LHC Jet Shapes in Dijet Events

Andrew Hornig LANL March 27, 2015

In collaboration with Yiannis Makris, Thomas Mehen

Motivation

❖ "measured jets" : probed with mass, angularity, etc $\mu_H = Q$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{2}{2}$ *}*

L^{*I*} μ ^{*z*} = *L*_{*L*} = *L*_{*I*} = *L*_{*II*} = *L*_{*IIIIs*} *Kunszt*, *Soper '91*, *'92*
See also: Yang-Ting Chien's talk

Ems, Runszt, Soper 91, 92
see also: Yang-Ting Chien's talk **Ellis, Kunszt, Soper '91, '92**

sections vetoing radiation with total energy greater than ⇤ in an- gular regions outside of found jets. Though ^a hard scale

sections vetoing radiation with total energy greater than ⇤ in an- gular regions outside of found jets. Though ^a hard scale

understanding

distribution

understanding

the

the
theory

ratio

ratio

see also: hang mig emen's tank
Andrew Hornig, LANL SCET 2015 Mar 27, 2015 **15 SCET 2015** *Q* ⁼ *^Hµ* P^2 2015 $\frac{2}{2}$, $\frac{2}{2}$

^µ^L ^m ⁼*^S* ²*/Q* ¹

$$
\mu_S^R = m_2^2 / Q_{\text{Indrew Hornig, LAML}} \qquad \text{SCET 2015} \qquad \text{Mar 27, 2015}
$$

¹ (purple).

SCET 2015 $\overline{2}$ $\overline{5}$ coefficient $\overline{2}$ in $\overline{2}$ $\overline{2}$ for $\overline{2}$ mass**i**CET 2015 Solution for a particular choice of the hierarchy
Solution for the hierarchy of the hierarchy of the hierarchy
Solution for the hierarchy of the hierarchy of the hierarchy of the hierarchy
Solution for the hierarchy of the 2 *m*
2 *m*
2 *m*
2 *m*
2 *m*
2 *m*
2 *m*
2 *m*
2 *m Mar 27, 2* $\frac{1}{2}$ Mar 27, 2015 1)
8)
8)
F
SCET 2015 Mar 27, 2015 1 **m**
2 *m*
2 *m
27, 2015 Mar 27, 2015* satisfies *Max*
15 Max *µ/µ* (In an extending from RG in the cross section of the cr

theory.

3 $\overline{3}$

^µ^L ^m ⁼*^S* ²*/Q* ¹ **Baumgart, Leibovich, Mehen, Rout for 1406.2295** ¹ (purple).

s = $\frac{1}{2}$ *Andrew Hornig, LANL* SCET 2015 Mar 27, 2015 *m* $\frac{1}{2}$ $\frac{1}{2}$ $CET 2015$ $\overline{2}$ $\overline{5}$ coefficient $\overline{2}$ in $\overline{2}$ $\overline{2}$ for $\overline{2}$ \overline{R} $\overline{2}/\overline{Q}$ $\mu_S^R = m_2^2 / Q$ _{nd rew} Hornig LANL massSCF **i**CET 2015 **s**
Mar 27, 2 masssatisfies *Max*
15 Max the fragmentation function evaluated at the jet scale *µ^J* = 2*E* tan(*R/*2).

µ/µ (

Motivation sections vetoing radiation with total energy greater than ⇤ in an- gular regions outside of found jets. Though ^a hard scale *^Q* appears in these ratios, we found in [21] that the NGLs still arise from considering both scales in the ratio to sections vetoing radiation with total energy greater than ⇤ in an- gular regions outside of found jets. Though ^a hard scale *^Q* appears in these ratios, we found in [21] that the NGLs still arise from considering both scales in the ratio to*^µ^H* ⁼ *^Q ^µ^H* ⁼ *^Q* $\sqrt{ }$

❖ quarkonia @ LHC: ² 2 Refs. [5, 19, 20] studied NGLs of ⇤*/Q* in cross sections veton with + *··· .* (2) \mathfrak{p} LF $\frac{1}{2}$ + *··· .* (2) Here *µ*1*,*2 are the scales at which soft radiation is probed in di↵erent sharply-divided regions. For the hemisphere HC. $\overline{}$

Right jet scale

Right jet scale

Soft scales

Soft scales

 $V = 011$ e step in gin of NGLs in e↵ective field theory. We considered the ecolusions producing back-to-back-to-back-to-back-to-back-to-back-to-back-to-back-to-back-to-back jets, and calculated the product of P *s*), as also in [22], the hemisphere soft func-* one st

* 1

* 1

* 1

* 1 $\begin{array}{ccccc}\n\ast & \text{one s} & \\
\ast & \ast \\
\ast & \ast\n\end{array}$ function on ratios of multiple soft scales, and revealed new subleading (single) NGLs and non-logarithmic non- $\begin{array}{c} \circledast \\ \circledast \end{array}$ These NGLs are organized into a multiplicative factor % one step
% mo
% um
- * Pvt $\begin{array}{c} \circ \text{ one step} \\ \circ \text{ m} \\ \circ \text{ m} \\ \circ \text{ m} \\ \circ \text{ P} \end{array}$ $\begin{array}{c} \circ \text{ one st} \\ \circ \text{ n} \\ \circ \text{ n} \\ \circ \text{ c} \end{array}$ ending the substantial states $\frac{1}{2}$ entering the total contract contract contract contract the total contract cont These New York are one ❖ one step further:

4

 $\overline{4}$

- μ *dred* μ *ets:* **e** *s*), as also in [22], the hemisphere soft func- $\overline{}$ $\mu_H = Q$ $*$ more info if "measured" jets? $=$ θ
-) in ² *, ^m*¹ *^m*(factorized dijet invariant mass distribution *^e*+*e* collisions producing back-to-back jets, and calcu- Hard scale $11nd$ arctood). These calculations clarified the origin of *^R , k ^Lk*(*S* tion \hat{N} in an EFT framework as the dependence of a software of a so $\frac{1}{2}$ *lred factory aw* α ^{*S*} well understod ❖ unmeasured factorization not as well understood
- *u*² *Leat* 101 *LIII* μ_S = Λ ² *^m* ⁼ *^J* λ \sim μ *<u>Rured</u>* case $\frac{1}{2}$ $\frac{1}{2}$ new subleading (single) NGLs and non-logarithmic non-* Pythia/Madonia not great for LHC J/ Here *µ*1*,*² are the scales at which soft radiation is probed $\mu_J^R = m_2$ (bye quarkonia, for $\partial^2 f$ $\mu_S^{\rm out} = \Lambda$ *m*2 *^Lµ^S* ⁼ *m*2 * unmeasur

* Pythia/M

→ setup σ

(bye q¹ \therefore unmeasu
 \therefore Pythia / N

→ setup (bye c wunneasu

• Pythia / I

→ setup

(bye * more 11

* unmea

* Pythia
 \rightarrow setu

(by → more in

→ unmeas

→ Pythia /

→ setup

(bye * more

* unme

* Pythia

→ setu

(b → mor

→ mor

→ Pyth

→ se ↵ective field theory. We considered the † one step

† mass mass mass mass of *m*

† Py

→ * unmeasured
* Pythia/Mad
→ setup σ fo
(bye qua • unmeasure
• Pythia/Ma
→ setup σ f
(bye qua → unmeasure

→ Pythia/Ma

→ setup σ

(bye qu * more into

* unmeasur

* Pythia / M

→ setup o

(bye q * more inf

* unmeas

* Pythia / *2*

→ setup

(bye <table>\n<tr>\n<td>∗ more i</td>\n</tr>\n<tr>\n<td>∗ unmea</td>\n</tr>\n<tr>\n<td>∗ Pythia</td>\n</tr>\n<tr>\n<td>→ setu</td>\n</tr>\n</table> → more info if "measured" jets?

→ unmeasured factorization not as well ur

→ Pythia/Madonia not great for LHC J/Ψ † one step fu
† more
† unmarian
† Pyth:
† → set ecolusions produces with the step for the step \ast p * one step
→ mo
→ un
→ Py ➔ setup σ for measured case

$$
\mu_S^R = m_2^2 / Q_{\text{ndrew Hornig, LAML}} \qquad \text{SCET 2015} \qquad \text{Mar 27, 2015}
$$

SCET 2015 $\overline{2}$ $\overline{5}$ coefficient $\overline{2}$ in $\overline{2}$ $\overline{2}$ for $\overline{2}$ mass**i**CET 2015 \int CET 2015 Mar 27, 2015 2 *m*
2 *m*
2 *m*
2 *m*
2 *m*
2 *m*
2 *m*
2 *m*
2 *m* mass γ ψ
scet 2015 Mar 27, 2015 **2**
2 *m*₂₇ *m*₂₇ *m*₂₇ *m*₂₇ *m*₂₇ *m*₂₇ *m*₂₇ *m*₂₇ *m*₂₉ *m*₂₇ *m*₂₉ satisfies *Max*
15 Max

be soft and later taking one of them to *^Q* in an inclusive

theory.

func- *^s*

ori- gin of NGLs in ^e↵ective field theory. We considered the factorized dijet invariant mass distribution

back-to-back

the factorized dijet invariant mass distribution

 $\frac{2}{2}$

calculations

multiplicative

with

framework

(single)

with

back-to-back

 calcu- lated to *^O* $\frac{22}{3}$

dependence

and

non-logarithmic

non-logarithmic

multiplicative

func- *^s*

calculations

of NGLs in an EFT framework as the dependence of ^a soft

understanding

be soft and later taking one of them to *^Q* in an inclusive

origin

and

in

in

understanding

calcu-

origin

s
Mar 27, 2 $\frac{1}{2}$ Mar 27, 2015 *µ/µ* (In an extending from RG in the cross section of the cr

). These calculations clarified the origin of *^R , k ^Lk*(*S* tion NGLs in an EFT framework as the dependence of ^a soft

 \mathbf{u} new subleading (single) NGLs and non-logarithmic non-

Setup

• fragmentation: "just" change $J(\tau) \rightarrow J(\tau, z)$

Andrew Hornig, LANL SCET 2015 Mar 27, 2015

Jet Algorithms ⇢*ij* = min*{*(*pⁱ T*) *T*) 2↵*} ^R*² *,* (2.1) JULISOIIUIIID *R***i**j **n** *R R R R R <i>R R* particle pairs and is less than both of the single particle metrics, i.e., ⇢*ij <* min*{*⇢*i,* ⇢*j}*.

*R*²

ij

- same for all k_T-type (k_T, C/A, anti-k_T) to $\mathcal{O}(\alpha_s)$ $\frac{R}{2}$ k_T-ty $\mathsf{p}\text{e}$ (k_T, C/A, and K_T) to $\mathcal{O}(\alpha_s)$ where in the first equality ✓*ij* and *ij* are measured with respect to the beam, and ✓*ij* \mathcal{I} saille for all κ _T-type $(\kappa$ _T, C/A, and κ _T/to $\mathcal{O}(\alpha_s)$ \therefore same for all k-type $(k_T C/A)$ anti-k- θ . impose an *e*+*e*-type polar angle restriction that particles are within a jet of size *R* and $\mathcal{O}(\alpha_s)$ ²
	- ❖ simple replacement ✓*ij* sin ✓*^J* \therefore cimple replacement

This latter constraint amounts to

$$
R \to \mathcal{R} \sin \theta_J = \frac{\mathcal{R}}{\cosh y_J} \qquad \left(\text{and } 2E \tan \frac{R}{2} \to p_T \mathcal{R} \right)
$$

2↵

,(*p^j*

Hadron Angularities although the details are beyond the scope of the present work. **In is helpful to re-write the angularities** collisions in terms of ingredients that are boost invariant, such as *p^T* and the right-hand H_n dron Angularition collisions in terms of ingredients that are boost invariant, such as *p^T* and the right-hand II_{α} do so, Λ_{α} and I_{α} *T* radion *i* with $\frac{1}{2}$ and collisions in terms of ingredients that are boost invariant, such as *p^T* and the right-hand side of Eq. (2.5). To do so, fig. (2.5). To d *yiJ* and transverse momenta *piJ ^T* of particles *with respect to the jet axis*,

It is helpful to re-write the angularity definition used in [37] in the context of *e*+*e*

side of Eq. (2.5). To do so, first recall the definition used in terms of the pseudo-rapidities of the pseudo-
To do so, file the pseudo-rapidities of the pseudo-rapidities of the pseudo-rapidities of the pseudo-rapiditie

 $*$ e⁺e⁻ definition: α ^{+e</sub> *a*} 2*E^J i*

 $\tau_a^{e^+e^-}=$ 1 $2E_J$ $\sqrt{ }$ *i* $\frac{1}{2}$ *i |piJ* $\sum_{i=1}^{n}$ $\tau^{e^+e^-} = \frac{1}{\tau^{10}} \sum |p_T^{iJ}|e^ \frac{1}{1}$ $\frac{1}{1}$ i

collisions in terms of ingredients that are boost invariant, such as *p^T* and the right-hand

 $|p_T^{iJ}|e^{-(1-a)|y_{iJ}|}$ *original event shape: Berger, Kucs, Sterman hepton <i>T this def'n for jets: Ellis, AH, Lee, Vermilion, Wals ^T [|]e*(1*a*)*|yiJ [|] .* (2.7) *this def'n for jets*: Ellis, AH, Lee, Vermilion, Walsh 1001.0014 **original** *event* **shape: Berger, Kucs, Sterman [hep-ph/0303051](http://arxiv.org/abs/hep-ph/0303051)**

$$
= (2E_J)^{-(2-a)} (p_T)^{1-a} \sum_i |p_T^i| \left(\frac{\theta_{iJ}}{\sin \theta_J}\right)^{2-a} \left(1 + \mathcal{O}(\theta_{iJ}^2)\right)
$$

boost inv. offender (overall factor!)

boos *<u>inv.</u> 0* From the discussion above the sum over particles are boost invariant. The sum over particles are boost invariant. The one of the one o oboa hadron collecture for the factor.)

another simple rescaling: *a* ⌘

out out the output of 2² $\frac{1}{2}$. The overall power of 2² . The canonical version and a boost in version and version an o↵ender is the overall power of 2*E^J* . Therefore, we can arrive at a boost invariant version $\left| \int \right|$ cancels offender (and dimensionless)

$$
\tau_a^{pp}\equiv\frac{1}{p_T}\sum_i|p_T^i|(\Delta\mathcal{R}_{iJ})^{2-a}=\left(\frac{2E_J}{p_T}\right)^{2-a}\tau_a^{e^+e^-}+\mathcal{O}(\tau_a^2)
$$

Scale Hierarchies

* for unmeasured jets & beams, need

cites substructure/jet mass/hadron event shape calculation event shape calculation in the calculation of the c
The calculation of the calculation

 $\mathcal{R}^2 \ll 1$ $e^{-y_{\text{cut}}} \ll 1$ $\frac{3}{3}$

*^ey*cut ⌧ ¹

- ❖ for measured jets, could get full R-dep numerically $\frac{1}{2}$ for measured is to could get full D done numer *The abused for could get full reach hume* $*$ for measured jets,
- ∗ to avoid non-global and kinematic logs anga^s example **R**

Reference numerically as in the second of the second $\frac{1}{2}$ as in $\frac{1}{2}$. The second $\frac{1}{2}$ as in $\frac{1}{2}$ as in $\frac{1}{2}$ and $\frac{1}{2}$ and $\frac{1}{2}$ and $\frac{1}{2}$ and $\frac{1}{2}$ and $\frac{1}{2}$ and $\$ *^T /p^J ^T* ⇠ ⌧*^a* ⌧ *^R*² ⌧ 1 (1.1) \ast to avoid non-global and kinematic logs see also: Shireen Gangal's talk;
- Focus is on the dynamical scale. The dynamical scale scale scale scale scale scale. We do not at the dynamical scale sca f. petries: (The Same of the S **Bauer, Dunn, AH [1102.4899](http://arxiv.org/abs/1102.4899) see also: Jim Talbert's talk see also: David Farhi's talk from `14 see also: Shireen Gangal's talk; Gangal, Stahlhofen, Tackmann 1412.4792**

 soft **SOIT SCALES:** $p_T^{\text{cut}}/p_T^{\text{}} \sim \tau_a^{\text{}} \sim \tau_a^2$
 See also: Duff Neill's talk; Larkoski, Moult, Neill 1501.04596 soft scales: $p_T^{\text{cut}}/p_T^J \sim \tau_a^1 \sim \tau_a^2$ **see also: Duff Neill's talk; Larkoski, Moult, Neill 1501.04596**

> $(\text{within } 10^{\pm 1})$ (within $10^{\pm 1}$)

hard scales: $p_T^J \sim \hat{s} \sim \hat{t} \sim \hat{u}$ The main dividend measurements and the main dividend measurements at *e*+*e* colliders and hadron colliders and hadron colliders is and hadron colliders in the colliders is in the collider of the colliders and the collider **formula; discuss Scenaris Scenaris: See also: Bauer, Tackmann, Walsh, Zuberi ("Ninja") 1106.6047**
see also: Piotr Pietrulewicz's talk *p*cut *^T /p^J* **see also: Piotr Pietrulewicz's talk** *Talsh, Zuberi ("Ninja")* 1106.604/
Talk

Andrew Hornig, LANL SCET 2015 Mar 27, 2015

8

Factorized Cross-Section **dy**₂^d *^N ^f*1(*x*1; *^µ*)*f*2(*x*2; *^µ*) Tr*{*H0S0*}* (3.3) where the tree-level of the tree-level hard and soft functions, respectively, \mathbf{r} is a parton distributions, \mathbf{r} is a parton distribution distribution distributions, \mathbf{r} Γ_{20} for particle Γ_{20} is a normalization associated with a normalization associated with a normalization and Γ_{20} racionized eross-becuon $\frac{1}{2}$ \int $\frac{1}{2}$ \bigcap *^u* ⁼ 2*p*² *^T ^ey/*² cosh *^y*

2

where H⁰ and S⁰ are the tree-level hard and soft functions, respectively, *fⁱ* is a parton dis-

The e ϵ radiative corrections to E radiative corrections to Eq. (3.3) is described in the soft and collinear ϵ

= *s t .* (3.2)

1

2

\bullet horn[.] T , radiative corrections to E radiative corrections to E and collinear \mathcal{L} and collinear \mathcal{L} The born cross-section can be written in the form born:

$$
\frac{d\sigma_{\text{born}}}{dy_1 dy_2 dp_T} = \frac{p_T x_1 x_2}{8\pi E_{\text{cm}}^4} \frac{1}{N} f_1(x_1; \mu) f_2(x_2; \mu) \text{Tr}\{\mathbf{H}_0 \mathbf{S}_0\}
$$

❖ unmeasured jets: λ anm α are d i.e., α . dimitabuted jet. $*$ unmeasured jets: over initial particle states (e.g., *N* = 4*N*²

*d*born

 $\overline{1}$

*p^T x*1*x*²

❖ measured jets: $d\sigma$ $dy_1 dy_2 dp_T$ 1eas11 re B ¹; *µ*¹ *µ*^{*p*}^{*measured* iet functions (different)} = $p_T x_1 x_2$ $8\pi E_{\rm cm}^4$ 1 *N* $B(x_1; \mu) \bar{B}(x_2; \mu) \text{Tr}\{\mathbf{H}(\mu) \mathbf{S}^{\text{unmeas}}(\mu)\} [J_1(\mu) J_2(\mu)]$ $\frac{d\sigma}{d\sigma} = p_T x_1 x_2 \frac{1}{R(x_1, y_1)} \bar{R}(x_2, y_2) \operatorname{Tr}(\mathbf{H}(y_1) \operatorname{sumeas}(y_1) \mathbf{I}(y_2) \mathbf{I}(y_2)$ $\frac{dy_1 dy_2 dp_T}{\sqrt{dx_1}}$ $8\pi E_{\rm cm}^4$ N jets are both "unmeasured" (in the terminology of [37]), i.e., are tagged with an algorithm but and when one of the contract with \mathbf{m} experiments are \mathbf{m} and \math angularity jet shape λ . When both jets are left unmediately unmediately are left unmeasured with an algorithm $d\sigma$ $p_T x_1 x_2 1$, the algebra cross-section takes the formulation a_1a_2 \cdot **measured** jets: unmeas./measured, jet functions (different) $W_{\rm eff}$ both jets are measured, the cross-section takes the form takes the form takes the form takes the form $d\sigma$ $dy_1 dy_2 dp_T d\tau_a^1 d\tau_a^2$ $\iota)J_2(\tau$ where the *Ji*(*µ*) are unmeasured jet functions and *S*unmeas is the unmeasured soft function. = *p^T x*1*x*² $8\pi E_{\rm cm}^4$ 1 *N* $B(x_1; \mu) \bar{B}(x_2; \mu) \operatorname{Tr}\{\mathbf{H}(\mu) \mathbf{S}(\tau_a^1, \tau_a^2; \mu)\} \otimes [J_1(\tau_a^1; \mu) J_2(\tau_a^2; \mu)]$

Unmeasured Beam Functions *^x*(1 *^x*) ln[*x*1*^a* + (1 *^x*) m *f* ✓²⁰ ²³*^a* **SULL** \overline{a} 0 d Beam Filnc \overline{v} 4.2 Unmeasured Beam Functions While the unmeasured beam function has not to our knowledge appeared in the literature, it is directly related to the unmerstally related to the unmerstally function of \sim \sim \sim \sim \sim

fragmenting jet function for a jet of energy *E* and (*e*+*e*) cone radius *R* can be written as

0

¹*a*]

* "unmeasured" fragmenting jet function: fragmenting jet function for a jet of energy *E* and (*e*+*e*) cone radius *R* can be written as Fod'' $f_{r, \alpha}$ *i n l*_{ill} *p*_{*i*} *iet* function:

dx (1 *^x*(1 *^x*))²

18

Z ¹

4.2 Unmeasured Beam Functions

 $\mathcal{G}(E,R,z;\mu)=\sum$ *i* $\int dz$ $\frac{dZ}{dz} \mathcal{J}_{ij}(E,R,z';\mu)D^h_j(z/z';\mu) + \mathcal{O}(\Lambda_{\rm QCD}^2/E^2)$ Procura, Waale *ⁱ* is a fragmentation function for parton *i* in hadron *h* and the *Jij* are matching $\mathcal{J}_{ij}(E,R,z',\mu) \equiv \mathcal{J}_{ij}(2E\tan\frac{r}{2},z',\mu)$ (F, P, w) $\sum \int dz \tau(F, P, v', v) D^h(v', v', v) + O(\Lambda^2 - \ell F^2)$ **Program Washaring 1110.0920** $\Delta(E, n, \lambda, \mu) = \sum_{i} \int \frac{1}{z} J_{ij}(E, n, \lambda, \mu) D_j(\lambda/\lambda, \mu) + O(\Lambda_{\text{QCD}}/E)$ Frocura, Waalewijn 1110.003. to *O*(↵*s*)) is such that we can write *R* $\frac{\pi}{2}, z', \mu$

Procura, Waalewijn [1110.0839](http://arxiv.org/abs/1110.0839)

- \therefore simple replacement: $2E \tan \frac{\pi}{2} \rightarrow$ *R* \cdot simple replacement: $2E \tan \frac{R}{2} \rightarrow E_{\rm cm} e^{-y_{\rm cut}}$ of \mathcal{A} (ToDo), it can be shown that an unmeasured beam function in a collider with TODO), it can be shown that \mathcal{A} of [49] (TODO), it can be shown that an unmeasured beam function in a collider with TODO c -reproducineire. ω_{com} γ ω_{cm} $2E\tan$ *R* 2 $\rightarrow E_{\rm cm}e^{-y_{\rm cut}}$
- $*$ unmeasured beam function: *a D*calli *L*uittuon

same as D at $\mathcal{O}(\alpha_s)$ $(\text{different at } \mathcal{O}(\alpha_s^2))$

$$
B_i(\mathbf{E}_{\rm cm}, y_{\rm cut}, x_i; \mu) = \sum_j \int \frac{dz}{z} \mathcal{J}_{ij}(\mathbf{E}_{\rm cm}e^{-y_{\rm cut}}, z', \mu) f_j(z/z', \mu) + \mathcal{O}(\Lambda_{\rm QCD}^2/E^2)
$$

evolves like unmeas. jet

Procura, Stewart 0911.4980

to θ of θ of θ is an θ is a beam with energy θ is a beam with energy θ is an θ is a beam with energy θ is a beam with energy θ is a beam with energy salite as above to $O(\alpha_s)$

10 Andrew Hornig, LANL SCET 2015 Mar 27, 2015

Jet Functions I , there were both "measured" in \mathcal{S} is functions, corresponding to jets functions, corresponding to j

 S tu $\overline{\mathcal{S}}$ here $\overline{\mathcal{S}}$ here $\overline{\mathcal{S}}$ here $\overline{\mathcal{S}}$ here $\overline{\mathcal{S}}$ here $\overline{\mathcal{S}}$ here $\overline{\mathcal{S}}$

❖ unmeasured: For measured jet functions, we need to apply the rescaling Eq. (2.9). The identity probed. The latter can be obtained using the hadron collider algorithms with the rescaling

$$
J_i \xrightarrow{E \tan \frac{R}{2} \to p_T \mathcal{R}} 1 + \frac{\alpha_s}{2\pi} \left[\left(\frac{C_i}{\epsilon^2} + \frac{\gamma_i}{\epsilon} \right) \left(\frac{\mu}{p_T \mathcal{R}} \right)^{2\epsilon} + d_J^{i, \text{alg}} \right]
$$
\nnow boost invariant!

 $*$ measured: definition of $J(\tau)$ and $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ implies that this rescaling can be accomplished to all orders via the transformation *Ji*(⌧*a*) = *Je*+*e ⁱ* (⌧*a*) $\sigma(\tau)$ and $\delta(\tau - A\hat{\tau}) = A^{-1}\delta(A^{-1}\tau - \hat{\tau})$

$$
J_i(\tau_a) = \left(\frac{p_T}{2E_J}\right)^{2-a} J_i^{e^+e^-} \left(\left(\frac{p_T}{2E_J}\right)^{2-a} \tau_a\right)
$$

= $\delta(\tau_a) - \frac{\alpha_s}{2\pi} \left[\left(\frac{\mu}{p_T}\right)^{2\epsilon} \left(\frac{1}{\tau_a}\right)^{1+\frac{2\epsilon}{2-a}} \left(\frac{1}{\epsilon} \frac{2C_i}{1-a} + \frac{\gamma_i}{1-a/2}\right) + \delta(\tau_a) f_i(a) \right]$
now boost invariant!

Andrew Hornig, LANL SCET 2015 Mar 27, 2015 α *a* α (4.9) α $\frac{1}{2}$ *ANI*

SCET 20 Mar 27, 2015

Soft Function(s) In general, we can write the bare soft function at *O*(↵*s*) for dijet production when both

jets have ⌧*^a* measured as

Soft Function(s) In general, we can write the bare soft function at *O*(↵*s*) for dijet production when both

jets have ⌧*^a* measured as

$$
\mathbf{S}(\tau_a^1, \tau_a^2) = \mathbf{S}^{\text{unmeas}} \delta(\tau_a^1) \delta(\tau_a^2) + [\mathbf{S}_0 S^{\text{meas}}(\tau_a^1) \delta(\tau_a^2) + (1 \leftrightarrow 2)] + \mathcal{O}(\alpha_s^2)
$$
\n
$$
\text{always present}
$$
\n
$$
\text{(can sub in favorite jet shape)}
$$

and we will distinguish the corresponding renormalized function with an explicit argument

Soft Function(s) In general, we can write the bare soft function at *O*(↵*s*) for dijet production when both

jets have ⌧*^a* measured as

$$
S(\tau_a^1, \tau_a^2) = S^{\text{unmeas}} \delta(\tau_a^1) \delta(\tau_a^2) + [S_0 S^{\text{meas}}(\tau_a^1) \delta(\tau_a^2) + (1 \leftrightarrow 2)] + \mathcal{O}(\alpha_s^2)
$$
\n
$$
\mathcal{A}
$$

orders (at least for the measured case), but since the PDFs and fragmentation functions di↵er perturbatively the *R*-dependent of the *R-dependent* of this expression takes the form (TOD) TODO use the form (TODO) TODO use the form boost invariant

Andrew Hornig, LANL SCET 2015 Mar 27, 2015

Soft Function(s) In general, we can write the bare soft function at *O*(↵*s*) for dijet production when both N clearly has the desired boost-invariant properties. For later \sim that the *R*-dependent divergent part of this expression takes the form (TODO) TODO use

(⌧ *ⁱ*

↵*sCⁱ*

⌧ *i a*

pT

^a) ln *R .* (4.16)

1 *a*

 \mathbf{r}

jets have ⌧*^a* measured as

✏

⇡

*S*meas(⌧ *ⁱ*

(1 ✏)

^a, µ)

$$
\mathbf{S}(\tau_a^1, \tau_a^2) = \mathbf{S}^{\text{unmeas}} \delta(\tau_a^1) \delta(\tau_a^2) + [\mathbf{S}_0 S^{\text{meas}}(\tau_a^1) \delta(\tau_a^2) + (1 \leftrightarrow 2)] + \mathcal{O}(\alpha_s^2)
$$

\n
$$
\mathbf{S}^{\text{unmeas}} = \mathbf{S}_0 + \left[\frac{\mathbf{S}_0}{2} \sum_{\langle i \neq j \rangle} \mathbf{T}_i \cdot \mathbf{T}_j \left(S_{ij}^{\text{inel}} + \sum_{k=1}^N S_{ij}^k \right) + \text{h.c.} \right]
$$

\n
$$
S_{ij}^{\text{inel}} = \frac{1}{\epsilon} \frac{\alpha_s}{2\pi} \left(\frac{\mu}{p_T^{\text{ent}}} \right)^{2\epsilon} \mathcal{I}_{ij}^{\text{inel}} = -g^2 \mu^{2\epsilon} \int \frac{d^d k}{(2\pi)^{d-1}} \frac{n_i \cdot n_j}{(n_i \cdot k)(n_j \cdot k)} \delta(k^2) \Theta(k^0) \Theta_{p_T}
$$

\n
$$
S_{ij}^k = \frac{1}{\epsilon} \frac{\alpha_s}{2\pi} \left(\frac{\mu}{p_T^{\text{out}}} \right)^{2\epsilon} \mathcal{I}_{ij}^k = g^2 \mu^{2\epsilon} \int \frac{d^d k}{(2\pi)^{d-1}} \frac{n_i \cdot n_j}{(n_i \cdot k)(n_j \cdot k)} \delta(k^2) \Theta(k^0) \Theta_{p_T} \Theta_k^k
$$

\n* **not simply related to anything calculated...**

 \blacksquare

generic *i, j* (either jet or beam). Do do this, we align the 1-direction (or "ˆ*z*") with direction

Andrew Hornig, LANL SCET 2015 Mar 27, 2015

Soft Function(s) \mathbf{M} Hunci ⇥*k ^R* ⌘ ⇥(*RkJ < R*)*,* (4.18) $\overline{\mathbf{a}}$ **and put the 12-plane, and the 12-plane, and the 12-plane, and the 12-plane, and 123-spatial particles** $\overline{\mathbf{a}}$ **in the 123-spatial particles** $\overline{\mathbf{a}}$ **in the 123-spatial particles** $\overline{\mathbf{a}}$ **in the 123-s** of *^d*-dimensional space. Using the shorthands *^cij* ⌘ ¹*nⁱ ·n^j* , *^sij* ⌘ (1*c*² *ij*)1*/*² , *cⁱ* ⌘ cos ✓*i*, and *sⁱ* ⌘ sin ✓*i*, the dot products of the gluon's momentum *k* take the form ↵*s* ✓ *µ* ◆2✏ Z *ddk*

ij ⁼ *g*2*µ*2✏

O(↵*s*). The di↵erence from [37] is that now each contribution involves a *p^T* veto instead

*^I*incl

$$
S_{ij}^{\text{incl}} \equiv \frac{1}{\epsilon} \frac{\alpha_s}{2\pi} \left(\frac{\mu}{p_T^{\text{cut}}}\right)^{2\epsilon} \mathcal{I}_{ij}^{\text{incl}} = -g^2 \mu^{2\epsilon} \int \frac{d^d k}{(2\pi)^{d-1}} \frac{n_i \cdot n_j}{(n_i \cdot k)(n_j \cdot k)} \delta(k^2) \Theta(k^0) \Theta_{p_T}
$$

~*nⁱ · k* = *c*¹

*p*cut

$$
\mathcal{I}_{ij}^{\text{incl}} = \frac{(1 - c_{ij})e^{\gamma_E \epsilon}}{2\sqrt{\pi}\Gamma(1/2 - \epsilon)} \int_0^{\pi} d\theta_1 \sin^{-2\epsilon} \theta_1 \int_0^{\pi} d\theta_2 \sin^{-2\epsilon} \theta_2 \frac{1}{1 - c_1} \frac{1}{1 - c_{ij}c_1 - s_{ij}s_1c_2}
$$
\n
$$
\times \left[\frac{\Gamma(1/2 - \epsilon)}{\sqrt{\pi}\Gamma(-\epsilon)} \int_0^{\pi} d\theta_3 \sin^{-1-2\epsilon} \theta_3 \left(1 - (n_{B1}c_1 + n_{B3}s_1c_2 + n_{B3}s_1s_2c_3)^2\right)^{\epsilon} \right].
$$

ij ⌘

Sk

✏

1

2⇡

$$
\begin{array}{c}\n n_j \\
 \hline\n n_j \\
 \hline\n \tilde{n}_j \cdot k = c_{ij}c_1 + s_{ij}s_1c_2 \\
 \hline\n \tilde{n}_B \cdot k = n_{B1}c_1 + n_{B2}s_1c_2 + n_{B3}s_1s_2c_3\n\end{array}
$$

(2⇡)*d*¹

(*nⁱ · k*)(*n^j · k*)

(*k*2)⇥(*k*0) ⇥*p^T* ⇥*^k*

nⁱ · n^j

Andrew Hornig, LANL SCET 2015 Mar 27, 2015 **d**1 sin2 sin2 Mar 27, 2015

$Soft$ **Function(s)** ~*n^j · k* = *cijc*¹ + *sijs*1*c*² ~*n^B · k* = *nB*1*c*¹ + *nB*2*s*1*c*² + *nB*3*s*1*s*2*c*³ (4.21) C_0 f *¹* E ₁₁ ω g ¹ ω ⁰ C_0 ft Funotial T parenthesis to the \mathcal{L}_1 power in the second line is the second line is the sine of the sine OOL **differential** *^a, µ*) $\overline{}$ 2✏ ↵*sCⁱ* \blacksquare \triangleright oft Function(s) 1 2✏ ↵*sCⁱ* ⇡ soft Function(s)

*S*incl

ij ⁺^X

⇣

1 *c*¹

1 *cijc*¹ *sijs*1*c*²

Sk ij⌘

and *sⁱ* ⌘ sin ✓*i*, the dot products of the gluon's momentum *k* take the form

that the *R*-dependent divergent part of this expression takes the form (TODO) TODO use

that the *R*-dependent divergent part of this expression takes the form (TODO) TODO use

The additional part of the soft function we require, Sunmer soft function we require, Sunmer Sunmer Sunmer Sun
The soft function we require, Sunmer Sun

The additional part of the soft function we require, Sunmer of the soft function we require, Sunmer as a summe
The soft function we require, Sunmer as a summer of the summer as a summer of the summer as a summer of the su

 $\overline{\mathbb{R}}$

$$
\mathcal{I}_{ij}^{\text{incl}} = \frac{(1 - c_{ij})e^{\gamma_E \epsilon}}{2\sqrt{\pi} \Gamma(1/2 - \epsilon)} \int_0^{\pi} d\theta_1 \sin^{1-2\epsilon} \theta_1 \int_0^{\pi} d\theta_2 \sin^{-2\epsilon} \theta_2 \frac{1}{1 - c_1} \frac{1}{1 - c_{ij}c_1 - s_{ij}s_1c_2}
$$

$$
\times \left[\frac{\Gamma(1/2 - \epsilon)}{\sqrt{\pi} \Gamma(-\epsilon)} \int_0^{\pi} d\theta_3 \sin^{-1-2\epsilon} \theta_3 \left(1 - (n_{B1}c_1 + n_{B3}s_1c_2 + n_{B3}s_1s_2c_3)^2\right)^{\epsilon} \right].
$$

 $B^2 - 4 \mathcal{I}^{\bar{B}}$

(different)

(different

$$
\[\cdots\] \xrightarrow{\text{planar}} (1 - (n_{B1}c_1 + n_{B2}s_1c_2)^2)^{\epsilon} \rightarrow \text{can boost for back-to-}
$$

0

È,

S0

 $\mathcal{I}_{B\bar{B}}^{\mathrm{incl}} + \mathcal{I}_{B\bar{B}}^{B} + \mathcal{I}_{B\bar{B}}^{\bar{B}}$

B

 \mathcal{I}_{BJ}^J

 \overline{T} ^{Iincl} + \overline{T} *B* + \overline{T} *J* \overline{Y}

+

 $\mathcal{I}_{BJ}^\mathrm{incl} + \mathcal{I}_{BJ}^B + \mathcal{I}_{BJ}^J$

, (4.23) ➔ can boost for back-to-back jets! Sunmeas = S⁰ + S0 described in [51, 52]. The 4!*/*(2!)² = 6 matrices ^T*ⁱ ·*T*^j* are of rank *^R*, the same as that

4

(1 ✏)

$$
\left[\dots\right] \xrightarrow{n_i=n_B} \sin^{2\epsilon} \theta_1 \longrightarrow \text{rapidity diverge}
$$

 $|1S| \leq R$

sum is IR finite

 $\overline{\mathcal{I}_{BJ}^\text{incl}} + \mathcal{I}_{BJ}^B + \mathcal{I}_{BJ}^J$

}

 $\frac{I_{BB}^{\text{incl}} + I_{B\bar{B}}^B + I_{B\bar{B}}^B}{I_{BB}^B + I_{B\bar{B}}^B}$

Im is IR fi

The quantity in parenthesis to the \mathcal{L}_1 power in the second line is the second line of the sine o

the *ij*-plane for all *i, j*) and the integration over ✓³ can be easily performed. The entire

Sunmeas

→ rapidity divergences… $div - d$: $cos \theta$ and color space formalism as the color space formalism as $sin \theta$ described in [51, 52]. The 4!*/*(2!)² = 6 matrices ^T*ⁱ ·*T*^j* are of rank *^R*, the same as that

alternative: rapidity regulate $\mathcal{I}_{BB}^{\rm incl}$ & $\mathcal{I}_{BJ}^{\rm incl}$ with $SCET_{II}$ (measured) beam functions for p_T resummation $\frac{1}{2}$ \hat{B} *BB*¯ 2*y*cut $\rule{1em}{0.5mm}$ *D* $\rule{1em}{0.5mm}$ *Wi* $\rule{1em}{0.5mm}$ with SCET_{II} (measu

Andrew Hornig, LANL SCET 2015 Mar 27, 2015 Mar 27, 2015 15 $\sqrt{2}$ $12¹$ $Andrew Hornig, L2$

^p⇡(1*/*² ✏)

0

Soft Function(s) *ⁱ* T*ⁱ* = 0), and that Ω *n*^{Γ} *l* \overline{L} = *y^J* ln(2 cosh *y^J*) ln *ⁿ^J · ⁿ*¯*^B* In general, we can write the bare soft function at *O*(↵*s*) for dijet production when both

jets have ⌧*^a* measured as

 \sim 1, 2, and \sim ln *ⁿ*¹ *· ⁿ*² $\left(\frac{a}{a}\right)^2$ $\left(\frac{a}{c}\right)^2$ $\mathbf{S}(\tau_a^1, \tau_a^2) = \mathbf{S}^{\text{unmeas}} \delta(\tau_a^1) \delta(\tau_a^2) + [\mathbf{S}_0 S^{\text{meas}}(\tau_a^1) \delta(\tau_a^2) + (1 \leftrightarrow 2)] + \mathcal{O}(\alpha_s^2)$

$$
\mathbf{S}^{\text{unmeas}} = \mathbf{S}_0 + \left\{ \frac{\mathbf{S}_0}{2} \left[\left(\frac{1}{\epsilon} + 2 \ln \frac{\mu}{p_T^{\text{cut}}} \right) \mathbf{S}^{\text{div}} - \frac{\alpha_s}{\pi} (C_1 + C_2) \ln^2 \mathcal{R} \right. \\ - \frac{2\alpha_s}{\pi} \mathbf{T}_1 \cdot \mathbf{T}_2 \ln \left(1 + e^{\Delta y} \right) \ln \left(1 + e^{-\Delta y} \right) \right] + \text{h.c.} \right\}
$$
 only one (new) matrix

= *y^J* ln(2 cosh *y^J*)*,* (4.27)

Soft Function(s) *ⁱ* T*ⁱ* = 0), and that Ω *n*^{Γ} *l* \overline{L} = *y^J* ln(2 cosh *y^J*) ln *ⁿ^J · ⁿ*¯*^B* In general, we can write the bare soft function at *O*(↵*s*) for dijet production when both where dependence on the left-hand side arises from enforcing and side are the polar side are the polar side and side are the polar side an $\textbf{UOLL} \perp \textbf{UIL} \textbf{UOLU} \perp \textbf{V}$

R

R

jets have ⌧*^a* measured as

 \sim 1, 2, and \sim ln *ⁿ*¹ *· ⁿ*² $\left(\frac{a}{a}\right)^2$ $\left(\frac{a}{c}\right)^2$ $\mathbf{S}(\tau_a^1, \tau_a^2) = \mathbf{S}^{\text{unmeas}} \delta(\tau_a^1) \delta(\tau_a^2) + [\mathbf{S}_0 S^{\text{meas}}(\tau_a^1) \delta(\tau_a^2) + (1 \leftrightarrow 2)] + \mathcal{O}(\alpha_s^2)$ -1) $\delta(\tau^2)$ $\langle \tau_a^2 \rangle + [\mathbf{S}_0 S^{\text{meas}}(\tau_a^1) \delta(\tau_a^2)]$ ln *ⁿ^J · ⁿ*¯*^B*

$$
\mathbf{S}^{\text{unmeas}} = \mathbf{S}_0 + \left\{ \frac{\mathbf{S}_0}{2} \left[\left(\frac{1}{\epsilon} + 2 \ln \frac{\mu}{p_{\text{cut}}^{\text{cut}}} \mathbf{S}^{\text{div}} \right) - \frac{\alpha_s}{\pi} (C_1 + C_2) \ln^2 \mathcal{R} \right. \\ - \left. \frac{2\alpha_s}{\pi} \mathbf{F}_1 \cdot \mathbf{T}_2 \ln \left(1 + e^{\Delta y} \right) \ln \left(1 + e^{-\Delta y} \right) \right] + \text{h.c.} \right\}
$$
\nonly one (new) matrix

\n
$$
\mathbf{S}^{\text{div}} = \Gamma(\alpha_s) \left(\frac{1}{2} \sum_{\langle i \neq j \rangle} \mathbf{T}_i \cdot \mathbf{T}_j \ln \frac{n_i \cdot n_j}{2} - y_{\text{cut}} (C_B + C_{\bar{B}}) + C_1 \ln \frac{\mathcal{R}}{2 \cosh y_1} + C_2 \ln \frac{\mathcal{R}}{2 \cosh y_2} \right)
$$

Andrew Hornig, LANL SCET 2015 Mar 27, 2015

= *y^J* ln(2 cosh *y^J*)*,* (4.27)

NLL' Cross-Section \blacksquare jets are both "unmeasured" (in the terminology of [37]), i.e., are tagged with an algorithm parameters of Table 2. To arrive at Γ \mathcal{P} dimensions to explicit \mathcal{P} unde quantities with bars to distinguish the corresponding measured measured measured measured measured measure **F** *F F F <i>POCC* **i** θ *AC* **il** parameters of Table 2. To arrive at Eq. (5.29), we used the consistency of the anomalous *^F* for *F* = *Ji,B,H* are given to NLL' in Eq. (B.16) in terms of the parameters of Table 2. To arrive at \mathcal{C} dimensions to explicitly cancel the *µ* dependence to all orders. Here and below, we denote under with bars to distinguish the corresponding measured measured measured measured measured measured measure

, (5.29)

When the angularity of one or more jets is measured, we need to include *S*meas(⌧ *ⁱ*

F =*H,B,B,J* ¯ ¹*,J*²

✓ *µ^F*

❖ unmeasured result: angularity jet shape ⌧*a*. When both jets are left unmeasured (i.e., tagged with an algorithm inmeasured result: $\overline{}$ When the angularity of one or more jets is measured, we need to include *S*meas(⌧ *ⁱ* \mathcal{A} (and it is corresponding anomalous dimension measuremeasur

over initial particle states (e.g., *N* = 4*N*²

 $\frac{1}{2}$ = $\frac{1}{2}$, $\frac{1}{2}$

 $= \frac{p_T x_1 x_2}{8 H}$ $8\pi E_{\rm cm}^4$ 1 *N* $B(x_1; \mu_B) \bar{B}(x_2; \mu_B) J_1(\mu_J) J_2(\mu_J) \, \Pi^{\text{unmeas}}(\bar{\mu}_S, \bar{\mu}_J, \mu_B, \mu_H)$ $\times \, {\rm Tr}\{{\bf H}(\mu_H){\bf U}^\dagger(\mu,\bar\mu_S,\mu_H){\bf S}^{\rm unmeas}(\mu_S){\bf U}(\mu,\bar\mu_S,\mu_H)\}$ $d\sigma$ *dy*1*dy*2*dp^T* $J) \, \Pi^{\text{unmeas}}(\bar{\mu}_S, \bar{\mu}_J, \mu_B, \mu_H)$ $\frac{d\sigma}{d\sigma} = \frac{p_T x_1 x_2}{p_T x_1 x_2} \frac{1}{B(x_1, \mu_B)} \overline{B(x_2, \mu_B)} I_1(\mu_I) I_2(\mu_I) \Pi^{\text{unmeas}}(\overline{\mu_B}, \overline{\mu_I}, \mu_B, \mu_I)$ $dy_1 dy_2 dp_T = 8\pi E_{\text{cm}}^4 N^{D(x_1,\mu_B)D(x_2,\mu_B)J_1(\mu_J)J_2(\mu_J)}$ ($\mu_S, \mu_J, \mu_B, \mu_H$) need to replace the unmeasured jet functions *Jⁱ* with measured ones *J*(⌧ *ⁱ* $J_B) \bar B(x_2; \mu_B) J_1(\mu_J) J_2(\mu_J) \, \Pi^{\rm unmeas}(\bar\mu_S, \bar\mu_J, \mu_B, \mu_H)$ part of the soft functions, \mathbf{r} is easier to first do the evolutions of μ , μ ₅, μ _H) μ (μ ₅) σ (μ , μ ₅, μ _H)]

 ω unmeasured to distinguish to distinguish the corresponding measured measured measured measured measured measured

eK^F (*µ^F , ^µ*¯*S*)

❖ measured result: where $\overline{1}$, and the resulting functions. And the resulting functions in Eq. (3.1), and the renormalized functions. And the renormalized functions. The resulting functions of $\overline{1}$, and the renormalized functions. Th for the case of two measured is the case of the ca $M₁$ are $\alpha \alpha \alpha$ *i*_{*i*}(α) and α α is the unmeasured some is the White and the cross-section tenders the cross-

U(*µ, µS, µH*) = U*S*(*µ, µS*)U*H*(*µ, µH*) (5.28) $T d\tau_a^1 d\tau_a^2$ $8\pi E_{\rm cm}^4$ N^{-(6.1, respectively, respectively, and respectively, respective} $\text{Tr}\left\{\textbf{H}(\mu_H)\textbf{U}^\dagger(\mu,\mu_S,\mu_H)\right\}$ $\frac{d\sigma}{d p_T d\tau_a^1 d\tau_a^2} = \frac{p_T x_1 x_2}{8\pi E_{\rm cm}^4} \frac{1}{N} B(x_1; \mu_B) \bar{B}(x_2; \mu_B) \left[\Pi^{\rm meas}(\tau_a^1, \tau_a^2; \mu_S, \bar{\mu}_S, \mu_J, \mu_B, \mu_H) \right] \text{Tr} \left\{ \mathbf{H}(\mu_H) \mathbf{U}^\dagger(\mu, \mu_S, \mu_H) \left[\mathbf{S}^{\rm unmeas}(\mu_S) \right] \right\}$ $\left[\frac{1}{I}(\tau_a^1; \omega_s^1, \mu_I) + (1 \leftrightarrow 2) \right]$ $\mathbf{U}(\mu, \mu_s, \mu_H)$ } 1 *N* $B(x_1; \mu_B) \bar{B}(x_2; \mu_B)$ $\sqrt{ }$ $\Pi^{\text{meas}}(\tau_a^1, \tau_a^2; \mu_S, \bar\mu_S, \mu_J, \mu_B, \mu_H) \text{ Tr}\left\{ \mathbf{H}(\mu_H) \mathbf{U}\right\}$ 8⇡*E*⁴ cm *N* $(\tau_a^1, \tau_a^2; \mu_S, \bar{\mu}_S, \mu_J, \mu_B, \mu_H)$ T $\mathrm{H}^{\mathsf{F}}\left\{\mathbf{H}(\mu_{H})\mathbf{U}^{\dagger}\right\}$ $+ S_0(f_S^1(\tau_a^1; \omega_S^1, \mu_S) + f_J^1(\tau_a^1; \omega_S^1, \mu_J) + (1 \leftrightarrow 2))$ $\bigg] \mathbf{U}(\mu, \mu_S, \mu_H)$ \mathfrak{d} $+$ $d\sigma$ $dy_1 dy_2 dp_T d\tau_a^1 d\tau_a^2$ $\bar u_S, \mu$ \mathbf{S}_0

When the angularity of one or more jets is measured, we need to include *S*meas(⌧ *ⁱ*

NLL' Cross-Section \blacksquare jets are both "unmeasured" (in the terminology of [37]), i.e., are tagged with an algorithm parameters of Table 2. To arrive at Γ \mathcal{P} dimensions to explicit \mathcal{P} unde quantities with bars to distinguish the corresponding measured measured measured measured measured measure **F** *F F F <i>POCC* **i** θ *AC* **il** parameters of Table 2. To arrive at Eq. (5.29), we used the consistency of the anomalous *^F* for *F* = *Ji,B,H* are given to NLL' in Eq. (B.16) in terms of the parameters of Table 2. To arrive at \mathcal{C} dimensions to explicitly cancel the *µ* dependence to all orders. Here and below, we denote under with bars to distinguish the corresponding measured measured measured measured measured measured measure

, (5.29)

When the angularity of one or more jets is measured, we need to include *S*meas(⌧ *ⁱ*

F =*H,B,B,J* ¯ ¹*,J*²

✓ *µ^F*

❖ unmeasured result: angularity jet shape ⌧*a*. When both jets are left unmeasured (i.e., tagged with an algorithm inmeasured result: $\overline{}$ When the angularity of one or more jets is measured, we need to include *S*meas(⌧ *ⁱ* \mathcal{A} (and it is corresponding anomalous dimension measuremeasur

over initial particle states (e.g., *N* = 4*N*²

 $\frac{1}{2}$ = $\frac{1}{2}$, $\frac{1}{2}$

 $= \frac{p_T x_1 x_2}{8 H}$ $8\pi E_{\rm cm}^4$ 1 *N* $B(x_1; \mu_B) \bar{B}(x_2; \mu_B) J_1(\mu_J) J_2(\mu_J) \, \Pi^{\text{unmeas}}(\bar{\mu}_S, \bar{\mu}_J, \mu_B, \mu_H)$ $\times \, {\rm Tr}\{{\bf H}(\mu_H){\bf U}^\dagger(\mu,\bar\mu_S,\mu_H){\bf S}^{\rm unmeas}(\mu_S){\bf U}(\mu,\bar\mu_S,\mu_H)\}$ $d\sigma$ *dy*1*dy*2*dp^T* $J) \, \Pi^{\text{unmeas}}(\bar{\mu}_S, \bar{\mu}_J, \mu_B, \mu_H)$ $\frac{d\sigma}{d\sigma} = \frac{p_T x_1 x_2}{p_T x_1 x_2} \frac{1}{B(x_1, \mu_B)} \overline{B(x_2, \mu_B)} I_1(\mu_I) I_2(\mu_I) \Pi^{\text{unmeas}}(\overline{\mu_B}, \overline{\mu_I}, \mu_B, \mu_I)$ $dy_1 dy_2 dp_T = 8\pi E_{\text{cm}}^4 N^{D(x_1,\mu_B)D(x_2,\mu_B)J_1(\mu_J)J_2(\mu_J)}$ ($\mu_S, \mu_J, \mu_B, \mu_H$) need to replace the unmeasured jet functions *Jⁱ* with measured ones *J*(⌧ *ⁱ* $J_B) \bar B(x_2; \mu_B) J_1(\mu_J) J_2(\mu_J) \, \Pi^{\rm unmeas}(\bar\mu_S, \bar\mu_J, \mu_B, \mu_H)$ part of the soft functions, \mathbf{r} is easier to first do the evolutions of μ , μ ₅, μ _H) μ (μ ₅) σ (μ , μ ₅, μ _H)]

 ω unmeasured to distinguish to distinguish the corresponding measured measured measured measured measured measured

eK^F (*µ^F , ^µ*¯*S*)

❖ measured result: where $\overline{1}$, and the resulting functions. And the resulting functions in Eq. (3.1), and the renormalized functions. And the renormalized functions. The resulting functions of $\overline{1}$, and the renormalized functions. Th for the case of two measured is the case of the ca $M₁$ are $\alpha \alpha \alpha$ *i*_{*i*}(α) and α α is the unmeasured some is the White and the cross-section tenders the cross-

U(*µ, µS, µH*) = U*S*(*µ, µS*)U*H*(*µ, µH*) (5.28) $T d\tau_a^1 d\tau_a^2$ $8\pi E_{\text{cm}}^4$ N^{-(6.1, respectively, respectively, and respectively, respectively, respectively, respectively, respectively, respectively, \mathbb{R}^4} *^S* (*µ, ^µ*¯*S*) ^Y ⇧*i ^B*(*µ, µB*) Y ⇧¯ *i ^J* (*µ, µ*¯*^J*) *i*=*B,B*¯ *i*=1*,*2 $\frac{y}{j}$ $\frac{y}{z}$ *x* μ ^{*j*} $\left[\frac{\partial ij}{\partial (1 - z)}\right]$ *,* (5.29) *^F* for *F* = *Ji,B,H* are given to NLL' in Eq. (B.16) in terms of the parameters of Table 2. To arrive at Eq. (5.29), we used the consistency of the anomalous dimensions to explicitly cancel the *µ* dependence to all orders. Here and below, we denote $\alpha;\tau_a,\Omega,\mu) = \delta_{ij}\delta(1-z)f_J(\tau_a,\Omega,\mu) + \bar P_{ji}(z)g_a^{ij}(z;\tau_a,\Omega,\mu) + c^{ij}(z)$ $\frac{1}{2}$ s to convolving measured fragmenting jet function for a=0) **FIGURE 1101.4953** $\text{Tr}\left\{\textbf{H}(\mu_H)\textbf{U}^\dagger(\mu,\mu_S,\mu_H)\right\}$ $\frac{d\sigma}{d p_T d\tau_a^1 d\tau_a^2} = \frac{p_T x_1 x_2}{8\pi E_{\rm cm}^4} \frac{1}{N} B(x_1; \mu_B) \bar{B}(x_2; \mu_B) \left[\Pi^{\rm meas}(\tau_a^1, \tau_a^2; \mu_S, \bar{\mu}_S, \mu_J, \mu_B, \mu_H) \right] \text{Tr} \left\{ \mathbf{H}(\mu_H) \mathbf{U}^\dagger(\mu, \mu_S, \mu_H) \left[\mathbf{S}^{\rm unmeas}(\mu_S) \right] \right\}$ $+ S\left(f_S^1(\tau_a^1; \omega_S^1, \mu_S) + f_I^1(\tau_a^1; \omega_S^1, \mu_I) + (1 \leftrightarrow 2)\right] U(\mu, \mu_S, \mu_H)\right\}$ where *fⁱ ^J* and *fⁱ ^S* are given in Eqs. (5.10) and (5.23), respectively, and we defined ⇧meas(⌧ ¹*,*² *^a* ; *µS, µ*¯*S, µ^J , µB, µH*) ⌘ ⇧unmeas(¯*µS, µ^J , µB, µH*) 1 *N* $B(x_1; \mu_B) \bar{B}(x_2; \mu_B)$ $\sqrt{ }$ $\Pi^{\text{meas}}(\tau_a^1, \tau_a^2; \mu_S, \bar\mu_S, \mu_J, \mu_B, \mu_H) \text{ Tr}\left\{ \mathbf{H}(\mu_H) \mathbf{U}\right\}$ *^S*(⌧ ¹ *^a* ; !¹ *^S, µS*) + *f* ¹ *^J* (⌧ ¹ *^a* ; !¹ *^S, µ^J*) + (1 \$ 2)ⁱ U(*µ, µS, µH*) o $\frac{1}{2}$ $f(x) = \frac{f(x)}{2}$ and $f(x) = \frac{f(x)}{2}$, $f(x) = \frac{f(x)}{2}$ *ⁱ*=1*,*² ⇧¯*ⁱ ^J* (*µ, µ^J*) $\overline{}$ *Ui ^J* (⌧ *ⁱ* = ⇧unmeas(¯*µS, µ^J , µB, µH*) $P(x) = \overline{P}(x) e^{i\hat{i}}(x; \tau, \Omega, \mu) + e^{i\hat{i}}(x)$ $\frac{1}{2}$ is the Eqs. (3.30) $\frac{1}{2}$ and $\frac{1}{2}$ and $\frac{1}{2}$ and $\frac{1}{2}$ and $\frac{1}{2}$ and $\frac{1}{2}$ and $\frac{1}{2}$ ances to convolving in asured tragineming jet function for $a=0$ 8⇡*E*⁴ cm *N* $(\tau_a^1, \tau_a^2; \mu_S, \bar{\mu}_S, \mu_J, \mu_B, \mu_H)$ T $\mathrm{H}^{\mathsf{F}}\left\{\mathbf{H}(\mu_{H})\mathbf{U}^{\dagger}\right\}$ $+ S\left(\int_{S}^{1}(\tau_{a}^{1}; \omega_{S}^{1}, \mu_{S}) + f_{J}^{1}(\tau_{a}^{1}; \omega_{S}^{1}, \mu_{J}) + (1 \leftrightarrow 2)\right)$ $\mathbf{U}(\mu, \mu_{S}, \mu_{H})$ \mathfrak{d} $+$ $\int f \, dx$ $\int f \, dx$ $\int g(x)$ $\int g(x) dx$, $\int g(x) dx$, $\int g(x) dx$ ⇧meas(⌧ ¹*,*² *^a* ; *µS, µ*¯*S, µ^J , µB, µH*) ⌘ ⇧unmeas(¯*µS, µ^J , µB, µH*) $\overline{}$ *ⁱ*=1*,*² ⇧¯*ⁱ* a_1 , a_2 , μ) in a_3 is a_1 in a_2 and a_3 are evaluated in Table 2 and are evaluated in Ta $d\sigma$ $dy_1 dy_2 dp_T d\tau_a^1 d\tau_a^2$ $\bar u_S, \mu$ $+ S6$ **Where represents convolutions of** $f_s + f_J \to \sum_{r} \int \frac{dx}{r} D_i(x) \left| \delta_{ij} \delta(r) \right|$ $j \frac{\partial}{\partial z}$ power corrections to Eqs. (3.4) and (3.5) can be included via matching to fixed order QCD. Resummation of logs of ⌧*^a* is achieved by RG evolution of each factorized component from it's canonical scale (cf. Table 2) to the common scale *µ*. Both the hard and soft function are in general hermitian matrices of rank *R* equal to the number of linearly independent color operators associated with the hard process (e.g., *R* = 2 for *qq* ! *qq*, 3 for *qq* ! *gg*, $f_J^{ij}(z;\tau_a,\Omega,\mu)=\delta_{ij}\delta(1-z)f_J(\tau_a,\Omega,\mu)+\bar P_{ji}(z)g_a^{ij}(z;\tau_a,\Omega,\mu)+c^{ij}(z)$ matrix RG equations. The fixed order calculation of the components in Eqs. (3.4) and $\overline{\text{S}}$ and the subject of the subject of the subject of the subject of the next section sections. (reduces to convolving measured fragmenting jet function for a=0) $f_S + f_J \rightarrow \sum$ *j* $\int dx$ *x* $D_i(x)$ \therefore fragmentation: $f_S + f_J \rightarrow \sum \int \frac{dx}{x} D_i(x) \left[\delta_{ij} \delta(1 - z/x) f_S + f_J^{ij}(z/x) \right]$ *z* 1

When the angularity of one or more jets is measured, we need to include *S*meas(⌧ *ⁱ*

NLL' Cross-Section \blacksquare jets are both "unmeasured" (in the terminology of [37]), i.e., are tagged with an algorithm parameters of Table 2. To arrive at Γ \mathcal{P} dimensions to explicit \mathcal{P} unde quantities with bars to distinguish the corresponding measured measured measured measured measured measure **F** *F F F <i>POCC* **i** θ *AC* **il** parameters of Table 2. To arrive at Eq. (5.29), we used the consistency of the anomalous *^F ,* !*ⁱ ^F* for *F* = *Ji,B,H* are given to NLL' in Eq. (B.16) in terms of the parameters of Table 2. To arrive at \mathcal{C} dimensions to explicitly cancel the *µ* dependence to all orders. Here and below, we denote under with bars to distinguish the corresponding measured measured measured measured measured measured measure

, (5.29)

When the angularity of one or more jets is measured, we need to include *S*meas(⌧ *ⁱ*

F =*H,B,B,J* ¯ ¹*,J*²

✓ *µ^F*

❖ unmeasured result: angularity jet shape ⌧*a*. When both jets are left unmeasured (i.e., tagged with an algorithm inmeasured result: $\overline{}$ When the angularity of one or more jets is measured, we need to include *S*meas(⌧ *ⁱ* \mathcal{A} (and it is corresponding anomalous dimension measuremeasur

over initial particle states (e.g., *N* = 4*N*²

 $\frac{1}{2}$ = $\frac{1}{2}$, $\frac{1}{2}$

 $= \frac{p_T x_1 x_2}{8 H}$ $8\pi E_{\rm cm}^4$ 1 *N* $B(x_1; \mu_B) \bar{B}(x_2; \mu_B) J_1(\mu_J) J_2(\mu_J) \, \Pi^{\text{unmeas}}(\bar{\mu}_S, \bar{\mu}_J, \mu_B, \mu_H)$ $\times \, {\rm Tr}\{{\bf H}(\mu_H){\bf U}^\dagger(\mu,\bar\mu_S,\mu_H){\bf S}^{\rm unmeas}(\mu_S){\bf U}(\mu,\bar\mu_S,\mu_H)\}$ $d\sigma$ *dy*1*dy*2*dp^T* $J) \, \Pi^{\text{unmeas}}(\bar{\mu}_S, \bar{\mu}_J, \mu_B, \mu_H)$ $\frac{d\sigma}{d\sigma} = \frac{p_T x_1 x_2}{p_T x_1 x_2} \frac{1}{B(x_1, \mu_B)} \overline{B(x_2, \mu_B)} I_1(\mu_I) I_2(\mu_I) \Pi^{\text{unmeas}}(\overline{\mu_B}, \overline{\mu_I}, \mu_B, \mu_I)$ $dy_1 dy_2 dp_T = 8\pi E_{\text{cm}}^4 N^{D(x_1,\mu_B)D(x_2,\mu_B)J_1(\mu_J)J_2(\mu_J)}$ ($\mu_S, \mu_J, \mu_B, \mu_H$) need to replace the unmeasured jet functions *Jⁱ* with measured ones *J*(⌧ *ⁱ* $J_B) \bar B(x_2; \mu_B) J_1(\mu_J) J_2(\mu_J) \, \Pi^{\rm unmeas}(\bar\mu_S, \bar\mu_J, \mu_B, \mu_H)$ part of the soft functions, \mathbf{r} is easier to first do the evolutions of μ , μ ₅, μ _H) μ (μ ₅) σ (μ , μ ₅, μ _H)]

 ω unmeasured to distinguish to distinguish the corresponding measured measured measured measured measured measured

eK^F (*µ^F , ^µ*¯*S*)

❖ measured result: where $\overline{1}$, and the resulting functions. And the resulting functions in Eq. (3.1), and the renormalized functions. And the renormalized functions. The resulting functions of $\overline{1}$, and the renormalized functions. Th for the case of two measured is the case of the ca $M₁$ are $\alpha \alpha \alpha$ *i*_{*i*}(α) and α α is the unmeasured some is the White and the cross-section tenders the cross-

$$
\frac{d\sigma}{dy_1 dy_2 dp_1 d\tau_a^1 d\tau_a^2} = \frac{p_T x_1 x_2}{8\pi E_{cm}^4} \frac{1}{N} B(x_1; \mu_B) \bar{B}(x_2; \mu_B) \left[\Pi^{\text{meas}}(\tau_a^1, \tau_a^2; \mu_S, \bar{\mu}_S, \mu_J, \mu_B, \mu_H) \text{Tr} \left\{ \mathbf{H}(\mu_H) \mathbf{U}^\dagger(\mu, \mu_S, \mu_H) \left[\mathbf{S}^{\text{unmeas}}(\mu_S) \right] \right\} \right] + \mathbf{S} \left(\int_S^1 (\tau_a^1; \omega_s^1, \mu_S) + f_J^\dagger(\tau_a^1; \omega_s^1, \mu_S) + f_J^\dagger(\tau_a^1; \omega_s^1, \mu_B) + (1 \leftrightarrow 2) \right) \left[\mathbf{U}(\mu, \mu_S, \mu_H) \right] \right]_+ \tag{PRELIMINARY} \mathbf{m} \mathbf{I} \mathbf{I}
$$

When the angularity of one or more jets is measured, we need to include *S*meas(⌧ *ⁱ*

Summary & Outlook

- ❖ most pp results can be obtained directly from e+e-
- ❖ new soft function for out-of-jet/beam radiation
- ❖ NLL' resummation of (boost inv.) jet angularity
- ❖ future:
	- ❖ comparisons to Pythia, etc
	- ❖ quarkonia pheno…
	- ❖ interplay between unmeas/meas beam approaches