Inclusive and Exclusive Accuracy in Resummed Cross Sections

Jonathan Walsh, UC Berkeley

work with Daniele Bertolini, Mikhail Solon, and Frank Tackmann

Resummed Cross Sections

resummation predicts two quantities:

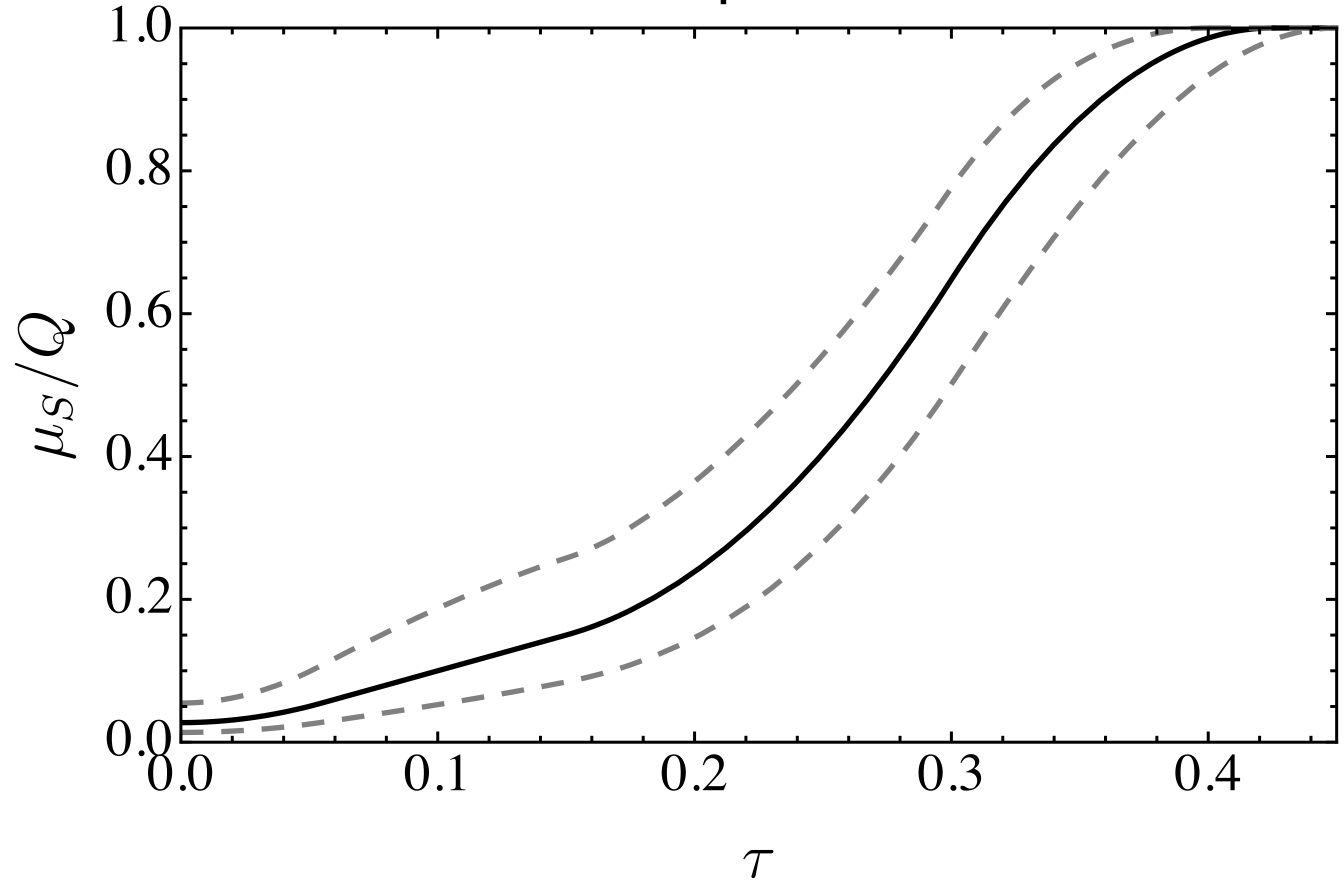
$$\frac{d\sigma}{d\tau}(\mu_i)$$: spectrum

$$\Sigma(\mu_i, \tau)$$: cumulant

- $\Sigma(\mu_i, \tau) = \int_0^\tau d\tau' \, \frac{d\sigma}{d\tau'}$

RGE, fixed order matching implemented by profile scales: $\mu_i = \mu_i(\tau)$

soft profile



profile scale variations used to assess scale uncertainties

I will take τ to be thrust for this talk, but generically it can be any jet resolution variable

Resummed Cross Sections

resummation predicts two quantities:

$$\frac{d\sigma}{d\tau}(\mu_i)$$: spectrum

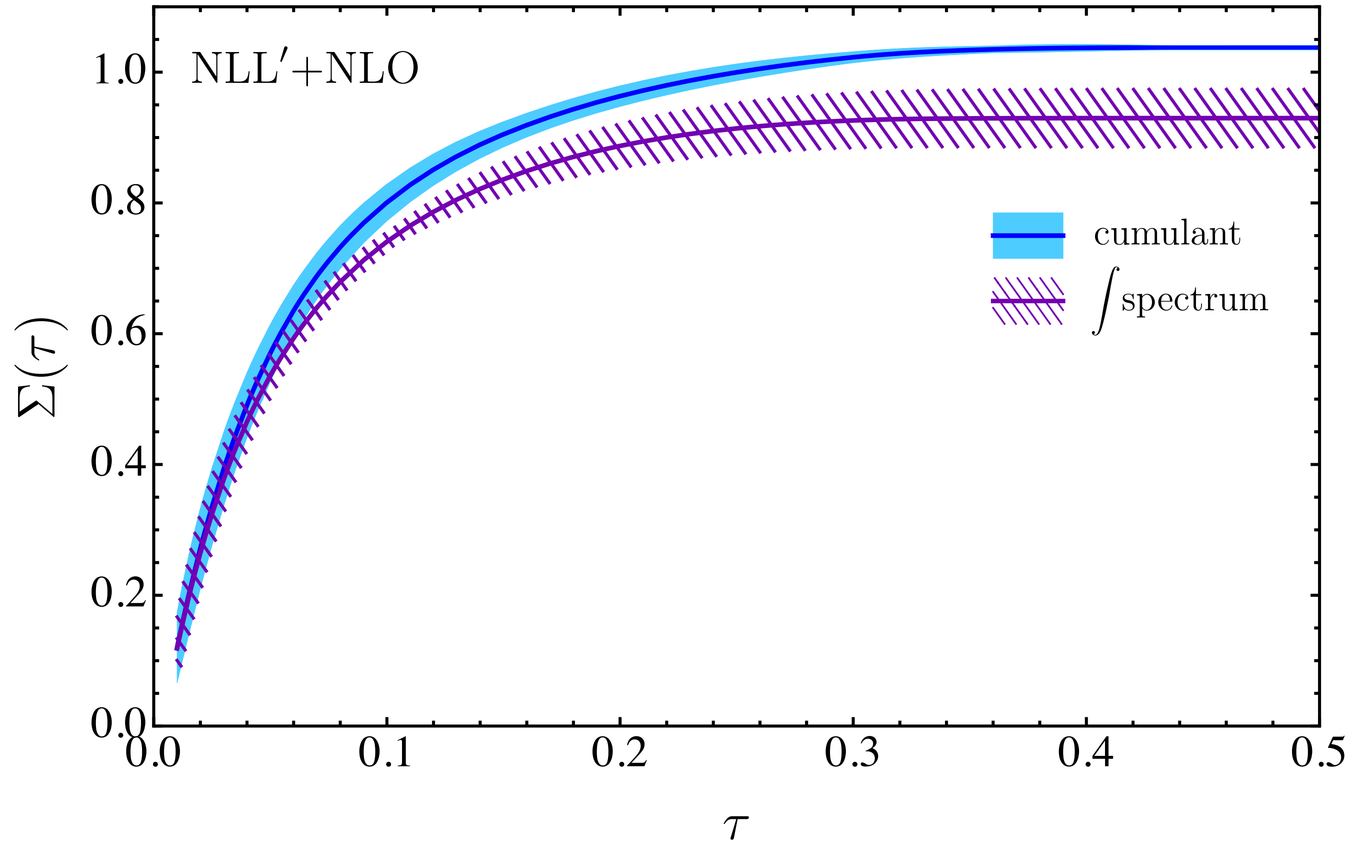
$$\Sigma(\mu_i, \tau)$$: cumulant

- $\Sigma(\mu_i, \tau) = \int_0^\tau d\tau' \, \frac{d\sigma}{d\tau'}$

RGE, fixed order matching implemented by profile scales: $\mu_i = \mu_i(\tau)$

simple problem: typically, these two predictions are inconsistent

cumulant vs. integrated spectrum



$$\Sigma(\mu_i(\tau), \tau) \quad \text{vs.} \quad \int_0^\tau d\tau' \, \frac{d\sigma}{d\tau'}(\mu_i(\tau'))$$

How does the Inconsistency Arise?

cumulant
with free scales

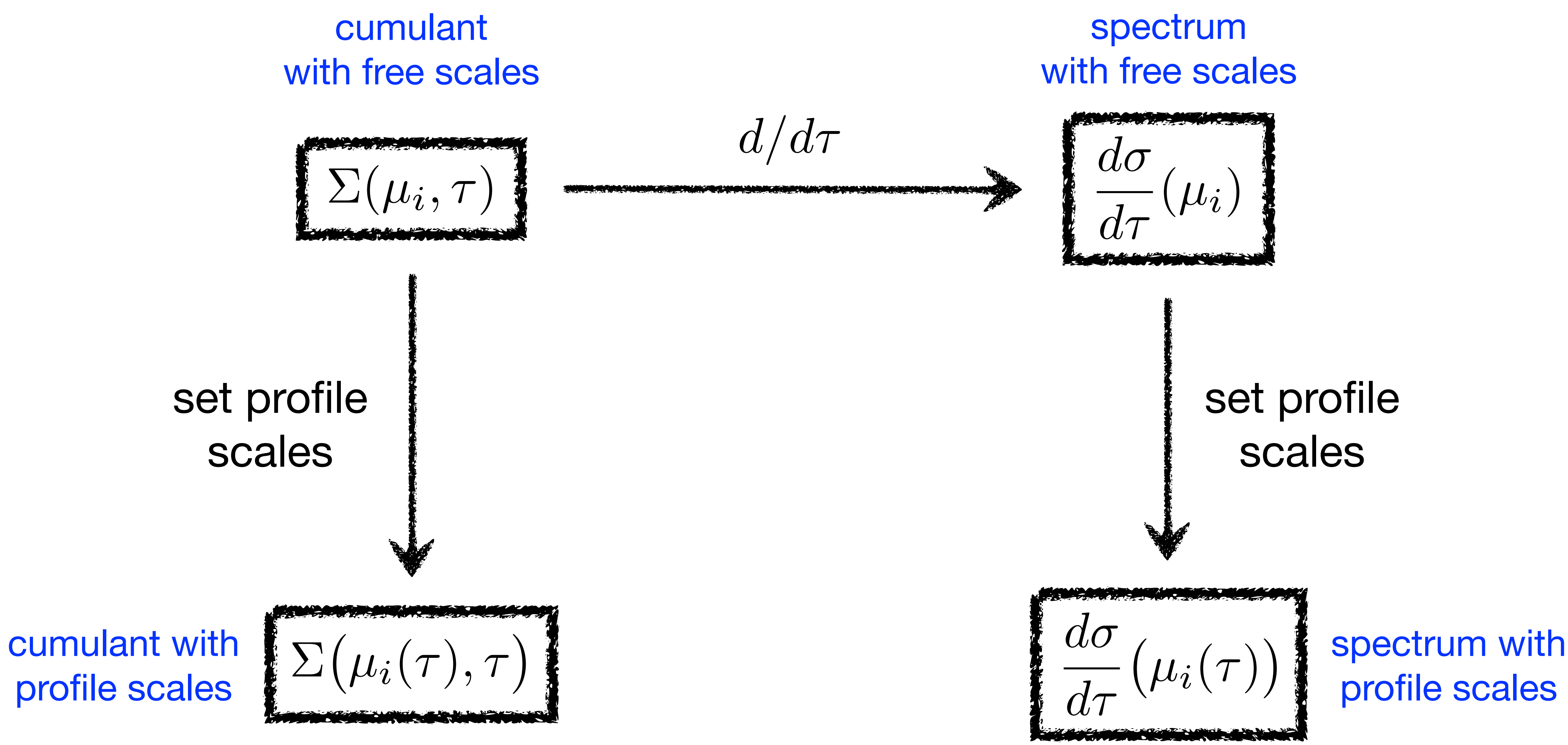
$$\Sigma(\mu_i, \tau)$$

$$\xrightarrow{d/d\tau}$$

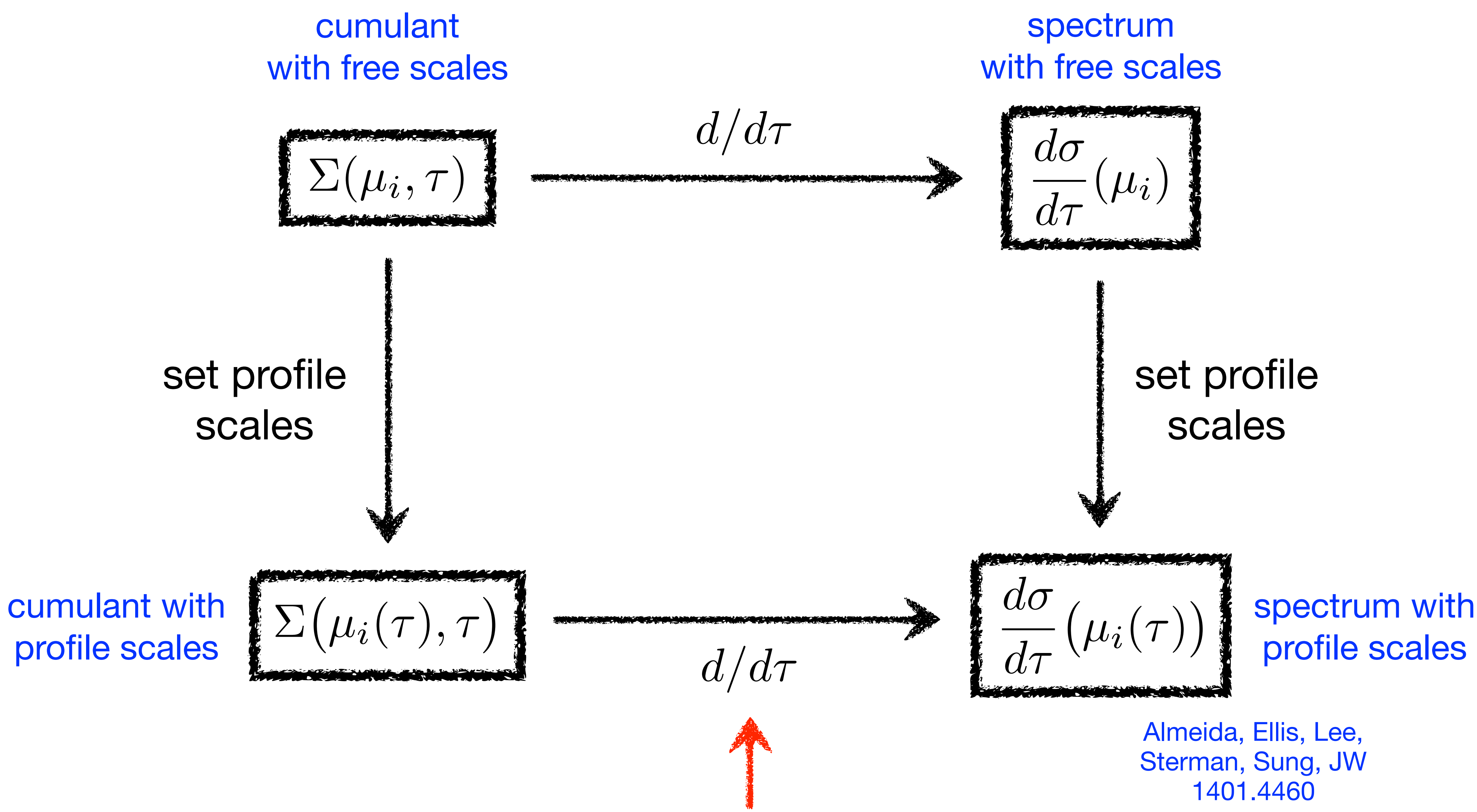
spectrum
with free scales

$$\frac{d\sigma}{d\tau}(\mu_i)$$

How does the Inconsistency Arise?

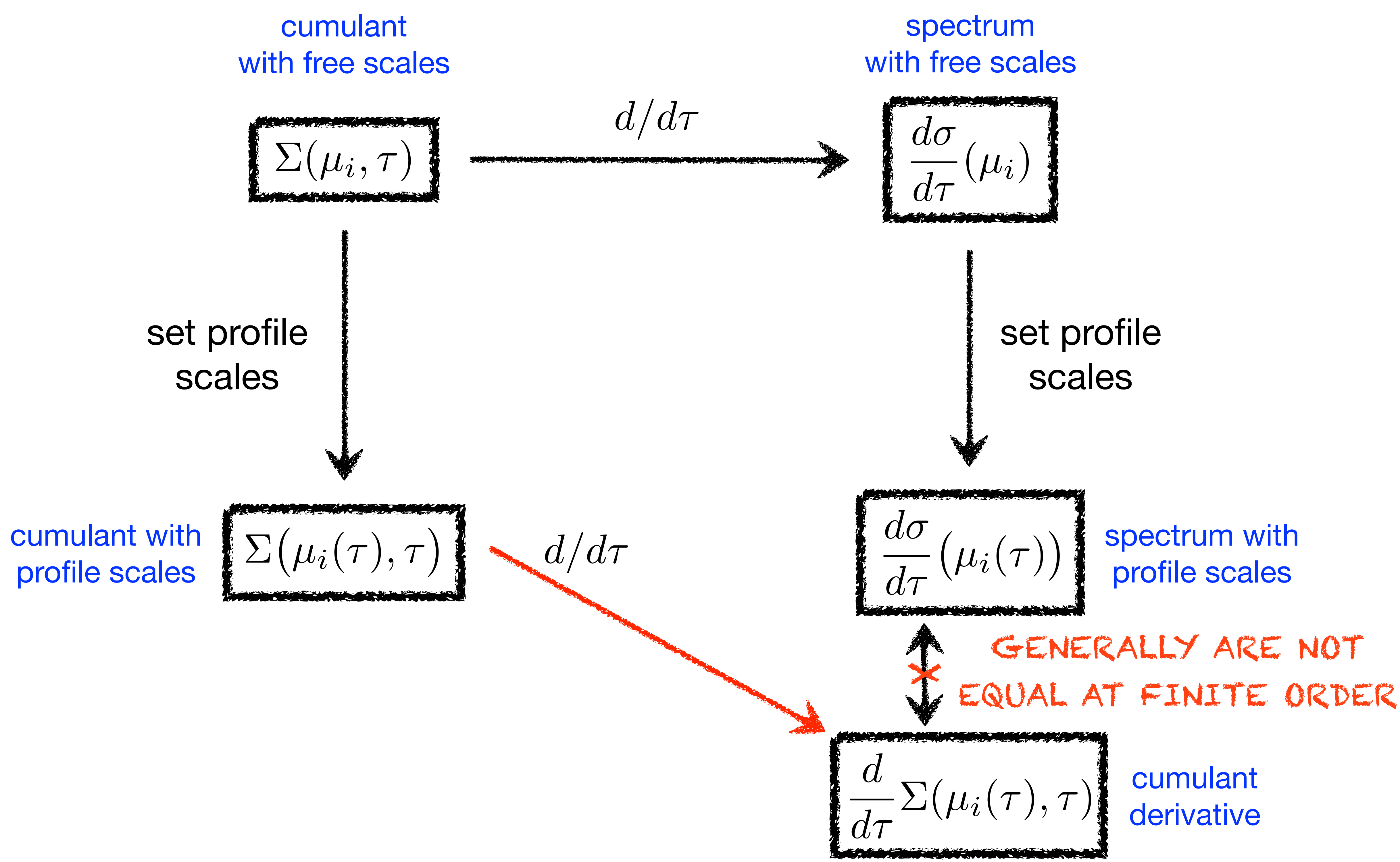


How does the Inconsistency Arise?

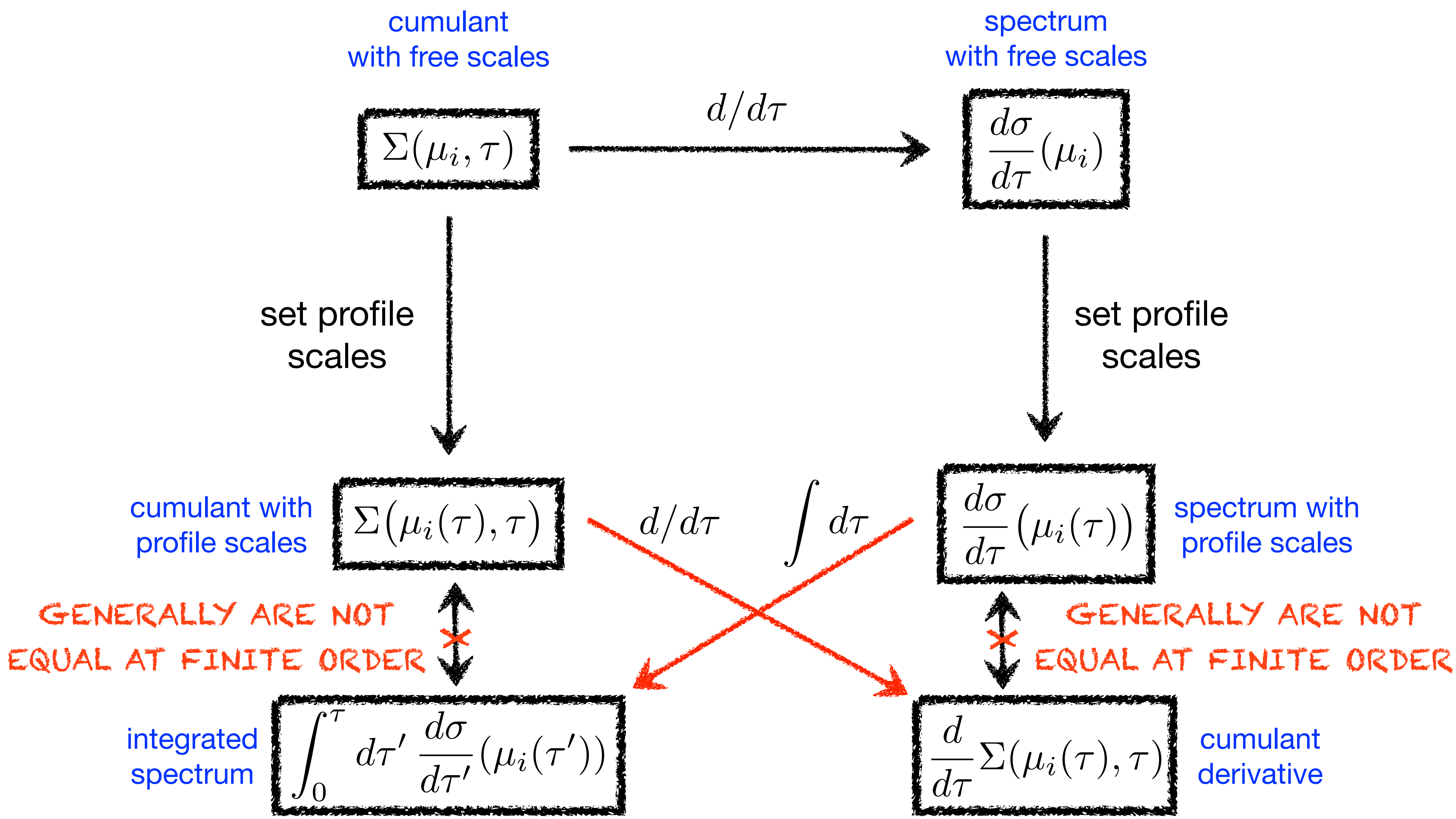


TRUE WHEN WORKING
TO ALL ORDERS IN RG

How does the Inconsistency Arise?



How does the Inconsistency Arise?



Two Paths to the Spectrum

$$\frac{d}{d\tau}\Sigma(\mu_i(\tau),\tau) \quad \text{vs.} \quad \frac{d\sigma}{d\tau}\big(\mu_i(\tau)\big)$$: difference probes the commutator $\big[\mu_i = \mu_i(\tau),\, d/d\tau\big]$

cumulant (free scales) → **spectrum** *(free scales)* → **spectrum (profiles)** $\frac{d}{d\tau}\times[\mu_i = \mu_i(\tau)]$

cumulant (free scales) → *cumulant* **(profiles)** → **spectrum (profiles)** $[\mu_i = \mu_i(\tau)]\times\frac{d}{d\tau}$

in terms of full/partial derivatives:

$$\frac{d}{d\tau} = \frac{\partial}{\partial\tau} + \frac{d\mu_i}{d\tau} \;\Rightarrow\; \left[\frac{d}{d\tau}\Sigma(\mu_i(\tau),\tau) - \frac{d\sigma}{d\tau}(\mu_i(\tau))\right] \propto \frac{d\mu_i}{d\tau}\times(\text{higher order})$$

Almeida, Ellis, Lee, Sterman, Sung, JW 1401.4460

Two Paths to the Spectrum

$$\frac{d}{d\tau}\Sigma(\mu_i(\tau),\tau) \quad \text{vs.} \quad \frac{d\sigma}{d\tau}\big(\mu_i(\tau)\big)$$: difference probes the commutator $\big[\mu_i = \mu_i(\tau),\, d/d\tau\big]$

cumulant (free scales) → **spectrum** *(free scales)* → **spectrum (profiles)** $\frac{d}{d\tau} \times [\mu_i = \mu_i(\tau)]$

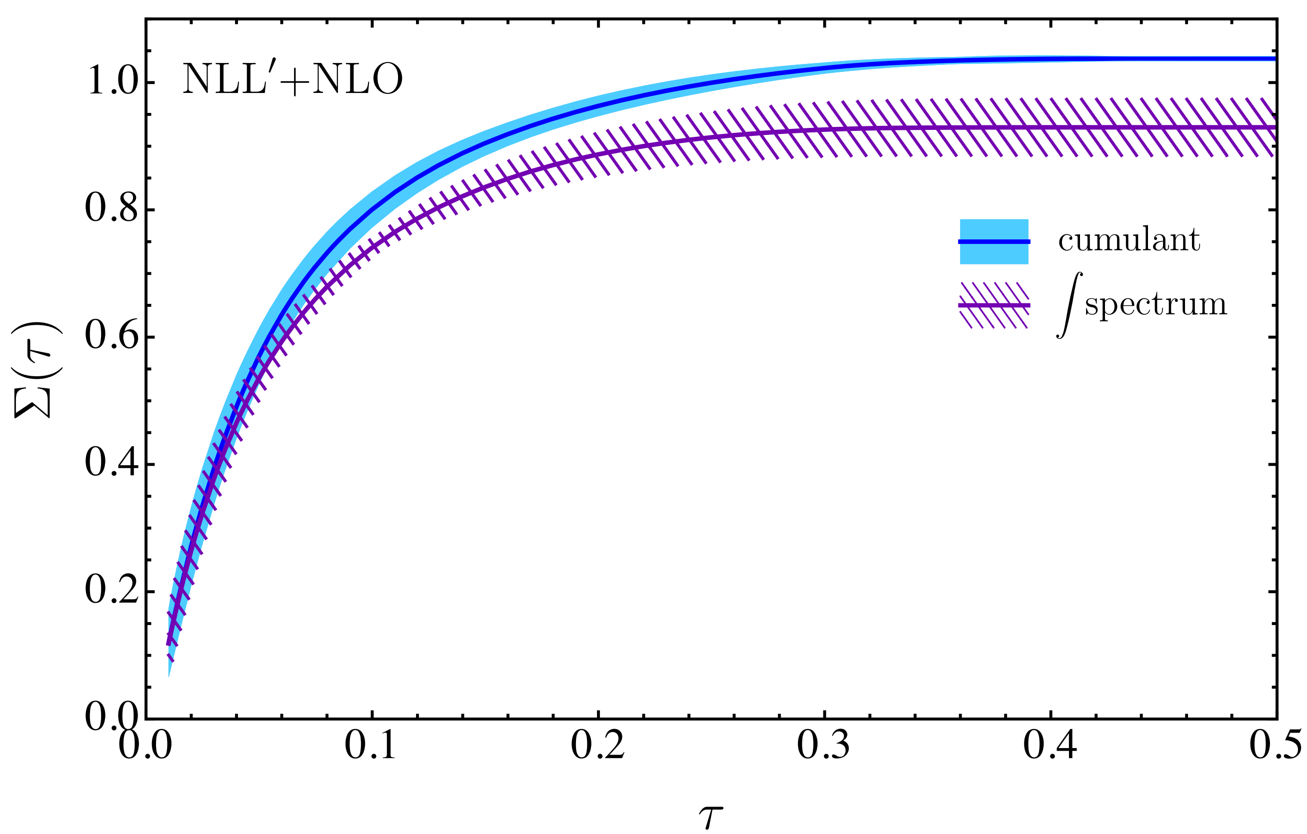
cumulant (free scales) → *cumulant* **(profiles)** → **spectrum (profiles)** $[\mu_i = \mu_i(\tau)] \times \frac{d}{d\tau}$

in terms of full/partial derivatives:

$$\frac{d}{d\tau} = \frac{\partial}{\partial\tau} + \frac{d\mu_i}{d\tau} \;\Rightarrow\; \left[\frac{d}{d\tau}\Sigma(\mu_i(\tau),\tau) - \frac{d\sigma}{d\tau}(\mu_i(\tau))\right] \propto \frac{d\mu_i}{d\tau} \times (\text{higher order})$$

Almeida, Ellis, Lee, Sterman, Sung, JW 1401.4460

cumulant vs. integrated spectrum



Integrated spectrum gets cumulant observables wrong:

- cumulant beyond small τ
- inclusive cross section $(\tau \to \tau_{\max})$
- uncertainties

Accuracy of Each Resummed Cross Section

$$\Sigma\big(\mu_i(\tau),\tau\big)$$

vs.

$$\frac{d\sigma}{d\tau}\big(\mu_i(\tau)\big)$$

+ inclusive cross section
+ correlations in uncertainties
- poor large τ behavior in spectrum
- poor point-by-point uncertainties in the spectrum

accurate in the *inclusive/integrated* sense

+ accurate shape in the transition/tail
+ robust point-by-point uncertainties
- inclusive cross section
- correlations in uncertainties

accurate in the *exclusive/differential* sense

Accuracy of Each Resummed Cross Section

$$\Sigma\big(\mu_i(\tau),\tau\big)$$

vs.

$$\frac{d\sigma}{d\tau}\big(\mu_i(\tau)\big)$$

- \+ inclusive cross section
- \+ correlations in uncertainties
- − poor large τ behavior in spectrum
- − poor point-by-point uncertainties in the spectrum

accurate in the *inclusive/integrated* sense

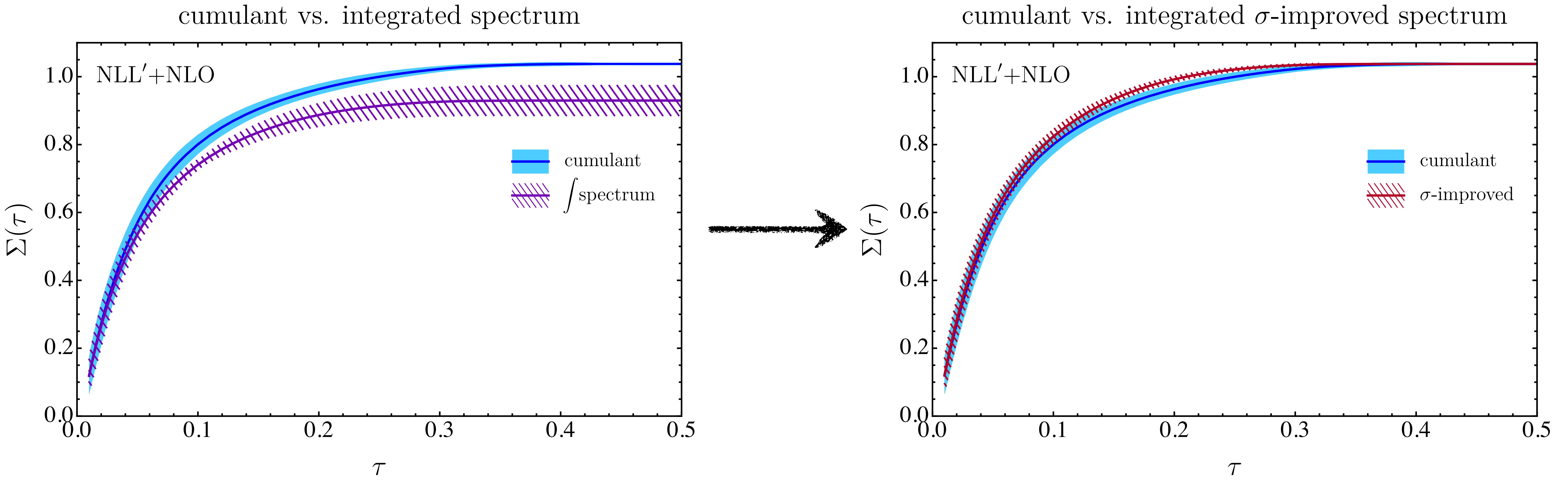
- \+ accurate shape in the transition/tail
- \+ robust point-by-point uncertainties
- − inclusive cross section
- − correlations in uncertainties

accurate in the *exclusive/differential* sense

I will describe a resummation method that gives a spectrum accurate both *exclusively* and *inclusively*

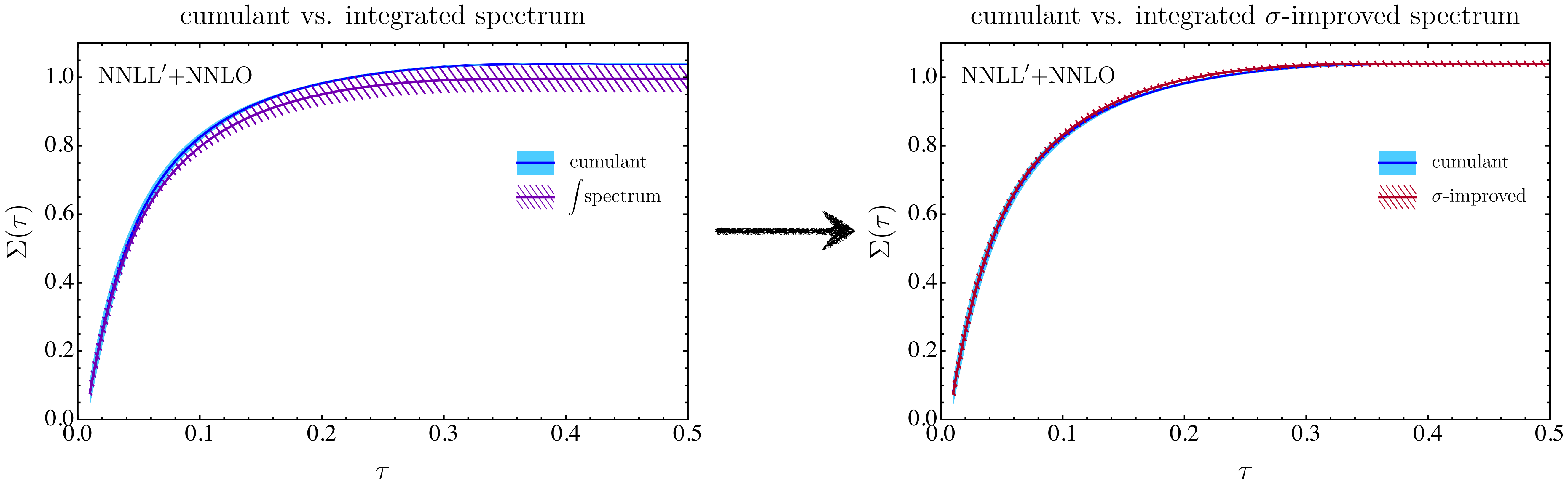
Accuracy of Resummed Cross Sections

I will describe a resummation framework
that gives a spectrum accurate both
exclusively and *inclusively*



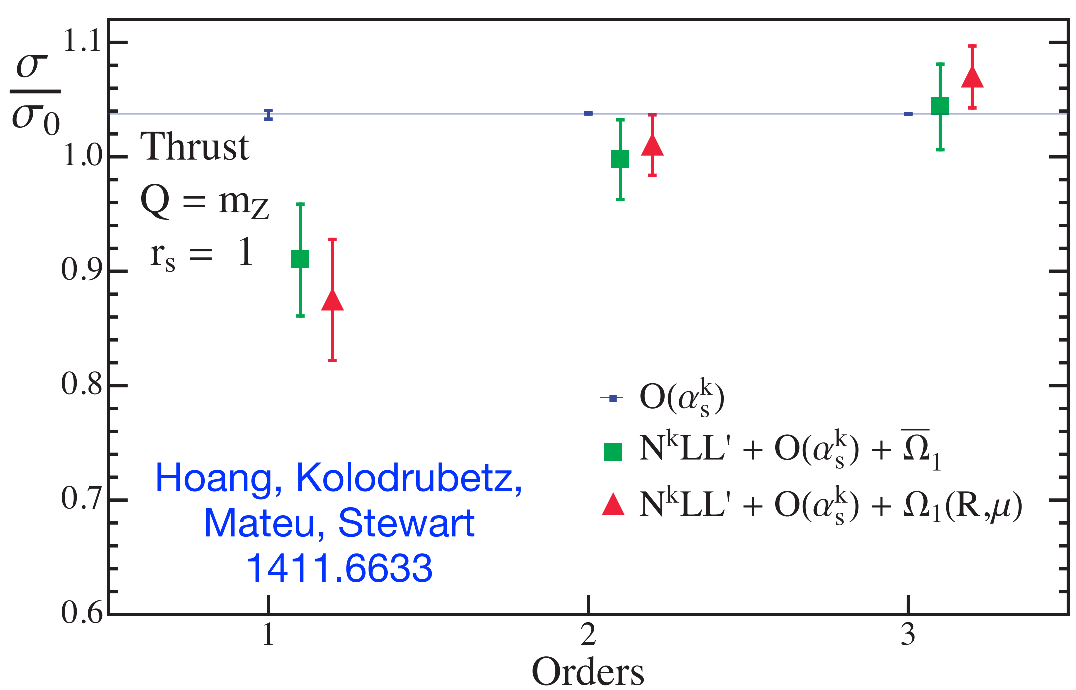
Accuracy of Resummed Cross Sections

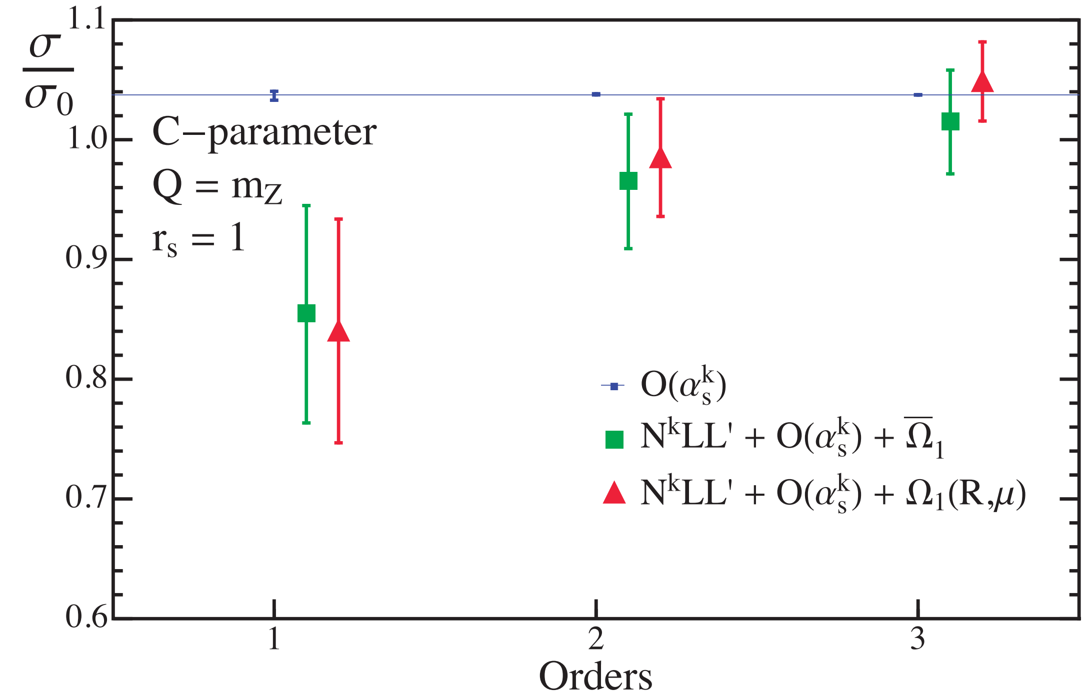
I will describe a resummation framework
that gives a spectrum accurate both
exclusively and *inclusively*



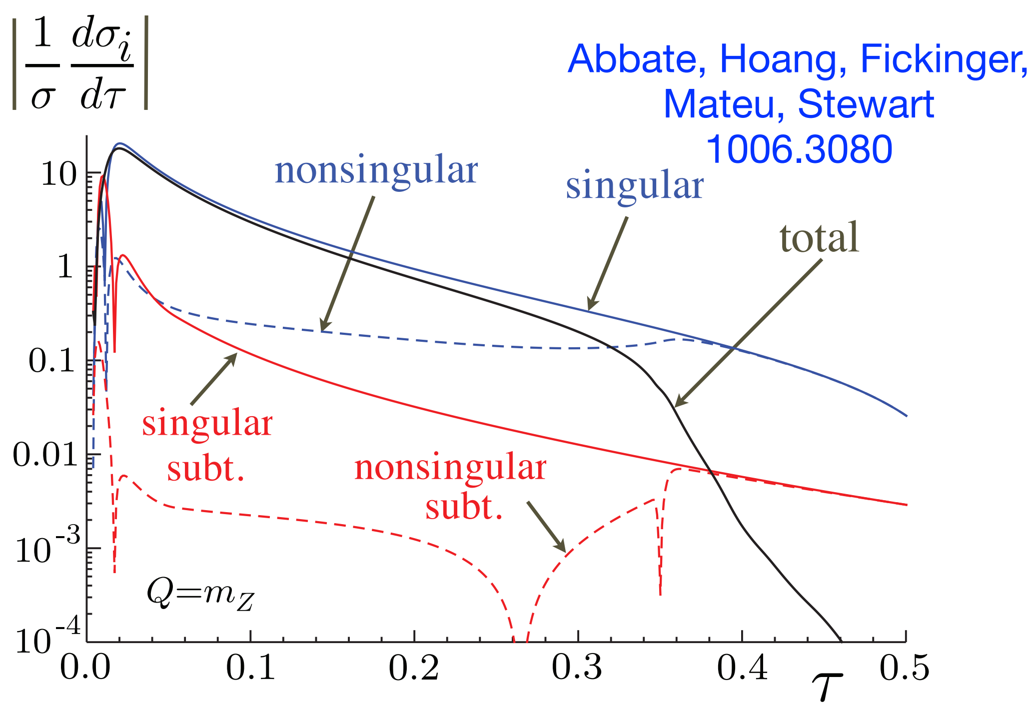
Connection to Known Problems

Spectra with the wrong inclusive cross section is a well-known problem





Intimately connected with the fact that the derivative of the cumulant has poor behavior in the transition/tail region



Connection to Known Problems

Want to associate scale variation to specific components of uncertainty

general covariance matrix decomposition:

$$C = C_{\rm y} + \sum_{i<j} C_{\rm cut}^{ij}$$

fully correlated ($C_{\rm y}$)

anti-correlated 2-by-2 blocks ($C_{\rm cut}^{ij}$)

consider the cross section for *N* jet bins
e.g. 2-jet and ≥3-jet bins for thrust

$$C_{\rm y} = \vec{\Delta}_{\rm y} \vec{\Delta}_{\rm y}^{\mathbf{T}}$$

$$C_{\rm cut}^{ij} = (\Delta_{\rm cut}^{ij})^2 \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}_{ij}$$

$\Delta_{\rm y}^{i}$: yield uncertainty for bin *i* fully correlated with total rate

$\Delta_{\rm cut}^{ij}$: migration uncertainty between bins *i* and *j*

Connection to Known Problems

Want to associate scale variation to specific components of uncertainty

general covariance matrix decomposition:

$$C = C_{\rm y} + \sum_{i<j} C^{ij}_{\rm cut}$$

consider the cross section for *N* jet bins
e.g. 2-jet and ≥3-jet bins for thrust

fully correlated ($C_{\rm y}$) — anti-correlated 2-by-2 blocks ($C^{ij}_{\rm cut}$)

$$C_{\rm y} = \vec{\Delta}_{\rm y} \vec{\Delta}^{\mathbf{T}}_{\rm y}$$

$$C^{ij}_{\rm cut} = (\Delta^{ij}_{\rm cut})^2 \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}_{ij}$$

$\Delta^i_{\rm y}$: yield uncertainty for bin *i*
fully correlated with total rate

$\Delta^{ij}_{\rm cut}$: migration uncertainty
between bins *i* and *j*

soft, jet scale variations should not change the inclusive cross section:

$\mu_J,\ \mu_S$ variations map directly onto migration uncertainties
***iff** they leave the inclusive cross section unchanged*

this is not the case for standard profile variations

The Main Idea

Define a resummation method with two novel features:

1. Add higher order terms to the spectrum that bring the inclusive cross section close to the fixed order value

   - must maintain a sensible distribution in the tail region

   - must be consistent across fixed order scales (convergence)

2. Use an algorithm to identify families of soft and jet profiles that preserve the total cross section

   - use these families to determine the soft, jet scale uncertainties

   - our algorithm can identify arbitrarily many profiles preserving the total cross section, and we can test its robustness

Step 1

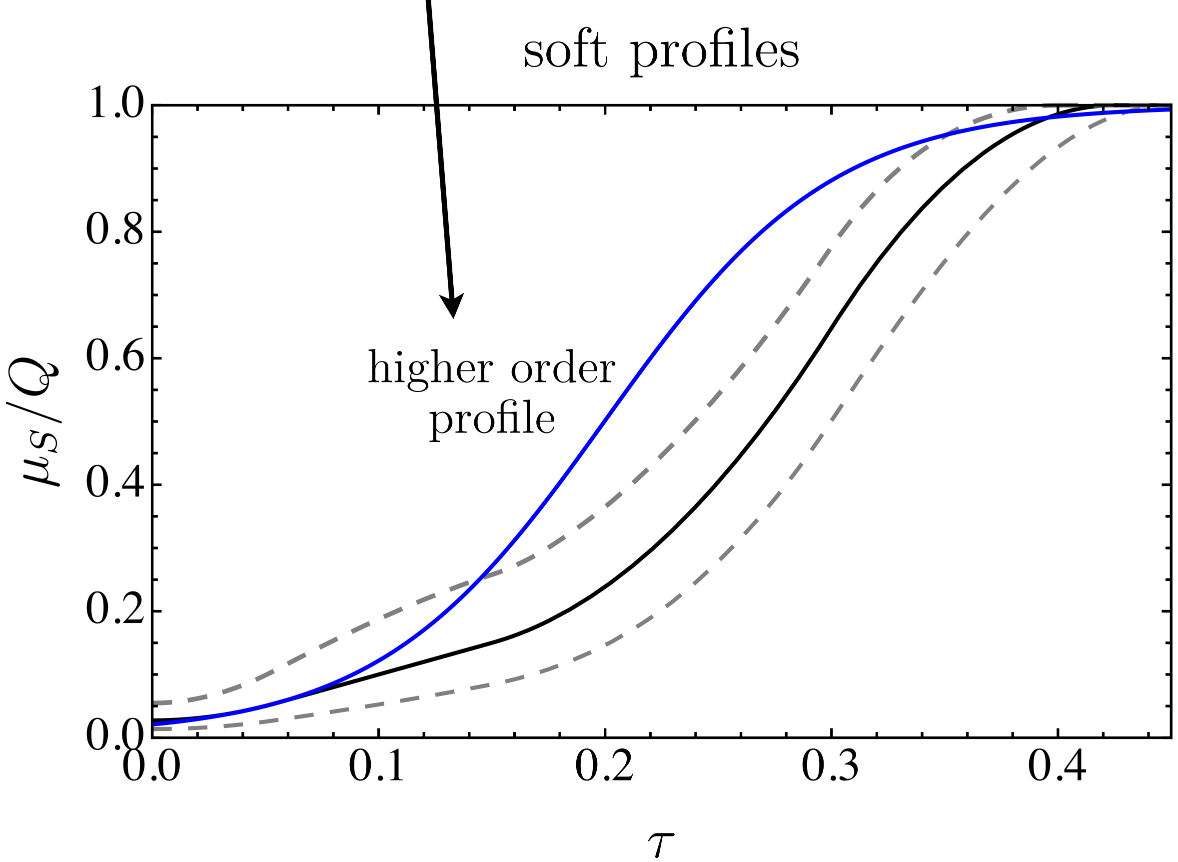
1. Add higher order terms to the spectrum that bring the inclusive cross section close to the fixed order value

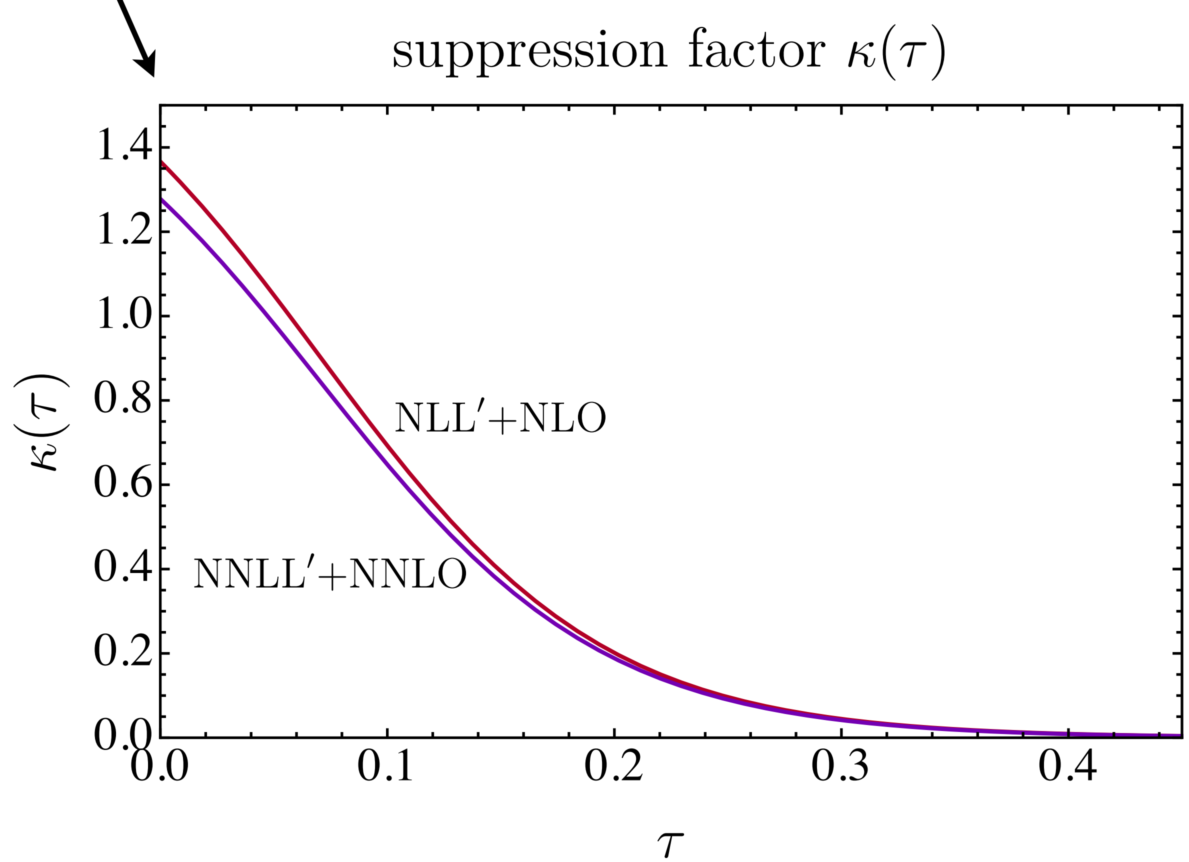
Use the fact that the cumulant derivative / spectrum difference is higher order:

add $$\left[\Sigma'(\tilde{\mu}_i(\tau),\tau) - \frac{d\sigma}{d\tau}(\tilde{\mu}_i(\tau))\right]\kappa(\tau)$$

use a special smooth profile so that $d\tilde{\mu}_i/d\tau$ is smooth

suppression factor to reduce effect in the tail





Step 1

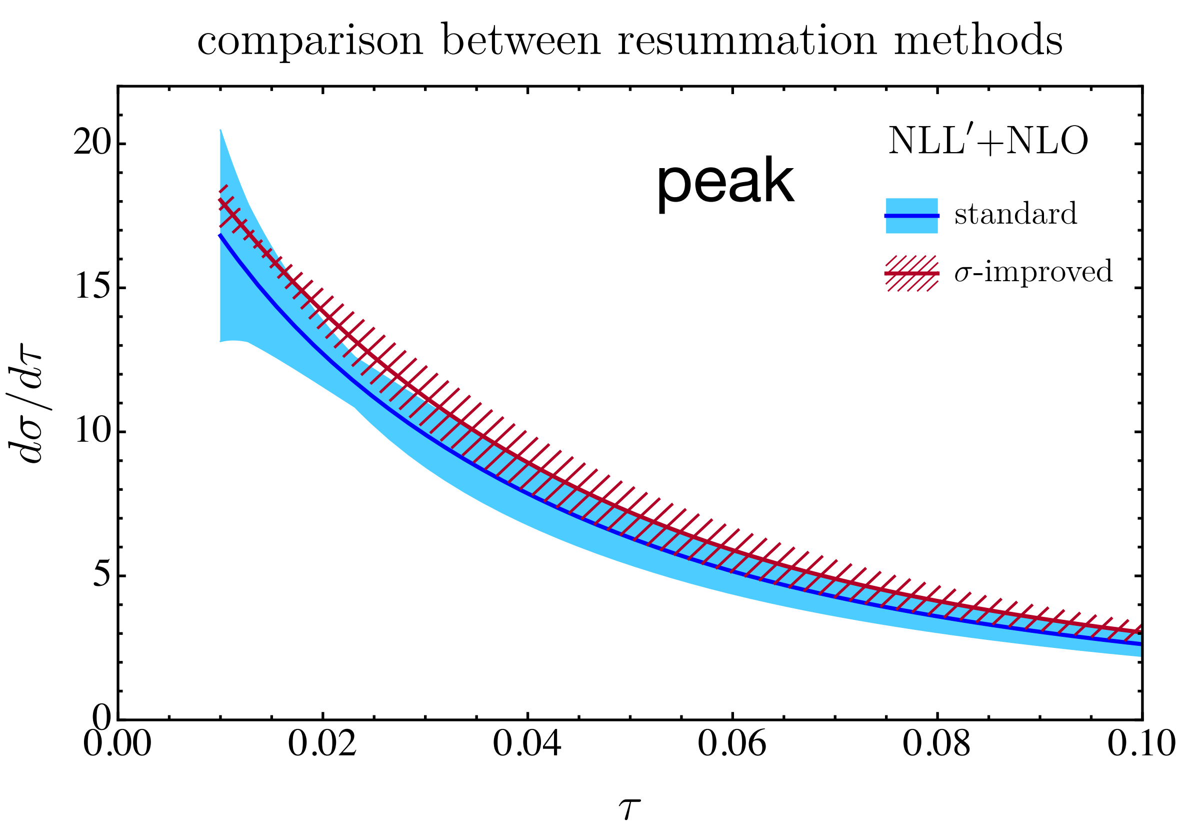
1. Add higher order terms to the spectrum that bring the inclusive cross section close to the fixed order value

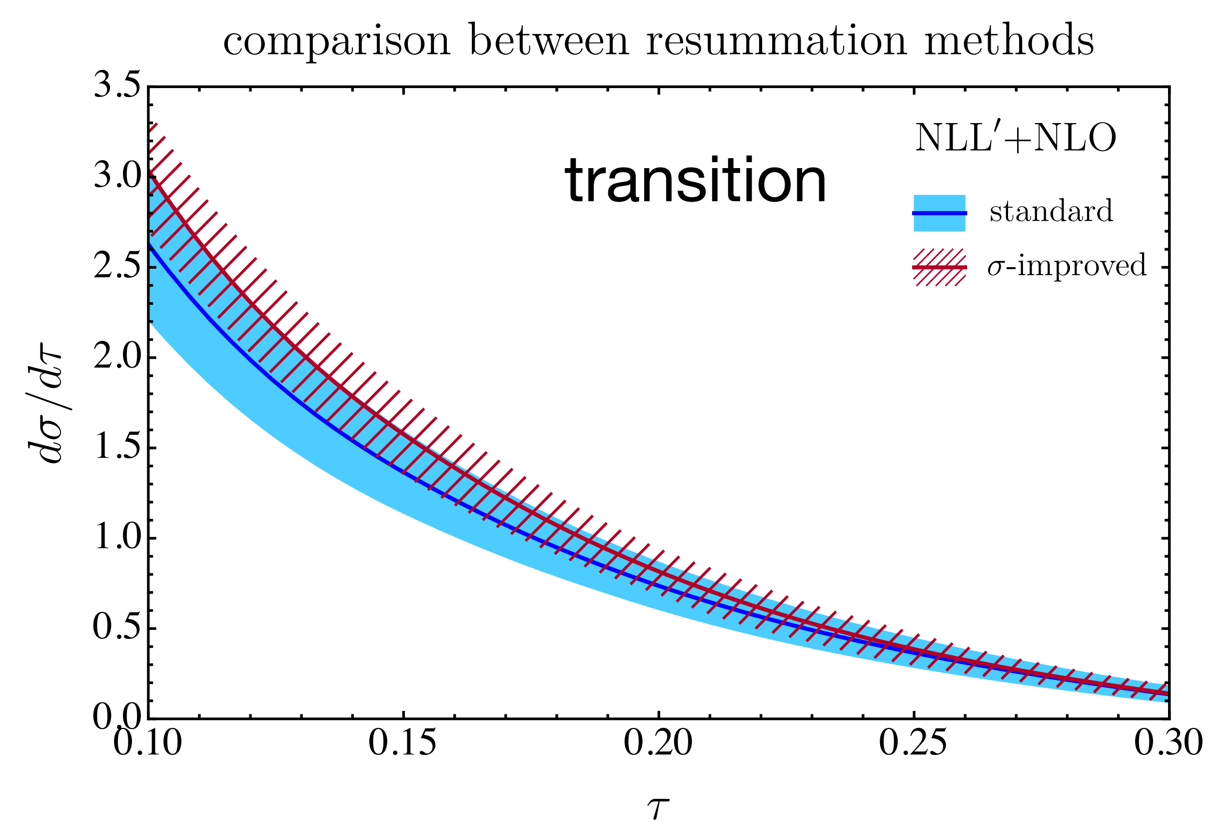
Use the fact that the cumulant derivative / spectrum difference is higher order:

add $$\left[\Sigma'(\tilde{\mu}_i(\tau),\tau) - \frac{d\sigma}{d\tau}(\tilde{\mu}_i(\tau))\right]\kappa(\tau)$$

use a special smooth profile so that $d\tilde{\mu}_i/d\tau$ is smooth

suppression factor to reduce effect in the tail





Step 1

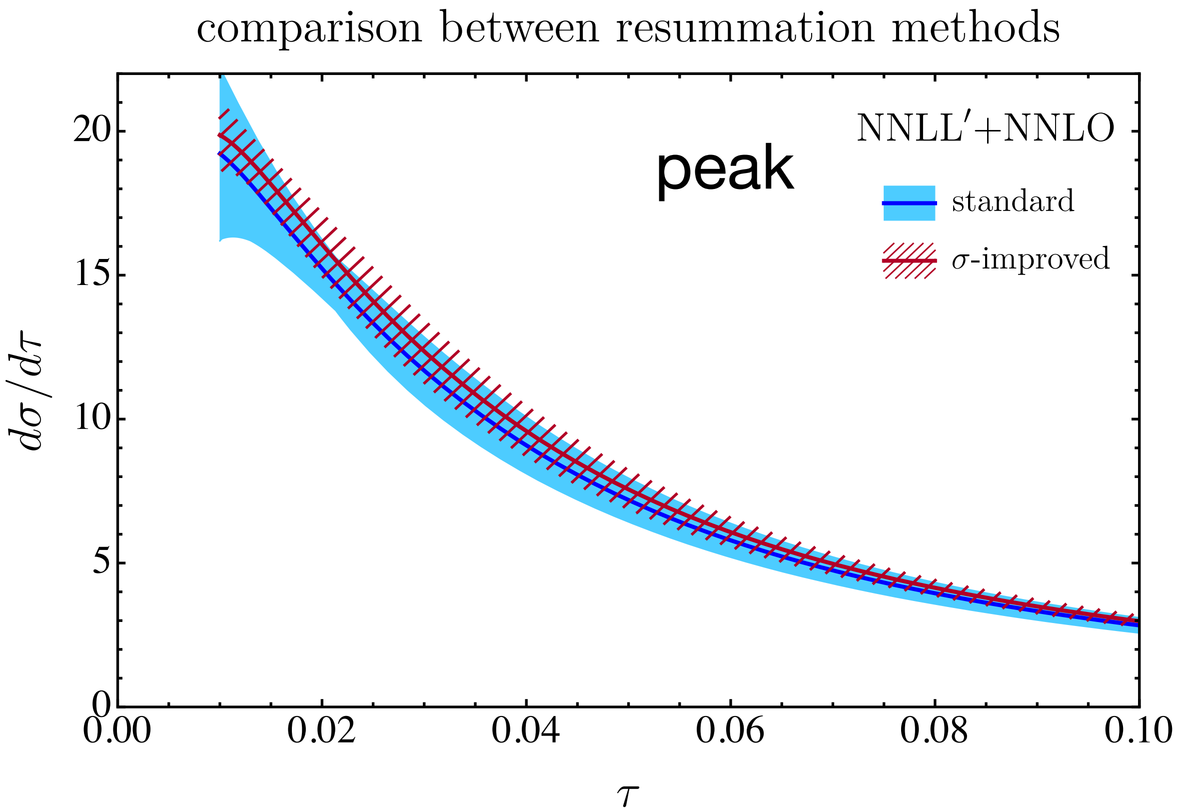
1. Add higher order terms to the spectrum that bring the inclusive cross section close to the fixed order value

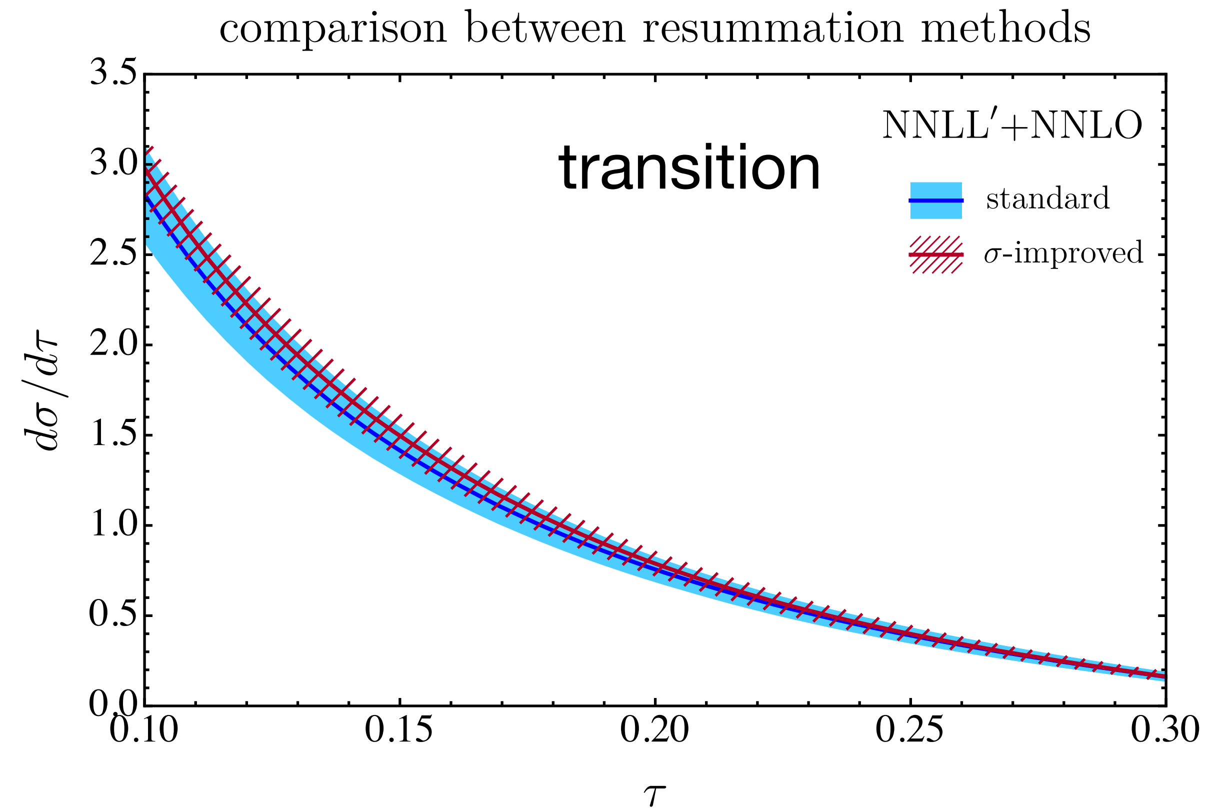
Use the fact that the cumulant derivative / spectrum difference is higher order:

add $\left[\Sigma'(\tilde{\mu}_i(\tau),\tau) - \frac{d\sigma}{d\tau}(\tilde{\mu}_i(\tau))\right]\kappa(\tau)$

use a special smooth profile so that $d\tilde{\mu}_i/d\tau$ is smooth

suppression factor to reduce effect in the tail



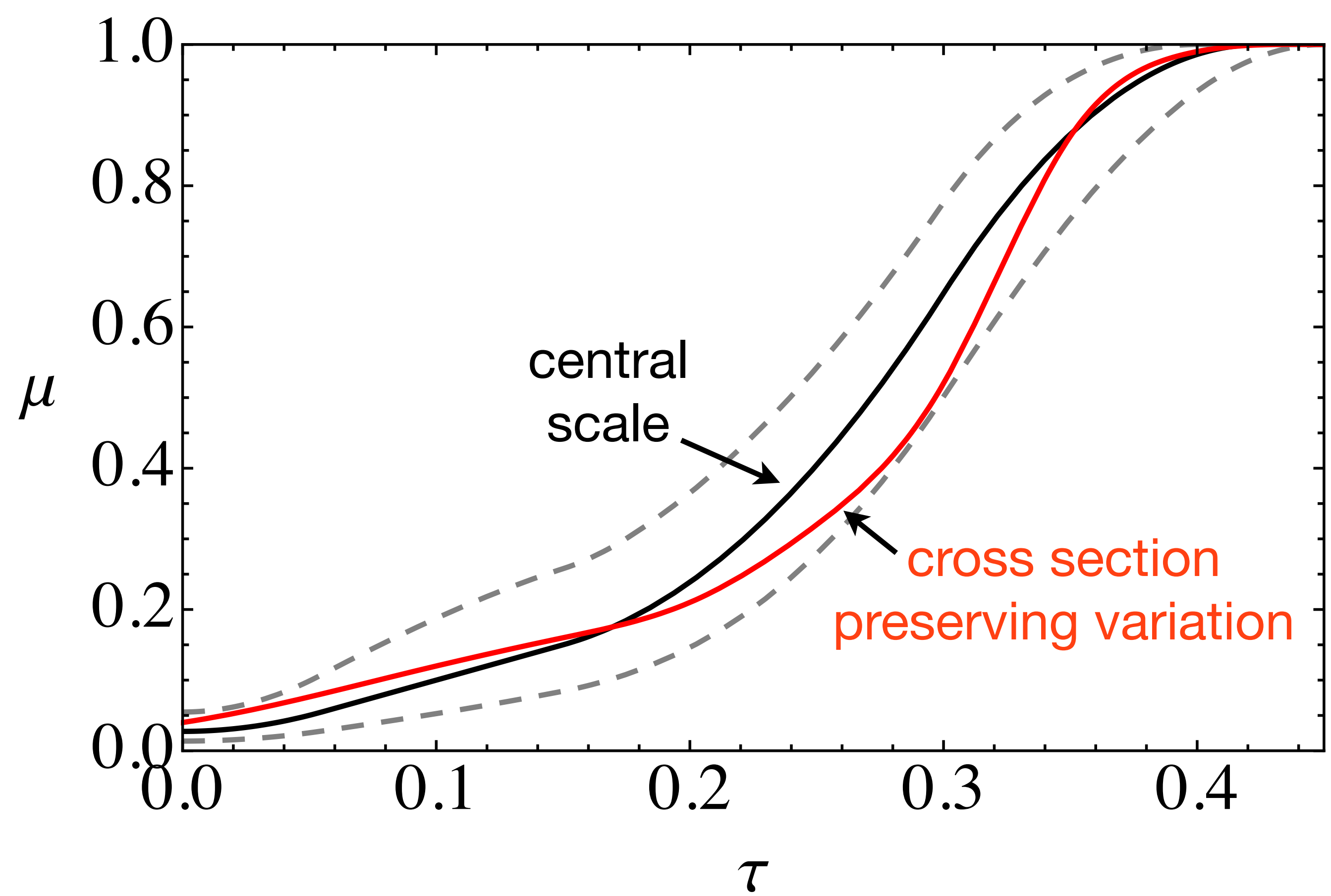


Step 2

2. Use an algorithm to identify families of soft and jet profiles that preserve the total cross section

we will attempt to *fill* the standard scale variation band with profiles that all preserve the inclusive cross section

preserve reliable point-by-point uncertainties *and* capture long-distance correlations



Step 2

2. Use an algorithm to identify families of soft and jet profiles that preserve the total cross section

This can be cast as a math problem: find $\mu(\tau)$ such that

$$\sigma_{\rm incl} = \int_0^{\tau_{\max}} d\tau \, \frac{d\sigma}{d\tau}(\mu(\tau), \tau)$$

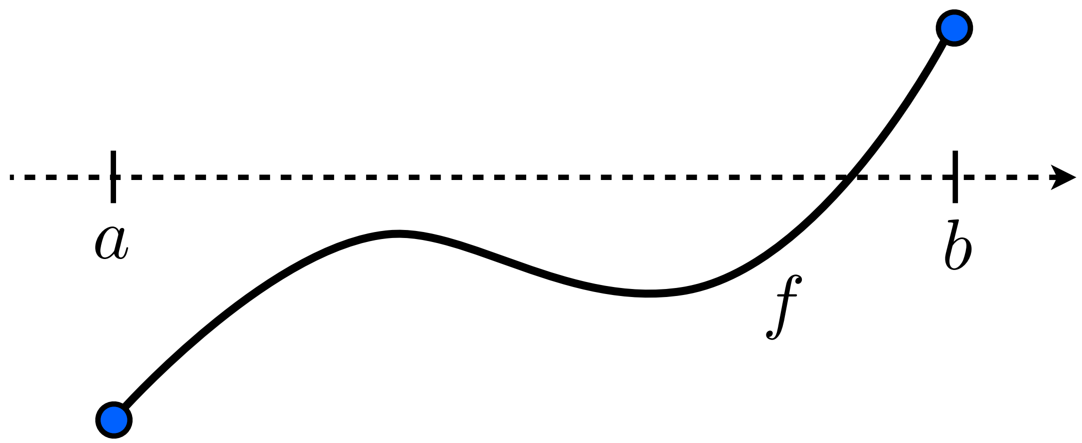
subject to some simple constraints on μ
(monotonicity, smoothness, fixed shape near endpoints)

This is a fairly generic problem,
and we have devised a generic algorithm to solve it

Quiz: what is Bolzano's Theorem?

The Intermediate Value Theorem
(Bolzano's Theorem)

for continuous functions:



$$f(a) < 0 < f(b) \quad \Rightarrow \quad \exists\, c \in [a, b] \text{ with } f(c) = 0$$

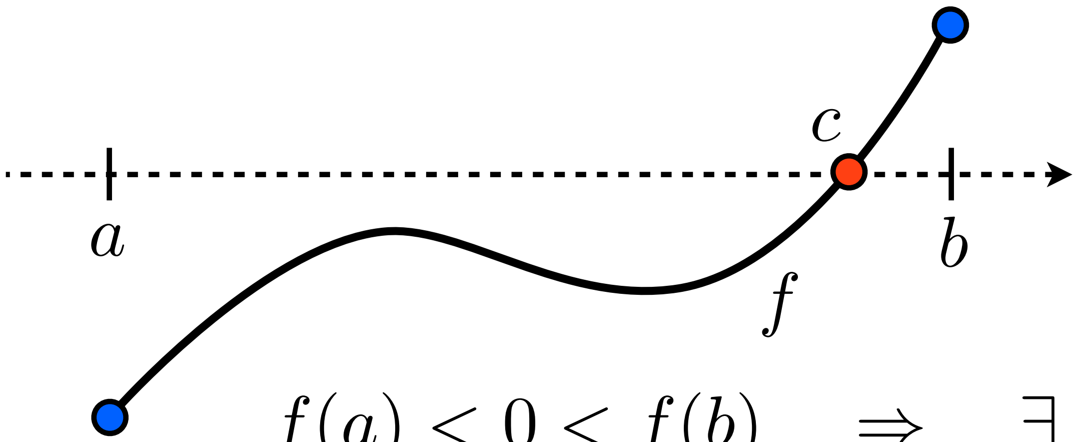
Bernard Bolzano
(1781-1848)

fun application:

At any time, on any great circle,
there are two points on opposite sides
of the Earth with the same temperature

$$T(p) - T(\bar{p})$$

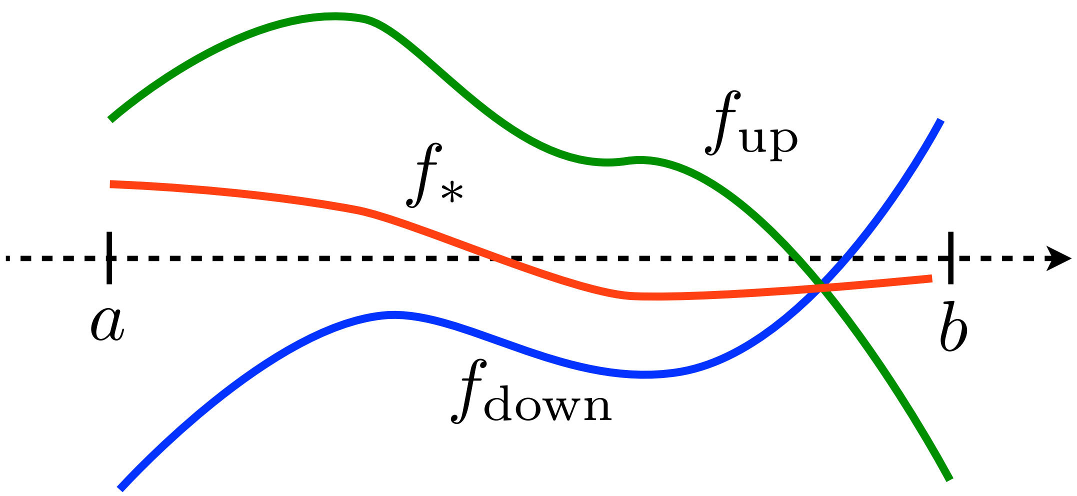
Extensions of Bolzano's Theorem



standard "single point" case

find a point where a function vanishes

$$f(a) < 0 < f(b) \quad \Rightarrow \quad \exists\, c \in [a,b] \text{ with } f(c) = 0$$



"line" case

find a function with a given integral

$$\int_a^b f_{\rm down} < 0 < \int_a^b f_{\rm up} \quad \Rightarrow \quad \int_a^b f_* = 0$$

$$\text{where } f_* = a_* f_{\rm up} + (1-a_*) f_{\rm down}\,, \quad a_* = \frac{-\int_a^b f_{\rm down}}{\int_a^b (f_{\rm up} - f_{\rm down})}$$

$$\forall\, x,\ f_{\rm down}(x) \le f_*(x) \le f_{\rm up}(x) \ \text{ or } \ f_{\rm down}(x) \ge f_*(x) \ge f_{\rm up}(x)$$

Extensions of Bolzano's Theorem

suppose we want to find *g(x)* satisfying

"parametric line" case (our case)

$$\int dx\, R[g(x), x] = A$$

where *A* is a non-extremal constant

take two *g* satisfying

$$\int dx\, R[g_{\rm down}(x), x] = A_{\rm down} < A$$

$$\int dx\, R[g_{\rm up}(x), x] = A_{\rm up} > A$$

then there is some *g* such that $\int dx\, R[g_*(x), x] = A$

for instance $g_* = a_* g_{\rm up} + (1 - a_*) g_{\rm down}\,, \quad a_* \in [0,1]$

in fact, there are infinitely many g_* such that

$$\forall\, x,\;\; g_{\rm down}(x) \leq g_*(x) \leq g_{\rm up}(x) \;\;\text{or}\;\; g_{\rm down}(x) \geq g_*(x) \geq g_{\rm up}(x)$$

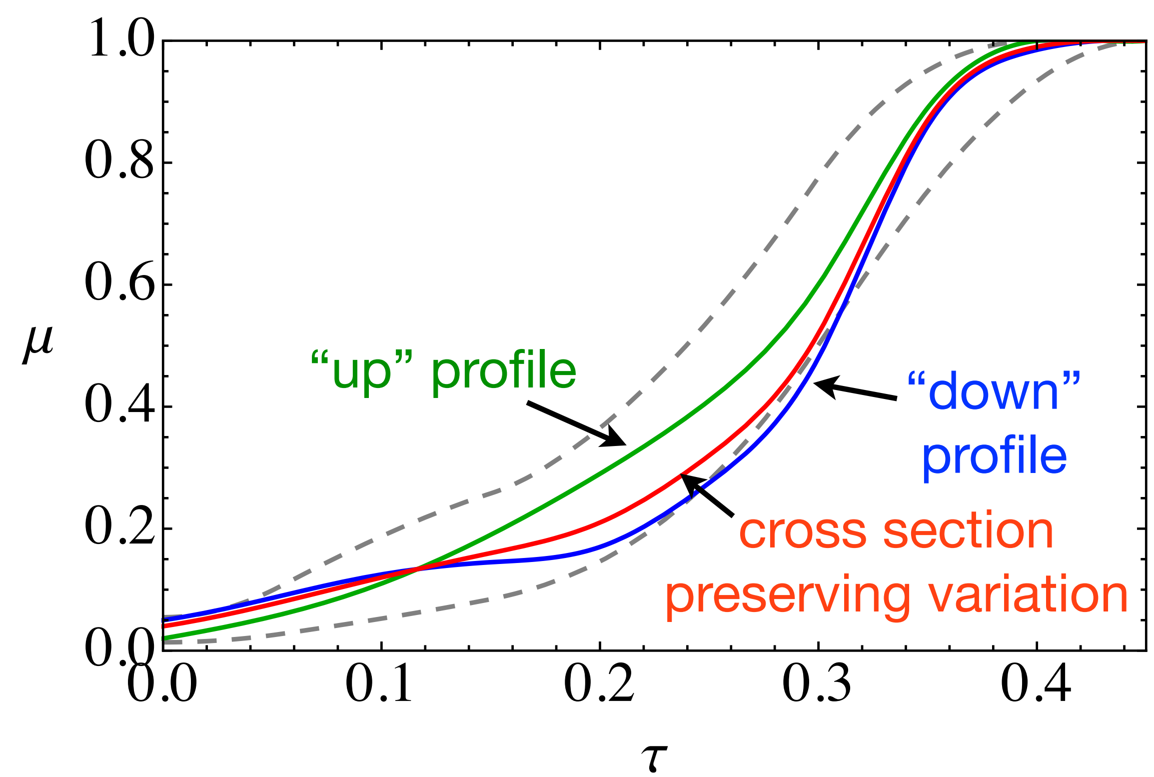
The Bolzano Algorithm

1. Identify a set of candidate profiles μ

2. Separate candidate profiles by whether or not they give an inclusive cross section less than or greater than the true inclusive cross section
$$\{\mu\} \;\rightarrow\; \{\mu_{\text{up}}\},\ \{\mu_{\text{down}}\}$$

3. On every pair of "down" and "up" profiles, find a_* such that
$$\mu_* = a_* \mu_{\text{up}} + (1 - a_*)\mu_{\text{down}}$$
has the correct inclusive cross section

4. Select all μ_* satisfying desired properties:
   - Monotonicity
   - Smoothness
   - $\forall \tau\,,\ \mu_{\text{down}}^{\text{vary}}(\tau) \leq \mu_*(\tau) \leq \mu_{\text{up}}^{\text{vary}}(\tau)$



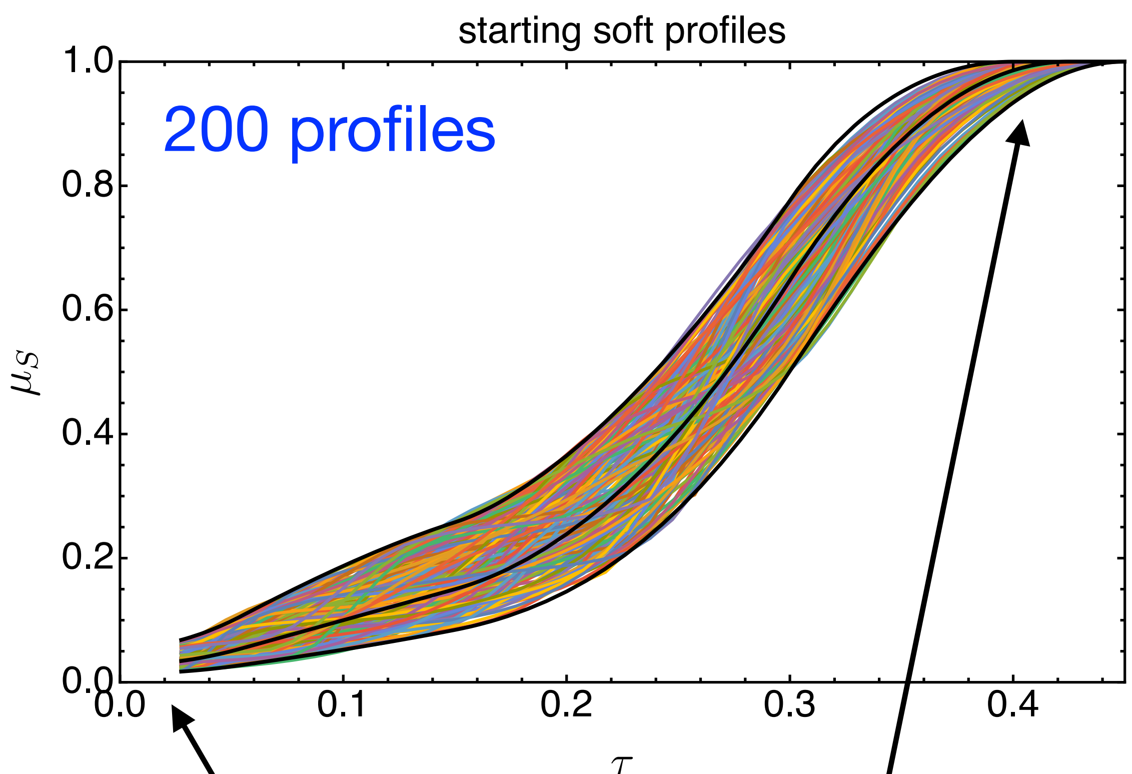
can replace step 3 with a spectrum-space solution:
$$\mu_* = \sigma^{-1}\left[\sigma_* = b_* \sigma(\mu_{\text{up}}) + (1 - b_*)\sigma(\mu_{\text{down}})\right]$$

N candidate profiles give ~$N^2/4$ solutions

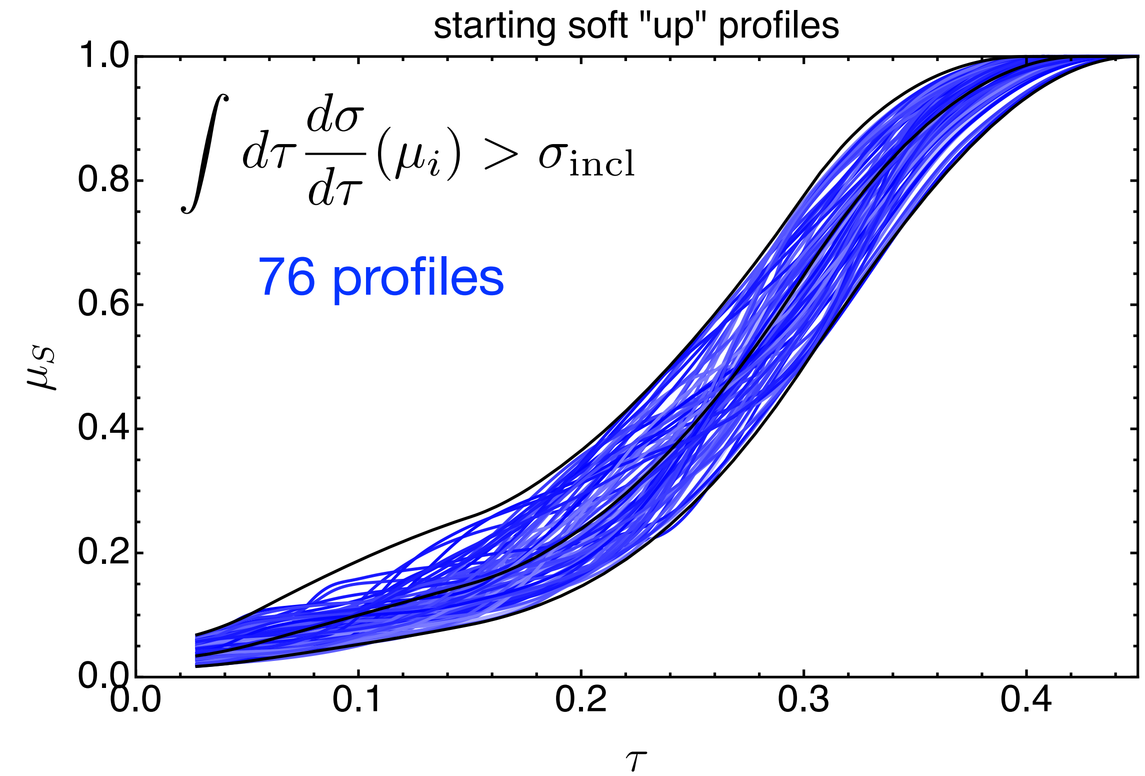
Profiles: Algorithm and Solutions

start with random profiles that
fill out the standard uncertainty band

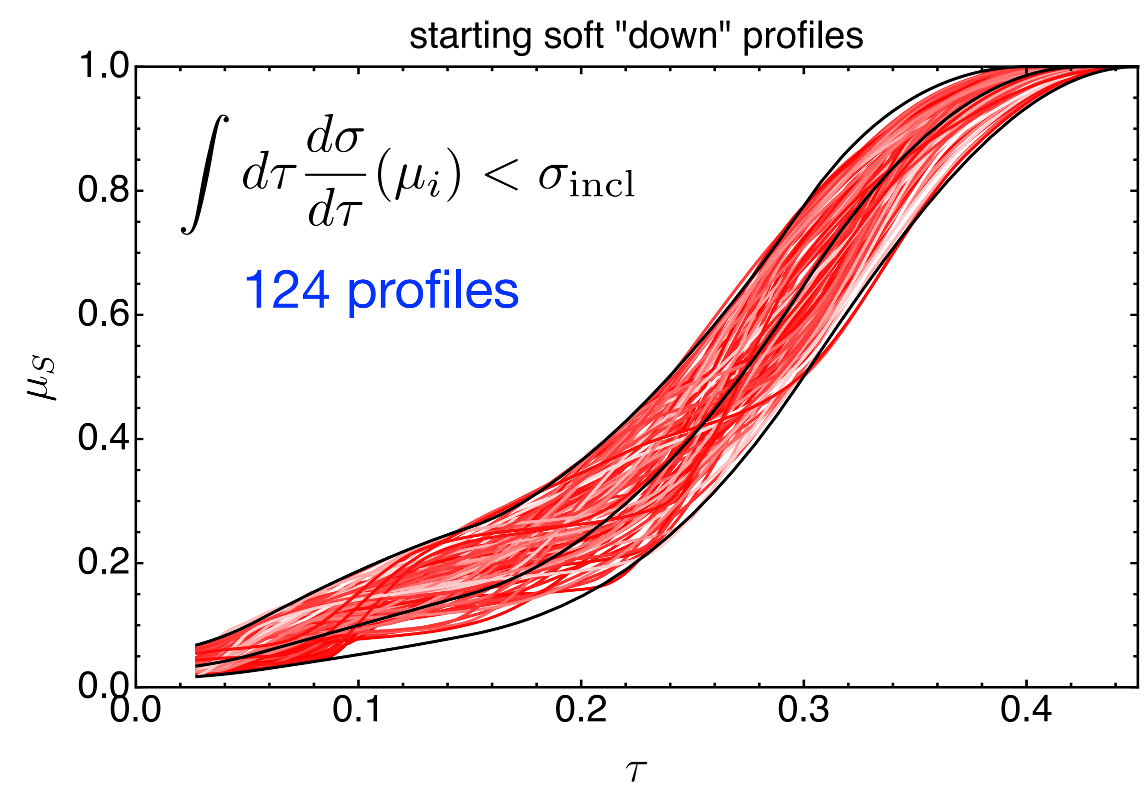
divide into groups by
the total cross section



up

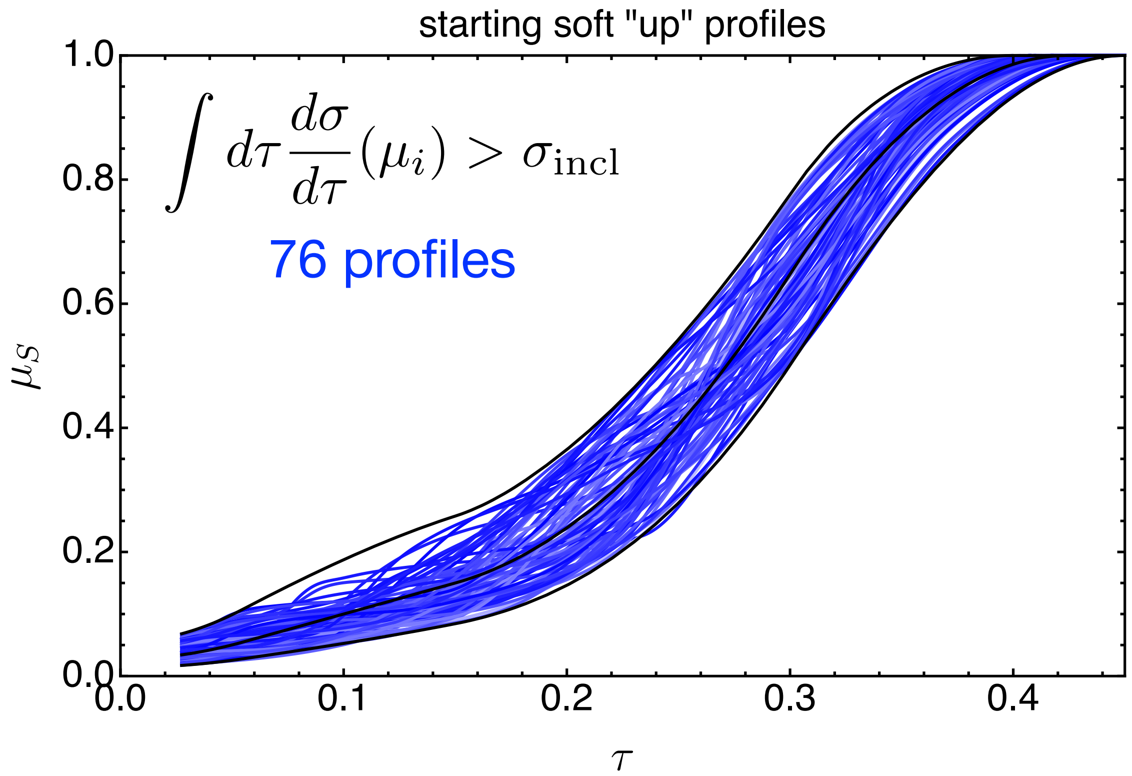


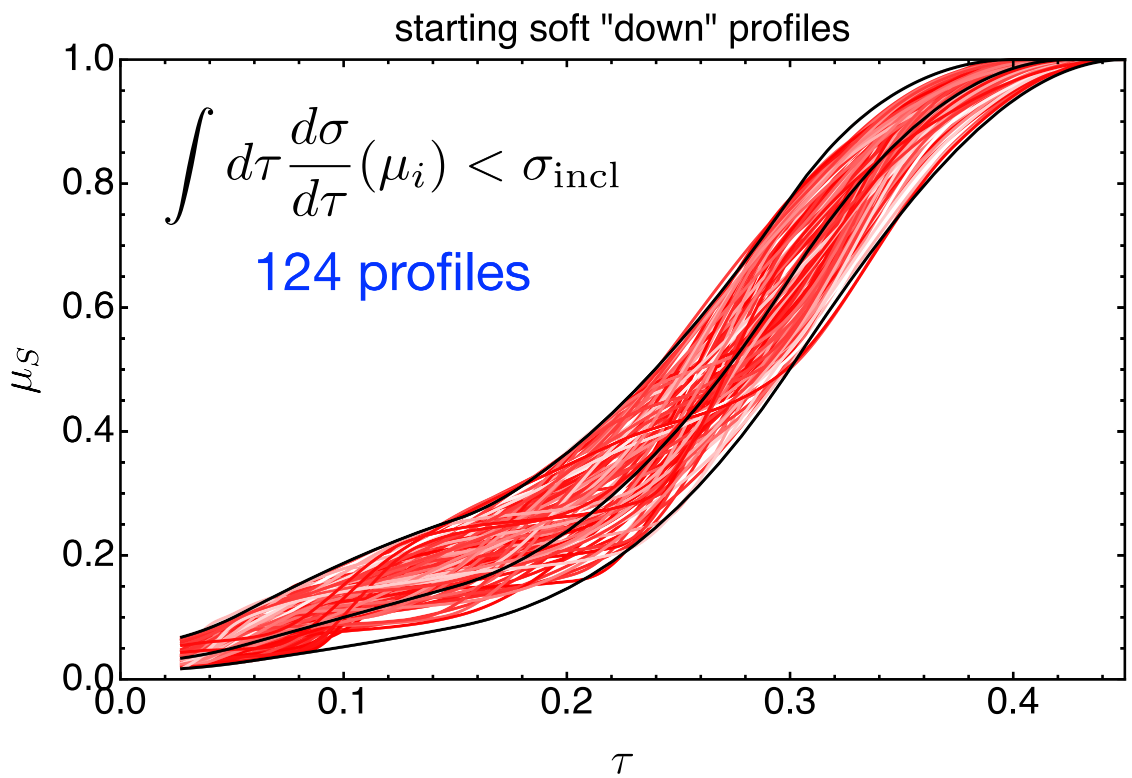
down



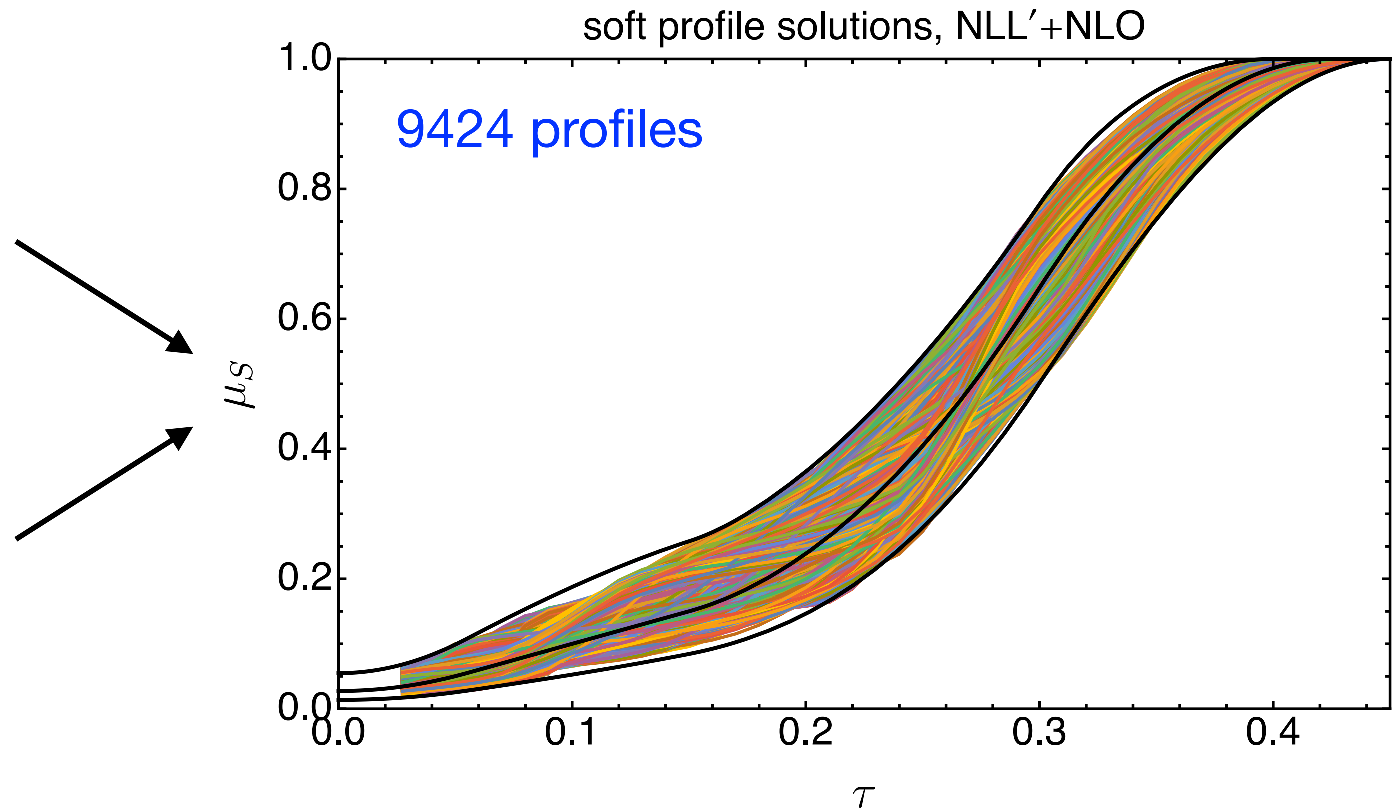
fix the shapes in the
small and large thrust regions

Profiles: Algorithm and Solutions



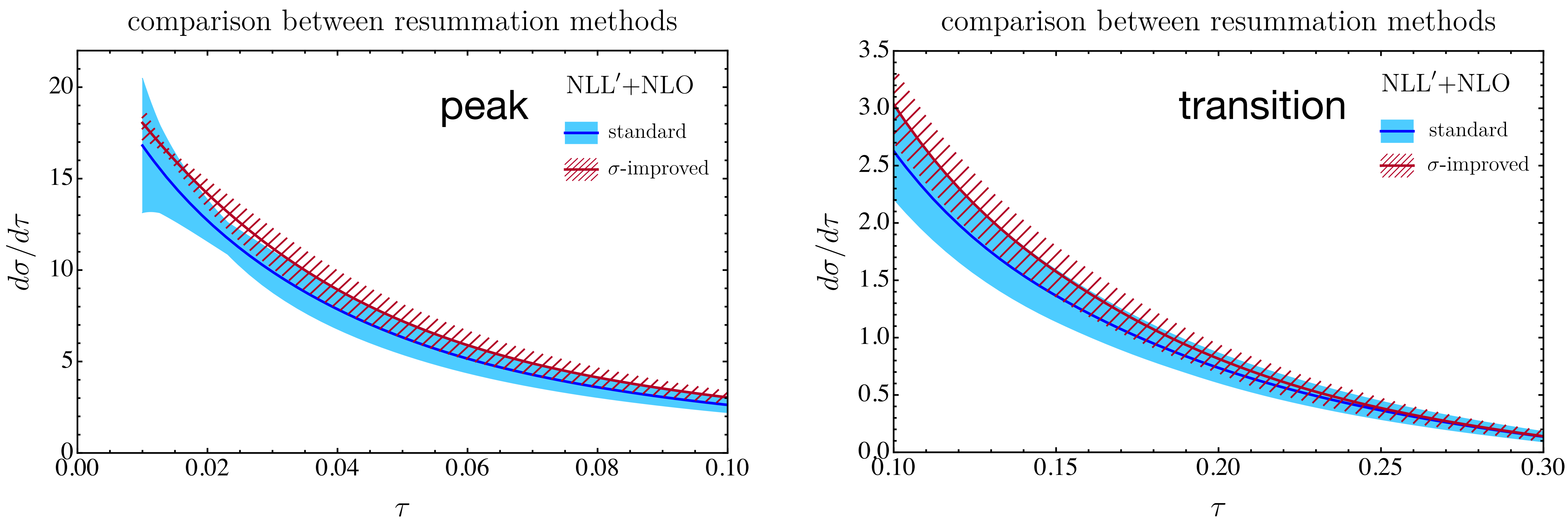


run Bolzano algorithm
to find solutions

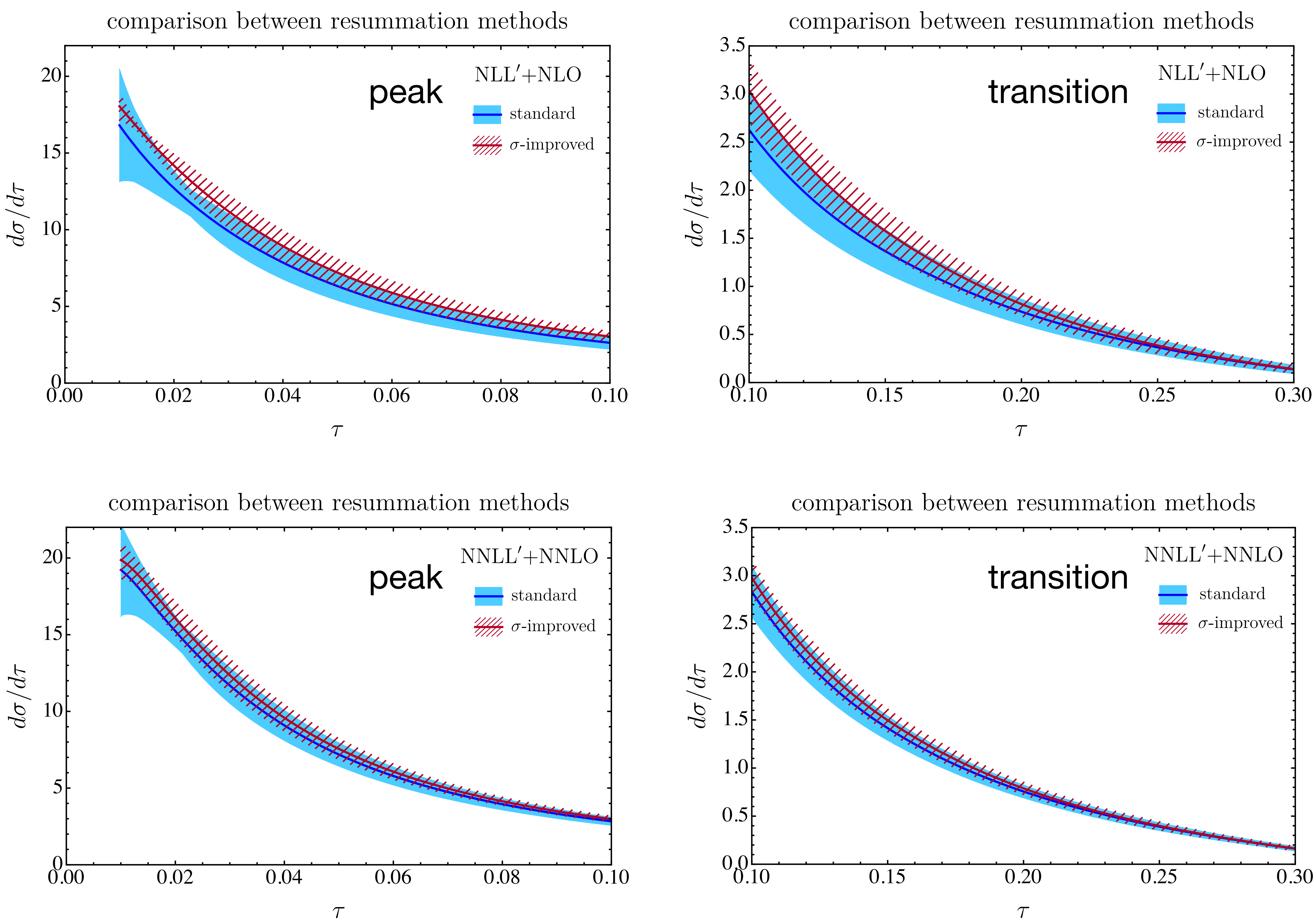


do the same thing for
the jet profiles

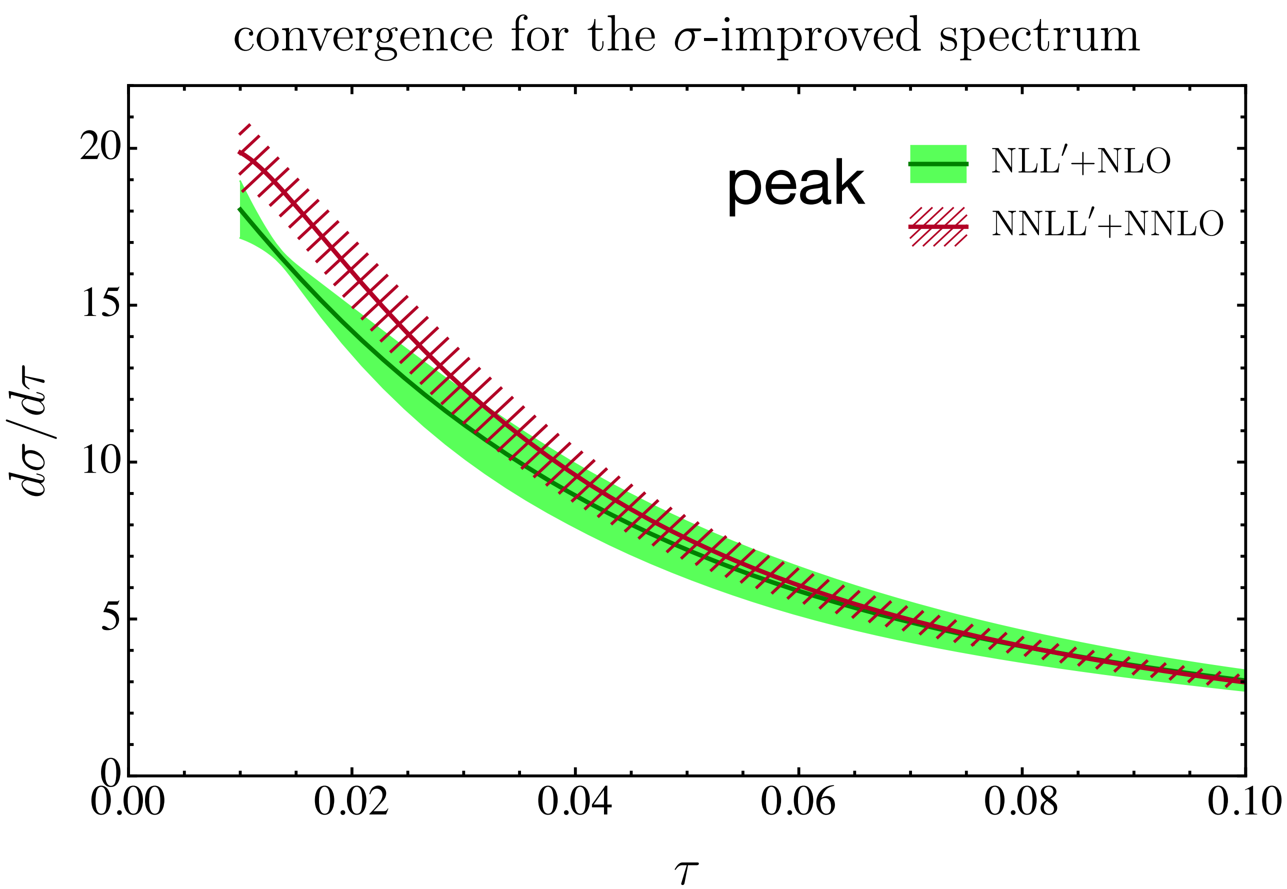
Comparison between Resummation Methods

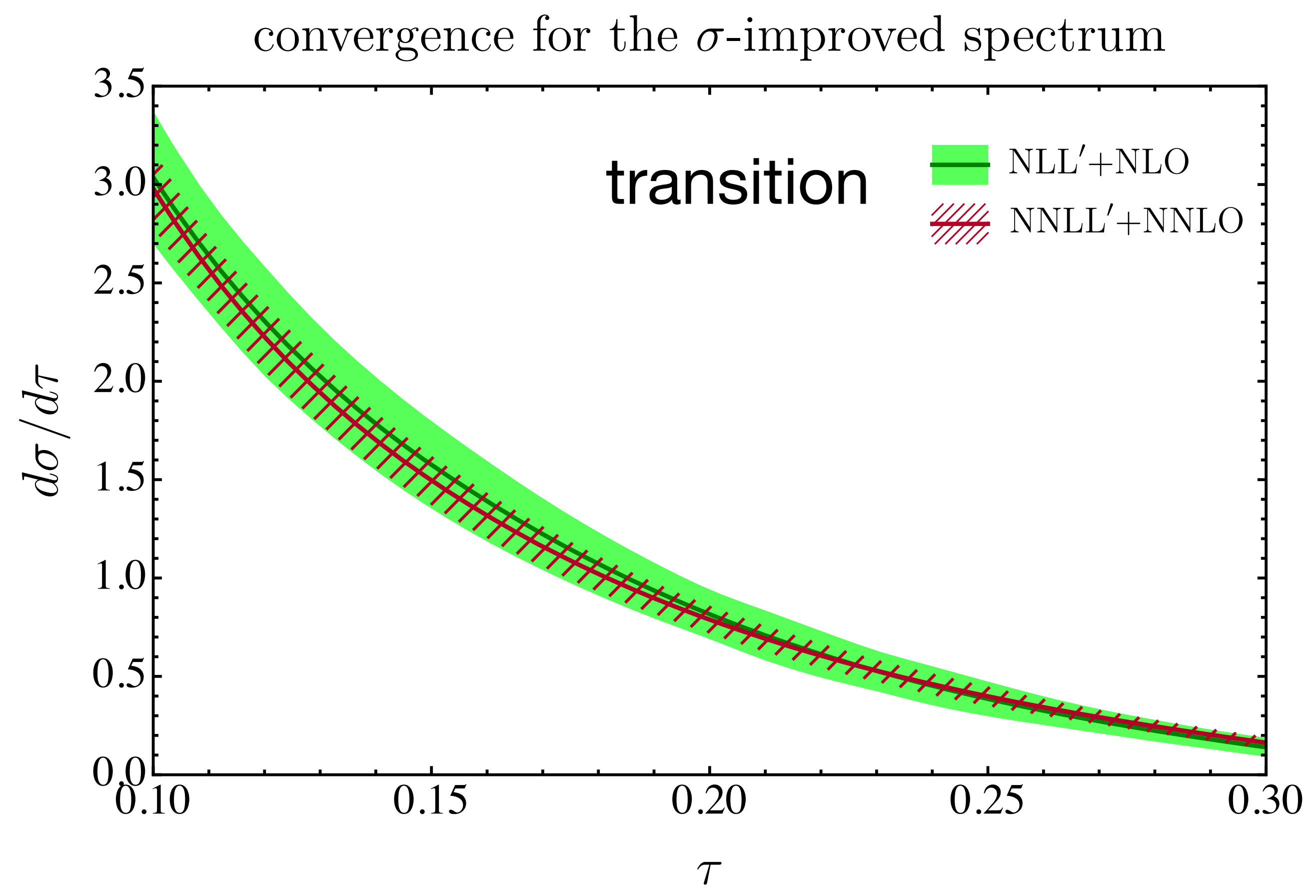


Comparison between Resummation Methods



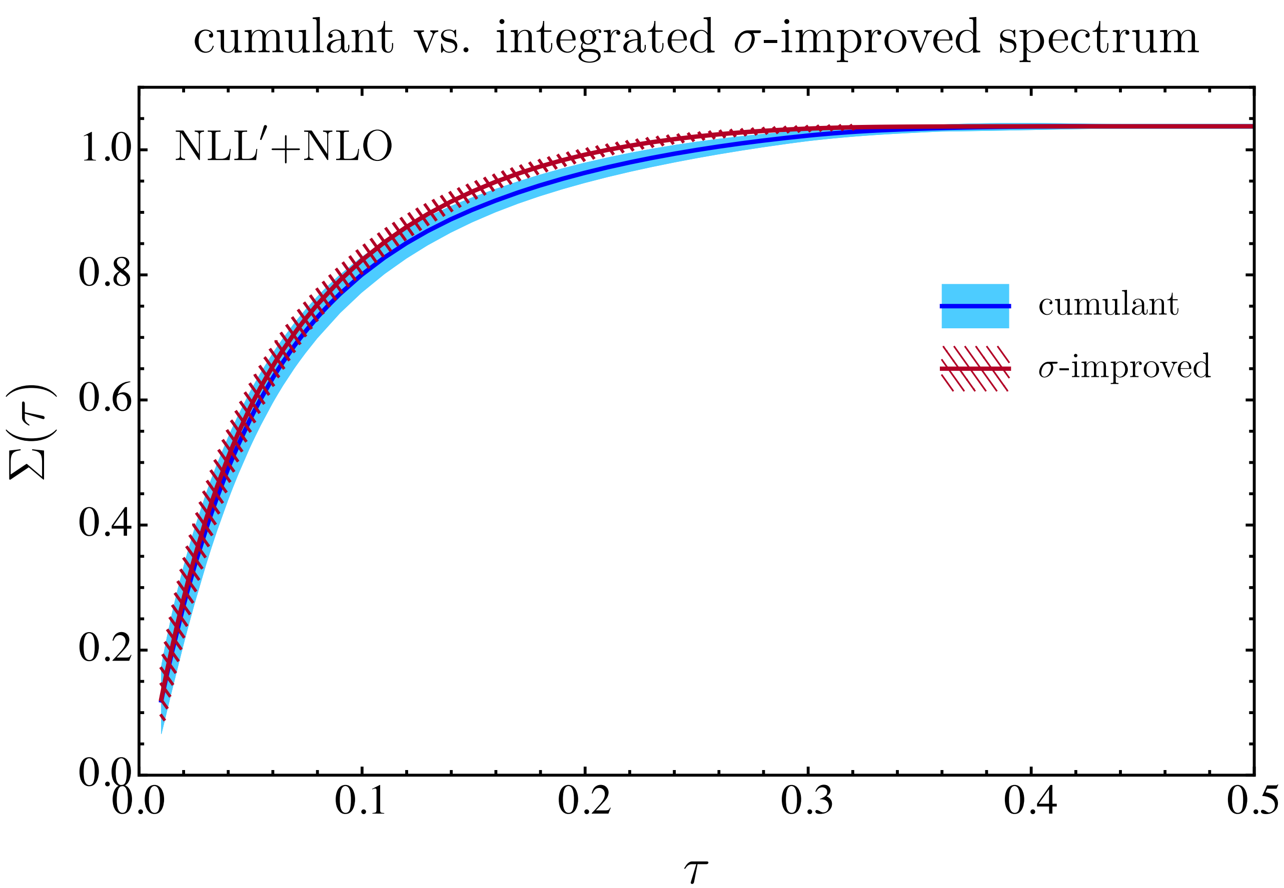
Convergence



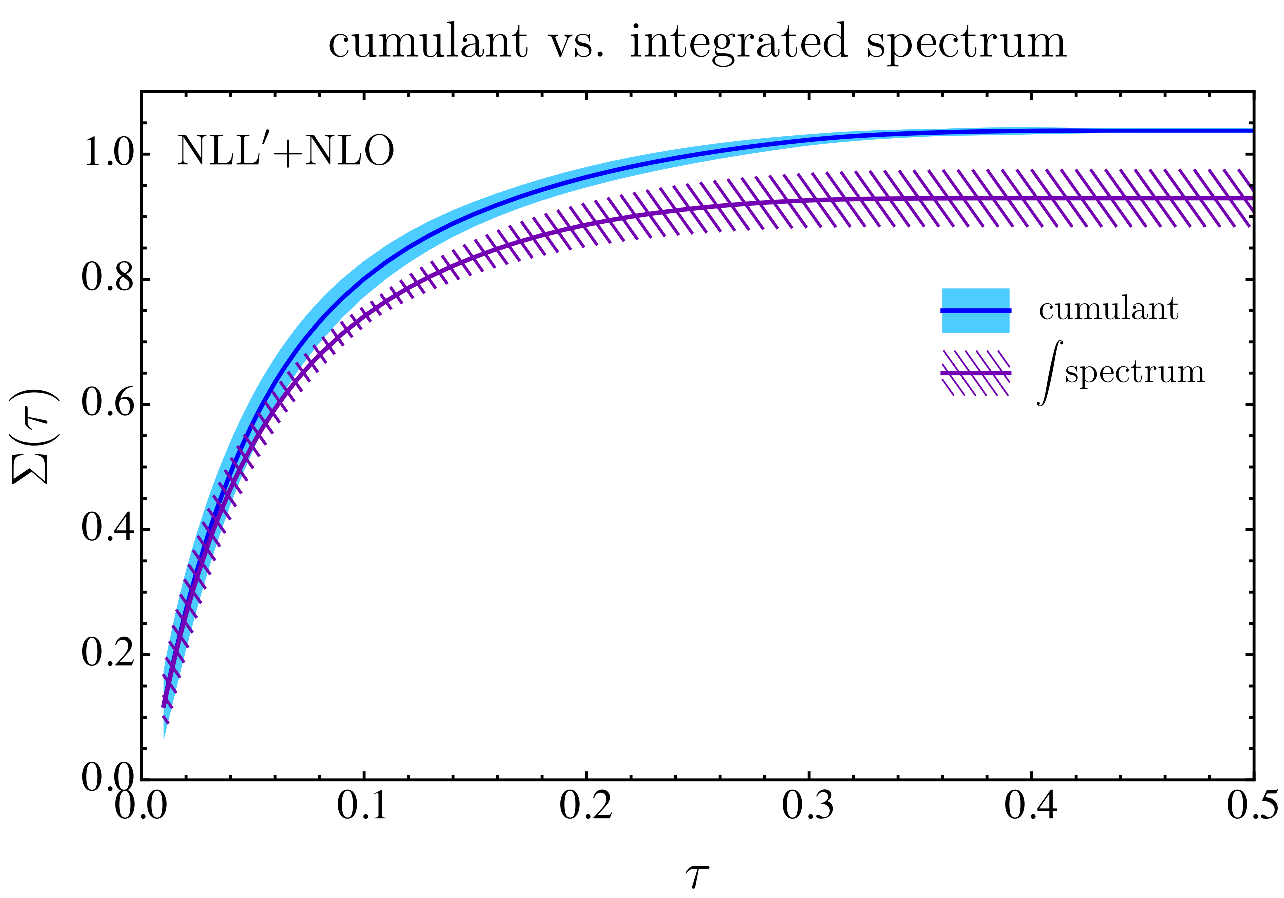


slight non-convergence in the peak region
exists also in the standard resummed spectrum
(artifact of pinching in resummation scale dependence)

Cumulant

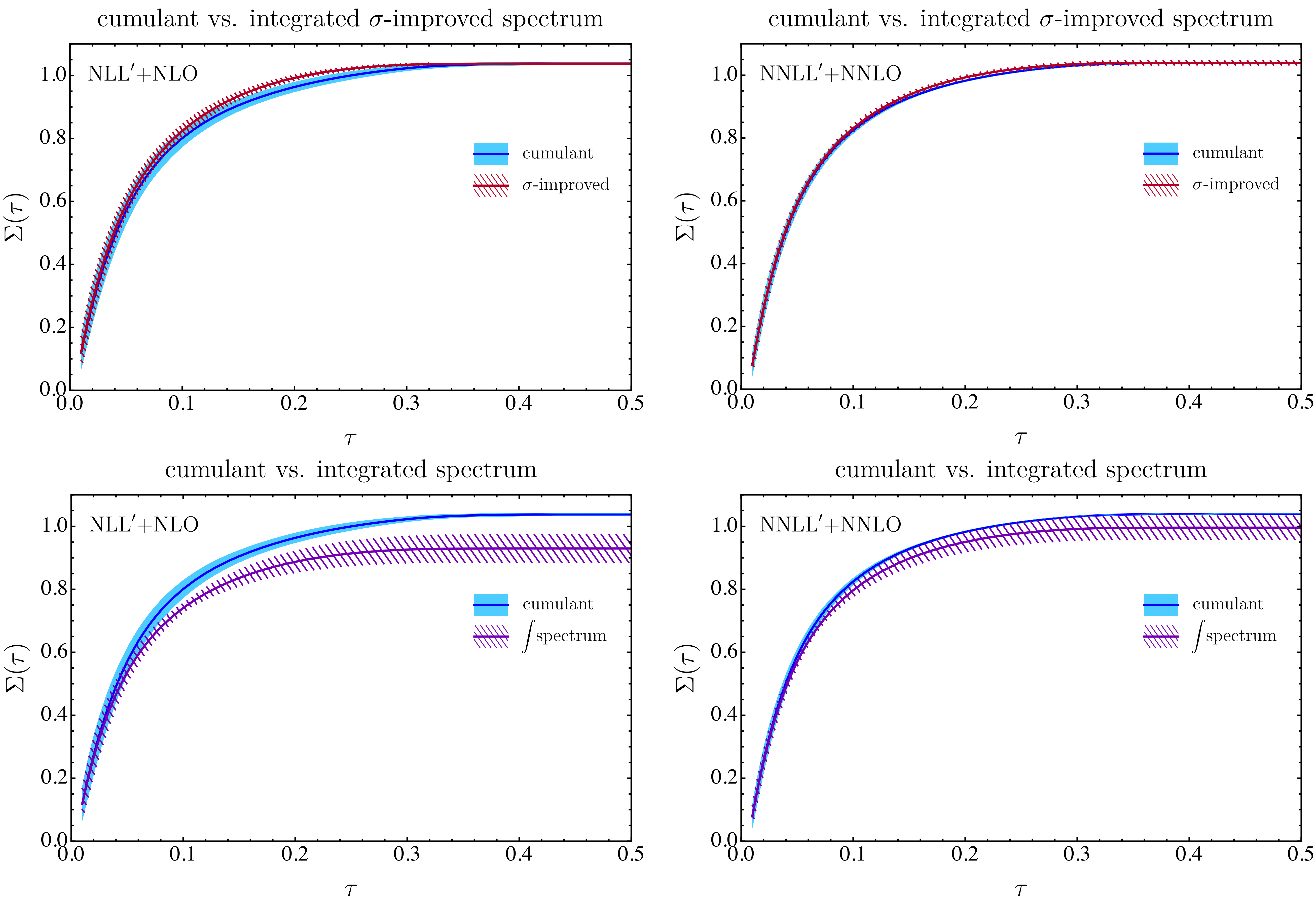


“σ-improved” spectrum resummation



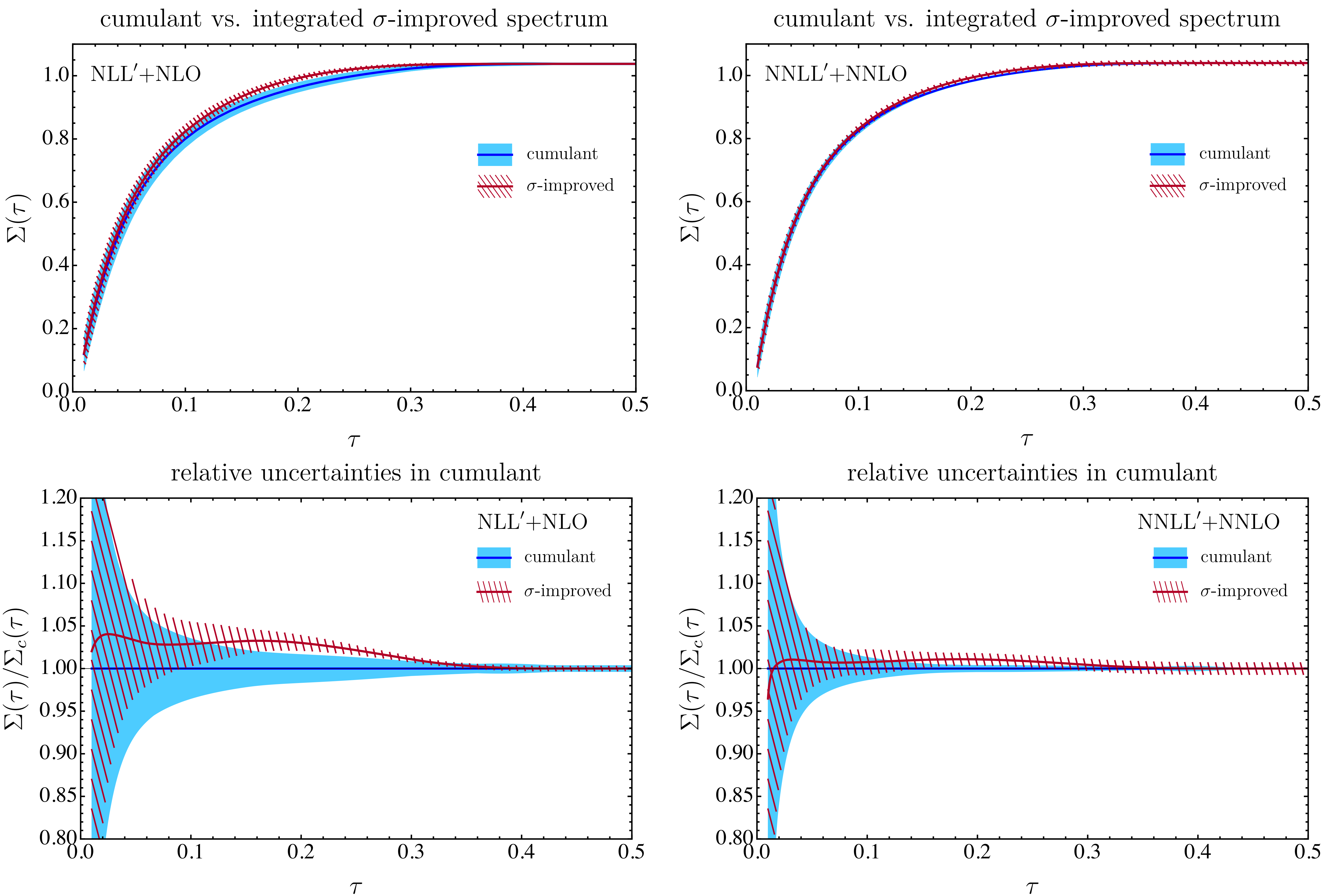
standard cumulant/spectrum resummation

Cumulant

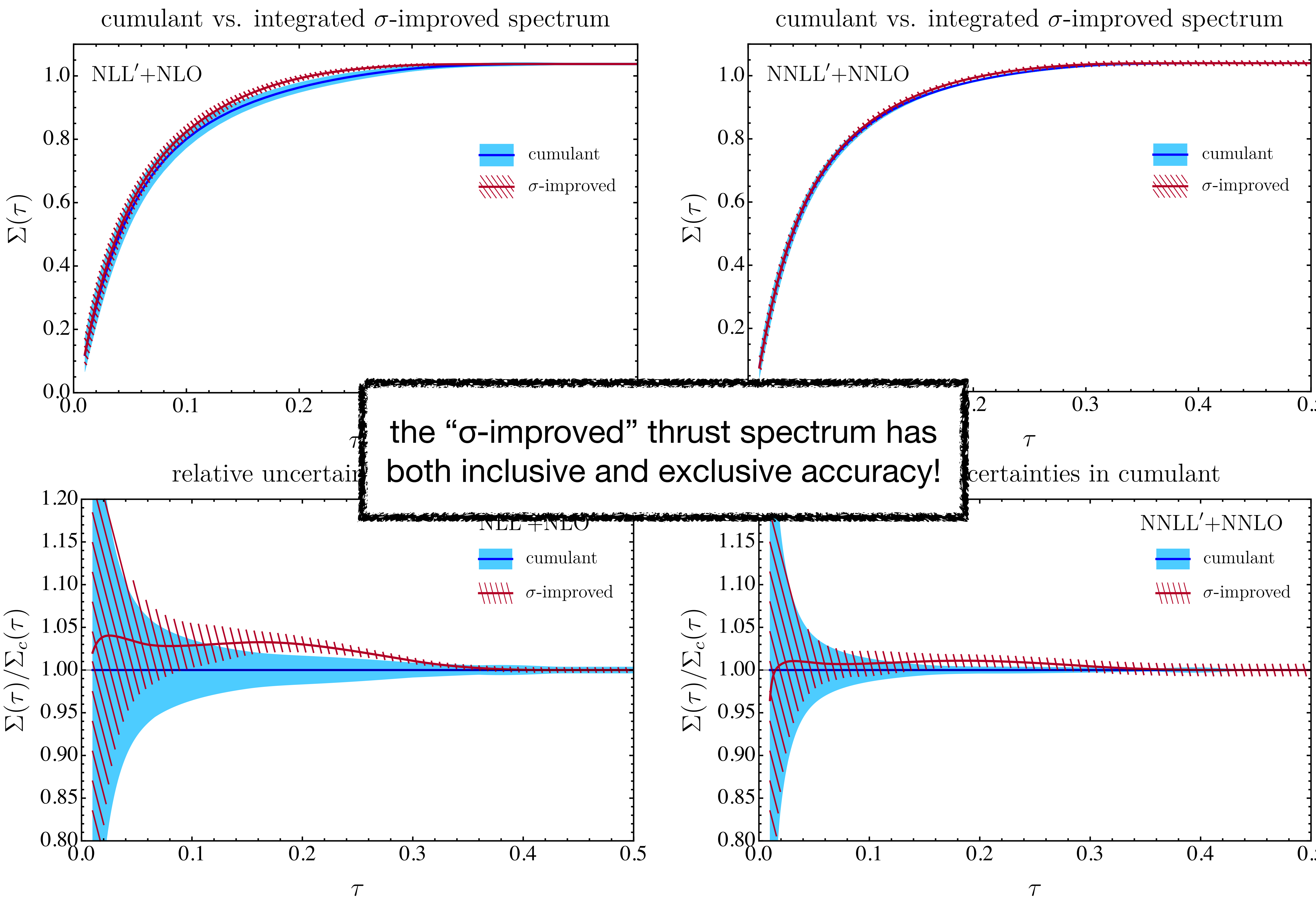


cumulant vs. integrated σ-improved spectrum
NLL′+NLO
cumulant
σ-improved
Σ(τ)
τ
cumulant vs. integrated σ-improved spectrum
NNLL′+NNLO
cumulant
σ-improved
Σ(τ)
τ
cumulant vs. integrated spectrum
NLL′+NLO
cumulant
∫spectrum
Σ(τ)
τ
cumulant vs. integrated spectrum
NNLL′+NNLO
cumulant
∫spectrum
Σ(τ)
τ

Cumulant


cumulant vs. integrated σ-improved spectrum
NLL′+NLO
cumulant
σ-improved
Σ(τ)
τ
cumulant vs. integrated σ-improved spectrum
NNLL′+NNLO
cumulant
σ-improved
Σ(τ)
τ
relative uncertainties in cumulant
NLL′+NLO
cumulant
σ-improved
Σ(τ)/Σ_c(τ)
τ
relative uncertainties in cumulant
NNLL′+NNLO
cumulant
σ-improved
Σ(τ)/Σ_c(τ)
τ

Cumulant

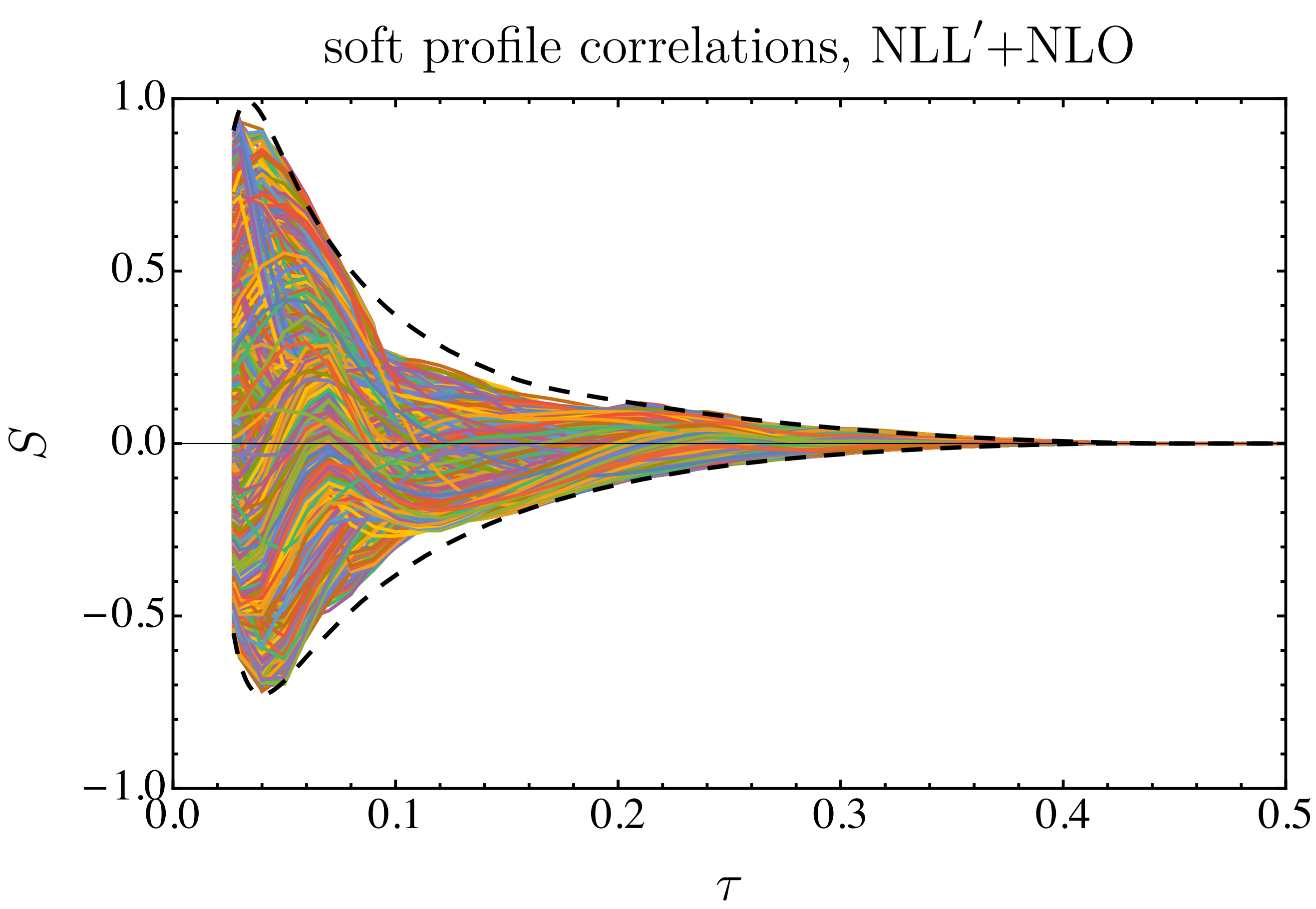


the "σ-improved" thrust spectrum has both inclusive and exclusive accuracy!

Correlations

can study correlations across τ:

$$S = \frac{d\sigma}{d\tau}(\mu_i) - \frac{d\sigma}{d\tau}(\mu_{\text{central}})$$

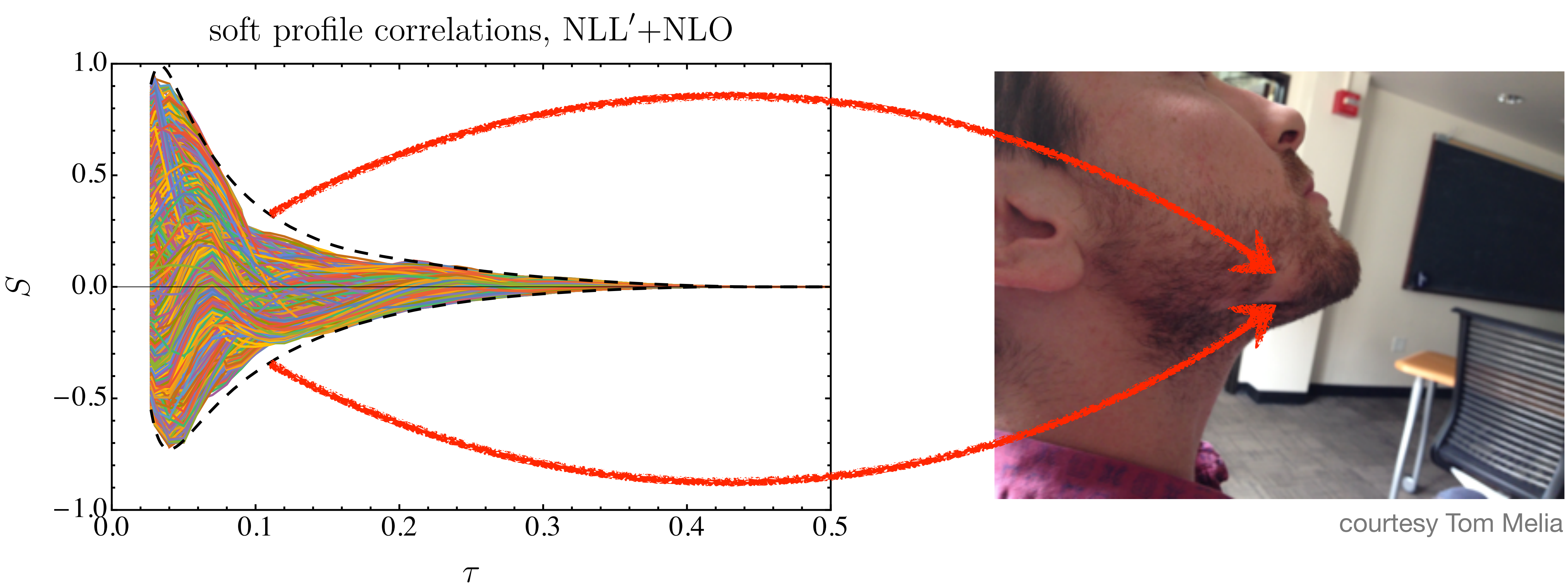


“gap” regions hard to fill, require precise correlations at small and large τ

Correlations

can study correlations across τ:

$$S = \frac{d\sigma}{d\tau}(\mu_i) - \frac{d\sigma}{d\tau}(\mu_{\text{central}})$$



courtesy Tom Melia

Two-Point Correlations

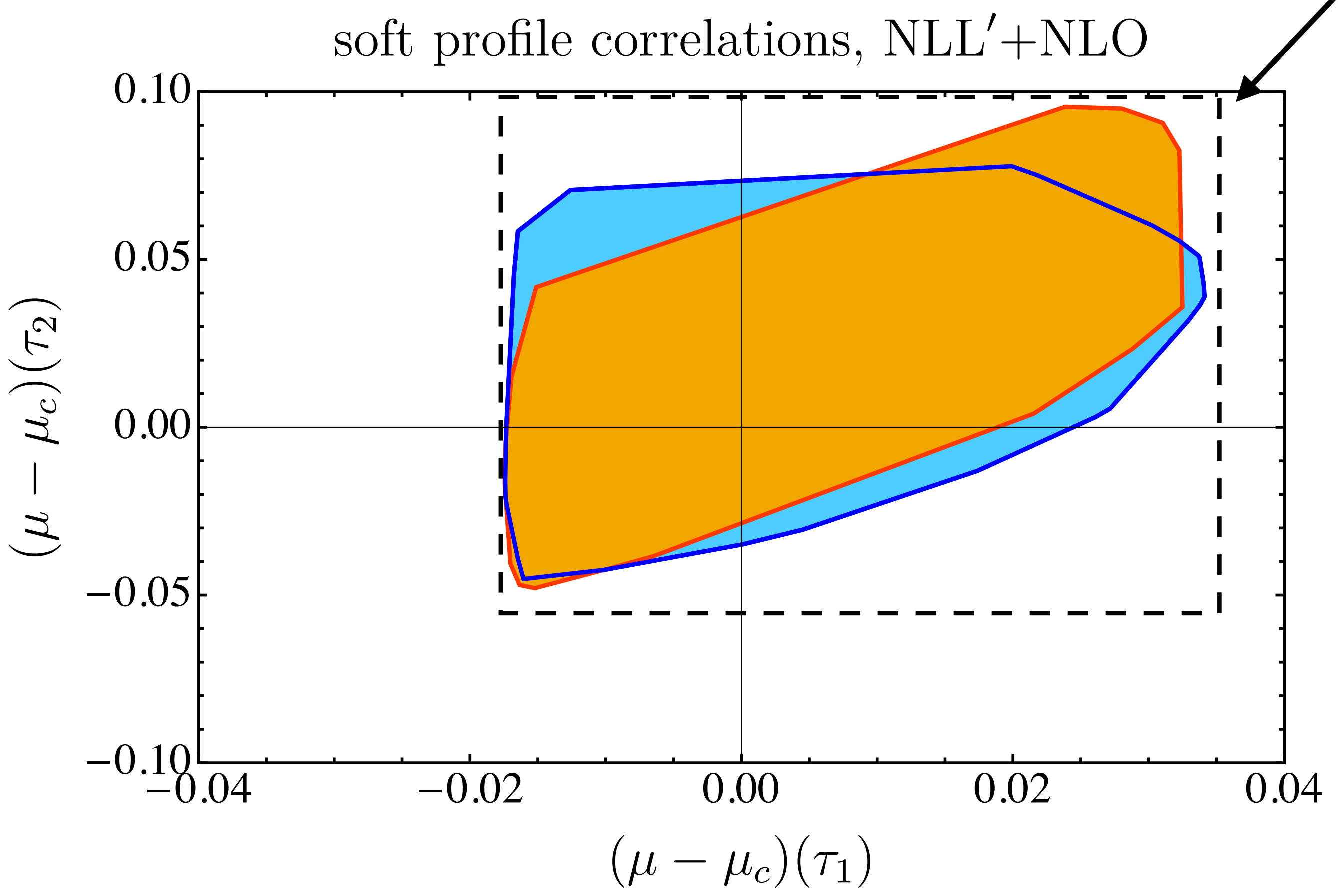
can study correlations across τ:

$$\frac{d\sigma}{d\tau}(\mu_i) - \frac{d\sigma}{d\tau}(\mu_{\text{central}})$$

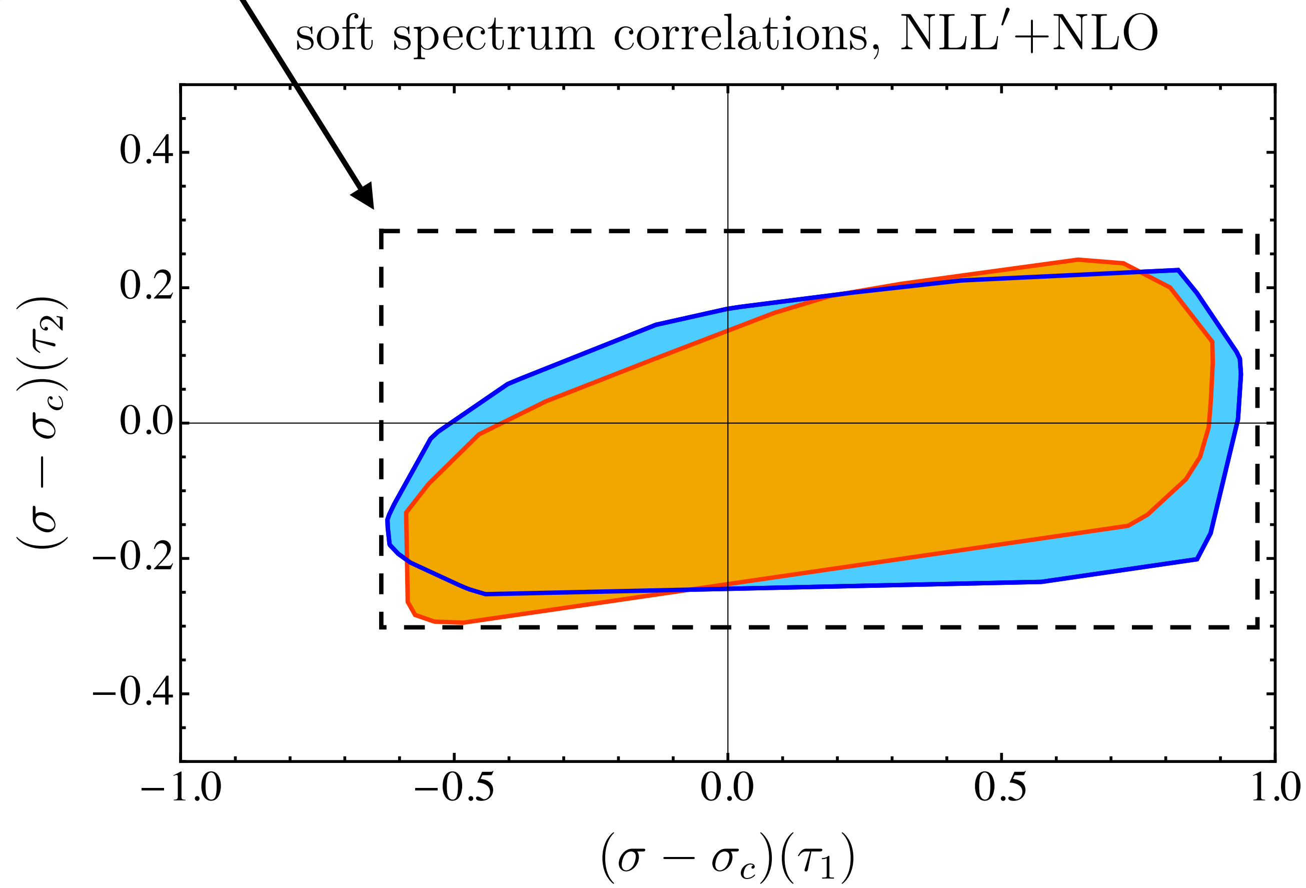
$\tau_1 = 0.03$, $\tau_2 = 0.12$

default region

profile



spectrum

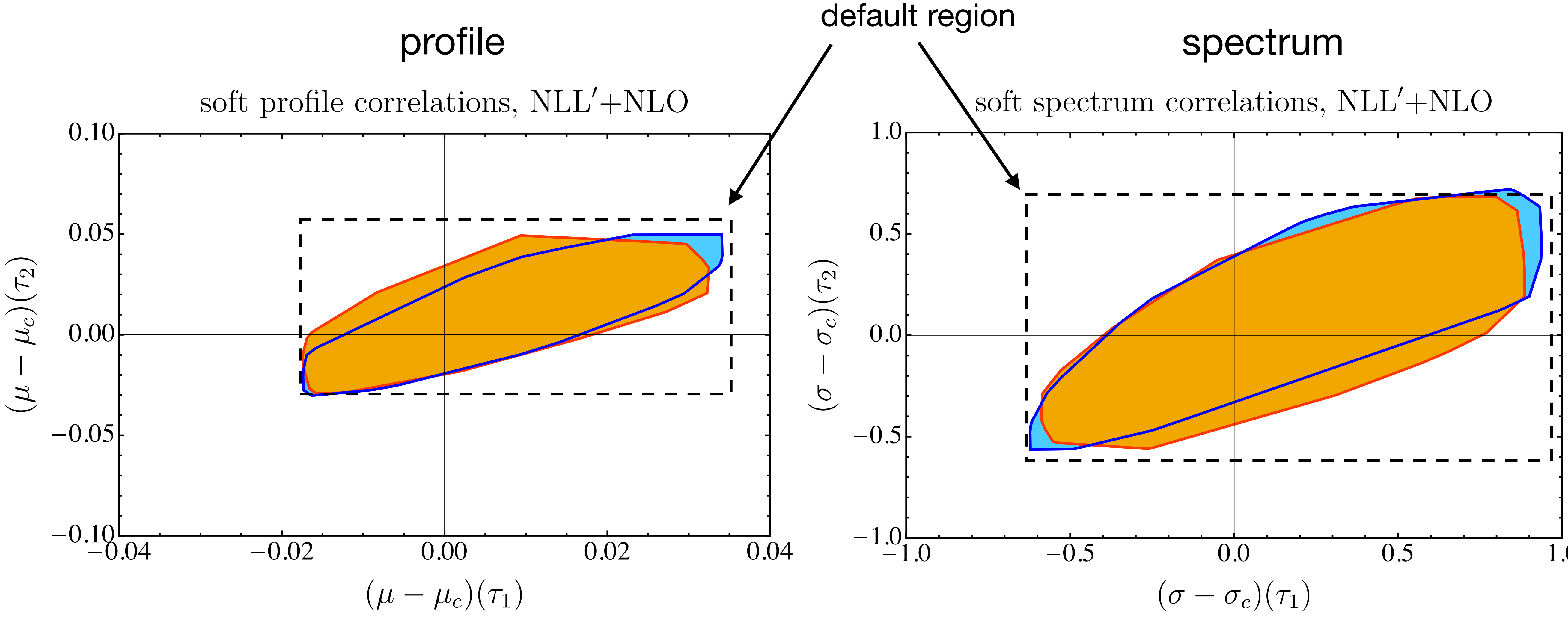


Two-Point Correlations

can study correlations across τ:

$$\frac{d\sigma}{d\tau}(\mu_i) - \frac{d\sigma}{d\tau}(\mu_{\mathrm{central}})$$

$\tau_1 = 0.03$, $\tau_2 = 0.06$

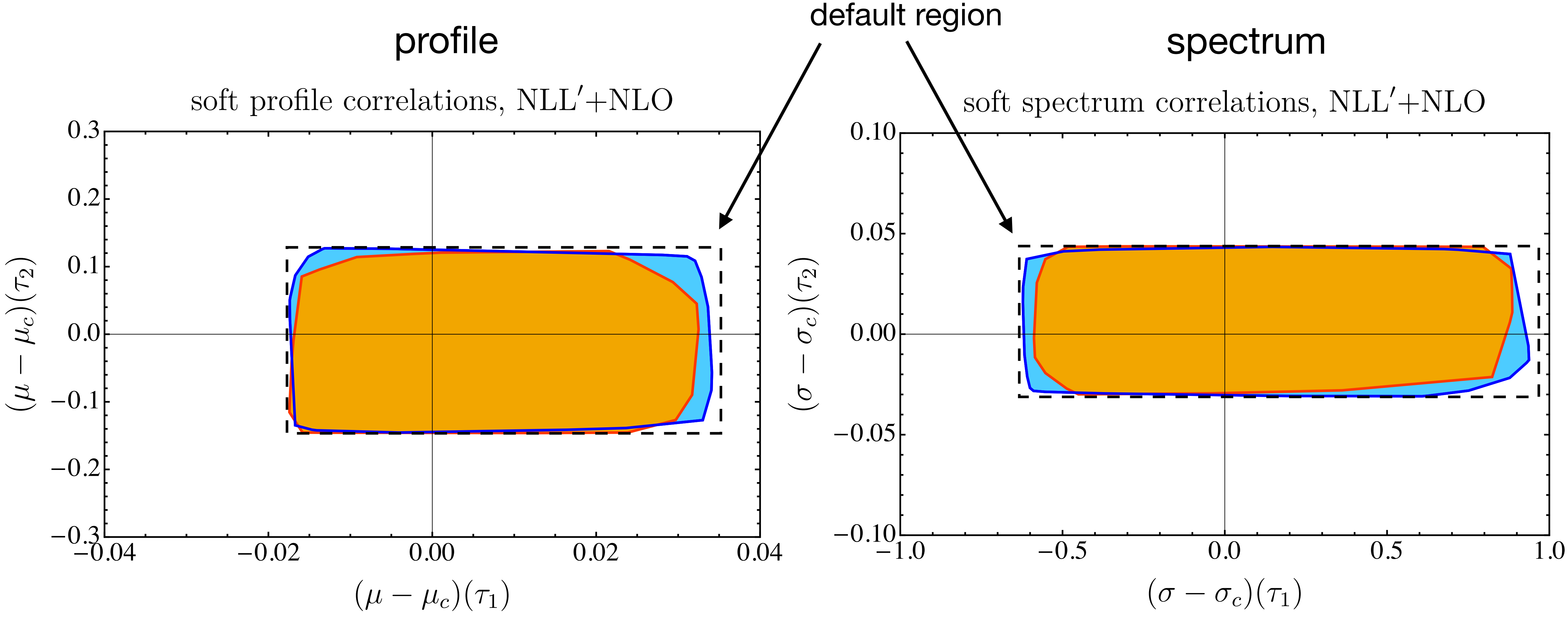


Two-Point Correlations

can study correlations across τ:

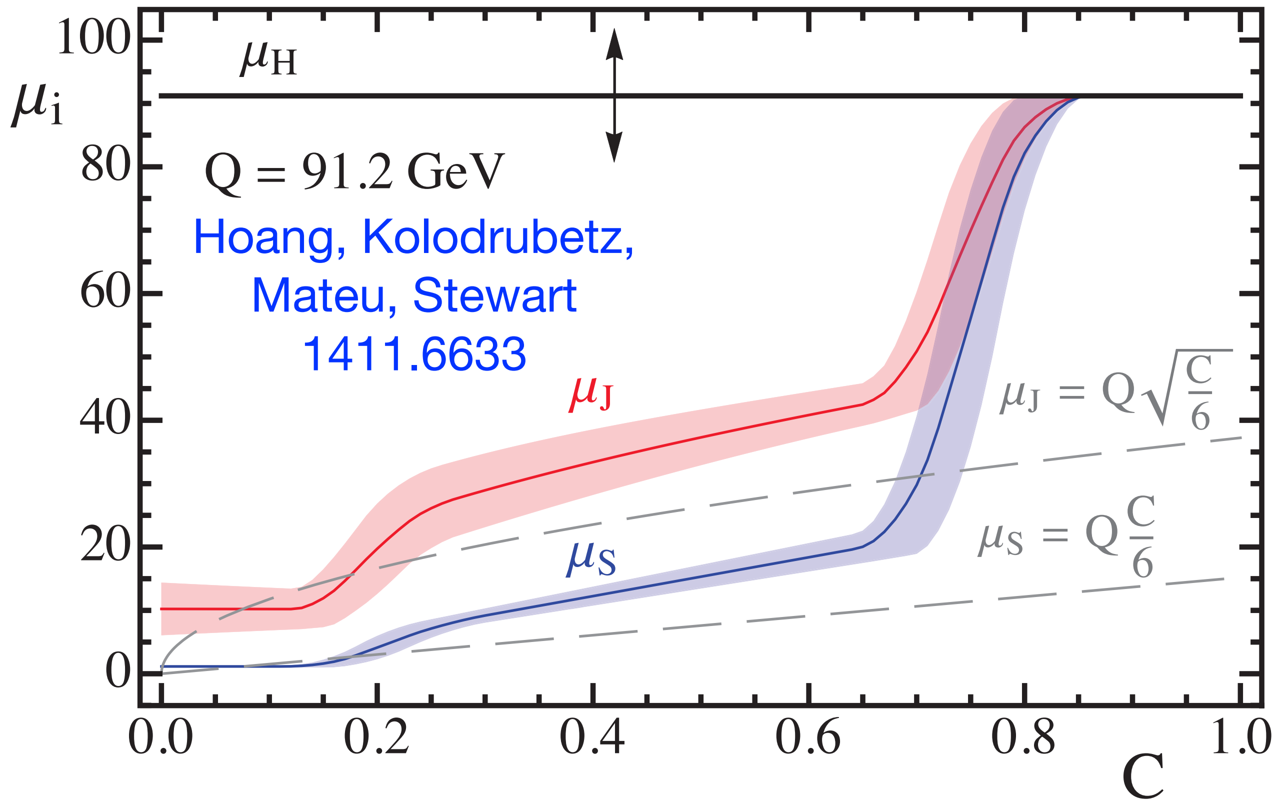
$$\frac{d\sigma}{d\tau}(\mu_i) - \frac{d\sigma}{d\tau}(\mu_{\rm central})$$

$\tau_1 = 0.03$, $\tau_2 = 0.3$



Future Directions

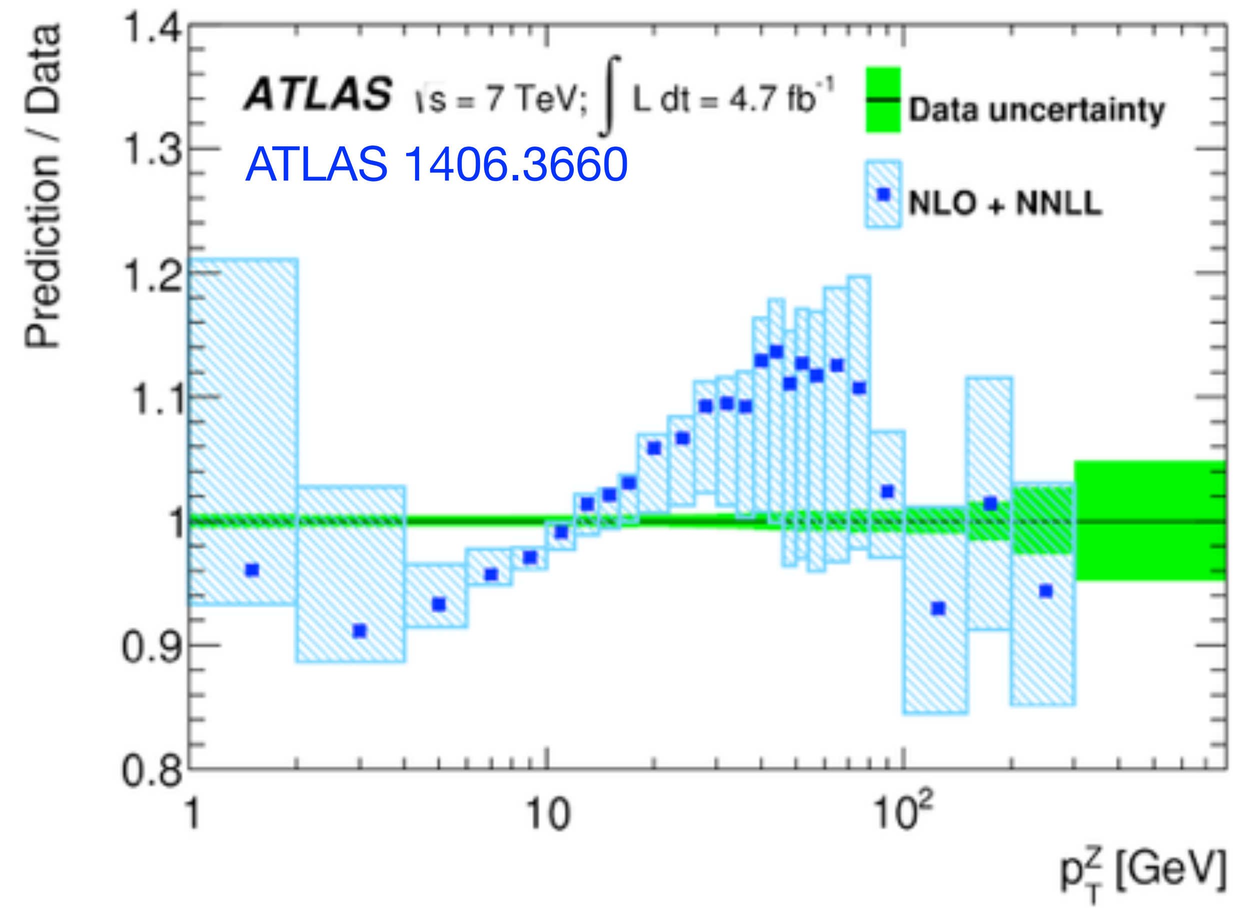
C-parameter in e^+e^-



C-parameter has very distinctive profile scales

large non-perturbative regime, large canonical regime

vector boson p_T distribution at LHC



persistent disagreement with data across theoretical predictions (resummation, Monte Carlo generators)

possible discrepancies in matching

Conclusions

- Resummation improves the accuracy of many exclusive cross sections, but loses accuracy in the corresponding inclusive cross section
- We have defined a method for resummation to preserve the accuracy at both the inclusive and exclusive level
  - An algorithm is used to find profile scales preserving the total cross section
  - Rigorously connects physical components of uncertainty with parameters of the factorization theorem
- Studies on thrust in e^+e^- are very promising
  - Follow-up studies for event shapes, especially hadronic collisions

Extra Slides

Uncertainties: Fixed Order

fixed order uncertainties estimated via variation of renormalization, factorization scales

$$\sigma(\mu_R, \mu_F) = \int \hat{\sigma}(\mu_R, \mu_F)\mathcal{L}(\mu_F)$$

consider two jet bins: Z + 0 jets, Z + ≥1 jets separated by a jet p_T veto

$$\sigma_0(p_T^{\rm cut}),\ \sigma_{\geq 1}(p_T^{\rm cut})$$

$$\sigma_{\rm incl} = \sigma_0(p_T^{\rm cut}) + \sigma_{\geq 1}(p_T^{\rm cut})$$

covariance matrix has the form:

$$C = \begin{pmatrix} \Delta_{\rm 0y}^2 & \Delta_{\rm 0y}\Delta_{\geq 1{\rm y}} \\ \Delta_{\rm 0y}\Delta_{\geq 1{\rm y}} & \Delta_{\geq 1{\rm y}}^2 \end{pmatrix} + \begin{pmatrix} \Delta_{\rm cut}^2 & -\Delta_{\rm cut}^2 \\ -\Delta_{\rm cut}^2 & \Delta_{\rm cut}^2 \end{pmatrix}$$

inclusive cross section constraint: $\sigma_{\rm incl} = \sigma_0 + \sigma_{\geq 1} \ \Rightarrow\ \Delta_{\rm incl} = \Delta_{\rm 0y} + \Delta_{\geq 1{\rm y}}$

$$C = \begin{pmatrix} \Delta_{\rm incl}^2 + \Delta_{\geq 1}^2 - 2\Delta_{\rm incl}\Delta_{\geq 1{\rm y}} & -\Delta_{\geq 1}^2 + \Delta_{\rm incl}\Delta_{\geq 1{\rm y}} \\ -\Delta_{\geq 1}^2 + \Delta_{\rm incl}\Delta_{\geq 1{\rm y}} & \Delta_{\geq 1}^2 \end{pmatrix}$$

2 degrees of freedom: $\Delta_{\geq 1{\rm y}},\ \Delta_{\geq 1} = \left(\Delta_{\geq 1{\rm y}}^2 + \Delta_{\rm cut}^2\right)^{1/2}$

unclear how to estimate parameters from scale variation: assumptions needed

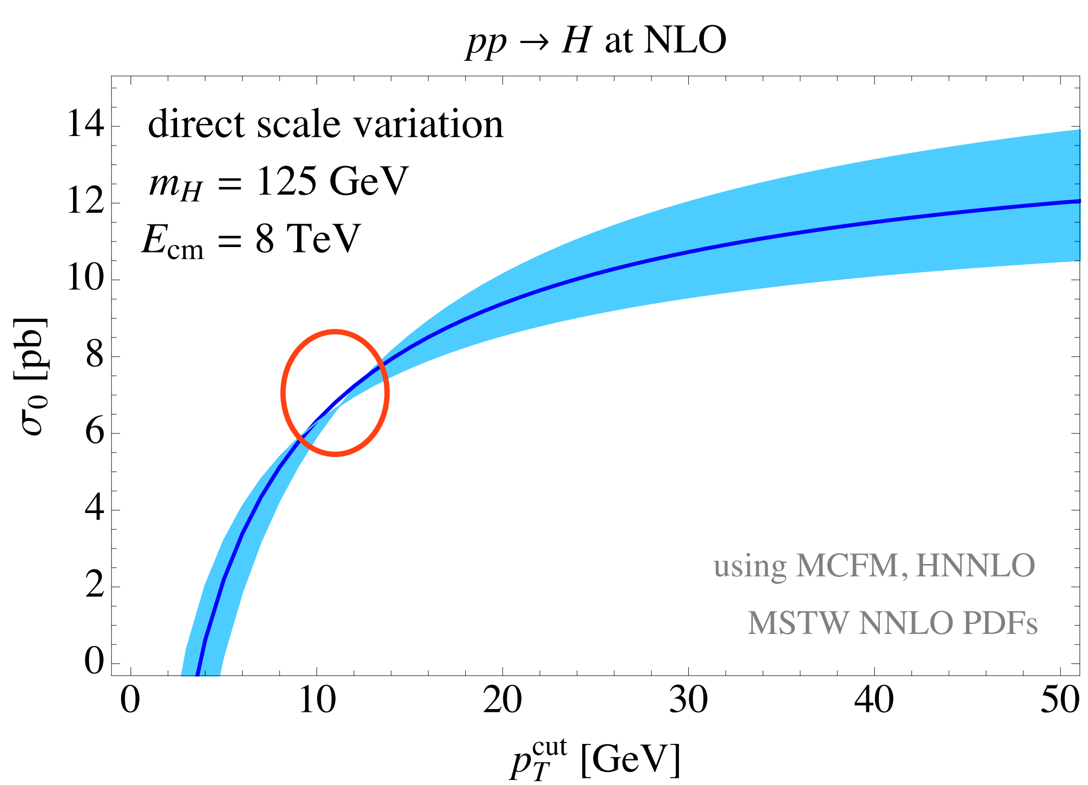
Uncertainties: Fixed Order

$$C = \begin{pmatrix} \Delta_{\rm incl}^2 + \Delta_{\geq 1}^2 - 2\Delta_{\rm incl}\Delta_{\geq 1y} & -\Delta_{\geq 1}^2 + \Delta_{\rm incl}\Delta_{\geq 1y} \\ -\Delta_{\geq 1}^2 + \Delta_{\rm incl}\Delta_{\geq 1y} & \Delta_{\geq 1}^2 \end{pmatrix}$$

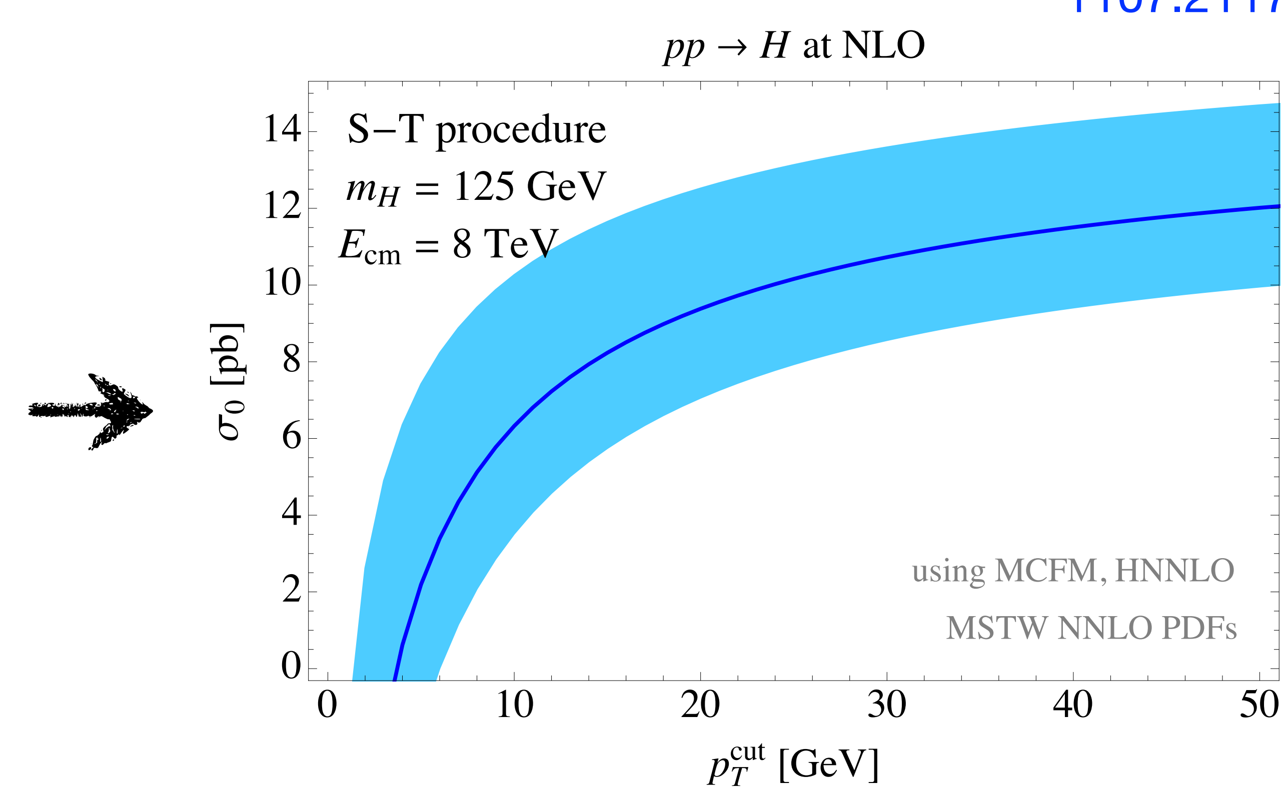
Stewart-Tackmann method: assume $\Delta_{\geq 1y} = 0 \;\Rightarrow\; C = \begin{pmatrix} \Delta_{\rm incl}^2 + \Delta_{\geq 1}^2 & -\Delta_{\geq 1}^2 \\ -\Delta_{\geq 1}^2 & \Delta_{\geq 1}^2 \end{pmatrix}$

i.e. inclusive cross sections of different multiplicity have uncorrelated uncertainties

Stewart, Tackmann
1107.2117



standard “direct” scale variation



Stewart-Tackmann procedure

Uncertainties: Fixed Order

$$C = \begin{pmatrix} \Delta_{\rm incl}^2 + \Delta_{\geq 1}^2 - 2\Delta_{\rm incl}\Delta_{\geq 1 \rm y} & -\Delta_{\geq 1}^2 + \Delta_{\rm incl}\Delta_{\geq 1 \rm y} \\ -\Delta_{\geq 1}^2 + \Delta_{\rm incl}\Delta_{\geq 1 \rm y} & \Delta_{\geq 1}^2 \end{pmatrix}$$

Stewart-Tackmann method: assume $\Delta_{\geq 1 \rm y} = 0 \Rightarrow C = \begin{pmatrix} \Delta_{\rm incl}^2 + \Delta_{\geq 1}^2 & -\Delta_{\geq 1}^2 \\ -\Delta_{\geq 1}^2 & \Delta_{\geq 1}^2 \end{pmatrix}$

i.e. inclusive cross sections of different multiplicity have uncorrelated uncertainties

Stewart, Tackmann
1107.2117

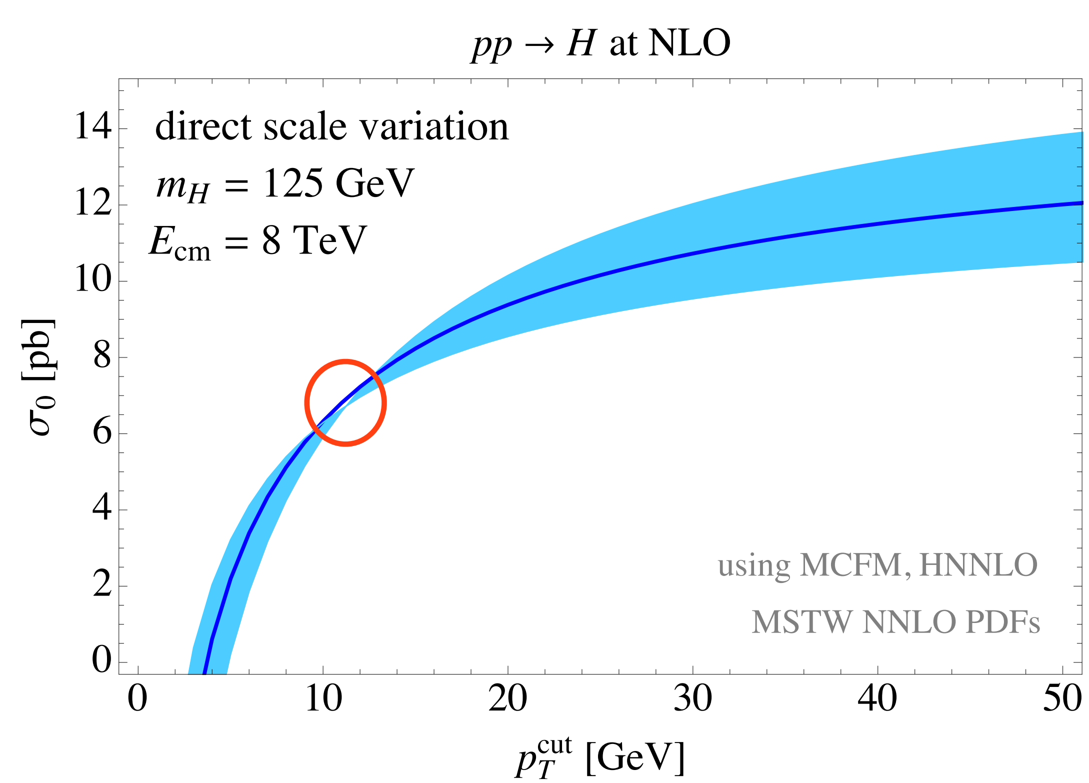
cause of uncertainty pinch:

$$\sigma_0(p_T^{\rm cut}) \propto \sigma_B \left[1 + \frac{\alpha_s}{\pi}\left(K_{\rm NLO} - 2C_A \ln^2 \frac{m_H}{p_T^{\rm cut}}\right) + \ldots\right]$$

unphysical
cancellation between large K-factor and logs

part of the total rate

bin cut between 0 jets, 1+ jets



Uncertainties: Fixed Order

$$C = \begin{pmatrix} \Delta^2_{\rm incl} + \Delta^2_{\geq 1} - 2\Delta_{\rm incl}\Delta_{\geq 1{\rm y}} & -\Delta^2_{\geq 1} + \Delta_{\rm incl}\Delta_{\geq 1{\rm y}} \\ -\Delta^2_{\geq 1} + \Delta_{\rm incl}\Delta_{\geq 1{\rm y}} & \Delta^2_{\geq 1} \end{pmatrix}$$

Stewart-Tackmann method: assume $\Delta_{\geq 1{\rm y}} = 0 \;\Rightarrow\; C = \begin{pmatrix} \Delta^2_{\rm incl} + \Delta^2_{\geq 1} & -\Delta^2_{\geq 1} \\ -\Delta^2_{\geq 1} & \Delta^2_{\geq 1} \end{pmatrix}$

i.e. inclusive cross sections of different multiplicity have uncorrelated uncertainties

Stewart, Tackmann
1107.2117

efficiency method: assume 0-jet efficiency, total cross section uncertainties uncorrelated

$$\Rightarrow\; \Delta_{0{\rm y}} = \Delta_{\rm incl}\epsilon_0,\; \Delta_{\geq 1{\rm y}} = \Delta_{\rm incl}(1-\epsilon_0),\; \Delta_{\rm cut} = \sigma_{\rm incl}\Delta_{\epsilon 0}$$

Banfi, Salam,
Zanderighi
1203.5773

both approaches physically well-motivated, although not always ideal

Uncertainties: Resummed

Uncertainties assessed by variation of factorization scales

$$\delta\{\mu, \mu_H, \mu_J, \mu_S\} \;\rightarrow\; \Delta(\tau)$$

standard approach: 3 types of scale variation

$\Delta_\mu(\tau)$: collective variation of all scales (fixes logarithms, probes fixed order scale)

$\Delta^J_{\rm res}(\tau)$: jet scale variation (probes logarithms, fixes fixed order scale)

$\Delta^S_{\rm res}(\tau)$: soft scale variation (probes logarithms, fixes fixed order scale)

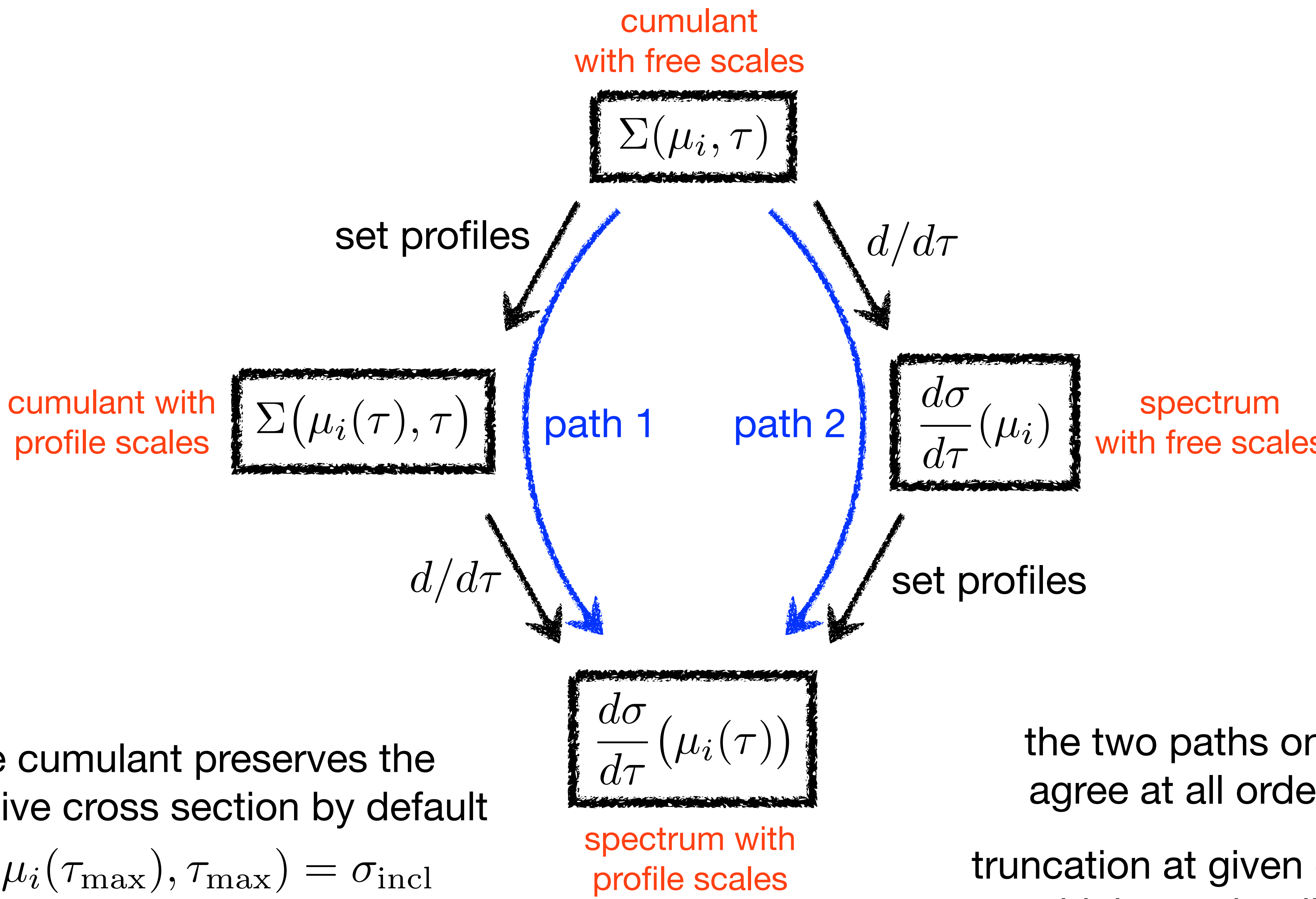
associate scale variation with components of uncertainty:

$$\Delta_\mu \;\rightarrow\; \Delta_{\rm y}$$

$$\Delta^J_{\rm res}, \Delta^S_{\rm res} \;\rightarrow\; \Delta_{\rm cut}$$

these assignments are physically well-motivated, although not necessarily valid

Two Paths to Resummation



the cumulant preserves the inclusive cross section by default

$$\Sigma(\mu_i(\tau_{\max}), \tau_{\max}) = \sigma_{\text{incl}}$$

the two paths only agree at all orders

truncation at given order creates higher order difference