Fractional Jet Multiplicity

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Outline

- New observable: fractional jet multiplicity
- This talk: discuss analytic predictions and unique features
  [DB, J.Thaler, and J.Walsh 1501.01965]

Fractional Jet Multiplicity

definition

$$\widetilde{N}_{\text{jet}} = \sum_{i \in \text{event}} \frac{E_i}{E_{iR}} \, \Theta(E_{iR} > E_{\text{cut}})$$

$$E_{iR} = \sum_j E_j \, \Theta(\Delta\theta_{ij} < R)$$

= energy in a cone around particle i

Fractional Jet Multiplicity

definition

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$$E_{iR} = \sum_j E_j \, \Theta(\Delta\theta_{ij} < R)$$

= energy in a cone around particle i

Each particle in the event can contribute to jet multiplicity

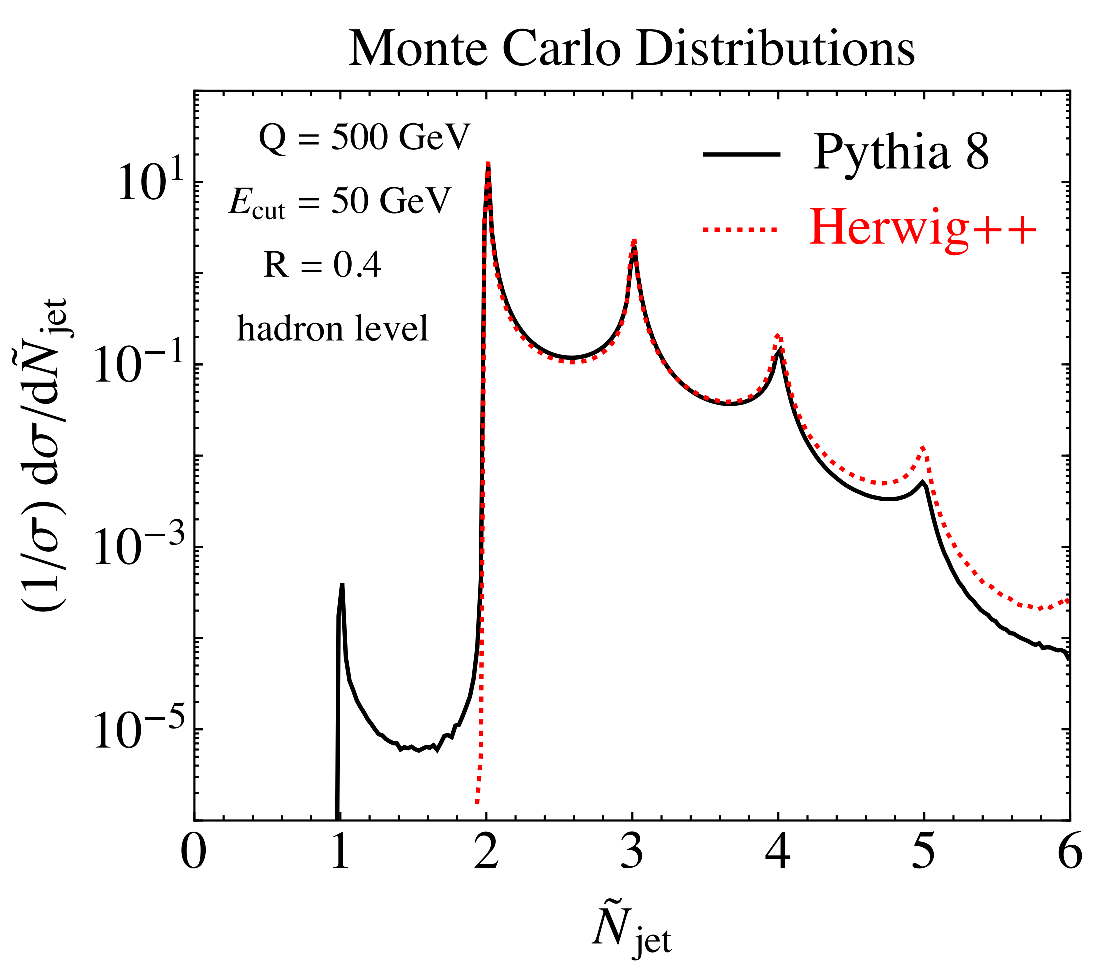
Apparent over-counting is fixed by the weight E_i/E_{iR}

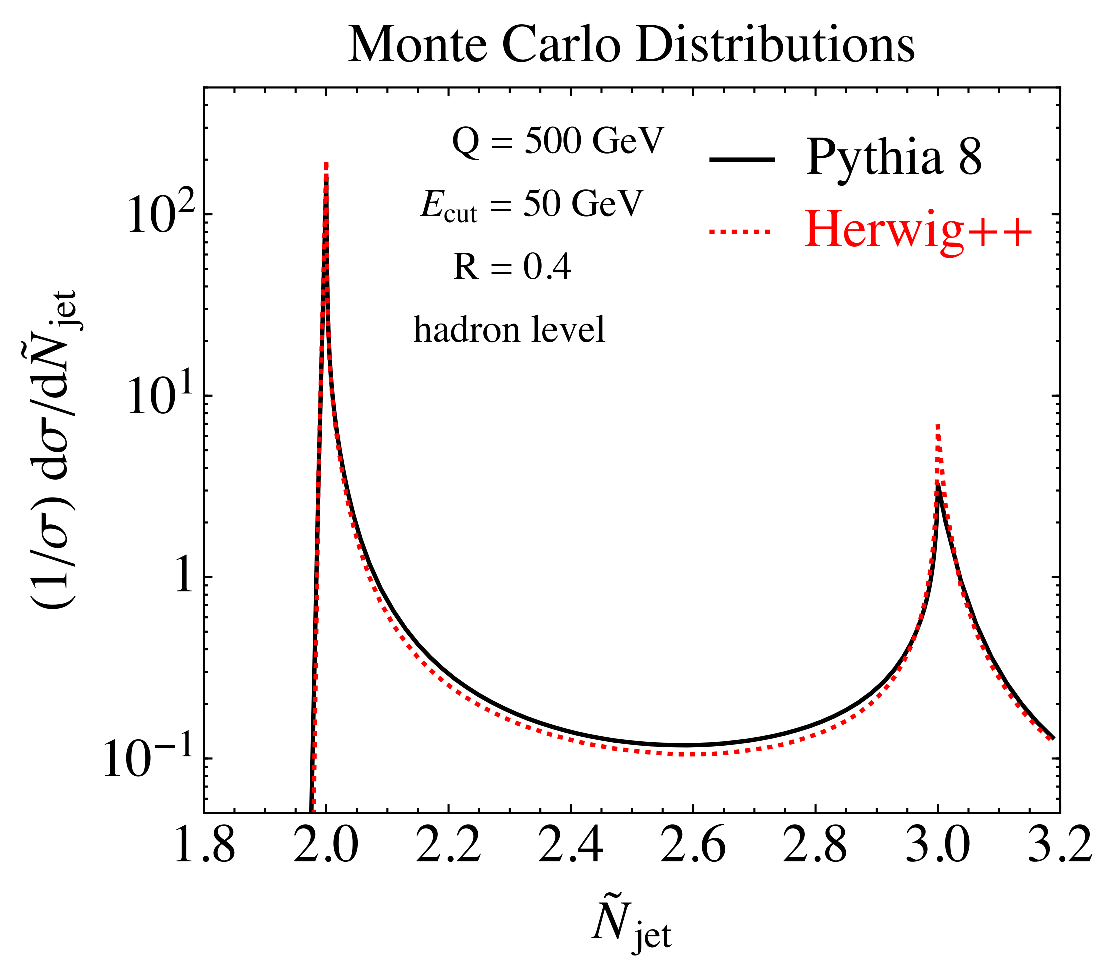
[Jets-Without-Jets
DB, T.Chan, and J.Thaler 1310.7584]

Fractional Jet Multiplicity

definition

Example of distributions in $e^+e^- \to \text{jets}$

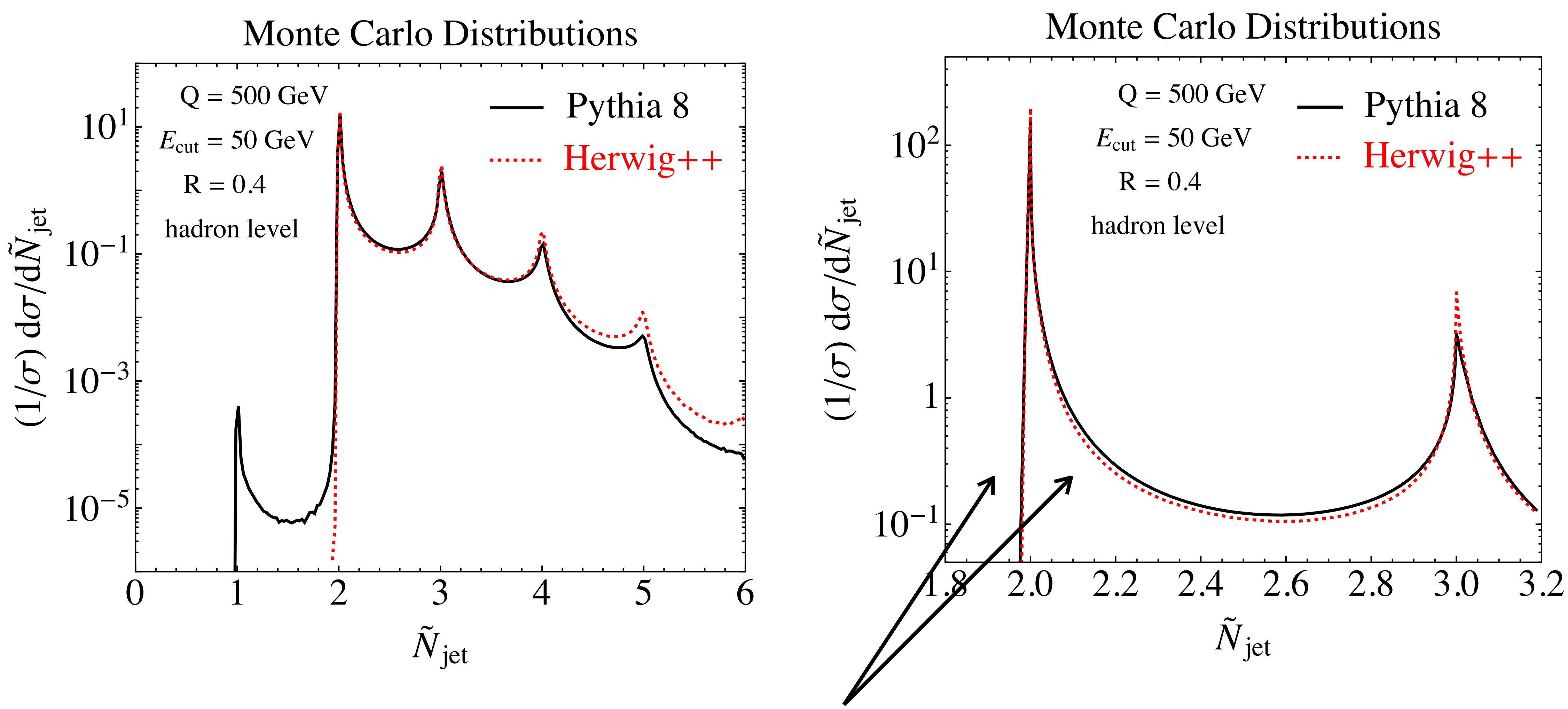




Fractional Jet Multiplicity

definition

Example of distributions in $e^+e^- \to \text{jets}$

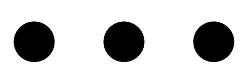


I will describe analytically the near-integer distribution

Fractional Jet Multiplicity

definition

Collimated particles give integer number of jets



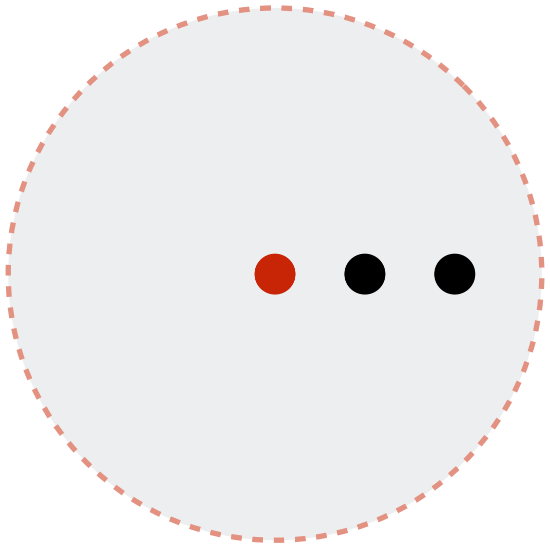
Consider three hard particles

$E_1=E_2=E_3>E_{cut}$

Fractional Jet Multiplicity

definition

Collimated particles give integer number of jets



Consider three hard particles
$E_1 = E_2 = E_3 > E_{cut}$

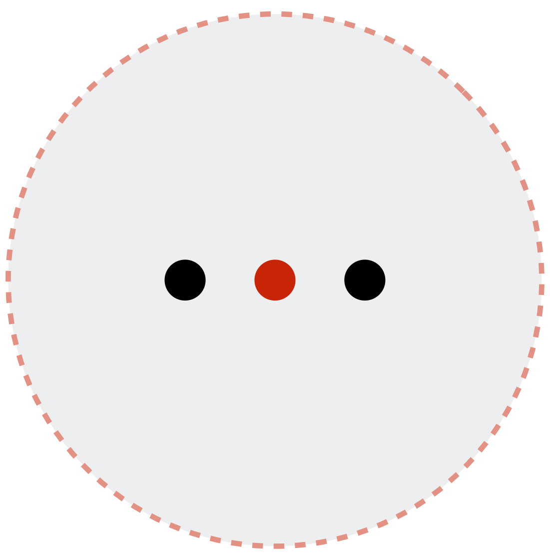
$$\widetilde{N}_{jet} = \Sigma_i \, E_i \,/\, E_{iR}$$

$$= 1/3$$

Fractional Jet Multiplicity

definition

Collimated particles give integer number of jets



Consider three hard particles
$E_1=E_2=E_3>E_{cut}$

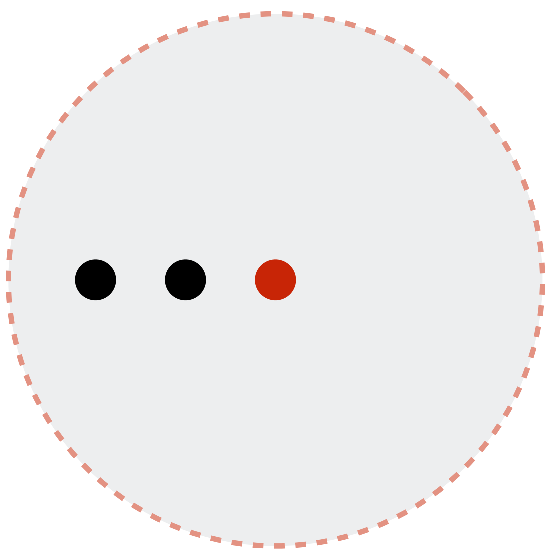
$$\tilde{N}_{jet} = \Sigma_i \; E_i \,/\, E_{iR}$$

$$= 1/3+1/3$$

Fractional Jet Multiplicity

definition

Collimated particles give integer number of jets



$\tilde{N}_{jet} = \Sigma_i E_i / E_{iR}$

$= 1/3 + 1/3 + 1/3 = 1$

Consider three hard particles
$E_1 = E_2 = E_3 > E_{cut}$

Fractional Jet Multiplicity

definition

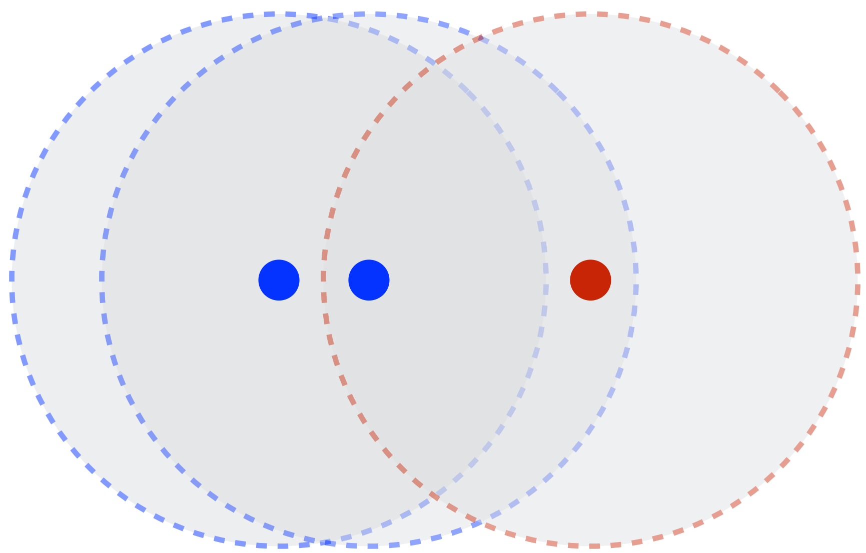
Less collimated particles give fractional number of jets

Consider three hard particles
$E_1=E_2=E_3>E_{cut}$

Fractional Jet Multiplicity

definition

Less collimated particles give fractional number of jets



Consider three hard particles
$E_1=E_2=E_3>E_{cut}$

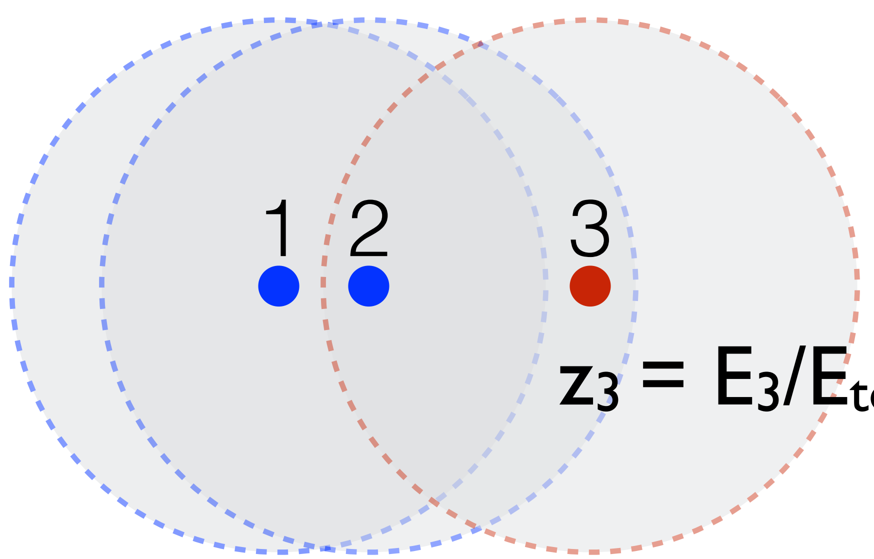
$$\widetilde{N}_{jet} = \Sigma_i \, E_i \,/\, E_{iR}$$

$$= 1/2+1/3+1/2=4/3$$

Fractional Jet Multiplicity

definition

Soft limit drives the approach to the integer value



$z_3 = E_3/E_{tot} = 1/3$

$\widetilde{N}_{jet} = \Sigma_i \, E_i \, / \, E_{iR}$

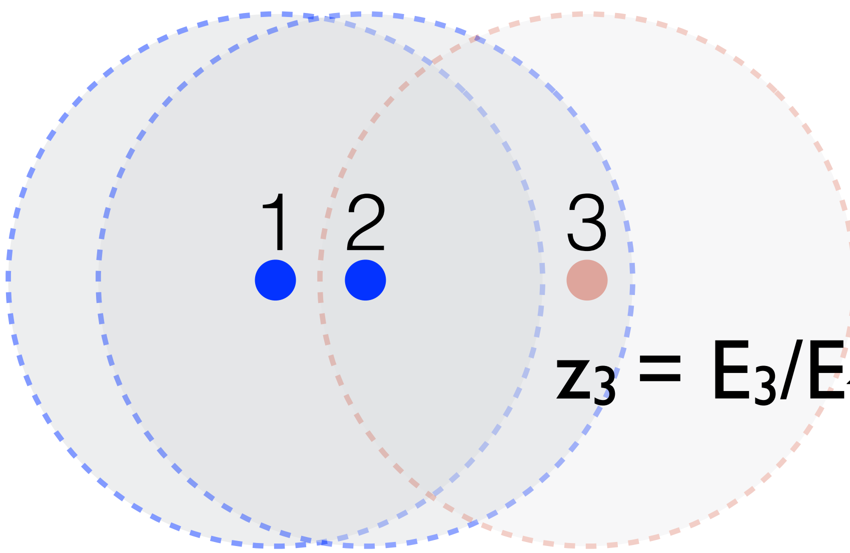
$= 4/3 \simeq 1.3$

Consider now one particle going soft
$E_1 = E_2 > E_{cut}$ and $E_3 \to 0$

Fractional Jet Multiplicity

definition

Soft limit drives the approach to the integer value



$z_3 = E_3/E_{tot} = 1/9$

$\tilde{N}_{jet} = \Sigma_i\, E_i / E_{iR}$

$\simeq 1.1$

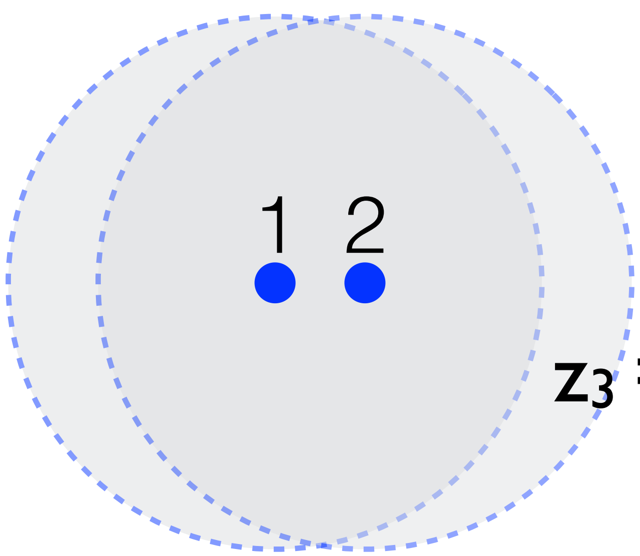
Consider now one particle going soft

$E_1=E_2>E_{cut}$ and $E_3 \to 0$

Fractional Jet Multiplicity

definition

Soft limit drives the approach to the integer value



$z_3 = E_3/E_{tot} = 0$

$\widetilde{N}_{jet} = \Sigma_i E_i / E_{iR}$

$= 1$

Consider now one particle going soft
$E_1=E_2>E_{cut}$ and $E_3 \rightarrow 0$

Fractional Jet Multiplicity
definition

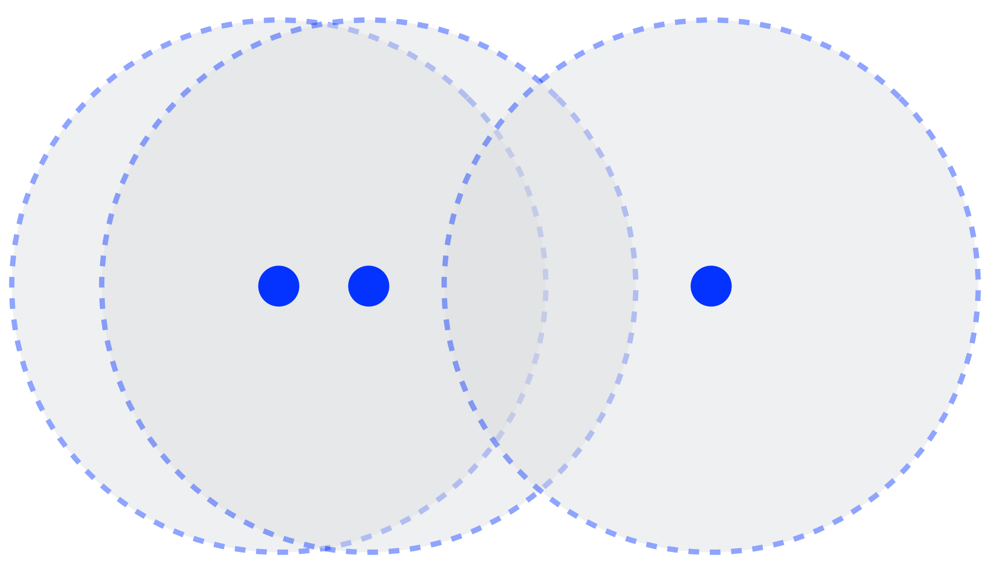
Well separated clusters of particles give integer number of jets

Consider three hard particles
$E_1=E_2=E_3>E_{cut}$

Fractional Jet Multiplicity

definition

Well separated clusters of particles give integer number of jets



Consider three hard particles
$E_1=E_2=E_3>E_{cut}$

$$\tilde{N}_{jet} = \Sigma_i \; E_i \,/\, E_{iR}$$

$$= 1/2+1/2+1=2$$

Fractional Jet Multiplicity

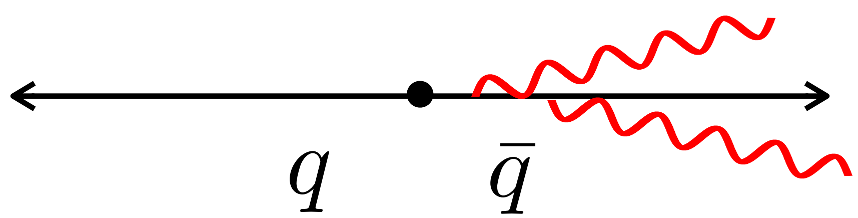
properties

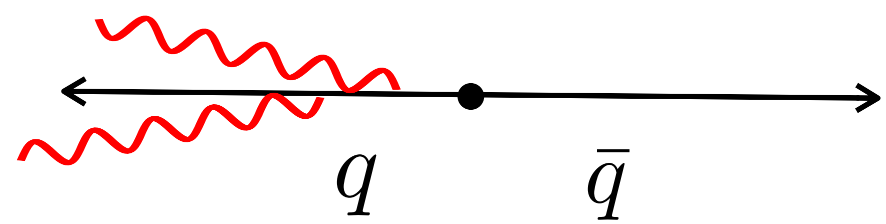
- To get non-integer behavior we need at least three particles.
  LO = $O(\alpha_s^2)$ in pQCD
- They have to be collimated (~ within 2R) and occupy special regions of phase space
- Near-integer behavior is driven by soft logarithms

Near-integer distribution

fixed order

Consider $e^+e^- \to \mathrm{jets}$





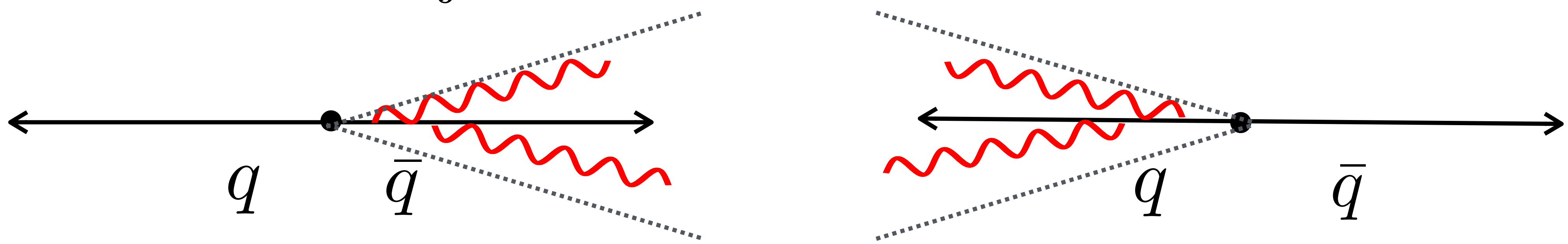
$$\Delta_{2-} = 2 - \widetilde{N}_{\mathrm{jet}}, \quad \Delta_{2+} = \widetilde{N}_{\mathrm{jet}} - 2, \quad \Delta_{3-} = 3 - \widetilde{N}_{\mathrm{jet}}$$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Delta_{2\pm}} = \int \mathrm{d}\Phi_4\, \mathcal{T}(e^+e^- \to 4 \;\; \mathrm{partons})\, \mathcal{F}(\Delta_{2\pm}, \Phi_4)$$

Near-integer distribution

fixed order

Consider $e^+e^- \to \text{jets}$



$$\Delta_{2-} = 2 - \widetilde{N}_{\text{jet}}, \quad \Delta_{2+} = \widetilde{N}_{\text{jet}} - 2, \quad \Delta_{3-} = 3 - \widetilde{N}_{\text{jet}}$$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Delta_{2\pm}} = \int \mathrm{d}\Phi_4 \, \underline{\mathcal{T}(e^+e^- \to 4 \;\; \text{partons})} \, \mathcal{F}(\Delta_{2\pm}, \Phi_4)$$

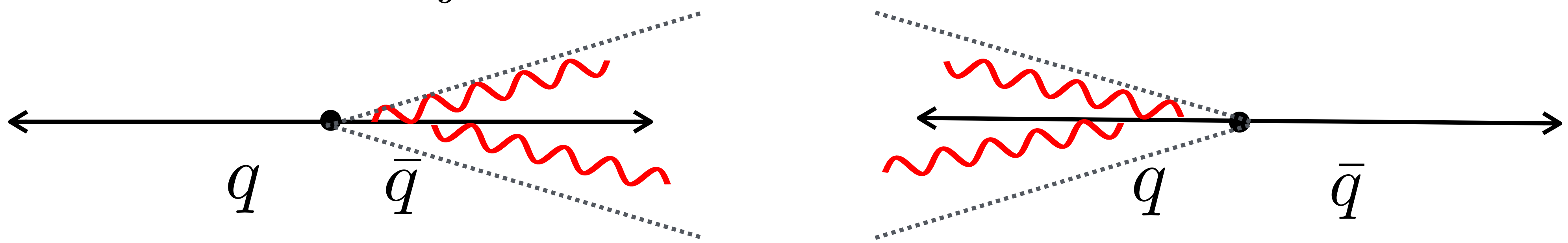
$$\mathcal{T}(e^+e^- \to 4 \;\; \text{partons}) \simeq \mathcal{T}(e^+e^- \to q\bar{q}) \cdot \sum_k \mathcal{T}_k^{\text{coll}}(1 \to 3)$$

$k \in \{q \to ggq \,, \bar{q} \to gg\bar{q} \,, q \to q'\bar{q}'q \,, \bar{q} \to q'\bar{q}'\bar{q}\}$

Near-integer distribution

fixed order

Consider $e^+e^- \to \text{jets}$



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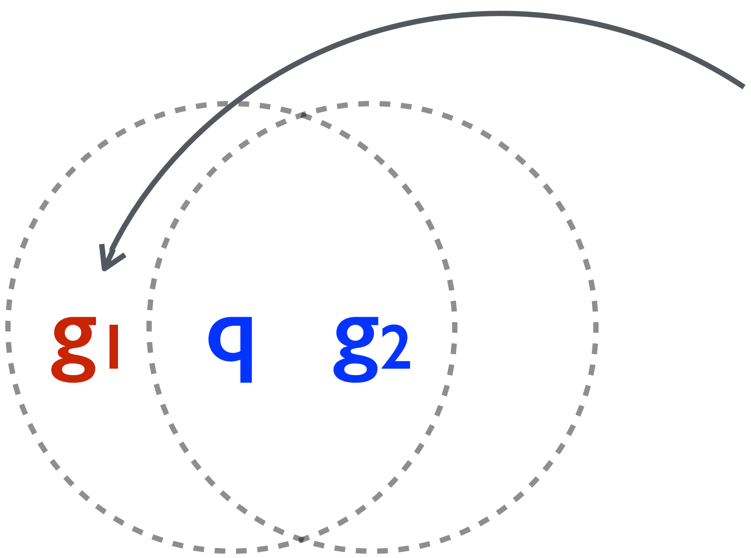
$$k \in \{q \to ggq \,, \bar{q} \to gg\bar{q} \,, q \to q'\bar{q}'q \,, \bar{q} \to q'\bar{q}'\bar{q}\}$$

measurement function

Near-integer distribution

soft logarithms and rapidity divergences

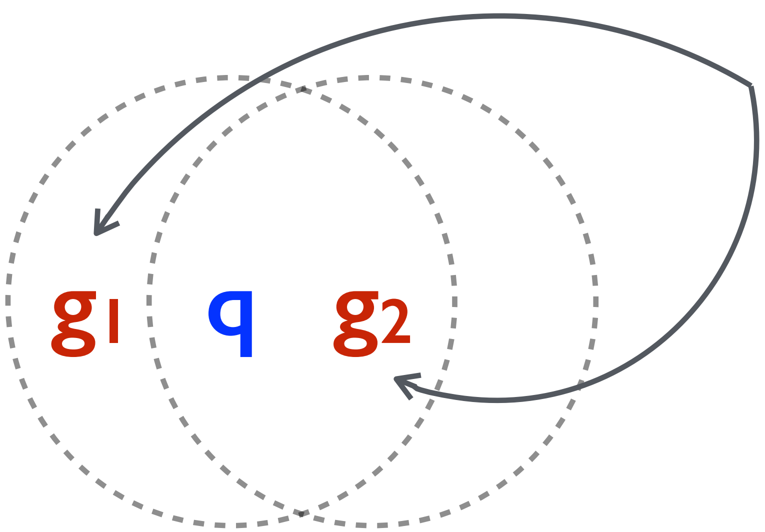


single soft limit

$$z_1 = E_{g_1}/E_{\text{tot}} \to 0$$

$$\Delta_{2+} \simeq z_1 f(z_2) \simeq \lambda^2, \quad (z_1, z_2) \simeq (\lambda^2, 1)$$

$$\mathrm{d}\sigma/\mathrm{d}\Delta_{2+} \simeq 1/\Delta_{2+}$$



double soft limit

$$z_{1,2} = E_{g_{1,2}}/E_{\text{tot}} \to 0$$

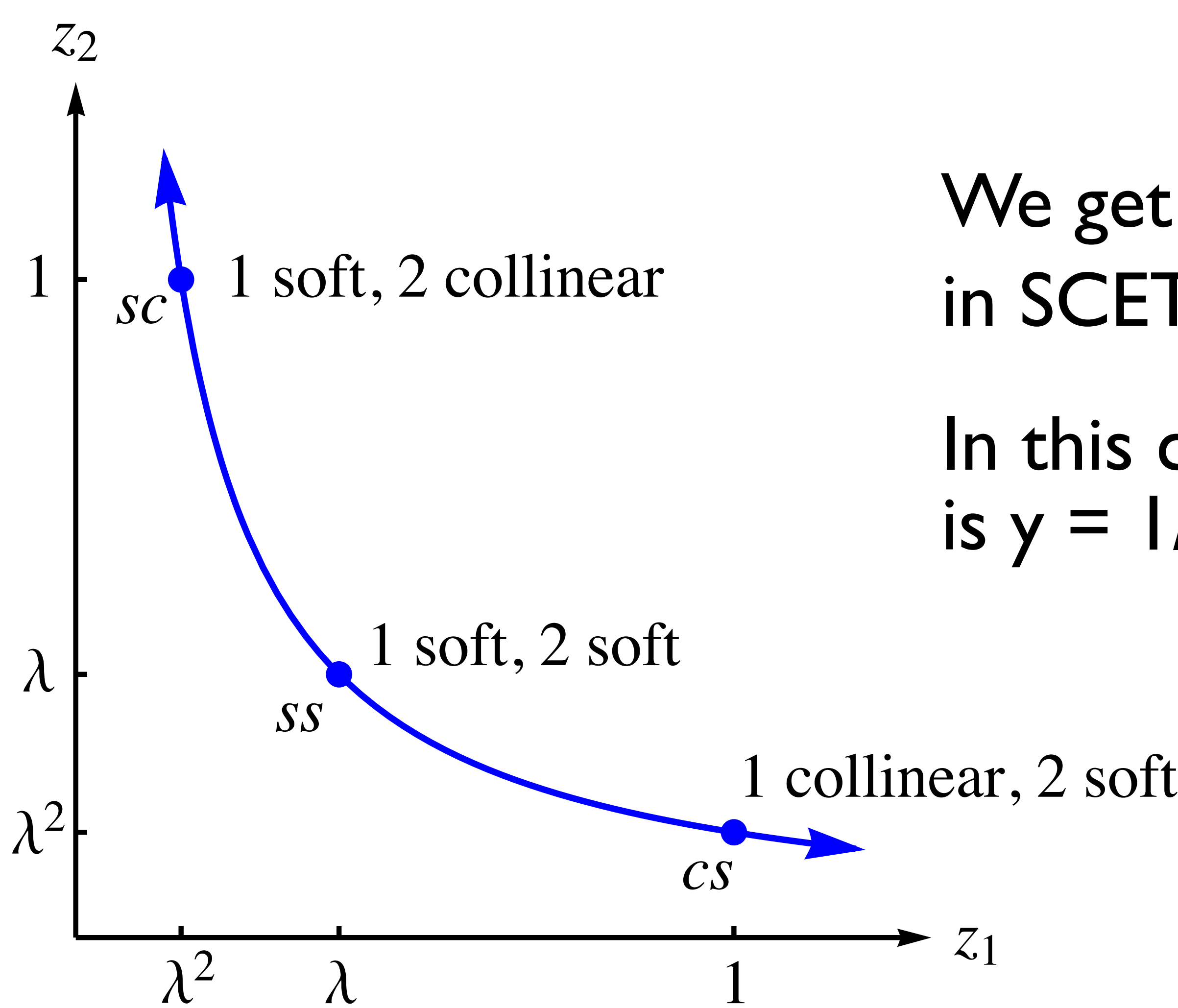
$$\Delta_{2+} \simeq 2 z_1 z_2 \simeq \lambda^2, \quad (z_1, z_2) \simeq (\lambda, \lambda)$$

$$\mathrm{d}\sigma/\mathrm{d}\Delta_{2+} \simeq \log \Delta_{2+}/\Delta_{2+} + 1/\Delta_{2+}$$

Near-integer distribution

soft logarithms and rapidity divergences

Modes controlling near-integer behavior



We get "rapidity-like" divergences like in $SCET_{II}$ and we use rapidity-regulators

In this case the rapidity-like variable is $y = 1/2 \log(z_1/z_2)$

Near-integer distribution

fixed order

Fixed order result

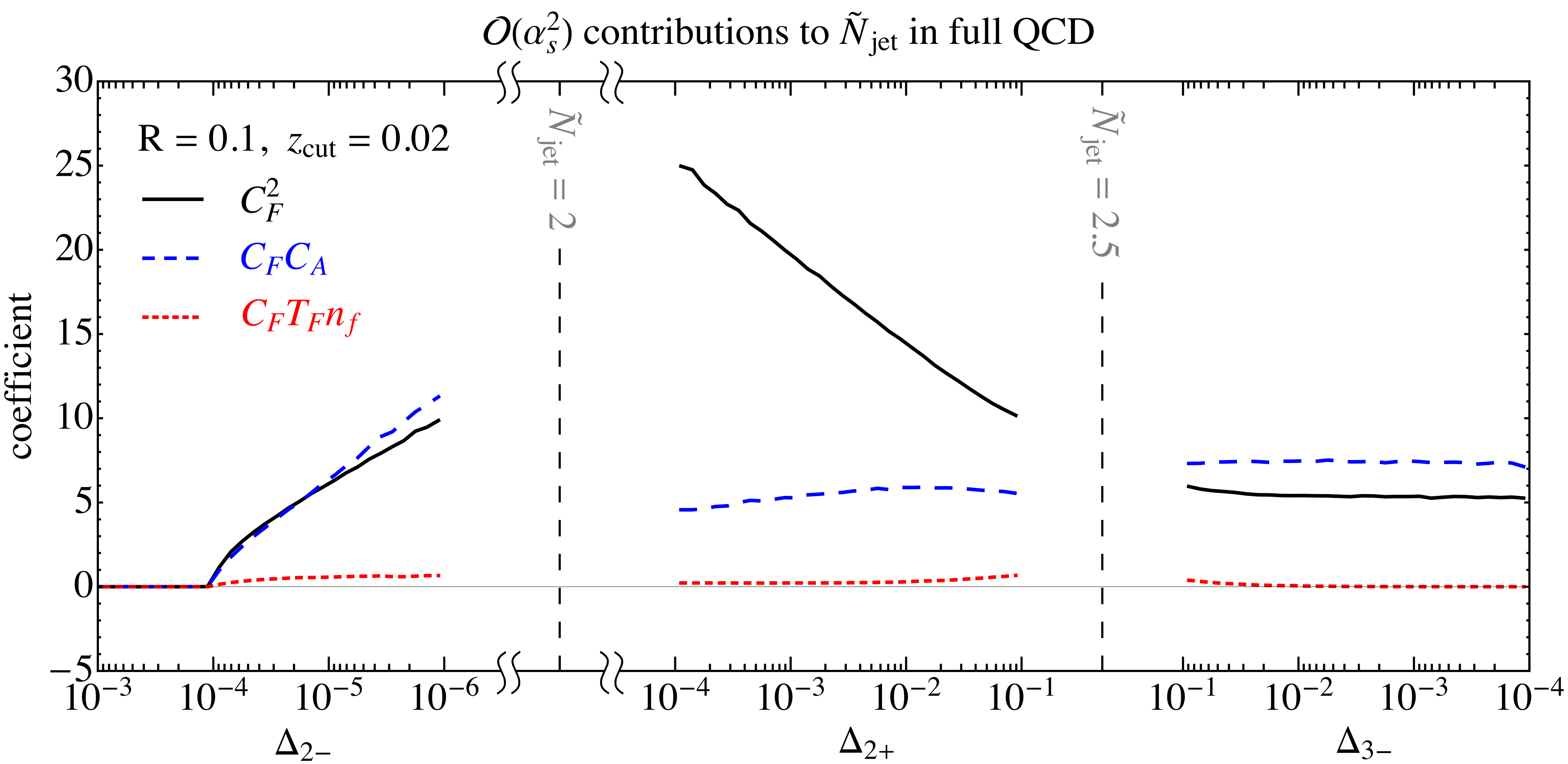
$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Delta_{2\pm}} = \sigma_0 \delta(\Delta_{2\pm})$$
$$+ \kappa_1 \left(\frac{\alpha_s}{\pi}\right)^2 \mathcal{L}_1(\Delta_{2\pm}) + \kappa_0 \left(\frac{\alpha_s}{\pi}\right)^2 \mathcal{L}_0(\Delta_{2\pm}) + \text{non-singular terms}$$

- We calculated k_1 and k_0 (which include different color structures and leading dependence on $z_{cut} = E_{cut} / E_{tot}$)
- Get non singular from Event2

Near-integer distribution

fixed order

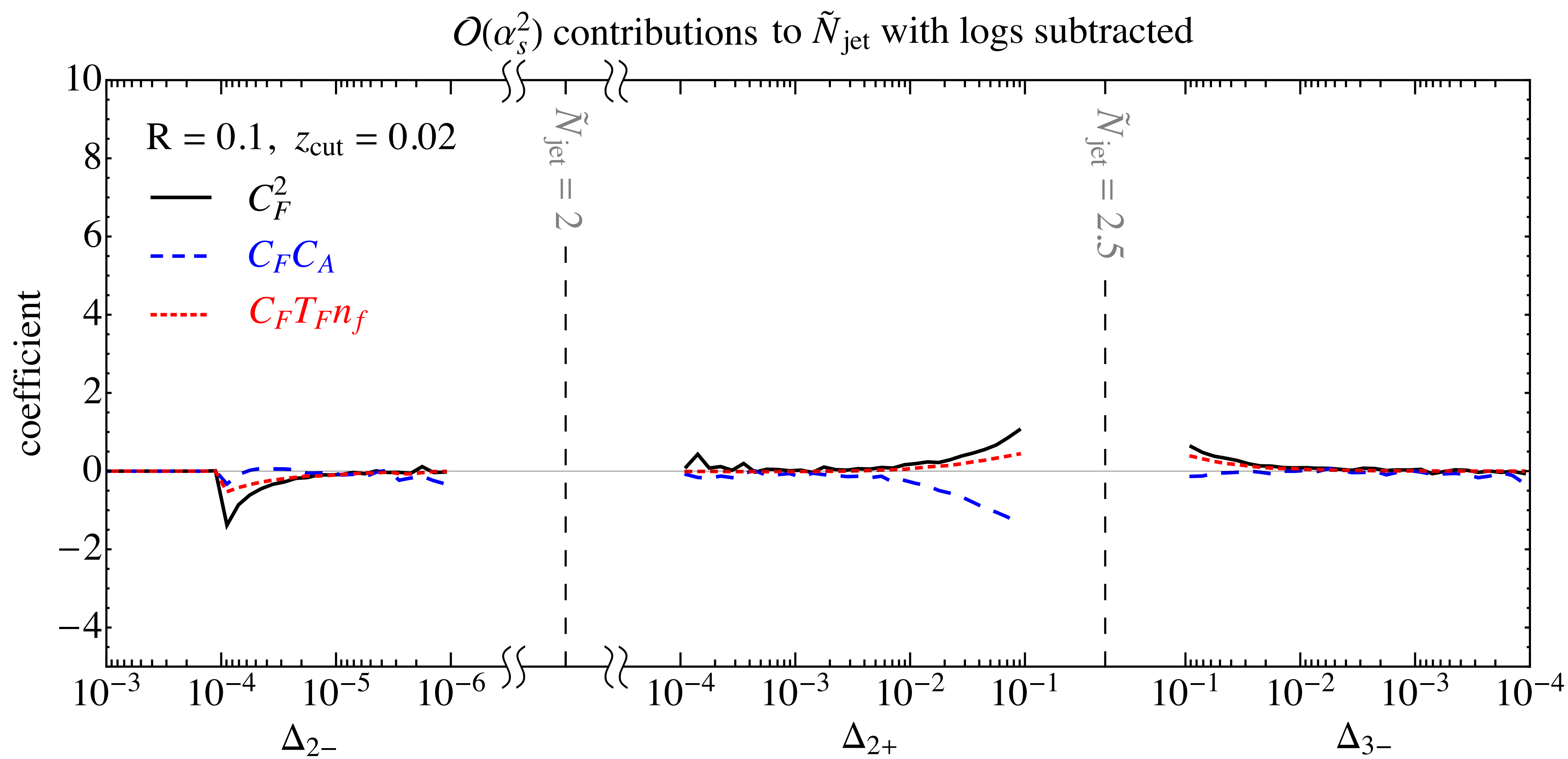
Event2 comparison:



Near-integer distribution

fixed order

Event2 comparison:



Near-integer distribution

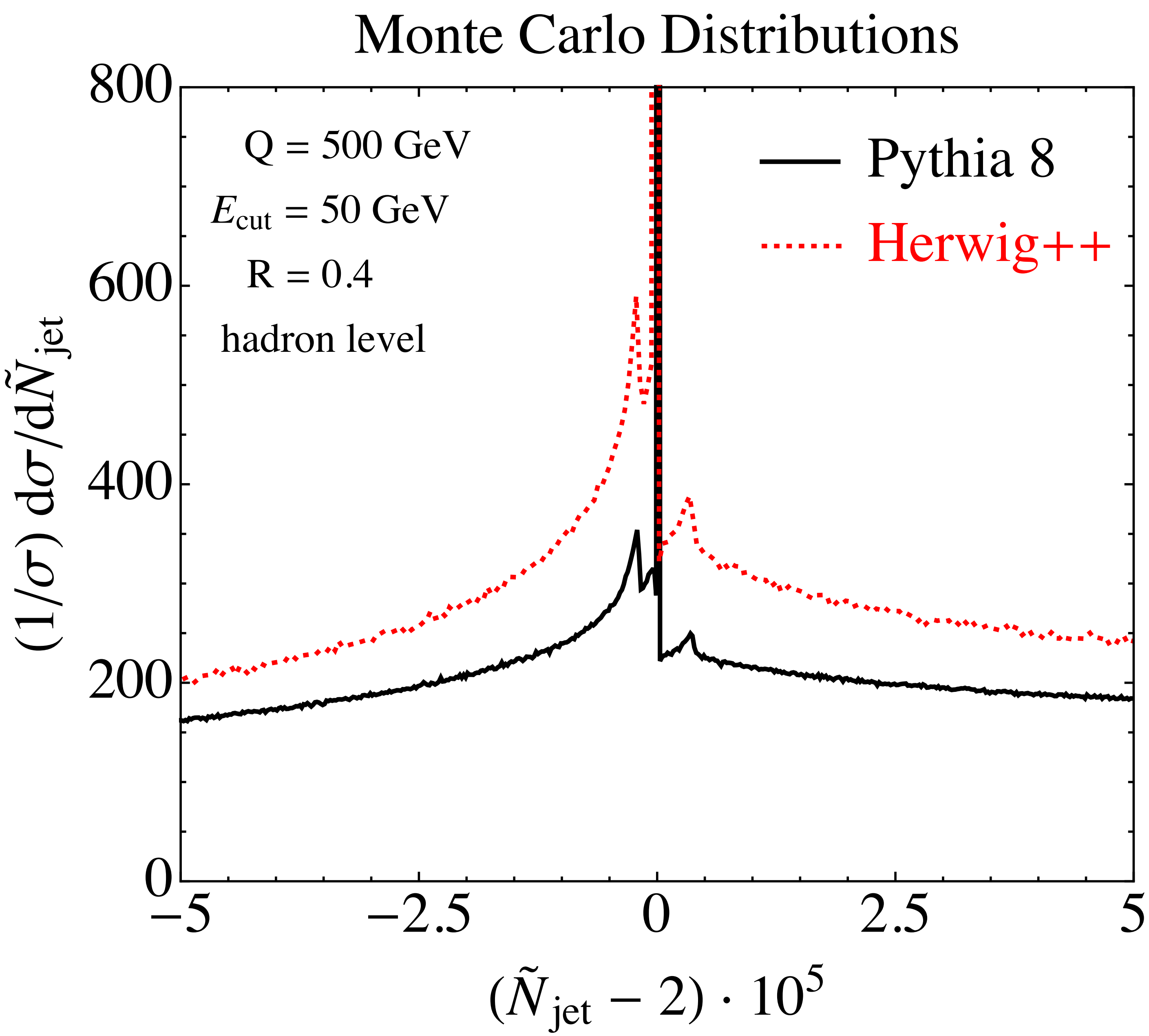
fixed order

Fractional jet multiplicity

- Near-integer driven by soft-logs only
- Hybrid event-shape / jet-algorithm behavior
- Non-additive / non-factorizable / non-global

Near-integer distribution

Hybrid event-shape / jet algorithm



Near-integer distribution

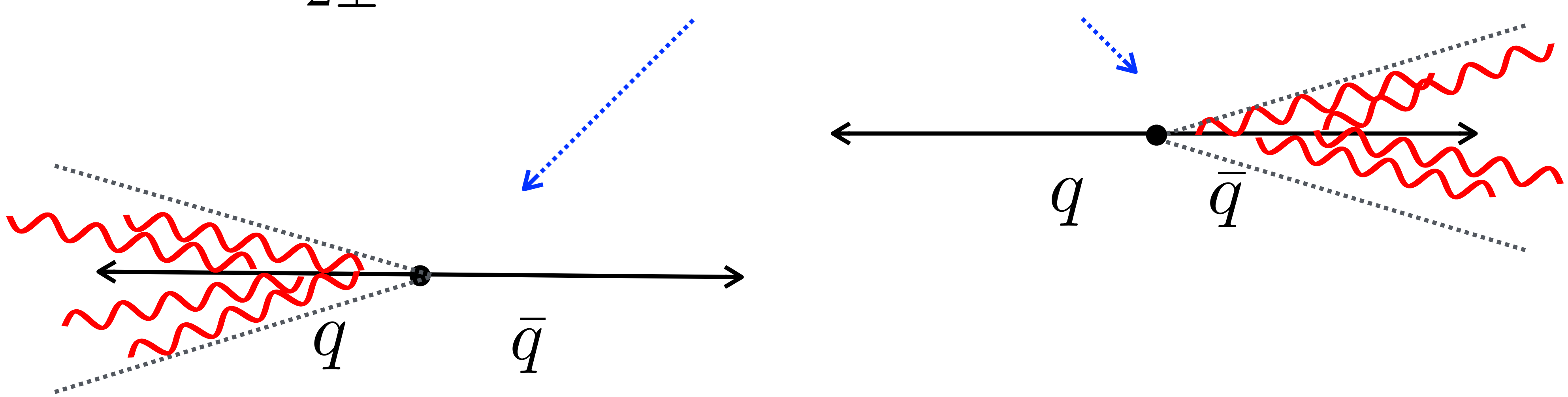
additivity / factorizability / non-global logs

Additive	Factorizable	Global Logs	
Yes	Yes	Yes	Thrust
Yes	Yes	No	Hemisphere Mass
Yes	No	Yes	Jade Algorithm Rate
Yes	No	No	$\sum_i E_i/Q\,\Theta(E_{iR} - E_{\rm cut})$
No	Yes	Yes	?
No	Yes	No	?
No	No	Yes	?
No	No	No	fractional jet multiplicity

Near-integer distribution

beyond FO, collinear functions

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Delta_{2\pm}} \simeq \sigma_0\, C_q(\Delta_{2\pm}) \otimes C_{\bar{q}}(\Delta_{2\pm})$$

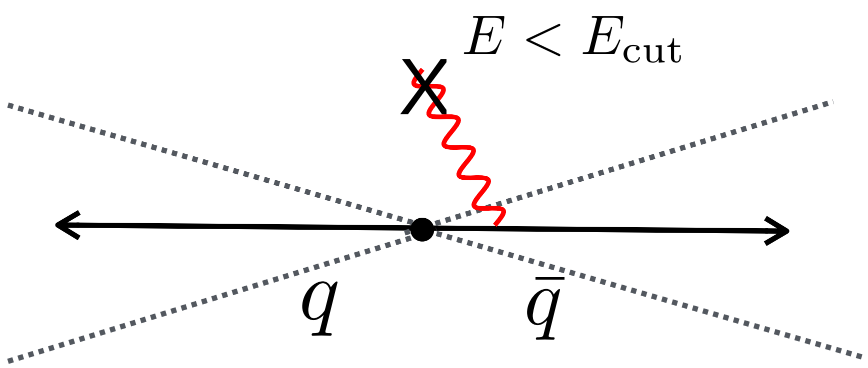


$$C_{q,\bar{q}}(\Delta_{2\pm}) = \delta(\Delta_{2\pm}) + \sum_{n=2}^{\infty}\left(\frac{\alpha_s}{\pi}\right)^n \sum_{k=-1}^{n-1}\left[\kappa_{k,+}^{(n)}\,\mathcal{L}_k(\Delta_{2\pm}) + \kappa_{k,-}^{(n)}\,\mathcal{L}_k(\Delta_{2\pm})\right]$$

Near-integer distribution

beyond FO, collinear functions

$$\frac{d\sigma}{d\Delta_{2\pm}} \simeq \sigma_0 \, C_q(\Delta_{2\pm}) \otimes C_{\bar{q}}(\Delta_{2\pm})$$

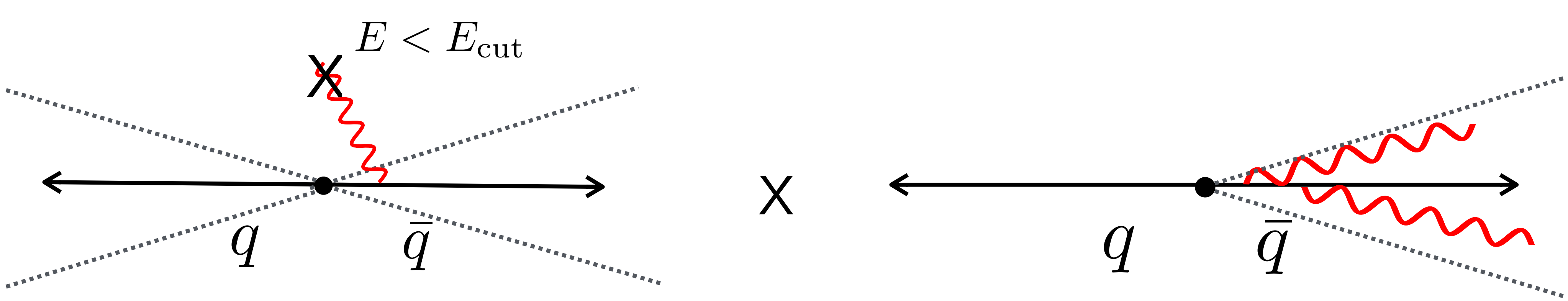


Wide angle soft emissions give enhanced logarithmic contributions to $\widetilde{N}_{\rm jet}$=2 cross-section

Near-integer distribution

beyond FO, collinear functions

$$\frac{d\sigma}{d\Delta_{2\pm}} \simeq \sigma_0\, C_q(\Delta_{2\pm}) \otimes C_{\bar{q}}(\Delta_{2\pm})$$

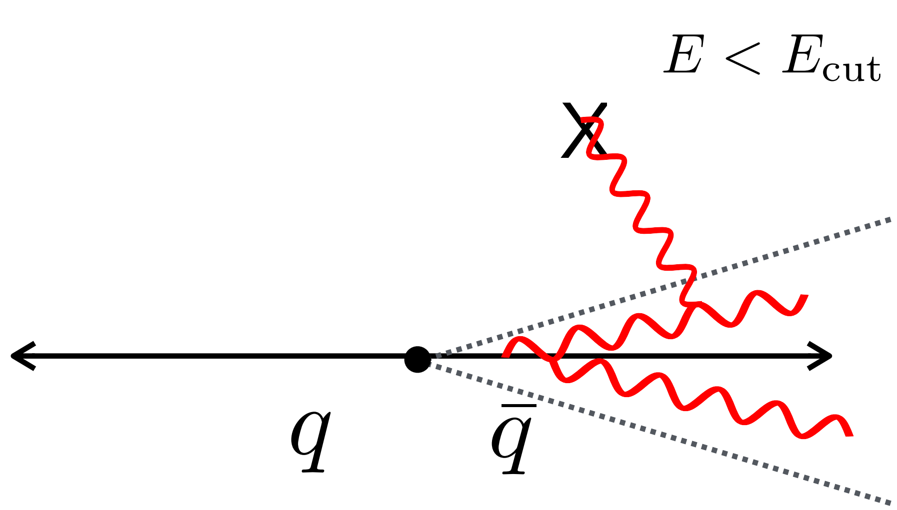


At higher orders they also enhance fractional $\widetilde{\mathrm{N}}_{\mathrm{jet}}$ cross-section

Near-integer distribution

beyond FO, collinear functions

$$\frac{d\sigma}{d\Delta_{2\pm}} \simeq \sigma_0\, C_q(\Delta_{2\pm}) \otimes C_{\bar{q}}(\Delta_{2\pm})$$



Note that this contribution is contained in C

Near-integer distribution

a candidate factorization theorem

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Delta_{2\pm}} \simeq \sigma(\widetilde{N}_{\mathrm{jet}} = 2)\,[C_q(\Delta_{2\pm}) \otimes C_{\bar{q}}(\Delta_{2\pm})]$$

Near-integer distribution

a candidate factorization theorem

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Delta_{2\pm}} \simeq \sigma(\widetilde{N}_{\text{jet}} = 2)\,[C_q(\Delta_{2\pm}) \otimes C_{\bar{q}}(\Delta_{2\pm})]$$

$$\begin{aligned}\sigma(\widetilde{N}_{\text{jet}} = 2) = \sigma_0 H_{q\bar{q}}(Q,\mu) J_q(Q, R, z_{\text{cut}}, \mu) J_{\bar{q}}(Q, R, z_{\text{cut}}, \mu) S_{q\bar{q}}(R, z_{\text{cut}}, \mu) \\ + \sigma_2^{\text{non-fac}}(Q, R, z_{\text{cut}}, \mu)\end{aligned}$$

Near-integer distribution

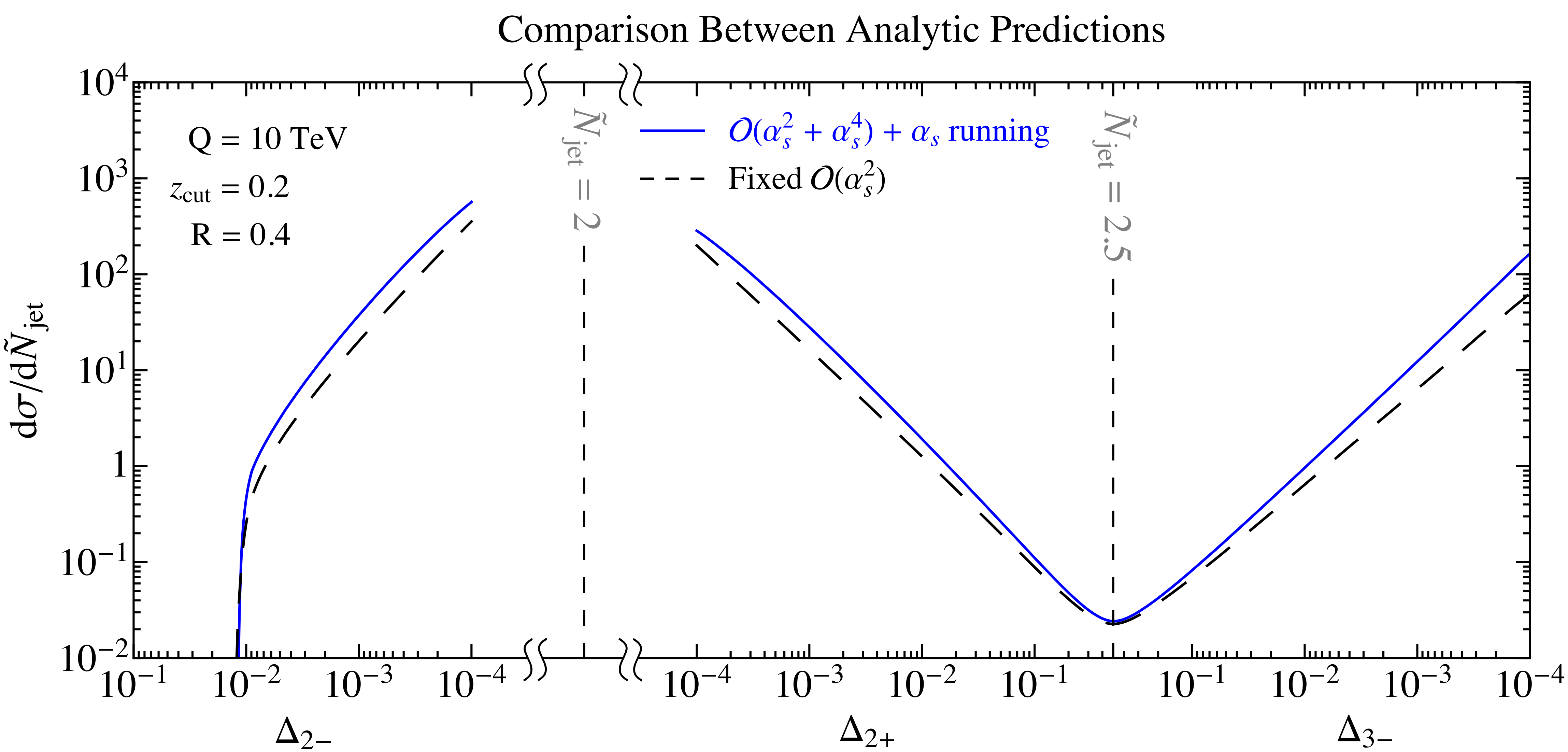
beyond FO, improved distributions

Improved distributions

- Include $O(\alpha_s^4)$ terms from convolutions
- Running coupling $\alpha_s(\mu), \mu = Q\sqrt{\Delta_{2\pm}}$

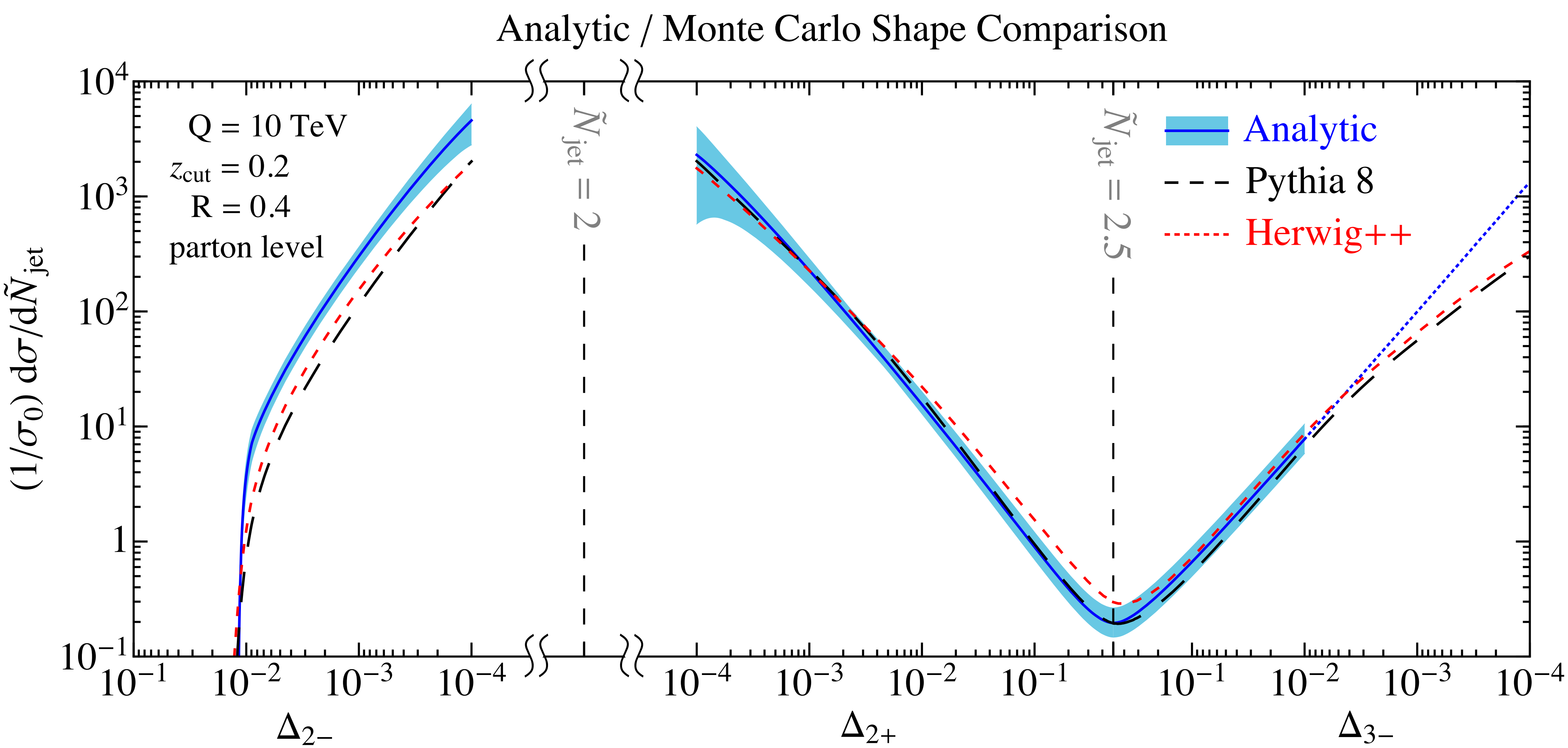
Near-integer distribution

beyond FO, improved distributions



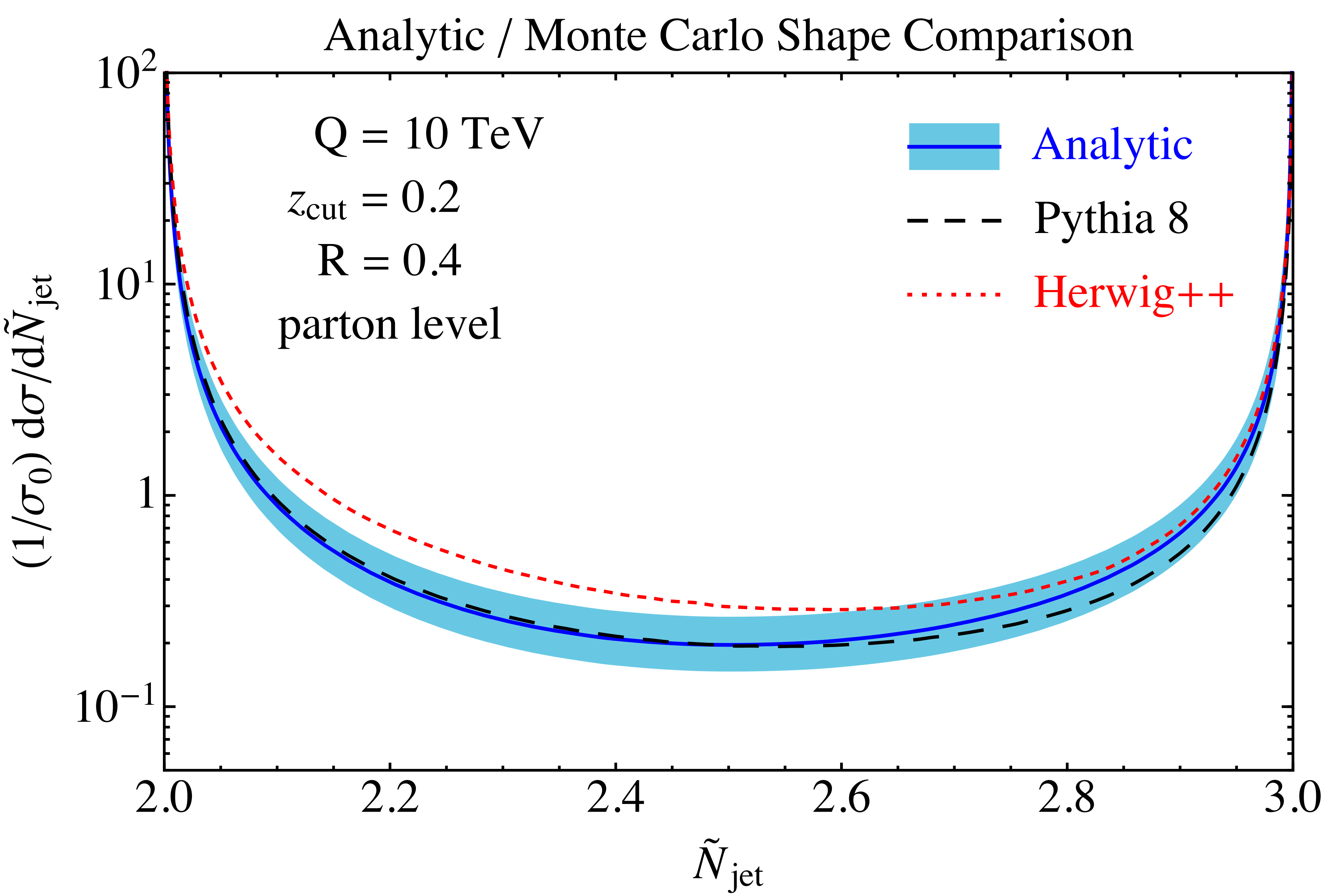
Near-integer distribution

beyond FO, improved distributions



Near-integer distribution

beyond FO, improved distributions



Conclusions

- Fractional jet multiplicity can be used as a novel and more powerful probe of jet formation. E.g. quark/gluon discrimination?

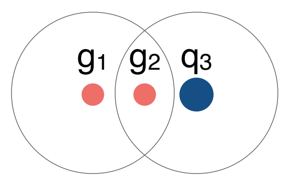
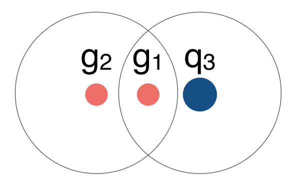
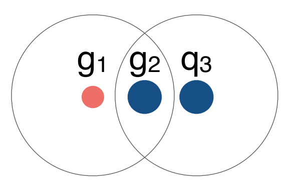
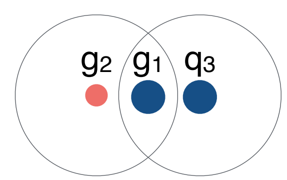
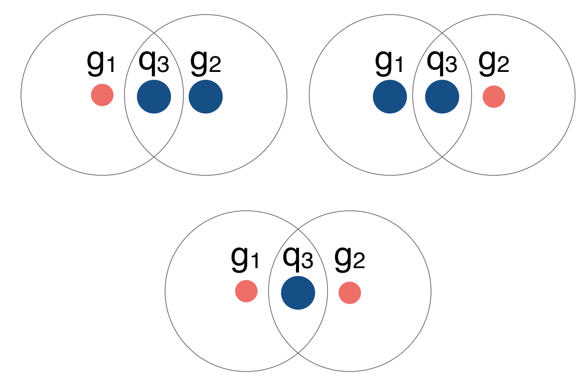
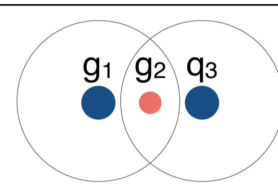
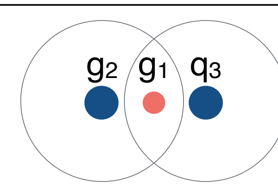
- It has peculiar analytic properties. However, I showed we still have very good analytic control, which in principle is improvable. E.g. generalize to LHC case?

- Wide dynamic range, potential test of matching/merging matrix-element/parton shower.
  Potential phenomenological applications e.g. in multijet final states?

Backup

near-integer phase space configurations

$$\Delta_{2-} = 2 - \widetilde{N}_{\rm jet}, \quad \Delta_{2+} = \widetilde{N}_{\rm jet} - 2, \quad \Delta_{3-} = 3 - \widetilde{N}_{\rm jet}$$

Observable	$\mathcal{R}_A$	$\mathcal{R}_B$	$\mathcal{R}_C$
Δ_{2-}			–
Δ_{2+}			
Δ_{3-}			–

Backup

near-integer phase space configurations

$$\Delta_{2-} = 2 - \widetilde{N}_{\rm jet}, \quad \Delta_{2+} = \widetilde{N}_{\rm jet} - 2, \quad \Delta_{3-} = 3 - \widetilde{N}_{\rm jet}$$

Observable	Region	Expression	Limit	Cuts
Δ_{2-}	$\mathcal{R}_A$	$z_1 z_2$	1, 2 soft	$z_1 + z_2 < z_{\rm cut}$
Δ_{2-}	$\mathcal{R}_B$	$z_1 z_2$	1, 2 soft	$z_1 + z_2 < z_{\rm cut}$
Δ_{2+}	$\mathcal{R}_A$	$z_1(1-z_2^2)/z_2$	1 soft	$z_2 > z_{\rm cut}$
Δ_{2+}	$\mathcal{R}_B$	$z_2(1-z_1^2)/z_1$	2 soft	$z_1 > z_{\rm cut}$
Δ_{2+}	$\mathcal{R}_C$	$z_1 z_2(2-z_2)/(1-z_2)$	1 soft	–
Δ_{2+}	$\mathcal{R}_C$	$z_1 z_2(2-z_1)/(1-z_1)$	2 soft	–
Δ_{2+}	$\mathcal{R}_C$	$2 z_1 z_2$	1, 2 soft	–
Δ_{3-}	$\mathcal{R}_A$	$z_2[1-z_1(1-z_1)]/[z_1(1-z_1)]$	2 soft	$z_1 > z_{\rm cut}$
Δ_{3-}	$\mathcal{R}_B$	$z_1[1-z_2(1-z_2)]/[z_2(1-z_2)]$	1 soft	$z_2 > z_{\rm cut}$

Backup

rapidity divergences

$$\Delta = z_1 z_2$$

$$\begin{aligned}
1\ \text{soft:}\quad I_{sc}(\Delta) &= \int_0^\infty \frac{\mathrm{d}z_1}{z_1} \int_0^1 \frac{\mathrm{d}z_2}{z_2}\, (z_1 z_2)^{-2\epsilon}\, \delta(\Delta - z_1 z_2) \\
2\ \text{soft:}\quad I_{cs}(\Delta) &= \int_0^1 \frac{\mathrm{d}z_1}{z_1} \int_0^\infty \frac{\mathrm{d}z_2}{z_2}\, (z_1 z_2)^{-2\epsilon}\, \delta(\Delta - z_1 z_2) \\
1,\,2\ \text{soft:}\quad I_{ss}(\Delta) &= \int_0^\infty \frac{\mathrm{d}z_1}{z_1} \int_0^\infty \frac{\mathrm{d}z_2}{z_2}\, (z_1 z_2)^{-2\epsilon}\, \delta(\Delta - z_1 z_2)
\end{aligned}$$

Backup

rapidity divergences

$\Delta = z_1 z_2$

$s = z_1 z_2, \quad y = 1/2 \log(z_1/z_2)$ ← energy-sharing "rapidity"

$$\text{1 soft:} \quad I_{sc}(\Delta) = \Delta^{-1-2\epsilon} \int_{-\infty}^{\infty} \mathrm{d}y \, \Theta\Big(-\frac{1}{2}\ln(1/\Delta) < y\Big)$$

$$\text{2 soft:} \quad I_{cs}(\Delta) = \Delta^{-1-2\epsilon} \int_{-\infty}^{\infty} \mathrm{d}y \, \Theta\Big(y < \frac{1}{2}\ln(1/\Delta)\Big)$$

$$\text{1, 2 soft:} \quad I_{ss}(\Delta) = \Delta^{-1-2\epsilon} \int_{-\infty}^{\infty} \mathrm{d}y$$

Backup

rapidity divergences

$\Delta = z_1 z_2$

$s = z_1 z_2, \quad y = 1/2 \log(z_1/z_2)$

energy-sharing "rapidity"

$$\text{1 soft:} \quad I_{sc}(\Delta) = \Delta^{-1-2\epsilon} \int_{-\infty}^{\infty} \mathrm{d}y \, \Theta\Big(-\frac{1}{2}\ln(1/\Delta) < y\Big) \left(\frac{\nu}{E_J}\right)^{\eta} s^{-\eta/2} e^{y\eta}$$

$$\text{2 soft:} \quad I_{cs}(\Delta) = \Delta^{-1-2\epsilon} \int_{-\infty}^{\infty} \mathrm{d}y \, \Theta\Big(y < \frac{1}{2}\ln(1/\Delta)\Big) \left(\frac{\nu}{E_J}\right)^{\eta} s^{-\eta/2} e^{-y\eta}$$

$$\text{1, 2 soft:} \quad I_{ss}(\Delta) = \Delta^{-1-2\epsilon} \int_{-\infty}^{\infty} \mathrm{d}y \left(\frac{\nu}{E_J}\right)^{\eta} s^{-\eta/2} |2\sinh y|^{-\eta}$$

rapidity regulators

$$I_{\mathrm{full}}(\Delta) = I_{sc}(\Delta) + I_{cs}(\Delta) + I_{ss}(\Delta)$$