Effective Field Theory of Forward Scattering and Factorization Violation

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Total Scattering Cross Sections in QFT

Consider the total scattering cross section for neutral particles

$$
\sigma(s) = \int \frac{d\sigma}{dt} dt
$$

some measure of traverse momentum transfer at intermediate stage of calculation

t

This integral is dominated by the region $t \ll s$

Power Counting Exercise

Break up integral into regions with distinct power counting parameters

1:
$$
s \sim t \gg \Lambda
$$
 $\lambda \equiv \Lambda/(t,s)$ SCET-I,II

2:
$$
t \ll s
$$
 $\lambda \equiv t/s$ SCETH-like

Consider region 1: There must exist some underlying hard event which must be integrated out generating some higher dimensional external operator

$$
S \sim C(s,t) \int d^4x \bar{\xi}_n \xi_{\bar{n}} \bar{\xi}_{n_1} \xi_{n_2} \sim \lambda^4
$$

Region 2: No underlying hard interaction, at the scale *t* generate the interaction **SCET FORMALISM IS A TREATMENT OF A LIGHTER AND IN A LIGHTER AND IN A LIGHTER ACTION I**

 $\delta(x) \sim \delta(x_+) \delta(x_-) \log(x_+^2)$ " $V'' \sim \delta(x_+) \delta(x_-) \log(x_+^2)$

is dressed by soft aluons analogy with NRQCD, Coulomb kernel is dressed by soft gluons $\binom{2}{\perp}$ Strong analogy with NRQCD, Coulomb kernel is dressed by soft gluons

> with large (+)-momentum. The number of such gluons must be vanishingly small. The first non-trivial population of such gluons μ gluons emanating from a nucleon moving with a large (+)-momentum, are those which scale as ^k [∼] [λ², ^λ², ^λ], which

q q AP y AP 1 23 y3 2 y1 y' y' y' y p'0 p0 q'1 q'2 q'3 q3 q2 q1 e + nucleus *e* + Jet(*k*⇥) + *X* FIG. 5: An order n diagram which contributes solely to transverse broadening. \mathcal{L} derived for Glauber gluons in section 2, the leading component of new \mathcal{L} If there were no hard interaction then the non-**|** Note that while this operator is at the $\frac{1}{2}$ is a phase in hard in $\frac{1}{2}$ is a phase in hard in hard in $\frac{1}{2}$ i itali **p** seed of doubt on factorization proofs. ו ויטופי
י י heart of region 2, it also exists at leading order in region 1, where it plants the

m=0

Note: no hard interactions to all orders in perturbation theory. The scale $s = (p_1 + p_2)^2$ plays no dynamical role. Consider the dressing of the Glauber kernel by soft gluons $M \sim$ *f*(*s/t, t/m*²) *t* The scale *s* can only show up in logs: RG can not hope to capture the logs, need RRG $\log(p_{+}/\nu) + \log(p_{-}/\nu) + \log(t/\nu^{2}) = \log(s/t)$ $F(s/t) \sim (s/t)^{\gamma}$ Regge behaviour k_+ ⁺ *Q* λQ \bar{n} -coll. soft

 $k⁺$

n-coll.

 $\lambda^2 Q$

 λQ

Matching onto the action with the section stattering process, so we take \mathcal{L} -polarization for the external gluon fields. Expanding in \mathcal{L}

Matching Soft-collinear Operators 22 *T ^Buⁿ* ~*q* 2 \mathcal{L} ih8⇡↵*sv*¯*^s* 2 θ forward scattering of soft with *n*-collinear particles we write operators with *n*-collinear and soft

FIG. 8. Tree level matching for the *nnss* Glauber operators. In a) we show the four full QCD graphs with *t*-changed singularity are matching results and the matching reading α M otobing is identical to the collinear pollinear forward scattering of soft with *n*-collinear particles we write operators with *n*-collinear and soft $\overline{\ }$ Matching is identical to the collinear-collinear case C_A

$$
O^{qq}_{ns}=\mathcal{O}^{qB}_{n}\frac{1}{\mathcal{P}^{2}_{\perp}}\mathcal{O}^{q_{n}B}_{s}\,,\ \ O^{qg}_{ns}=\mathcal{O}^{qB}_{n}\frac{1}{\mathcal{P}^{2}_{\perp}}\mathcal{O}^{g_{n}B}_{s}\,,\ \ O^{gq}_{ns}=\mathcal{O}^{gB}_{n}\frac{1}{\mathcal{P}^{2}_{\perp}}\mathcal{O}^{q_{n}B}_{s}\,,\ \ O^{gg}_{ns}=\mathcal{O}^{gB}_{n}\frac{1}{\mathcal{P}^{2}_{\perp}}\mathcal{O}^{g_{n}B}_{s}
$$

$$
\begin{split} \mathcal{O}^{q_nB}_s &= 8\pi\alpha_s \left(\bar{\psi}^n_S\, T^B \frac{\rlap{\hspace{0.1cm}/}{n}}{2} \psi^n_S \right), \\ \mathcal{O}^{q_nB}_s &= 8\pi\alpha_s \left(\frac{i}{2} f^{BCD} \mathcal{B}^{nC}_{S\perp\mu}\, n \cdot (\mathcal{P} \!+\! \mathcal{P}^\dagger) \mathcal{B}^{nD\mu}_{S\perp} \right). \end{split}
$$

^S? can be found in Eqs. (12) and (16). Once again with our conventions Final Glauber action

The form of the collinear operators are fixed but the soft can have a much more general form coecients *C*1*,...,*¹⁰ in Eq. (87). S of S and S of S

$$
\mathcal{O}_{s}^{AB}=8\pi\alpha_{s}\sum_{i}C_{i}O_{i}^{AB}
$$

 $O_1 = \mathcal{P}_{\perp}^{\mu} \mathcal{S}_n^T \mathcal{S}_{\bar{n}} \mathcal{P}_{\perp \mu}$ $O_2 = \mathcal{P}_{\perp}^{\mu} \mathcal{S}_{\bar{n}}^T \mathcal{S}_n \mathcal{P}_{\perp \mu}$ $O_3 = \mathcal{P}_\perp \cdot (g\widetilde{\mathcal{B}}_{S\perp}^n)(\mathcal{S}_n^T\mathcal{S}_{\bar{n}}) + (\mathcal{S}_n^T\mathcal{S}_{\bar{n}})(g\widetilde{\mathcal{B}}_{S\perp}^{\bar{n}}) \cdot \mathcal{P}_\perp$ $O_4 = \mathcal{P}_\perp \cdot (g\widetilde{\mathcal{B}}_{S\perp}^{\bar{n}})(\mathcal{S}_{\bar{n}}^T\mathcal{S}_n) + (\mathcal{S}_{\bar{n}}^T\mathcal{S}_n)(g\widetilde{\mathcal{B}}_{S\perp}^n) \cdot \mathcal{P}_\perp$ $O_5 = \mathcal{P}^{\perp}_{\mu}(\mathcal{S}^T_n \mathcal{S}_{\bar{n}})(g\widetilde{\mathcal{B}}^{ \bar{n} \mu}_{S \perp}) + (g\widetilde{\mathcal{B}}^{n \mu}_{S \perp})(\mathcal{S}^T_n \mathcal{S}_{\bar{n}})\mathcal{P}^{\perp}_{\mu}$ $O_6 = \mathcal{P}^{\perp}_{\mu}(\mathcal{S}_{\bar{n}}^T \mathcal{S}_n)(g\widetilde{\mathcal{B}}_{S\perp}^{n\mu}) + (g\widetilde{\mathcal{B}}_{S\perp}^{\bar{n}\mu})(\mathcal{S}_{\bar{n}}^T \mathcal{S}_n)\mathcal{P}^{\perp}_{\mu}$ $O_7 = (g\mathcal{B}_{S\perp}^{n\mu})S_n^T S_{\bar{n}}(g\mathcal{B}_{S\perp\mu}^{\bar{n}})$ $O_8 = (g\mathcal{B}_{S\perp}^{\bar{n}\mu})S_{\bar{n}}^T S_n (g\mathcal{B}_{S\perp\mu}^n)$ $O_9 = S_n^T n_\mu \bar{n}_\nu (ig \widetilde{G}_s^{\mu\nu}) S_{\bar{n}}$ $O_{10} = S_{\bar{n}}^T n_{\mu} \bar{n}_{\nu} (ig \widetilde{G}^{\mu\nu}_s) S_n$

Need to match up to 2 gluons to fix all of the coefficients F. Matching with up to Two Soft Gluons (NEW)

Not at all obvious that one collinear emission can be matched given that there are non-local TOP's which contribute in the EFT

37

 n -collituon. b) entiminated after uning for the four quark operator with one *n*-collinear with one *n*-c non-locality only eliminated after using the on-shell condition $k \cdot A = 0$

milor motobing worke for aluon anaratore \mathbf{F} 12, is the attached to one of the gluons of the extra \mathbf{F} similar matching works for gluon operators

Matching Soft Operator

 \blacksquare Full theory graphs. B) EFT Lipatov Operator graphs. B) EFT Lipatov Operator graph with one soft gluon, shown by two equivalent gluon, shown by two equivalent gluon, shown by two equivalent gluon, shown by two equiv Matching all polarizations w/o using on shell (simplifies 2 gluon matching) conditions at 1-gluon

First row is reproduced by TOP's in EFT

$$
\frac{C_2 = C_4 = C_5 = C_6 = C_8 = C_{10} = 0},
$$

$$
C_1 = -C_3 = -C_7 = +1, \qquad C_9 = -\frac{1}{2}.
$$

$$
\mathcal{O}_{s}^{BC} = 8\pi\alpha_{s} \left\{ \mathcal{P}_{\perp}^{\mu} \mathcal{S}_{n}^{T} \mathcal{S}_{\bar{n}} \mathcal{P}_{\perp\mu} - \mathcal{P}_{\mu}^{\perp} g \widetilde{\mathcal{B}}_{S\perp}^{n\mu} \mathcal{S}_{n}^{T} \mathcal{S}_{\bar{n}} - \mathcal{S}_{n}^{T} \mathcal{S}_{\bar{n}} g \widetilde{\mathcal{B}}_{S\perp}^{\bar{n}\mu} \mathcal{P}_{\mu}^{\perp} - g \widetilde{\mathcal{B}}_{S\perp}^{n\mu} \mathcal{S}_{n}^{T} \mathcal{S}_{\bar{n}} g \widetilde{\mathcal{B}}_{S\perp\mu}^{\bar{n}} - \frac{n_{\mu} \bar{n}_{\nu}}{2} \mathcal{S}_{n}^{T} ig \widetilde{G}_{s}^{\mu\nu} \mathcal{S}_{\bar{n}} \right\}^{BC}.
$$

At one gluon level this operator reproduces the Lipatov vertex and generalizes it to arbitrary number of gluons. hard corrections to the theory. The form is uniquely fixed to all loops as there are no IV. ONE LOOP MATCHING CALCULATION CALCULATIONS AND A CONTINUES.

Matching at one loop the section. We will see the internal section. We detail a ratio of μ to yield well defined results for the various possible contractions of two operators which induce a

Glauber Loop Exegesis: Two insertions of $\mathcal{O}_{ns\bar{n}}$ by considering the iteration of two *Oqq* ${\cal O}_{n s \bar n}$

$$
I_{\rm Gbox} = \int \frac{d^{d-2}k_{\perp} \, dk^{+} \, dk^{-}}{2(\vec{k}_{\perp}^{2})(\vec{k}_{\perp} + \vec{q}_{\perp})^{2} \Big(k^{+} + p_{3}^{+} - (\vec{k}_{\perp} + \vec{q}_{\perp}/2)^{2}/p_{2}^{-} + i0\Big) \Big(-k^{-} + p_{4}^{-} - (\vec{k}_{\perp} + \vec{q}_{\perp}/2)^{2}/p_{1}^{+} + i0\Big)},
$$

$$
I_{\rm Gcbox} = \int \frac{d^{d-2}k_{\perp} \, dk^{+} \, dk^{-}}{2(\vec{k}_{\perp}^{2})(\vec{k}_{\perp} + \vec{q}_{\perp})^{2} \Big(k^{+} + p_{3}^{+} - (\vec{k}_{\perp} + \vec{q}_{\perp}/2)^{2}/p_{2}^{-} + i0\Big) \Big(+k^{-} + p_{1}^{-} - (\vec{k}_{\perp} + \vec{q}_{\perp}/2)^{2}/p_{1}^{+} + i0\Big)}.
$$

 T is illustrated the presence of additional singularities that occur in the presence of Glaubert in the pre region is wen domisen in the *r* to stand mine the sent of the two mesgrate is we ϵ is to calculate the C_A please we heed the individual diagrams to be well to However to calculate the $_{\,CA}\;$ piece we need the individual diagrams to be well defined Neither integral is well defined. In the Abelian limit the sum of the two integrals is well defined.

duce a rapidity requ sd_{1100} aromidit u_{100} regulator is Introduce a rapidity regulator $|2q_3/\nu|^{-\eta}$

$$
\mid 2q_3/\nu\mid^{-\eta}
$$

$$
I_{\rm Gebox}=\int \! \frac{d^{d-2}k_\perp \ d k^0 \ d k^z \ |k^2|^{-2\eta} \ (\nu/2)^{2\eta}}{(\vec{k}_\perp^2)(\vec{k}_\perp+\vec{q}_\perp)^2 \Big(k^0-k^z+p_3^+-(\vec{k}_\perp+\frac{\vec{q}_\perp}{2})^2/p_2^-+i0\Big) \Big(k^0+k^z+p_1^--(\vec{k}_\perp+\frac{\vec{q}_\perp}{2})^2/p_1^+ +i0\Big)}
$$

$$
I_{\rm Gbox} = \int \frac{d^{d-2}k_{\perp} \ d k^{0} \ d k^{z} \ |k^{z}|^{-2\eta} \ (\nu/2)^{2\eta}}{(\vec{k}_{\perp}^{2})(\vec{k}_{\perp} + \vec{q}_{\perp})^{2} \left(k^{0} - k^{z} + p_{3}^{+} - (\vec{k}_{\perp} + \frac{\vec{q}_{\perp}}{2})^{2}/p_{2}^{-} + i0\right) \left(-k^{0} - k^{z} + p_{4}^{-} - (\vec{k}_{\perp} + \frac{\vec{q}_{\perp}}{2})^{2}/p_{1}^{+} + i0\right)}
$$

$$
= \left(\frac{-i}{4\pi}\right) \int \frac{d^{d-2}k_{\perp}}{(\vec{k}_{\perp}^2)(\vec{k}_{\perp} + \vec{q}_{\perp})^2} \left[-i\pi + \mathcal{O}(\eta)\right].
$$

Glauber Phase 1 1 *.* (48) First term in build up of

 α thore μ *k* μ *k* α *a* Non-Abelian contribution to Yes coming from soft loops **i**s there a Non-Abelian contribution lo there a Nan Abolian contribution to the phase? Is there a Non-Abelian contribution to the phase?

= ⇣*i* ،
ر ⌘ Z *d d*2*k*? e llan ph ϵ e is universal as it does not care about the the spin of the collinear lines. **1 rie Abellan priase is universal as il does not care** τ_{left} Λ_{left} the space the spacetime constraints so that the spacetime constraints so that the box diagram constraints so that the box diagram constraints so that the box diagram constraints so that the box diagr alone is alone is like the two Glauber potentials the two Glauber potentials over all times, which the cross over a The Abelian phase is universal as it does not care

We can also consider soft-collinear forward scattering

FIG. 11. One loop iterations of the Glauber potential for *n*–soft forward scattering of a *qq*¯ pair. carry $(\lambda^2, \lambda, \lambda^2)$ Just the boost of the previous case, yields the same result but now the the glaubers

with crossed α in the same manner as the same manner as the same manner as the same manner as the crossed box above, α and the same guidaller alone $\alpha + \beta + \beta$ is the noninsure homogenous scaling in the non-loop exact. In the non- $| k_{+} - \beta k_{-} |^{n}$ Can tweak regulator to $\qquad k_{+} - \beta k_{-}$ |ⁿ

Gives same result as and the full set of such diagrams will be computed in Sec. IV A and App. E. collinear-collinear

Other source of rapidity divergences are the Wilson lines which need to be regulated n-collinear gluons respectively and the second rapidity of th with a coupled on tapienty divergence on a and the

$$
S_n = \sum_{\text{perms}} \exp\left\{\frac{-g}{n \cdot \mathcal{P}} \left[\frac{w|2\mathcal{P}^z|^{-\eta/2}}{\nu^{-\eta/2}} n \cdot A_s\right] \right\}, \quad S_{\bar{n}} = \sum_{\text{perms}} \exp\left\{\frac{-g}{\bar{n} \cdot \mathcal{P}} \left[\frac{w|2\mathcal{P}^z|^{-\eta/2}}{\nu^{-\eta/2}} \bar{n} \cdot A_s\right] \right\},
$$

$$
W_n = \sum_{\text{perms}} \exp\left\{\frac{-g}{\bar{n} \cdot \mathcal{P}} \left[\frac{w^2|\bar{n} \cdot \mathcal{P}|^{-\eta}}{\nu^{-\eta}} \bar{n} \cdot A_n\right] \right\}, \quad W_{\bar{n}} = \sum_{\text{perms}} \exp\left\{\frac{-g}{n \cdot \mathcal{P}} \left[\frac{w^2|n \cdot \mathcal{P}|^{-\eta}}{\nu^{-\eta}} n \cdot A_{\bar{n}}\right] \right\}
$$
(

Note we regulate every gluon to be consistent with the Glaubers (important for zero bin cancellation)

Zero Bin Subtractions

Soft, Collinear and Glauber all overlap lapping modes are Glauber, soft, and collinear. Therefore the structure of these subtractions for one-loop soft and collinear graphs is Soft, Collinear and Glauber, all overlan

$$
S = S - S(G) \t Cn = Cn - Cn(S) - Cn(G) + Cn(GS)
$$

$$
S^{(G)}=G
$$

Glauber in hard matching

Consider first the full QCD graphs shown in Fig. 18 which we number from a) to j). These bin diagrams vanish but crucially not all as in additional box-type graphs obtained by rotating Fig. 18a, but neither of the set of these graphs of these graphs of these graphs of the set of these graphs graphs are computed the computed then the EFT limit with the EFT limit with the EFT limit with $\frac{1}{2}$ additional protects obtained by an and the set of the s
The disconsister but or the fill not all as in this example. Using the rapidity regulator many of the zerobin diagrams vanish but crucially not all as in

Soft Total

\n
$$
= \frac{i\alpha_s^2}{t} S_3^{n\bar{n}} \left\{ -\frac{4}{\eta} h(\epsilon, \mu^2/m^2) - \frac{4}{\eta} g(\epsilon, \mu^2/t) - 4\ln\left(\frac{\mu^2}{4\nu^2}\right) \ln\left(\frac{m^2}{-t}\right) - 2\ln^2\left(\frac{\mu^2}{m^2}\right) + 2\ln^2\left(\frac{\mu^2}{-t}\right) - 2\frac{\pi^2}{3} + \frac{22}{3}\ln\frac{\mu^2}{-t} + \frac{134}{9} \right\} + \frac{i\alpha_s^2}{t} S_4^{n\bar{n}} \left[-\frac{8}{3}\ln\left(\frac{\mu^2}{-t}\right) - \frac{40}{9} \right].
$$
\n**Notice:** no UV poles

\nNote: no UV poles

\n**Note:** no UV poles

\n**EXECUTE:**

\n**Note:** no UV poles

$$
\text{Collinear} \quad = \frac{i\alpha_s^2}{t} \mathcal{S}_3^{n\bar{n}} \left\{ \frac{4}{\eta} h(\epsilon, \mu^2/m^2) + \frac{4}{\eta} g(\epsilon, \mu^2/t) + 4\ln\left(\frac{4\nu^2}{s}\right) \ln\left(\frac{-t}{m^2}\right) + 2\ln^2\left(\frac{m^2}{-t}\right) + 4 + \frac{4\pi^2}{3} \right\} + \frac{i\alpha_s^2}{t} \mathcal{S}_2^{n\bar{n}} \left[-4\ln^2\left(\frac{m^2}{-t}\right) - 12\ln\left(\frac{m^2}{-t}\right) - 14 \right].
$$
\n(112)

\ncombining collinear sectors

$$
\ln s/\nu^2 = \ln(n \cdot p/\nu) + \ln(\bar{n} \cdot p/\nu)
$$

\nTotal EFT
$$
\therefore = \frac{i\alpha_s^2}{t} S_1^{n\bar{n}} \left[8i\pi \ln\left(\frac{-t}{m^2}\right) \right] + \frac{i\alpha_s^2}{t} S_2^{n\bar{n}} \left[-4\ln^2\left(\frac{m^2}{-t}\right) - 12\ln\left(\frac{m^2}{-t}\right) - 14 \right]
$$

$$
+ \frac{i\alpha_s^2}{t} S_3^{n\bar{n}} \left\{ -4\ln\left(\frac{s}{-t}\right) \ln\left(\frac{-t}{m^2}\right) + \frac{22}{3} \ln\frac{\mu^2}{-t} + \frac{170}{9} + \frac{2\pi^2}{3} \right\} = \text{full, no hard matching!}
$$

$$
+ \frac{i\alpha_s^2}{t} S_4^{n\bar{n}} \left[-\frac{8}{3} \ln\left(\frac{\mu^2}{-t}\right) - \frac{40}{9} \right]. \qquad \text{running of Glauber}_{(113)}
$$
potential

Even though the rapidity divergences cancel we must renormalize the factorized operators distinctly *A* for the bare and renormalized operators. The same decomposition applies for the ¯*n*-collinear sector with *n* ! *n*¯ for all terms, which we write out just to be definite *ⁿ*¯ (⌫*, µ*)*, ^V*ˆ*On*¯ ⁼ 0 1 0 1 \overline{a} *n a i stinctly* agn the rapidity divergences cancel we \overline{C} 1 + *V qq ^sⁿ V qg* $\frac{1}{2}$

Collinear mixing: which we write out in the definite out of all terms of the definite out of the defin

$$
\vec{O}_{\bar{n}}^{Bbare} = \hat{V}_{\mathcal{O}_{\bar{n}}} \cdot \vec{O}_{\bar{n}}^{B}(\nu, \mu) , \qquad \hat{V}_{\mathcal{O}_{\bar{n}}} = \begin{pmatrix} 1 + \delta V_{\bar{n}}^{qq} & \delta V_{\bar{n}}^{qg} \\ \delta V_{\bar{n}}^{qq} & 1 + \delta V_{\bar{n}}^{gg} \end{pmatrix} , \qquad \vec{O}_{\bar{n}}^{B} = \begin{pmatrix} \mathcal{O}_{\bar{n}}^{qB} \\ \mathcal{O}_{\bar{n}}^{gB} \end{pmatrix} .
$$

Soft single index ops: Soft single index ops:

$$
\vec{\mathcal{O}}_{s_n}^{A\text{bare}} = \hat{V}_{\mathcal{O}_{s_n}} \cdot \vec{\mathcal{O}}_{s_n}^{A}(\nu,\mu) \, , \qquad \hat{V}_{\mathcal{O}_{s_n}} = \begin{pmatrix} 1 + \delta V_{s_n}^{qq} & \delta V_{s_n}^{qg} \\ & \\ \delta V_{s_n}^{gq} & 1 + \delta V_{s_n}^{gg} \end{pmatrix} \, , \qquad \vec{\mathcal{O}}_{s_n}^{A} = \begin{pmatrix} \mathcal{O}_{s}^{q_n A} \\ \mathcal{O}_{s}^{g_n A} \end{pmatrix} \, ,
$$

 \mathbf{S} of do \mathbf{S} in do \mathbf{S} and \mathbf{S} \mathbf{S} AB \mathbf{S} \mathbf{S} AB \mathbf{S} Soft double index ops: $\vec{\mathcal{O}}_s^{ABbare} = \hat{V}_{\mathcal{O}_s} \cdot \vec{\mathcal{O}}_s^{AB}(\nu,\mu)$,

$$
\hat{V}_{\mathcal{O}_s} = \begin{pmatrix}\n1 + \delta V_s & 0 & 0 & 0 & 0 \\
\delta V_s^{Tqq} & & & & \\
\delta V_s^{Tgq} & & & & \\
\delta V_s^{Tgg} & & & & \\
\end{pmatrix}, \qquad \vec{O}_s^{AB} = \begin{pmatrix}\n\mathcal{O}_s^{AB} \\
i \int d^4 x \, T \, \mathcal{O}_s^{q_n A}(x) \mathcal{O}_s^{q_n B}(0) \\
i \int d^4 x \, T \, \mathcal{O}_s^{q_n A}(x) \mathcal{O}_s^{q_n B}(0) \\
i \int d^4 x \, T \, \mathcal{O}_s^{q_n A}(x) \mathcal{O}_s^{q_n B}(0)\n\end{pmatrix}
$$

^s the index *A* couples to an

.

Re-sum the logs in parton-parton scattering, simplest to run collinear sectors (no TOPs) collinear loop diagrams for this operator. The fact that there is no nontrivial Wilson coecient between the quark and gluon operators in either the *n*-collinear or soft sectors also implies that d^{*i*} *ij*=*q,g* \log *n* barton-responding *P*2 ? (*Oqn^A ^s* ⁺ *^Ogn^A* between the quark and gluon operators in either the *n*-collinear or soft sectors also implies that This equation indicates that the one-loop rapidity divergences cancel between the soft and *n*- \mathcal{L} must not mix into a different combination combination \mathcal{L}

Structure is Fixed *d*

 ν *d* $\frac{d}{d\nu}(\mathcal{O}_n^{qA} + \mathcal{O}_n^{gA}) = \gamma_{n\nu}(\mathcal{O}_n^{qA} + \mathcal{O}_n^{gA})\,, \qquad \nu$ *d* $\frac{d}{d\nu}(\mathcal{O}^{q_{n}A}_{s}+\mathcal{O}^{g_{n}A}_{s})=\gamma_{s\nu}(\mathcal{O}^{q_{n}A}_{s}+\mathcal{O}^{g_{n}A}_{s})\,.$ d^{*do a company of the comp}* $U_n^{q+1} + U_n^{q+1} = \gamma_{n\nu} (U_n^{q+1} + U_n^{q+1}), \qquad \nu_{\overline{d_{1\nu}}} (U_s^{q^{n+1}} + U_s^{q^{n+1}}) = \gamma_{s\nu} (U_s^{q^{n+1}} + U_s^{q^{n+1}})$

 $\overline{\Omega}$

mixing leads to universality

 $\gamma_{n\nu} \equiv \gamma_{n\nu}^{qq} + \gamma_{n\nu}^{gg} = \gamma_{n\nu}^{gg} + \gamma_{n\nu}^{gg}$, $\frac{qg}{n\nu}$,

n So all we need is γ_n^{ν} *is*

 \sim \sim \sim \sim Which we can extract I we need is λ/n from matching calculation The result in Eq. (146) constrains the sum of the sum of the sum of the columns of $\frac{1}{\sqrt{2}}$ **O**ⁿ to be equal. The fact that $\frac{1}{2}$

$$
M(\nu = \sqrt{s}) \sim (s/t)^{\gamma} \qquad \gamma = \frac{\alpha_s C_A}{2\pi} Log(-t/m^2)
$$

Gluon Reggeization

Sectors for **O**_{ij}a Due to the connection between the rapidity cuto↵s in the neighbouring soft and *n*-collinear

Summing Logs in IR safe quantities In this section we consider how to include the Glauber Lagrangian into a factorized analysis ourning Logo in in out quantities

Consider observables for which the soft radiation is not measured. For this class the RRG equation will reduce to the factorization in the time expanding the time equation BKFL equation Of course we can not count perturbative Glauber exchange to give the correct result when *t <* ⇤, however we should be able to capture all of the large logs at any given order by running in measured. For this class the RRG equation will reduce to the

Consider the total cross section for hadron scattering try to capture s dependence Schematical and total pross section for nadion-seationing try to
Capture s dependence $\frac{1}{2}$ \mathcal{X} al cross section for hadron scatt
conture a dependence *k*! *k*0 ! *d*ependence *ⁿ*)(*xi*)

$$
\sigma_{pp} = \int \frac{d\sigma}{dt} dt \sim \int dt \sum_{X_s, X_n, X_{\bar{n}}} \langle p_n \bar{p}_n | X_s, X_n, X_{\bar{n}} \rangle O(t(\mathcal{P}_{\perp}^n, \mathcal{P}_{\perp}^{\bar{n}})) \langle X_s, X_n, X_{\bar{n}} | p_n \bar{p}_n \rangle,
$$

insert \mathcal{L}_G^H

<u>I</u> α *<i>i* α *i* α *i ⁿ*? *|* (177) Running the soft function of Glauber operator *^s*(1*,*1)(*x*1*, x*10) = 4(*x*¹ *^x*10)*OAB ^Oqn^A ^s* (*x*1) + *^Ogn^A* ⇤⇥*Oqn*¯*^A ^s* (*x*10) + *^Ogn*¯*^A ^s* (*x*1) + ⇥ *^s* (*x*1)

$$
O^{AB}_{s(1,1)}(q_\perp,q'_\perp)=\mathcal{O}^{AB}_s(q_\perp,q'_\perp)+\int\!\! d^4x\;T\left[\mathcal{O}^{q_nA}_s(q_\perp)+\mathcal{O}^{g_nA}_s(q_\perp)\right]\left[\mathcal{O}^{q_{\bar n}A}_s(q'_\perp)+\mathcal{O}^{g_{\bar n}A}_s(q'_\perp)\right].
$$

$$
G^{ABA'B'}(q_\perp,q'_\perp)=\sum_X \big\langle 0 \big|{\cal O}^{AB}_{s(1,1)}(q_\perp,q'_\perp)\big|X\big\rangle \big\langle X \big|{\cal O}^{A'B'}_{s(1,1)}(q_\perp,q'_\perp)\big|0\big\rangle
$$

S *q'* S *q'* $\tilde{G}(l_\perp,l'_\perp)\nleftrightarrow l^2_\perp G\left(l_\perp,l'_\perp\right)l'_\perp$ \pm $2₂$ Here used: U oro upodu $\tilde{\gamma}$ ulting $\tilde{\gamma}$ q' the space. That is, the factor of q'' is the factor sits between collinear si

$$
\tilde{G}^{ren}(\vec{l}_{\perp}, \vec{l}'_{\perp}) = \int d^2q_{\perp} Z(l_{\perp}, q_{\perp}) \tilde{G}^{bare}(q_{\perp}, l'_{\perp})
$$

$$
\nu \frac{d}{d\nu} \tilde{G}^{ren}(q_{\perp}, l'_{\perp}) = \int d^2q'_{\perp} \gamma(q_{\perp}, q'_{\perp}) \tilde{G}^{ren}(q'_{\perp}, l'_{\perp})
$$

$$
\cdots \qquad \cdots \qquad \cdots
$$

$$
\gamma(q_{\perp}, q'_{\perp}) = \int d^2 k_{\perp} Z(q_{\perp}, k_{\perp}) \nu \frac{d}{d\nu} Z^{-1}(k_{\perp}, l'_{\perp}) = \frac{\alpha_s C_A}{\pi^2} \left(\frac{1}{(q_{\perp} - q'_{\perp})} - \delta^2 (q_{\perp} - q'_{\perp}) \int d^2 l_{\perp} \frac{q_{\perp}^2}{2l_{\perp}^2 (l_{\perp} - q_{\perp})^2} \right)
$$
\n**BFKL equation**

\n(187)

Soft functions evolution is given by the for situation situations where the Glauber exchange is in portant and does not cancel out, such as forwards where the Glauber exchange is in portant and does not cancel out, such as forwards where the Glauber exchange is i scattering. Since the Glauber Lagrangian couples together soft and collinear modes we can only

Open Directions:

Indenumerable