Effective Field Theory of Forward Scattering and Factorization Violation

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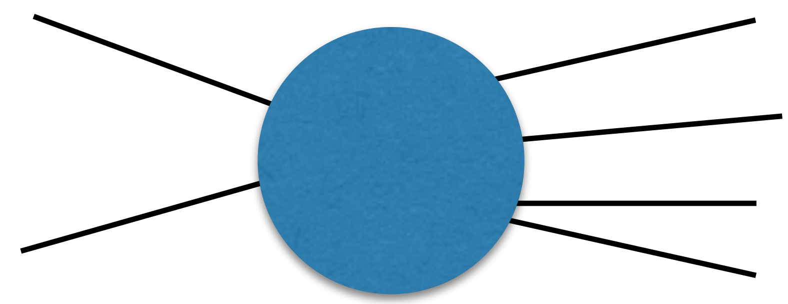
Total Scattering Cross Sections in QFT

Consider the total scattering cross section for neutral particles

$$\sigma(s) = \int \frac{d\sigma}{dt} dt$$



t some measure of traverse momentum transfer at intermediate stage of calculation

This integral is dominated by the region $t \ll s$

Power Counting Exercise

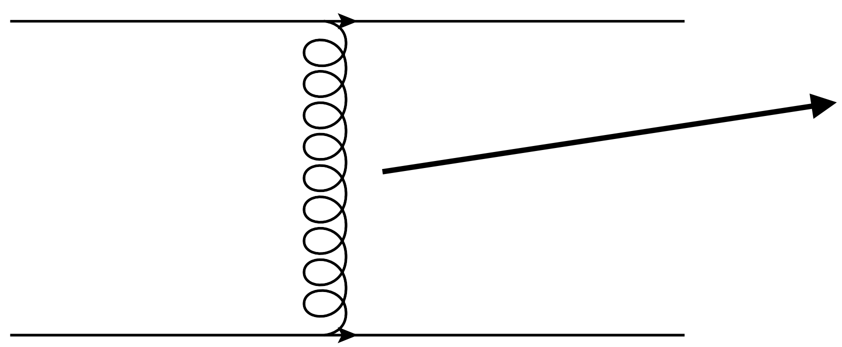
Break up integral into regions with distinct power counting parameters

1:	$s \sim t \gg \Lambda$	$\lambda \equiv \Lambda/(t,s)$	SCET-I,II
2:	$t \ll s$	$\lambda \equiv t/s$	SCETII-like

Consider region 1: There must exist some underlying hard event which must be integrated out generating some higher dimensional external operator

$$S \sim C(s,t) \int d^4x \bar{\xi}_n \xi_{\bar{n}} \bar{\xi}_{n_1} \xi_{n_2} \sim \lambda^4$$

Region 2: No underlying hard interaction, at the scale t generate the interaction



$(\lambda^n, \lambda^m, \lambda) : (m+n) \geq 3$

"Glauber" mode

$$S \sim \int d^4x \;\; \bar{\xi}_n \xi_{\bar{n}} \frac{1}{t} \bar{\xi}_n \xi_{\bar{n}} \sim \lambda^{-2}\lambda^2\lambda^{-2}\lambda^2 \sim 1$$

$$\text{"}V\text{"} \sim \delta(x_+)\delta(x_-)\log(x_\perp^2)$$

Strong analogy with NRQCD, Coulomb kernel is dressed by soft gluons

Note that while this operator is at the heart of region 2, it also exists at leading order in region 1, where it plants the seed of doubt on factorization proofs.

Note: no hard interactions to all orders in perturbation theory. The scale

$$s = (p_1 + p_2)^2$$

plays no dynamical role.

Consider the dressing of the Glauber kernel by soft gluons

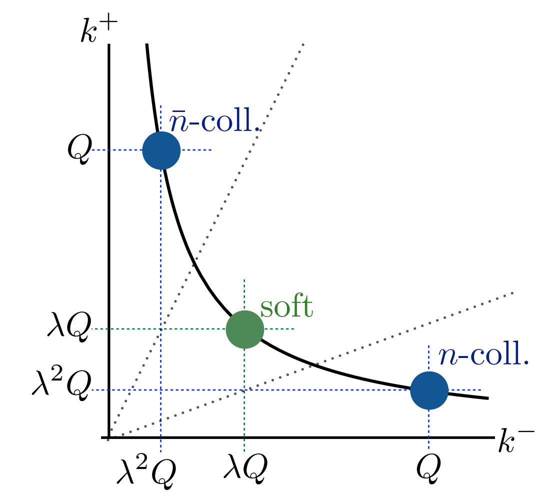
$$M \sim \frac{f(s/t, t/m^2)}{t}$$

The scale s can only show up in logs:

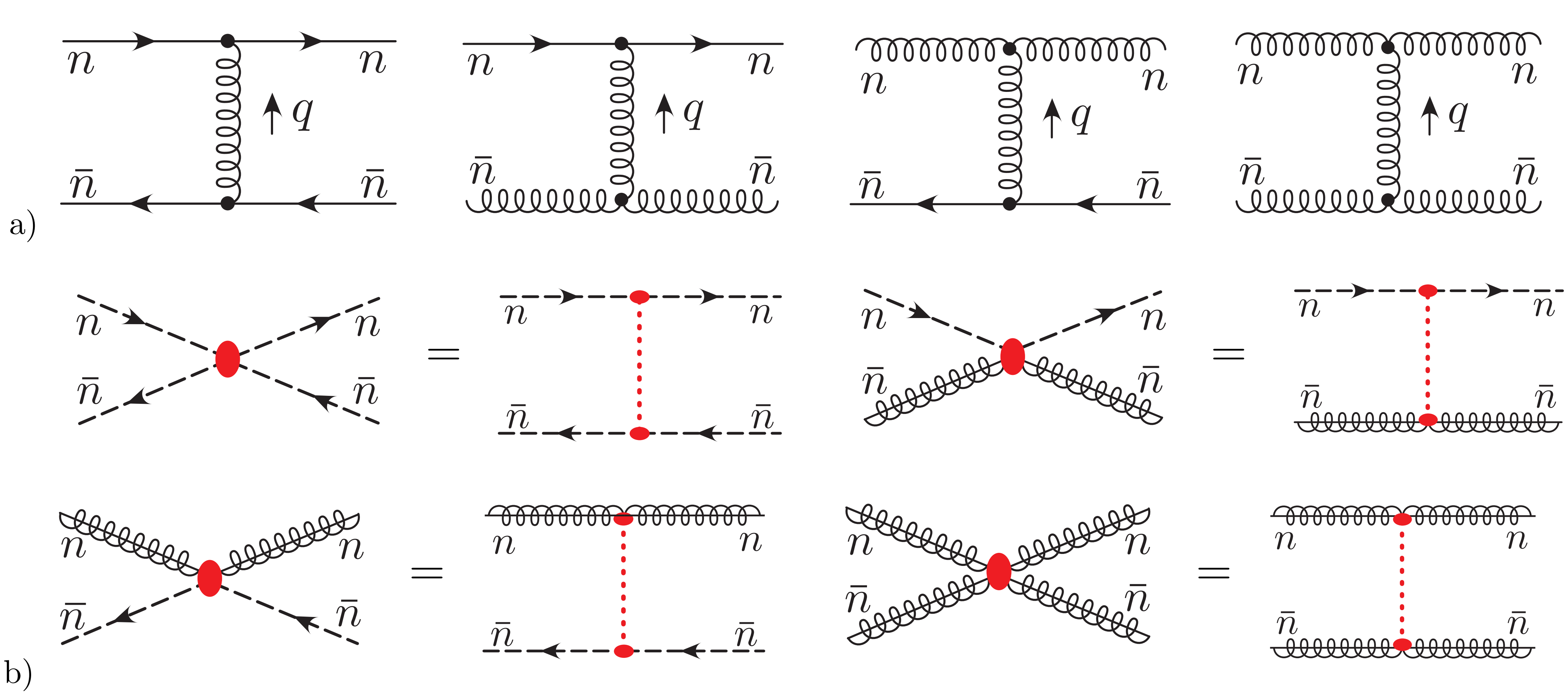
$$\log(p_+/\nu) + \log(p_-/\nu) + \log(t/\nu^2) = \log(s/t)$$

RG can not hope to capture the logs, need RRG

$$F(s/t) \sim (s/t)^\gamma$$ Regge behaviour



Matching onto the action



Operator basis:

$$O^{qq}_{ns\bar{n}} = \mathcal{O}^{qB}_{n}\frac{1}{\mathcal{P}_\perp^2}\mathcal{O}^{BC}_{s}\frac{1}{\mathcal{P}_\perp^2}\mathcal{O}^{qC}_{\bar{n}}\,, \qquad O^{gq}_{ns\bar{n}} = \mathcal{O}^{gB}_{n}\frac{1}{\mathcal{P}_\perp^2}\mathcal{O}^{BC}_{s}\frac{1}{\mathcal{P}_\perp^2}\mathcal{O}^{qC}_{\bar{n}}\,,$$

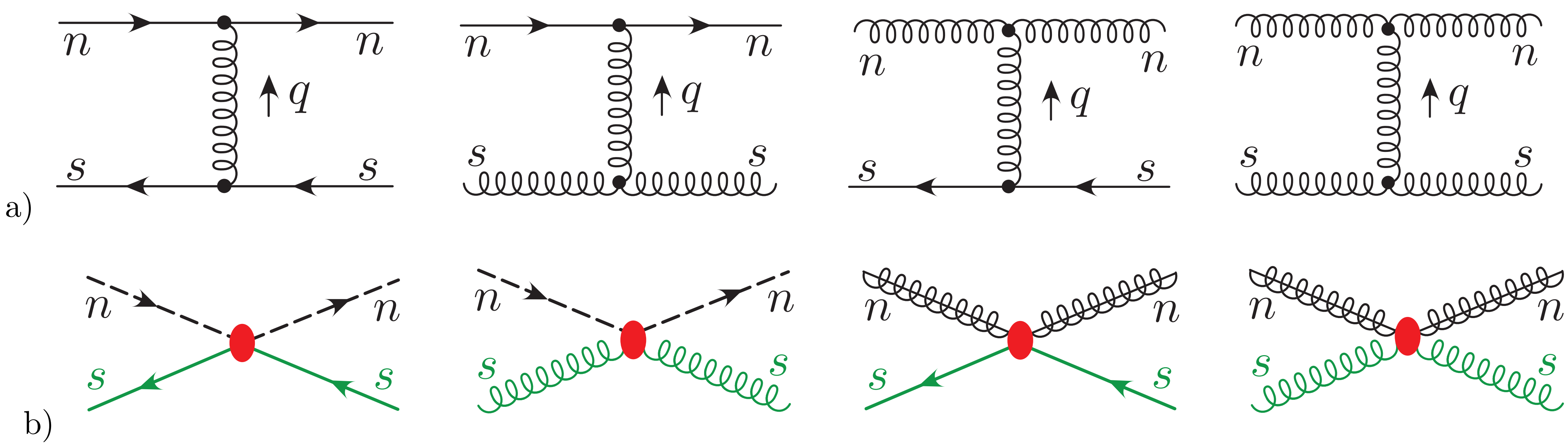
$$O^{qg}_{ns\bar{n}} = \mathcal{O}^{qB}_{n}\frac{1}{\mathcal{P}_\perp^2}\mathcal{O}^{BC}_{s}\frac{1}{\mathcal{P}_\perp^2}\mathcal{O}^{gC}_{\bar{n}}\,, \qquad O^{gg}_{ns\bar{n}} = \mathcal{O}^{gB}_{n}\frac{1}{\mathcal{P}_\perp^2}\mathcal{O}^{BC}_{s}\frac{1}{\mathcal{P}_\perp^2}\mathcal{O}^{gC}_{\bar{n}}\,.$$

Must allow for soft emission

$$\mathcal{O}^{BC}_{s} = 8\pi\alpha_s\mathcal{P}_\perp^2\delta^{BC} + \ldots$$

$$\mathcal{O}^{qB}_{n} = \overline{\chi}_{n,\omega}T^B\frac{\bar{\not{n}}}{2}\,\chi_{n,\omega}\,, \qquad \mathcal{O}^{gB}_{n} = \frac{i}{2}f^{BCD}\mathcal{B}^{C}_{n\perp\mu,-\omega}\,\bar{n}\cdot(\mathcal{P}+\mathcal{P}^\dagger)\mathcal{B}^{D\mu}_{n\perp,\omega}\,.$$

Matching Soft-collinear Operators



Matching is identical to the collinear-collinear case

$$O^{qq}_{ns} = \mathcal{O}^{qB}_{n} \frac{1}{\mathcal{P}_\perp^2} \mathcal{O}^{q_n B}_{s}\,, \quad O^{qg}_{ns} = \mathcal{O}^{qB}_{n} \frac{1}{\mathcal{P}_\perp^2} \mathcal{O}^{g_n B}_{s}\,, \quad O^{gq}_{ns} = \mathcal{O}^{gB}_{n} \frac{1}{\mathcal{P}_\perp^2} \mathcal{O}^{q_n B}_{s}\,, \quad O^{gg}_{ns} = \mathcal{O}^{gB}_{n} \frac{1}{\mathcal{P}_\perp^2} \mathcal{O}^{g_n B}_{s}$$

$$\mathcal{O}^{q_n B}_{s} = 8\pi\alpha_s \left(\bar{\psi}^n_S \, T^B \frac{\not{n}}{2} \psi^n_S \right),$$

$$\mathcal{O}^{g_n B}_{s} = 8\pi\alpha_s \left(\frac{i}{2} f^{BCD} \mathcal{B}^{nC}_{S\perp\mu} \, n \cdot (\mathcal{P} + \mathcal{P}^\dagger) \mathcal{B}^{nD\mu}_{S\perp} \right).$$

Final Glauber action

$$
\begin{aligned}
\mathcal{L}_G^{\mathrm{II}(0)} &= \sum_{n,\bar{n}} \sum_{i,j=q,g} O_{ns\bar{n}}^{ij} + \sum_{n} \sum_{i,j=q,g} O_{ns}^{ij} \\
&\equiv \sum_{n,\bar{n}} \sum_{i,j=q,g} \mathcal{O}_n^{iB} \frac{1}{\mathcal{P}_\perp^2} \mathcal{O}_s^{BC} \frac{1}{\mathcal{P}_\perp^2} \mathcal{O}_{\bar{n}}^{jC} + \sum_{n} \sum_{i,j=q,g} \mathcal{O}_n^{iB} \frac{1}{\mathcal{P}_\perp^2} \mathcal{O}_s^{j_n B} .
\end{aligned}
$$

Three rapidity sectors

Two rapidity sectors

The form of the collinear operators are fixed but the soft can have a much more general form

$$\mathcal{O}_s^{AB} = 8\pi\alpha_s \sum_i C_i O_i^{AB}$$

$$O_1 = \mathcal{P}_\perp^\mu \mathcal{S}_n^T \mathcal{S}_{\bar{n}} \mathcal{P}_{\perp\mu}$$

$$O_2 = \mathcal{P}_\perp^\mu \mathcal{S}_{\bar{n}}^T \mathcal{S}_n \mathcal{P}_{\perp\mu}$$

$$O_3 = \mathcal{P}_\perp \cdot (g\widetilde{\mathcal{B}}_{S\perp}^n)(\mathcal{S}_n^T \mathcal{S}_{\bar{n}}) + (\mathcal{S}_n^T \mathcal{S}_{\bar{n}})(g\widetilde{\mathcal{B}}_{S\perp}^{\bar{n}}) \cdot \mathcal{P}_\perp$$

$$O_4 = \mathcal{P}_\perp \cdot (g\widetilde{\mathcal{B}}_{S\perp}^{\bar{n}})(\mathcal{S}_{\bar{n}}^T \mathcal{S}_n) + (\mathcal{S}_{\bar{n}}^T \mathcal{S}_n)(g\widetilde{\mathcal{B}}_{S\perp}^n) \cdot \mathcal{P}_\perp$$

$$O_5 = \mathcal{P}_\mu^\perp (\mathcal{S}_n^T \mathcal{S}_{\bar{n}})(g\widetilde{\mathcal{B}}_{S\perp}^{\bar{n}\mu}) + (g\widetilde{\mathcal{B}}_{S\perp}^{n\mu})(\mathcal{S}_n^T \mathcal{S}_{\bar{n}})\mathcal{P}_\mu^\perp$$

$$O_6 = \mathcal{P}_\mu^\perp (\mathcal{S}_{\bar{n}}^T \mathcal{S}_n)(g\widetilde{\mathcal{B}}_{S\perp}^{n\mu}) + (g\widetilde{\mathcal{B}}_{S\perp}^{\bar{n}\mu})(\mathcal{S}_{\bar{n}}^T \mathcal{S}_n)\mathcal{P}_\mu^\perp$$

$$O_7 = (g\mathcal{B}_{S\perp}^{n\mu}) S_n^T S_{\bar{n}} (g\mathcal{B}_{S\perp\mu}^{\bar{n}})$$

$$O_8 = (g\mathcal{B}_{S\perp}^{\bar{n}\mu}) S_{\bar{n}}^T S_n (g\mathcal{B}_{S\perp\mu}^n)$$

$$O_9 = S_n^T n_\mu \bar{n}_\nu (ig\widetilde{G}_s^{\mu\nu}) S_{\bar{n}}$$

$$O_{10} = S_{\bar{n}}^T n_\mu \bar{n}_\nu (ig\widetilde{G}_s^{\mu\nu}) S_n$$

Need to match up to 2 gluons to fix all of the coefficients

Not at all obvious that one collinear emission can be matched given that there are non-local TOP's which contribute in the EFT

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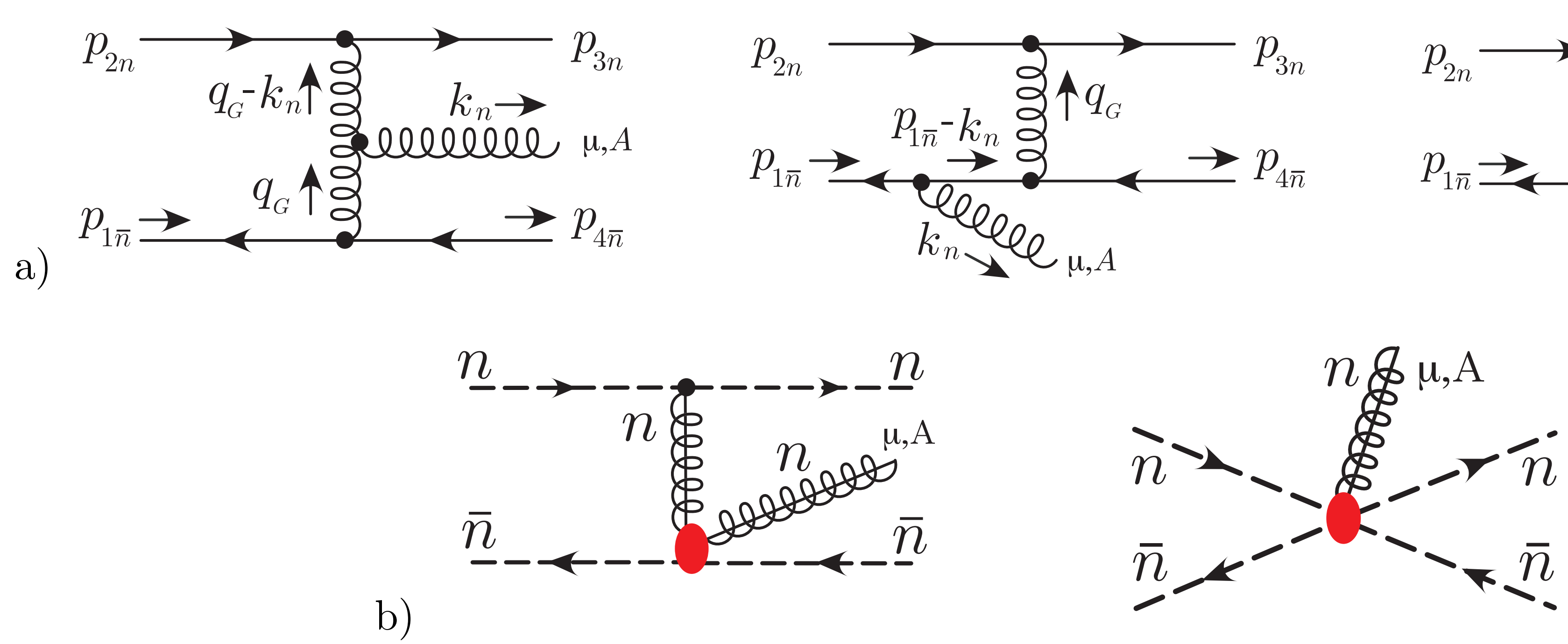


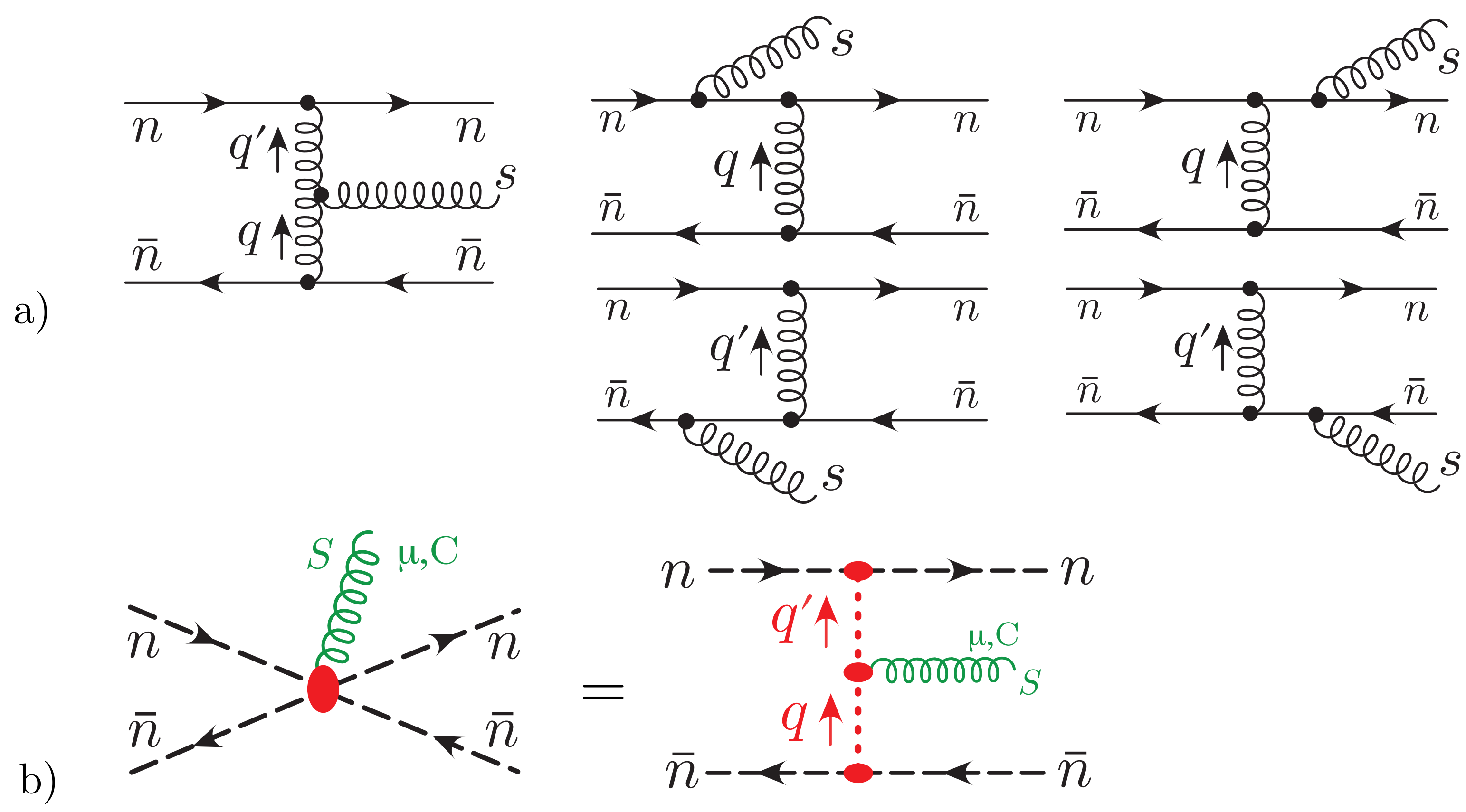
from
Wilson line

non-locality only eliminated after using the on-shell condition

$$k \cdot A = 0$$

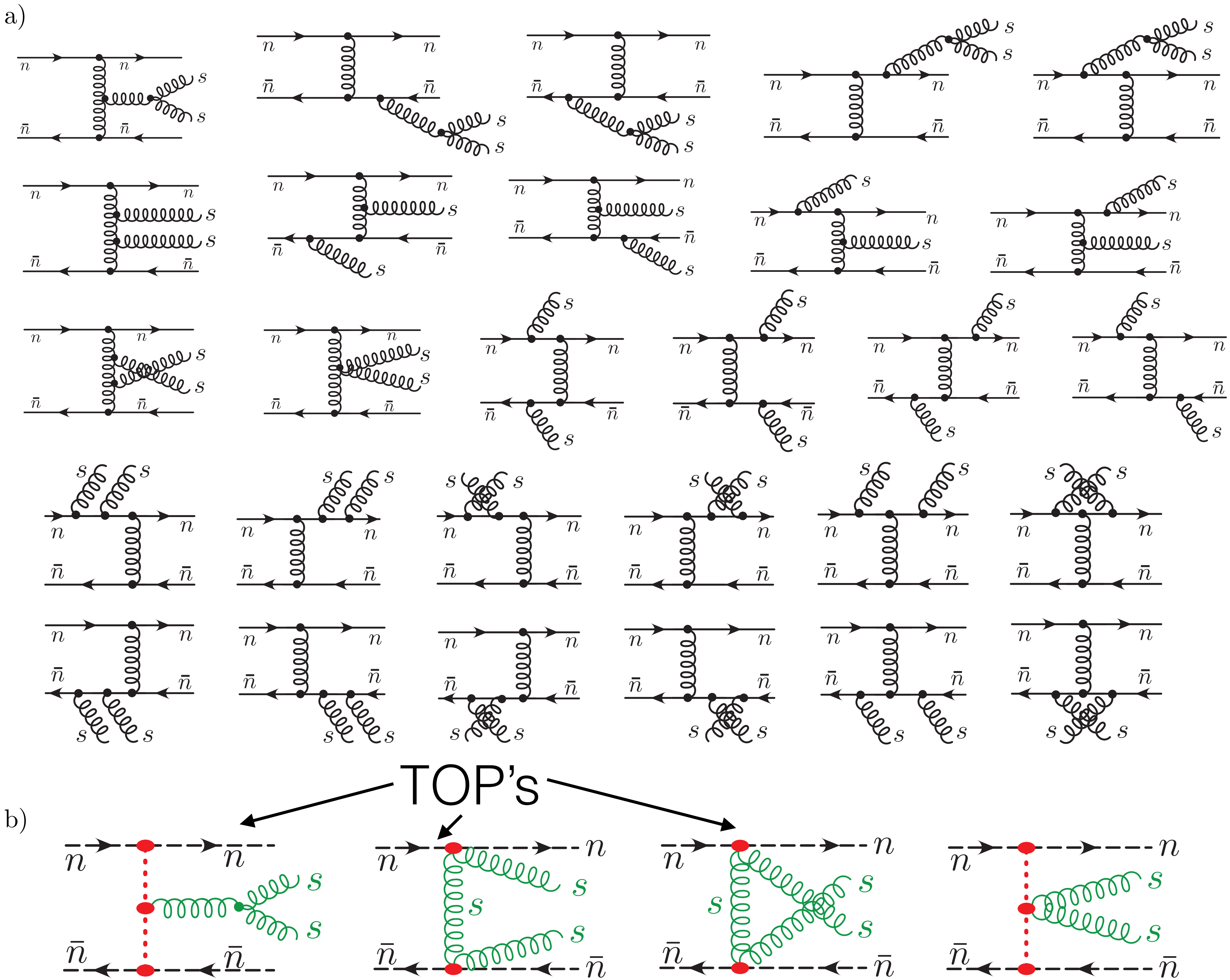
similar matching works for gluon operators

Matching Soft Operator



Matching all polarizations w/o using on shell conditions at 1-gluon
(simplifies 2 gluon matching)

First row is reproduced by TOP’s in EFT



$$\boxed{C_2 = C_4 = C_5 = C_6 = C_8 = C_{10} = 0}\,,$$

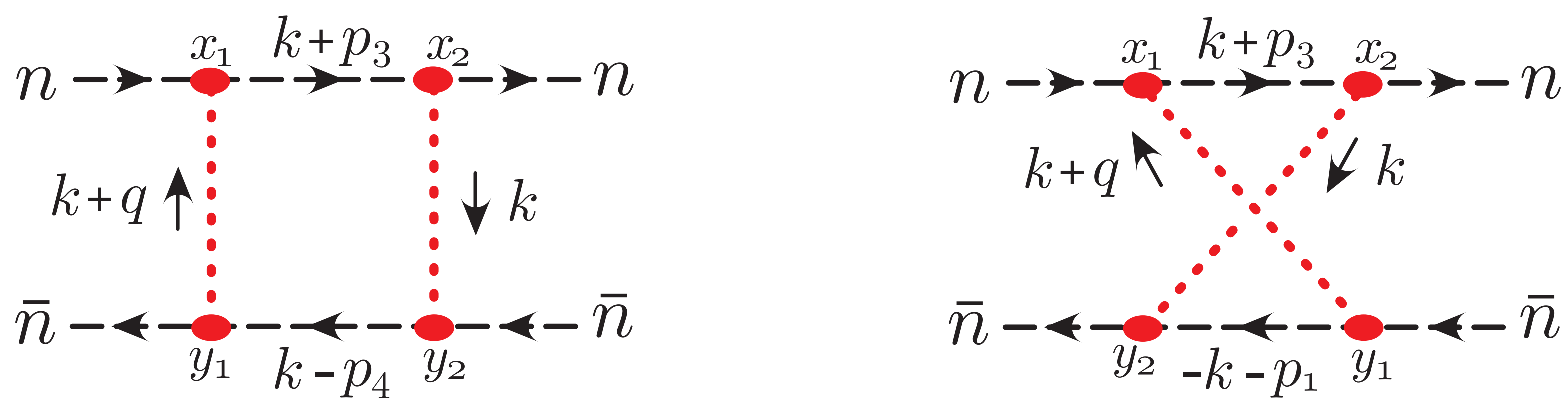
$$\boxed{C_1 = -C_3 = -C_7 = +1\,, \qquad C_9 = -\frac{1}{2}}\,.$$

$$\boxed{\begin{aligned} \mathcal{O}_s^{BC} = 8\pi\alpha_s \Big\{ & \mathcal{P}_\perp^\mu \mathcal{S}_n^T \mathcal{S}_{\bar{n}} \mathcal{P}_{\perp\mu} - \mathcal{P}_\mu^\perp g \widetilde{\mathcal{B}}_{S\perp}^{n\mu} \mathcal{S}_n^T \mathcal{S}_{\bar{n}} - \mathcal{S}_n^T \mathcal{S}_{\bar{n}} g \widetilde{\mathcal{B}}_{S\perp}^{\bar{n}\mu} \mathcal{P}_\mu^\perp - g \widetilde{\mathcal{B}}_{S\perp}^{n\mu} \mathcal{S}_n^T \mathcal{S}_{\bar{n}} g \widetilde{\mathcal{B}}_{S\perp\mu}^{\bar{n}} \\ & - \frac{n_\mu \bar{n}_\nu}{2} \mathcal{S}_n^T ig \widetilde{G}_s^{\mu\nu} \mathcal{S}_{\bar{n}} \Big\}^{BC}\,. \end{aligned}}$$

At one gluon level this operator reproduces the Lipatov vertex and generalizes it to arbitrary number of gluons. The form is uniquely fixed to all loops as there are no hard corrections to the theory.

Matching at one loop

Glauber Loop Exegesis: Two insertions of $\mathcal{O}_{ns\bar{n}}$



$$I_{\rm Gbox} = \int \frac{đ^{d-2}k_\perp \; đk^+ \; đk^-}{2(\vec{k}_\perp^{\,2})(\vec{k}_\perp+\vec{q}_\perp)^2\Big(k^+ + p_3^+ - (\vec{k}_\perp+\vec{q}_\perp/2)^2/p_2^- + i0\Big)\Big(-k^- + p_4^- - (\vec{k}_\perp+\vec{q}_\perp/2)^2/p_1^+ + i0\Big)}\,,$$

$$I_{\rm Gcbox} = \int \frac{đ^{d-2}k_\perp \; đk^+ \; đk^-}{2(\vec{k}_\perp^{\,2})(\vec{k}_\perp+\vec{q}_\perp)^2\Big(k^+ + p_3^+ - (\vec{k}_\perp+\vec{q}_\perp/2)^2/p_2^- + i0\Big)\Big(+k^- + p_1^- - (\vec{k}_\perp+\vec{q}_\perp/2)^2/p_1^+ + i0\Big)}\,.$$

Neither integral is well defined. In the Abelian limit the sum of the two integrals is well defined. However to calculate the C_A piece we need the individual diagrams to be well defined

Introduce a rapidity regulator $\quad | \, 2q_3/\nu \, |^{-\eta}$

$$I_{\rm Gcbox} = \int \frac{\text{đ}^{d-2}k_\perp \;\, \text{đ}k^0 \;\, \text{đ}k^z \;\, |k^z|^{-2\eta}\,(\nu/2)^{2\eta}}{(\vec{k}_\perp^{\,2})(\vec{k}_\perp+\vec{q}_\perp)^2 \Big(k^0 - k^z + p_3^+ - (\vec{k}_\perp + \frac{\vec{q}_\perp}{2})^2/p_2^- + i0\Big)\Big(k^0 + k^z + p_1^- - (\vec{k}_\perp + \frac{\vec{q}_\perp}{2})^2/p_1^+ + i0\Big)}$$

$$I_{\rm Gbox} = \int \frac{\text{đ}^{d-2}k_\perp \;\, \text{đ}k^0 \;\, \text{đ}k^z \;\, |k^z|^{-2\eta}\,(\nu/2)^{2\eta}}{(\vec{k}_\perp^{\,2})(\vec{k}_\perp+\vec{q}_\perp)^2 \Big(k^0 - k^z + p_3^+ - (\vec{k}_\perp + \frac{\vec{q}_\perp}{2})^2/p_2^- + i0\Big)\Big(-k^0 - k^z + p_4^- - (\vec{k}_\perp + \frac{\vec{q}_\perp}{2})^2/p_1^+ + i0\Big)}$$

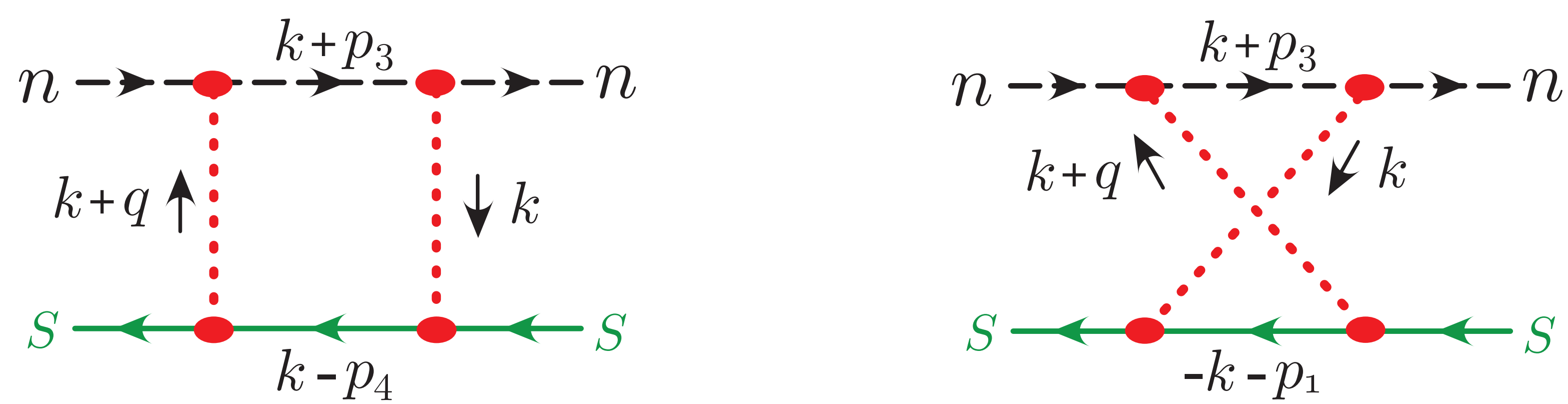
$$= \Big(\frac{-i}{4\pi}\Big) \int \frac{\text{đ}^{d-2}k_\perp}{(\vec{k}_\perp^{\,2})(\vec{k}_\perp+\vec{q}_\perp)^2} \Big[-i\pi + \mathcal{O}(\eta)\Big] \,.$$

First term in build up of Glauber Phase

Is there a Non-Abelian contribution to the phase? Yes coming from soft loops

The Abelian phase is universal as it does not care about the the spin of the collinear lines.

We can also consider soft-collinear forward scattering



Just the boost of the previous case, yields the same result but now the the glaubers carry $(\lambda^2, \lambda, \lambda^2)$

Can tweak regulator to insure homogenous scaling $\mid k_+ - \beta k_- \mid^\eta$

Gives same result as collinear-collinear

Other source of rapidity divergences are the Wilson lines which need to be regulated

$$S_n = \sum_{\text{perms}} \exp\left\{\frac{-g}{n\cdot\mathcal{P}}\left[\frac{w|2\mathcal{P}^z|^{-\eta/2}}{\nu^{-\eta/2}} n\cdot A_s\right]\right\}, \qquad S_{\bar n} = \sum_{\text{perms}} \exp\left\{\frac{-g}{\bar n\cdot\mathcal{P}}\left[\frac{w|2\mathcal{P}^z|^{-\eta/2}}{\nu^{-\eta/2}} \bar n\cdot A_s\right]\right\},$$

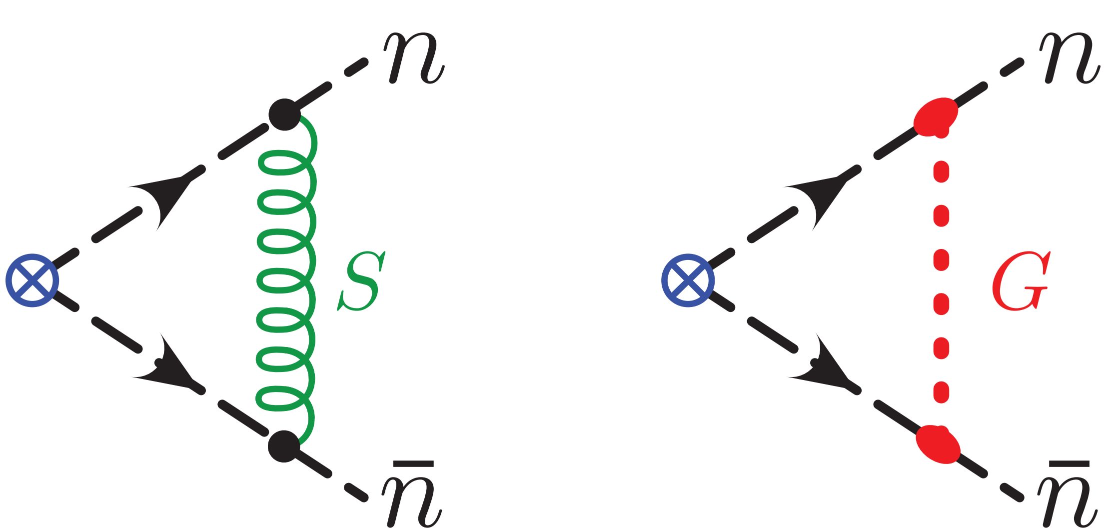
$$W_n = \sum_{\text{perms}} \exp\left\{\frac{-g}{\bar n\cdot\mathcal{P}}\right]\frac{w^2|\bar n\cdot\mathcal{P}|^{-\eta}}{\nu^{-\eta}} \bar n\cdot A_n\Big]\Big\}, \qquad W_{\bar n} = \sum_{\text{perms}} \exp\left\{\frac{-g}{n\cdot\mathcal{P}}\left[\frac{w^2|n\cdot\mathcal{P}|^{-\eta}}{\nu^{-\eta}} n\cdot A_{\bar n}\right]\right\}$$

Note we regulate every gluon to be consistent with the Glaubers (important for zero bin cancellation)

Zero Bin Subtractions

Soft, Collinear and Glauber all overlap

$$S = S - S^{(G)} \qquad C_n = C_n - C_n^{(S)} - C_n^{(G)} + C_n^{(GS)}$$



$$S^{(G)} = G$$

This is why we don't see the Glauber in hard matching

Using the rapidity regulator many of the zero-bin diagrams vanish but crucially not all as in this example.

One loop matching

Full Theory Graphs

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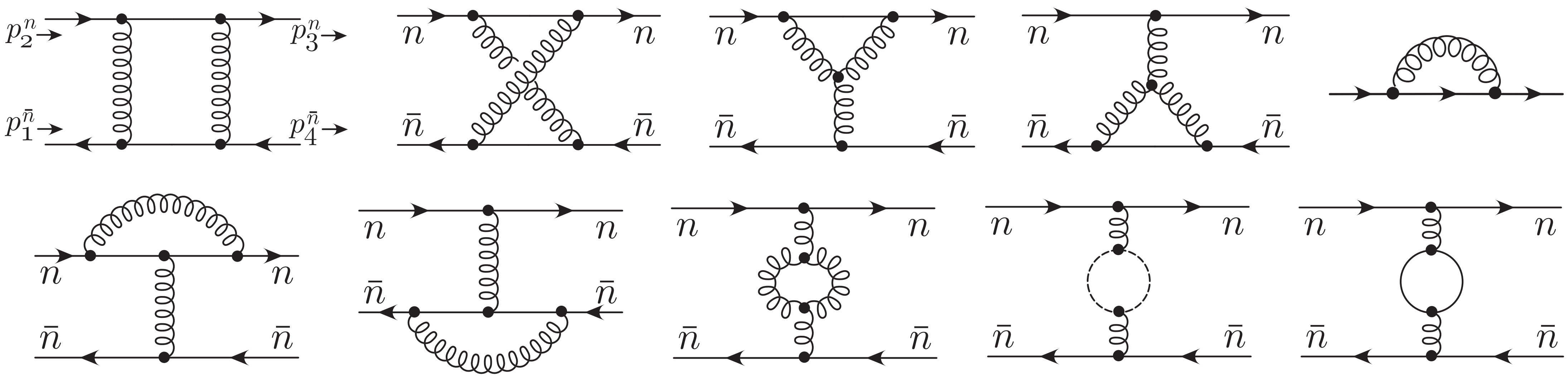


$$\text{Figs. } 18 + Z_g \text{ c.t.} = \frac{i\alpha_s^2}{t}\mathcal{S}_1^{n\bar n}\left[8i\pi\ln\left(\frac{-t}{m^2}\right)\right] + \frac{i\alpha_s^2}{t}\mathcal{S}_2^{n\bar n}\left[-4\ln^2\left(\frac{m^2}{-t}\right) - 12\ln\left(\frac{m^2}{-t}\right) - 14\right]$$

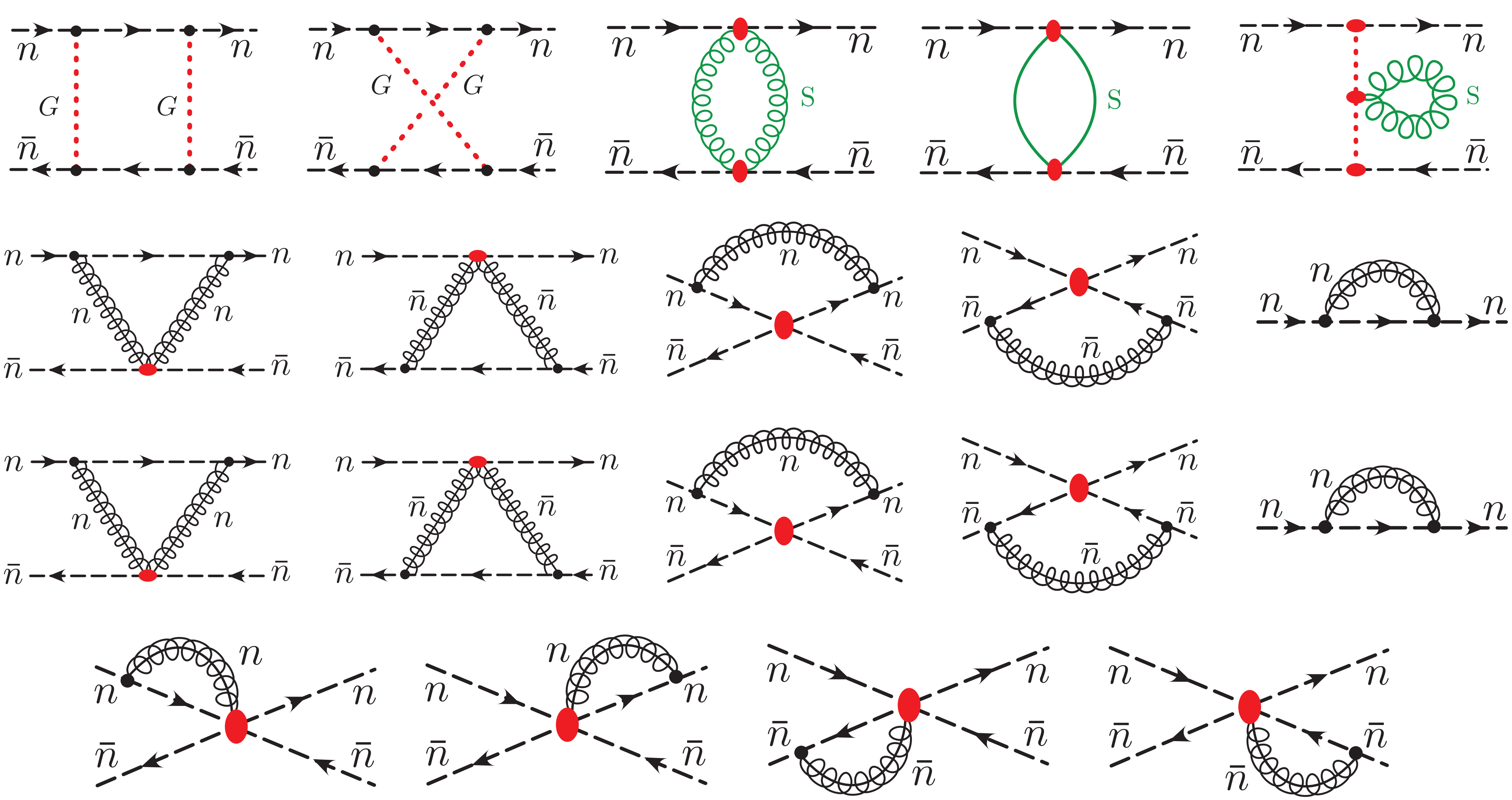
$$+\frac{i\alpha_s^2}{t}\mathcal{S}_3^{n\bar n}\left[-4\ln\left(\frac{s}{-t}\right)\ln\left(\frac{-t}{m^2}\right) + \frac{22}{3}\ln\left(\frac{\mu^2}{-t}\right) + \frac{170}{9} + \frac{2\pi^2}{3}\right]$$

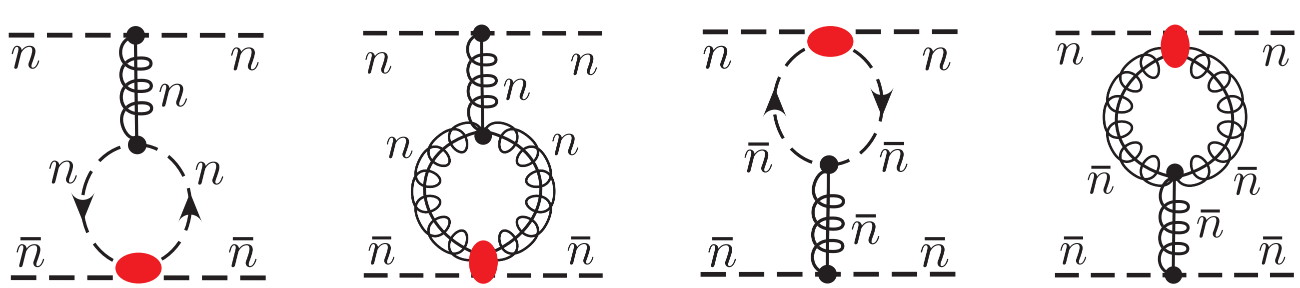
$$+\frac{i\alpha_s^2}{t}\mathcal{S}_4^{n\bar n}\left[-\frac{8}{3}\ln\left(\frac{\mu^2}{-t}\right) - \frac{40}{9}\right].$$

$$\mathcal{S}_1^{n\bar n} = \left[\bar u_n T^A T^B \frac{\bar{n}\!\!\!/}{2} u_n\right]\left[\bar v_{\bar n} T^B T^A \frac{n\!\!\!/}{2} v_{\bar n}\right], \qquad \mathcal{S}_2^{n\bar n} = C_F\left[\bar u_n T^A \frac{\bar{n}\!\!\!/}{2} u_n\right]\left[\bar v_{\bar n} T^A \frac{n\!\!\!/}{2} v_{\bar n}\right],$$

$$\mathcal{S}_3^{n\bar n} = C_A\left[\bar u_n T^A \frac{\bar{n}\!\!\!/}{2} u_n\right]\left[\bar v_{\bar n} T^A \frac{n\!\!\!/}{2} v_{\bar n}\right], \qquad \mathcal{S}_4^{n\bar n} = T_F n_f\left[\bar u_n T^A \frac{\bar{n}\!\!\!/}{2} u_n\right]\left[\bar v_{\bar n} T^A \frac{n\!\!\!/}{2} v_{\bar n}\right].$$

EFT Diagrams





Killed by soft zero-bin subtractions

Soft Total

$$
= \frac{i\alpha_s^2}{t}\,\mathcal{S}_3^{n\bar n}\bigg\{-\frac{4}{\eta}h(\epsilon,\mu^2/m^2)-\frac{4}{\eta}\,g(\epsilon,\mu^2/t)-4\ln\Big(\frac{\mu^2}{4\nu^2}\Big)\ln\Big(\frac{m^2}{-t}\Big)
-2\ln^2\Big(\frac{\mu^2}{m^2}\Big)+2\ln^2\Big(\frac{\mu^2}{-t}\Big)-\frac{2\pi^2}{3}+\frac{22}{3}\ln\frac{\mu^2}{-t}+\frac{134}{9}\bigg\}
+\frac{i\alpha_s^2}{t}\,\mathcal{S}_4^{n\bar n}\bigg[-\frac{8}{3}\ln\Big(\frac{\mu^2}{-t}\Big)-\frac{40}{9}\bigg]. \tag{108}
$$

Includes counter term for α

Note: no UV poles

Collinear total

$$
= \frac{i\alpha_s^2}{t}\,\mathcal{S}_3^{n\bar n}\bigg\{\frac{4}{\eta}h(\epsilon,\mu^2/m^2)+\frac{4}{\eta}\,g(\epsilon,\mu^2/t)+4\ln\Big(\frac{4\nu^2}{s}\Big)\ln\Big(\frac{-t}{m^2}\Big)+2\ln^2\Big(\frac{m^2}{-t}\Big)+4+\frac{4\pi^2}{3}\bigg\}
+\frac{i\alpha_s^2}{t}\,\mathcal{S}_2^{n\bar n}\bigg[-4\ln^2\Big(\frac{m^2}{-t}\Big)-12\ln\Big(\frac{m^2}{-t}\Big)-14\bigg]. \tag{112}
$$

combining collinear sectors

$$\ln s/\nu^2 = \ln(n\cdot p/\nu)+\ln(\bar n\cdot p/\nu)$$

Total EFT

$$
= \frac{i\alpha_s^2}{t}\,\mathcal{S}_1^{n\bar n}\bigg[8i\pi\ln\Big(\frac{-t}{m^2}\Big)\bigg]+\frac{i\alpha_s^2}{t}\,\mathcal{S}_2^{n\bar n}\bigg[-4\ln^2\Big(\frac{m^2}{-t}\Big)-12\ln\Big(\frac{m^2}{-t}\Big)-14\bigg]
+\frac{i\alpha_s^2}{t}\,\mathcal{S}_3^{n\bar n}\bigg\{-4\ln\Big(\frac{s}{-t}\Big)\ln\Big(\frac{-t}{m^2}\Big)+\frac{22}{3}\ln\frac{\mu^2}{-t}+\frac{170}{9}+\frac{2\pi^2}{3}\bigg\}
+\frac{i\alpha_s^2}{t}\,\mathcal{S}_4^{n\bar n}\bigg[-\frac{8}{3}\ln\Big(\frac{\mu^2}{-t}\Big)-\frac{40}{9}\bigg]. \tag{113}
$$

=full, no hard matching!

running of Glauber potential

Even though the rapidity divergences cancel we must renormalize the factorized operators distinctly

Collinear mixing:

$$\vec{\mathcal{O}}_{\bar{n}}^{B\text{bare}} = \hat{V}_{\mathcal{O}_{\bar{n}}} \cdot \vec{\mathcal{O}}_{\bar{n}}^{B}(\nu,\mu)\,, \qquad \hat{V}_{\mathcal{O}_{\bar{n}}} = \begin{pmatrix} 1+\delta V_{\bar{n}}^{qq} & \delta V_{\bar{n}}^{qg} \\ \delta V_{\bar{n}}^{gq} & 1+\delta V_{\bar{n}}^{gg} \end{pmatrix}, \qquad \vec{\mathcal{O}}_{\bar{n}}^{B} = \begin{pmatrix} \mathcal{O}_{\bar{n}}^{qB} \\ \mathcal{O}_{\bar{n}}^{gB} \end{pmatrix}.$$

Soft single index ops:

$$\vec{\mathcal{O}}_{s_n}^{A\text{bare}} = \hat{V}_{\mathcal{O}_{s_n}} \cdot \vec{\mathcal{O}}_{s_n}^{A}(\nu,\mu)\,, \qquad \hat{V}_{\mathcal{O}_{s_n}} = \begin{pmatrix} 1+\delta V_{s_n}^{qq} & \delta V_{s_n}^{qg} \\ \delta V_{s_n}^{gq} & 1+\delta V_{s_n}^{gg} \end{pmatrix}, \qquad \vec{\mathcal{O}}_{s_n}^{A} = \begin{pmatrix} \mathcal{O}_{s}^{q_n A} \\ \mathcal{O}_{s}^{g_n A} \end{pmatrix},$$

Soft double index ops:

$$\vec{\mathcal{O}}_{s}^{AB\text{bare}} = \hat{V}_{\mathcal{O}_{s}} \cdot \vec{\mathcal{O}}_{s}^{AB}(\nu,\mu)\,,$$

$$\hat{V}_{\mathcal{O}_s} = \begin{pmatrix} 1+\delta V_s & 0 & 0 & 0 & 0 \\ \delta V_s^{Tqq} & & & & \\ \delta V_s^{Tgq} & & \hat{V}_{\mathcal{O}_{s_n}} \otimes \hat{V}_{\mathcal{O}_{s_{\bar{n}}}} & & \\ \delta V_s^{Tgg} & & & & \\ \delta V_s^{Tqg} & & & & \end{pmatrix}, \qquad \vec{\mathcal{O}}_{s}^{AB} = \begin{pmatrix} \mathcal{O}_s^{AB} \\ i\int d^4x\, T\, \mathcal{O}_s^{q_n A}(x)\mathcal{O}_s^{q_{\bar{n}} B}(0) \\ i\int d^4x\, T\, \mathcal{O}_s^{g_n A}(x)\mathcal{O}_s^{q_{\bar{n}} B}(0) \\ i\int d^4x\, T\, \mathcal{O}_s^{g_n A}(x)\mathcal{O}_s^{g_{\bar{n}} B}(0) \\ i\int d^4x\, T\, \mathcal{O}_s^{q_n A}(x)\mathcal{O}_s^{g_{\bar{n}} B}(0) \end{pmatrix}.$$

Re-sum the logs in parton-parton scattering, simplest to run collinear sectors (no TOPs)

Structure is Fixed

$$\nu \frac{d}{d\nu}(\mathcal{O}_n^{qA} + \mathcal{O}_n^{gA}) = \gamma_{n\nu}(\mathcal{O}_n^{qA} + \mathcal{O}_n^{gA}) \,, \qquad \nu \frac{d}{d\nu}(\mathcal{O}_s^{q_n A} + \mathcal{O}_s^{g_n A}) = \gamma_{s\nu}(\mathcal{O}_s^{q_n A} + \mathcal{O}_s^{g_n A}) \,.$$

mixing leads to universality

$$\gamma_{n\nu} \equiv \gamma_{n\nu}^{qq} + \gamma_{n\nu}^{gq} = \gamma_{n\nu}^{gg} + \gamma_{n\nu}^{qg} \,,$$

So all we need is γ_n^ν

Which we can extract from matching calculation

$$M(\nu = \sqrt{s}) \sim (s/t)^\gamma \qquad \gamma = \frac{\alpha_s C_A}{2\pi} Log(-t/m^2)$$

Gluon Reggeization

Summing Logs in IR safe quantities

Consider observables for which the soft radiation is not measured. For this class the RRG equation will reduce to the BKFL equation

Consider the total cross section for hadron scattering try to capture s dependence

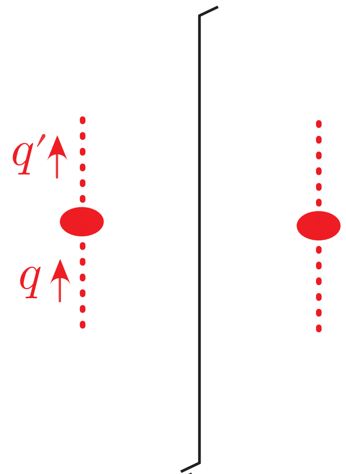
$$\sigma_{pp} = \int \frac{d\sigma}{dt} dt \sim \int dt \sum_{X_s, X_n, X_{\bar{n}}} \langle p_n \bar{p}_n \mid X_s, X_n, X_{\bar{n}} \rangle O(t(\mathcal{P}_\perp^n, \mathcal{P}_\perp^{\bar{n}})) \langle X_s, X_n, X_{\bar{n}} \mid p_n \bar{p}_n \rangle,$$

insert $\mathcal{L}_G^{II}$

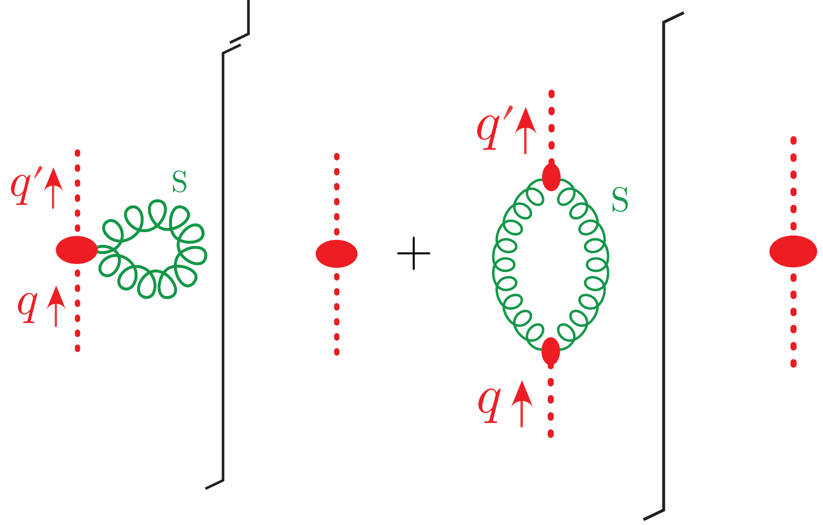
Running the soft function of Glauber operator

$$O_{s(1,1)}^{AB}(q_\perp, q'_\perp) = \mathcal{O}_s^{AB}(q_\perp, q'_\perp) + \int d^4x \; T \left[\mathcal{O}_s^{q_n A}(q_\perp) + \mathcal{O}_s^{g_n A}(q_\perp)\right] \left[\mathcal{O}_s^{q_{\bar{n}} A}(q'_\perp) + \mathcal{O}_s^{g_{\bar{n}} A}(q'_\perp)\right].$$

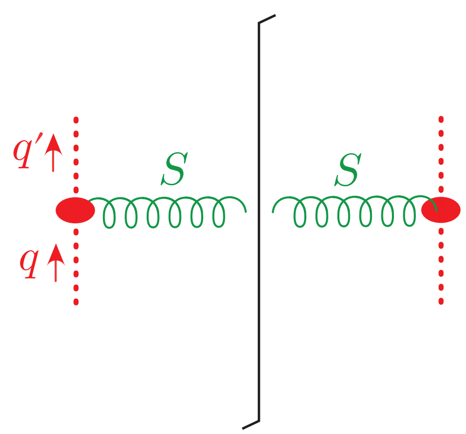
$$G^{ABA'B'}(q_\perp, q'_\perp) = \sum_X \langle 0 | O_{s(1,1)}^{AB}(q_\perp, q'_\perp) | X \rangle \langle X | O_{s(1,1)}^{A'B'}(q_\perp, q'_\perp) | 0 \rangle$$



$$= G^{(0)abcd}(l_\perp, l'_\perp) \equiv O_s^{ab} \mid 0\rangle\langle 0 \mid O_s^{cd} = \left(\frac{8\pi\alpha}{l_\perp^2}\right)^2 \delta^{ab}\delta^{cd}\delta^2(l_\perp - l'_\perp)\,,$$



$$= -\frac{\alpha}{2\pi^2} a C_A \Gamma[\eta] \int d^2 q_\perp \frac{G^{(0)}(l_\perp, l'_\perp)\, l_\perp^2}{q_\perp^2 (q_\perp - l_\perp)^2}$$



$$= (2)^8 \alpha^3 \pi^2 f^{abe} f^{cde} \Gamma[\eta] \int \frac{[d^2 q_\perp]}{(q_\perp - l_\perp)^2 l_\perp^2 q_\perp^2} \delta^2(q_\perp - l'_\perp)$$

$$= \frac{\alpha}{\pi^2} C_A \Gamma[\eta] \int \frac{[d^2 q_\perp]\, q_\perp^2}{(q_\perp - l_\perp)^2 l_\perp^2} G^{(0)abcd}(q_\perp, l'_\perp)\,,$$

Here used: $\tilde{G}(l_\perp, l'_\perp) = l_\perp^2 G(l_\perp, l'_\perp) {l'_\perp}^2$

$$\tilde{G}^{ren}(\vec{l}_\perp, \vec{l}'_\perp) = \int d^2 q_\perp Z(l_\perp, q_\perp) \tilde{G}^{bare}(q_\perp, l'_\perp)$$

$$\nu \frac{d}{d\nu} \tilde{G}^{ren}(q_\perp, l'_\perp) = \int d^2 q'_\perp \gamma(q_\perp, q'_\perp) \tilde{G}^{ren}(q'_\perp, l'_\perp)$$

$$\gamma(q_\perp, q'_\perp) = \int d^2 k_\perp Z(q_\perp, k_\perp) \nu \frac{d}{d\nu} Z^{-1}(k_\perp, l'_\perp) = \frac{\alpha_s C_A}{\pi^2} \left(\frac{1}{(q_\perp - q'_\perp)} - \delta^2(q_\perp - q'_\perp) \int d^2 l_\perp \frac{q_\perp^2}{2 l_\perp^2 (l_\perp - q_\perp)^2} \right) \tag{187}$$

BFKL equation

Soft functions evolution is given by the BFKL equation

Open Directions:

Indenumerable