The role of Glauber exchange in SCET and the BFKL equation

Sean Fleming, University of Arizona

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The Glauber interaction in SCET

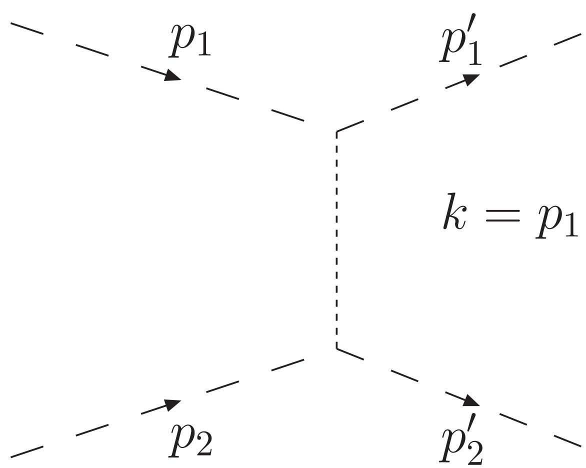
- Include Glauber interactions in SCET Lagrangian

F. Liu & J. P. Ma, arXiv:0802.2973; A. Idilbi & A. Majumder, Phys. Rev. D 80 (2009) 054022; S. Fleming Phys.Lett. B735 (2014) 266; I. Z. Rothstein & I. W. Stewart (in progress)

quark forward scattering $s \gg t\,, \lambda \sim \sqrt{\frac{s}{t}}$

$$p_1^\mu = \frac{\sqrt{s}}{2} n^\mu$$

$$p_1'^\mu \approx \frac{\sqrt{s}}{2}\, n^\mu + \frac{\vec{k}_\perp^2}{2\sqrt{s}} \bar{n}^\mu - k_\perp^\mu$$



$$k = p_1 - p_1' \approx k_\perp^\mu \sim \sqrt{t}$$

$$p_2^\mu = \frac{\sqrt{s}}{2} \bar{n}^\mu$$

$$p_2'^\mu \approx \frac{\vec{k}_\perp^2}{2\sqrt{s}}\, n^\mu + \frac{\sqrt{s}}{2}\, \bar{n}^\mu + k_\perp^\mu$$

The Glauber interaction in SCET

I. Z. Rothstein & I. W. Stewart (in progress)

Tree-level Matching:

$$\mathcal{A}_{QCD} = -\frac{g^2}{\vec{k}_\perp^2}\bar{u}(p_1')T^a\gamma^\mu u(p_1)\bar{u}(p_2')T^a\gamma_\mu u(p_2) \approx -\frac{n\cdot\bar{n}\,g^2}{\vec{k}_\perp^2}\bar{\xi}_n T^a\frac{\bar{n}\!\!\!/}{2}\xi_n\bar{\xi}_{\bar{n}}T^a\frac{n\!\!\!/}{2}\xi_{\bar{n}}$$

Tree-level SCET operator:

$$\mathcal{O}_G^{n\bar{n}} = -\frac{2\,g^2}{\vec{k}_\perp^2}\bar{\xi}_{p_1',n}T^a\frac{\bar{n}\!\!\!/}{2}\xi_{p_1,n}\bar{\xi}_{p_2',\bar{n}}T^a\frac{n\!\!\!/}{2}\xi_{p_2,\bar{n}}$$

SCET Operator with Wilson Lines:

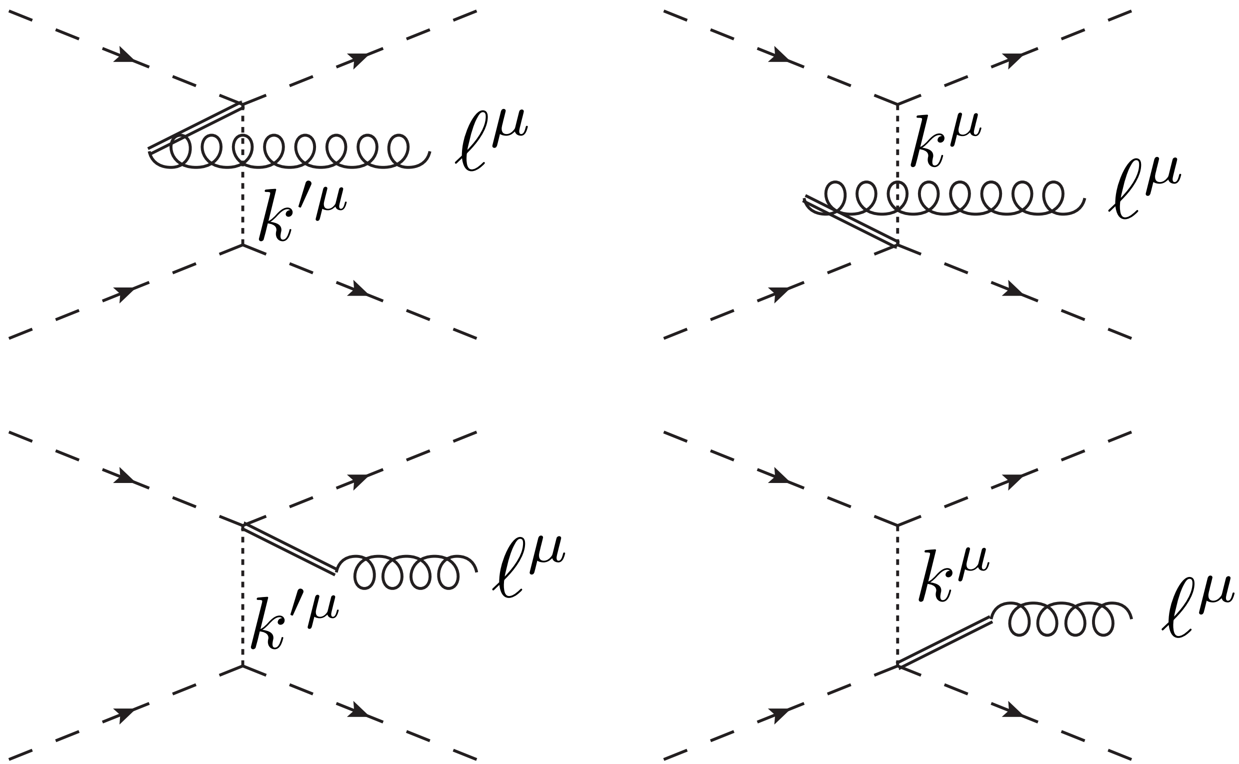
$$\mathcal{L}_G^{n\bar{n}} = -8\pi\,\alpha_s(\mu)\,\bar{\xi}_{p_2',\bar{n}}W_{\bar{n}}S_{\bar{n}}^\dagger T^a\frac{n\!\!\!/}{2}S_{\bar{n}}W_{\bar{n}}^\dagger\xi_{p_2,\bar{n}}\frac{1}{\vec{\mathcal{P}}_\perp^2}\bar{\xi}_{p_1',n}W_nS_n^\dagger T^a\frac{\bar{n}\!\!\!/}{2}S_nW_n^\dagger\xi_{p_1,n}$$

Not gauge invariant!

The Glauber interaction in SCET

- Ward identity not satisfied in the soft sector:

emission from
soft Wilson line



$$\ell^\mu = \frac{1}{2}\bar{n}\cdot k\, n^\mu - \frac{1}{2} n\cdot k'\,\bar{n}^\mu + (k-k')^\mu_\perp$$

$$m^\mu \propto \left(\frac{n^\rho}{n\cdot k'}\frac{1}{k'^2_\perp} + \frac{\bar{n}^\rho}{\bar{n}\cdot k'}\frac{1}{k^2_\perp} \right)$$

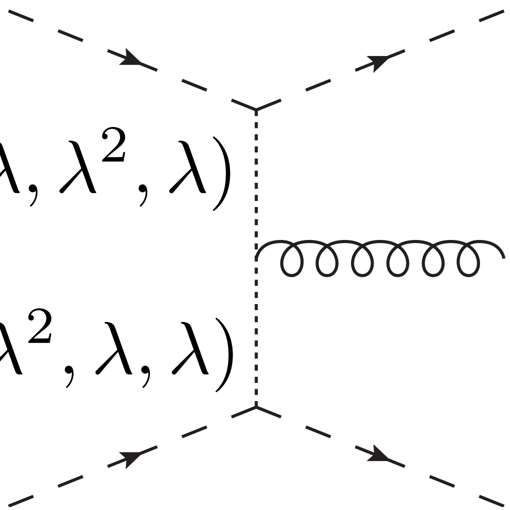
$$\longrightarrow \quad \ell\cdot m \propto \frac{1}{k'^2_\perp} - \frac{1}{k^2_\perp} \neq 0$$

The Glauber interaction in SCET

- Real emission from Glauber

$$k^\mu \sim (\lambda, \lambda^2, \lambda)$$

$$k'^\mu \sim (\lambda^2, \lambda, \lambda)$$



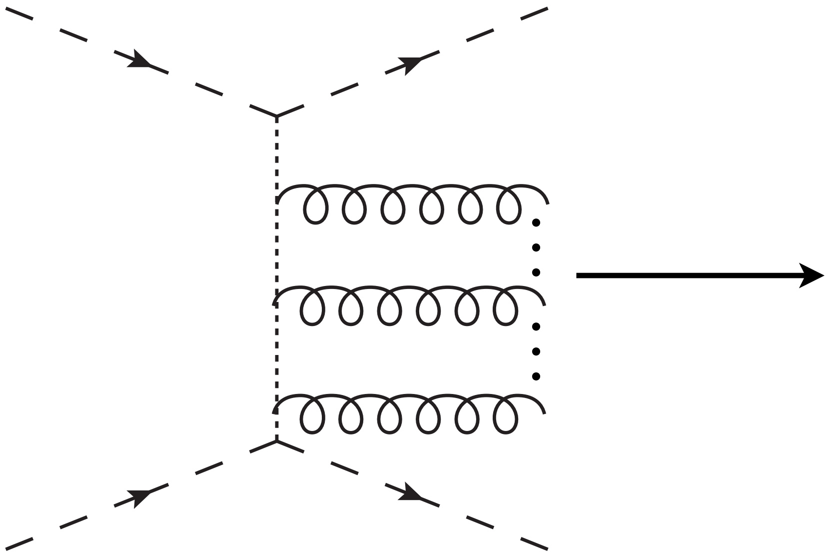
$$\ell^\mu = \frac{1}{2}\bar{n}\cdot k\, n^\mu - \frac{1}{2} n\cdot k'\, \bar{n}^\mu + (k - k')^\mu_\perp$$

- Add all soft emission to get the gauge invariant Lipton vertex

$$m^\rho_{\rm tot} \propto \left[\frac{1}{k_\perp^2}\frac{1}{k_\perp'^2}\left(k_\perp^\rho + k_\perp'^{\rho} - \frac{1}{2}\bar{n}^\rho n\cdot k' - \frac{1}{2} n^\rho \bar{n}\cdot k\right) + \left(\frac{n^\rho}{n\cdot k'}\frac{1}{k_\perp'^2} + \frac{\bar{n}^\rho}{\bar{n}\cdot k'}\frac{1}{k_\perp^2}\right)\right]$$

The Glauber interaction in SCET

- Implies real soft gluons from Glauber should be summed in $\mathcal{L}_G^{n\bar{n}}$



$$\mathcal{L}_G^{n\bar{n}\,\mathrm{summed}} = \quad ?$$

gauge invariant

The Glauber interaction in SCET

- Can matrix elements of the summed operator factor?

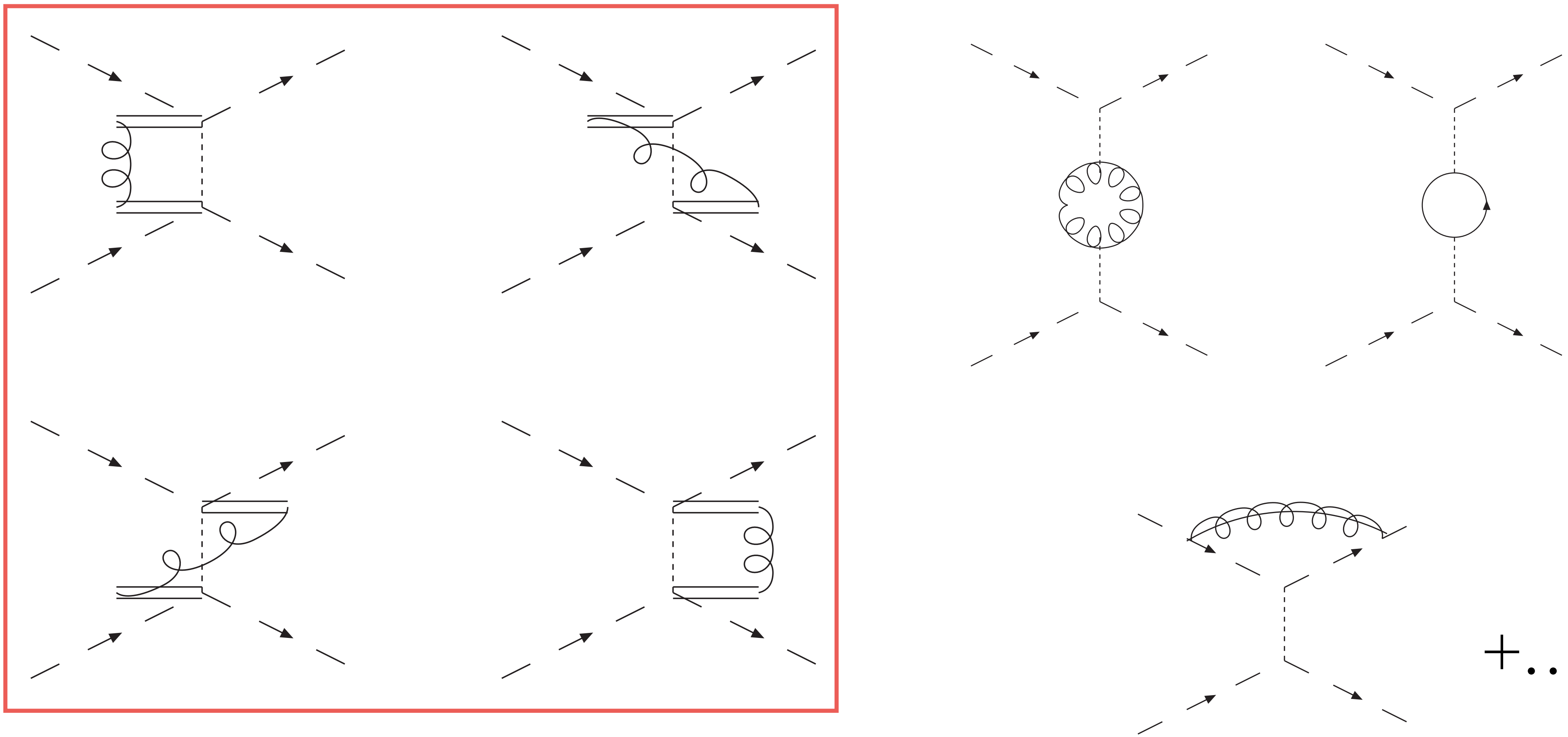
$$\langle \mathcal{L}_G^{n\bar{n}\,\mathrm{summed}} \rangle \overset{?}{=} \langle n\text{-collinear} \rangle \otimes \langle \bar{n}\text{-collinear} \rangle \otimes \langle \mathrm{soft} \rangle$$

- BFKL indicates yes…

The Glauber interaction in SCET and the BFKL equation

S. Fleming ; J. F. Donoghue, B. K. El-Menoufi, and G. Ovanesyan (2014), arXiv:1405.1731

- Renormalization of the Glauber interaction $\mathcal{L}_G^{n\bar{n}}$



The Glauber interaction in SCET and the BFKL equation

$$\mathcal{A} = -8\pi\alpha_s(\mu)\,\bar{\xi}_n T^a \frac{\bar{n}\!\!\!/}{2}\xi_n \bar{\xi}_{\bar{n}} T^a \frac{n\!\!\!/}{2}\xi_{\bar{n}} \left[i N_c \alpha_s(\mu) \mathcal{I}(\vec{k}_\perp) \right] + \text{counterterms}$$

$$\mathcal{I}(\vec{k}_\perp) = \int \frac{dq^-}{q^-} \int \frac{d^2 q_\perp}{(2\pi)^2} \frac{1}{\vec{q}_\perp^{\,2}} \frac{1}{(\vec{q}+\vec{k})_\perp^2}$$ Gluon Regge Trajectory

- Introduce rapidity regulator J.-Y. Chiu, A. Jain, D. Neill, and I. Z. Rothstein, Phys. Rev. Lett. 108, 151601 (2012) & JHEP 1205, 084 (2012)

$$\mathcal{I}(\vec{k}_\perp) = \frac{-2i}{(4\pi)^2} \frac{w(\nu)^2}{\vec{k}_\perp^2} \left[\frac{1}{\eta} \ln\left(\frac{\vec{k}_\perp^2}{m_g^2}\right) + \ln\left(\frac{\vec{k}_\perp^2}{4\nu}\right) \ln\left(\frac{\vec{k}_\perp^2}{m_g^2}\right) - \frac{1}{4}\ln^2\left(\frac{\vec{k}_\perp^2}{m_g^2}\right) + i\pi \ln\left(\frac{\vec{k}_\perp^2}{m_g^2}\right) \right]$$

- The counter term subtracts the $\frac{1}{\eta}$ pole

The Glauber interaction in SCET and the BFKL equation

- Renormalization of rapidity divergence gives rise to a rapidity RGE (R-RGE)

$$\frac{d}{d\ln\nu}\mathcal{L}_G^{n\bar{n}}(\nu) = \gamma(\nu)\mathcal{L}_G^{n\bar{n}}(\nu)$$

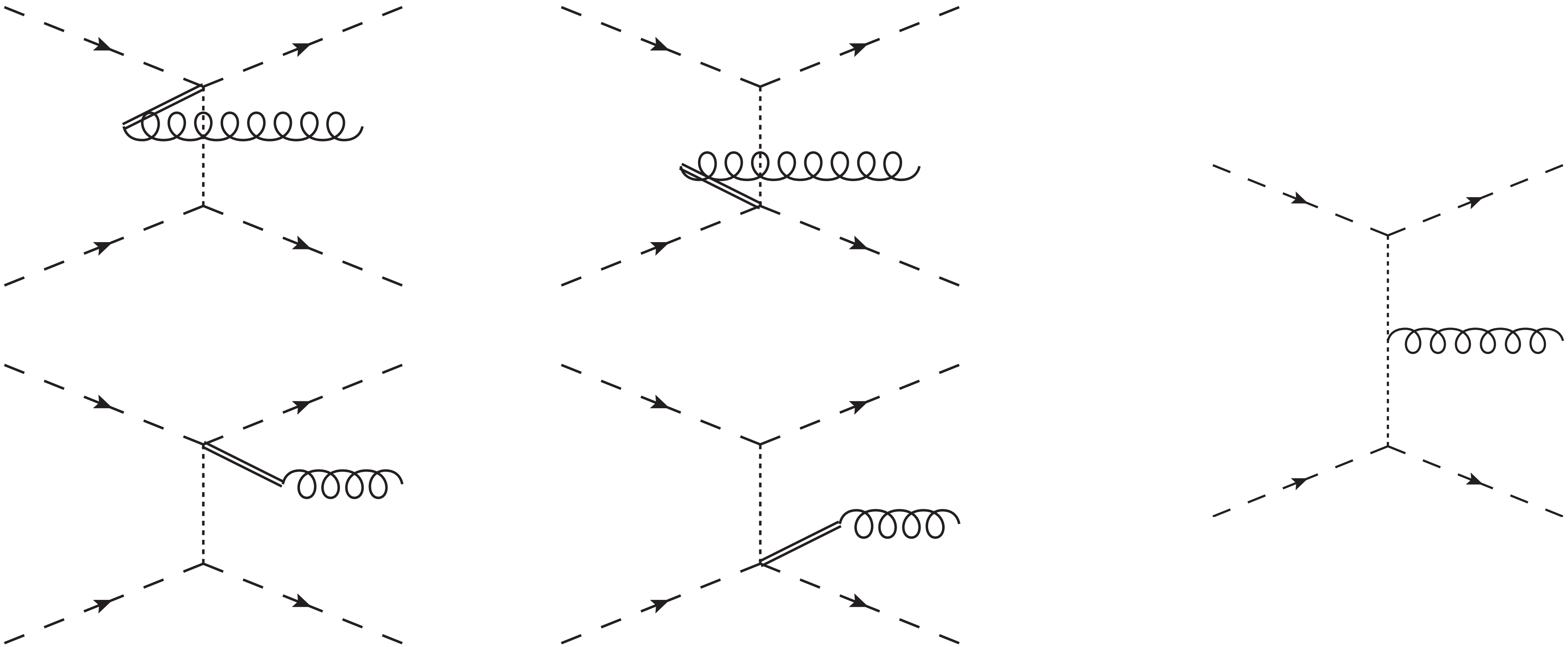
with $$\gamma(\nu) = -\frac{\alpha_s N_c}{4\pi^2}\ln\left(\frac{k_\perp^2}{m_g^2}\right)$$

solving $$\mathcal{L}_G^{n\bar{n}}(\nu) = \mathcal{L}_G^{n\bar{n}}(\nu_0)\left(\frac{\nu}{\nu_0}\right)^{-\frac{\alpha_s N_c}{4\pi^2}\ln\left(\frac{k_\perp^2}{m_g^2}\right)}$$

reggeized gluon

The Glauber interaction in SCET and the BFKL equation

- Real contributions



The Glauber interaction in SCET and the BFKL equation

- The NLO expression determined from all diagrams is

$$\sigma = \frac{2\alpha_s^2 C_F}{N_c} \int \frac{d^2\vec{k}_\perp}{\vec{k}_\perp^2} \int \frac{d^2\vec{k}_\perp}{\vec{k}_\perp'^2} \Bigg\{ \delta^{(2)}(\vec{k}_\perp - \vec{k}'_\perp) + \left(\frac{\alpha_s N_c}{\pi^2}\right) \frac{\Gamma(\eta)\Gamma\left(\frac{1}{2} - \eta\right)}{\sqrt{\pi}} \nu^{2\eta} w(\nu)^2$$
$$\times \int \frac{d^2 q_\perp}{[(\vec{q}_\perp - \vec{k}_\perp)^2]^{1+\eta}} \left[\delta^{(2)}(\vec{q}_\perp - \vec{k}'_\perp) - \frac{\vec{k}_\perp^2}{2\vec{q}_\perp^{\,2}} \delta^{(2)}(\vec{k}_\perp - \vec{k}'_\perp) \right] \Bigg\} .$$

- Isolating the rapidity divergent term:

$$\sigma = \frac{2\alpha_s^2 C_F}{N_c} \int \frac{d^2\vec{k}_\perp}{\vec{k}_\perp^2} \int \frac{d^2\vec{k}_\perp}{\vec{k}_\perp'^2} \Bigg\{ \delta^{(2)}(\vec{k}_\perp - \vec{k}'_\perp)$$
$$+ \left(\frac{\alpha_s N_c}{\pi^2}\right) \frac{w(\nu)^2}{\eta} \int \frac{d^2 q_\perp}{(\vec{q}_\perp - \vec{k}_\perp)^2} \left[\delta^{(2)}(\vec{q}_\perp - \vec{k}'_\perp) - \frac{\vec{k}_\perp^2}{2\vec{q}_\perp^{\,2}} \delta^{(2)}(\vec{k}_\perp - \vec{k}'_\perp) \right] + \ldots \Bigg\}$$

The Glauber interaction in SCET and the BFKL equation

- Assume the cross section factorizes into soft and collinear

$$\sigma = J_n \otimes J_{\bar{n}} \otimes G$$

where $\quad G(\vec{k}_\perp - \vec{k}'_\perp) \equiv \langle O_G^{\rm soft} \rangle$

not known

- Identify the two-dimension Dirac delta function in transverse-momentum space as the leading order value of G:

$$\begin{aligned} G(\vec{k}_\perp - \vec{k}'_\perp, \nu) &= \int d^2\ell_\perp \mathcal{Z}^{-1}(\vec{k}_\perp - \vec{\ell}_\perp; \eta, \nu) G(\vec{\ell}_\perp - \vec{k}'_\perp; \nu)^{(0)} \\ &= \int d^2\ell_\perp \mathcal{Z}^{-1}(\vec{k}_\perp - \vec{\ell}_\perp; \eta, \nu) \delta^{(2)}(\vec{\ell}_\perp - \vec{k}'_\perp) \\ &= \delta^{(2)}(\vec{k}_\perp - \vec{k}'_\perp) + \text{counterterms}\,, \end{aligned}$$

The Glauber interaction in SCET and the BFKL equation

- Derive R-RGE for G

$$\frac{d}{d\ln\nu}G(\vec{k}_\perp - \vec{k}'_\perp;\nu) = \int d^2\ell_\perp \gamma_\nu(\vec{k}_\perp - \vec{\ell}_\perp)G(\vec{\ell}_\perp - \vec{k}'_\perp;\nu)$$

with

$$\gamma_\nu(\vec{k}_\perp - \vec{k}'_\perp) = \left(\frac{\alpha_s N_c}{\pi^2}\right)\left[\frac{1}{(\vec{k}_\perp - \vec{k}'_\perp)^2} - \frac{1}{2}\delta^{(2)}(\vec{k}_\perp - \vec{k}'_\perp)\int \frac{d^2q_\perp}{(\vec{q}_\perp - \vec{k}_\perp)^2}\frac{\vec{k}^2_\perp}{\vec{q}^{\,2}_\perp}\right]$$

This is the BFKL equation

?