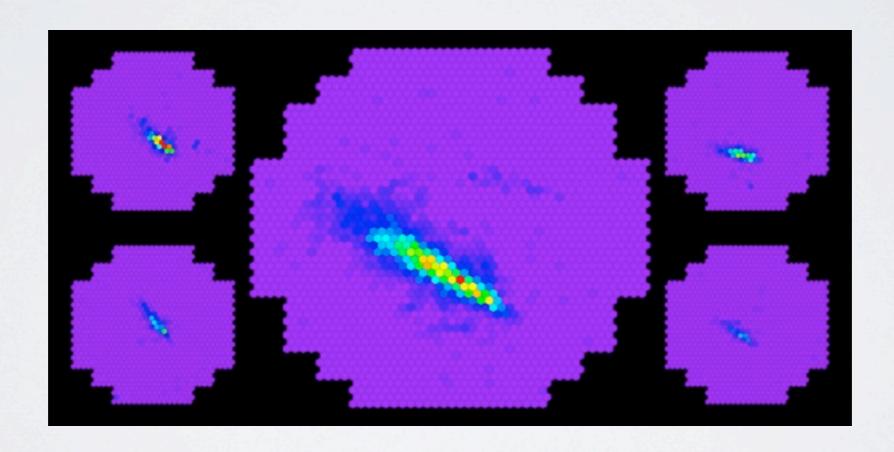
ONTHE ANNIHILATION OF WIMPS

Matthew Baumgart (Carnegie Mellon U.)



SCET Workshop 2015 - Santa Fe 3/27/2015

DARK MATTER

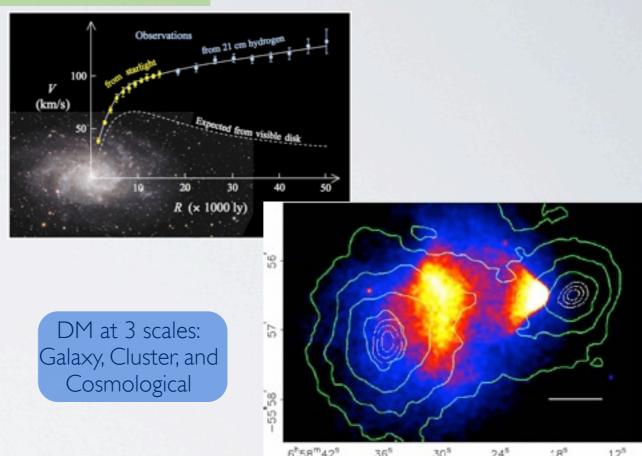
Dark Matter (DM) posited since 1930s (Zwicky, Oort, Rubin...) to explain variety of astrophysical anomalies

- Key questions about missing 27% of universe's energy
 - Does it interact with us nongravitationally?
 - What are its quantum numbers?
- Variety of motivated candidates

 (axions, asymmetric, sterile
 neutrinos...), but we focus on:

 Weakly-Interacting Massive Particles

 (WIMPs)



Parameter	Planck	
	Best fit	68% limits
$\Omega_b h^2$	0.022068	0.02207 ± 0.00033
$\Omega_c h^2$	0.12029	0.1196 ± 0.0031
100θ _{MC}	1.04122	1.04132 ± 0.00068
τ	0.0925	0.097 ± 0.038
n _s	0.9624	0.9616 ± 0.0094
$\ln(10^{10}A_s)$	3.098	3.103 ± 0.072
Ω_{Λ}	0.6825	0.686 ± 0.020
$\Omega_{\rm m}$	0.3175	0.314 ± 0.020

WHY WIMPS?

- · General consideration of particle DM in cosmology lead to "WIMP miracle"
- · Particle freezes out when annihilation rate drops below cosmic expansion

$$n(T_f)\langle \sigma v \rangle \sim \left(H \sim \frac{T_f^2}{M_{Pl}} \right)$$

· Frozen out particle has density ratio to photons today as at freezeout

$$\frac{n_{X0}}{T_0^3} \simeq \frac{n(T_f)}{T_f^3}$$

This lets us calculate relic density of dark matter...

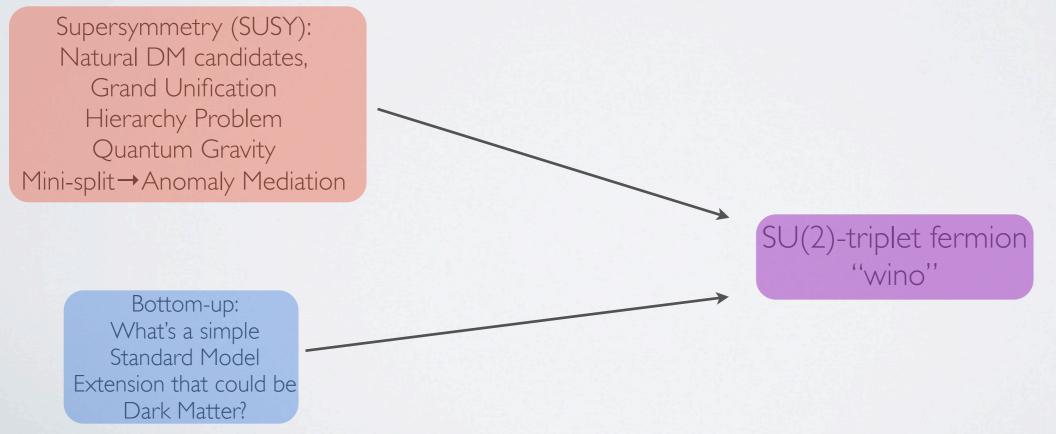
$$\Omega_X h^2 \equiv \frac{\rho_{X0}}{\rho_c} h^2 \sim \frac{M_X}{T_f} \frac{1}{3 \times 10^4} \frac{1}{\langle \sigma v \rangle} \frac{1}{T_0 M_{Pl}} \sim \frac{1}{10^3 \langle \sigma v \rangle} \frac{1}{\text{TeV}^2}$$

• ...and expected mass given a perturbative annihilation process $\langle \sigma v \rangle \sim C \alpha^2/M_X^2$

$$M_X \sim \text{TeV}\left(10\sqrt{C}\alpha\right)\sqrt{\frac{\Omega_X h^2}{0.12}}$$

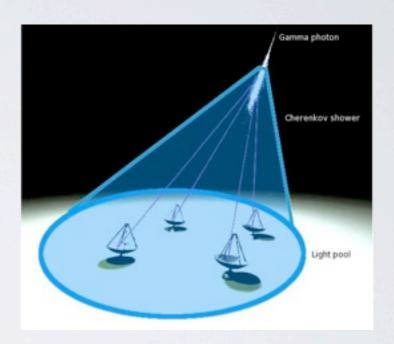
FROM WIMP TO WINO

- Without any input of electroweak physics, cosmology gives us Weak-scale (TeV) with weak couplings
- Tempting to connect Dark Matter to Hierarchy Problem as in SUSY
- What are motivated candidates and how do we find/exclude them?





- Probing TeV-scale winos in 2015 requires indirect detection (xx→observable)
- High Energy Steroscopic System (HESS)
- Atmospheric Cherenkov telescope
- Constrains photon lines. Our main interest...





Schematic of air shower observed by Cherenkov Telescope (spie.org)

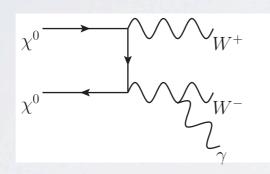


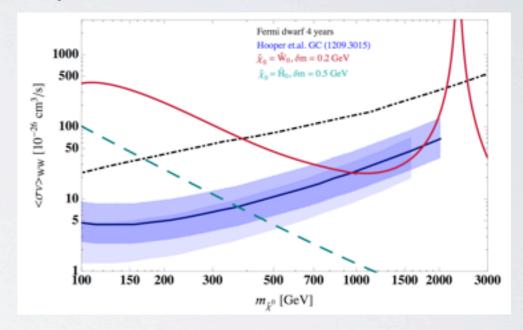
HESS in Namibia

INDIRECT DETECTION

• Continuum ($\chi\chi\to W^+W^-\to photons$) useful for sub-TeV limits

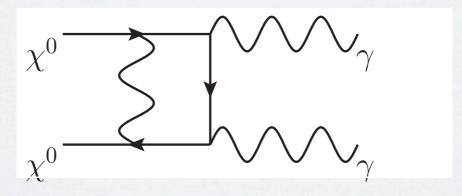
(Fermi)





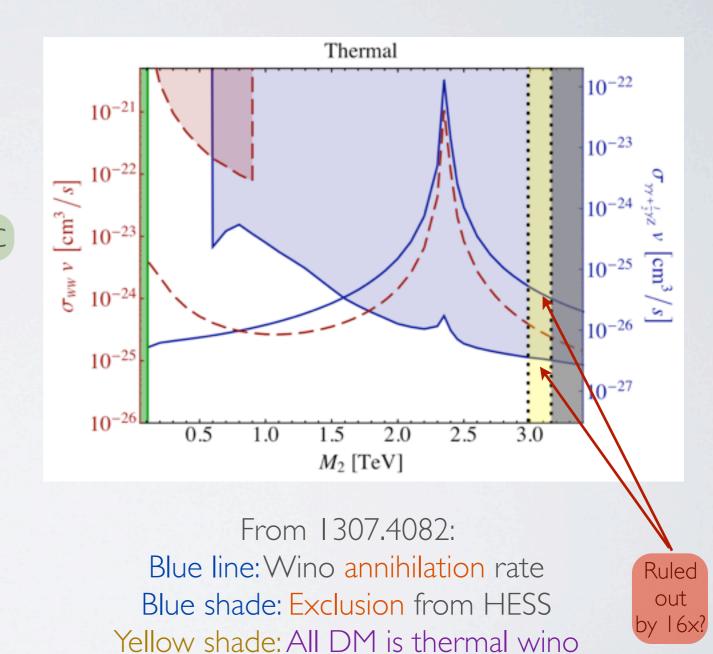
• Line $(\chi \chi \rightarrow \gamma + X)$ extends to multi-TeV range

J. Fan & M. Reece: 1307.4400



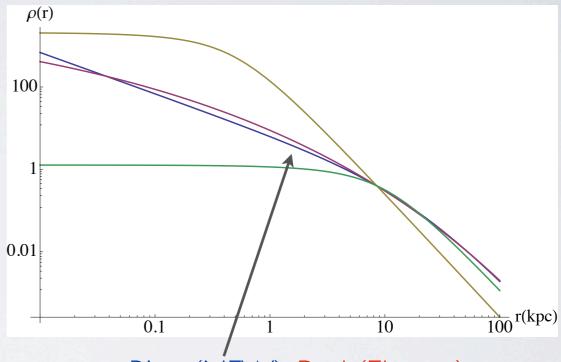
"WINO DARK MATTER UNDER SIEGE"*

- SM gauge couplings + Boltzmann Equation→M≈3 TeV thermal relic
- Has our beautiful, simple model been shot dead by HESS?
- Navarro-Frank-White (cusped) halo profile assumed

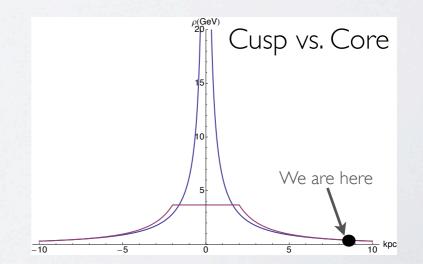


THE HALO LOOPHOLE

- Indirect detection searches suffer from large astrophysical uncertainties from unknown form of DM halo.
- Observation only constrains possible core < 10 kpc (Nesti & Salucci 1304.5127)
- Flux of photons observed by HESS proportional to integrated ρ^2
- Dwarf galxies evince corings, but simulation, even with baryons, finds cusps down to I kpc*



Blue (NFW), Red (Einasto), Yellow (Burkert-0.5), Green (Burkert-10) fixed to local density (8.5 kpc) of 0.4 GeV/cm³



WHAT COULD SAVETHE WINO?

Halo Model

Radiative Corrections

Flatten distribution in galactic center (core)

Claim in literature of 75% reduction at NLO

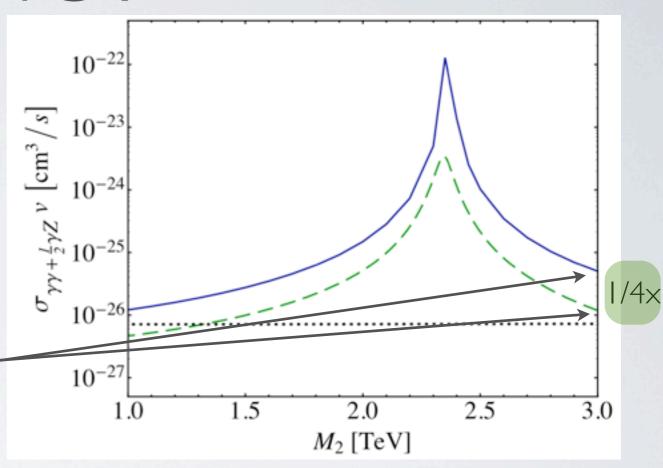
Core needed:

1.5 kpc

Core needed:

0.5 kpc

Simulations with baryons show cusped profiles down to I kpc (1208.4844, 1305.5360, 1306.0898)



From Cohen et al. 1307.4082

Factor of few at stake,

need state of the art calculation to determine

MB, Ira Rothstein, & Varun Vaidya: 1409.4415 (PRL) & 1412.8698 (JHEP)

SOMMERFELD ENHANCEMENT

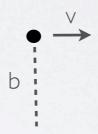
- Wino annihilation subject to 2
 effects beyond perturbation theory*
 - Sommerfeld Enhancement
 - Sudakov double logs
- Quantum-Mechanically, shortdistance annihilation rate modified by wavefunction-at-origin in presence of potential:

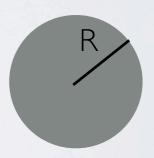
$$\langle \sigma v \rangle = |\psi(0)|^2 \Gamma_{\text{pert.}}$$





In absence of gravitation, capture radius is geometric, R

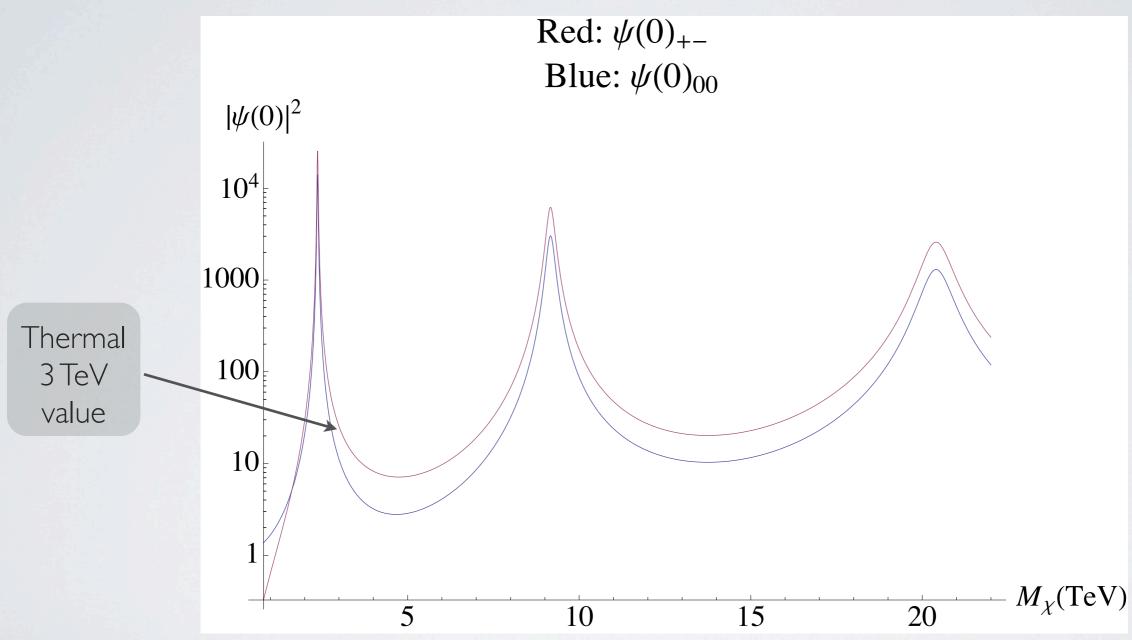




Turning on gravity, cross section grows for slower projectile: $b_{\rm capture} = R \sqrt{1 + \frac{2GM}{v^2 R}}$

^{*}See also 1409.7392: M. Bauer et al.; 1409.8294: G. Ovanesyan et al.

WINO SOMMERFELD FACTORS



Solving Schrodinger equation for $v = 10^{-3}$ gives us resonance regions with $O(10^4)$ enhancement

MODES & FACTORIZATION

- Large logs \rightarrow disparate scales ($\lambda \sim m_W/M_{wimp}$) \rightarrow factorization
- Factorization lets us separate 2 nontrivial behaviors (Sommerfeld and Sudakov) in hybrid NRQCD/SCET-2 theory*
- Setting up EFT requires list of relevant modes

Soft Higgs
couplings
are powersuppressed;
Collinear Higgses
only run
Fragmentation
Functions

WIMPs
$$(\chi): (E \sim \lambda^2, p \sim \lambda)$$

Potential:
$$(E \sim \lambda^2, p \sim \lambda) \leftarrow$$

Collinear
$$(B^{\mu\perp}): (k_+ \sim 1, k_- \sim \lambda^2, k_\perp \sim \lambda) \longleftarrow$$

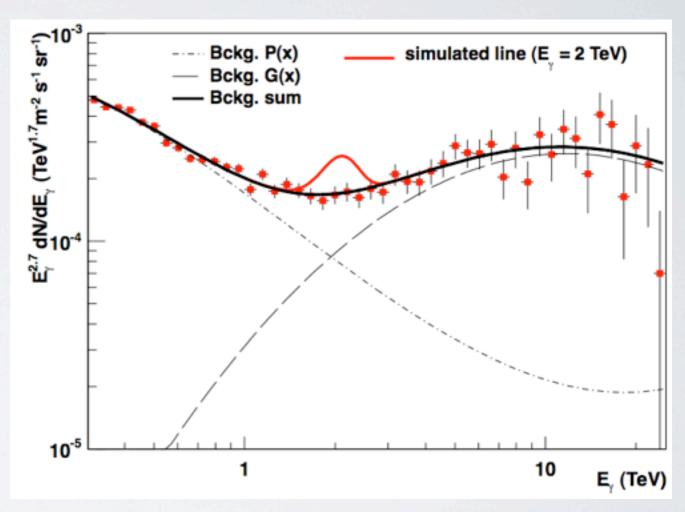
Soft
$$(S_{ab}): (k_+ \sim \lambda, k_- \sim \lambda, k_\perp \sim \lambda)$$

These are all just W field in full theory

^{*}See also Fleming, Leibovich, Mehen hep-ph/0607121 for photon spectrum for quarkonium decays

SEMI-INCLUSIVE ANNIHILATION

- Our interest is in setting limits from indirect detection
- HESS is an air Cherenkov telescope that observes photons colliding with the atmosphere
- Therefore, we compute $\chi\chi \rightarrow \gamma + X$



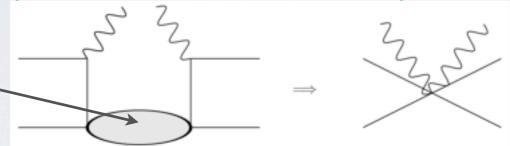
From HESS collaboration 1301.1173 at 3 TeV, energy resolution is ~400 GeV

 E_Y (soft W)= M_X - $m_W/2$ E_Y (collinear Ws) = M_X - m_W^2/M_X

OPERATOR BASIS

• Since our interest is semi-inclusive processes, it is useful to work with the Operator Product Expansion (OPE)

General recoil state has virtuality λM_X^2 so we integrate out



Matching tree level with one-loop running, we generate:

$$O_{1} = (\bar{\chi}\gamma^{5}\chi) |0\rangle\langle 0| (\bar{\chi}\gamma^{5}\chi) B^{\mu A \perp} B^{A \perp}_{\mu}$$

$$O_{2} = \frac{1}{2} \left\{ (\bar{\chi}\gamma^{5}\chi) |0\rangle\langle 0| (\bar{\chi}_{A}\gamma^{5}\chi_{B}) + (\bar{\chi}_{A}\gamma^{5}\chi_{B}) |0\rangle\langle 0| (\bar{\chi}\gamma^{5}\chi) \right\} B^{\perp A}_{\mu} B^{\mu B \perp}$$

$$O_{3} = (\bar{\chi}_{C}\gamma^{5}\chi_{D}) |0\rangle\langle 0| (\bar{\chi}_{D}\gamma^{5}\chi_{C}) B^{\mu A \perp} B^{A \perp}_{\mu}$$

$$O_{4} = (\bar{\chi}_{A}\gamma^{5}\chi_{C}) |0\rangle\langle 0| (\bar{\chi}_{C}\gamma^{5}\chi_{B}) B^{\perp A}_{\mu} B^{\mu B \perp}$$

Vacuum projectors for WIMPs guarantee large momentum in process needed for OPE

We implicitly work with SU(2) adjoint, so basis reduced by having Majorana fermion

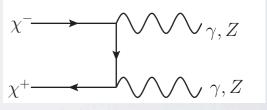
Spin-singlets by Fermi statistics up to O(v) corrections

Collinear boson fields in symmetric limit

WILSON COEFFICIENTS

- We have four operators, but controlled by one tree-level matching coefficient
 - Just two dimension-five operators in "square root" of OPE, $\mathbf{XX}\mathbf{B}_{n}\mathbf{B}_{\underline{n}}$ and $\mathbf{X}^{C}\mathbf{X}^{D}\,\mathbf{B}_{\underline{n}}{}^{C}\mathbf{B}_{\underline{n}}{}^{D}$ and
 - Their coefficients equal and opposite to cancel, e.g.

$$X^3X^3 \rightarrow W^3W^3$$



· Squaring in OPE gives trivial color contraction, so

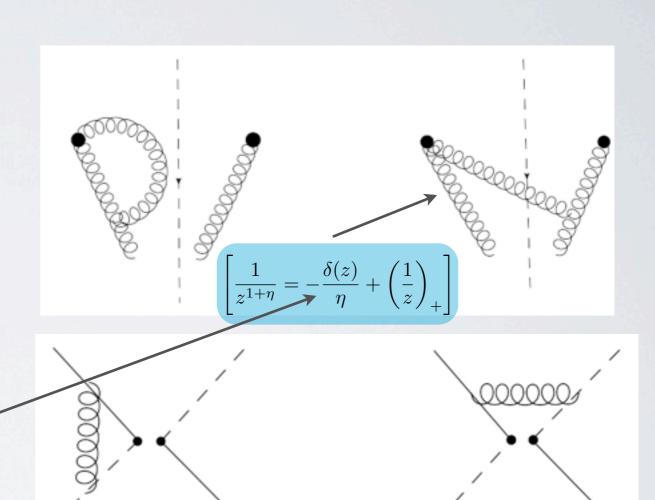
$$C_1 \equiv C \quad C_2 = -2C$$

$$C_3 = 0 \quad C_4 = C$$

$$C(M_{\chi}) = \frac{\pi \alpha_W^2 \sin^2 \theta_W}{2M_{\chi}^3}$$

ANOMALOUS DIMENSIONS

- Compute soft & collinear anomalous dimensions separately
- Real process only contributes double log for zero-energy emission
- Thus, real & virtual corrections to xx→y+X will appear as loops



Above: Collinear contributions O_c^a (real & virtual)

Below: Interactions between

soft Wilson lines O_s^a (solid - timelike; dashed - lightlike) $O_{1,3}$ have trivial color structure and thus 0-anomalous dimension

ANOMALOUS DIMENSION RESULTS

 For the collinear and soft sectors of our operators, (a nonsinglet and b - singlet)

$$\begin{split} \gamma^c_{aa} &= \frac{3g^2}{4\pi^2}\log(\frac{\nu^2}{4M_\chi^2}), \quad \gamma^s_{aa} = \frac{-3g^2}{4\pi^2}\log(\frac{\nu^2}{\mu^2}), \quad \text{Note } \mathbf{v} \\ \gamma^c_{ba} &= \frac{-g^2}{4\pi^2}\log(\frac{\nu^2}{4M_\chi^2}), \quad \gamma^s_{ba} = \frac{g^2}{4\pi^2}\log(\frac{\nu^2}{\mu^2}) \end{split}$$

 Resum logs from Renormalization Group equation (2,4 nonsinglet and 1,3 - singlet)

$$\mu \frac{d}{d\mu} C_{2,4}(\mu) = -(\gamma_{aa}^c + \gamma_{aa}^s) C_{2,4}$$
$$\mu \frac{d}{d\mu} C_{1,3}(\mu) = -(\gamma_{ba}^c + \gamma_{ba}^s) C_{2,4}.$$

Logs minimized for (μ, ν) soft: (m_W, m_W) collinear: (M_X, m_W)

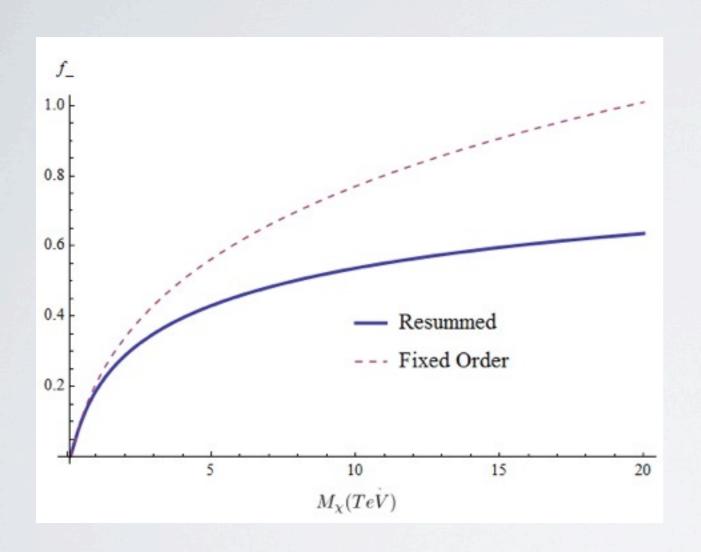
TOTAL RATE

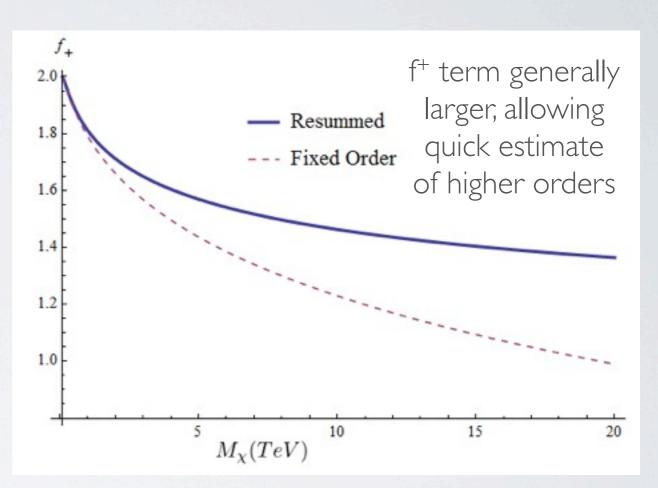
$$\frac{1}{E_{\gamma}} \frac{d\sigma}{dE_{\gamma}} = \frac{C_1(\mu = E_{\gamma})}{4M_{\chi}^2 v} \delta(E_{\gamma} - M_{\chi}) \left[\frac{2}{3} f_{-} |\psi_{00}(0)|^2 + 2f_{+} |\psi_{+-}(0)|^2 + \frac{2}{3} f_{-} (\psi_{00} \psi_{+-} + \text{h.c.}) \right]$$

- The ψ -factors quantify the Sommerfeld enhancement for annihilating state
- The f-factors arise from running Wilson coefficients and resum Sudakov double-logs

$$f_{\pm} \equiv 1 \pm \exp\left[-\frac{3\alpha_W}{\pi}\log^2\left(\frac{M_W}{E_{\gamma}}\right)\right]$$

RESUMMATION

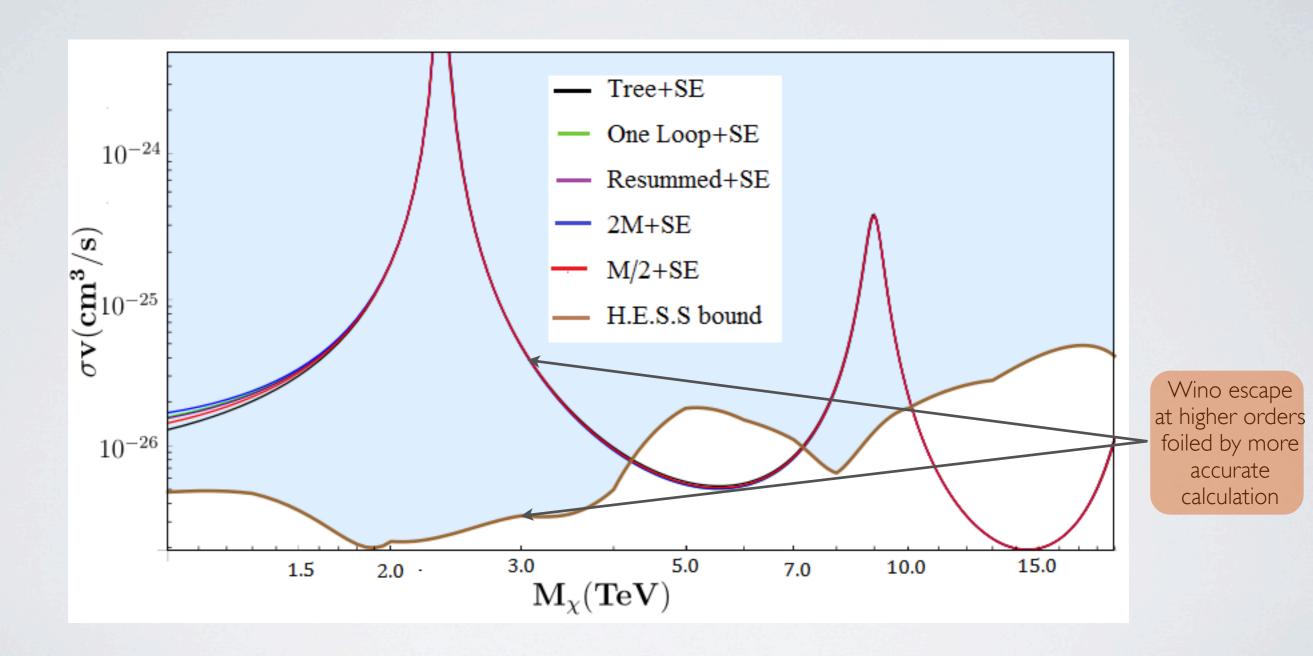




Sudakov factor vs. Dark Matter mass Resummation is modest ~5% affect for thermal Wino (3 TeV)

More important effect that we looked at semi-inclusive instead of two-body annihilation

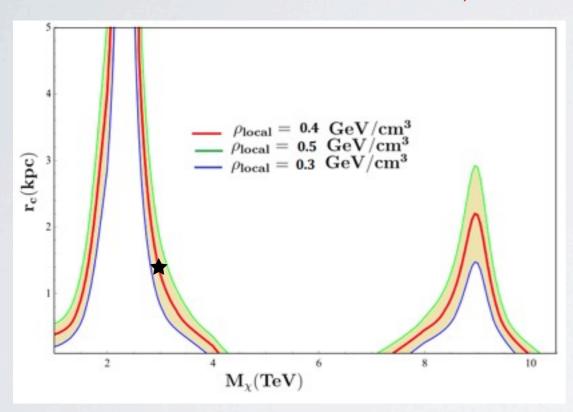
TOTAL RATE & EXCLUSION



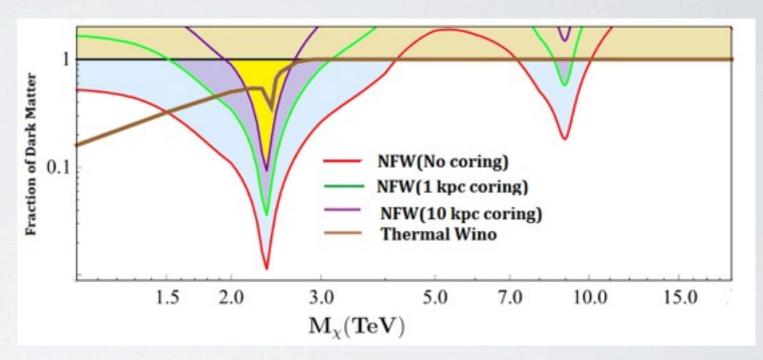
Exclusion curve taken from Ovanesyan, Slatyer, and Stewart (1409.8294); HESS data with NFW profile

WINOVIABILITY

Initial motivation for Wino stressed its simplicity, but perhaps its role in Dark Matter is more involved, including a non-thermal history, multi-component DM, mixing with other EW states



Imposing a constant density below a given radius for NFW (core), at what point does wino become viable total DM?



Possibility for the wino to make up some fraction of DM with NFW profile flattened to a constant core at some radius

CONCLUSION

- When computing observationally relevant semi-inclusive rate, NLO effects are small (~few%)
- Semi-Inclusive \rightarrow Leading Log operator mixing \rightarrow $1 \pm Exp[-Log^2]$.
- Viability of thermal wino dark matter requires profile in tension with simulation (core ~ 1.5 kpc)
- Discovery of wino would have huge implications for astrophysical Dark Matter

SUSY & DARK MATTER

- The Minimal Supersymmetric Standard Model (MSSM) comes with several natural WIMP dark matter candidates
 - Bino: Generic LSP in many SUSY-breaking scenarios. Overcloses universe unless nearly degenerate slepton provides co-annihilation (Δ M/M \sim 5%)
 - Wino: Generic LSP in Anomaly Mediated Supersymmetry Breaking (AMSB)
 - Higgsino: Pure Higgsino ruled out by direct detection, but bino/wino admixture possible*
 - · Sneutrino: Ruled out by direct detection, but could be possible in extension
 - Gravitino: LSP of low-scale Gauge Mediation, very light O(100 keV), with overclosure and Big Bang Nucleosynthesis problems

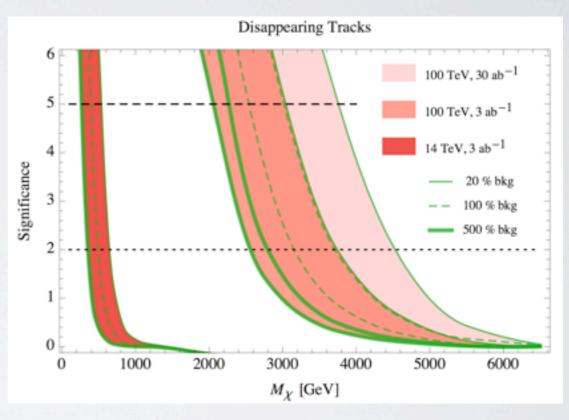
TO FIND ATEV WINO

 LHC: Small mass splitting between \mathbf{X}^{\pm} and \mathbf{X}^{0} (Δ M=165 MeV) makes disappearing track O(10cm) Current limit M_x>270 GeV χ_1^{\pm} decaying into $\chi_1^{\vee} + \pi^{\pm}$ high-p_T charged particle interacting with TRT material low-p_T charged particle scattered in materials resulting in badly measured track p_T reconstructed track

TRT

Pixel

SCT



From Cirelli et al. 1407.7058

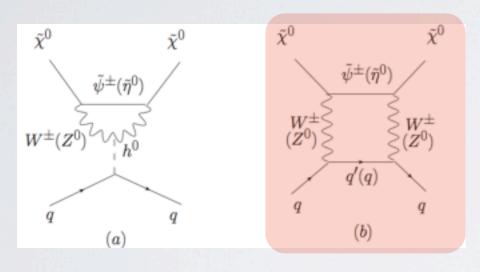
1202.4847 (ATLAS)

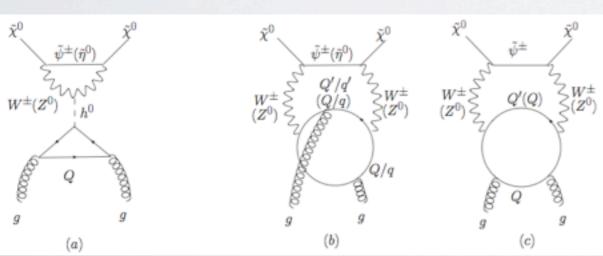
24

true particle track

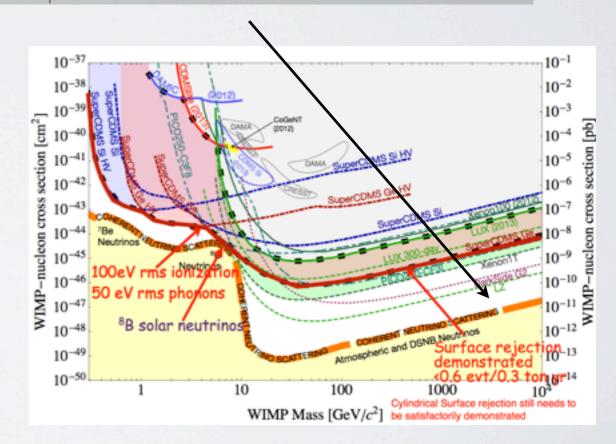
TO FIND ATEV WINO

Direct Detection: Wino has no renormalizable couplings to Z-boson or Higgs Cross section proceeds through loops, which partially cancel for $m_h = 125$ GeV





Highlighted diagram gives negative contribution, all others positive, $\sigma \sim 10^{-47}$ cm²



WINOS BEYOND PERTURBATION THEORY*

- There are reasons to think wino annihilation in the presentday may receive large higher-order corrections:
 - Slowly-moving (v~10⁻³) winos' annihilation rate subject to **Sommerfeld Enhancement**. Higher perturbative order brings in new channel.
 - Sudakov double-logarithms: Large in electroweak physics, even in inclusive observables. Can't trust fixed order. We must resum logs.

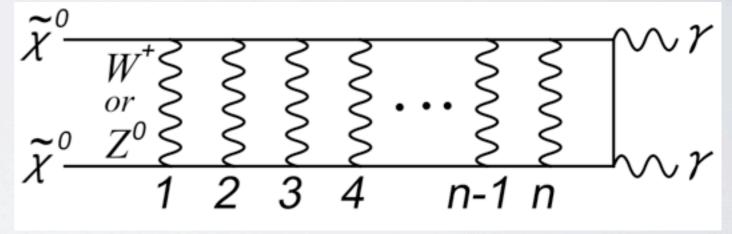
*MB, Rothstein, I and Vaidya, V: 1409.4415 & 1412.8698

SOMMERFELD & FIELD THFORY

Corrections at $O(v^2)$ to potential and $O(\alpha)$ from running couplings

- In nonrelativistic regime, summing infinite ladder exchange → solving Schrodinger equation for appropriate potential
- W-exchanges flip between $\mathbf{X}^0\mathbf{X}^0$ and $\mathbf{X}^+\mathbf{X}^-$ states
- Ladder exchanges unsuppressed

$$\mathcal{A}_n \simeq \alpha \left(\frac{\alpha_2 M}{m_W}\right)^n$$



From Hisano et al. hep-ph/0412403, see also Arkani-Hamed et al.: 0810.0713, Slatyer: 0910.5713

ELECTROWEAK POTENTIAL*

• Potential accounts for Yukawa exchange of Ws, Zs, Coulomb exchange of γ s, and χ^+ - χ^0 mass splitting = (0.17 GeV)

$$\begin{pmatrix}
2\delta M - \frac{\alpha}{r} - \alpha_W c_W^2 \frac{e^{-m_Z r}}{r} & -\sqrt{2}\alpha_W \frac{e^{-m_W r}}{r} \\
-\sqrt{2}\alpha_W \frac{e^{-m_W r}}{r} & 0
\end{pmatrix}$$

Mass splitting means +- state decays exponentially

• Between the two-body |So states with interpolating fields:

$$\frac{1}{\sqrt{2}}\chi^{+}(\vec{x}-\vec{r}/2)\chi^{-}(\vec{x}-\vec{r}/2) \qquad \frac{1}{2}\chi^{0}(\vec{x}-\vec{r}/2)\chi^{0}(\vec{x}-\vec{r}/2)$$

*See also Hisano et al.: hep-ph/04 | 2403

BLOCH-NORDSIECK THEOREM VIOLATION*

• Electroweak physics has infrared divergences, even in fully inclusive observables

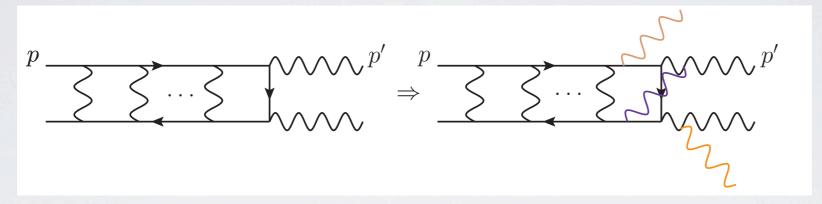


σ_{AB} ≠ σ_{AD}, virtual corrections only cancel emission upon color averaging

- We avoid pathology because
 - QED: Emission doesn't change particle identity → real/virtual cancellation → Bloch-Nordsieck
 - QCD: Singlets let us average over initial colors, factorization isolates IR sensitivity (e.g. PDFs)
 - Electroweak: Gauge boson masses cut off divergence, but allow for log(Q²/m_W²)²

SUDAKOV LOGARITHMS

• Emission of soft or collinear radiation can lead to infrared divergences



 Accounting for extra emission enters multiplicatively as Sudakov double logarithm

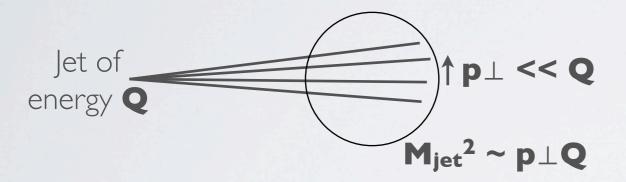
$$d\sigma(p \to p' + \gamma) = d\sigma(p \to p') \frac{\alpha}{\pi} \log(Q^2/\mu^2) \log(Q^2/m^2)$$

• For wino annihilation at the thermal mass (3 TeV), another challenge for perturbation theory $\frac{\alpha_W}{\pi} \log(M_{\rm wino}^2/m_W^2)^2 \approx 0.6$

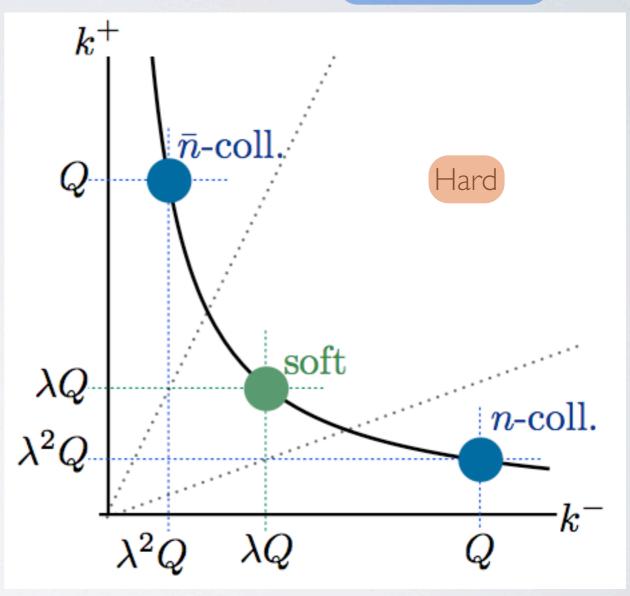
SOFT-COLLINEAR EFFECTIVE THEORY

Lightcone momenta $k^+ = k^0 + k^3$ $k^- = k^0 - k^3$

 Large scale-hierarchies can arise within one field



 We can use Renormalization Group to resum kinematic logs



Integrate out hard gauge boson modes, but keep those collinear to null directions and soft fields

METHOD OF DESCENT

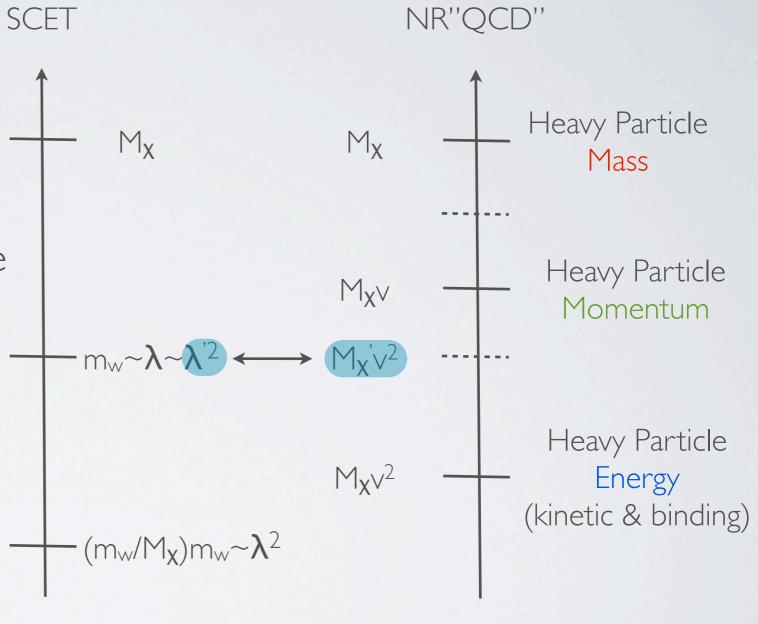
 Interactions in full theory couple SCET soft modes to WIMPs, collinear Ws, and NR ultrasofts

• Instead of matching directly, we raise M_X to fictitious value, $M_X^{'}$

• On SCET side, we work in SCET-I with $\lambda' = (m_w/M_X)^{1/2}$

• After field redefinition, we send M_X back to its value of interest, and lower to SCET-2 with $\lambda = m_w/M_X$

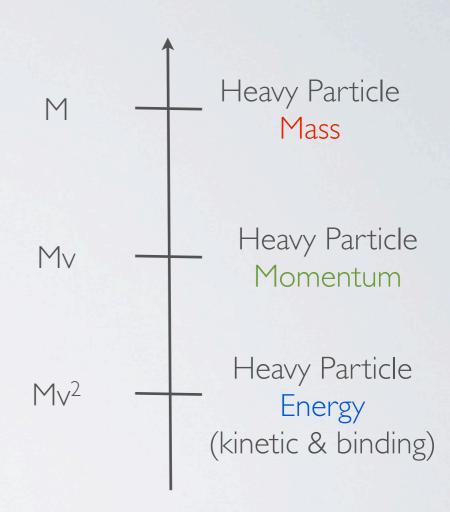
$$\chi \to S_v \chi$$
 $B \to S_n B$



*Original development for SCET-I to SCET-II matching hep-ph/02 I 1069: Bauer, Pirjol, Stewart

NR"QCD" FOR WIMPS

- Nonrelativistic EFTs handle separation of scales in bound-state problems*
- Unsuppressed ladder-exchange of potential bosons (E~Mv², p~Mv) → Sommerfeld
- WIMPs heavy, but Renormalization Group resums soft-scale logs



Hierarchy of scales for nonrelativistic particles interacting with potential

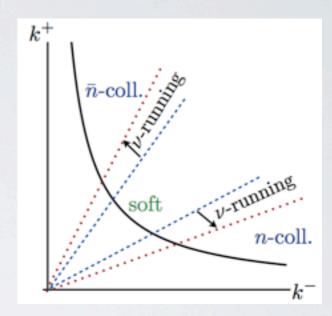
RAPIDITY RG

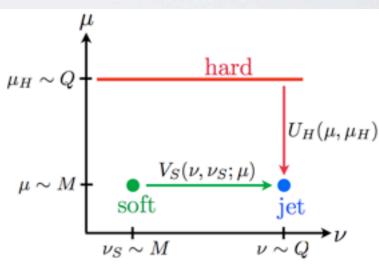
SCET is a "modal" theory

$$A_{\mu} = A_{\mu}^{c,n} + A_{\mu}^{c,\bar{n}} + A_{\mu}^{soft} + \dots$$

- We can get divergences when integrals invade other sectors.
 Soft-collinear overlap requires boost-violating regulator
- Regulating sets up RG for resumming these rapidity logs

$$W_n = \sum_{\text{perms}} \exp \left[-\frac{g}{\bar{n} \cdot P} \frac{\nu^{\eta}}{|\bar{n} \cdot P|^{\eta}} \bar{n} \cdot A_n \right]$$





From Chiu et al. 1202.0814: In SCETII, soft and collinear modes have same virtuality v-running lets us minimize log between soft & collinear scales

WILSON LINES, GAUGE INVARIANCE, SOFTS

 SCET has rich gauge structure (separate for soft and collinear sectors) and collinear fields contain implicit collinear Wilson lines

$$B_{\mu}^{a\perp} \equiv f^{abc} W_n^T (D_{\mu}^{\perp})^{bc} W_n$$

We have soft Wilson lines for both WIMP and collinear fields

$$S_{(v,n)} = P[e^{ig \int_{-\infty}^{0} (v,n) \cdot A((v,n)\lambda) d\lambda}]$$

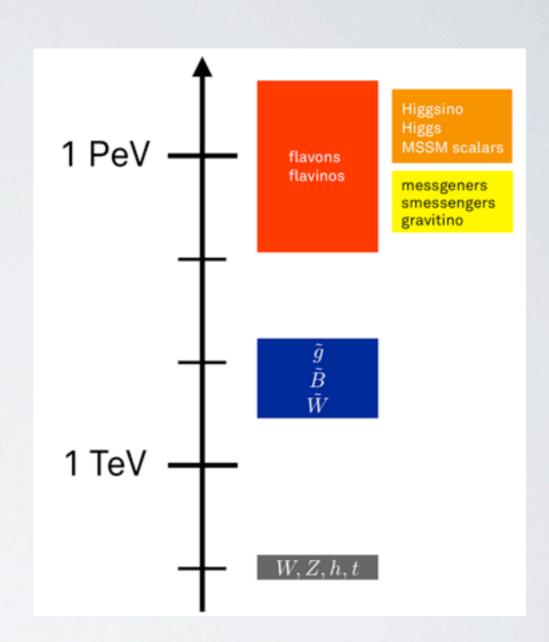
Soft&Collinear-gauge-boson structure is therefore

$$O_s^a = S_{vA'A}^T S_{vBB'} S_{n\tilde{A}A}^T S_{nB\tilde{B}} \qquad O_s^b = \mathbf{1} \, \delta_{\tilde{A}\tilde{B}} \delta_{A'B'}$$

$$O_c^a = B_{\tilde{A}}^{\perp} B_{\tilde{B}}^{\perp} \qquad O_c^b = B^{\perp} \cdot B^{\perp} \delta_{\tilde{A}\tilde{B}}$$

POST-HIGGS SUSY

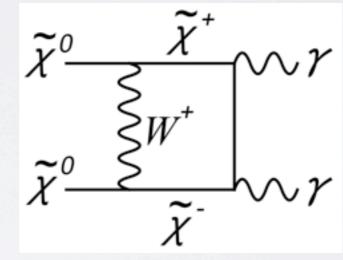
- For all its virtues, SUSY is famously unobserved
- Higgs + nothing else means?
 - SUSY to eliminate most fine-tuning? 10⁻⁸ instead of 10⁻³²
 - Higgs mass (125 GeV > m_Z) and flavor physics point to sfermions at 100-1000 TeV
 - Simpler SUSY model building (Gravity +Anomaly mediation)
 - Gauginos (w/ wino LSP) at right scale for WIMP Miracle



From 1403.6118: MB, Stolarski, and Zorawski Mini-split SUSY allows a radiative generation of flavor hierarchies and allows for thermal WIMP-DM

CHARGED & NEUTRAL STATES

 Perturbative annihilation proceeds through either charged (tree) or neutral (one-loop) channels



$$\sigma_{\text{one-loop}}v = \frac{4\pi\alpha^2\alpha_W^2}{m_W^2}$$

• For asymptotic neutral state (Dark Matter), we compute amplitude for charged or neutral state at origin

$$\psi_{00} = \langle 0|\bar{\chi}^0 \gamma^5 \chi^0 | \chi^0 \chi^0 \rangle_S$$

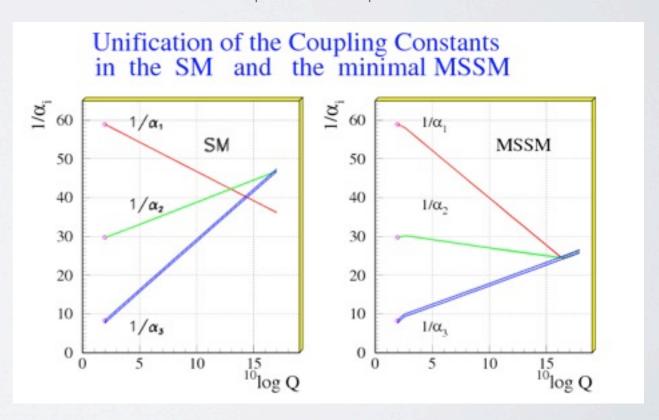
$$\psi_{\pm} = \langle 0|\bar{\chi}^+ \gamma^5 \chi^+ | \chi^0 \chi^0 \rangle_S$$

SUPERSYMMETRY

- Supersymmetry: Unique extension of global spacetime symmetries of special relativity*
- Posits bosonic partner for all fermions with same quantum numbers (and vice versa)
- Discrete symmetry to protect proton (R-parity) provides a new stable particle (LSP)
- Connecting SUSY to weak scale thus gives natural WIMP dark matter



SUSY ties superpartner masses and weak scale in most of its parameter space



Strongest indirect evidence for weak-scale SUSY possibly the unification of gauge couplings

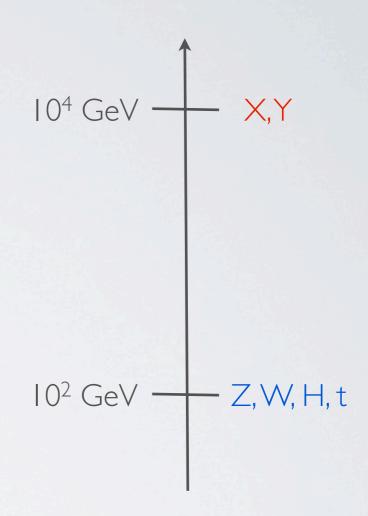
EFFECTIVE FIELD THEORY

- Stuck with large log, we must resum $\alpha \log^2$ to $\exp[C \alpha \log^2]$
- Essential tool of the modern, phenomenological field theorist
- Systematically decouple high-energy degrees of freedom to answer low-energy questions as in Wilsonian-RG

$$Z[\phi]_{\Lambda-\Delta\Lambda} = \int_{k \in (\Lambda-\Delta\Lambda,\Lambda)} \mathcal{D}\phi \exp[iS(\phi)]$$

• Match amplitudes as expansion in $1/M_X^2$, α

Resum logs log(Mx/mt) by running in EFT



Schematic for simple EFT, where X,Y integrated out to generate new interactions for SM fields

