LHC Jet Shapes in Dijet Events

Andrew Hornig LANL March 27, 2015

In collaboration with Yiannis Makris, Thomas Mehen

Motivation



* "measured jets" : probed with mass, angularity, etc $\mu_H = Q$

> "jet shapes" (not *the* jet shape $\Psi(r/R)$) Ellis, Kunszt, Soper '91, '92

see also: Yang-Ting Chien's talk





$$\mu_S^R = m_2^2/Q_{\text{Andrew Hornig, LANL}}$$

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Baumgart, Leibovich, Mehert, Rothstein 1406.2295

$$\mu_S^R = m_2^2/Q_{\text{Andrew Hornig, LANL}}$$
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Motivation

* quarkonia @ LHC:



* one step further:

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- * more info if "measured" jets?
- unmeasured factorization not as well understood
- * Pythia/Madonia not great for LHC J/Ψ
 μ_J^R = m₂
 → setup σ for measured case
 (bye quarkonia,μfor²for²fow)
 μ_S^{out} = Λ

$$\mu_S^R = m_2^2/Q_{\text{Andrew Hornig, LANL}}$$

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Setup



* fragmentation: "just" change $J(\tau) \rightarrow J(\tau, z)$

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Jet Algorithms



- * same for all k_T-type (k_T, C/A, anti-k_T) to $\mathcal{O}(\alpha_s)$
- simple replacement

$$R \to \mathcal{R} \sin \theta_J = \frac{\mathcal{R}}{\cosh y_J} \qquad \left(\text{and } 2E \tan \frac{R}{2} \to p_T \mathcal{R} \right)$$

Hadron Angularities

e⁺e⁻ definition:

 $\tau_a^{e^+e^-} = \frac{1}{2E_J} \sum_{i} |p_T^{iJ}| e^{-(1-a)|y_{iJ}|}$

original event shape: Berger, Kucs, Sterman hep-ph/0303051 this def'n for jets: Ellis, AH, Lee, Vermilion, Walsh 1001.0014

$$= (2E_J)^{-(2-a)} (p_T)^{1-a} \sum_i |p_T^i| \left(\frac{\theta_{iJ}}{\sin \theta_J}\right)^{2-a} \left(1 + \mathcal{O}(\theta_{iJ}^2)\right)$$

bo

another simple rescaling: •

cancels offender (and dimensionless)

$$\tau_{a}^{pp} \equiv \frac{1}{p_{T}} \sum_{i} |p_{T}^{i}| (\Delta \mathcal{R}_{iJ})^{2-a} = \left(\frac{2E_{J}}{p_{T}}\right)^{2-a} \tau_{a}^{e^{+}e^{-}} + \mathcal{O}(\tau_{a}^{2})$$

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Scale Hierarchies

for unmeasured jets & beams, need

 $\mathcal{R}^2 \ll 1$ $e^{-y_{\rm cut}} \ll 1$

- * for measured jets, could get full R-dep numerically
- to avoid non-global and kinematic logs
- Bauer, Dunn, AH 1102.4899 see also: Jim Talbert's talk see also: Shireen Gangal's talk; Gangal, Stahlhofen, Tackmann 1412.4792 see also: David Farhi's talk from `14

soft scales: $p_T^{\rm cut}/p_T^J \sim \tau_a^1 \sim \tau_a^2$ see also: Duff Neill's talk; Larkoski, Moult, Neill 1501.04596

(within $10^{\pm 1}$)

hard scales: $p_T^J \sim \hat{s} \sim \hat{t} \sim \hat{u}$ see also: Bauer, Tackmann, Walsh, Zuberi ("Ninja") 1106.6047 see also: Piotr Pietrulewicz's talk

Factorized Cross-Section

* born:

$$\frac{d\sigma_{\text{born}}}{dy_1 dy_2 dp_T} = \frac{p_T x_1 x_2}{8\pi E_{\text{cm}}^4} \frac{1}{N} f_1(x_1;\mu) f_2(x_2;\mu) \operatorname{Tr}\{\mathbf{H}_0 \mathbf{S}_0\}$$

* unmeasured jets:

 $\frac{d\sigma}{dy_1 dy_2 dp_T} = \frac{p_T x_1 x_2}{8\pi E_{\rm cm}^4} \frac{1}{N} B(x_1;\mu) \bar{B}(x_2;\mu) \operatorname{Tr} \{\mathbf{H}(\mu) \mathbf{S}^{\rm unmeas}(\mu)\} [J_1(\mu) J_2(\mu)]$ measured jets: $\frac{d\sigma}{dy_1 dy_2 dp_T d\tau_a^1 d\tau_a^2} = \frac{p_T x_1 x_2}{8\pi E_{\rm cm}^4} \frac{1}{N} B(x_1;\mu) \bar{B}(x_2;\mu) \operatorname{Tr} \{\mathbf{H}(\mu) \mathbf{S}(\tau_a^1,\tau_a^2;\mu)\} \otimes [J_1(\tau_a^1;\mu) J_2(\tau_a^2;\mu)]$

Unmeasured Beam Functions

"unmeasured" fragmenting jet function:

$$\mathcal{G}(E, R, z; \mu) = \sum_{i} \int \frac{dz}{z} \mathcal{J}_{ij}(E, R, z'; \mu) D_j^h(z/z'; \mu) + \mathcal{O}(\Lambda_{\text{QCD}}^2/E^2)$$
$$\mathcal{J}_{ij}(E, R, z', \mu) \equiv \mathcal{J}_{ij}(2E \tan \frac{R}{2}, z', \mu)$$

Procura, Waalewijn 1110.0839

- simple replacement: $2E \tan \frac{R}{2} \to E_{\rm cm} e^{-y_{\rm cut}}$ *
- unmeasured beam function:

same as D at $\mathcal{O}(\alpha_s)$ (different at $\mathcal{O}(\alpha_s^2)$)

$$B_{i}(E_{cm}, y_{cut}, x_{i}; \mu) = \sum_{j} \int \frac{dz}{z} \mathcal{J}_{ij}(E_{cm}e^{-y_{cut}}, z', \mu) f_{j}(z/z', \mu) + \mathcal{O}(\Lambda_{QCD}^{2}/E^{2})$$

evolves like unmeas. jet same as above to $\mathcal{O}(\alpha_{s})$

Procura, Stewart 0911.4980

Jet Functions

* unmeasured:

$$J_{i} \xrightarrow{E \tan \frac{R}{2} \to p_{T} \mathcal{R}} 1 + \frac{\alpha_{s}}{2\pi} \left[\left(\frac{C_{i}}{\epsilon^{2}} + \frac{\gamma_{i}}{\epsilon} \right) \left(\frac{\mu}{p_{T} \mathcal{R}} \right)^{2\epsilon} + d_{J}^{i, \text{alg}} \right]$$

now boost invariant!

* measured: definition of $J(\tau)$ and $\delta(\tau - A\hat{\tau}) = A^{-1}\delta(A^{-1}\tau - \hat{\tau})$



$$\begin{split} \mathbf{S}(\tau_a^1, \tau_a^2) &= \mathbf{S}^{\text{unmeas}} \delta(\tau_a^1) \delta(\tau_a^2) + [\mathbf{S}_0 S^{\text{meas}}(\tau_a^1) \delta(\tau_a^2) + (1 \leftrightarrow 2)] + \mathcal{O}(\alpha_s^2) \\ & \swarrow \\ always \text{ present} & \text{add for each meas jet} \\ & \text{(can sub in favorite jet shape)} \end{split}$$

$$\mathbf{S}(\tau_{a}^{1},\tau_{a}^{2}) = \mathbf{S}^{\text{unmeas}}\delta(\tau_{a}^{1})\delta(\tau_{a}^{2}) + [\mathbf{S}_{0}S^{\text{meas}}(\tau_{a}^{1})\delta(\tau_{a}^{2}) + (1\leftrightarrow 2)] + \mathcal{O}(\alpha_{s}^{2})$$

$$always \text{ present} \qquad \text{add for each meas jet} \\ (\text{can sub in favorite jet shape}) \\ \mathbf{rescaling} \quad \tau_{a}^{e^{+}e^{-}} \to \tau_{a}^{pp} \\ \mathbf{S}^{\text{meas}}(\tau_{a}^{i}) = \sum_{\langle i\neq j \rangle} \left(\frac{p_{T}}{2E_{J}}\right)^{2-a} S_{ij}^{\text{meas}}\left(\left(\frac{p_{T}}{2E_{J}}\right)^{2-a} \tau_{a}^{i}\right) \\ = \frac{1}{\epsilon} \frac{\alpha_{s}C_{i}}{\pi} \frac{e^{\gamma_{E}\epsilon}}{\Gamma(1-\epsilon)} \frac{1}{1-a} \left(\frac{1}{\tau_{a}^{i}}\right)^{1+2\epsilon} \left(\frac{\mu}{p_{T}}\right)^{2\epsilon} \mathcal{R}^{2\epsilon(1-a)} \\ \mathbf{R}^{2\epsilon(1-a)}$$

boost invariant

$$\mathbf{S}(\tau_{a}^{1},\tau_{a}^{2}) = \mathbf{S}^{\text{unmeas}}\delta(\tau_{a}^{1})\delta(\tau_{a}^{2}) + [\mathbf{S}_{0}S^{\text{meas}}(\tau_{a}^{1})\delta(\tau_{a}^{2}) + (1\leftrightarrow 2)] + \mathcal{O}(\alpha_{s}^{2})$$

$$\mathbf{S}^{\text{unmeas}} = \mathbf{S}_{0} + \left[\frac{\mathbf{S}_{0}}{2}\sum_{\langle i\neq j\rangle}\mathbf{T}_{i}\cdot\mathbf{T}_{j}\left(S_{ij}^{\text{incl}} + \sum_{k=1}^{N}S_{ij}^{k}\right) + \text{h.c.}\right]$$

$$\mathbf{S}^{\text{incl}}_{ij} = \frac{1}{\epsilon}\frac{\alpha_{s}}{2\pi}\left(\frac{\mu}{p_{T}^{\text{cut}}}\right)^{2\epsilon}\mathcal{I}^{\text{incl}}_{ij} = -g^{2}\mu^{2\epsilon}\int\frac{d^{d}k}{(2\pi)^{d-1}}\frac{n_{i}\cdot n_{j}}{(n_{i}\cdot k)(n_{j}\cdot k)}\delta(k^{2})\Theta(k^{0})\Theta_{pr}$$

$$S_{ij}^{k} = \frac{1}{\epsilon}\frac{\alpha_{s}}{2\pi}\left(\frac{\mu}{p_{T}^{\text{cut}}}\right)^{2\epsilon}\mathcal{I}^{k}_{ij} = g^{2}\mu^{2\epsilon}\int\frac{d^{d}k}{(2\pi)^{d-1}}\frac{n_{i}\cdot n_{j}}{(n_{i}\cdot k)(n_{j}\cdot k)}\delta(k^{2})\Theta(k^{0})\Theta_{pr}\Theta_{R}^{k}$$
not simply related to anything calculated...

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*

$$S_{ij}^{\text{incl}} \equiv \frac{1}{\epsilon} \frac{\alpha_s}{2\pi} \left(\frac{\mu}{p_T^{\text{cut}}}\right)^{2\epsilon} \mathcal{I}_{ij}^{\text{incl}} = -g^2 \mu^{2\epsilon} \int \frac{d^d k}{(2\pi)^{d-1}} \frac{n_i \cdot n_j}{(n_i \cdot k)(n_j \cdot k)} \delta(k^2) \Theta(k^0) \Theta_{p_T}$$

$$\begin{aligned} \mathcal{I}_{ij}^{\text{incl}} &= \frac{(1-c_{ij})e^{\gamma_E\epsilon}}{2\sqrt{\pi}\Gamma(1/2-\epsilon)} \int_0^{\pi} d\theta_1 \, \sin^{1-2\epsilon} \theta_1 \int_0^{\pi} d\theta_2 \, \sin^{-2\epsilon} \theta_2 \frac{1}{1-c_1} \frac{1}{1-c_{ij}c_1-s_{ij}s_1c_2} \\ &\times \left[\frac{\Gamma(1/2-\epsilon)}{\sqrt{\pi}\Gamma(-\epsilon)} \int_0^{\pi} d\theta_3 \, \sin^{-1-2\epsilon} \theta_3 \left(1-(n_{B1}c_1+n_{B3}s_1c_2+n_{B3}s_1s_2c_3)^2 \right)^{\epsilon} \right]. \end{aligned}$$



$$\vec{n}_i \cdot k = c_1$$

 $\vec{n}_j \cdot k = c_{ij}c_1 + s_{ij}s_1c_2$
 $\vec{n}_B \cdot k = n_{B1}c_1 + n_{B2}s_1c_2 + n_{B3}s_1s_2c_3$

$$\mathcal{I}_{ij}^{\text{incl}} = \frac{(1-c_{ij})e^{\gamma_E\epsilon}}{2\sqrt{\pi}\Gamma(1/2-\epsilon)} \int_0^{\pi} d\theta_1 \, \sin^{1-2\epsilon}\theta_1 \int_0^{\pi} d\theta_2 \, \sin^{-2\epsilon}\theta_2 \frac{1}{1-c_1} \frac{1}{1-c_{ij}c_1-s_{ij}s_1c_2} \\ \times \left[\frac{\Gamma(1/2-\epsilon)}{\sqrt{\pi}\Gamma(-\epsilon)} \int_0^{\pi} d\theta_3 \, \sin^{-1-2\epsilon}\theta_3 \left(1-(n_{B1}c_1+n_{B3}s_1c_2+n_{B3}s_1s_2c_3)^2\right)^{\epsilon}\right].$$

$$\left[\cdots\right] \xrightarrow{\text{planar}} \left(1 - (n_{B1}c_1 + n_{B2}s_1c_2)^2\right)^{\alpha}$$

(different coordinates)

$$\left[\cdots\right] \xrightarrow{n_i = n_B} \sin^{2\epsilon} \theta_1$$

 $\mathcal{I}_{B\bar{B}}^{\text{incl}} + \mathcal{I}_{B\bar{B}}^{B} + \mathcal{I}_{B\bar{B}}^{\bar{B}}$

sum is IR finite

 $\mathcal{I}_{B,I}^{\text{incl}} + \mathcal{I}_{B,I}^B + \mathcal{I}_{B,I}^J$

→ rapidity divergences...

alternative: rapidity regulate $\mathcal{I}_{B\bar{B}}^{incl}$ & \mathcal{I}_{BJ}^{incl} with SCET_{II} (measured) beam functions for p_T resummation

 $\mathbf{S}(\tau_a^1, \tau_a^2) = \mathbf{S}^{\text{unmeas}} \delta(\tau_a^1) \delta(\tau_a^2) + [\mathbf{S}_0 S^{\text{meas}}(\tau_a^1) \delta(\tau_a^2) + (1 \leftrightarrow 2)] + \mathcal{O}(\alpha_s^2)$

$$\mathbf{S}^{\text{unmeas}} = \mathbf{S}_0 + \left\{ \frac{\mathbf{S}_0}{2} \left[\left(\frac{1}{\epsilon} + 2 \ln \frac{\mu}{p_T^{\text{cut}}} \right) \mathbf{S}^{\text{div}} - \frac{\alpha_s}{\pi} (C_1 + C_2) \ln^2 \mathcal{R} - \frac{2\alpha_s}{\pi} \mathbf{T}_1 \cdot \mathbf{T}_2 \ln \left(1 + e^{\Delta y} \right) \ln \left(1 + e^{-\Delta y} \right) \right] + \text{h.c.} \right\}$$
only one (new) matrix

 $\mathbf{S}(\tau_a^1, \tau_a^2) = \mathbf{S}^{\text{unmeas}} \delta(\tau_a^1) \delta(\tau_a^2) + [\mathbf{S}_0 S^{\text{meas}}(\tau_a^1) \delta(\tau_a^2) + (1 \leftrightarrow 2)] + \mathcal{O}(\alpha_s^2)$

$$\mathbf{S}^{\text{unmeas}} = \mathbf{S}_{0} + \left\{ \frac{\mathbf{S}_{0}}{2} \left[\left(\frac{1}{\epsilon} + 2 \ln \frac{\mu}{p_{T}^{\text{cut}}} \right) \mathbf{S}^{\text{div}} - \frac{\alpha_{s}}{\pi} (C_{1} + C_{2}) \ln^{2} \mathcal{R} - \frac{2\alpha_{s}}{\pi} \mathbf{T}_{1} \cdot \mathbf{T}_{2} \ln (1 + e^{\Delta y}) \ln (1 + e^{-\Delta y}) \right] + \text{h.c.} \right\}$$

$$\mathbf{s}^{\text{div}} = \Gamma(\alpha_{s}) \left(\frac{1}{2} \sum_{\langle i \neq j \rangle} \mathbf{T}_{i} \cdot \mathbf{T}_{j} \ln \frac{n_{i} \cdot n_{j}}{2} - y_{\text{cut}} (C_{B} + C_{\bar{B}}) + C_{1} \ln \frac{\mathcal{R}}{2 \cosh y_{1}} + C_{2} \ln \frac{\mathcal{R}}{2 \cosh y_{2}} \right)$$

NLL' Cross-Section

* unmeasured result:

 $\frac{d\sigma}{dy_1 dy_2 dp_T} = \frac{p_T x_1 x_2}{8\pi E_{\rm cm}^4} \frac{1}{N} B(x_1; \mu_B) \bar{B}(x_2; \mu_B) J_1(\mu_J) J_2(\mu_J) \Pi^{\rm unmeas}(\bar{\mu}_S, \bar{\mu}_J, \mu_B, \mu_H) \\ \times \operatorname{Tr}\{\mathbf{H}(\mu_H) \mathbf{U}^{\dagger}(\mu, \bar{\mu}_S, \mu_H) \mathbf{S}^{\rm unmeas}(\mu_S) \mathbf{U}(\mu, \bar{\mu}_S, \mu_H)\}$

* measured result:

 $\frac{d\sigma}{dy_1 dy_2 dp_T d\tau_a^1 d\tau_a^2} = \frac{p_T x_1 x_2}{8\pi E_{\rm cm}^4} \frac{1}{N} B(x_1;\mu_B) \bar{B}(x_2;\mu_B) \left[\Pi^{\rm meas}(\tau_a^1,\tau_a^2;\mu_S,\bar{\mu}_S,\mu_J,\mu_B,\mu_H) \operatorname{Tr} \left\{ \mathbf{H}(\mu_H) \mathbf{U}^{\dagger}(\mu,\mu_S,\mu_H) \left[\mathbf{S}^{\rm unmeas}(\mu_S) + \mathbf{S}_0 \left(f_S^1(\tau_a^1;\omega_S^1,\mu_S) + f_J^1(\tau_a^1;\omega_S^1,\mu_J) + (1\leftrightarrow 2) \right) \right] \mathbf{U}(\mu,\mu_S,\mu_H) \right\} \right]_{\perp}$

NLL' Cross-Section

* unmeasured result:

 $\frac{d\sigma}{dy_1 dy_2 dp_T} = \frac{p_T x_1 x_2}{8\pi E_{\rm cm}^4} \frac{1}{N} B(x_1; \mu_B) \bar{B}(x_2; \mu_B) J_1(\mu_J) J_2(\mu_J) \Pi^{\rm unmeas}(\bar{\mu}_S, \bar{\mu}_J, \mu_B, \mu_H) \\ \times \operatorname{Tr}\{\mathbf{H}(\mu_H) \mathbf{U}^{\dagger}(\mu, \bar{\mu}_S, \mu_H) \mathbf{S}^{\rm unmeas}(\mu_S) \mathbf{U}(\mu, \bar{\mu}_S, \mu_H)\}$

* measured result:

 $\frac{d\sigma}{dy_{1}dy_{2}dp_{T}d\tau_{a}^{1}d\tau_{a}^{2}} = \frac{p_{T}x_{1}x_{2}}{8\pi E_{cm}^{4}} \frac{1}{N} B(x_{1};\mu_{B}) \bar{B}(x_{2};\mu_{B}) \left[\Pi^{\text{meas}}(\tau_{a}^{1},\tau_{a}^{2};\mu_{S},\bar{\mu}_{S},\mu_{J},\mu_{B},\mu_{H}) \operatorname{Tr} \left\{ \mathbf{H}(\mu_{H})\mathbf{U}^{\dagger}(\mu,\mu_{S},\mu_{H}) \left[\mathbf{S}^{\text{unmeas}}(\mu_{S}) + \mathbf{S}(f_{S}^{1}(\tau_{a}^{1};\omega_{S}^{1},\mu_{S}) + f_{J}^{1}(\tau_{a}^{1};\omega_{S}^{1},\mu_{J}) + (1\leftrightarrow 2)) \right] \mathbf{U}(\mu,\mu_{S},\mu_{H}) \right\} \right]_{+}$ $\ast \text{ fragmentation: } f_{S} + f_{J} \rightarrow \sum_{j} \int_{z}^{1} \frac{dx}{x} D_{i}(x) \left[\delta_{ij}\delta(1-z/x)f_{S} + f_{J}^{ij}(z/x) \right]$ $f_{J}^{ij}(z;\tau_{a},\Omega,\mu) = \delta_{ij}\delta(1-z)f_{J}(\tau_{a},\Omega,\mu) + \bar{P}_{ji}(z)g_{a}^{ij}(z;\tau_{a},\Omega,\mu) + c^{ij}(z)$ (reduces to convolving measured fragmenting jet function for a=0)
Jain, Procura, Waalewijn 1101.4953

NLL' Cross-Section

* unmeasured result:

 $\frac{d\sigma}{dy_1 dy_2 dp_T} = \frac{p_T x_1 x_2}{8\pi E_{\rm cm}^4} \frac{1}{N} B(x_1; \mu_B) \bar{B}(x_2; \mu_B) J_1(\mu_J) J_2(\mu_J) \Pi^{\rm unmeas}(\bar{\mu}_S, \bar{\mu}_J, \mu_B, \mu_H) \\ \times \operatorname{Tr}\{\mathbf{H}(\mu_H) \mathbf{U}^{\dagger}(\mu, \bar{\mu}_S, \mu_H) \mathbf{S}^{\rm unmeas}(\mu_S) \mathbf{U}(\mu, \bar{\mu}_S, \mu_H)\}$

* measured result:

$$\frac{d\sigma}{dy_{1}dy_{2}dp_{T}d\tau_{a}^{1}d\tau_{a}^{2}} = \frac{p_{T}x_{1}x_{2}}{8\pi E_{cm}^{4}} \frac{1}{N} B(x_{1};\mu_{B}) \bar{B}(x_{2};\mu_{B}) \left[\Pi^{\text{meas}}(\tau_{a}^{1},\tau_{a}^{2};\mu_{S},\bar{\mu}_{S},\mu_{J},\mu_{B},\mu_{H}) \operatorname{Tr} \left\{ \mathbf{H}(\mu_{H})\mathbf{U}^{\dagger}(\mu,\mu_{S},\mu_{H}) \left[\mathbf{S}^{\text{unmeas}}(\mu_{S}) + \mathbf{S}(f_{S}^{1}(\tau_{a}^{1};\omega_{S}^{1},\mu_{S}) + f_{J}^{1}(\tau_{a}^{1};\omega_{S}^{1},\mu_{J}) + (1\leftrightarrow 2)) \right] \mathbf{U}(\mu,\mu_{S},\mu_{H}) \right\} \right]_{+}$$

$$\frac{1}{2} \operatorname{fragmentation:} f_{S} + f_{J} \rightarrow \sum_{j} \int_{z}^{1} \frac{dx}{x} D_{i}(x) \left[\delta_{ij}\delta(1-z/x)f_{S} + f_{J}^{ij}(z/x) \right]$$

$$\frac{1}{\sigma} \propto \mathbf{D}(z, \mu = \tau^{1/(2-a)})$$

$$\frac{1}{\beta_{J}}(z;\tau_{a},\Omega,\mu) = \delta_{ij}\delta(1-z)f_{J}(\tau_{a},\Omega,\mu) + \bar{P}_{ji}(z)g_{a}^{ij}(z;\tau_{a},\Omega,\mu) + c^{ij}(z)$$

$$(\text{reduces to convolving measured fragmenting jet function for a=0)}$$

$$\operatorname{Jain, Procura, Waalewijn 1101.4953}$$

Summary & Outlook

- * most pp results can be obtained directly from e⁺e⁻
- new soft function for out-of-jet/beam radiation
- NLL' resummation of (boost inv.) jet angularity
- * future:
 - * comparisons to Pythia, etc
 - * quarkonia pheno...
 - interplay between unmeas/meas beam approaches