Regge behavior in effective field theory

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SCET 2015, Santa Fe, New Mexico

March 25, 2015

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- In collaboration with J. Donoghue and G. Ovanesyan (arXiv:1405.1731).
- Related work done by S. Fleming (arXiv:1404.5672).

Outline

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T. Regge (1931-2014)

- Regge investigated the non-relativistic scattering off a Yukawa potential using the theory of complex angular momentum.[1]
- Before the advent of QCD, these ideas were carried over trying to describe relativistic hadron scattering.

[1] **T. Regge: Nuovo Cim. 14 951 (1959)**.

- At very high energies, one interesting prediction of Regge theory is the emergence of power-law behavior in scattering amplitudes in the forward limit.

Regge behavior

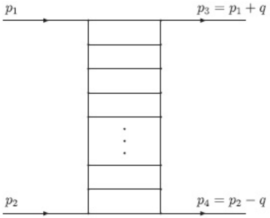
As $s \to \infty, t$ fixed, the **asymptotic** behavior of the amplitude grows as $\mathcal{M} \sim s^{\alpha(t)}$.

- Our interest in this work is to describe Regge behavior in an EFT framework.

Motivation

Regge behavior turns logarithms to a power-law.
How does Regge behavior arise in SCET?

- Using a scalar theory with a cubic interaction, J. C. Polkinghorne demonstrated that Regge behavior emerges in perturbation theory.[2]

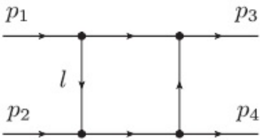


$$s = (p_1 + p_2)^2, \quad t = (p_3 - p_1)^2, \quad s \gg -t, m^2.$$

- Seemingly very complicated, the Regge limit of the ladder graphs can nevertheless be analyzed and summed.

[2] **J. C. Polkinghorne: J. Math. Phys. 4 503 (1963)**.

- Let us analyze the box graph:



- Integrating over the loop momentum:

$$i\mathcal{M}_{box} = i\frac{g^4}{16\pi^2}\int_{FP}\frac{\delta(1-x_1-x_2-y_1-y_2)}{(x_1x_2s+y_1y_2t-m^2D(x,y))^2}$$

- The leading behavior of this graph precisely comes from the corner $x_i \sim 0$:

$$i\mathcal{M}_{box} = ig^2\beta(t)\frac{1}{-s}\ln(-s-i0)$$

- The function $\beta(t)$ is what eventually defines the Regge exponent.

$$\beta(t) = \frac{g^2}{16\pi^2}\int dy_1dy_2\frac{\delta(1-y_1-y_2)}{[m^2-y_1y_2t]} = \frac{g^2}{4\pi}\int\frac{d^2\boldsymbol{l}_\perp}{(2\pi)^2}\frac{1}{[\boldsymbol{l}_\perp^2+m^2][(\boldsymbol{l}_\perp+\boldsymbol{q}_\perp)^2+m^2]}.$$

- The cross-box: send $s \to u \approx -s$.

Note

In the sum, only the imaginary part of the box survives:

$$\mathcal{M}^{box\ +\ crossed} = -g^2\beta(t)\left[\frac{1}{s}\ln(-s) + \frac{1}{-s}\ln(s)\right] = \frac{i\pi}{s}g^2\beta(t).$$

- At higher loops:

$$\mathcal{M}^{(n)}_{Total} = \frac{i\pi g^2}{s}\frac{\beta^n(t)}{(n-1)!}\ln^{n-1}(s)$$

- Regge behavior emerges:

$$\mathcal{M}_{Regge} = i\pi g^2\beta(t)s^{-1+\beta(t)}$$

Lesson

Regge Logs come from the imaginary part of the ladder graphs which corresponds to the s-channel cut associated with all the rungs of the ladder becoming on-shell.

Outline

The previous analysis does not lead to any insight into the **momentum modes** that make up the Reggeon.

- To answer the above question: use Cutkosky rule.
- Counting parameter:

$$\lambda = \sqrt{\frac{-t}{s}}$$

- On-shell final states:

$$q \sim \sqrt{s}(\lambda^2, \lambda^2, \lambda), \quad p^\mu = (p \cdot n, p \cdot \bar{n}, \boldsymbol{p}_\perp)$$

- The Reggeon is a Glauber object.

- The answer is not that simple: strong ordering $\rightarrow$ Regge logs

$$|l_1^+| \ll |l_2^+| \ldots. \ll |l_k^+| \ll |l_{k+1}^+| \ll, \ldots \ll |l_N^+|,$$

$$l_1^- \gg l_2^-, \ldots. \gg l_k^- \gg l_{k+1}^- \gg, \ldots \gg l_N^-.$$

- Donoghue & Wyler[3] found:

Feature

To connect an n-collinear particle with an $\bar{n}$-collinear one, at least one of the loop momenta needs to be in the Glauber region.

Question

How can we understand strong ordering in EFT?

[3] J. F. Donoghue and D. Wyler: Phys. Rev. D 81, 114023 (2010).

Outline

Goal

Our goal is to employ the method of regions[a] to study the ladder graphs and isolate the modes reposnsible for generating Regge behavior.

[a] **M. Beneke and V. A. Smirnov Nucl. Phys. B522**, **321 (1998)**.

- The most important aspect: **pinch analysis**.
- The box graph: **only** collinear modes are pinched.

$$n-\text{collinear}: l \sim \sqrt{s}(\lambda^2, 1, \lambda), \quad \bar{n}-\text{collinear}: l \sim \sqrt{s}(1, \lambda^2, \lambda)$$

- Hard and ultra-soft modes: sub-leading.

- Smirnov reproduced the full result of the **massless** box diagram[4].
- Collinear modes using an **analytic regulator**:

Feature 1

The Glauber graph vansihes.

Feature 2

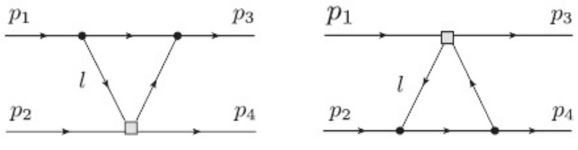
The overlap integrals are scaleless and therefore vanish.

Feature 3

The analytic regulator parameter cancels in the final answer.

[4] **V. A. Smirnov: Springer Tracts Mod. Phys. 177, 1 (2002)**.

- The computation is correct: conforms to the result of the pinch analysis.
- Conceptual problem: **unitarity**.
- Look back at collinear graphs:



Perhaps the imaginary part comes from a **subregion** of the collinear graphs. We need to employ a regulator that gives us the required insight.

- Regulator: massive internal lines with $m^2 \sim \lambda^2$.
- Matching: need full QCD result.
- The answer contains a substantial number of dilogarithms.
- Nevertheless, when expanded in the Regge limit:

$$\mathcal{M}^{(1)}_{\text{QCD}} = \frac{i\pi g^2 \beta(t)}{s}$$

Note

The exact computation did not show any finite pieces accompanying the Regge log at one loop. This is not the case at two loops.

- Matching in SCET:

$$\mathcal{M}_n^{(1)} = \mathcal{M}_{\bar{n}}^{(1)} = \mathcal{M}_{\mathrm{QCD}}^{(1)}.$$

- The result is **twice** the QCD result: matching fails? Zero-bin subtraction[5].

Note

The overlap integrals are no longer vanishing with our regulator.

- There is one overlap in this case:

$$\mathcal{M}_{n/\bar{n}}^{(1)} = \mathcal{M}_{\mathrm{QCD}}^{(1)}.$$

- Including the overlap, matching works in SCET:

$$\mathcal{M}_{\mathrm{SCET}}^{(1)} = \mathcal{M}_n^{(1)} + \mathcal{M}_{\bar{n}}^{(1)} - \mathcal{M}_{n/\bar{n}}^{(1)} = \mathcal{M}_{\mathrm{QCD}}^{(1)}.$$

[5] **A. V. Manohar and I. W. Stewart: Phys. Rev. D 76**, **074002 (2007)**,
B. Jantzen: J. High Energy Phys. 12 (2011) 076.

Lesson 1

The overlap integral being equal to the QCD result hints that the Regge log is coming from a subregion.

- Use Cutkosky rule to pin down the true mode:

$$\text{Im}\,\mathcal{M}^{(1)}_{\text{QCD}} = \frac{g^4}{16\pi^2\sqrt{s}} \int d^4l \, \frac{\delta((p_1 - l)^2 - m^2)\delta(l^0)}{\sqrt{s}(l^0 - l_z)(\sqrt{s}(l^0 - l_z) + q^2 - 2\vec{l}\cdot\vec{q})}$$

Recall

$l^+ = l^0 - l_z, \quad l^- = l^0 + l_z$

- Delta function forces $l^0 = 0$: the true mode has $|l^+| = |l^-|$.

- The integal over l_z contains two roots. The smaller root is Taylor expanded:

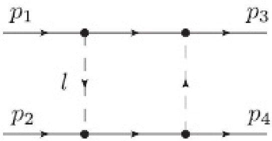
$$l_z^{\text{small}} = \frac{\Delta}{\sqrt{s}} \left(1 + \frac{\Delta}{s} + \ldots \right), \quad \Delta = \boldsymbol{l}_\perp^2 + m^2$$

- The leading piece yields the Regge log.

Lesson 2

The true region that gives the imaginary part has a large transverse component.

- The Glauber mode indeed plays a major role with our regulator.



- The Glauber graph:

$$\mathcal{M}_G^{(1)} = \mathcal{M}_{\text{QCD}}^{(1)}$$

- The overlaps proliferate. All being equal to QCD:

$$\mathcal{M}^{(1)}_{n/G} = \mathcal{M}^{(1)}_{\bar{n}/G} = \mathcal{M}^{(1)}_{n/\bar{n}/G} = \mathcal{M}^{(1)}_{\text{QCD}} \quad .$$

- The full overlap formula can be organized as follows:

$$\begin{aligned}\mathcal{M}^{(1)}_{SCET_G} &= \mathcal{M}^{(1)}_{n} + \mathcal{M}^{(1)}_{\bar{n}} + \mathcal{M}^{(1)}_{G} - \mathcal{M}^{(1)}_{n/\bar{n}} - \mathcal{M}^{(1)}_{n/G} - \mathcal{M}^{(1)}_{\bar{n}/G} + \mathcal{M}^{(1)}_{n/\bar{n}/G} \\ &= \mathcal{M}^{(1)}_{G} = \mathcal{M}^{(1)}_{\text{QCD}}.\end{aligned}$$

Lesson 3

The genuine collinear contributions **vanish** with our regulator.

Lesson 4

At one loop, the Glauber mode is the true mode underlying Regge physics. In coformation with unitarity, it is the only mode forcing the intermediate states to be on-shell.

Presciprtion

Use SCET$_G$ and identify the graphs with on-shell intermediate states.

- From now onwards, we focus only on the imaginary part. At two loops, we find:

$$\text{Im}\,\mathcal{M}^{(2)}_{\text{QCD}}(t=0) = \frac{g^6}{256\pi^5 s}\int d^2\boldsymbol{l}_1 d^2\boldsymbol{l}_2 \frac{1}{(\Delta_1\Delta_2)^2}\left[\ln\frac{s}{\Delta_{12}} - 2\right]$$
$$\theta\left(s - \left(\sqrt{\Delta_1} + \sqrt{\Delta_2} + \sqrt{\Delta_{12}}\right)^2\right).$$

$$\Delta_1 = \boldsymbol{l}^2_{1\perp} + m^2, \quad \Delta_2 = \boldsymbol{l}^2_{2\perp} + m^2, \quad \Delta_{12} = (\boldsymbol{l}_{1\perp} - \boldsymbol{l}_{2\perp})^2 + m^2$$

Note

There are extra pieces at leading order which come with the log.

- The modes of interest at two loops are: $(n, G), (G, \bar{n}), (G, G)$.

Note

Notice the appearence of the Glauber connectors.

- The (G, G) mode is sub-leading.
- The contribution of the above modes is computable modulo one caveat.

Caveat

The internal masses do not suffice to regulate the infrared any more. We had to relax strict power counting in the arguments of the step functions to regulate the integrals.

- The result for the (n, G) mode is:

$$\text{Im}\,\mathcal{M}^{(2)}_{nG} = \frac{g^6}{256\pi^5} \int \frac{d^2\boldsymbol{l}_{1\perp} d^2\boldsymbol{l}_{2\perp}}{s\,\Delta_1\Delta_2\Delta_{1q}\Delta_{2q}} \left[\ln\frac{s}{\Delta_{12}} + \frac{1}{2}\ln\frac{\Delta_{1q}}{\Delta_1} - \frac{\arctan U}{U}\right].$$

$U = \sqrt{\frac{4\Delta_1\Delta_{1q}}{(t+\Delta_1+\Delta_{1q})^2} - 1} \geq 0$. The answer is well behaved.

Note

In the limit $t = 0$, the arctangent piece is equal to 1.

- Similar to the one loop case, the $(G, \bar{n})$ mode yields an identical result. The overlap shows up once again!

- The overlap integral must contain the Regge log. Indeed:

$$\text{Im}\,\mathcal{M}^{(2)}_{nG/G\bar{n}} = \frac{g^6}{256\pi^5}\frac{1}{s}\int \frac{d^2\boldsymbol{l}_1 d^2\boldsymbol{l}_2}{\Delta_1\Delta_2\Delta_{1q}\Delta_{2q}}\ \ln\frac{s}{\Delta_{12}}$$

- Matching works. We have the correct set of modes:

$$\text{Im}\,\mathcal{M}^{(2)}_{\text{SCET}_\text{G}}(t=0) = \frac{g^6}{256\pi^5}\int \frac{d^2\boldsymbol{l}_1 d^2\boldsymbol{l}_2}{s\ (\Delta_1\Delta_2)^2}\left[\ln\frac{s}{\Delta_{12}} - 2\right]$$
$$= \text{Im}\mathcal{M}^{(2)}_{\text{QCD}}(t=0)$$

Lesson 5

Regge physics lives in the n-fold overlap of the graphs with on-shell intermediate states. At n loops, we have a total of n graphs with a **Glauber cell** connecting n-collinear momenta to $\bar{n}$-collinear ones.

- Let us move to the 3-loop ladder and test this program.

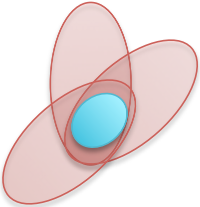
- At three loops, we have three graphs $(G,\bar{n},\bar{n}),(n,G,\bar{n}),(n,n,G)$.
- The 3-fold overlap between the three modes is rather simple:

$$\text{Im}\,\mathcal{M}^{(3)}_{nnG/nG\bar{n}/G\bar{n}\bar{n}} = \frac{g^8}{512\pi^8}\frac{1}{2^3}\frac{1}{s}\int d^2\boldsymbol{l}_{1\perp}\,d^2\boldsymbol{l}_{2\perp}\,d^2\boldsymbol{l}_{3\perp}\,\frac{I_{l_1^- l_2^-}}{\Delta_1\Delta_2\Delta_3\Delta_{1q}\Delta_{2q}\Delta_{3q}}.$$

$$I_{l_1^- l_2^-} = \int_{\Delta/\sqrt{s}}^{\sqrt{s}}\frac{dl_2^-}{l_2^-}\int_{l_2^-}^{\sqrt{s}}\frac{dl_1^-}{l_1^-} \approx \frac{1}{2}\ln^2 s.$$

- The logic generalizes in a straightforward way to n-loops. The expanded propagators become transverse, and the remaining nested integral yields the Regge log with the correct numerical factor.

- The **strongly ordered** region is a common subregion in any mode that forces the intermediate states on-shell:



- The overlap of all these modes; the (**Regge mode**), is precisely what yields the Regge logs and thus this is where Regge behavior lives.

Outline

- Regge behavior in field theory is very important from a phenomenology standpoint as it converts logarithms into powers of the energy.
- So far, Regge behavior has not been easy to understand within SCET.
- Using the method of regions, we were able to isolate the momentum modes responsible for Regge behavior in a scalar toy theory. Albeit technical details concerning regularization, we formed a good picture of the underlying physics.
- We found that Glauber modes are very **efficient** in describing Regge behavior.
- In the future, we hope to work with a full EFT appropriate for describing Regge behavior and apply it to phenomenological applicaions.

Thank you for listening.