Very rare, exclusive radiative decays of W and Z bosons in QCD factorization

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PRISMA
Cluster of Excellence
Precision Physics, Fundamental Interactions and Structure of Matter

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Motivation

One of the main challenges to particle physics is to obtain rigorous control about non-perturbative physics in QCD.

For hard exclusive processes with final-state hadrons:

“QCD factorization”

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Factorization into partonic rates convoluted with light-cone distribution amplitudes (LCDAs)

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Factorization into partonic rates convoluted with light-cone distribution amplitudes (LCDAs)

Amplitudes will be organized in an expansion in the scale separation

$$\lambda \sim \frac{\Lambda_{\mathrm{QCD}}}{E_M}$$

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$\rightarrow$ power-corrections expected to be small!

Price to pay: Low branching ratios, experimentally extremely challenging to identify

Motivation

But: Large rates of electroweak gauge bosons are expected at the HL-LHC and future machines, opening up the possibility to conduct such studies:

- high-luminosity LHC ($3000\,\mathrm{fb}^{-1}$): $\sim 10^{11}$ Z bosons, $\sim 5 \cdot 10^{11}$ W bosons
- TLEP, dedicated run at Z pole: $\sim 10^{12}$ Z bosons per year
- LHC: large samples of W bosons in dedicated runs at WW or $t\bar{t}$ thresholds

[Mangano, Melia (*2014*), arXiv:1410.7475]

Motivation

Our interest was raised by recent studies of $h \to V\gamma$ decays as probes for non-standard Yukawa couplings

[Isidori, Manohar, Trott (*2013*), Phys. Lett. B 728, 131]

[Bodwin, Petriello, Stoynev, Velasco (*2013*), Phys. Rev. D 88, no. 5, 053003]

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Based on:

Exclusive Radiative Decays of W and Z Bosons in QCD Factorization

Yuval Grossman, MK, Matthias Neubert

arXiv:1501.06569

Outline

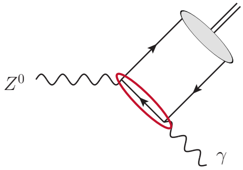
1. QCD factorization
   - The factorization formula
   - Light cone distributions for mesons

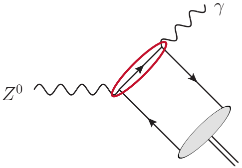
2. Decays of electroweak gauge bosons
   - Radiative hadronic decays of Z bosons
   - Radiative hadronic decays of W bosons
   - Z decays as BSM probes
   - Weak radiative Z decays to M + W

3. Conclusions, summary and outlook

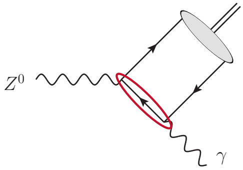
QCD factorization
The factorization formula

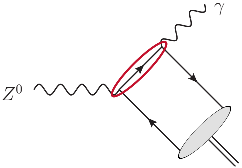
The factorization formula





- In the decays considered, the intermediate fermion propagator is highly virtual





- In the decays considered, the intermediate fermion propagator is highly virtual
- Soft collinear effective theory allows seperation of scales into
  $\to$ the hard scale E
  $\to$ and the hadronic scale μ_0

[Bauer et al. (*2001*), Phys. Rev. D 63, 114020]
[Bauer Pirjol, Stewart (*2002*), Phys. Rev. D 65, 054022]
[Beneke, Chapovsky, Diehl, Feldmann (*2002*), Nucl. Phys. B 643, 431]

The factorization formula

- Final state meson moving along the direction n^μ described by collinear quark, anti-quark and gluon fields

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- Scaling of the collinear momenta p_c:

$$\left(n \cdot p_c, \bar{n} \cdot p_c, p_c^\perp\right) \sim E\left(\lambda^2, 1, \lambda\right)$$

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- Collinear quark and gluon fields:

$$\mathcal{X}_c = \frac{\not{n}\not{\bar{n}}}{4} W_c^\dagger q \qquad \mathcal{A}_{c\perp}^\mu = W_c^\dagger \left(iD_{c\perp}^\mu W_c\right)$$

with $W_c(x) = \mathbf{P} \exp\left(ig \int_{-\infty}^{0} dt\ \bar{n} \cdot A_c(x + t\bar{n})\right)$

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- At leading order, the decay amplitude $\mathcal{A}_{V\to M\gamma}$ can be written as:

$$\mathcal{A} = \sum_i \int dt\ C_i(t,\mu) \langle M(k)| \bar{\mathcal{X}}_c(t\bar{n}) \frac{\not{\bar{n}}}{2} \Gamma_i \mathcal{X}_c(0) |0\rangle + \dots$$

$$= \sum_i \int dt\ C_i(t,\mu) \langle M(k)| \bar{q}(t\bar{n}) \frac{\not{\bar{n}}}{2} \Gamma_i [t\bar{n},0] q(0) |0\rangle + \dots$$

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- $\langle M|\dots|0\rangle = -if_M E\int_0^1 dx\, e^{ixt\bar{n}\cdot k}\phi_M(x,\mu)$ defines the light-cone distribution amplitude

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- The factorization formula now reads:

$$\mathcal{A} = -if_M E \int_0^1 dx\, H_M(x,\mu)\phi_M(x,\mu) + \begin{array}{c}\text{power}\\\text{corrections}\end{array}$$

The factorization formula

Define: Projectors M_M, can be applied to partonic amplitudes directly.

In a practical calculation each Feynman diagram gives an expression of the form:

$$\bar{u}(k_1)A(q, k_1, k_2)v(k_2) = \mathrm{Tr}\left[v(k_2)\bar{u}(k_1)A(q, k_1, k_2)\right]$$

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The projection is then:

$$\bar{u}(k_1)A(q,k_1,k_2)v(k_2) \to \int_0^1 dx\ \mathrm{Tr}\left[M_M(k,x,\mu)\,A(q,k_1,k_2)\right]$$

The projector M_M depends on the type of meson (pseudoscalar, vector meson [longitudinal/tranverse polarization]).

The factorization formula

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For a pseudoscalar meson, the projector to twist-3-order is given by:

$$M_P(k,x,\mu) = \frac{if_P}{4}\left\{\not{k}\gamma_5\phi_P(x,\mu) - \mu_P(\mu)\gamma_5\left[\phi_p(x,\mu) - i\sigma_{\mu\nu}\frac{k^\mu \bar{n}^\nu}{k\cdot\bar{n}}\frac{\phi'_\sigma(x,\mu)}{6} + i\sigma_{\mu\nu}k^\mu\frac{\phi_\sigma(x\mu)}{6}\frac{\partial}{\partial k_{\perp\nu}}\right] + \text{3-part.}\right\}$$

where

$$\phi_p(x,\mu) = 1 \qquad \phi_\sigma(x,\mu) = 6x(1-x)$$

when three-particle LCDAs are neglected (Wandzura-Wilczek approximation).

[Wandzura, Wilczek (*1977*), Phys. Lett. B 72, 195]

QCD factorization

Light cone distributions for mesons

Gegenbauer expansion of the LCDAs

- The LCDA can be interpreted as the amplitude for finding a quark with longitudinal momentum fraction x

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- Defined by local matrix element (here example for pseudo-scalar)

$$\langle P(k)|\, \bar{q}(t\bar{n}) \frac{\bar{\not{n}}}{2} \gamma^5 \, [t\bar{n}, 0]\, q(0)\, |0\rangle = -i f_M E \int_0^1 dx\, e^{ixt\bar{n}\cdot k} \phi_M(x,\mu)$$

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- For light mesons information about the LCDAs has to be extracted from lattice QCD or sum rules. For mesons containing a heavy quark (or for heavy quarkonia), this can be addressed with HQET (or NRQCD).

Gegenbauer expansion of the LCDAs

We expand the LCDAs in the basis of Gegenbauer polynomials:

$$\phi_M(x,\mu) = 6x(1-x)\left[1 + \sum_{n=1}^{\infty} a_n^M(\mu)\, C_n^{(3/2)}(2x-1)\right]$$

where $C_n^{(\alpha)}(x)$ are the Gegenbauer polynomials. The scale-dependence of the LCDA is in the Gegenbauer moments $a_n^M(\mu)$

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$\rightarrow$ RG evolution important AND works in our favor

- The Gegenbauer expansion yields a diagonal scale-evolution of the coefficients:

$$a_n^M(\mu) = \left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{\gamma_n/2\beta_0} a_n^M(\mu_0)$$

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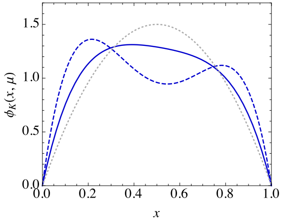
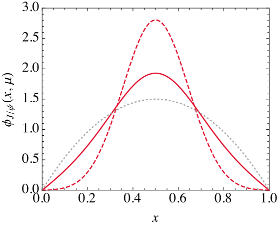
- Every anomalous dimension γ_n is strictly positive

$$\Rightarrow a_n^M(\mu \to \infty) \to 0$$
$$\Rightarrow \phi_M(x, \mu \to \infty) \to 6x(1-x)$$

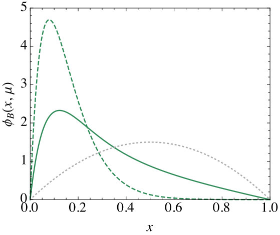
RG evolution of the LCDAs



a) K LCDA



b) J/ψ LCDA



c) B LCDA

LCDAs for mesons at different scales, dashed lines: $\phi_M(x,\mu=\mu_0)$, solid lines: $\phi_M(x,\mu=m_Z)$, grey dotted lines: $\phi_M(x,\mu\to\infty)$

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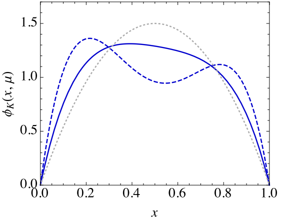

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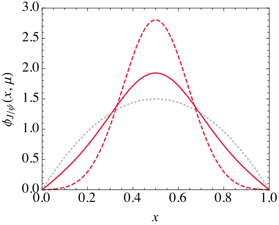

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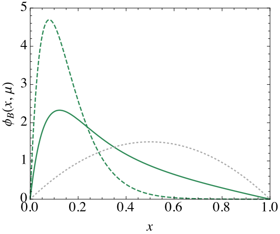

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At high scales compared to Λ_{QCD} (e.g. $\mu\sim m_Z$) the sensitivity to poorly-known a_n^M is greatly reduced!

Heavy mesons: quarkonia

For heavy quarkonium states $M \sim (Q\bar{Q})$ the LCDA peaks at $x = 1/2$. In the limit of $m_Q \to \infty$, the width of the LCDA vanishes and $\phi_M \to \delta(x - \frac{1}{2})$.

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Using NRQCD, the LCDA can be related to a local matrix element

[Caswell, Lepage (*1986*), Phys. Lett. B 167, 437]

[Bodwin, Braaten, Lepage (*1995*), Phys. Rev. D 51, 1125]

One finds:

$$\int_0^1 dx\, (2x-1)^2 \phi_M(x,\mu_0) = \frac{\langle v^2 \rangle_M}{3} + \mathcal{O}(v^4)$$

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Our model at the low scale:

$$\phi_M(x,\mu_0) = x(1-x)\exp\left[-\frac{6(x-\frac{1}{2})^2}{\langle v^2 \rangle}\right] \times \text{normalization}$$

Heavy mesons: heavy-light states

For heavy-light mesons $M \sim (q\bar{Q})$, one defines:

$$\int_0^1 dx\, \frac{\phi_M(x,\mu_0)}{x} = \frac{m_M}{\lambda_M(\mu_0)} + \dots$$

[Beneke, Buchalla, Neubert, Sachrajda (*1999*), Phys. Rev. Lett. 83, 1914]

where m_M is the meson mass and the parameter λ_M is a (poorly known) hadronic parameter and we have to use estimates.

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[Grozin, Neubert (*1997*), Phys. Rev. D 55, 272]

Heavy meson LCDAs and RGE

Heavy meson LCDAs at the low scale $\mu_0 = 1\,\mathrm{GeV}$:

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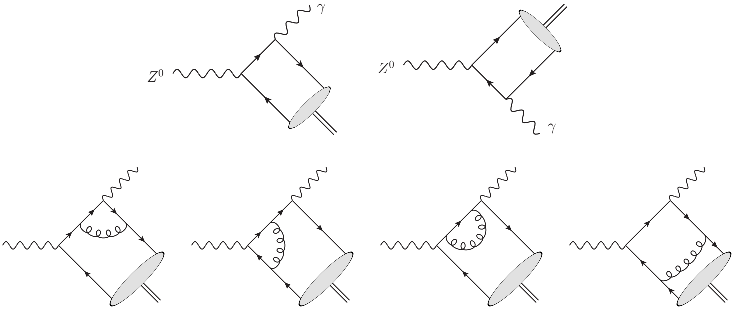
For light mesons, only the first few moments are known (we use up to $n=2$). For heavy mesons, we calculate the first 20 Gegenbauer moments to resolve the peak structure of the LCDAs.

Decays of electroweak gauge bosons

The $Z \to M + \gamma$ decay amplitude

Diagrams at $\mathcal{O}(\alpha_s)$:



$+$ analogous QCD corrections for second graph

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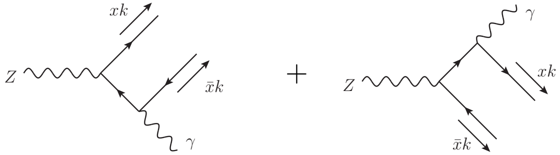
- Compute the hard interactions at desired loop-order:



$$i\mathcal{A} \propto \bar{q}(xk)\left[\gamma^\nu\left(v_q - a_q\gamma^5\right)\not{p}\gamma^\mu\right]q(\bar{x}k)\frac{\kappa(x)}{x} + \frac{\kappa(\bar{x})}{\bar{x}}\bar{q}(xk)\left[\gamma^\mu\not{p}'\gamma^\nu\left(v_q - a_q\gamma^5\right)\right]q(\bar{x}k)$$

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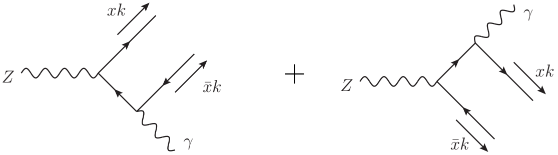

contains $\mathcal{O}(\alpha_s)$ corrections

$$i\mathcal{A} \propto \bar{q}(xk)\left[\gamma^\nu\left(v_q - a_q\gamma^5\right)\not{p}\gamma^\mu\right] q(\bar{x}k)\frac{\kappa(x)}{x} + \frac{\kappa(\bar{x})}{\bar{x}}\bar{q}(xk)\left[\gamma^\mu\not{p}'\gamma^\nu\left(v_q - a_q\gamma^5\right)\right] q(\bar{x}k)$$

The $Z \to M + \gamma$ decay amplitude

- Dirac structure of the amplitude is of the form:

$$\Gamma = v_q \gamma^\nu \not{p} \gamma^\mu - a_q \gamma^\nu \not{p} \gamma^\mu \gamma^5$$

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$$M_P = i\frac{f_P}{4}\phi_P(x,\mu)\,\not{k}\gamma^5$$

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The $Z \to M + \gamma$ decay amplitude

JG|U

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- At leading twist only P and $V_\|$ allowed! (recall: projecting involves $\mathrm{Tr}[M\,\Gamma]$)
  Subleading twist contributions **strongly** power-suppressed!

The $Z \to M + \gamma$ decay amplitude

JG|U

At the end of the day, we find:

$$i\mathcal{A} = \pm \frac{egf_M}{2\cos\theta_W} \left[i\epsilon_{\mu\nu\alpha\beta} \frac{k^\mu q^\nu \varepsilon_Z^\alpha \varepsilon_\gamma^{*\beta}}{k \cdot q} F_1^M - \left(\varepsilon_Z \cdot \varepsilon_\gamma^* - \frac{q \cdot \varepsilon_Z k \cdot \varepsilon_\gamma^*}{k \cdot q} \right) F_2^M \right]$$

with the form factors

$$F_1^M = \frac{\mathcal{Q}_M}{6} [I_+^M(m_Z) + \bar{I}_+^M(m_Z)] = \quad \mathcal{Q}_M \sum_{n=0}^{\infty} C_{2n}^{(+)}(m_Z, \mu) a_{2n}^M(\mu)$$

$$F_2^M = \frac{\mathcal{Q}'_M}{6} [I_-^M(m_Z) + \bar{I}_-^M(m_Z)] = -\mathcal{Q}'_M \sum_{n=0}^{\infty} C_{2n+1}^{(-)}(m_Z, \mu) a_{2n+1}^M(\mu)$$

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+ for pseudoscalar, - for vector

with the form factors

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quark couplings to photon and Z boson

The $Z \to M + \gamma$ decay amplitude

JG|U

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Convolution of LCDA with the hard function:

$$I_\pm^M(m_V) = \int_0^1 dx\, H_\pm(x, m_V, \mu) \phi_M(x, \mu)$$

The $Z \to M + \gamma$ decay amplitude

JG|U

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Sums over even and odd Gegenbauer moments and a coefficient function $C_n^{(\pm)}(m_V, \mu)$

The $Z \to M + \gamma$ decay amplitude

JG|U

Coefficient functions:

$$C_n^{(\pm)}(m_V, \mu) = 1 + \frac{C_F \alpha_s(\mu)}{4\pi} c_n^{(\pm)}\left(\frac{m_V}{\mu}\right) + \mathcal{O}(\alpha_s^2)$$

with:

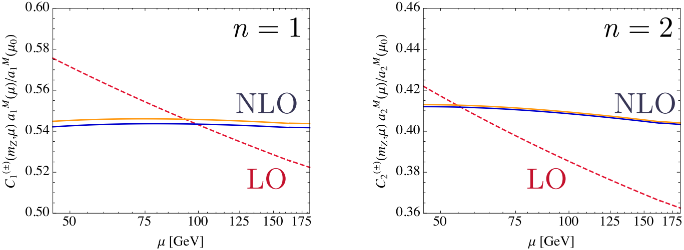
$$c_n^{(\pm)}\left(\frac{m_V}{\mu}\right) = \left[\frac{2}{(n+1)(n+2)} - 4H_{n+1} + 3\right]\left(\log\frac{m_V^2}{\mu^2} - i\pi\right) + 4H_{n+1}^2 - \frac{4\,(H_{n+1} - 1) \pm 1}{(n+1)(n+2)} + \frac{2}{(n+1)^2(n+2)^2} - 9$$

Large logs are resummed to all orders by choosing $\mu \sim m_Z$!

The $Z \to M + \gamma$ decay amplitude

JG|U

The combination $C_n^{(\pm)}(m_V, \mu) a_n^M(\mu)$ is formally scale independent!



The form factors become:

$$\mathrm{Re}F_1^M = \mathcal{Q}_M \left[0.94 + 1.05 a_2^M(m_Z) + 1.15 a_4^M(m_Z) + 1.22 a_6^M(m_Z) + \ldots\right]$$
$$= \mathcal{Q}_M \left[0.94 + 0.41 a_2^M(\mu_0) + 0.29 a_4^M(\mu_0) + 0.23 a_6^M(\mu_0) + \ldots\right]$$
$$F_2^M = 0$$

The $Z \to M + \gamma$ decay amplitude

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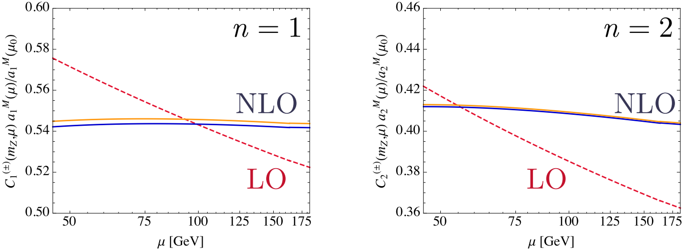

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moments at the high scale

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The $Z \to M + \gamma$ decay amplitude

JG|U

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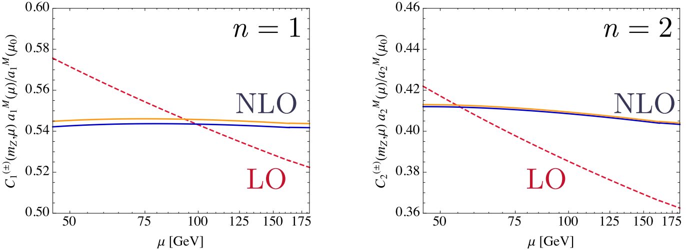

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$$F_2^M = 0$$

$\rightarrow$ **sensitivity strongly reduced!**

Results for $Z \to M\gamma$

For the branching ratios $\mathrm{BR}(Z \to M\gamma)$ we find:

$Z \to \dots$	Branching ratio	asym.	LO
$\pi^0\gamma$	$(9.80\ {}^{+0.09}_{-0.14}{}_{\mu}\ \pm 0.03_f\ \pm 0.61_{a_2} \pm 0.82_{a_4})\ \cdot 10^{-12}$	7.71	14.67
$\rho^0\gamma$	$(4.19\ {}^{+0.04}_{-0.06}{}_{\mu}\ \pm 0.16_f\ \pm 0.24_{a_2} \pm 0.37_{a_4})\ \cdot 10^{-9}$	3.63	5.68
$\omega\gamma$	$(2.89\ {}^{+0.03}_{-0.05}{}_{\mu}\ \pm 0.15_f\ \pm 0.29_{a_2} \pm 0.25_{a_4})\ \cdot 10^{-8}$	2.54	3.84
$\phi\gamma$	$(8.63\ {}^{+0.08}_{-0.13}{}_{\mu}\ \pm 0.41_f\ \pm 0.55_{a_2} \pm 0.74_{a_4})\ \cdot 10^{-9}$	7.12	12.31
$J/\psi\,\gamma$	$(8.02\ {}^{+0.14}_{-0.15}{}_{\mu}\ \pm 0.20_f\ {}^{+0.39}_{-0.36}{}_{\sigma})\ \cdot 10^{-8}$	10.48	6.55
$\Upsilon(1S)\,\gamma$	$(5.39\ {}^{+0.10}_{-0.10}{}_{\mu}\ \pm 0.08_f\ {}^{+0.11}_{-0.08}{}_{\sigma})\ \cdot 10^{-8}$	7.55	4.11
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scale dependence

decay constant

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JG|U

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scale dependence ↑ (the μ column)

decay constant ↑ (the f column)

LCDA shape ↑ (the a_2, a_4 and σ columns)

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obtained when using only asymptotic form of LCDA

$$\phi_M(x) = 6x(1-x)$$

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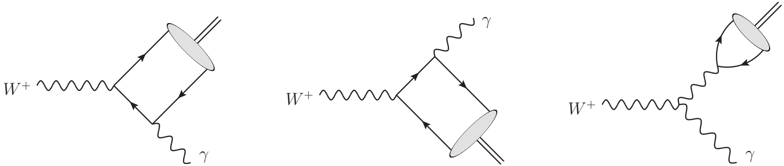
obtained when using only LO hard functions

JG|U

The $\mathbf{W} \to \mathrm{M} + \gamma$ decay amplitude

The decay $W \to M + \gamma$ is similar to the $Z \to M + \gamma$ decay, except for an additional local contribution:



The form factor decomposition now looks as follows:

$$i\mathcal{A}(W^+ \to M^+\gamma) = \pm\frac{egf_M}{4\sqrt{2}}\,V_{ij}\left(i\epsilon_{\mu\nu\alpha\beta}\frac{k^\mu\,q^\nu\,\varepsilon_W^\alpha\varepsilon_\gamma^{*\beta}}{k\cdot q}F_1^M - \varepsilon_W^\perp\cdot\varepsilon_\gamma^{\perp *}F_2^M\right)$$

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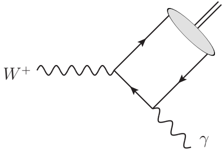

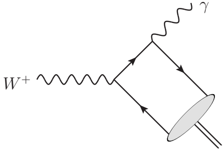

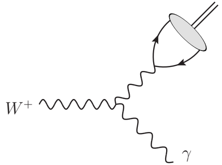

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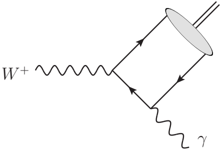
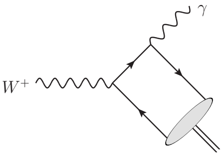
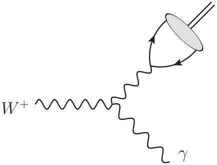
$W \to M + \gamma$

The decay $W \to M + \gamma$ is similar to the $Z \to M + \gamma$ decay, except for an additional local contribution:







The form factor decomposition now looks as follows:

$$i\mathcal{A}(W^+ \to M^+ \gamma) = \pm \frac{e g f_M}{4\sqrt{2}} V_{ij} \left(i\epsilon_{\mu\nu\alpha\beta} \frac{k^\mu q^\nu \varepsilon_W^\alpha \varepsilon_\gamma^{*\beta}}{k \cdot q} F_1^M - \varepsilon_W^\perp \cdot \varepsilon_\gamma^{\perp *} F_2^M \right)$$

+ for pseudoscalar, - for vector

For the branching ratios $W^\pm \to M^\mp \gamma$, we find:

mode	Branching ratio	asym.	LO
$\pi^\pm\gamma$	$(4.00\,^{+0.06}_{-0.11}\,{}_\mu \pm 0.01_f \pm 0.49_{a_2} \pm 0.66_{a_4}) \cdot 10^{-9}$	2.45	8.09
$\rho^\pm\gamma$	$(8.74\,^{+0.17}_{-0.26}\,{}_\mu \pm 0.33_f \pm 1.02_{a_2} \pm 1.57_{a_4}) \cdot 10^{-9}$	6.48	15.12
$K^\pm\gamma$	$(3.25\,^{+0.05}_{-0.09}\,{}_\mu \pm 0.03_f \pm 0.24_{a_1} \pm 0.38_{a_2} \pm 0.51_{a_4}) \cdot 10^{-10}$	1.88	6.38
$K^{*\pm}\gamma$	$(4.78\,^{+0.09}_{-0.14}\,{}_\mu \pm 0.28_f \pm 0.39_{a_1} \pm 0.66_{a_2} \pm 0.80_{a_4}) \cdot 10^{-10}$	3.18	8.47
$D_s\gamma$	$(3.66\,^{+0.02}_{-0.07}\,{}_\mu \pm 0.12_{\rm CKM} \pm 0.13_f\,^{+1.47}_{-0.82}\,{}_\sigma) \cdot 10^{-8}$	0.98	8.59
$D^\pm\gamma$	$(1.38\,^{+0.01}_{-0.02}\,{}_\mu \pm 0.10_{\rm CKM} \pm 0.07_f\,^{+0.50}_{-0.30}\,{}_\sigma) \cdot 10^{-9}$	0.32	3.42
$B^\pm\gamma$	$(1.55\,^{+0.00}_{-0.03}\,{}_\mu \pm 0.37_{\rm CKM} \pm 0.15_f\,^{+0.68}_{-0.45}\,{}_\sigma) \cdot 10^{-12}$	0.09	6.44

Results for $W \to M + \gamma$

For the branching ratios $W^{\pm} \to M^{\mp}\gamma$, we find:

mode	Branching ratio	asym.	LO
$\pi^{\pm}\gamma$	$(4.00\,^{+0.06}_{-0.11}\,{}_{\mu} \pm 0.01_f \pm 0.49_{a_2} \pm 0.66_{a_4}) \cdot 10^{-9}$	2.45	8.09
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flavour off-diagonal mesons allowed

Results for $W \to M + \gamma$

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introduces uncertainties from CKM elements

Decays of electroweak gauge bosons
Z decays as BSM probes

$Z \to M + \gamma$ decays as BSM probes

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Our analysis can straight-forwardly be generalized to the case of non-SM Z boson couplings to quarks!

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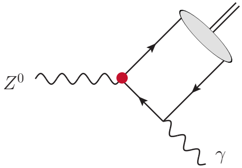

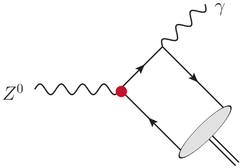

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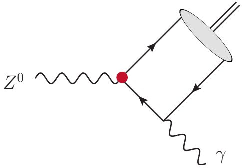

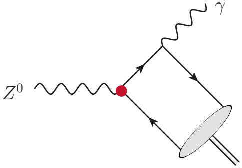

At LEP, $|a_b|$ and $|a_c|$ have been measured to 1%, using our predictions, $|a_s|$, $|a_d|$ and $|a_u|$ could be measured to $\sim 6\%$

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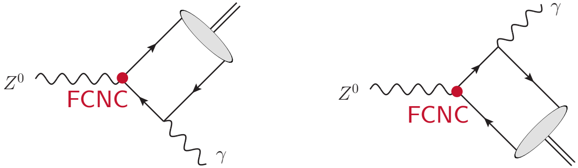

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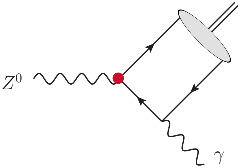
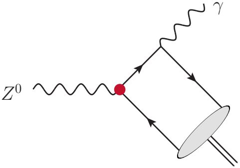
Introducing FCNC couplings allows the production of flavor off-diagonal mesons

$Z \to M + \gamma$ decays as FCNC probes





Model independent predictions for flavor off-diagonal mesons:

Decay mode	Branching ratio	SM background
$Z^0 \to K^0\gamma$	$\left[(7.70 \pm 0.83)\,\lvert v_{sd}\rvert^2 + (0.01 \pm 0.01)\,\lvert a_{sd}\rvert^2\right] \cdot 10^{-8}$	$\frac{\lambda}{\sin^2\theta_W}\,\frac{\alpha}{\pi} \sim 2 \cdot 10^{-3}$
$Z^0 \to D^0\gamma$	$\left[(5.30\,^{+\,0.67}_{-\,0.43})\,\lvert v_{cu}\rvert^2 + (0.62\,^{+\,0.36}_{-\,0.23})\,\lvert a_{cu}\rvert^2\right] \cdot 10^{-7}$	$\frac{\lambda}{\sin^2\theta_W}\,\frac{\alpha}{\pi} \sim 2 \cdot 10^{-3}$
$Z^0 \to B^0\gamma$	$\left[(2.08\,^{+\,0.59}_{-\,0.41})\,\lvert v_{bd}\rvert^2 + (0.77\,^{+\,0.38}_{-\,0.26})\,\lvert a_{bd}\rvert^2\right] \cdot 10^{-7}$	$\frac{\lambda^3}{\sin^2\theta_W}\,\frac{\alpha}{\pi} \sim 8 \cdot 10^{-5}$
$Z^0 \to B_s\gamma$	$\left[(2.64\,^{+\,0.82}_{-\,0.52})\,\lvert v_{bs}\rvert^2 + (0.87\,^{+\,0.51}_{-\,0.33})\,\lvert a_{bs}\rvert^2\right] \cdot 10^{-7}$	$\frac{\lambda^2}{\sin^2\theta_W}\,\frac{\alpha}{\pi} \sim 4 \cdot 10^{-4}$

$Z \to M + \gamma$ decays as FCNC probes

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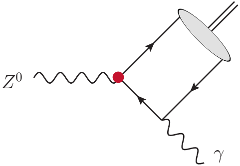

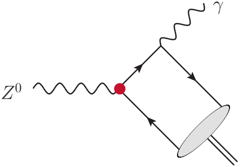

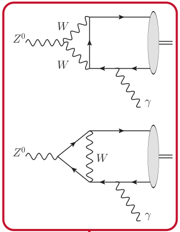

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$Z \to M + \gamma$ decays as FCNC probes

FCNCs would induce tree-level neutral-meson mixing, strongly constrained:

$\left\|\text{Re}\left[(v_{sd} \pm a_{sd})^2\right]\right\|$	$< 2.9 \cdot 10^{-8}$	$\left\|\text{Re}\left[(v_{sd})^2 - (a_{sd})^2\right]\right\|$	$< 3.0 \cdot 10^{-10}$
$\left\|\text{Im}\left[(v_{sd} \pm a_{sd})^2\right]\right\|$	$< 1.0 \cdot 10^{-10}$	$\left\|\text{Im}\left[(v_{sd})^2 - (a_{sd})^2\right]\right\|$	$< 4.3 \cdot 10^{-13}$
$\left\|(v_{cu} \pm a_{cu})^2\right\|$	$< 2.2 \cdot 10^{-8}$	$\left\|(v_{cu})^2 - (a_{cu})^2\right\|$	$< 1.5 \cdot 10^{-8}$
$\left\|(v_{bd} \pm a_{bd})^2\right\|$	$< 4.3 \cdot 10^{-8}$	$\left\|(v_{bd})^2 - (a_{bd})^2\right\|$	$< 8.2 \cdot 10^{-9}$
$\left\|(v_{bs} \pm a_{bs})^2\right\|$	$< 5.5 \cdot 10^{-7}$	$\left\|(v_{bs})^2 - (a_{bs})^2\right\|$	$< 1.4 \cdot 10^{-7}$

[Bona et al. (*2007*), JHEP 0803, 049]

[Bertone et al. (*2012*), JHEP 1303, 089]

[Carrasco et al. (*2013*), JHEP 1403, 016]

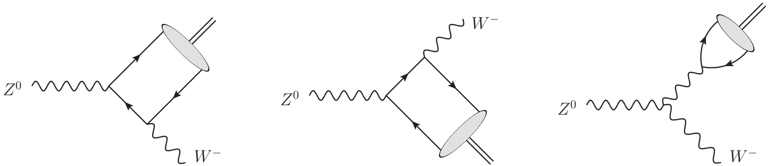
These bounds push our branching ratios down to 10^{-14}, rendering them unobservable.

Decays of electroweak gauge bosons

Weak radiative Z decays to M + W

The $Z \to M + W$ decay

The contributing diagrams in this case look similar to the $W \to M\gamma$ decays:



Form factor decomposition:

$$i\mathcal{A}(Z \to M^+ W^-) = \pm \frac{g^2 f_M}{4\sqrt{2} c_W} V_{ij} \left(1 - \frac{m_W^2}{m_Z^2}\right) \times \left(i\epsilon_{\mu\nu\alpha\beta} \frac{k^\mu q^\nu \varepsilon_Z^\alpha \varepsilon_W^{*\beta}}{k \cdot q} F_1^M - \varepsilon_Z \cdot \varepsilon_W^* F_2^M + \frac{q \cdot \varepsilon_Z \, k \cdot \varepsilon_W^*}{k \cdot q} F_3^M \right)$$

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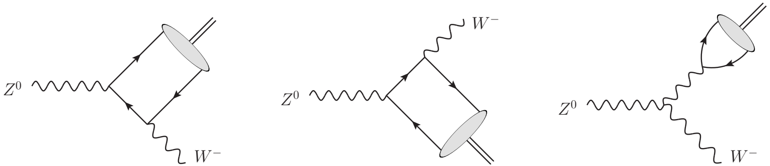

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now allowed because W can be longitudinally polarized

The $Z \to M + W$ decay

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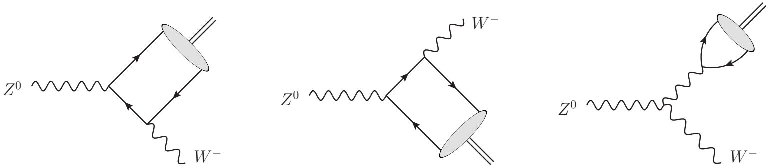

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Allows the QCD factorization approach to be tested at lower scale $(m_Z - m_W) \approx 10\,\mathrm{GeV}$!

Results for $Z \to M + W$

For the decay rates, we find:

$$\Gamma(Z \to M^+ W^-) = \frac{\pi \alpha^2(m_Z) f_M^2}{48 m_Z} \, |V_{ij}|^2 \, \frac{s_W^2}{c_W^2} \left(\frac{3}{2} + \frac{3}{2} s_W^2 + \frac{227}{180} s_W^4 + 0.003 a_1^M + \dots \right)$$

Our predictions for the branching ratios are:

Decay mode	Branching ratio
$Z^0 \to \pi^\pm W^\mp$	$(1.51 \pm 0.005_f) \cdot 10^{-10}$
$Z^0 \to \rho^\pm W^\mp$	$(4.00 \pm 0.15_f) \cdot 10^{-10}$
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$Z^0 \to K^{*\pm} W^\mp$	$(1.96 \pm 0.12_f) \cdot 10^{-11}$
$Z^0 \to D_s W^\mp$	$(6.04 \pm 0.20_{\text{CKM}} \pm 0.22_f) \cdot 10^{-10}$
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very small sensitivity to LCDA

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The $\mathcal{O}(\alpha_s)$ corrections to this are an interesting project left for future work, in particular the scale dependence of the result.

Conclusions, summary and outlook

Conclusions, summary and outlook

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To summarize:

Decay mode	Branching ratio	Decay mode	Branching ratio
$Z^0 \to \pi^0\gamma$	$(9.80 \pm 1.03) \cdot 10^{-12}$	$W^\pm \to \pi^\pm\gamma$	$(4.00 \pm 0.83) \cdot 10^{-9}$
$Z^0 \to \rho^0\gamma$	$(4.19 \pm 0.47) \cdot 10^{-9}$	$W^\pm \to \rho^\pm\gamma$	$(8.74 \pm 1.91) \cdot 10^{-9}$
$Z^0 \to \omega\gamma$	$(2.89 \pm 0.41) \cdot 10^{-8}$	$W^\pm \to K^\pm\gamma$	$(3.25 \pm 0.69) \cdot 10^{-10}$
$Z^0 \to \phi\gamma$	$(8.63 \pm 1.01) \cdot 10^{-9}$	$W^\pm \to K^{*\pm}\gamma$	$(4.78 \pm 1.15) \cdot 10^{-10}$
$Z^0 \to J/\psi\,\gamma$	$(8.02 \pm 0.45) \cdot 10^{-8}$	$W^\pm \to D_s\gamma$	$(3.66\,^{+1.49}_{-0.85}) \cdot 10^{-8}$
$Z^0 \to \Upsilon(1S)\,\gamma$	$(5.39 \pm 0.16) \cdot 10^{-8}$	$W^\pm \to D^\pm\gamma$	$(1.38\,^{+0.51}_{-0.33}) \cdot 10^{-9}$
$Z^0 \to \Upsilon(4S)\,\gamma$	$(1.22 \pm 0.13) \cdot 10^{-8}$	$W^\pm \to B^\pm\gamma$	$(1.55\,^{+0.79}_{-0.60}) \cdot 10^{-12}$

- For $Z \to V\gamma \to \mu^+\mu^-\gamma$, one can trigger on muons and the photon

Conclusions, summary and outlook

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- For $Z \to V\gamma \to \mu^+\mu^-\gamma$, one can trigger on muons and the photon
- We expect $\mathcal{O}(100)$ $J/\psi\,\gamma$ events at the LHC

Conclusions, summary and outlook

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To summarize:

Decay mode	Branching ratio	Decay mode	Branching ratio
$Z^0 \to \pi^0\gamma$	$(9.80 \pm 1.03) \cdot 10^{-12}$	$W^\pm \to \pi^\pm\gamma$	$(4.00 \pm 0.83) \cdot 10^{-9}$
$Z^0 \to \rho^0\gamma$	$(4.19 \pm 0.47) \cdot 10^{-9}$	$W^\pm \to \rho^\pm\gamma$	$(8.74 \pm 1.91) \cdot 10^{-9}$
$Z^0 \to \omega\gamma$	$(2.89 \pm 0.41) \cdot 10^{-8}$	$W^\pm \to K^\pm\gamma$	$(3.25 \pm 0.69) \cdot 10^{-10}$
$Z^0 \to \phi\gamma$	$(8.63 \pm 1.01) \cdot 10^{-9}$	$W^\pm \to K^{*\pm}\gamma$	$(4.78 \pm 1.15) \cdot 10^{-10}$
$Z^0 \to J/\psi\,\gamma$	$(8.02 \pm 0.45) \cdot 10^{-8}$	$W^\pm \to D_s\gamma$	$(3.66\,^{+\,1.49}_{-\,0.85}) \cdot 10^{-8}$
$Z^0 \to \Upsilon(1S)\,\gamma$	$(5.39 \pm 0.16) \cdot 10^{-8}$	$W^\pm \to D^\pm\gamma$	$(1.38\,^{+\,0.51}_{-\,0.33}) \cdot 10^{-9}$
$Z^0 \to \Upsilon(4S)\,\gamma$	$(1.22 \pm 0.13) \cdot 10^{-8}$	$W^\pm \to B^\pm\gamma$	$(1.55\,^{+\,0.79}_{-\,0.60}) \cdot 10^{-12}$

- For $Z \to V\gamma \to \mu^+\mu^-\gamma$, one can trigger on muons and the photon
- We expect $\mathcal{O}(100)$ $J/\psi\,\gamma$ events at the LHC
- Ideas for reconstructing (ρ, ω and ϕ) $+\,\gamma$ exist

[Kagan et al. (*2014*), arXiv:1406.1722]

Conclusions, summary and outlook

JG|U

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- Reconstructing W decays at the LHC is more challenging

[Mangano, Melia (*2014*), arXiv:1410.7475]

Conclusions and outlook

JG|U

A few things that I did not talk about today, but are featured in the paper:

- In some older papers, the authors speculated about a "possible huge enhancement" of the decays $W, Z \to P\gamma$ coming from an unsuppressed contribution from the axial anomaly.

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- We have derived decay constants for several mesons from updated experimental data, decreasing the uncertainty of our predictions.

Conclusions and outlook

We have derived predictions for the decay rates of exclusive radiative decays $V \to M + \gamma$ in the framework of QCD factorization. The branching ratios are small, between $\mathcal{O}(10^{-12})$ to $\mathcal{O}(10^{-9})$.

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Future work: Lots! The approach can be (and has been) applied to Higgs boson decays. A careful NLO analysis of these decays is work in progress. Also possible: decays with multiple mesons (i.e. $W, Z, h \to M_1 M_2$)

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Thank you for your attention!

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