The role of Glauber exchange in SCET and the BFKL equation

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SCET2015, March 2015

The Glauber interaction in SCET behavior in SCET from Glauber interactions between collinear particles.

Include Glauber interactions in SCET Lagrangian \blacksquare Include Glouber interactions in SCETI egrancies \Box Include Glauber interactions in SCET Lagrangian α soft gluon from the Glauber interaction and derive the Lipatov vertex. With the Lipatov ver

F. Liu & J. P. Ma, arXiv:0802.2973; A. Idilbi & A. Majumder, Phys. Rev. D 80 (2009) 054022; S. Fleming Phys.Lett. B735 $(2014) 266$; I. Z. Rothstein & I. W. Stewart (in progress)

quark forward scattering operator arises in SCET with \sqrt{a} \overline{a} diagram is given in Fig. 1(a). For the same of \overline{a} . For the same of \overline{a} qualk forward scaliering $s \gg v, \Lambda \sim \sqrt{\frac{1}{f}}$ incoming momentum can be expressed in terms of two light-like vectors \mathbf{v} and \mathbf{v} and \mathbf{v} \sqrt{a} $\mathbf{C} = \mathbf{C} \mathbf{C}$ from Glauber interactions between collinear particles. qualk forward scattering $s \gg v, \Lambda \sim \sqrt{\frac{1}{t}}$ V with V and small mass such that mass such small mass such sma k forward scattering $\epsilon \gg t \lambda_{\text{out}}/2$ In the form \bigvee is the form \bigvee in the set of \bigvee is the set of $\sqrt{2}$ $\sqrt{2$ $s \gg t$, $\lambda \sim$ \sqrt{s} *t*

2 h e Glai ilau ber interact 2 $\frac{1}{2}$ ⊥ \sin $C₁$ $\overline{1}$ 2 √s 2 The Glauber interaction in SCET

I. Z. Rothstein & I. W. Stewart (in progress)

where ^k² ≈ −⃗ $\mathbf{1} \cdot \mathbf{K}$. $\mathbf{1} \cdot \mathbf{K}$ Tree-level Matching:

$$
\mathcal{A}_{QCD} = -\frac{g^2}{\vec{k}_{\perp}^2} \bar{u}(p_1') T^a \gamma^\mu u(p_1) \bar{u}(p_2') T^a \gamma_\mu u(p_2) \approx -\frac{n \cdot \bar{n} g^2}{\vec{k}_{\perp}^2} \bar{\xi}_n T^a \frac{\hbar}{2} \xi_n \bar{\xi}_n T^a \frac{\hbar}{2} \xi_{\bar{n}}
$$

Tree-level SCET operator: 1 and 1 direction respectively. This amplitude is reproduced by the SCET operator first is rep

$$
\mathcal{O}_G^{n\bar{n}} = -\frac{2\,g^2}{\vec{k}_{\perp}^2}\bar{\xi}_{p_1',n}T^a\frac{\rlap{\hspace{0.02cm}/}{\bar{n}}}{2}\xi_{p_1,n}\bar{\xi}_{p_2',\bar{n}}T^a\frac{\rlap{\hspace{0.02cm}/}{\bar{n}}}{2}\xi_{p_2,\bar{n}}
$$

SCET Operator with Wilson Lines:

$$
\mathcal{L}_G^{n\bar{n}} = -8\pi \alpha_s(\mu) \bar{\xi}_{p'_2,\bar{n}} W_{\bar{n}} S_{\bar{n}}^{\dagger} T^a \frac{\hbar}{2} S_{\bar{n}} W_{\bar{n}}^{\dagger} \xi_{p_2,\bar{n}} \frac{1}{\vec{\mathcal{P}}_1^2} \bar{\xi}_{p'_1,n} W_n S_n^{\dagger} T^a \frac{\hbar}{2} S_n W_n^{\dagger} \xi_{p_1,n}
$$

Not gauge invariant! $N₀$ and \mathcal{A} = \mathcal{A} = \mathcal{A} auge invariant! In addition, so ft gluons with momentum that scales as $\frac{1}{2}$

Ward identity not satisfied in the soft sector:

`*µ* k^{μ} Ω lectoro, $\rho\mu$ $k^{\prime \mu}$ emission from soft Wilson line ℓ^{μ} k^{μ} ℓ^{μ} $k^{\prime \mu}$ $\overline{0000}$

$$
\ell^{\mu} = \frac{1}{2}\bar{n} \cdot k n^{\mu} - \frac{1}{2}n \cdot k' \bar{n}^{\mu} + (k - k')^{\mu}_{\perp}
$$
\n
$$
m^{\mu} \propto \left(\frac{n^{\rho}}{n \cdot k'} \frac{1}{k_{\perp}^{'2}} + \frac{\bar{n}^{\rho}}{\bar{n} \cdot k'} \frac{1}{k_{\perp}^{2}}\right) \longrightarrow \ell \cdot m \propto \frac{1}{k_{\perp}^{'2}} - \frac{1}{k_{\perp}^{2}} \neq 0
$$

Real emission from Glauber

 \mathcal{A} dd all soft amission to got the gauge interior \mathcal{I} inte Wilson din son the Glauber gluon. Add all soft emission to get the gauge invariant Lipton vertex

$$
m_{\text{tot}}^{\rho} \propto \left[\frac{1}{k_{\perp}^2} \frac{1}{k_{\perp}^{'2}} \left(k_{\perp}^{\rho} + k_{\perp}^{'\rho} - \frac{1}{2} \bar{n}^{\rho} n \cdot k' - \frac{1}{2} n^{\rho} \bar{n} \cdot k \right) + \left(\frac{n^{\rho}}{n \cdot k'} \frac{1}{k_{\perp}^{'2}} + \frac{\bar{n}^{\rho}}{\bar{n} \cdot k'} \frac{1}{k_{\perp}^2} \right) \right]
$$

Implies real soft gluons from Glauber should be summed in $\mathcal{L}_G^{n\bar{n}}$

gauge invariant

Can matrix elements of the summed operator factor?

$$
\langle \mathcal{L}_G^{n\bar{n} \text{ summed}} \rangle \stackrel{?}{=} \langle n\text{-collinear} \rangle \otimes \langle \bar{n}\text{-collinear} \rangle \otimes \langle \text{soft} \rangle
$$

BFKL indicates yes…

Renormalization of the Glauber interaction $\mathcal{L}_G^{n\bar{n}}$ S. Fleming ; J. F. Donoghue, B. K. El-Menoufi, and G. Ovanesyan (2014), arXiv:1405.1731

The Glauber interaction in SCET and the BFKL equation diagrams in Fig 3(a). The sum of these four diagrams gives \cup .
.
] and the BFKL equation

$$
\mathcal{A} = -8\pi\alpha_s(\mu)\bar{\xi}_n T^a \frac{\bar{\psi}}{2} \xi_n \bar{\xi}_n T^a \frac{\psi}{2} \xi_{\bar{n}} \left[iN_c \alpha_s(\mu) \mathcal{I}(\vec{k}_\perp) \right] + \text{counterterms}
$$

$$
\mathcal{I}(\vec{k}_\perp) = \int \frac{dq^-}{q^-} \int \frac{d^2q_\perp}{(2\pi)^2} \frac{1}{\vec{\sigma}^2} \frac{1}{(\vec{\sigma} + \vec{k})^2} \text{ Gluon Regge Trajectories}
$$

$$
\mathcal{I}(k_{\perp}) = \int \frac{dq}{q^{-}} \int \frac{d^{2}q}{(2\pi)^{2}} \frac{1}{\vec{q}_{\perp}^{2}} \frac{1}{(\vec{q} + \vec{k})_{\perp}^{2}} \text{ Gluon Regge Trajectories}
$$

 \overline{a} \blacksquare Introduce rapidity regulator \blacksquare chive lain D Neill and L7 Rothstein $\begin{array}{ccccc} \text{I} & \text{I} & \text{I} \end{array}$ and $\begin{array}{ccccc} \text{Phys. Rev.} & \text{Lett.} & \text{108, 151601 (2012) & \text{B.} & \text{HEP 1205, 084 (2012)} \end{array}$ and the first two diagrams in Figure 100 and the intersection. eulator 1.−y. \bigcup ⊥
Chiu, A. Ja Introduce rapidity regulator J.-Y. Chiu, A. Jain, D. Neill, and I. Z. Rothstein,

$$
\mathcal{I}(\vec{k}_{\perp}) = \frac{-2i}{(4\pi)^2} \frac{w(\nu)^2}{\vec{k}_{\perp}^2} \left[\frac{1}{\eta} \ln \left(\frac{\vec{k}_{\perp}^2}{m_g^2} \right) + \ln \left(\frac{\vec{k}_{\perp}^2}{4\nu} \right) \ln \left(\frac{\vec{k}_{\perp}^2}{m_g^2} \right) - \frac{1}{4} \ln^2 \left(\frac{\vec{k}_{\perp}^2}{m_g^2} \right) + i\pi \ln \left(\frac{\vec{k}_{\perp}^2}{m_g^2} \right) \right]
$$

The counter term subtracts the $\dot{-}$ pole T

Renormalization of rapidity divergence gives rise to a rapidity RGE (R-RGE)

$$
\frac{d}{d\ln \nu} \mathcal{L}_G^{n\bar{n}}(\nu) = \gamma(\nu) \mathcal{L}_G^{n\bar{n}}(\nu)
$$

with
$$
\gamma(\nu) = -\frac{\alpha_s N_c}{4\pi^2} \ln\left(\frac{k_\perp^2}{m_g^2}\right)
$$

solving
$$
\mathcal{L}_G^{n\bar{n}}(\nu) = \mathcal{L}_G^{n\bar{n}}(\nu_0) \left(\frac{\nu}{\nu_0}\right)^{-\frac{\alpha_s N_c}{4\pi^2} \ln\left(\frac{k_\perp^2}{m_g^2}\right)}
$$

reggeized gluon

Real contributions

vanishes. Adding these up we arrive at an expression for the forward scattering cross section for **• The NLO expression determined from all diagrams is**

$$
\sigma = \frac{2\alpha_s^2 C_F}{N_c} \int \frac{d^2 \vec{k}_{\perp}}{\vec{k}_{\perp}^2} \int \frac{d^2 \vec{k}_{\perp}}{\vec{k}_{\perp}^{'2}} \left\{ \delta^{(2)}(\vec{k}_{\perp} - \vec{k}_{\perp}') + \left(\frac{\alpha_s N_c}{\pi^2} \right) \frac{\Gamma(\eta)\Gamma(\frac{1}{2} - \eta)}{\sqrt{\pi}} \nu^{2\eta} w(\nu)^2 \right. \\ \times \int \frac{d^2 q_{\perp}}{[(\vec{q}_{\perp} - \vec{k}_{\perp})^2]^{1+\eta}} \left[\delta^{(2)}(\vec{q}_{\perp} - \vec{k}_{\perp}') - \frac{\vec{k}_{\perp}^2}{2\vec{q}_{\perp}^2} \delta^{(2)}(\vec{k}_{\perp} - \vec{k}_{\perp}') \right] \right\}.
$$

 $\overline{1}$ \mathcal{C} is divergent terms of we can isolate the rapidity divergent terms of \mathcal{C} Isolating the rapidity divergent term:

$$
\sigma = \frac{2\alpha_s^2 C_F}{N_c} \int \frac{d^2 \vec{k}_{\perp}}{\vec{k}_{\perp}^2} \int \frac{d^2 \vec{k}_{\perp}}{\vec{k}_{\perp}^{'2}} \left\{ \delta^{(2)}(\vec{k}_{\perp} - \vec{k}_{\perp}')
$$

$$
+ \left(\frac{\alpha_s N_c}{\pi^2}\right) \frac{w(\nu)^2}{\eta} \int \frac{d^2 q_{\perp}}{(\vec{q}_{\perp} - \vec{k}_{\perp})^2} \left[\delta^{(2)}(\vec{q}_{\perp} - \vec{k}_{\perp}') - \frac{\vec{k}_{\perp}^2}{2\vec{q}_{\perp}^2} \delta^{(2)}(\vec{k}_{\perp} - \vec{k}_{\perp}') \right] + \dots \right\}
$$

Assume the cross section factorizes into soft and collinear

$$
\sigma = J_n \otimes J_{\bar{n}} \otimes G
$$

where $G(\vec{k}_{\perp} - \vec{k}'_{\perp}) \equiv \langle O_G^{\text{soft}} \rangle$
not known

Identify the two-dimension Dirac delta function in transversemomentum space as the leading order value of G :

$$
G(\vec{k}_{\perp} - \vec{k}'_{\perp}, \nu) = \int d^2 \ell_{\perp} \mathcal{Z}^{-1}(\vec{k}_{\perp} - \vec{\ell}_{\perp}; \eta, \nu) G(\vec{\ell}_{\perp} - \vec{k}'_{\perp}; \nu)^{(0)}
$$

=
$$
\int d^2 \ell_{\perp} \mathcal{Z}^{-1}(\vec{k}_{\perp} - \vec{\ell}_{\perp}; \eta, \nu) \delta^{(2)}(\vec{\ell}_{\perp} - \vec{k}'_{\perp})
$$

=
$$
\delta^{(2)}(\vec{k}_{\perp} - \vec{k}'_{\perp}) +
$$
counterterms,

The Glauber interaction in SCET and the BFKL equation uber interaction in \blacksquare \blacks

Derive R-RGE for *G* \mathcal{L} \mathbb{R}^n terms in \mathbb{R}^n terms that does not vanish that do not UIN RENDENCE OF GENERAL AND COMMENT

$$
\frac{d}{d\ln\nu}G(\vec{k}_{\perp}-\vec{k}'_{\perp};\nu) = \int d^2\ell_{\perp}\gamma_{\nu}(\vec{k}_{\perp}-\vec{\ell}_{\perp})G(\vec{\ell}_{\perp}-\vec{k}'_{\perp};\nu)
$$

with with $\frac{1}{2}$ and $\frac{1}{$

$$
\gamma_{\nu}(\vec{k}_{\perp} - \vec{k}'_{\perp}) = \left(\frac{\alpha_s N_c}{\pi^2}\right) \left[\frac{1}{(\vec{k}_{\perp} - \vec{k}'_{\perp})^2} - \frac{1}{2} \delta^{(2)}(\vec{k}_{\perp} - \vec{k}'_{\perp}) \int \frac{d^2 q_{\perp}}{(\vec{q}_{\perp} - \vec{k}_{\perp})^2} \frac{\vec{k}_{\perp}^2}{\vec{q}_{\perp}^2}\right]
$$

This is the BFKL equation = າ≀
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19 [∂]w² (29)

