SUBLEADING SCET HELICITY OPERATORS

More Power To You

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with Ilya Feige, Ian Moult and Iain Stewart (1504.SOON)

Also includes work from “Employing Helicity Amplitudes for Resummation" (1504.SOONER) by Ian Moult, Iain Stewart, Frank Tackmann, and Wouter Waalewijn

OUTLINE

- Motivation
- Introduction to Spinor Helicity Formalism
- Helicity Operator Bases in SCET
- Subleading Dijets

MOTIVATION

- To factorize a process in SCET:
  - Construct a complete basis of operators that contribute, including all Lorentz and color structures
- Can also look at factorization beyond leading power in λ
  - Power suppressed operators and Lagrangians have been used in B-physics

    Bauer, Becher, Benzke, Bosch, Hill, Lee, Mantry, Neubert, Paz, Pirjol, Stewart…
  - Subleading factorization for e^+e^- event shapes recently introduced

    Freedman, Goerke (1303.1558, 1408.6240)
- For high multiplicity processes, constructing complete basis of operators is extremely laborious
- Similar issues even for low multiplicity when working beyond leading power in λ (e.g. more fields per sector)
- Can helicity formalism help?

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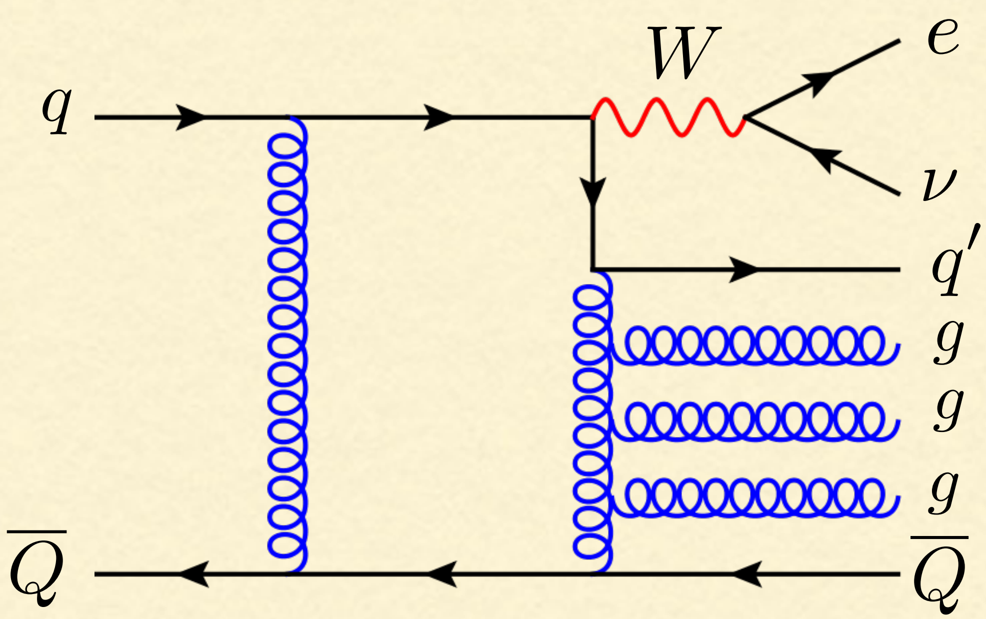
- Can helicity formalism help? YES!

HELICITY FORMALISM

- Immense progress in calculation of FO perturbative QCD amplitudes with spinor helicity formalism

  Bern, Dixon, Dunbar, Kosower, Parke, Taylor, …

- Calculate on shell amplitudes for fixed external helicities



- Strip color structure from amplitude

- Give compact expressions that can be squared and summed at the end

- NLO predictions available for high multiplicity final states (e.g. $pp \to W + 5\,\mathrm{jets}$ - BLACKHAT 1304.1253)

HELICITY FORMALISM

- Simplified notation for helicity spinors

$$|p\pm\rangle = \frac{1 \pm \gamma_5}{2} u(p) \quad , \quad \langle pq\rangle = \langle p - | q + \rangle \, , \, [pq] = \langle p + | q - \rangle$$

$$\varepsilon^\mu_+(p,k) = \frac{[p|\gamma^\mu|k\rangle}{\sqrt{2}\langle kp\rangle} \quad , \quad \varepsilon^\mu_-(p,k) = -\frac{\langle p|\gamma^\mu|k]}{\sqrt{2}[kp]} \quad ,$$ k is a reference vector

- Convenient to capture singularity structure

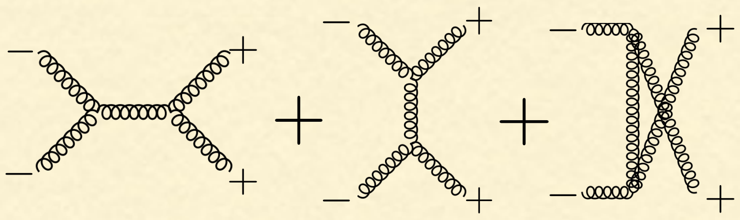
$$\langle pq\rangle = \sqrt{s_{pq}} e^{i\phi_{pq}} \quad , \quad [pq] = \sqrt{s_{pq}} e^{-i\phi_{pq}} \quad , \quad s_{pq} = (p+q)^2$$

- Nice symmetry properties
  - Crossing symmetry
  - $\mathrm{P} : \langle\,\rangle \leftrightarrow [\,]$

HELICITY FORMALISM EXAMPLE

- Look at $gg \to gg$ at tree level

$$\mathcal{A}(1^-2^-3^+4^+) = g^2 \sum_{\sigma \in S_n/\mathbb{Z}_n} \mathrm{Tr}(T^{a_{\sigma(1)}} \ldots T^{a_{\sigma(4)}}) A(\sigma(1^-), \ldots, \sigma(4^+)) =$$



Color structure

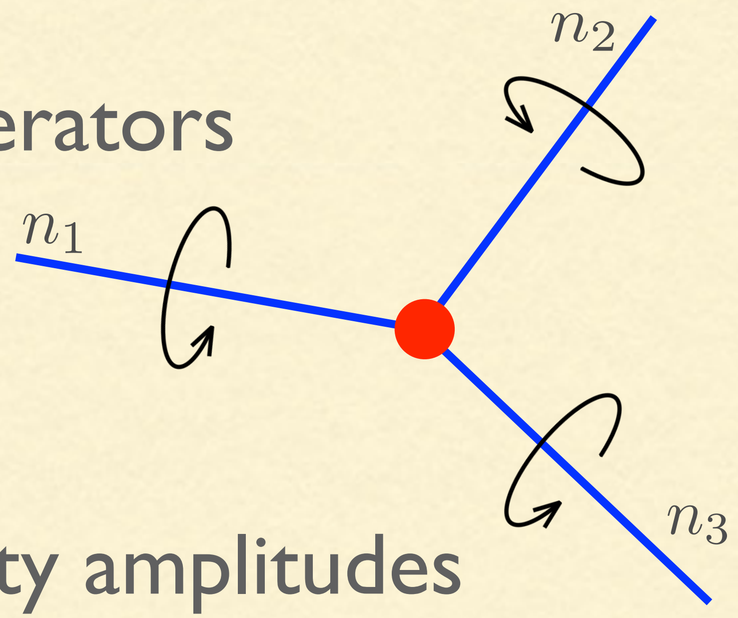
Color stripped amplitude

$$A(1^-, 2^-, 3^+, 4^+) = \frac{\langle 12 \rangle^4}{\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 41 \rangle}$$

- Many relations between amplitudes
  - e.g. $A(1^-, 2^+, 3^+, 4^-) = A(1^-, 2^-, 3^+, 4^+)$ from cyclic properties of the trace

COMBINING WITH SCET

- In SCET, collinear sectors give us natural directions to define helicities (n_i)
- Collinear gauge invariant fields behave like onshell fields
  - $\mathcal{B}^{\mu}_{n_i\perp}$ corresponds to two physical degrees of freedom
- Construct a SCET basis out of fixed helicity operators
  - Simplifies basis structure, scalar objects eliminate tedious tracking of Lorentz indices
- Matching facilitated by using known QCD helicity amplitudes



CONSTRUCTING HELICITY BASIS

- Standard collinear SCET building blocks

$$\mathcal{B}^{\mu}_{n_i\perp},\ \chi_{n_i},\ \mathcal{P}^{\mu}_{n_i\perp}$$

$\mathcal{O}(\lambda^1)$
$\mathcal{O}(\lambda^2)$
$\mathcal{O}(\lambda^3)$
$\mathcal{O}(\lambda^{>3})$

  - Already collinear gauge invariant

- Trivial to project to helicity states

$$\mathcal{B}^{a}_{i\pm} = -\varepsilon_{\mp\mu}(n_i,\bar{n}_i)\mathcal{B}^{a\mu}_{n_i,\omega_i\perp_i}\ ,\quad \chi^{\alpha}_{i\pm} = \frac{1\pm\gamma_5}{2}\chi^{\alpha}_{n_i,-\omega_i}$$

$$\mathcal{P}^{\pm}_{i\perp} = -\varepsilon_{\mp\mu}(n_i,\bar{n}_i)\cdot\mathcal{P}^{\mu}_{n_i\perp}$$

  - For $n_i^\mu = (1,0,0,1)$, $\bar{n}_i^\mu = (1,0,0,-1)$ we simply have

$$\mathcal{B}^{a}_{i+} = \frac{1}{\sqrt{2}}(\mathcal{B}^{a,1}_{n_i,\omega_i\perp_i} + i\mathcal{B}^{a,2}_{n_i,\omega_i\perp_i})\ ,\ \mathcal{B}^{a}_{i-} = \frac{1}{\sqrt{2}}(\mathcal{B}^{a,1}_{n_i,\omega_i\perp_i} - i\mathcal{B}^{a,2}_{n_i,\omega_i\perp_i})$$

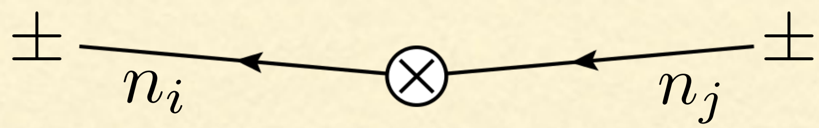
- Can simply insert $\mathcal{B}_{i\pm}$, $P^{\pm}_{i\perp}$ into operators

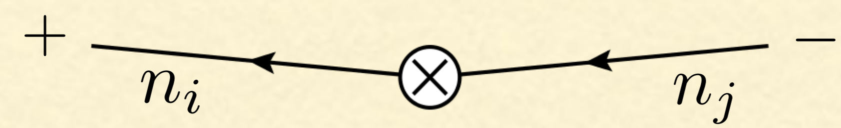
COLLINEAR BILINEARS

$\mathcal{O}(\lambda^1)$
$\mathcal{O}(\lambda^2)$
$\mathcal{O}(\lambda^3)$
$\mathcal{O}(\lambda^{>3})$

- Combine quarks into bilinears
  - Always come in quark-antiquark pairs
- Two quarks in different sectors

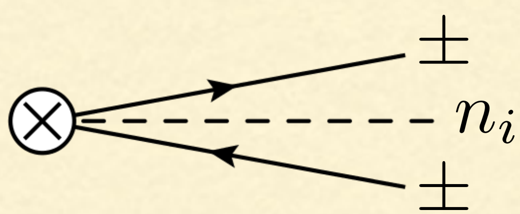
$$J_{ij\pm}^{\bar{\alpha}\beta} = \mp\varepsilon_{\mp}^{\mu}(n_i, n_j)\bar{\chi}_{i\pm}^{\bar{\alpha}}\gamma_\mu\chi_{j\pm}^{\beta} \quad , \quad J_{ijS}^{\bar{\alpha}\beta} = \bar{\chi}_{i+}^{\bar{\alpha}}\chi_{j-}^{\beta}$$

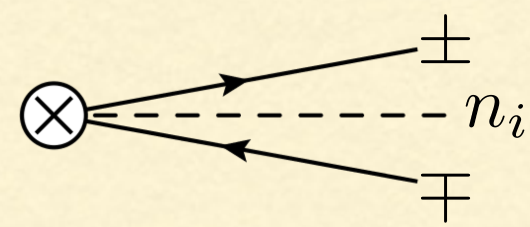




- Two quarks in same sector

$$J_{i\pm}^{\bar{\alpha}\beta} = \bar{\chi}_{i\pm}^{\bar{\alpha}}\bar{\not{n}}_i\chi_{i\pm}^{\beta} \quad , \quad J_{iS\pm}^{\bar{\alpha}\beta} = \mp\epsilon_{\mp}^{\mu}(n_i, \bar{n}_i)\bar{\chi}_{i\pm}^{\bar{\alpha}}\gamma_\mu\bar{\not{n}}_i\chi_{i\mp}^{\beta}$$



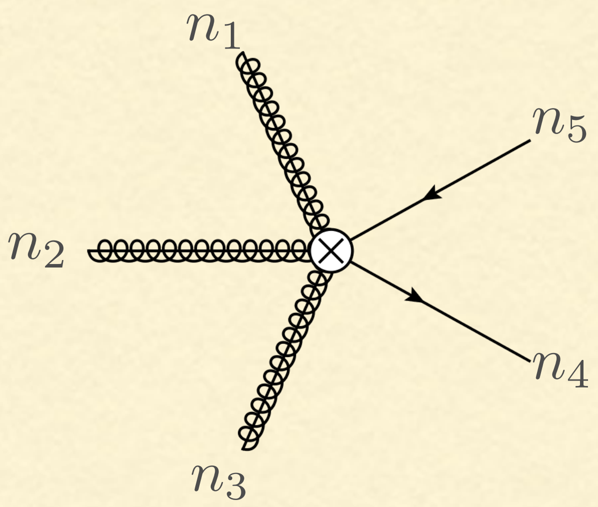


- Have simple Feynman rules

OPERATOR EXAMPLE

- High multiplicity process $pp \to 3\,\text{jets}$
  - For $gggq\bar{q}$ channel, traditional operator is

$$\mathcal{O}^{abc\bar{\alpha}\beta} = \mathcal{B}^{\mu a}_{n_1\perp}\mathcal{B}^{\nu b}_{n_2\perp}\mathcal{B}^{\sigma c}_{n_3\perp}\bar{\chi}^{\bar{\alpha}}_{n_4}\Gamma_{\mu\nu\sigma}\chi^{\beta}_{n_5}$$



  - Writing minimal complete basis for $\Gamma_{\mu\nu\sigma}$ is difficult

- Can immediately write down all operators in helicity basis

$$\mathcal{O}^{abc\bar{\alpha}\beta}_{+++(\pm)} = \frac{1}{3!}\mathcal{B}^a_{1+}\mathcal{B}^a_{2+}\mathcal{B}^a_{3+}J^{\bar{\alpha}\beta}_{45\pm}\,,\quad \mathcal{O}^{abc\bar{\alpha}\beta}_{++-(\pm)} = \frac{1}{2}\mathcal{B}^a_{1+}\mathcal{B}^a_{2+}\mathcal{B}^a_{3-}J^{\bar{\alpha}\beta}_{45\pm}$$

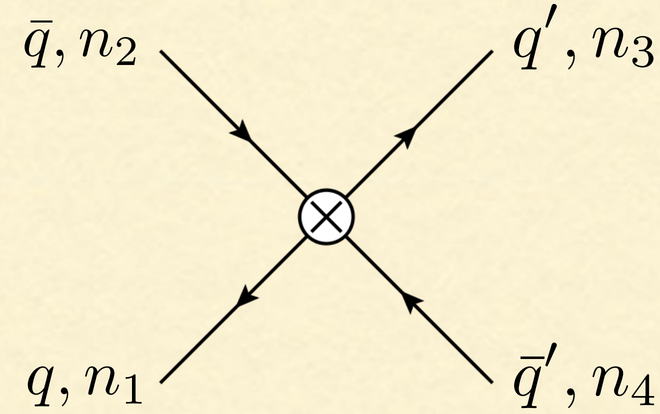
$$\mathcal{O}^{abc\bar{\alpha}\beta}_{--+(\pm)} = \frac{1}{2}\mathcal{B}^a_{1-}\mathcal{B}^a_{2-}\mathcal{B}^a_{3+}J^{\bar{\alpha}\beta}_{45\pm}\,,\quad \mathcal{O}^{abc\bar{\alpha}\beta}_{---(\pm)} = \frac{1}{3!}\mathcal{B}^a_{1-}\mathcal{B}^a_{2-}\mathcal{B}^a_{3-}J^{\bar{\alpha}\beta}_{45\pm}$$

COLOR STRUCTURE

- Want to use color techniques from the study of amplitudes
  - Need to separate color structure from operator
- Contract collinear operator with vector of color structures

$$\vec{O}^{\dagger}_{+\cdot\cdot(\cdot\cdot-)} = O^{\alpha_1\cdots a_n}_{+\cdot\cdot(\cdot\cdot-)}\,\vec{T}^{\dagger\,\alpha_1\cdots a_n}$$

  - $\vec{T}^{\dagger}$ spans color conserving subspace
  - For example, four well separated quarks



$$O^{\bar{\alpha}\beta\bar{\gamma}\delta}_{(\pm;\pm)} = J^{\bar{\alpha}\beta}_{12\pm}\,J^{\bar{\gamma}\delta}_{34\pm}\ ,\quad \vec{T}^{\dagger\,\alpha\bar{\beta}\gamma\bar{\delta}} = \left(\delta_{\alpha\bar{\delta}}\,\delta_{\gamma\bar{\beta}}\,,\ \delta_{\alpha\bar{\beta}}\,\delta_{\gamma\bar{\delta}}\right)$$

SOFTS

- Traditional building blocks not soft gauge invariant
  - e.g. $i\mathcal{D}^{\mu}_{us} = i\partial^{\mu}_{us} + gA^{\mu}_{us}$
  - Stops us from treating the color in same fashion as collinear case
- Motivates moving to post BPS redefined fields
  - $\vec{T}^{\dagger\, a_1 \cdots \alpha_n} \to \vec{T}^{\dagger\, a_1 \cdots \alpha_n}_{BPS} = (Y_1^{\dagger} \cdots Y_i^{\dagger})\vec{T}^{\dagger\, a_1 \cdots \alpha_n}(Y_{i+1} \cdots Y_m)$
  - Now includes soft Wilson lines
- Trade manifest locality for gauge invariance

SOFT GLUON FIELD

$\mathcal{O}(\lambda^1)$
$\mathcal{O}(\lambda^2)$
$\mathcal{O}(\lambda^3)$
$\mathcal{O}(\lambda^{>3})$

- Decompose soft covariant derivative, choosing arbitrary n_i

$$Y_{n_i}^{(r)\,\dagger} i\mathcal{D}_{us}^{(r)\,\mu} Y_{n_i}^{(r)} = i\partial_{us}^{\mu} + T_{(r)}^{a} g\mathcal{B}_{us(i)}^{a\mu}$$

- Define gauge invariant soft gluon helicity field

$$\mathcal{B}_{us(i)\pm}^{a} = -\varepsilon_{\mp\mu}(n_i,\bar{n}_i)\,\mathcal{B}_{us(i)}^{a\mu}\ ,\ \ \mathcal{B}_{us(i)0}^{a} = \bar{n}_{i\mu}\mathcal{B}_{us(i)}^{a\mu}$$

  - No preferred direction, so we include three projections of two degrees of freedom

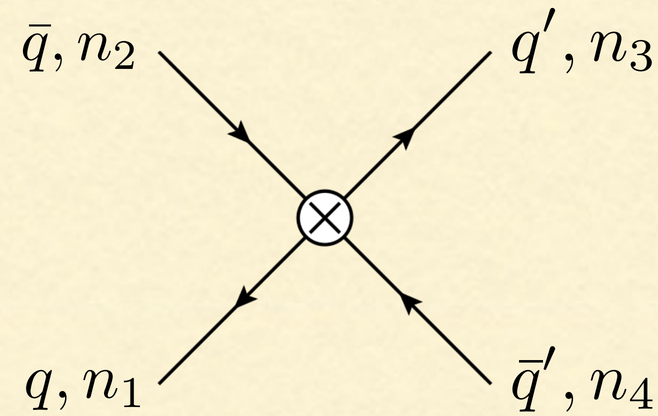
- $i\partial_{us}^{\mu} \to i\partial_{us(i)\pm}\,,\ i\bar{n}_{i\mu}\partial_{us(i)}^{\mu}$ (EOM remove $in_{i\mu}\partial_{us(i)}^{\mu}$)
- Can now take advantage of color organization techniques!

FORMING HELICITY OPERATORS

- Now have a complete set of building blocks for operators
  - Quark bilinears, $\mathcal{B}^a_{i\pm}$, $\mathcal{P}^{\pm}_{i\perp}$, $\mathcal{B}^a_{us(i)\pm,0}$, $\partial_{us(i)\pm,0}$

$\mathcal{O}(\lambda^1)$
$\mathcal{O}(\lambda^2)$
$\mathcal{O}(\lambda^3)$
$\mathcal{O}(\lambda^{>3})$

- Soft Wilson line structure is determined by BPS
  - Don't need $Y^\dagger_{n_1} Y_{n_2}$ independently
  - Example, four separated quarks

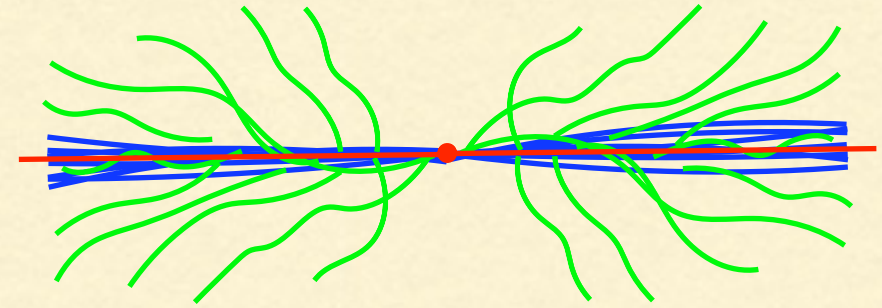


$$\mathcal{O}^{\bar{\alpha}\beta\bar{\gamma}\delta}_{(\pm;\pm)} = J^{\bar{\alpha}\beta}_{12\pm}\, J^{\bar{\gamma}\delta}_{34\pm}$$

$$\vec{T}^{\alpha\bar{\beta}\gamma\bar{\delta}} = \left(\delta_{\alpha\bar{\delta}}\delta_{\gamma\bar{\beta}}, \delta_{\alpha\bar{\beta}}\delta_{\gamma\bar{\delta}}\right)$$

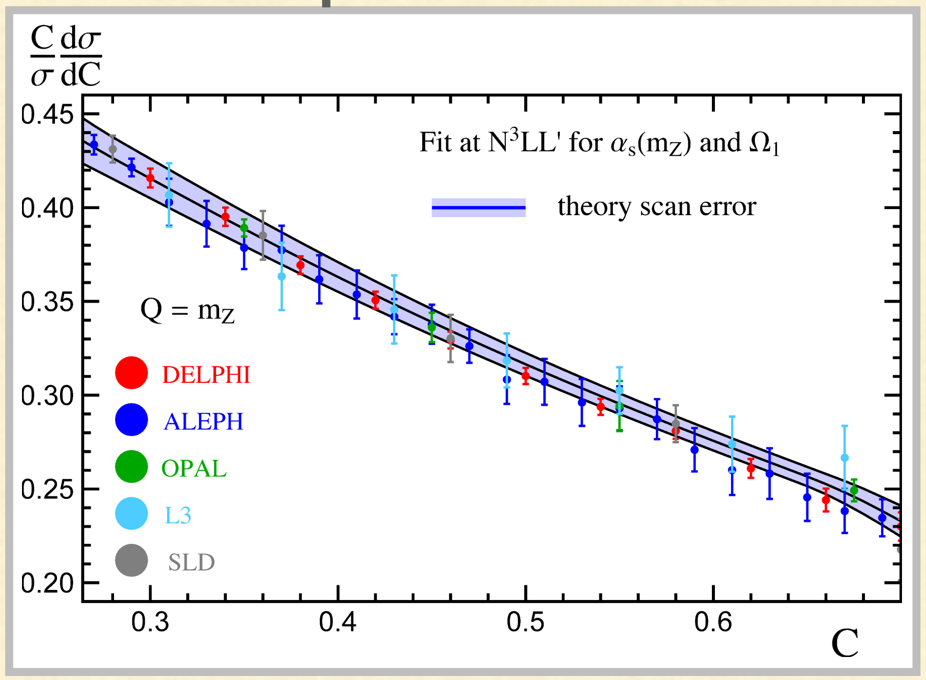
$$\xrightarrow{BPS} \vec{T}^{\dagger\,\alpha\bar{\beta}\gamma\bar{\delta}}_{BPS} = \left(\left[Y^\dagger_{n_1} Y_{n_4}\right]_{\alpha\bar{\delta}} \left[Y^\dagger_{n_3} Y_{n_2}\right]_{\gamma\bar{\beta}} , \left[Y^\dagger_{n_1} Y_{n_2}\right]_{\alpha\bar{\beta}} \left[Y^\dagger_{n_3} Y_{n_4}\right]_{\gamma\bar{\delta}} \right)$$

DIJETS



- Event shapes in $e^+e^- \to \text{dijets}$ are a good starting point for using these techniques
- Well studied in the literature
- Theoretically clean
- Minimum number of collinear sectors

C-parameter



Hoang, Kolodrubetz, Mateu, Stewart 1501.04111

DIJETS IN HELICITY BASIS

- Analyze $e^+e^- \to \text{dijets}$ with helicity
  - Work in center of mass frame
- Leading power operator:

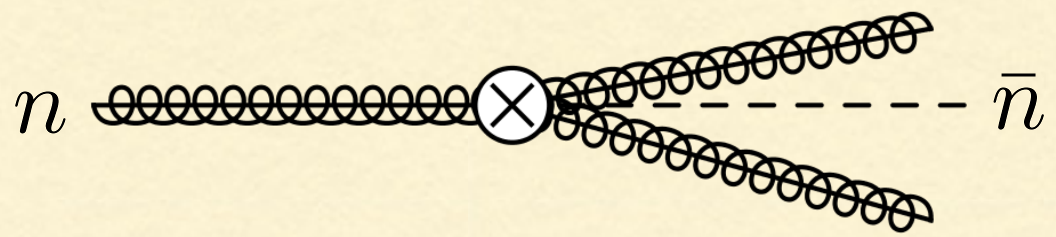
$$O^{\bar{\alpha}\beta}_{(\pm;\lambda)} = J_{e\pm} J^{\bar{\alpha}\beta}_{n\bar{n}\lambda} \ , \ \lambda = \pm$$

- Color basis:

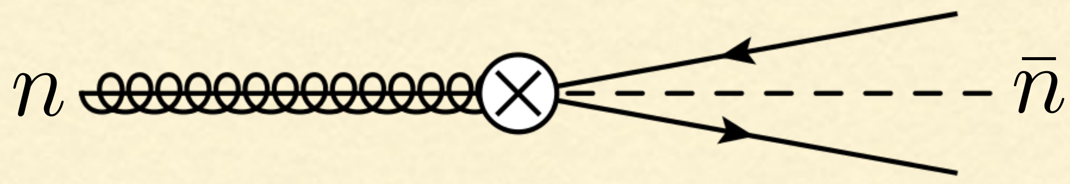
$$\vec{T}^{\alpha\bar{\beta}} = (\delta_{\alpha\bar{\beta}}) \to \vec{T}^{\alpha\bar{\beta}}_{BPS} = \left([Y_n^\dagger Y_{\bar{n}}]_{\alpha\bar{\beta}}\right)$$

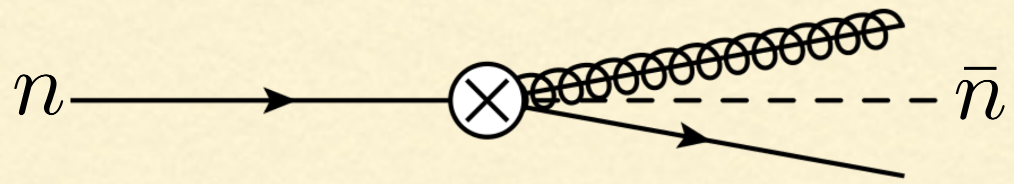
SUBLEADING DIJETS

- Physical helicity and color final states map one-to-one to operators
  - No Lorentz structure complication
- Types of operators suppressed by $\mathcal{O}(\lambda^1)$
  - Three gluons



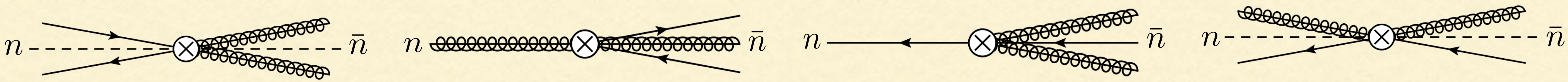
  - Two quarks and a gluon



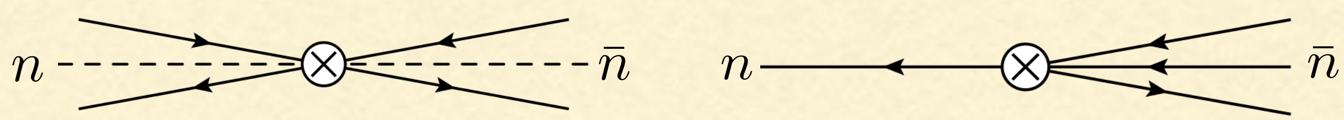


SUBLEADING DIJETS

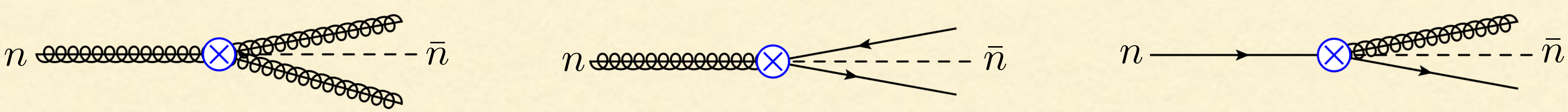
- Types of operators suppressed by $\mathcal{O}(\lambda^2)$
  - Two quarks, two gluons



  - Four quarks

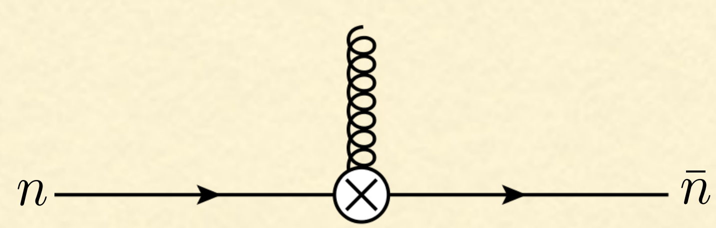


  - $\mathcal{P}_\perp$ insertions



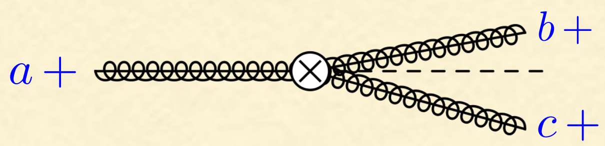
$J_{e\pm}\, J^{\bar{\alpha}\beta}_{n\bar{n}\,\pm}\, \left[\mathcal{P}^{\pm}_{\perp} \mathcal{B}^{a}_{\bar{n}\pm}\right]$

  - Ultrasoft gluon

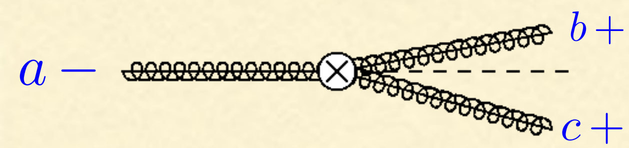


SUBLEADING DIJETS

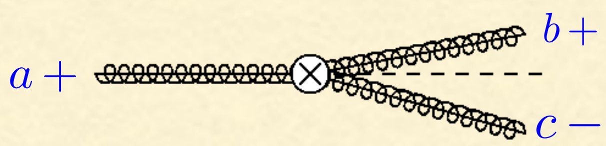
- Easily construct operators from these diagrams
- Three gluon example



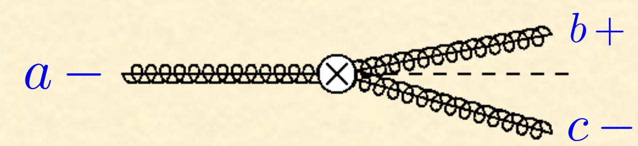
$$\leftrightarrow \frac{1}{2} J_{e\pm} \mathcal{B}^a_{n+} \mathcal{B}^b_{\bar{n}+} \mathcal{B}^c_{\bar{n}+}$$



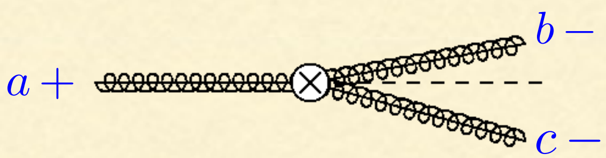
$$\leftrightarrow \frac{1}{2} J_{e\pm} \mathcal{B}^a_{n-} \mathcal{B}^b_{\bar{n}+} \mathcal{B}^c_{\bar{n}+}$$



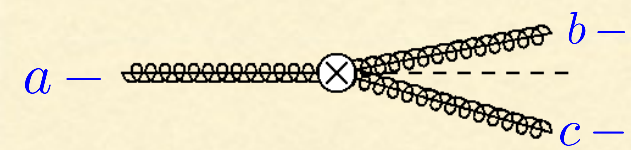
$$\leftrightarrow J_{e\pm} \mathcal{B}^a_{n+} \mathcal{B}^b_{\bar{n}+} \mathcal{B}^c_{\bar{n}-}$$



$$\leftrightarrow J_{e\pm} \mathcal{B}^a_{n-} \mathcal{B}^b_{\bar{n}+} \mathcal{B}^c_{\bar{n}-}$$



$$\leftrightarrow \frac{1}{2} J_{e\pm} \mathcal{B}^a_{n+} \mathcal{B}^b_{\bar{n}-} \mathcal{B}^c_{\bar{n}-}$$



$$\leftrightarrow \frac{1}{2} J_{e\pm} \mathcal{B}^a_{n-} \mathcal{B}^b_{\bar{n}-} \mathcal{B}^c_{\bar{n}-}$$

$$\vec{T}^{\dagger\, abc} = \left(i f^{abc} \,,\, d^{abc} \right)$$

- Complements work of Freedman and Goerke with the construction of a guaranteed complete basis

SUBLEADING DIJETS

- Extend to subleading power

$$O_{QCD} = O^{(0)} + O^{(1)} + O^{(2)} + \cdots \ , \ \mathcal{L}_{SCET} = \mathcal{L}^{(0)} + \mathcal{L}^{(1)} + \mathcal{L}^{(2)} + \cdots$$

$$\hat{\mathcal{M}} = \hat{\mathcal{M}}^{(0)} + \hat{\mathcal{M}}^{(1)} + \hat{\mathcal{M}}^{(2)} + \cdots$$

- Cross section will include different types of subleading terms

$$\sigma \sim \langle O_{QCD}\, \hat{\mathcal{M}}\, O_{QCD} \rangle$$

$$= \langle O^{(0)}\, \hat{\mathcal{M}}^{(0)}\, O^{(0)} \rangle \longleftarrow \mathcal{O}(\lambda^0)$$

$$\mathcal{O}(\lambda^1) \longrightarrow + \langle O^{(0)}\, \hat{\mathcal{M}}^{(0)}\, O^{(1)} \rangle + \langle O^{(0)}\, \hat{\mathcal{M}}^{(0)}\, \mathcal{L}^{(1)} O^{(0)} \rangle + \langle O^{(0)}\, \hat{\mathcal{M}}^{(1)}\, O^{(0)} \rangle$$

$$\mathcal{O}(\lambda^2) \longrightarrow + \langle O^{(0)}\, \hat{\mathcal{M}}^{(0)}\, O^{(2)} \rangle + \langle O^{(0)}\, \hat{\mathcal{M}}^{(0)}\, \mathcal{L}^{(1)} O^{(1)} \rangle + \langle O^{(1)}\, \hat{\mathcal{M}}^{(0)}\, O^{(1)} \rangle$$

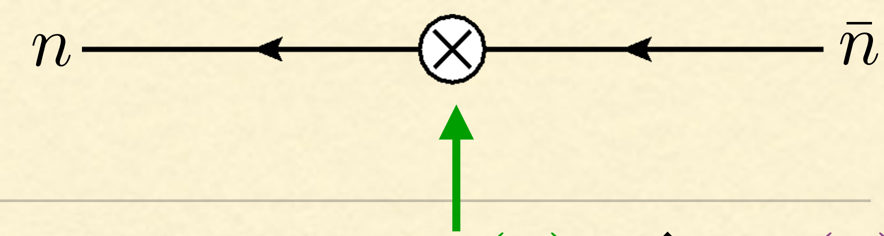
$$+ \langle O^{(0)}\, \hat{\mathcal{M}}^{(0)}\, \mathcal{L}^{(2)} O^{(0)} \rangle + \langle O^{(0)}\, \hat{\mathcal{M}}^{(1)}\, O^{(1)} \rangle + \langle O^{(0)}\, \hat{\mathcal{M}}^{(1)}\, \mathcal{L}^{(1)} \rangle$$

$$+ \langle O^{(0)}\, \hat{\mathcal{M}}^{(2)}\, O^{(0)} \rangle + \langle O^{(0)}\, \hat{\mathcal{M}}^{(0)}\, \mathcal{L}^{(1)} \mathcal{L}^{(1)} O^{(0)} \rangle$$

REDUCING THE BASIS

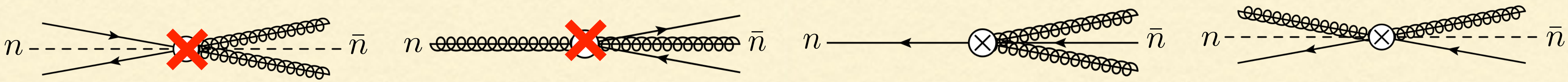
- Can easily see vanishing of Jet and Soft function at $\mathcal{O}(\lambda^1)$
  - Look at insertion of $\mathcal{O}(\lambda^1)$ operators against leading operator
    $$\langle O^{(0)}\,\hat{\mathcal{M}}^{(0)}\,O^{(1)}\rangle$$
  - Use conservation of fermion number and rotational invariance
  - e.g. $\langle 0|(J^{\bar{\alpha}\beta}_{n\bar{n}\lambda_1}(x))^\dagger \widehat{\mathcal{M}}^{(0)} J^{\bar{\delta}\gamma}_{n\bar{n}\lambda_2}(0)\mathcal{B}^a_{(n,\bar{n})\lambda_3}(0)|0\rangle = 0$
    because $\lambda_1 + \lambda_2 + \lambda_3 \neq 0$
  - Subleading Lagrangian insertions at $\mathcal{O}(\lambda^1)$ also disappear for similar reasons
    $$\langle O^{(0)}\,\hat{\mathcal{M}}^{(0)}\,\mathcal{L}^{(1)} O^{(0)}\rangle$$

SUBLEADING DIJETS

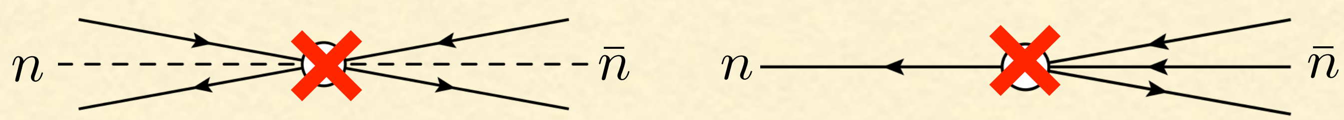


$\langle \mathcal{O}^{(0)} \, \hat{\mathcal{M}} \, \mathcal{O}^{(2)} \rangle$

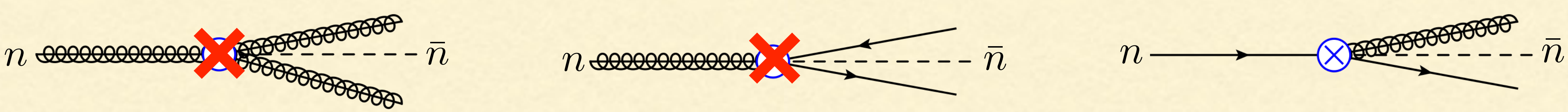
- Similar arguments reduce operators at $\mathcal{O}(\lambda^2)$
  - Two quarks, two gluons



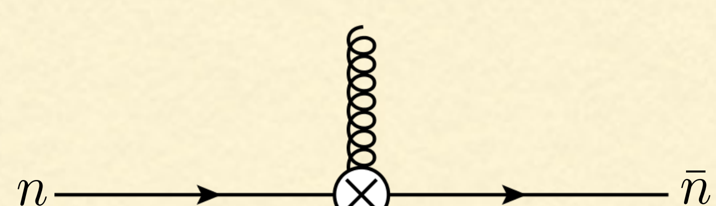
  - Four quarks



  - $\mathcal{P}_\perp$ insertions



  - Ultrasoft gluon



SUBLEADING FACTORIZATION

- Algebraic manipulations for factorization are simpler in this framework
  - e.g. $O^{a\,\bar{\alpha}\beta}_{\bar{n}(\pm;\lambda_1)\lambda_2} = J_{e\pm}\, J^{\bar{\alpha}\beta}_{\bar{n}\lambda_1}\, \mathcal{B}^a_{n\lambda_2}$

$$\langle O_{\bar{n}(\pm;\lambda_1)\lambda_2} \hat{\mathcal{M}}^{(0)} O^{a\,\bar{\alpha}\beta}_{\bar{n}(\pm;\lambda_3)\lambda_4} \rangle$$

$$\propto \langle\, J^{\bar{\alpha}\beta}_{\bar{n}\lambda_1}\, \hat{\mathcal{M}}^{(0)}_{\bar{n}}\, J^{\bar{\alpha}\beta}_{\bar{n}\lambda_3}\, \rangle \langle \mathcal{B}^a_{n\lambda_2}\, \hat{\mathcal{M}}^{(0)}_{n}\, \mathcal{B}^a_{n\lambda_4} \rangle$$

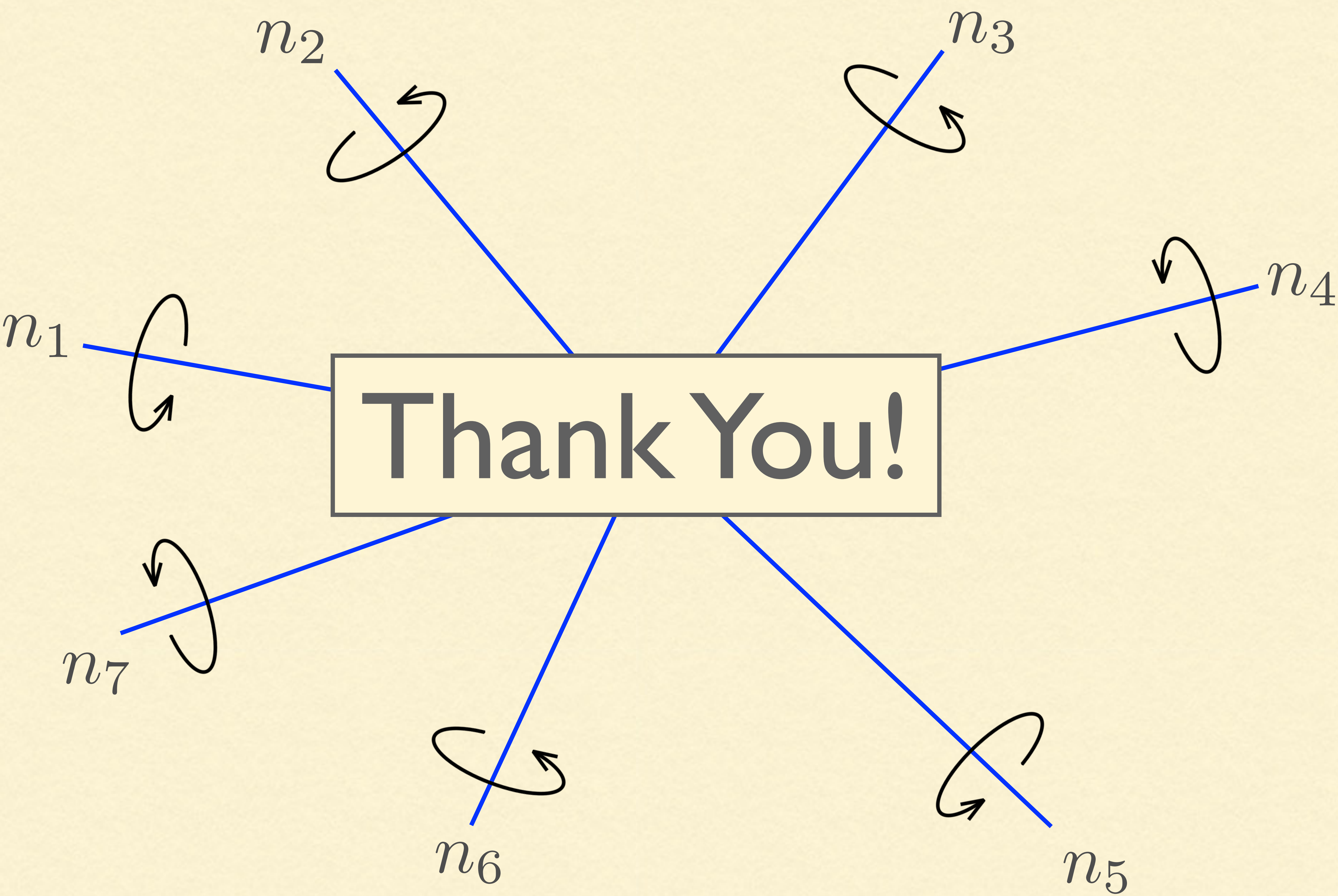
- Color structure and soft functions already separated and organized post-BPS

$$\vec{T}^{\alpha\bar{\beta}\gamma\bar{\delta}} = \left(\delta_{\alpha\bar{\delta}}\delta_{\gamma\bar{\beta}}, \delta_{\alpha\bar{\beta}}\delta_{\gamma\bar{\delta}}\right)$$

$$\xrightarrow{BPS} \vec{T}^{\dagger\,\alpha\bar{\beta}\gamma\bar{\delta}}_{BPS} = \left(\left[Y^\dagger_{n_1} Y_{n_4}\right]_{\alpha\bar{\delta}} \left[Y^\dagger_{n_3} Y_{n_2}\right]_{\gamma\bar{\beta}} , \left[Y^\dagger_{n_1} Y_{n_2}\right]_{\alpha\bar{\beta}} \left[Y^\dagger_{n_3} Y_{n_4}\right]_{\gamma\bar{\delta}} \right)$$

CONCLUSION

- Spinor helicity formalism pairs naturally with collinear sector labels of SCET
- Helicity fields in SCET provide a powerful tool for treating high multiplicity and subleading processes
- Methods work for any SCET process
- Complete subleading operator bases are now painless to write down
- Simpler treatment of subleading factorization!


Thank You!
n_1
n_2
n_3
n_4
n_5
n_6
n_7

Backup Slides

TABLE OF DIJET OPERATORS

	Category	Operators
Leading Power : $\mathcal{O}(\lambda^2)$	$e\bar{e}q\bar{q}$	$O^{\bar{\alpha}\beta}_{(\pm;\lambda)} = J_{e\pm} J^{\bar{\alpha}\beta}_{n\bar{n}\lambda}$
Subleading Power : $\mathcal{O}(\lambda^3)$	$e\bar{e}ggg$	$O^{abc}_{(\pm)\lambda_1\lambda_2\lambda_3} = S\, J_{e\pm}\, \mathcal{B}^a_{n\lambda_1}\, \mathcal{B}^b_{\bar{n}\lambda_2}\, \mathcal{B}^c_{\bar{n}\lambda_3}$
	$e\bar{e}q\bar{q}g$	$O^{a\,\bar{\alpha}\beta}_{(\pm;\lambda_1)\lambda_2} = J_{e\pm}\, J^{\bar{\alpha}\beta}_{n\bar{n}\,\lambda_1}\, \mathcal{B}^a_{n\lambda_2}$
		$O^{a\,\bar{\alpha}\beta}_{\bar{n}(\pm;\lambda_1)\lambda_2} = J_{e\pm}\, J^{\bar{\alpha}\beta}_{\bar{n}\lambda_1}\, \mathcal{B}^a_{n\lambda_2}$
Subsubleading Power : $\mathcal{O}(\lambda^4)$	$e\bar{e}q\bar{q}Q\bar{Q}$	$O^{\bar{\alpha}\beta\bar{\gamma}\delta}_{qQ1(\pm;\lambda_1;\lambda_2)} = J_{e\pm}\, J^{\bar{\alpha}\beta}_{qn\lambda_1}\, J^{\bar{\gamma}\delta}_{Q\bar{n}\lambda_2}$
		$O^{\bar{\alpha}\beta\bar{\gamma}\delta}_{qQ2(\pm;\lambda;\lambda)} = J_{e\pm}\, J^{\bar{\alpha}\beta}_{q\bar{Q}n\lambda}\, J^{\bar{\gamma}\delta}_{\bar{q}Q\bar{n}\lambda}$
		$O^{\bar{\alpha}\beta\bar{\gamma}\delta}_{qQ2(\pm;\lambda;-\lambda)} = J_{e\pm}\, J^{\bar{\alpha}\beta}_{q\bar{Q}nS\lambda}\, J^{\bar{\gamma}\delta}_{\bar{q}Q\bar{n}S\,-\lambda}$
		$O^{\bar{\alpha}\beta\bar{\gamma}\delta}_{qQ3(\pm;\lambda_1;+)} = J_{e\pm}\, J^{\bar{\alpha}\beta}_{qn\bar{n}\lambda_1}\, J^{\bar{\gamma}\delta}_{Qn\bar{n}\lambda_2}$
		$O^{\bar{\alpha}\beta\bar{\gamma}\delta}_{qQ4(\pm;\lambda_1;\lambda_2)} = J_{e\pm}\, J^{\bar{\alpha}\beta}_{qn\bar{n}\,\lambda_1}\, J^{\bar{\gamma}\delta}_{Q\bar{n}\lambda_2}$
		$O^{\bar{\alpha}\beta\bar{\gamma}\delta}_{qQ5(\pm;\lambda_1;\lambda_2)} = J_{e\pm}\, J^{\bar{\alpha}\beta}_{q\bar{n}n\lambda_1}\, J^{\bar{\gamma}\delta}_{Q\bar{n}\lambda_2}$
	$e\bar{e}q\bar{q}gg$	$O^{ab\,\bar{\alpha}\beta}_{\mathcal{B}1(\mathcal{B}2)(\pm;\lambda_1)\lambda_2\lambda_3} = S J_{e\pm}\, J^{\bar{\alpha}\beta}_{n\bar{n}\,\lambda_1} \mathcal{B}^a_{n(\bar{n})\lambda_2}\, \mathcal{B}^b_{n(\bar{n})\lambda_3}$
		$O^{ab\,\bar{\alpha}\beta}_{\mathcal{B}3(\pm;\lambda_1)\lambda_2\lambda_3} = J_{e\pm}\, J^{\bar{\alpha}\beta}_{n\bar{n}\,\lambda_1} \mathcal{B}^a_{n\lambda_2}\, \mathcal{B}^b_{\bar{n}\lambda_3}$
		$O^{ab\,\bar{\alpha}\beta}_{\mathcal{B}4(\mathcal{B}5)(\pm;\lambda_1)\lambda_2\lambda_3} = J_{e\pm}\, J^{\bar{\alpha}\beta}_{n\,\lambda_1} \mathcal{B}^a_{n(\bar{n})\lambda_2}\, \mathcal{B}^b_{\bar{n}\lambda_3}$
	$\mathcal{P}_\perp$	$O^{abc}_{\mathcal{P}\mathcal{B}\lambda_1(\pm)\lambda_2\lambda_3\lambda_4} = S\, J_{e\pm}\, \mathcal{B}^a_{n\lambda_2}\, \mathcal{B}^b_{\bar{n}\lambda_3}\, \left[\mathcal{P}^{\lambda_1}_\perp \mathcal{B}^c_{\bar{n}\lambda_4}\right]$
		$O^{a\,\bar{\alpha}\beta}_{\mathcal{P}1\lambda_1(\pm;\lambda_2)\lambda_3} = J_{e\pm}\, J^{\bar{\alpha}\beta}_{n\bar{n}\,\lambda_2}\, \left[\mathcal{P}^{\lambda_1}_\perp \mathcal{B}^a_{n\lambda_3}\right]$
		$O^{a\,\bar{\alpha}\beta}_{\mathcal{P}2\lambda_1(\pm;\lambda_2)\lambda_3} = J_{e\pm}\, J^{\bar{\alpha}\beta}_{\bar{n}\,\lambda_2(\mathcal{P}^{\lambda_1}_\perp)}\, \mathcal{B}^a_{n\lambda_3}$
	Ultrasoft	$O^{a\,\bar{\alpha}\beta}_{(us)(\pm;\lambda_1)\lambda_2} = J_{e\pm}\, J^{\bar{\alpha}\beta}_{n\bar{n}\,\lambda_1}\, \mathcal{B}^a_{us(n)\lambda_2}$
		$O^{a\,\bar{\alpha}\beta}_{(us)(\pm;\lambda)0} = J_{e\pm}\, J^{\bar{\alpha}\beta}_{n\bar{n}\,\lambda}\, \mathcal{B}^a_{us(n)0}$

TABLE OF RELEVANT DIJET OPERATORS

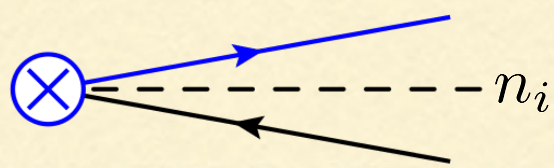
	Category	Operators
Leading Power : $\mathcal{O}(\lambda^2)$	$e\bar{e}q\bar{q}$	$O^{\bar{\alpha}\beta}_{(\pm;\lambda)} = J_{e\pm} J^{\bar{\alpha}\beta}_{n\bar{n}\lambda}$
Subleading Power : $\mathcal{O}(\lambda^3)$	$e\bar{e}ggg$	$O^{abc}_{(\pm)\lambda_1\lambda_2\lambda_3} = S\, J_{e\pm}\, \mathcal{B}^a_{n\lambda_1}\, \mathcal{B}^b_{\bar{n}\lambda_2}\, \mathcal{B}^c_{\bar{n}\lambda_3}$
	$e\bar{e}q\bar{q}g$	$O^{a\,\bar{\alpha}\beta}_{(\pm;\lambda_1)\lambda_2} = J_{e\pm}\, J^{\bar{\alpha}\beta}_{n\bar{n}\,\lambda_1}\, \mathcal{B}^a_{n\lambda_2}$ $O^{a\,\bar{\alpha}\beta}_{\bar{n}(\pm;\lambda_1)\lambda_2} = J_{e\pm}\, J^{\bar{\alpha}\beta}_{\bar{n}\lambda_1}\, \mathcal{B}^a_{n\lambda_2}$
Subsubleading Power : $\mathcal{O}(\lambda^4)$	$e\bar{e}q\bar{q}gg$	$O^{ab\,\bar{\alpha}\beta}_{\mathcal{B}1(\mathcal{B}2)(\pm;\lambda_1)\lambda_2\lambda_3} = S J_{e\pm}\, J^{\bar{\alpha}\beta}_{n\bar{n}\,\lambda_1}\, \mathcal{B}^a_{n(\bar{n})\lambda_2}\, \mathcal{B}^b_{n(\bar{n})\lambda_3}$ $O^{ab\,\bar{\alpha}\beta}_{\mathcal{B}3(\pm;\lambda_1)\lambda_2\lambda_3} = J_{e\pm}\, J^{\bar{\alpha}\beta}_{n\bar{n}\,\lambda_1}\, \mathcal{B}^a_{n\lambda_2}\, \mathcal{B}^b_{\bar{n}\lambda_3}$
	$\mathcal{P}_\perp$	$O^{a\,\bar{\alpha}\beta}_{\mathcal{P}1\lambda_1(\pm;\lambda_2)\lambda_3} = J_{e\pm}\, J^{\bar{\alpha}\beta}_{n\bar{n}\,\lambda_2}\, \left[\mathcal{P}^{\lambda_1}_\perp \mathcal{B}^a_{n\lambda_3}\right]$
	Ultrasoft	$O^{a\,\bar{\alpha}\beta}_{(us)(\pm;\lambda)0} = J_{e\pm}\, J^{\bar{\alpha}\beta}_{n\bar{n}\,\lambda}\, \mathcal{B}^a_{us(n)0}$

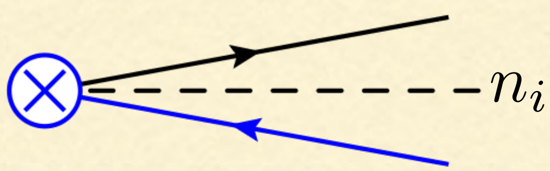
COLLINEAR BILINEARS

$\mathcal{O}(\lambda^1)$
$\mathcal{O}(\lambda^2)$
$\mathcal{O}(\lambda^3)$
$\mathcal{O}(\lambda^{>3})$

- $\mathcal{P}_\perp$ insertions examples
  - Insertion into same sector quark bilinears

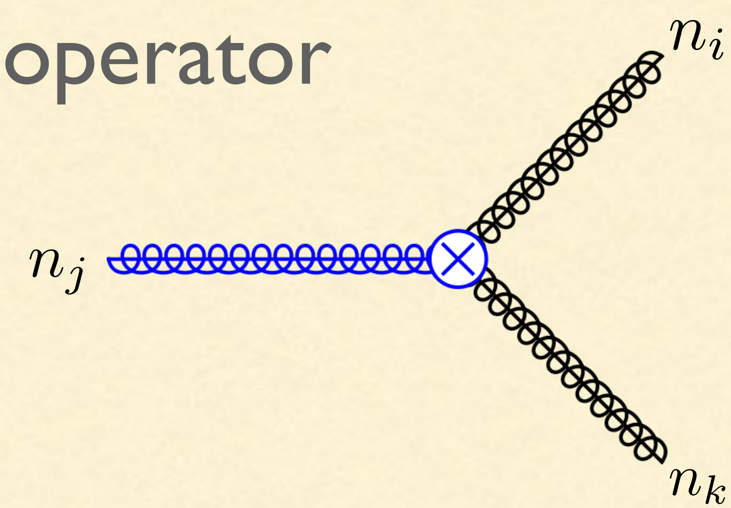
$$J^{\bar\alpha\beta}_{i(\mathcal{P}_\perp^\pm)\pm} = \left[\mathcal{P}_\perp^\pm \bar\chi^{\bar\alpha}_{i\pm}\right] \bar{n}\!\!\!/_i \chi^\beta_{i\pm} \;,\quad J^{\bar\alpha\beta}_{i\pm(\mathcal{P}_\perp^\pm)} = \bar\chi^{\bar\alpha}_{i\pm} \bar{n}\!\!\!/_i \left[\mathcal{P}_\perp^\pm \chi^\beta_{i\pm}\right]$$





  - Simply act on gluon within an operator

$$\mathcal{B}_{i+}\left[\mathcal{P}_\perp^\pm \mathcal{B}_{j-}\right]\mathcal{B}_{k+}$$



FULL LIST OF HELICITY BUILDING BLOCKS

- Building Blocks

$\mathcal{O}(\lambda^1)$
$\mathcal{O}(\lambda^2)$
$\mathcal{O}(\lambda^3)$
$\mathcal{O}(\lambda^{>3})$

$$\mathcal{B}^a_{i\pm}\ ,\ \mathcal{P}^{\pm}_{i\perp}\ ,\ J^{\bar{\alpha}\beta}_{ij\pm}\ ,\ J^{\bar{\alpha}\beta}_{ijS}\ ,\ J^{\bar{\alpha}\beta}_{i\pm}\ ,\ J^{\bar{\alpha}\beta}_{iS\pm}$$

$$\mathcal{B}^a_{us(i)\pm}\ ,\ \mathcal{B}^a_{us(i)0}\ ,\ \partial_{us(i)\pm}\ ,\ \partial_{us(i)0}\ ,\ J^{\bar{\alpha}\beta}_{i(us)\pm}\ ,\ J^{\bar{\alpha}\beta}_{i(us)S\pm}$$

MATCHING

- Match QCD onto an effective hard SCET Lagrangian
  - Expand in powers of λ: $\mathcal{L}_{\text{hard}} = \mathcal{L}_{\text{hard}}^{(0)} + \mathcal{L}_{\text{hard}}^{(1)} + \cdots$

$$\mathcal{L}_{\text{hard}}^{(j)} = \sum_{\{n_1,n_2\}} \sum_{l=2}^{2+j} \int \prod_{i=1}^{l} \mathrm{d}\omega_i \, \vec{O}^{(j)\dagger}_{+\cdot\cdot(\cdot\cdot-)}(n_1, n_2; \omega_1, \ldots, \omega_l) \vec{C}^{(j)}_{+\cdot\cdot(\cdot\cdot-)}(n_1, n_2; \omega_1, \ldots, \omega_l)$$

Wilson Coefficient is a vector in color space

- Match to tree level amplitude to extract Wilson coefficients
  - Need to consider subleading Lagrangian insertions

$$i(\mathcal{A}^{tree})^{(\lambda^p)} = -C^{(p)} \langle x| \vec{O}^{(p)\dagger} |0\rangle^{tree}_{\mathcal{L}^0_{SCET}} - \sum_{n=1} \langle x| \mathcal{L}^n_{SCET} \mathcal{L}^{p-n}_{\text{hard}} |0\rangle^{tree}_{\mathcal{L}^0_{SCET}}$$

Color stripped helicity amplitude expanded to $\mathcal{O}(\lambda^p)$

Easily evaluated using rules for helicity operators

Insertions disappear at leading power

SOFT QUARKS

$\mathcal{O}(\lambda^1)$
$\mathcal{O}(\lambda^2)$
$\mathcal{O}(\lambda^3)$
$\mathcal{O}(\lambda^{>3})$

- Ultrasoft gauge invariant ultrasoft quarks

$$\psi_{us(i)\pm} = Y_{n_i}^\dagger q_{us\pm}$$

- Form mixed collinear-us bilinears

$$J_{i(us)\pm}^{\bar\alpha\beta} = \mp\varepsilon_\mp^\mu(n_i,\bar n_i)\bar\chi_{i\pm}^{\bar\alpha}\,\gamma_\mu\psi_{us(i)\pm}^\beta\ ,\ J_{i(\overline{us})\pm}^{\bar\alpha\beta} = \mp\varepsilon_\mp^\mu(\bar n_i,n_i)\bar\psi_{us(i)\pm}^{\bar\alpha}\,\gamma_\mu\chi_{i\pm}^\beta$$

$$J_{i(us)S\pm}^{\bar\alpha\beta} = \bar\chi_{i\pm}^{\bar\alpha}\psi_{us(i)\mp}^\beta$$

  - Appear at leading power for single collinear sector factorization

- Can also write down extremely suppressed bilinears with two ultrasoft quarks

TREE LEVEL FEYNMAN RULES

- Collinear gluon

$$\langle g_\pm^a(p)|\mathcal{B}_{i\pm}^b|0\rangle = \delta^{ab}\tilde{\delta}(\tilde{p}_i - p)$$

$$\langle 0|\mathcal{B}_{i\pm}^b|g_\mp^a(-p)\rangle = \delta^{ab}\tilde{\delta}(\tilde{p}_i - p)$$

- Collinear quark bilinears

$$\langle q_\pm^{\alpha_1}(p_1)\bar{q}_\mp^{\bar{\alpha}_2}(p_2)|J_{12\pm}^{\bar{\beta}_1\beta_2}|0\rangle = \langle n_1 \mp|\bar{n}_1\pm\rangle\langle n_2 \pm|\bar{n}_2\mp\rangle \frac{\langle \bar{n}_1 \pm |p_1\mp\rangle\langle \bar{n}_2 \mp |p_2\pm\rangle}{8\sqrt{\bar{n}_1\cdot p_1 \bar{n}_2\cdot p_2}}\delta^{\alpha_1\bar{\beta}_1}\delta^{\beta_2\bar{\alpha}_2}\tilde{\delta}(\tilde{p}_1 - p_1)\tilde{\delta}(\tilde{p}_2 - p_2)$$

$$\langle q_+^{\alpha_1}(p_1)\bar{q}_+^{\bar{\alpha}_2}(p_2)|J_{12S}^{\bar{\beta}_1\beta_2}|0\rangle = \langle \bar{n}_1 n_1\rangle\langle n_2\bar{n}_2\rangle \frac{[p_1\bar{n}_1][\bar{n}_2 p_2]}{8\sqrt{\bar{n}_1\cdot p_1 \bar{n}_2\cdot p_2}}\delta^{\alpha_1\bar{\beta}_1}\delta^{\beta_2\bar{\alpha}_2}\tilde{\delta}(\tilde{p}_1 - p_1)\tilde{\delta}(\tilde{p}_2 - p_2)$$

$$\langle q_-^{\alpha_1}(p_1)\bar{q}_-^{\bar{\alpha}_2}(p_2)|(J^\dagger)_{12S}^{\bar{\beta}_1\beta_2}|0\rangle = [\bar{n}_1 n_1][n_2\bar{n}_2] \frac{\langle p_1\bar{n}_1\rangle\langle \bar{n}_2 p_2\rangle}{8\sqrt{\bar{n}_1\cdot p_1 \bar{n}_2\cdot p_2}}\delta^{\alpha_1\bar{\beta}_1}\delta^{\beta_2\bar{\alpha}_2}\tilde{\delta}(\tilde{p}_1 - p_1)\tilde{\delta}(\tilde{p}_2 - p_2)$$

$$\langle q_\pm^{\alpha_1}(p_1)\bar{q}_\mp^{\bar{\alpha}_2}(p_2)|J_{i\pm}^{\bar{\beta}_1\beta_2}|0\rangle = \frac{1}{2}\frac{\langle p_1 \pm|\bar{n}_i\mp\rangle\langle \bar{n}_i \mp|p_2\pm\rangle}{\sqrt{\bar{n}_i\cdot p_1 \bar{n}_i\cdot p_2}}\delta^{\alpha_1\bar{\beta}_1}\delta^{\beta_2\bar{\alpha}_2}\tilde{\delta}(\tilde{p}_1 - p_1)\tilde{\delta}(\tilde{p}_2 - p_2)$$

$$\langle q_\pm^{\alpha_1}(p_1)\bar{q}_\pm^{\bar{\alpha}_2}(p_2)|J_{iS\pm}^{\bar{\beta}_1\beta_2}|0\rangle = \frac{1}{2}\frac{\langle p_1 \pm|\bar{n}_i\mp\rangle\langle \bar{n}_i \pm|p_2\mp\rangle}{\sqrt{\bar{n}_i\cdot p_1 \bar{n}_i\cdot p_2}}\delta^{\alpha_1\bar{\beta}_1}\delta^{\beta_2\bar{\alpha}_2}\tilde{\delta}(\tilde{p}_1 - p_1)\tilde{\delta}(\tilde{p}_2 - p_2)$$

SUBLEADING LAGRANGIAN

$$\langle O^{(0)}\,\hat{\mathcal{M}}^{(0)}\,\mathcal{L}^{(1)} O^{(0)}\rangle$$

- Subleading SCET Lagrangian at $\mathcal{O}(\lambda^1)$

$$\mathcal{L}^{(1)}_{\chi_n\,\lambda} = \bar{\chi}^{\lambda}_n \Big(i\not{D}^{\mp}_{us\perp} \frac{1}{i\bar{n}\cdot\partial_n} i\not{\mathcal{D}}^{\pm}_{n\perp} \Big) \frac{\bar{\not{n}}}{2} \chi^{\lambda}_n + h.c.$$

$$\mathcal{L}^{(1)}_{\chi_n q_{us}\lambda} = \bar{\chi}^{\lambda}_n \frac{1}{i\bar{n}\cdot\partial_n} g\not{\mathcal{B}}^{\pm}_{n\perp} q^{\lambda}_{us} + \text{h.c.}$$

$$\mathcal{L}^{(1)}_{A_n} = \frac{2}{g^2}\text{Tr}\Big(\big[i\mathcal{D}^{\mu}_{nus}, i\mathcal{D}^{\pm}_{n\perp}\big]\big[i\mathcal{D}_{nus\mu}, iD^{\mp}_{us\perp}\big] \Big) + \mathcal{L}^{(1)}_{A_n,\text{gf}}$$

$$\mathcal{L}^{(1)}_{A_n,\text{gf}} = \frac{2}{\alpha}\text{Tr}\Big(\big[iD^{\pm}_{us\perp}, A^{\mp}_{n\perp}\big]\big[i\partial^{\nu}_{nus}, A_{n\nu}\big] \Big) + \text{ghosts}$$