Summing logarithms with a parton shower

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The prequel

- Parton shower event generators are an important tool for physics.

- Zoltan Nagy (DESY) and I have a parton shower event generator, DEDUCTOR.

Deductor

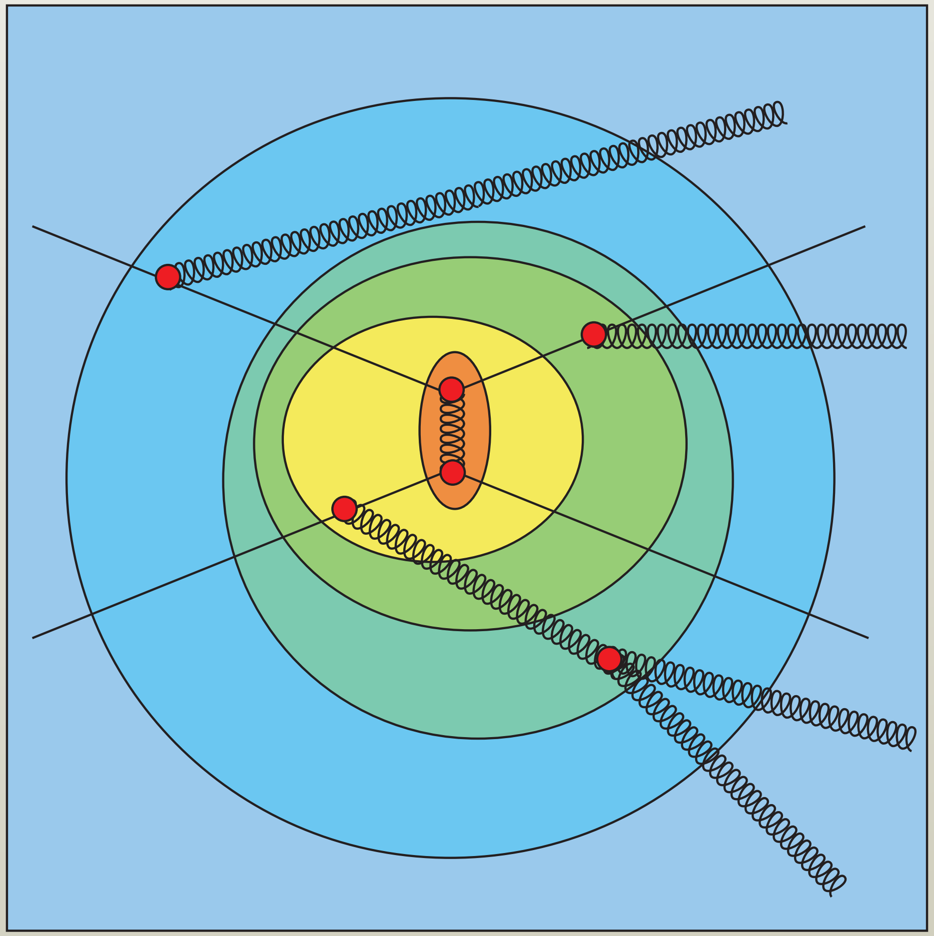
- http://pages.uoregon.edu/soper/deductor/
- Dipole shower.
- In principle, uses quantum density matrix in color & spin.
- LC+ approximation for color.
- Non-zero b and c quark masses.
- See M. Czakon, H. B. Hartanto, M. Kraus and M. Worek "Matching the Nagy-Soper parton shower at next-to-leading order."

Coming in Deductor

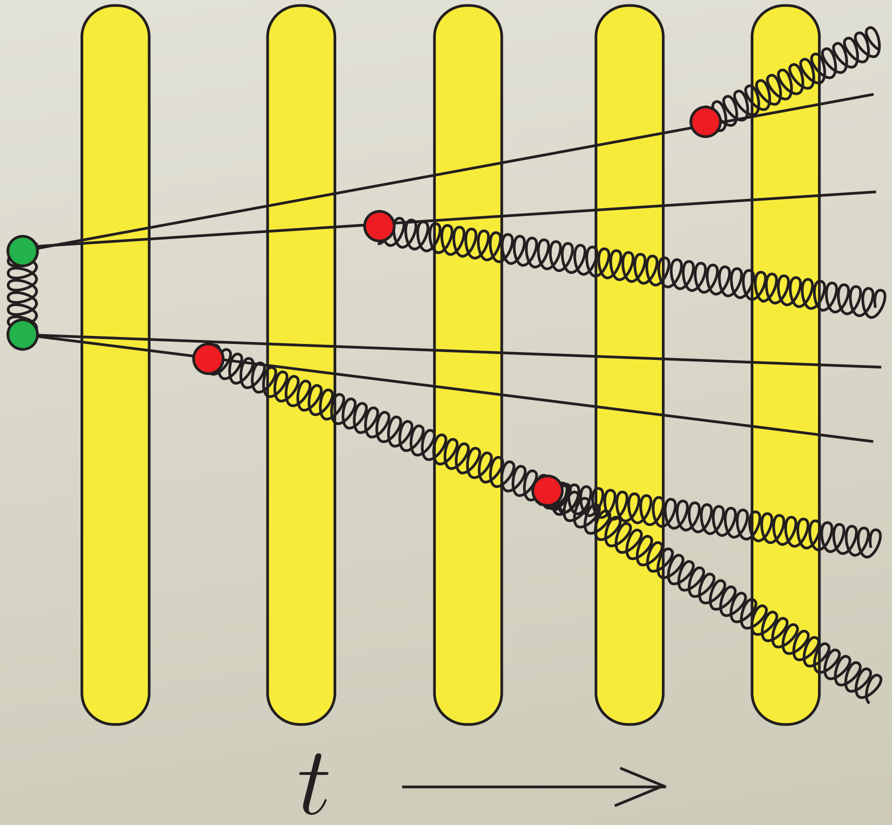
- Perturbative improvement to LC+ approximation.
- Quantum spin.
- Threshold logs (this talk).
- Choices, e.g. for definition of ordering variable.
- Interface to hadronization model.

Shower evolution

- Showers develop in "shower time."
- Hardest interactions first.



Real time picture



Shower time picture

Shower ordering variable

- Originally, PYTHIA used virtuality to order splittings.
- Now, PYTHIA and SHERPA use "k_T."
- DEDUCTOR uses Λ,

$$\Lambda_i^2 = \frac{p_i^2 - m_i^2}{2\, p_i \cdot Q_0}\, Q_0^2 \qquad \text{(final state)}$$

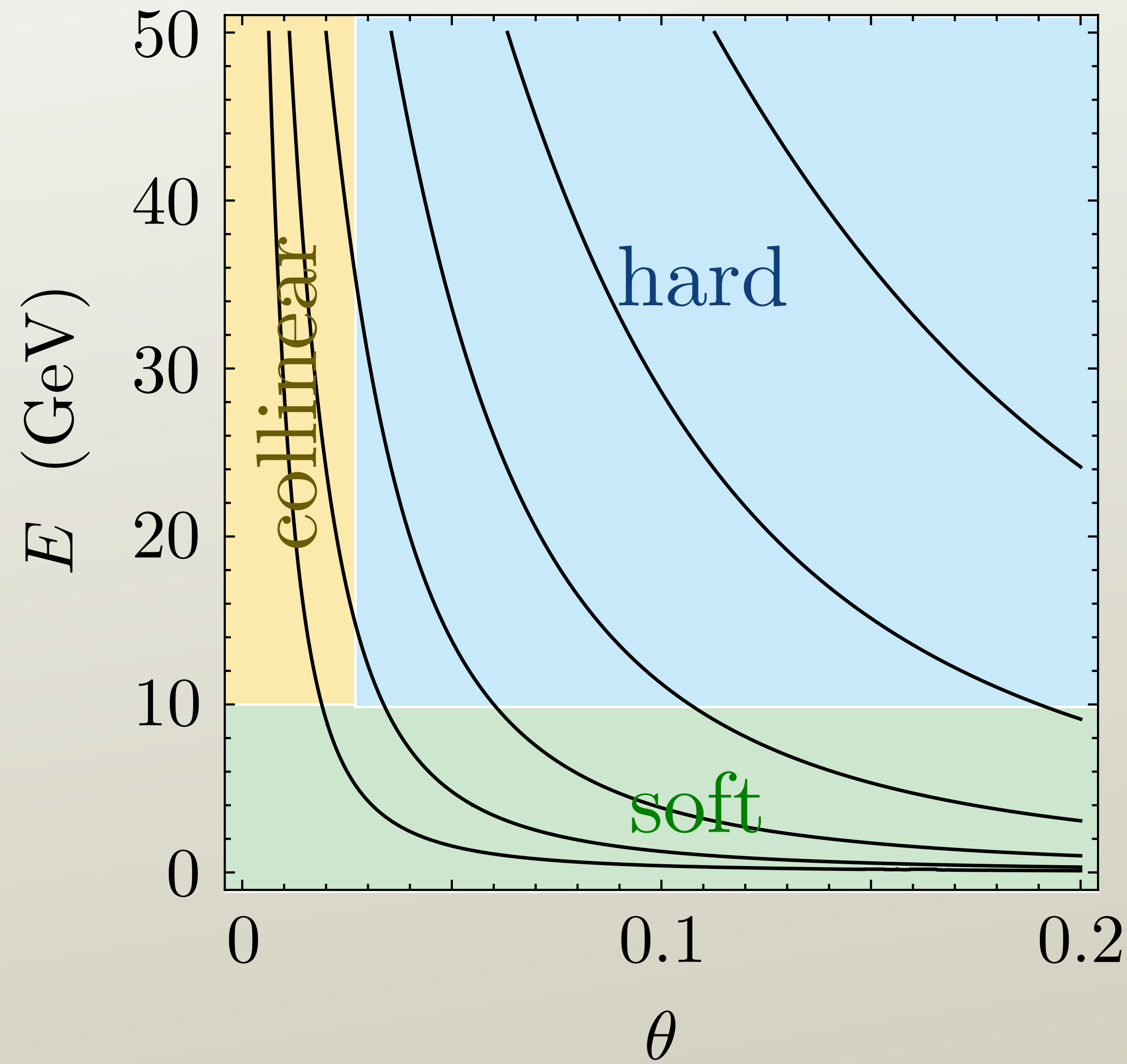
$$\Lambda_i^2 = \frac{|p_i^2 - m_i^2|}{2\eta_i\, p_\mathrm{A} \cdot Q_0}\, Q_0^2 \qquad \text{(initial state)}$$

where
Q_0 is a fixed timelike vector;
p_A is the incoming hadron momentum;
η_i is the parton momentum fraction.

Contrast with SCET

- SCET divides gluon emissions into hard, collinear to hadron A, collinear to hadron B, and soft.
- Each region gets its own special treatment.
- Since the boundaries between regions should not matter, we get differential equations to solve.

- In a parton shower, we have just two regions: hard and everything else.
- We solve a differential equation in the hardness variable that sets the boundary between hard and everything else.
- We count on having a good approximation to sort out collinear regions from the soft region.



Evolution equation

The shower state evolves in shower time.

$$\big|\rho(t)\big) = \mathcal{U}_{\mathcal{V}}(t,t')\big|\rho(t')\big)$$

$$\frac{d}{dt}\mathcal{U}_{\mathcal{V}}(t,t') = [\mathcal{H}_{\rm I}(t) - \mathcal{V}(t)]\mathcal{U}_{\mathcal{V}}(t,t')$$

$$\mathcal{H}_{\rm I}(t) = \text{splitting operator}$$

$$\mathcal{V}(t) = \text{no-splitting operator}$$

An obvious question

- Is this going to sum large logarithms?

Logarithms of $p_\perp$

- Consider $A + B \to Z + X$
- Measure the $p_\perp$ of the Z-boson for $p_\perp^2 \ll M_Z^2$,

$$\frac{d\sigma}{dp_\perp dY}$$

- There are large logarithms $\log(M_Z^2/p_\perp^2)$.

- We know how to sum these in QCD.

The QCD answer,

$$\frac{d\sigma}{d\boldsymbol{p}_\perp dY} \approx \int \frac{d^2\boldsymbol{b}}{(2\pi)^2}\, e^{i\boldsymbol{b}\cdot\boldsymbol{p}_\perp}$$

$$\times \sum_{a,b}\int_{x_{\mathrm{a}}}^{1}\frac{d\eta_{\mathrm{a}}}{\eta_{\mathrm{a}}}\int_{x_{\mathrm{b}}}^{1}\frac{d\eta_{\mathrm{b}}}{\eta_{\mathrm{b}}}\, f_{a/A}(\eta_{\mathrm{a}}, C^2/\boldsymbol{b}^2)\ f_{b/B}(\eta_{\mathrm{b}}, C^2/\boldsymbol{b}^2)$$

$$\times \exp\left(-\int_{C^2/\boldsymbol{b}^2}^{M^2}\frac{d\boldsymbol{k}_\perp^2}{\boldsymbol{k}_\perp^2}\left[A(\alpha_s(\boldsymbol{k}_\perp^2))\log\left(\frac{M^2}{\boldsymbol{k}_\perp^2}\right) + B(\alpha_s(\boldsymbol{k}_\perp^2))\right]\right)$$

$$\times \sum_{a',b'} H^{(0)}_{a'b'}\ C_{a'a}\left(\frac{x_{\mathrm{a}}}{\eta_{\mathrm{a}}}, \alpha_s\left(\frac{C^2}{\boldsymbol{b}^2}\right)\right) C_{b'b}\left(\frac{x_{\mathrm{b}}}{\eta_{\mathrm{b}}}, \alpha_s\left(\frac{C^2}{\boldsymbol{b}^2}\right)\right) .$$

$$A(\alpha_s) = 2\,C_{\mathrm{F}}\,\frac{\alpha_s}{2\pi} + 2\,C_{\mathrm{F}}\left\{C_{\mathrm{A}}\left[\frac{67}{18} - \frac{\pi^2}{6}\right] - \frac{5\,n_{\mathrm{f}}}{9}\right\}\left(\frac{\alpha_s}{2\pi}\right)^2 + \cdots ,$$

$$B(\alpha_s) = -4\,\frac{\alpha_s}{2\pi} + \left[-\frac{197}{3} + \frac{34 n_{\mathrm{f}}}{9} + \frac{20\pi^2}{3} - \frac{8 n_{\mathrm{f}}\pi^2}{27} + \frac{8\zeta(3)}{3}\right]\left(\frac{\alpha_s}{2\pi}\right)^2 + \cdots ,$$

$$C_{a'a}(z,\alpha_s) = \delta_{a'a}\delta(1-z) + \frac{\alpha_s}{2\pi}\left[\delta_{a'a}\left\{\frac{4}{3}(1-z) + \frac{2}{3}\delta(1-z)\left(\pi^2 - 8\right)\right\} + \delta_{ag}\, z(1-z)\right]$$

$$x_A = \sqrt{\frac{M^2}{s}}e^{Y} \qquad x_B = \sqrt{\frac{M^2}{s}}e^{-Y} \qquad C = 2e^{-\gamma_E}$$

Analytical approach

- From Nagy and me (2010).

- Start with the Fourier transform of the cross section.

$$\big(\boldsymbol{b}, Y\big|\rho(t)\big) = \int \frac{d\boldsymbol{p}_\perp}{(2\pi)^2}\, e^{i\boldsymbol{p}_\perp\cdot\boldsymbol{b}}\big(\boldsymbol{p}_\perp, Y\big|\rho(t)\big)$$

- Use the shower evolution equation.

$$\frac{d}{dt}\big(\boldsymbol{b}, Y\big|\rho(t)\big) = \big(\boldsymbol{b}, Y\big|\mathcal{H}_{\rm I}(t) - \mathcal{V}(t)\big|\rho(t)\big)$$

- Use what we know about the operators involved.

Result

✓ Exponentiation

$$\frac{d\sigma}{d\boldsymbol{p}_\perp dY} \approx \int \frac{d^2\boldsymbol{b}}{(2\pi)^2}\, e^{\mathrm{i}\boldsymbol{b}\cdot\boldsymbol{p}_\perp}$$

$$\times \sum_{a,b} \int_{x_\mathrm{a}}^1 \frac{d\eta_\mathrm{a}}{\eta_\mathrm{a}} \int_{x_\mathrm{b}}^1 \frac{d\eta_\mathrm{b}}{\eta_\mathrm{b}}\; f_{a/A}\big(\eta_\mathrm{a}, C^2/\boldsymbol{b}^2\big)\; f_{b/B}\big(\eta_\mathrm{b}, C^2/\boldsymbol{b}^2\big)$$

$$\times \exp\left(- \int_{C^2/\boldsymbol{b}^2}^{M^2} \frac{d\boldsymbol{k}_\perp^2}{\boldsymbol{k}_\perp^2} \left[A(\alpha_\mathrm{s}(\boldsymbol{k}_\perp^2)) \log\left(\frac{M^2}{\boldsymbol{k}_\perp^2}\right) + B(\alpha_\mathrm{s}(\boldsymbol{k}_\perp^2)) \right] \right)$$

$$\times \sum_{a',b'} H^{(0)}_{a'b'}\; C_{a'a}\left(\frac{x_\mathrm{a}}{\eta_\mathrm{a}}, \alpha_\mathrm{s}\left(\frac{C^2}{\boldsymbol{b}^2}\right)\right) C_{b'b}\left(\frac{x_\mathrm{b}}{\eta_\mathrm{b}}, \alpha_\mathrm{s}\left(\frac{C^2}{\boldsymbol{b}^2}\right)\right) .$$

✓ ✓

$$A(\alpha_\mathrm{s}) = 2\, C_\mathrm{F}\, \frac{\alpha_\mathrm{s}}{2\pi} + 2\, C_\mathrm{F} \left\{ C_\mathrm{A} \left[\frac{67}{18} - \frac{\pi^2}{6} \right] - \frac{5\, n_\mathrm{f}}{9} \right\} \left(\frac{\alpha_\mathrm{s}}{2\pi}\right)^2 + \cdots ,$$

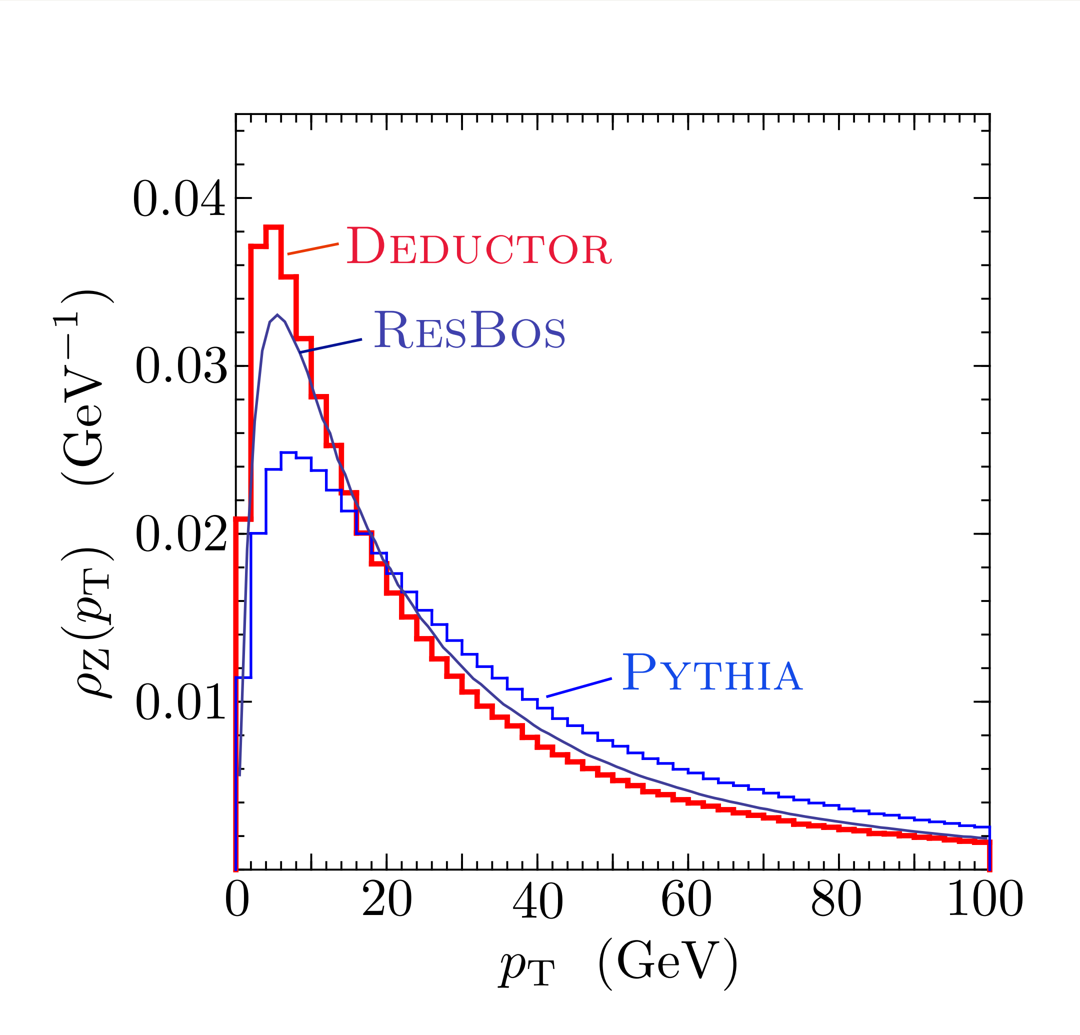
✓

$$B(\alpha_\mathrm{s}) = -4\, \frac{\alpha_\mathrm{s}}{2\pi} + \left[-\frac{197}{3} + \frac{34 n_\mathrm{f}}{9} + \frac{20\pi^2}{3} - \frac{8 n_\mathrm{f}\pi^2}{27} + \frac{8\zeta(3)}{3} \right] \left(\frac{\alpha_\mathrm{s}}{2\pi}\right)^2 + \cdots ,$$

$$C_{a'a}(z, \alpha_\mathrm{s}) = \delta_{a'a}\delta(1-z) + \frac{\alpha_\mathrm{s}}{2\pi} \left[\delta_{a'a} \left\{ \frac{4}{3}(1-z) + \frac{2}{3}\delta(1-z)(\pi^2 - 8) \right\} + \delta_{ag}\, z(1-z) \right]$$

Numerical approach with Deductor

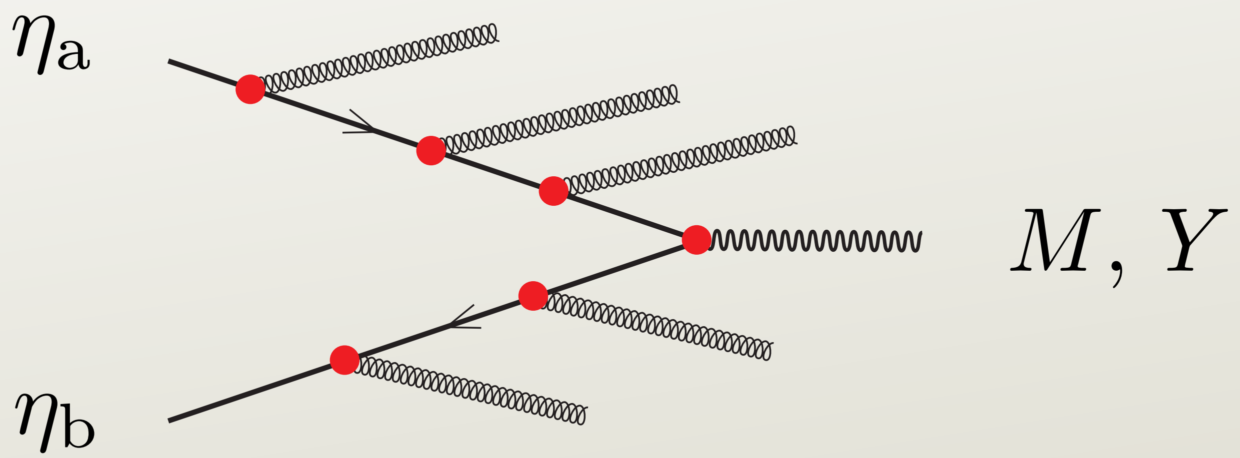
- Look at distribution of P_{T} of e^+e^- pairs with $M > 400$ GeV.
- $\int_0^{100\ \mathrm{GeV}} dp_{\mathrm{T}}\,\rho(p_{\mathrm{T}}) = 1.$
- A parton shower should get this right except for soft effects at $P_{\mathrm{T}} < 10$ GeV.



- We compare DEDUCTOR, PYTHIA, and the analytic log summation in RESBOS.
- DEDUCTOR appears to do well [Nagy and me (2014)].

Threshold logarithms

- Consider the Drell-Yan process with dimuon rapidity Y and mass M.



- There are logarithms of $(1-z)$ where

$$\hat{\eta} = \frac{\eta}{z}$$

- There is a large literature on summing these logarithms starting with Sterman (1987).

- These "threshold logs" are important when the parton distribution functions are steeply falling.

- They affect the cross section

$$\frac{d\sigma}{dM^2\, dY}$$

- A typical parton shower fixes the cross section at the Born cross section.

- Therefore the threshold logarithms are not included.

Including threshold logs

- A parton shower can sum logarithms if you let it.
- We propose to do that, at a leading log level.
- This is work in progress, not yet implemented in DEDUCTOR.
- I can show you the main idea.

What not to do

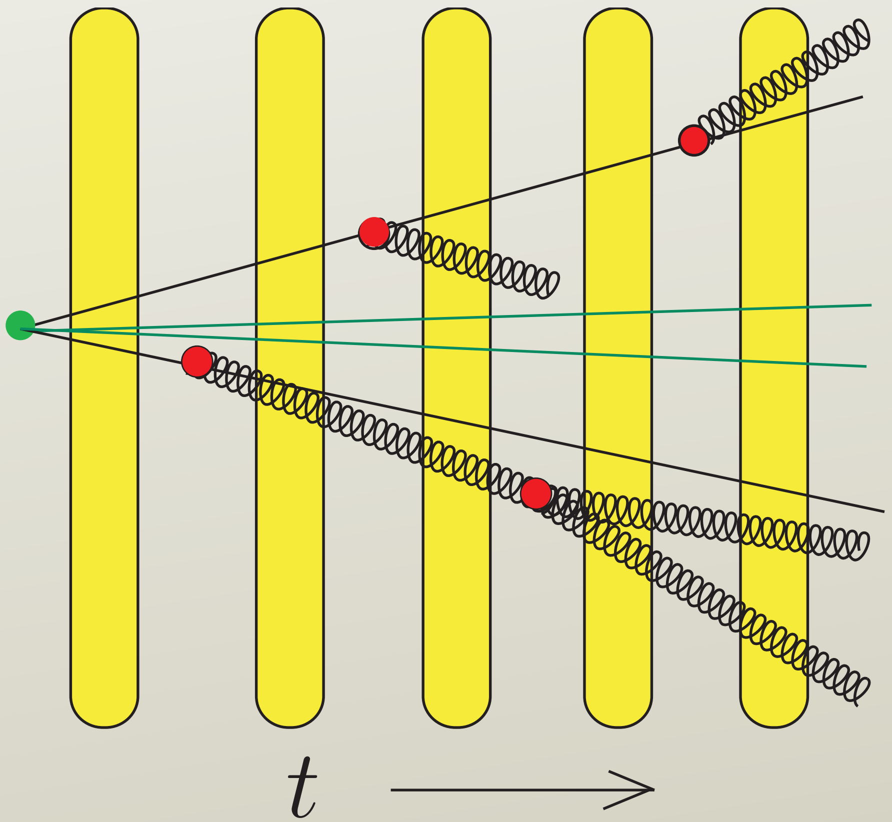
- The shower state evolves in shower time.

$$\big|\rho(t)\big) = \mathcal{U}_{\mathcal{V}}(t,t')\big|\rho(t')\big)$$

$$\frac{d}{dt}\mathcal{U}_{\mathcal{V}}(t,t') = [\mathcal{H}_{\rm I}(t) - \mathcal{V}(t)]\mathcal{U}_{\mathcal{V}}(t,t')$$

$$\mathcal{H}_{\rm I}(t) = \text{splitting operator}$$

$$\mathcal{V}(t) = \text{no-splitting operator}$$



- We calculate $\mathcal{V}(t)$ from $\mathcal{H}_I(t)$ so that the inclusive cross section does not change during the shower.

What to do

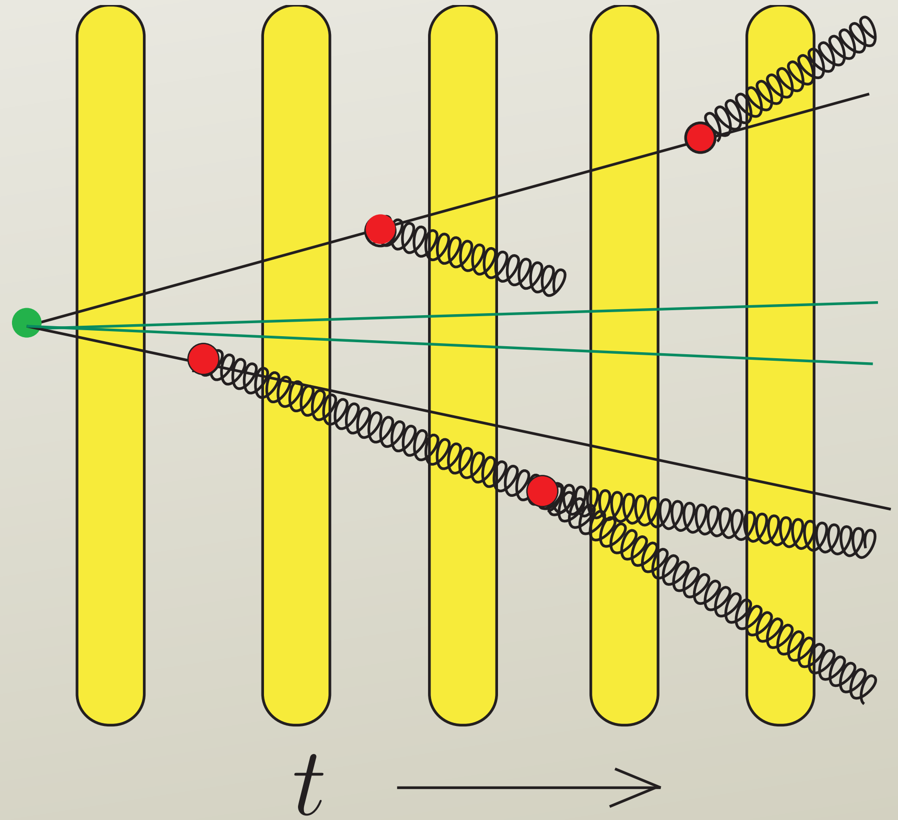
- The shower state evolves in shower time.

$$\big|\rho(t)\big) = \mathcal{U}_{\mathcal{A}}(t,t')\big|\rho(t')\big)$$

$$\frac{d}{dt}\mathcal{U}_{\mathcal{A}}(t,t') = [\mathcal{H}_{\mathrm{I}}(t) - \mathcal{A}(t)]\mathcal{U}_{\mathcal{A}}(t,t')$$

$$\mathcal{H}_{\mathrm{I}}(t) = \text{splitting operator}$$

$$\mathcal{A}(t) = \text{virtual splitting operator}$$



- We simply calculate $\mathcal{A}(t)$ from one loop virtual graphs plus parton evolution.

What happens

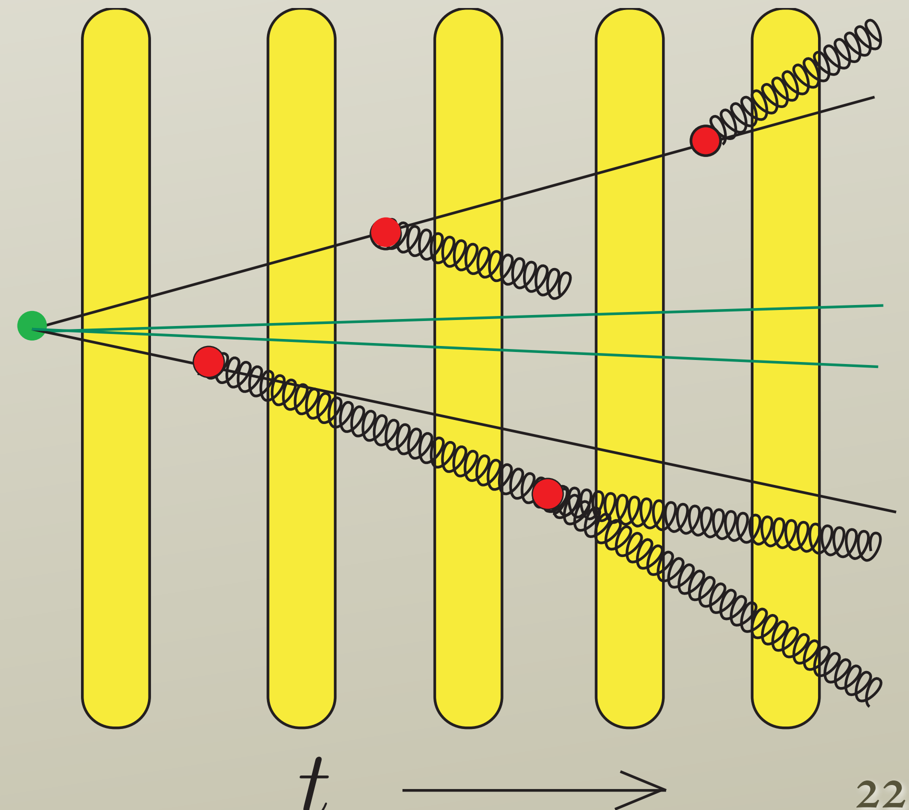
$$\mathcal{U}_{\mathcal{A}}(t,t_0) = \mathcal{N}_{\mathcal{A}}(t,t_0) + \int_{t_0}^{t} d\tau\, \mathcal{U}_{\mathcal{A}}(t,\tau)\mathcal{H}_I(\tau)\mathcal{N}_{\mathcal{A}}(\tau,t_0)$$

$$\mathcal{N}_{\mathcal{A}}(t_2,t_1) = \mathbb{T}\exp\left[\int_{t_1}^{t_2} d\tau\; [-\mathcal{V}(\tau) + (\mathcal{V}(\tau) - \mathcal{A}(\tau))]\right]$$

- Within the LC+ approximation, the operators commute.
- There is an extra factor

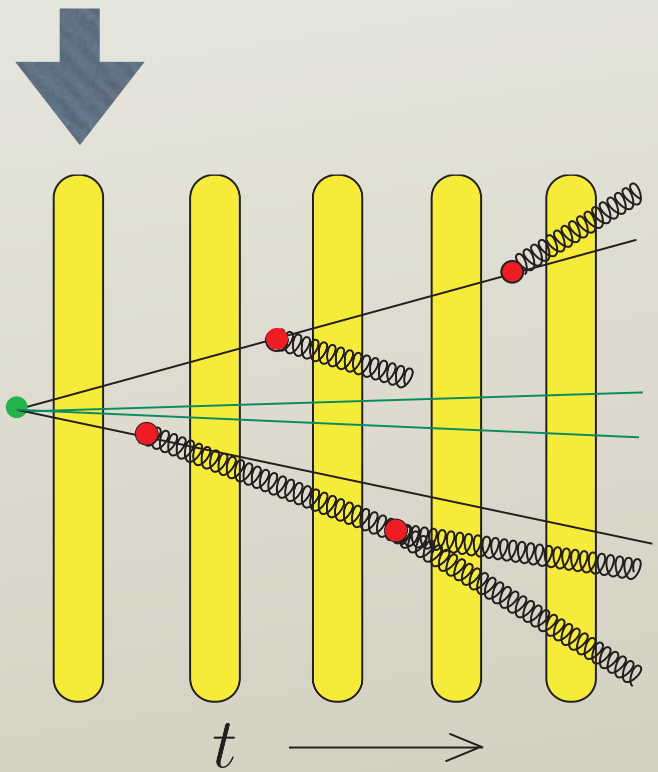
$$\exp\left[\int_{t_1}^{t_2} d\tau\; (\mathcal{V}(\tau) - \mathcal{A}(\tau))\right]$$

that changes the cross section.



The most important term

- Look at the Drell-Yan process.
- Look at the factor for line “a” just after the hard interaction.
- Assume that no real gluons have been emitted yet.



- Use y = dimensionless virtuality variable (with $y \ll 1$) and z = momentum fraction.

- Result: almost everything cancels.

- Two terms do not quite cancel.

$$
\begin{aligned}
&[\mathcal{V}_{\rm a}(t) - \mathcal{A}_{\rm a}(t)]\big|\{p,f,s',c',s,c\}_m\big) = \\
&\Bigg\{ \int_0^{1/(1+y)} \frac{dz}{z} \sum_{\hat a} \frac{\alpha_s}{2\pi} \left(\frac{f_{\hat a/A}(\eta_{\rm a}/z, Q^2 y/z)}{f_{a/A}(\eta_{\rm a}, Q^2 y)} P_{a\hat a}(z) - \delta_{a\hat a} \frac{2 C_a z}{1-z} \right) [1 \otimes 1] \\
&\ - \int_0^{1} \frac{dz}{z} \sum_{\hat a} \frac{\alpha_s}{2\pi} \left(\frac{f_{\hat a/A}(\eta_{\rm a}/z, Q^2 y/z)}{f_{a/A}(\eta_{\rm a}, Q^2 y)} P_{a\hat a}(z) - \delta_{a\hat a} \frac{2 C_a z}{1-z} \right) [1 \otimes 1] \\
&\ + \cdots \Bigg\}\big|\{p,f,s',c',s,c\}_m\big)
\end{aligned}
$$

- $z < 1/(1+y)$ comes from splitting kinematics.

- $z < 1$ comes from parton evolution.

- This leaves an integration over a tiny range of z:

$$[\mathcal{V}_{\rm a}(t) - \mathcal{A}_{\rm a}(t)]\big|\{p,f,s',c',s,c\}_m\big) =$$

$$\Bigg\{ \int_{1/(1+y)}^{1} \frac{dz}{z} \sum_{\hat a} \frac{\alpha_s}{2\pi} \left(\delta_{a\hat a} \frac{2C_a z}{1-z} - \frac{f_{\hat a/A}(\eta_{\rm a}/z, Q^2 y/z)}{f_{a/A}(\eta_{\rm a}, Q^2 y)} P_{a\hat a}(z) \right) [1 \otimes 1]$$

$$+ \cdots \Bigg\} \big|\{p,f,s',c',s,c\}_m\big)$$

- for $(1-z) < y/(1+y) \ll 1$ use $z \approx 1$ and

$$P_{a\hat a}(z) \approx \delta_{a\hat a} \frac{2C_a}{1-z}$$

- This gives

$$
\begin{aligned}
&[\mathcal{V}_{\rm a}(t) - \mathcal{A}_{\rm a}(t)]\big|\{p,f,s',c',s,c\}_m\big) = \\
&\left\{\int_{1/(1+y)}^1 dz\ \frac{\alpha_s}{2\pi}\,\frac{2C_a}{1-z}\left(1 - \frac{f_{a/A}(\eta_{\rm a}/z, Q^2 y)}{z f_{a/A}(\eta_{\rm a}, Q^2 y)}\right)[1\otimes 1]\right. \\
&\quad \left. + \cdots\right\}\big|\{p,f,s',c',s,c\}_m\big)
\end{aligned}
$$

- The $1/(1-z)$ factor creates the "threshold log."

- But the parton factor contains a factor $(1-z)$ so there is no actual log.

- For $y \ll 1$, this contribution is suppressed by a factor of y.

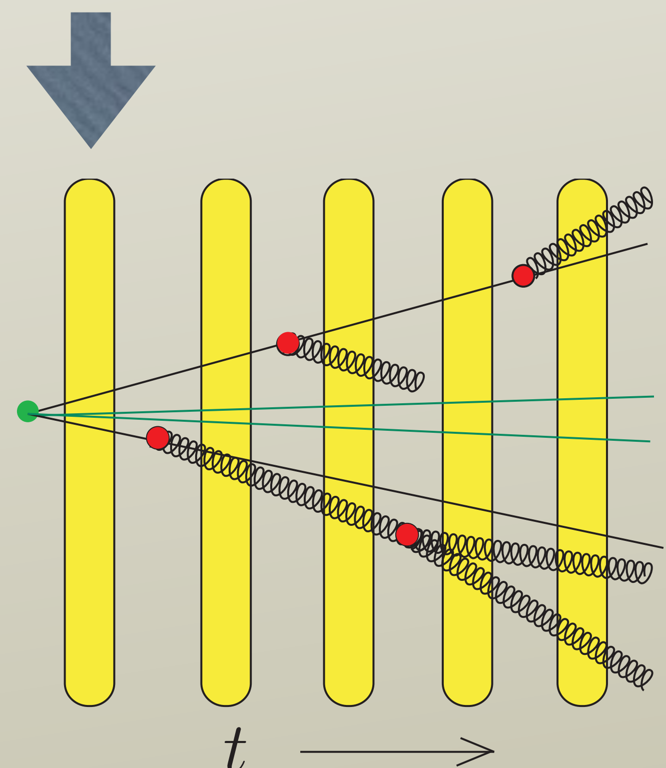
- But, the parton factor can be large, so we keep this.

- Look at the production of $\mu^+\mu^-$ with a large mass M.
- We should multiply the Born cross section by a factor

$$Z_{\rm a} Z_{\rm b} K$$

where $Z_{\rm a}$ and $Z_{\rm b}$ (not discussed here) convert the $\overline{\rm MS}$ parton distributions to virtuality based parton distributions and

$$K = \exp\left(\kappa_{\rm a}(a) + \kappa_{\rm b}(\bar{a})\right)$$



- K is the "extra" Sudakov factor for the first step in the shower.
- For the moment, assume that there are no more steps.

- Then K is given by

$$K = \exp\left(\kappa_{\rm a}(a) + \kappa_{\rm b}(\bar{a})\right)$$

$$\begin{aligned}\kappa_{\rm a}(a) = \int_0^1 \frac{dy}{y} \int_{1/(1+y)}^1 dz\ \frac{\alpha_{\rm s}(y(1-z)M^2)}{2\pi} \\ \times \left[1 - \frac{f_{a/A}(\eta_{\rm a}/z, yM^2)}{z f_{a/A}(\eta_{\rm a}, yM^2)}\right] \\ \times \frac{2C_{\rm F}}{1-z}\,\theta(m_\perp^2 < y(1-z)M^2)\end{aligned}$$

- $m_\perp = 1$ GeV provides an infrared cutoff.

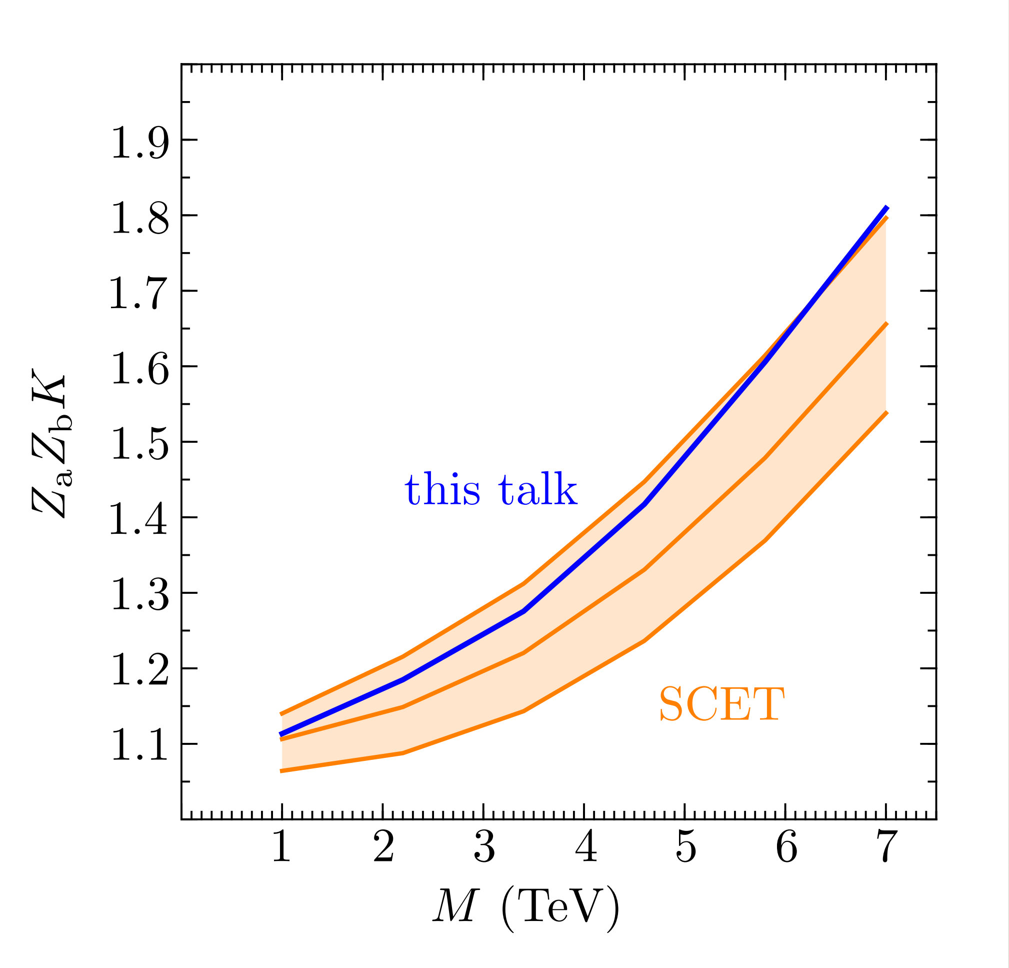
Numerical result

- Examine $p + p \to \mu^+ + \mu^- + X$ at $\sqrt{s} = 14$ TeV.

- Plot $Z_{\rm a} Z_{\rm b} K$ vs. the mass M of the dimuon at $Y = 0$.

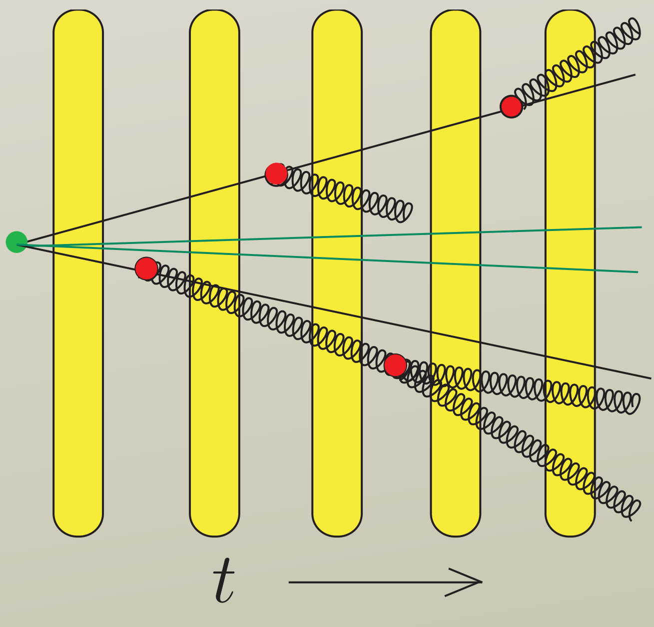
- Compare to SCET formulas from Becher, Neubert, & Xu (2008).

- For SCET, I use the "LO" result with a range of scale choices.



Conclusion on threshold logs

- We find simple and intuitive leading order formulas.
- This is in the context of a leading order parton shower not “NLO” or “NNLO.”
- The numerical results for the main factors seem sensible.
- We expect to implement this as part of DEDUCTOR.
- The summation will apply to all hard processes.
- It remains to be seen what happens.



General conclusion

- Parton shower event generators can sum logarithms.
- They are leading order, so not as precise as SCET.
- But they are useful because they are more general.
- Summing threshold logs with a parton shower seems possible.