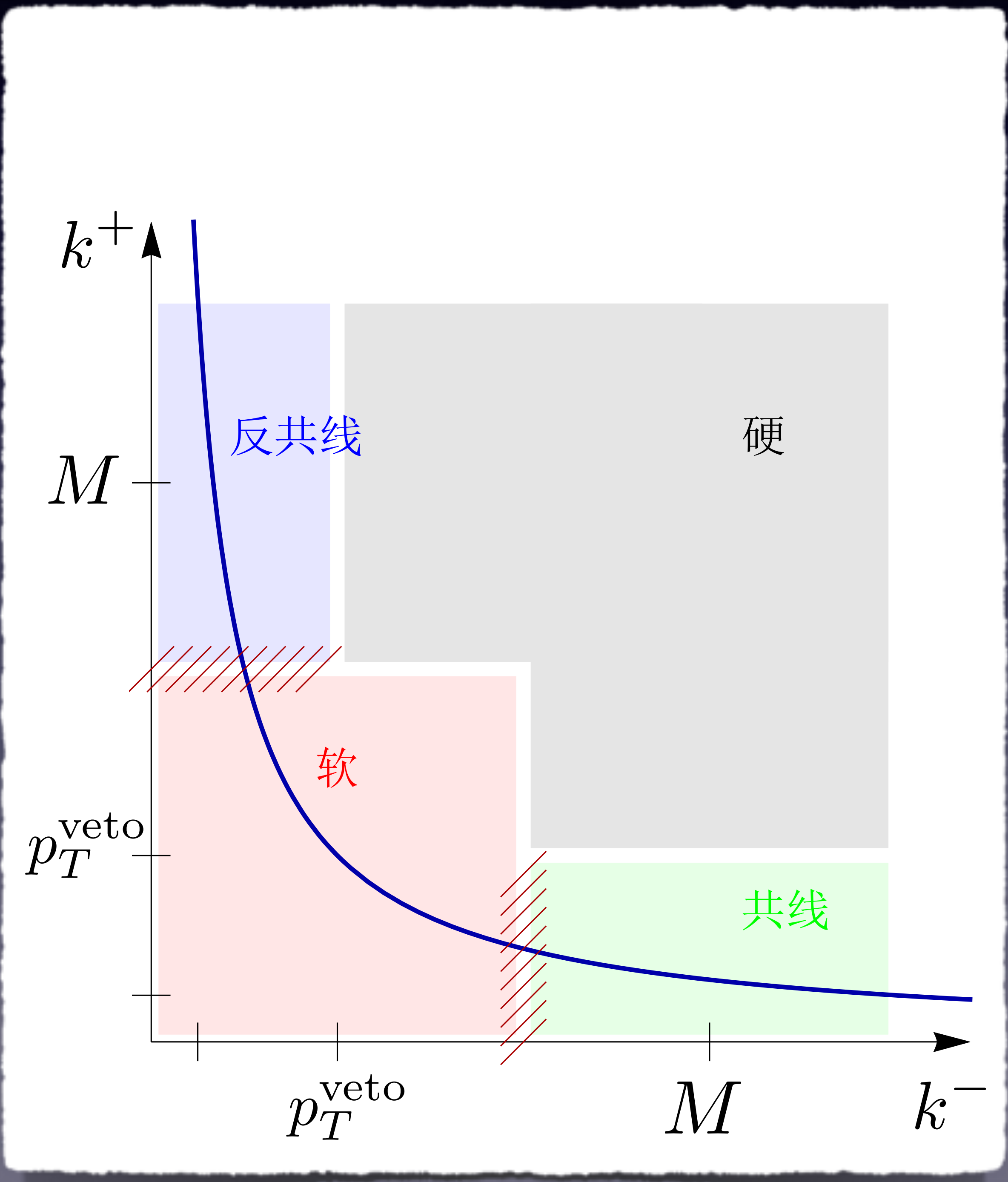
Veto ttbar

Thank you!

Backup Slides

Jet Definition



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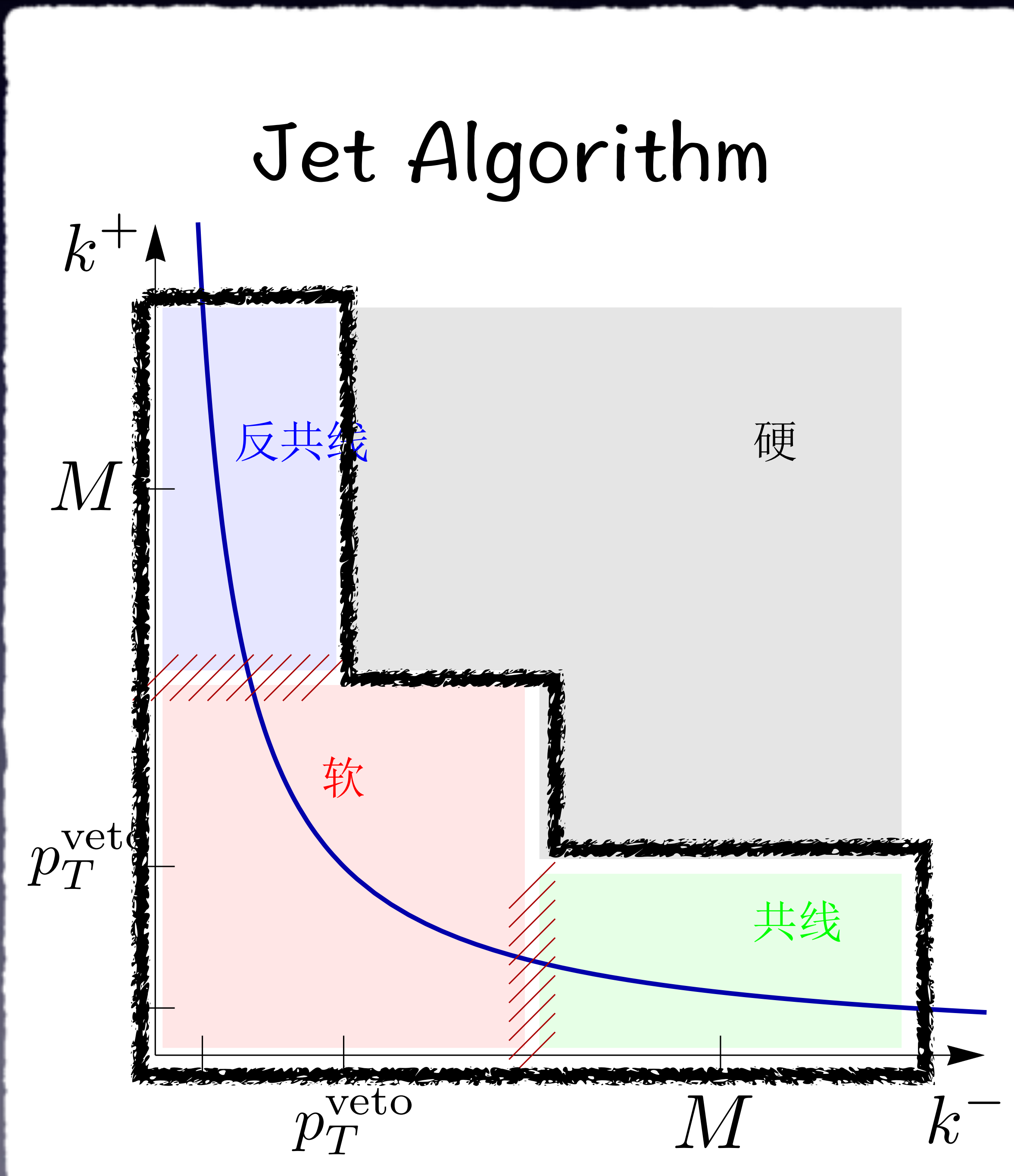

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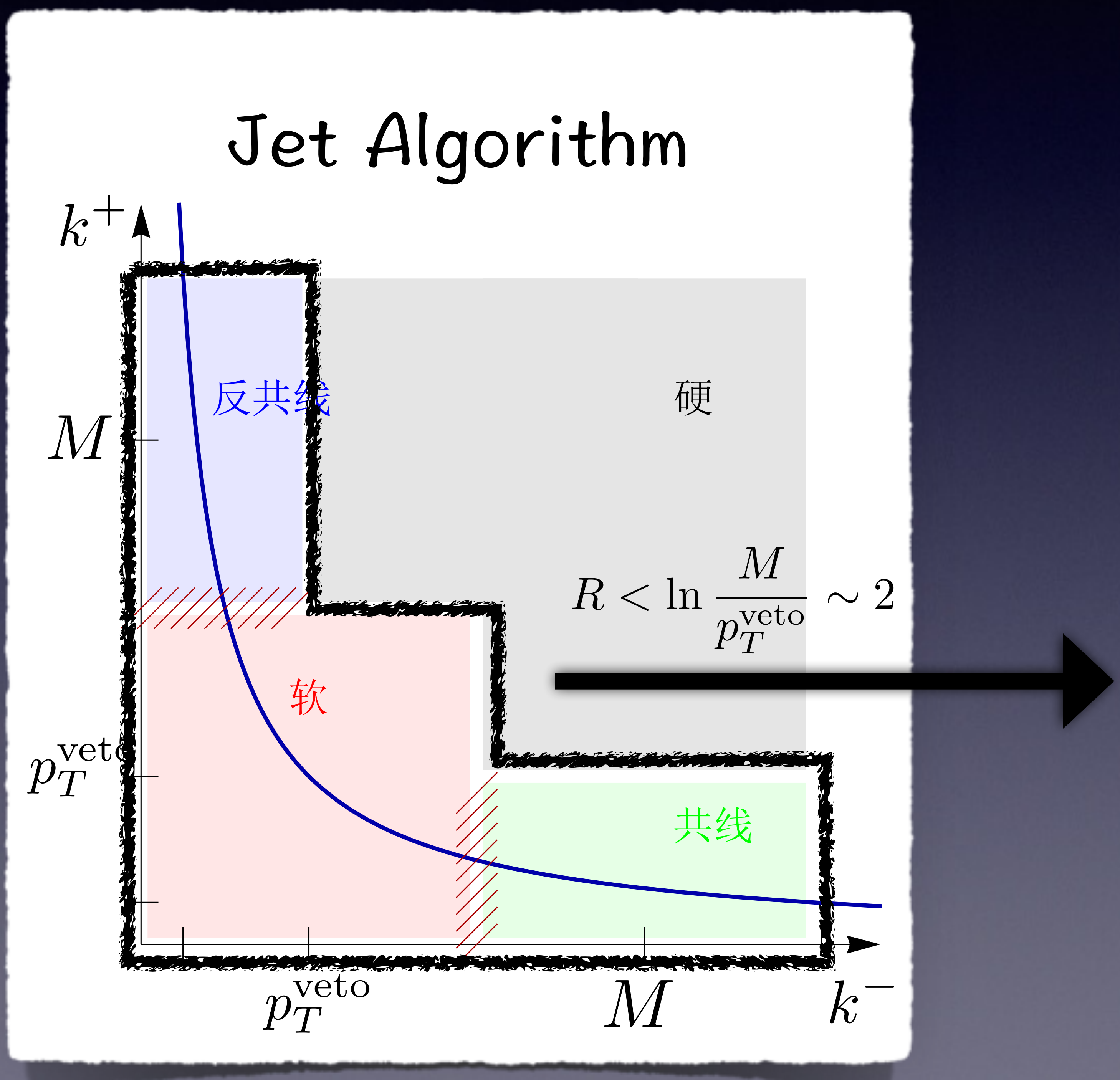
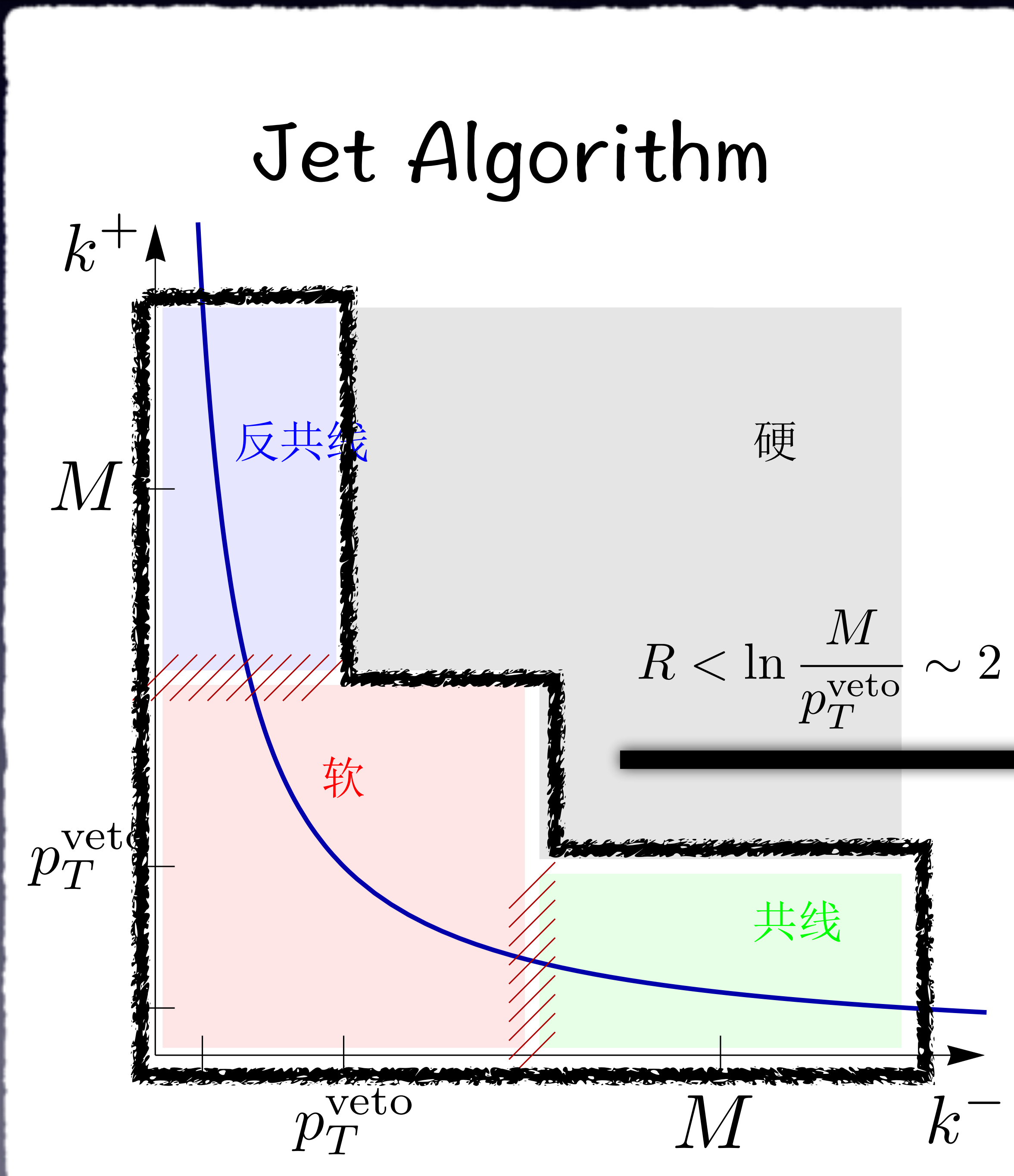
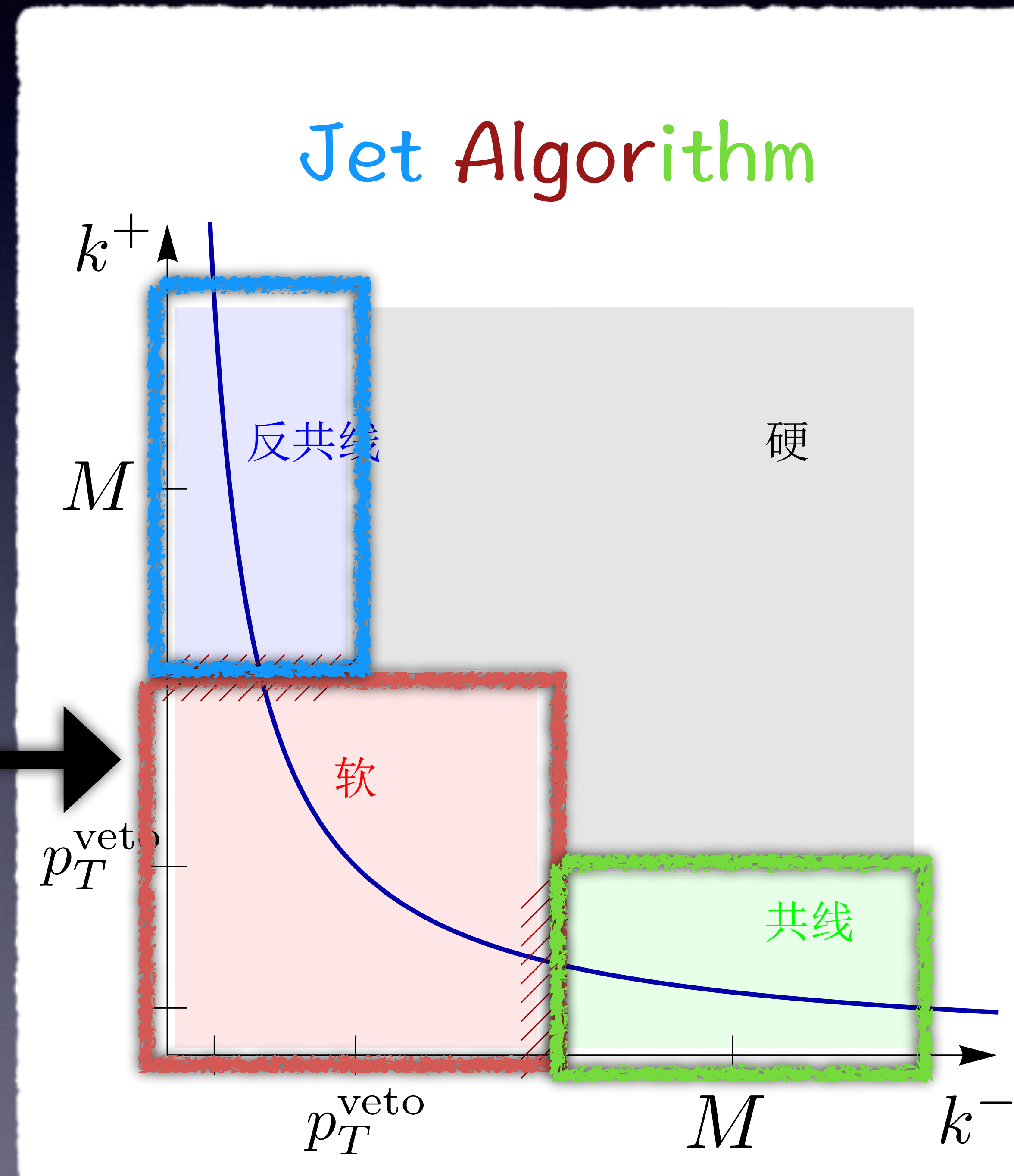


Jet Definition



$R < \ln \frac{M}{p_T^{\rm veto}} \sim 2$



Factorization formalism

The differential cross section:

$$\frac{d\sigma}{dydMd\cos\theta} = \frac{\beta_t}{3\pi^2 sM}\int d\phi_t d^2q_\perp d^2x_\perp e^{-iq_\perp\cdot x_\perp}\Bigg\{$$
$$4\mathcal{B}^{\mu\rho}_{g/N_1}(\zeta_1,x_\perp,p_T^{\text{veto}},\mu)\mathcal{B}^{\nu\sigma}_{g/N_2}(\zeta_2,x_\perp,p_T^{\text{veto}},\mu)\mathbf{Tr}\left[\mathcal{H}^{\mu\nu\rho\sigma}_{gg}(M,m_t,v_3,\mu)\mathcal{W}_{gg}(x_\perp,p_T^{\text{veto}},\mu)\right]$$
$$+\mathcal{B}_{q/N_1}(\zeta_1,x_\perp,p_T^{\text{veto}},\mu)\mathcal{B}_{\bar{q}/N_2}(\zeta_2,x_\perp,p_T^{\text{veto}},\mu)\mathbf{Tr}\left[\mathcal{H}_{q\bar{q}}(M,m_t,\cos\theta,\mu)\mathcal{W}_{q\bar{q}}(x_\perp,p_T^{\text{veto}},\mu)\right]$$
$$+(q\leftrightarrow\bar{q})\Bigg\}.$$

After integrating out q_T

$$\frac{d\sigma}{dydMd\cos\theta} = \frac{8\pi\beta_t}{3sM}\int_0^{2\pi}\frac{d\phi_t}{2\pi}\Bigg\{$$
$$4\mathcal{B}^{\mu\rho}_{g/N_1}(\zeta_1,0,p_T^{\text{veto}},\mu)\mathcal{B}^{\nu\sigma}_{g/N_2}(\zeta_2,0,p_T^{\text{veto}},\mu)\mathbf{Tr}\left[\mathcal{H}^{\mu\nu\rho\sigma}_{gg}(M,m_t,v_3,\mu)\mathcal{W}_{gg}(0,p_T^{\text{veto}},\mu)\right]$$
$$+\mathcal{B}_{q/N_1}(\zeta_1,0,p_T^{\text{veto}},\mu)\mathcal{B}_{\bar{q}/N_2}(\zeta_2,0,p_T^{\text{veto}},\mu)\mathbf{Tr}\left[\mathcal{H}_{q\bar{q}}(M,m_t,\cos\theta,\mu)\mathcal{W}_{q\bar{q}}(0,p_T^{\text{veto}},\mu)\right]$$
$$+(q\leftrightarrow\bar{q})\Bigg\}.$$

$$F_{i\bar{i}}(p_T^{\text{veto}}, \mu_f) = a_s \left[\Gamma_0^i L_\perp + d_1^{i,\text{veto}}(R)\right] + a_s^2 \left[\Gamma_0^i \beta_0 \frac{L_\perp^2}{2} + \Gamma_1^i L_\perp + d_2^{i,\text{veto}}(R)\right], \tag{A.5}$$

where the anomaly coefficient $d^{\text{veto}}(R)$ can be extracted from fixed order calculations of beam function. The function $h_i(p_T^{\text{veto}}, \mu_f)$ is given by

$$h_i(p_T^{\text{veto}}, \mu_f) = a_s \left(\Gamma_0^i \frac{L_\perp^2}{4} - \gamma_0^i L_\perp\right), \tag{A.6}$$

where the normalization condition of $h_i(p_T^{\text{veto}}, p_T^{\text{veto}}) \equiv 0$ is chosen. After calculating complete one loop function $\mathcal{I}_{i\leftarrow a}(z, p_T^{\text{veto}}, \mu)$, we have

$$d_1^{q(g),\text{veto}}(R) = 0, \tag{A.7}$$

$$\mathcal{R}_{q\leftarrow q}(z) = C_F \left[2(1-z) - \frac{\pi^2}{6}\delta(1-z)\right], \tag{A.8}$$

$$\mathcal{R}_{q\leftarrow g}(z) = 4T_F z(1-z), \tag{A.9}$$

$$\mathcal{R}_{g\leftarrow g}(z) = -C_A \frac{\pi^2}{6}\,\delta(1-z), \tag{A.10}$$

$$\mathcal{R}_{g\leftarrow q}(z) = 2C_F z. \tag{A.11}$$

$$\boldsymbol{S}_{i\bar{i}}^{(1)} = 4L_{\perp}\left(2\boldsymbol{w}_{i\bar{i}}^{13}\ln\frac{-t_1}{m_t M} + 2\boldsymbol{w}_{i\bar{i}}^{23}\ln\frac{-u_1}{m_t M} + \boldsymbol{w}_{i\bar{i}}^{33}\right)$$
$$- 4(\boldsymbol{w}_{i\bar{i}}^{13} + \boldsymbol{w}_{i\bar{i}}^{23})\mathrm{Li}_2\left(1 - \frac{t_1 u_1}{m_t^2 M^2}\right) + 4\boldsymbol{w}_{i\bar{i}}^{33}\ln\frac{t_1 u_1}{m_t^2 M^2}$$
$$- 2\boldsymbol{w}_{i\bar{i}}^{34}\frac{1+\beta_t^2}{\beta_t}[L_{\perp}\ln x_s + f_{34}], \tag{7}$$

where $x_s = (1-\beta_t)/(1+\beta_t)$ and

$$f_{34} = -\mathrm{Li}_2\left(-x_s\tan^2\frac{\theta}{2}\right) + \mathrm{Li}_2\left(-\frac{1}{x_s}\tan^2\frac{\theta}{2}\right)$$
$$+ 4\ln x_s \ln\cos\frac{\theta}{2}. \tag{8}$$

At the NNLL level the dependence of the RG invariant hard function $\overline{H}(M, p_T^{\rm veto})$ on the jet radius parameter R is caused from the two loop anomaly coefficient $d_2^{\rm veto}(R)$. The R dependence term has the form as

$$\exp\left[0.54\frac{d_2^{\rm veto}(R)}{d_2^q}\alpha_s^2(\mu)\ln\frac{M}{p_T^{\rm veto}}\right],$$