

# Glauber Gluons and Multiple Parton Interactions.

Jonathan Gaunt, DESY

SCET 2015, Sante Fe, NM, 25 March 2015



Based on **JHEP 1407 (2014) 110**

And **work in progress** together with

Markus Diehl, Daniel Ostermeier, Peter Plössl and Andreas Schäfer

# Outline

- Brief discussion of Glaubers and traditional Collins-Soper-Sterman method of factorisation analysis.
- Analysis of Glauber gluons for the observable  $E_T$ , with  $E_T \ll Q$ , and demonstration of lack of cancellation at the level of two Glauber gluon exchanges between spectators (plus review of successful cancellation for  $p_T$ ).
- Connection of these noncancelled Glauber exchange diagrams to MPI.
- Other MPI sensitive variables for which 'standard' factorisation fails.
- Discussion of Glauber modes in double parton scattering (DPS) – do Glauber modes cancel here?



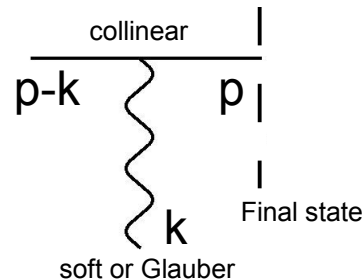
# Glauber modes

Glauber modes are soft modes satisfying:

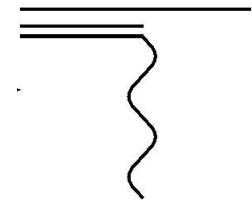
$$|k^+ k^-| \ll \mathbf{k}_T^2 \ll Q^2$$

They can potentially give leading power contributions to observables, and they cause problems for factorisation:

Simple example:



We want to strip the soft attachment from the collinear and replace by a Wilson line:



Look at this propagator denominator:  $(p - k)^2 = -2p \cdot k + k^2$

▲  
Eikonal piece

If gluon is soft, it is legitimate to only take eikonal piece, but this is not possible if it is Glauber

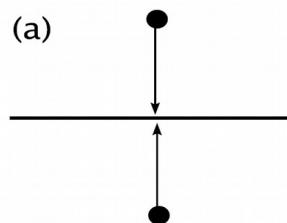
# CSS Factorisation Analysis

Will analyse Glauber contribution to several observables using traditional Collins Soper Sterman (CSS) methodology

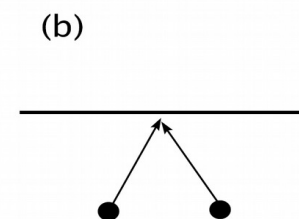
Brief review:

Want to identify non-UV leading power regions of Feynman graphs – i.e. small regions around the points at which certain particles go on shell, which despite being small are leading due to propagator denominators blowing up.

More precisely, need to find regions around **pinch singularities** – these are points where propagator denominators pinch the contour of the Feynman integral.

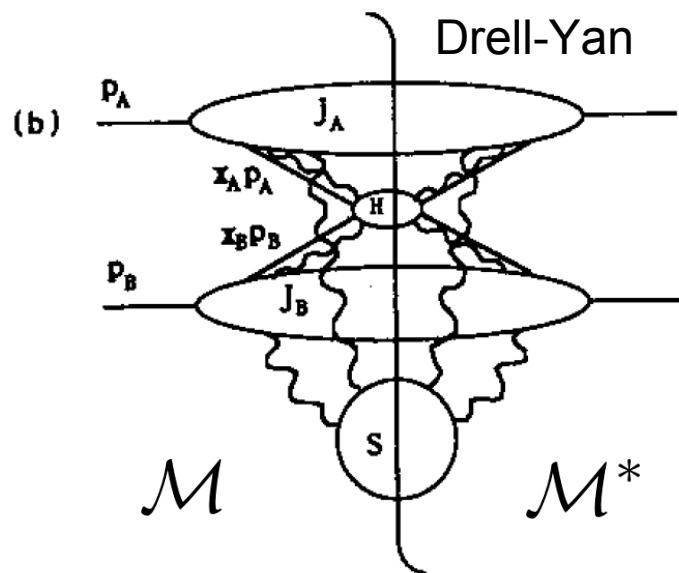


Pinched



Non-pinched

# CSS Factorisation Analysis

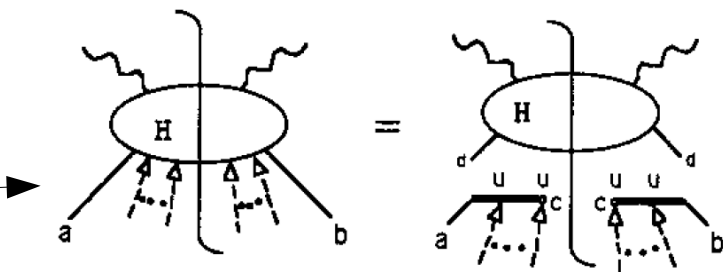


Pinch singularities in Feynman graphs correspond to physically (classically) allowed processes.

## Coleman-Norton theorem

Also need to do a power-counting analysis to determine if region around a pinch singularity is leading

Decouple all the connections between sectors using appropriate projectors (for physically polarised partons) and Ward identities.



Can't do this for Glauber's of course – either must provide an argument to show that they cancel (CSS) or find a novel way to include them in a factorisation framework (see previous talks).

# Hadronic Transverse Energy

First observable to consider: hadronic transverse energy  $E_T = \sum_{i \in X} \sqrt{p_{Ti}^2 + m_i^2}$  in  $pp \rightarrow V + X$

Resummation/factorisation formula for  $E_T$  in absence of Glaubers obtained by Papaefstathiou, Smillie, Webber:

$$\left[ \frac{d\sigma_H}{dQ^2 dE_T} \right]_{\text{res.}} = \frac{1}{2\pi} \sum_{a,b} \int_0^1 dx_1 \int_0^1 dx_2 \int_{-\infty}^{+\infty} d\tau e^{-i\tau E_T} f_{a/h_1}(x_1, \mu) f_{b/h_2}(x_2, \mu) \cdot W_{ab}^H(x_1 x_2 s; Q, \tau, \mu)$$

$$W_{ab}^H(s; Q, \tau, \mu) = \int_0^1 dz_1 \int_0^1 dz_2 C_{ga}(\alpha_S(\mu), z_1; \tau, \mu) C_{gb}(\alpha_S(\mu), z_2; \tau, \mu) \delta(Q^2 - z_1 z_2 s) \cdot \sigma_{gg}^H(Q, \alpha_S(Q)) S_g(Q, \tau) \cdot$$

Papaefstathiou, Smillie, Webber JHEP 1004 (2010) 084  
Grazzini, Papaefstathiou, Smillie, Webber arXiv:1403.3394

Contains effects of **collinear**, **soft**, **hard radiation**, could be rearranged into SCET-style **HxJxS form**:

$$\frac{d\sigma}{dE_T} = \sigma_0 H_{gg}(m_H, \mu) \int dY \int dk_a dk_b \times B_g(m_H, k_a, x_a, \mu, \nu) B_g(m_H, k_b, x_b, \mu, \nu) \times S_T^{gg}(E_T - k_a - k_b, \mu, \nu) \cdot$$

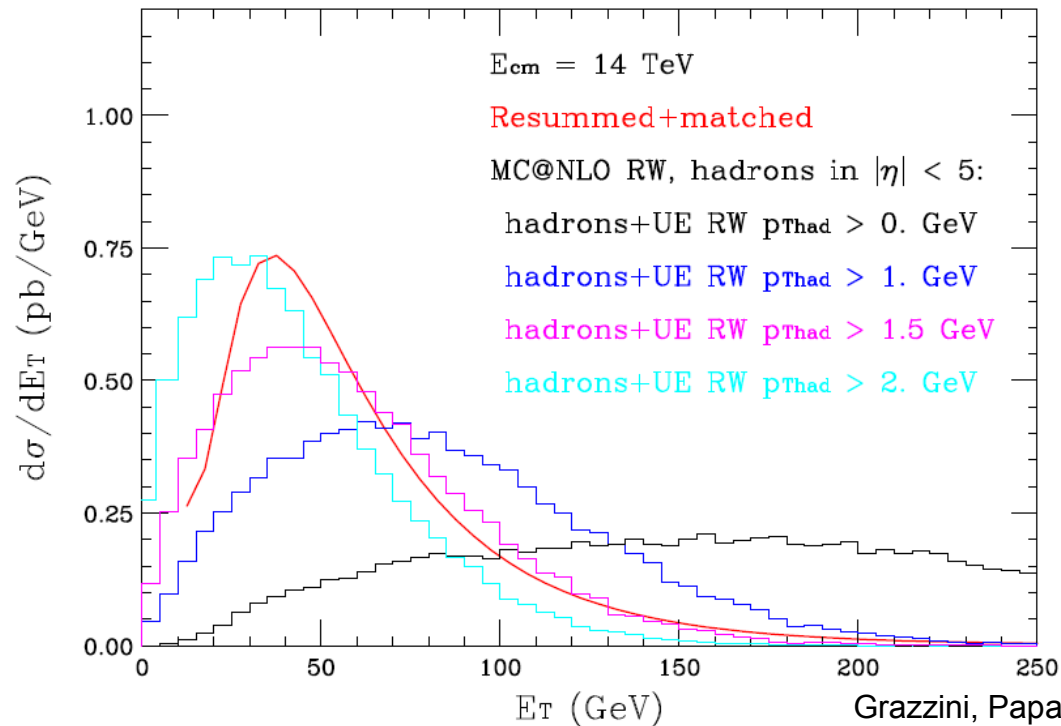
Tackmann, Walsh, Zuberi Phys. Rev. D 86 (2012) 053011

Does this give the full **leading contribution** (including all modes)?



# Hadronic Transverse Energy

Monte Carlo study with Herwig++:



Grazzini, Papaefstathiou, Smillie, Webber  
arXiv:1403.3394

Apparently not! Once we turn on UE, event shape **completely changes**

Can we see this from a factorisation point of view? Must be related to **Glauber gluons**.



# Model Setup

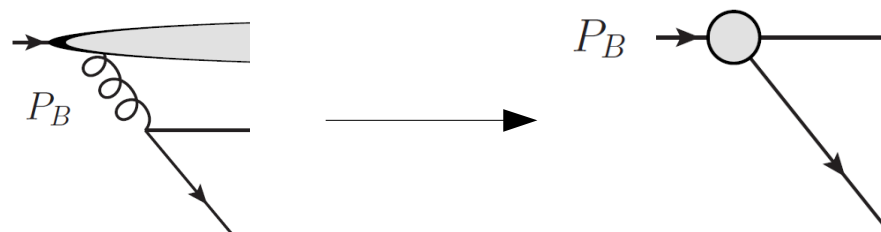
We will see if the effects of Glauber gluons can be cancelled for  $E_T$ , when  $E_T \ll Q$ . Will consider in parallel the observable  $p_T$  for which Glauber cancellation has been shown by CSS. Collins Soper Serman Nucl. Phys. B308 (1988) 833.

Model setup: Each 'proton' is composed from a **quark-antiquark pair**

Central 'hard process' is  $q\bar{q} \rightarrow V$  with  $V$  **colourless** and associated scale  $Q$

Assume momentum of proton A mainly along  $p_+$  direction and that of B mainly along  $p_-$  direction, but with small masses. All partons taken to be **massless**.

We assume little about the coupling of the quark-antiquark pair to the 'proton' – could either represent some soft nonperturbative coupling (appropriate when  $E_T \sim \Lambda$ ) or the perturbative quark-antiquark-gluon coupling (appropriate when  $E_T$  is perturbative)

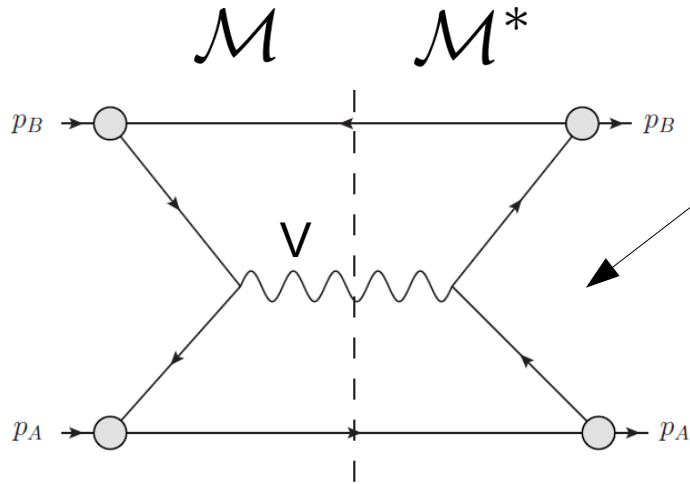


Calculation follows closely discussion in Collins pQCD book



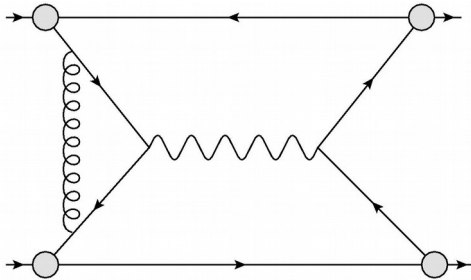


# One-Glauber Exchange

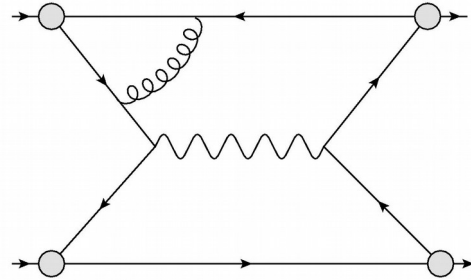


Lowest order 'parton model' process for  $p + p \rightarrow V + X$

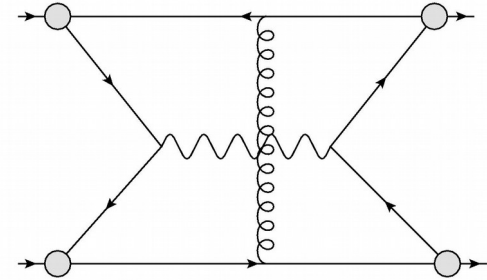
Consider one gluon corrections to this picture. Various possibilities:



Active-active interaction

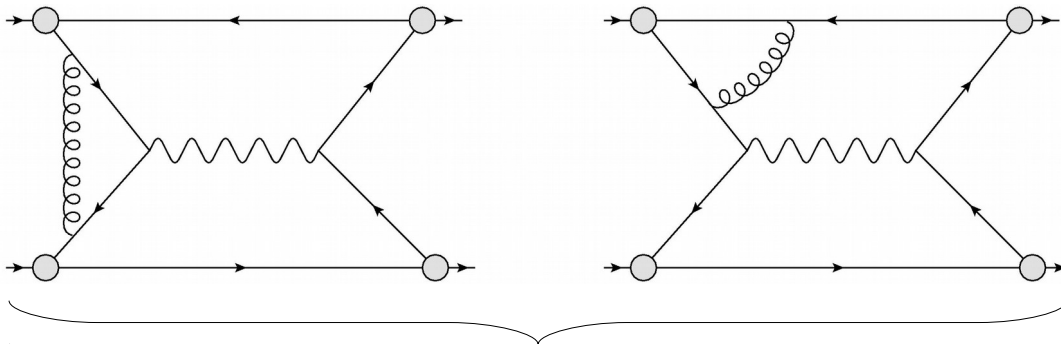


Active-spectator interaction



Spectator-spectator interaction

# One-Glauber Exchange



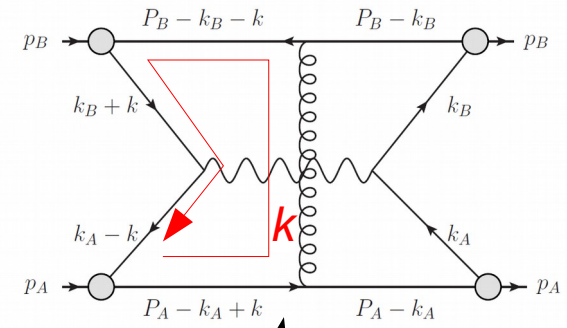
For these graphs, gluon momentum is not trapped in the Glauber region however → can deform loop integration contour and handle these using usual soft/collinear/hard regions.

Collins, Phys.Rev. D57 (1998) 3051–3056  
Collins, Metz, Phys.Rev.Lett. 93 (2004) 252001

$$\int \frac{dk^+ dk^-}{(2\pi)^2} \frac{\text{numerator}}{2k^+ k^- - \mathbf{k}_T^2 + i0}$$

$$\times \frac{1}{[-2k^+(P_B^- - k_B^-) + \dots + i0][2k^+ k_B^- + \dots + i0]}$$

$$\times \frac{1}{[-2k^- k_A^+ + \dots + i0][2k^-(P_A^+ - k_A^+) + \dots + i0]}$$



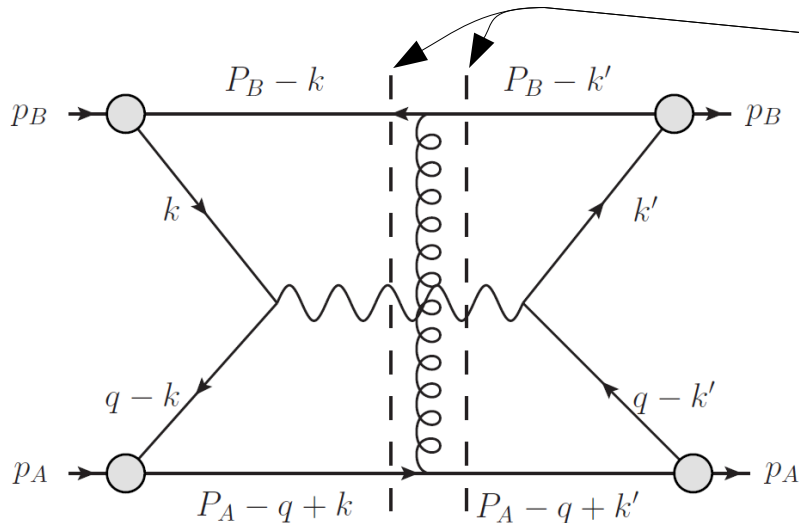
In this graph gluon is trapped in Glauber region

Traps  $k^+$  small

Traps  $k^-$  small



# One-Glauber Exchange



Two possible cuts of graph that leave gluon in Glauber region (cut through gluon forces it into central soft)

Consider case where cut is to the right, and consider  $k^+$  integral.

In top half of graph can ignore  $q^- - k^-$  compared to large components ( $q^-, P_B^- - q^-$ )

In bottom half of graph can ignore  $k^+$  compared to large components ( $q^+, P_A^+ - q^+$ )

In gluon propagator can ignore lightcone components of  $k$  compared to transverse components

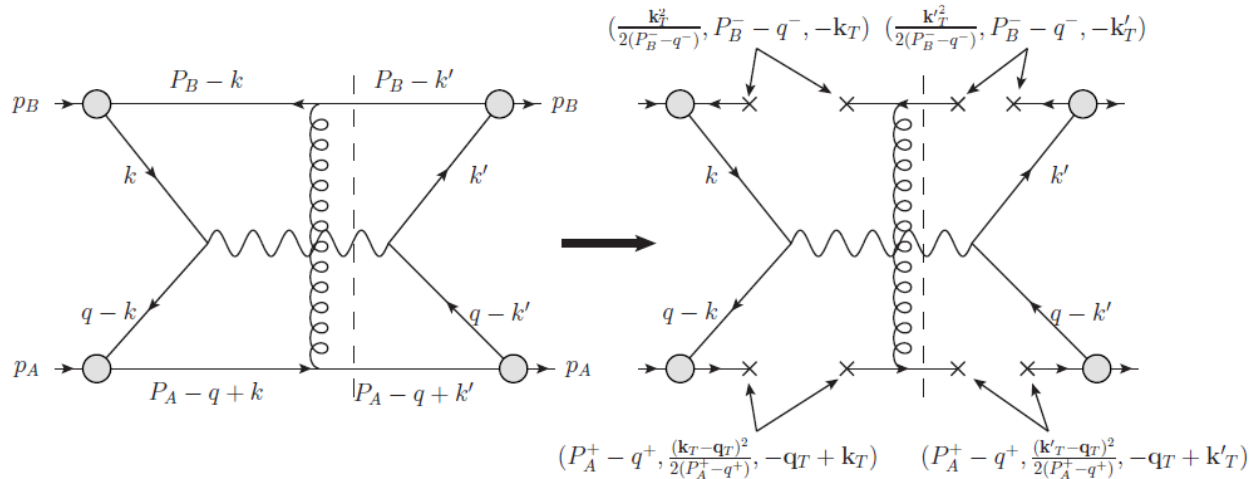
$$\begin{aligned} & \int \frac{dk^+}{2\pi} \frac{i}{(2q^- k^+ - \mathbf{k}_T^2 + i0)} \frac{i}{(2(-k^+ + P_B^+))(P_B^- - q^-) - \mathbf{k}_T^2 + i0} \\ &= \frac{i}{2(P_B^- - q^-)} \frac{i}{2q^- k^+_{\text{on-shell}} - \mathbf{k}_T^2 + i0} \\ &= \int \frac{dk^+}{2\pi} \frac{i}{(2q^- k^+ - \mathbf{k}_T^2 + i0)} 2\pi \delta(2(-k^+ + P_B^+)(P_B^- - q^-) - \mathbf{k}_T^2 + i0) \end{aligned}$$

Net effect – set  $P_B - k$  line on shell!



# One-Glauber Exchange

Repeat with  $k^-$ ,  $k'^+$ ,  $k'^-$  integrations:



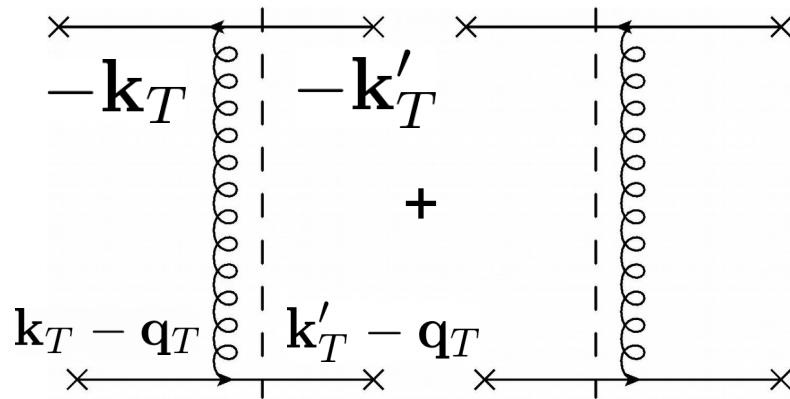
Factor out on-shell Glauber exchange graph!

Can do a similar procedure when cut is on left.

# Measured Transverse Momentum

Consider case where we measure  $p_T$  of  $V$ . For given momenta in the two decomposed graphs, the value of the measurement is the **same**.

Therefore, we can factor out the parton model graph and measurement and add together the two Glauber subgraphs:



$$\mathcal{M} = -iL \left\{ \begin{array}{l} \text{Graph level: } L(\mathbf{k}_T \rightarrow \mathbf{k}'_T) + L^*(\mathbf{k}'_T \rightarrow \mathbf{k}_T) \\ \text{Amplitude level: } i\mathcal{M}(\mathbf{k}_T \rightarrow \mathbf{k}'_T) - i\mathcal{M}^*(\mathbf{k}'_T \rightarrow \mathbf{k}_T) \end{array} \right\} = 0$$

**CUTKOSKY RULE**



# Measured Hadronic Transverse Energy

For  $E_T$ , can't do the same thing – it equals  $|\mathbf{k}_T| + |\mathbf{q}_T - \mathbf{k}_T|$  for cut to left and  $|\mathbf{k}'_T| + |\mathbf{q}_T - \mathbf{k}'_T|$  for cut to right. Can still arrange cancellation by change of transverse variables:



Parton model graph

$$\int d^{d-2}\mathbf{k}_T d^{d-2}\mathbf{k}'_T f_P(\mathbf{k}_T) f_P^*(\mathbf{k}'_T) L(\mathbf{k}_T \rightarrow \mathbf{k}'_T) \delta(E_T = |\mathbf{k}'_T| + |\mathbf{k}'_T - \mathbf{q}_T|) +$$

$$\int d^{d-2}\mathbf{k}_T d^{d-2}\mathbf{k}'_T f_P(\mathbf{k}_T) f_P^*(\mathbf{k}'_T) L^*(\mathbf{k}'_T \rightarrow \mathbf{k}_T) \delta(E_T = |\mathbf{k}_T| + |\mathbf{k}_T - \mathbf{q}_T|)$$

↓ Relabel  $\mathbf{k}_T \leftrightarrow \mathbf{k}'_T$  in second term

$$\int d^{d-2}\mathbf{k}_T d^{d-2}\mathbf{k}'_T \delta(E_T = |\mathbf{k}'_T| + |\mathbf{k}'_T - \mathbf{q}_T|) \times$$

$$[f_P(\mathbf{k}_T) f_P^*(\mathbf{k}'_T) L(\mathbf{k}_T \rightarrow \mathbf{k}'_T) + f_P(\mathbf{k}'_T) f_P^*(\mathbf{k}_T) L^*(\mathbf{k}_T \rightarrow \mathbf{k}'_T)]$$

↓  $L(\mathbf{k}_T \rightarrow \mathbf{k}'_T) \propto 1/(\mathbf{k}_T - \mathbf{k}'_T)^2 = L(\mathbf{k}'_T \rightarrow \mathbf{k}_T)$  &  $f_P = -f_P^*$

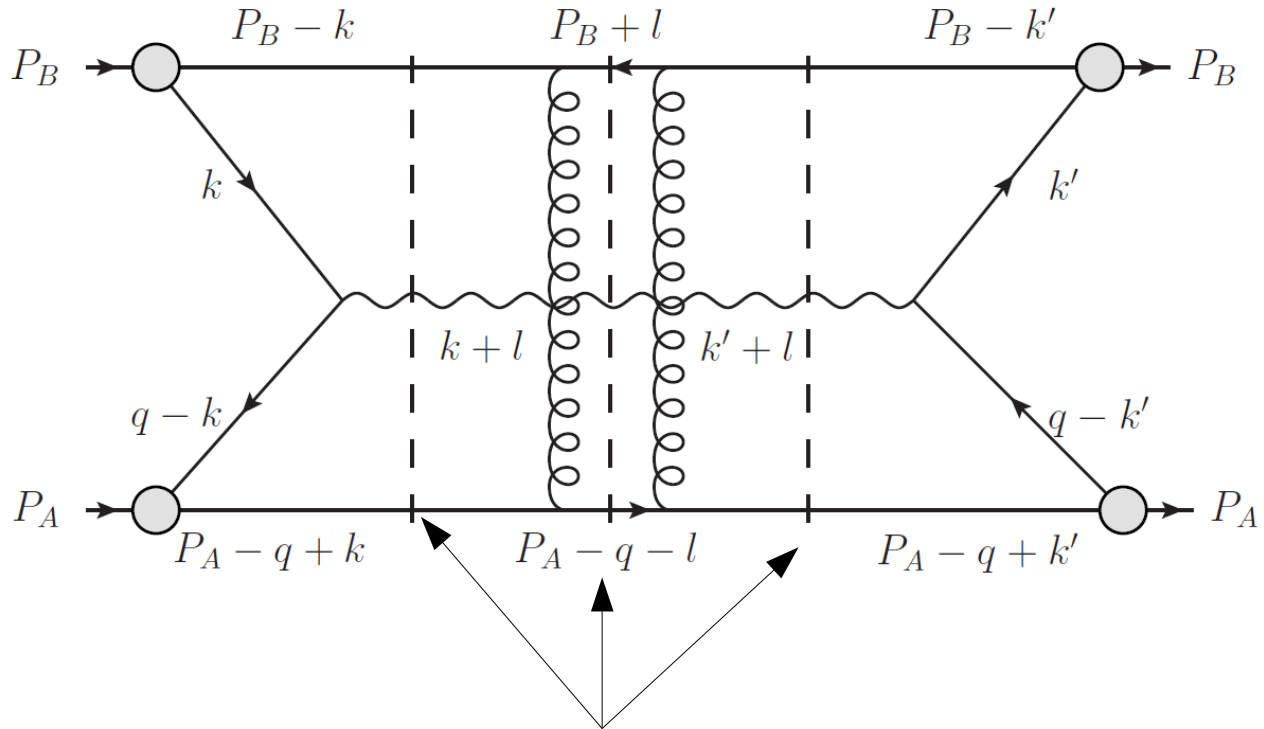
$$\int d^{d-2}\mathbf{k}_T d^{d-2}\mathbf{k}'_T \delta(E_T = |\mathbf{k}'_T| + |\mathbf{k}'_T - \mathbf{q}_T|) \times$$

$$f_P(\mathbf{k}_T) f_P^*(\mathbf{k}'_T) [L(\mathbf{k}_T \rightarrow \mathbf{k}'_T) + L^*(\mathbf{k}'_T \rightarrow \mathbf{k}_T)] \longrightarrow \mathbf{0}$$



# Two-Glauber Exchange

Now add in one more (Glauber) gluon between spectators:

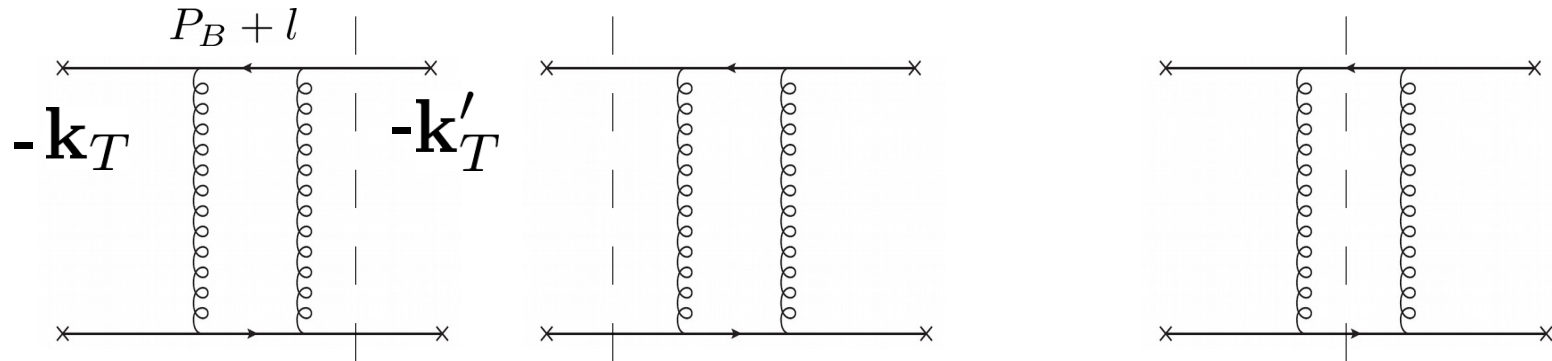


Now 3 cuts that leave both gluons in Glauber region

Factor out Glauber subgraphs as before.

# Measured Transverse Momentum

$p_T$  measured: Again, for given momenta in (decomposed) graphs, measurement is the same – factor measurement and parton model graphs, and combine Glauber subgraphs:



$$L(\mathbf{k}_T \rightarrow \mathbf{k}'_T; l) + L^*(\mathbf{k}'_T \rightarrow \mathbf{k}_T; l) + \int \Phi_2 L(\mathbf{k}_T \rightarrow \mathbf{l}_T) L^*(\mathbf{k}'_T \rightarrow \mathbf{l}_T)$$

$$i\mathcal{M}(\mathbf{k}_T \rightarrow \mathbf{k}'_T; l) - i\mathcal{M}^*(\mathbf{k}'_T \rightarrow \mathbf{k}_T; l) + \int \Phi_2 \mathcal{M}(\mathbf{k}_T \rightarrow \mathbf{l}_T) \mathcal{M}^*(\mathbf{k}'_T \rightarrow \mathbf{l}_T)$$

-Imaginary Part

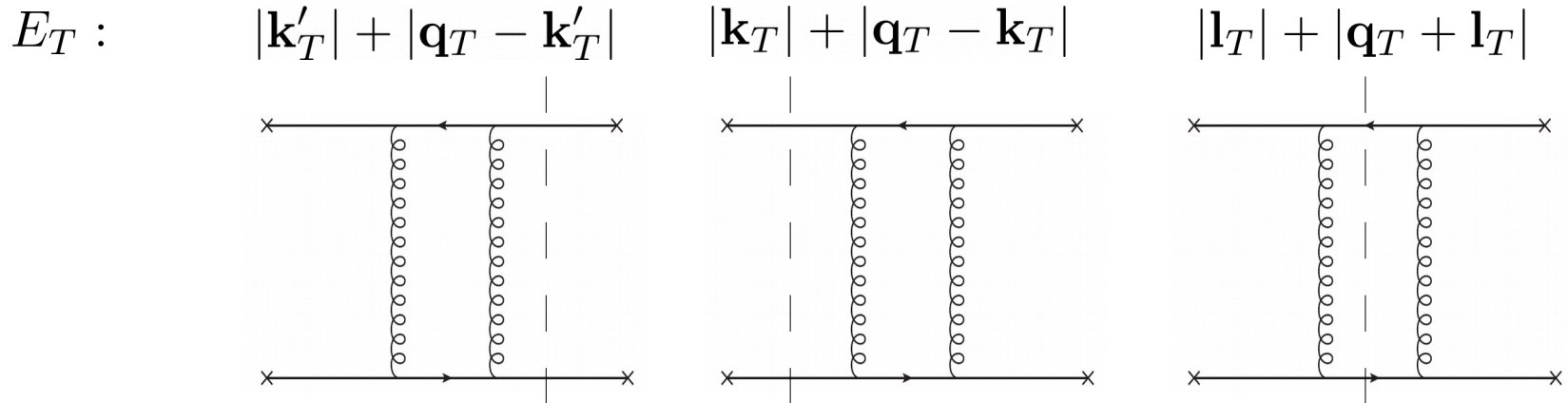
Sum over (1) cuts

=0 using Cutkosky rules



# Measured Hadronic Transverse Energy

For  $E_T$  cancellation of Glauber subgraphs fails, because  $E_T$  for central/real cut depends on loop momentum, whilst same is not true for external/absorptive cuts:



Maybe some change of loop and external variables is possible to arrange cancellation?

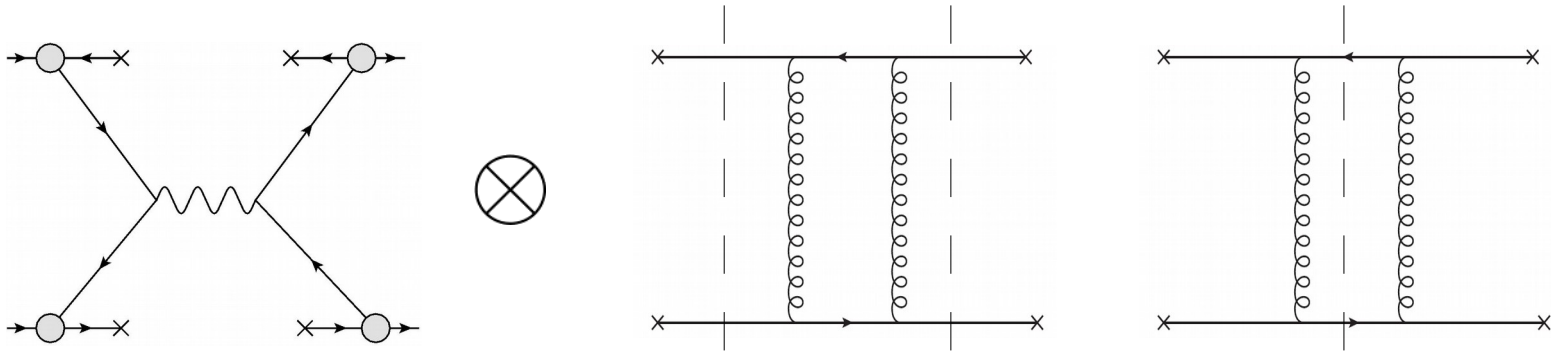
I argue no such change of variables is possible. Cutkosky cancellation is one that occurs **point by point** in spatial momentum – should match parameterisation of loop momentum between graphs.

Sterman, hep-ph/9606312



# Glauber Gluons and MPI

The factorisation breaking for  $E_T$  is associated with:



But this can be interpreted as:

**Primary hard interaction**

**Secondary low scale absorptive process**  
(‘MPI did not occur’)

**Secondary low scale scattering**  
(‘MPI occurred’)

Factorisation breaking effects **are due to MPI**, as was also found in MC studies. Close connection between **Glauber gluons and MPI!**

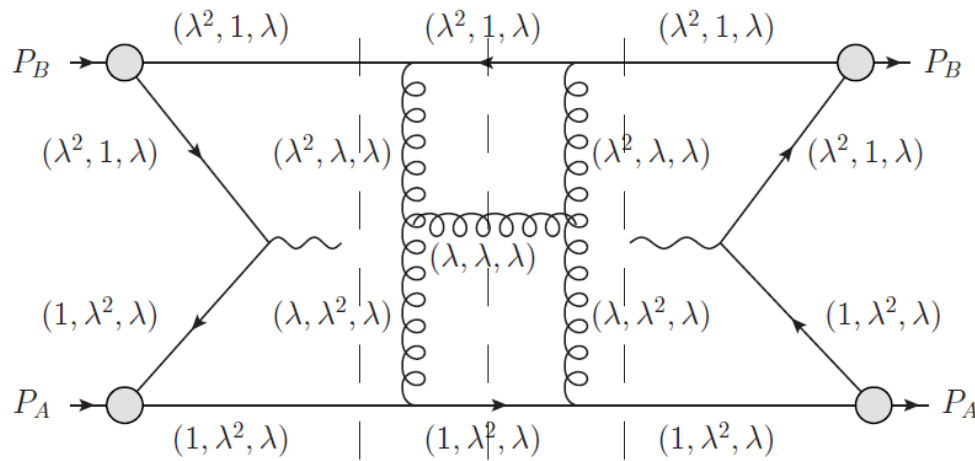
Absorptive process included in MCs via unitarity constraints.

NOTE: Contribution from Glauber region cancels for  $p_T$  because  $p_T$  of  $V$  is **insensitive** to whether extra interactions occurred or not, NOT because MPI is cancelled.

# Glauber Gluons and Regge Physics

See also talks by I. Rothstein, B. Mahmoud El-Melnoufi, and S. Fleming

Also achieve a leading power contribution by inserting central soft rung between Glauber verticals:



3 cuts again don't cancel for  $E_T$

This graph suppressed by additional power of  $\alpha_s$  compared to zero-rung graph, but **enhanced by rapidity (BFKL) logarithm**. Can insert arbitrary number of rungs (forming **Pomeron** type object) and still be at leading log order in BFKL sense.

→ should need good control of BFKL effects in MPI to describe  $E_T$  well.

# Other observables

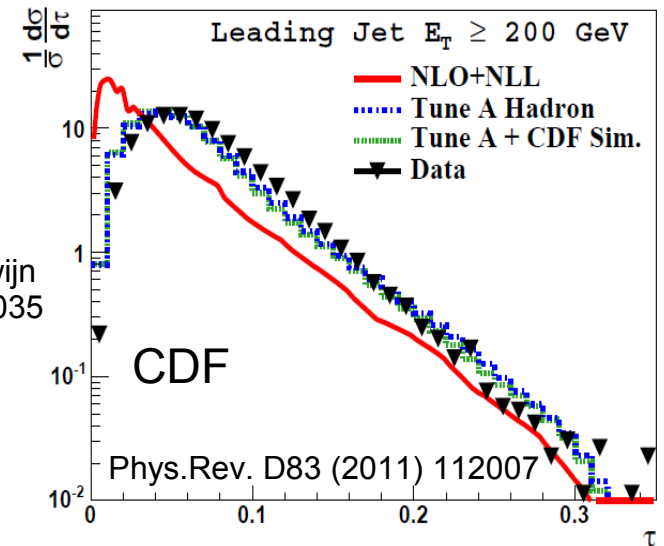
Same effect should occur for any other observable which is sensitive to whether an additional interaction occurred or not (**MPI sensitive observables**)

For example:

$$\text{Beam thrust } B_a = \sqrt{2} \sum_{i \in X, a} p_i \cdot p \quad B_b = \sqrt{2} \sum_{i \in X, b} p_i \cdot n$$

Stewart, Tackmann, Waalewijn  
Phys. Rev. D 81 (2010) 094035

$$\text{Transverse thrust } \max_{\mathbf{n}_{iT}} \sum_i |\mathbf{q}_{iT} \cdot \mathbf{n}_{iT}| / \sum_i |\mathbf{q}_{iT}|$$



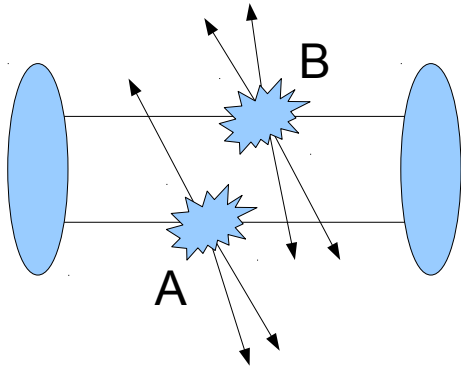
Usually global observables. Jet observables much less MPI sensitive as particles from MPI only collected up over jet radius  $\rightarrow$  MPI suppressed by radius  $R$ .

It was noted earlier that underlying event, or MPI effects are suppressed by jet radius  $R$

M. Dasgupta, L. Magnea, and G. P. Salam, JHEP 0802 (2008) 055  
Tackmann, Waalewijn, Stewart, Phys. Rev. Lett. 114, 092001 (2015)



# Double Parton Scattering



Double parton scattering (DPS) is where we have two hard interactions in one proton-proton collision.

DPS can be a background to rare (e.g. new physics) signals, and reveals new information about proton – i.e. correlations between partons.

Measure inclusive cross section or cross section differential in  $p_T$ s

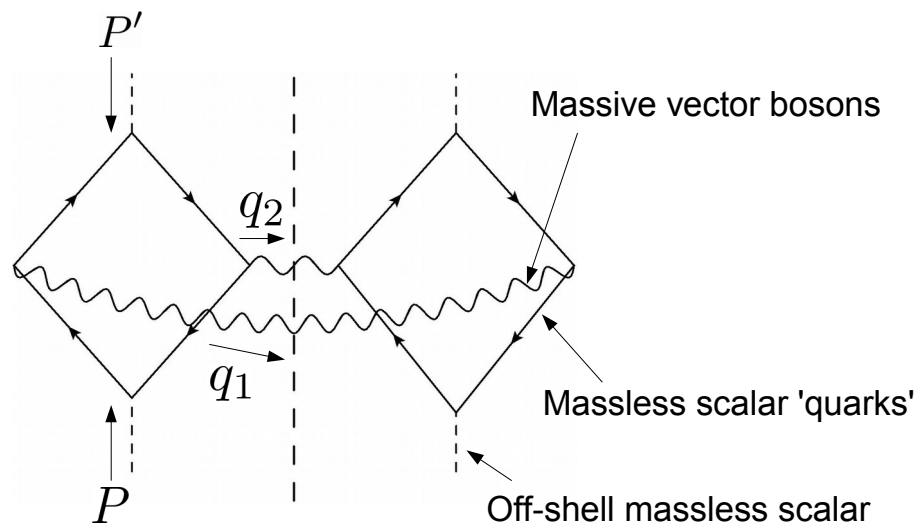
Of course there is also a (large) contribution to such observables from single scattering (SPS), plus some interplay between the two mechanisms.

This process is (funnily enough) MPI insensitive – doesn't care if further (soft) interactions occurred or not. Do Glauber effects cancel for this observable?

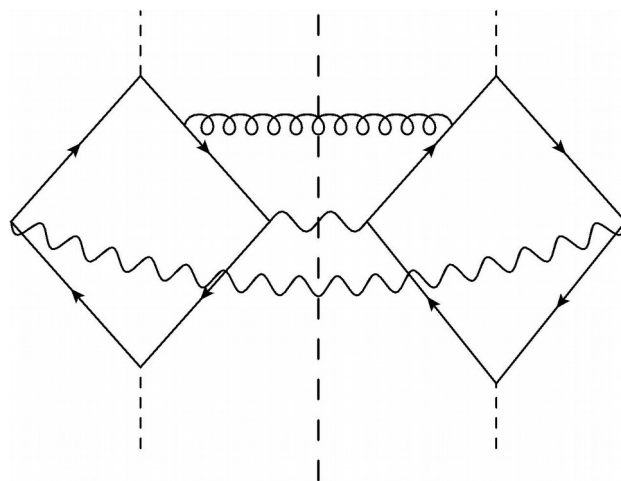
# Glauber in DPS – one loop model calculation

One loop model calculation

'Parton-model' process:



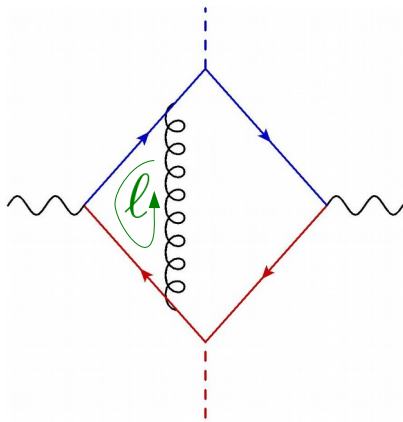
Real corrections:



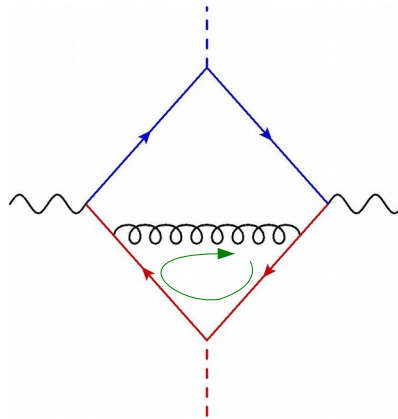
$$\propto [Tr(t^A)]^2 = 0$$

# Glauber in DPS – one loop model calculation

Virtual corrections:

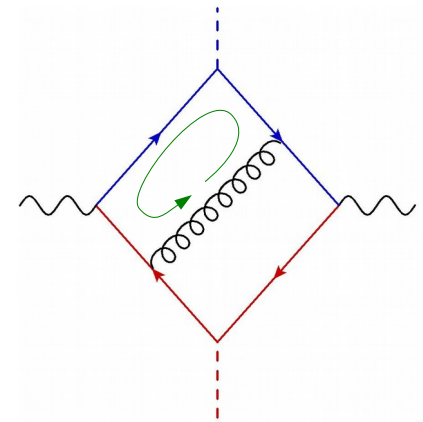


Neither  $l^+$  nor  $l^-$  is trapped small

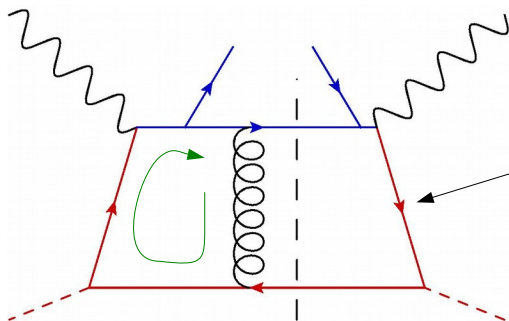


$l^-$  only is trapped small –  $l^+$  can be freely deformed away from origin (into region where  $l$  is collinear to  $P$ ).

'Topologically factored graph'



$l^+$  only is trapped small –  $l^-$  can be freely deformed away from origin (into region where  $l$  is collinear to  $P$ ).



Very similar to situation in SIDIS – no Glauber contribution there too.

More detailed checks that Glauber contributions are absent in the one-loop calculation will be in the upcoming paper.

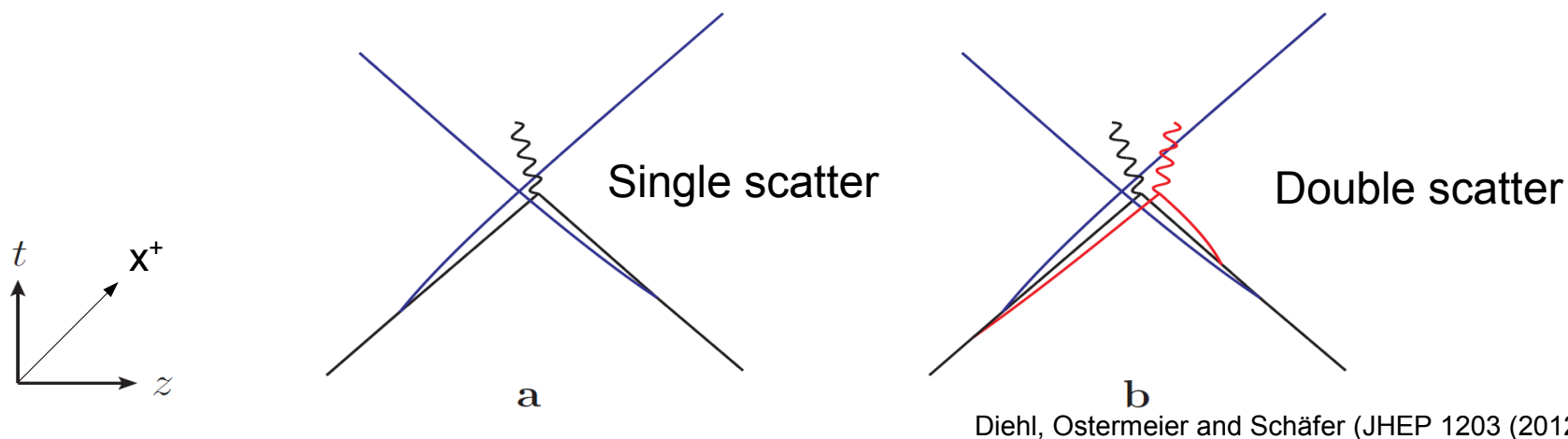


# Glauber in DPS – all-order analysis

All-order analysis can be done using the same method as used by CSS for single scattering – namely using by using light-cone perturbation theory (similar to old-fashioned time ordered perturbation theory, except applied along the direction of one of the beam jets.

Collins Soper Serman Nucl. Phys. B308 (1988) 833.

We find Glaubers also cancel for double scattering. Basic reason for this – spacetime structure of pinch surfaces for single and double scattering are rather similar:



More details will be in upcoming paper

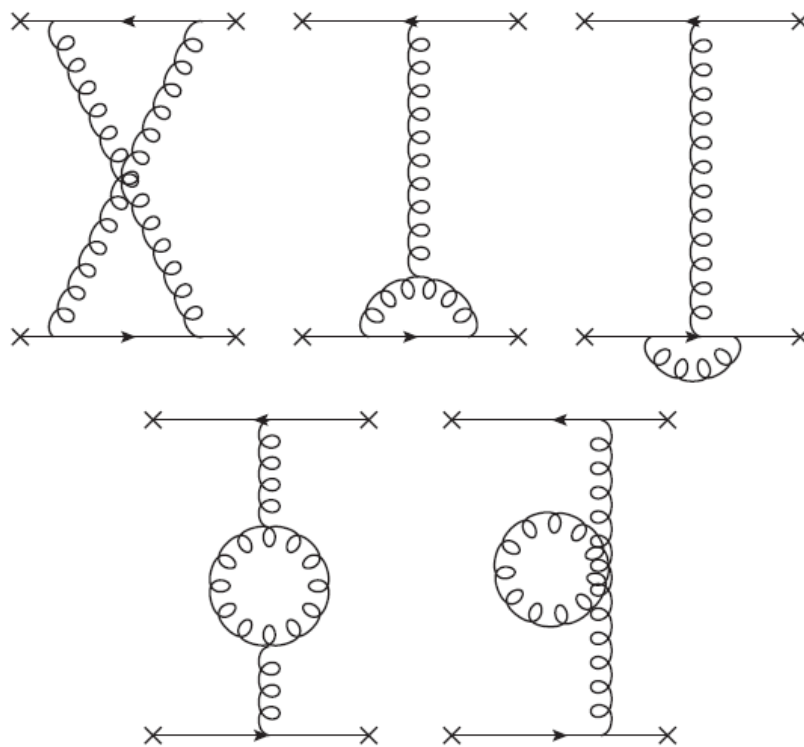


# Conclusions

- CSS-style cancellation of Glauber region fails for  $E_T \ll Q$ , at the level of **two Glauber gluons** exchanged between spectators.
- Can connect these diagrams to events with additional soft scatterings – connection between **Glauber gluons and MPI**.
- Standard factorisation with only collinear, soft and hard functions also fails for a wider class of MPI sensitive observables – e.g. beam thrust, transverse thrust.
- Double parton scattering observables are not MPI sensitive. Glauber cancellation appears to go through for double Drell-Yan similarly as for single Drell-Yan.



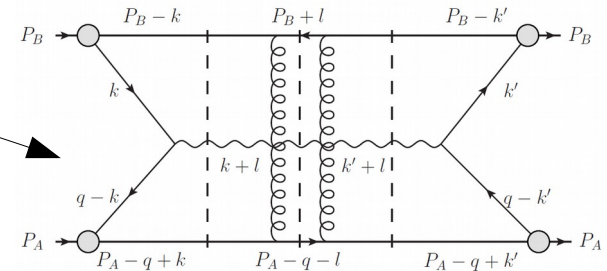
# Other two-gluon exchange graphs



# MPI sensitive variable of order of hard scale

What about when MPI sensitive observable  $O_S$  is of order of the hard scale  $Q$ ?  
 Then for the cumulant of  $O_S$ , are we inclusive enough that standard factorisation formula can be used?

Miscancellation of cuts in this graph now only smears observable by some power suppressed amount



The trouble is that we can have Glauber miscancellations on multiple spectator legs adding up to produce a big smearing of  $O_S$ , even when  $O_S \sim Q$

