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Jet functions, jet algorithm

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Jet function

- Factorized cross section

$$\sigma = H(Q^2, \mu) \otimes J_n \otimes J_{\overline{n}} \otimes S$$

final-state collinear parts

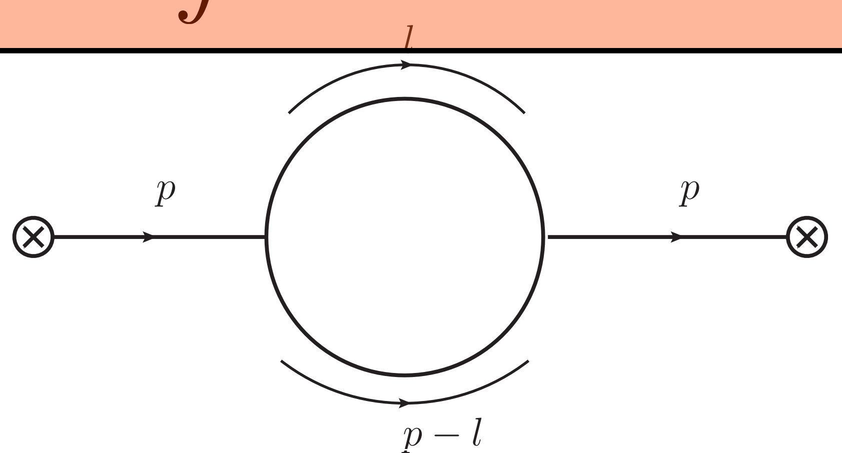
- Each part should be IR finite, or is it?

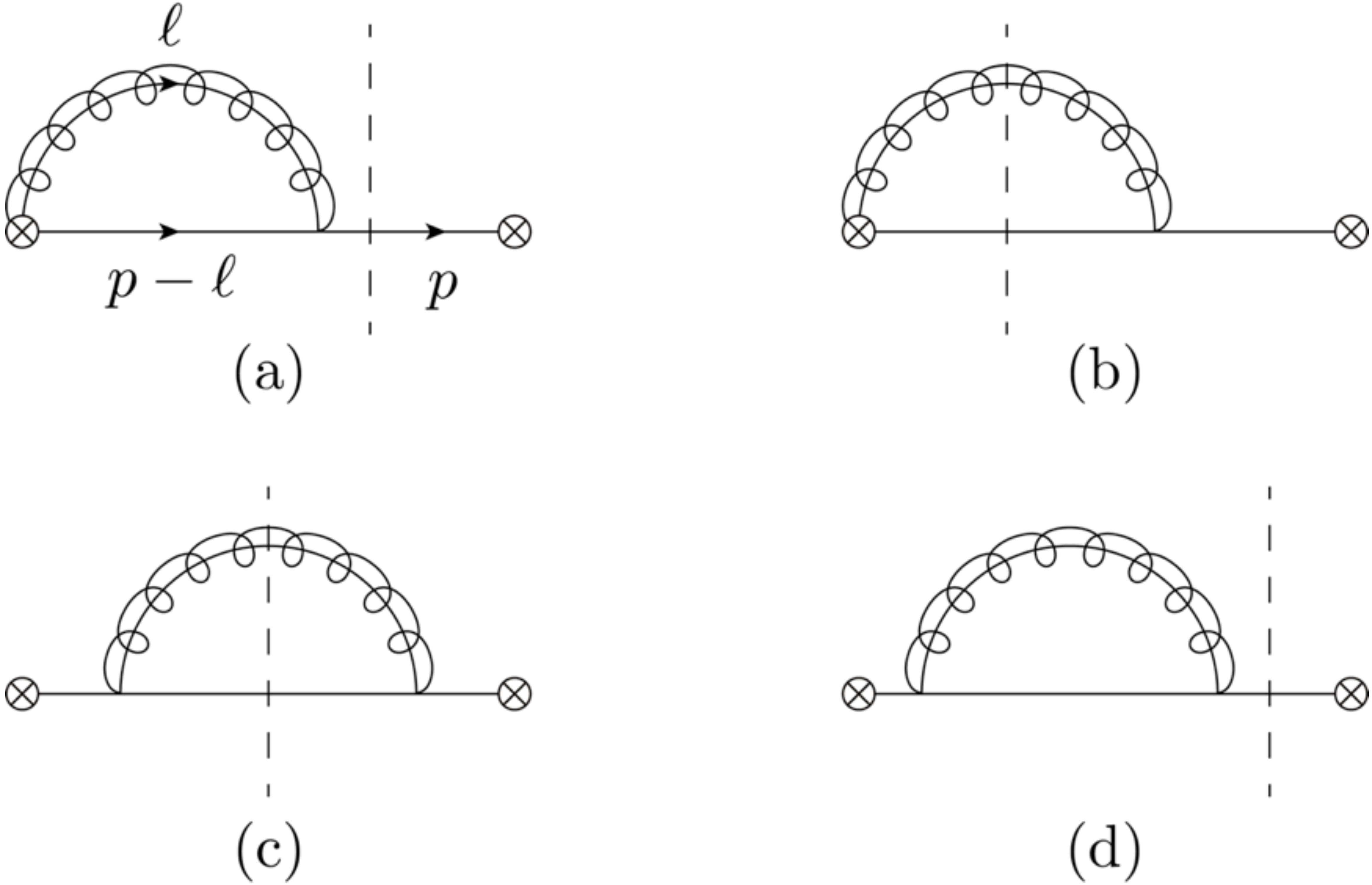
Jet function

Inclusive $$\sum_{X_n} \chi_n |X_n\rangle\langle X_n|\overline{\chi}_n = \int \frac{d^4 p_{X_n}}{(2\pi)^4}\frac{\not{n}}{2} J_n(p_{X_n})$$

Jet algorithm $$\sum_{X_n} \chi_n |X_n\rangle \Theta_{\rm alg} \langle X_n|\overline{\chi}_n = \int \frac{d^4 p_{X_n}}{(2\pi)^4}\frac{\not{n}}{2} J_n^{\rm jet}(p_{X_n})$$

$$J_{\rm int}(tQ,\mu) \overset{???}{=} \int dp^2 J_{\rm unint}(tQ,\mu;p^2)$$





ℓ
$p - \ell$
p
(a)
(b)
(c)
(d)

Sterman-Weinberg algorithm

$$\theta < 2\delta \longrightarrow \quad \Theta_J = \Theta\Big(t^2 > \frac{Q^2 l_+}{l_-(Q-l_-)^2}\Big)$$

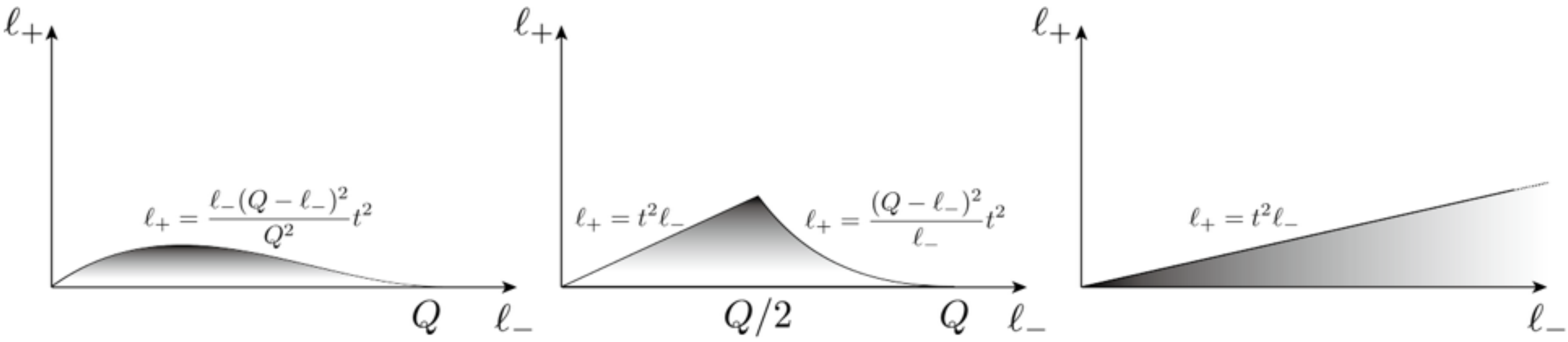
cone-type algorithm

$$\theta_q,\ \theta_g < R \longrightarrow \ \Theta_{\rm cone} = \Theta\Big(t^2 > \frac{l_+}{l_-}\Big)\Theta\Big(t^2 > \frac{p^2/Q - l_+}{Q - l_-}\Big)$$

$$t = \delta \text{ or } \tan R/2$$

We choose $t \sim \lambda.$

zero-bin $$\Theta^{(0)} = \Theta\Big(t^2 > \frac{l_+}{l_-}\Big)$$



(a) (b) (c)

SW | Cone-type | Zero bin

Pure dimensional regularization

$$\int_0^\infty du u^{-1-\epsilon} = \frac{1}{\epsilon_{\rm UV}} - \frac{1}{\epsilon_{\rm IR}}$$

$$\int_0^\infty du \int_0^\infty dv (uv)^{-1-\epsilon} = \Big(\int_0^\infty du u^{-1-\epsilon}\Big)^2 = \Big(\frac{1}{\epsilon_{\rm UV}} - \frac{1}{\epsilon_{\rm IR}}\Big)^2$$

The following integral does not make sense as it is.

$$\int_0^\infty du u^{-1-\epsilon} \int_0^{t^2 u} dv v^{-1-\epsilon}$$

$$M_a = \tilde{M}_a - M_a^0 = \frac{\alpha_s C_F}{2\pi}\Big(\frac{1}{\epsilon_{\rm UV}} - \frac{1}{\epsilon_{\rm IR}}\Big)\Big(\frac{1}{\epsilon_{\rm UV}} + 1 + \ln\frac{\mu}{Q}\Big)$$

Integrated jet function

$$
\begin{aligned}
\tilde{M}_b &= \frac{\alpha_s C_F}{2\pi}\frac{(e^{\gamma_{\rm E}}\mu^2)^\epsilon}{\Gamma(1-\epsilon)}\int_0^Q dl_-\frac{Q-l_-}{Q}l_-^{-1-\epsilon}\int_0^{l_-t^2(Q-l_-)^2/Q^2} dl_+l_+^{-1-\epsilon}\\
&= \frac{\alpha_s C_F}{2\pi}\Big[\frac{1}{2\epsilon_{\rm IR}^2}+\frac{1}{\epsilon_{\rm IR}}\Big(1+\frac{1}{2}\ln\frac{\mu^2}{Q^2t^2}\Big)+\ln\frac{\mu^2}{Q^2t^2}+\frac{1}{4}\ln^2\frac{\mu^2}{Q^2t^2}+4-\frac{3}{8}\pi^2\Big]
\end{aligned}
$$

$$
M_b^0 = \frac{\alpha_s C_F}{2\pi}\frac{(e^{\gamma_{\rm E}}\mu^2)^\epsilon}{\Gamma(1-\epsilon)}\int_0^\infty dl_-l_-^{-1-\epsilon}\int_0^{t^2l_-} dl_+l_+^{-1-\epsilon}
$$

$$
\begin{aligned}
&\int_0^\infty dl_-l_-^{-1-\epsilon}\int_0^{t^2l_-} dl_+l_+^{-1-\epsilon} = \Big(\int_0^\eta dl_-l_-^{-1-\epsilon}+\int_\eta^\infty dl_-l_-^{-1-\epsilon}\Big)\int_0^{t^2l_-} dl_+l_+^{-1-\epsilon}\\
&= \int_0^\eta dl_-l_-^{-1-\epsilon}\int_0^{t^2l_-} dl_+l_+^{-1-\epsilon}+\int_\eta^\infty dl_-l_-^{-1-\epsilon}\Big(\int_0^\infty dl_+l_+^{-1-\epsilon}-\int_{t^2l_-}^\infty dl_+l_+^{-1-\epsilon}\Big),
\end{aligned}
$$

If we use the rapidity regulator only in the collinear Wilson line,

$$W_n = \sum_{\text{perm}} \exp\Big[-\frac{g}{\overline{n}\cdot\mathcal{P}+\Delta+i0}\overline{n}\cdot A_n\Big],$$

we obtain the final result as

$$M_{\text{coll}} = \frac{\alpha_s C_F}{2\pi}\Big[\frac{1}{\epsilon_{\text{UV}}^2}+\frac{1}{\epsilon_{\text{UV}}}\Big(\frac{3}{2}+\ln\frac{\mu^2}{Q^2t^2}\Big)+\frac{3}{2}\ln\frac{\mu^2}{Q^2t^2}+\frac{1}{2}\ln^2\frac{\mu^2}{Q^2t^2}+\frac{13}{2}-\frac{3}{4}\pi^2\Big]$$

Unintegrated jet function

$$J_{\text{int}}(tQ,\mu) = \int dp^2 J_{\text{unint}}(tQ,\mu;p^2)$$

$$\tilde{M}_b = \frac{\alpha_s C_F}{2\pi}\frac{(e^{\gamma_E}\mu^2)^\epsilon}{\Gamma(1-\epsilon)}\frac{1}{p^2}\int dl_+ dl_- \delta((l-p)^2)(l_+l_-)^{-\epsilon}\frac{Q-l_-}{l_-}\Theta_{\rm cone}$$

$$= \frac{\alpha_s C_F}{2\pi}\frac{(e^{\gamma_E}\mu^2)^\epsilon}{\Gamma(1-\epsilon)}\frac{1}{(p^2)^{1+\epsilon}}\int_{p^2/(p^2+Q^2t^2)}^{Q^2t^2/(p^2+Q^2t^2)} dy y^{-1-\epsilon}(1-y)^{1-\epsilon} \qquad (y = l_-/Q)$$

$$= A\delta(p^2) + [B(p^2)]_{M^2}$$

$$= \frac{\alpha_s C_F}{2\pi}\Big\{\delta(p^2)\Big[\frac{1}{2\epsilon_{\rm IR}^2} + \frac{1}{\epsilon_{\rm IR}}\Big(1+\frac{1}{2}\ln\frac{\mu^2}{Q^2t^2}\Big) + \frac{1}{4}\ln^2\frac{\mu^2}{Q^2t^2} + 2 - \frac{5}{4}\pi^2$$

$$+ \ln\frac{\mu^2}{M^2} - \frac{1}{2}\ln^2\frac{Q^2t^2}{M^2} - 2\ln\frac{Q^2t^2}{Q^2t^2+M^2}\Big] + \Big[\frac{1}{p^2}\Big(-\frac{Q^2t^2-p^2}{Q^2t^2+p^2} + \ln\frac{Q^2t^2}{p^2}\Big]_{M^2}\Big\}$$

collinear: $(l_-, l_\perp, l_+) \sim Q(1, \lambda, \lambda^2)$

zero bin: $(l_-, l_\perp, l_+) \sim Q(\lambda^2, \lambda^2, \lambda^2)$

$0 < p^2 < Q^2 t^2 \qquad \longrightarrow p^2 << Q^2 \mathcal{O}(\lambda^2)$

$$M_b^0 = \frac{\alpha_s C_F}{2\pi} \frac{(e^{\gamma_E}\mu^2)^\epsilon}{\Gamma(1-\epsilon)} \frac{1}{(p^2)^{1+\epsilon}} \int_{p^2/Q^2t^2}^{\infty} dy y^{-1-\epsilon}$$

$$\rightarrow C\delta(p^2)$$

$$\int_0^\infty \frac{dp^2}{(p^2)^{1+\epsilon}} \int_{p^2/Q^2t^2}^{\infty} dy y^{-1-\epsilon}$$

$$= \int_0^{\eta^2} \frac{dp^2}{(p^2)^{1+\epsilon}} \Big[\int_0^\infty dy y^{-1-\epsilon} - \int_0^{p^2/Q^2t^2} dy y^{-1-\epsilon} \Big] + \int_{\eta^2}^{\infty} \frac{dp^2}{(p^2)^{1+\epsilon}} \int_{p^2/Q^2t^2}^{\infty} dy y^{-1-\epsilon}$$

$$M_b^0 = \frac{\alpha_s C_F}{2\pi}\delta(p^2)\Big[\frac{1}{2}\Big(\frac{1}{\epsilon_{\rm UV}} - \frac{1}{\epsilon_{\rm IR}}\Big)^2 + \frac{1}{2}\Big(\frac{1}{\epsilon_{\rm UV}} - \frac{1}{\epsilon_{\rm IR}}\Big) + \ln t^2\Big]$$

$$\begin{aligned} M_b \quad &= \quad \tilde{M}_b - M_b^0 = \frac{\alpha_s C_F}{2\pi}\Big\{\delta(p^2)\Big[-\frac{1}{2\epsilon_{\rm UV}^2} + \frac{1}{\epsilon_{\rm UV}\epsilon_{\rm IR}} + \frac{1}{\epsilon_{\rm IR}}\Big(1 + \ln\frac{\mu}{Q}\Big) - \frac{1}{\epsilon_{\rm UV}}\ln t \\ &\quad + \ln\frac{\mu^2}{Q^2t^2} + \frac{1}{4}\ln^2\frac{\mu^2}{Q^2t^2} + +2 + 2\ln 2 - \frac{5}{24}\pi^2\Big] \\ &\quad + \Big[\frac{1}{p^2}\Big(-\frac{Q^2t^2 - p^2}{Q^2t^2 + p^2} + \ln\frac{Q^2t^2}{p^2}\Big)\Big]_{M^2}\Big\} \end{aligned}$$

$$
\begin{aligned}
M_n^{\rm uninteg}(p^2,\mu) &= 2(M_a+M_b)+M_c+M_d \\
&= \frac{\alpha_s C_F}{2\pi}\Big\{\delta(p^2)\Big(\frac{1}{\epsilon_{\rm UV}^2}+\frac{1}{\epsilon_{\rm UV}}\Big(\frac{3}{2}+\ln\frac{\mu^2}{Q^2t^2}\Big)+\frac{3}{2}\ln\frac{\mu^2}{Q^2t^2}+\frac{1}{2}\ln^2\frac{\mu^2}{Q^2t^2} \\
&\quad +\frac{7}{2}+3\ln 2-\frac{5\pi^2}{12}\Big)+\Big[\frac{2}{p^2}\ln\frac{Q^2t^2}{p^2}+\frac{1}{p^2}\Big(\frac{3}{2}-\frac{3Q^2t^2}{p^2+Q^2t^2}\Big)\Big]_{M^2=Q^2t^2}\Big\}.
\end{aligned}
$$

Integrated result

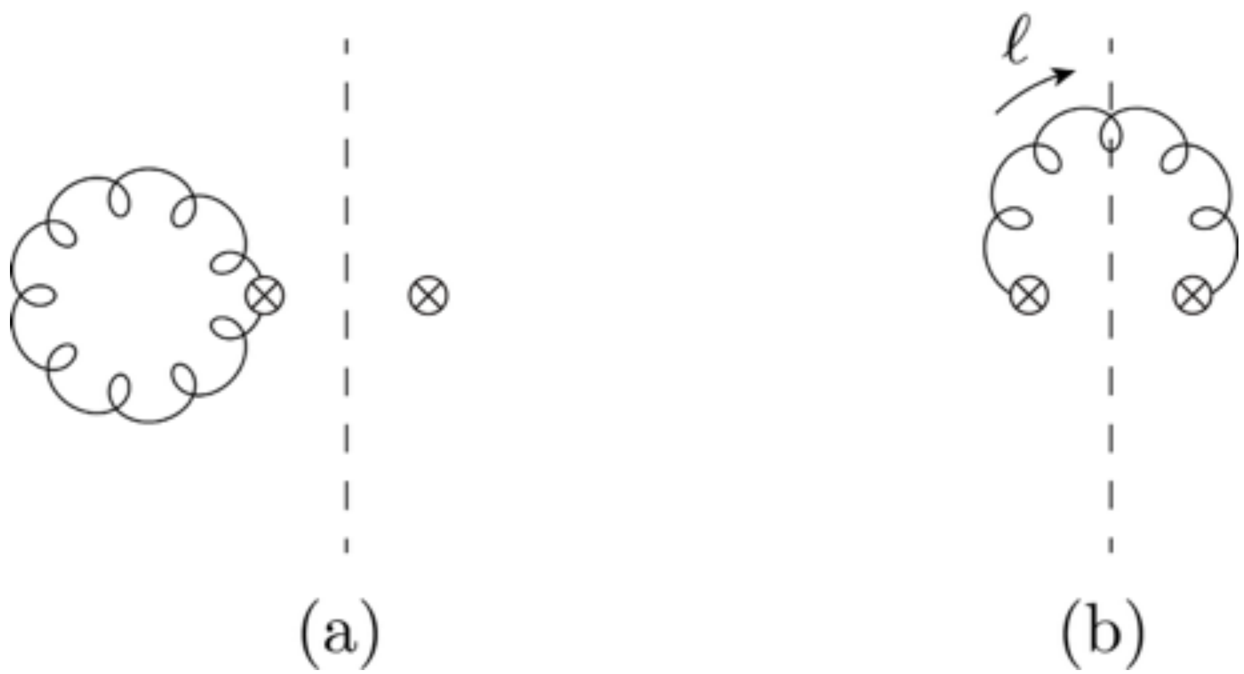
$$
\begin{aligned}
M_{\rm coll} &= 2(M_a+M_b)+M_c+M_d \\
&= \frac{\alpha_s C_F}{2\pi}\Big[\frac{1}{\epsilon_{\rm UV}^2}+\frac{1}{\epsilon_{\rm UV}}\Big(\frac{3}{2}+\ln\frac{\mu^2}{Q^2t^2}\Big)+\frac{3}{2}\ln\frac{\mu^2}{Q^2t^2}+\frac{1}{2}\ln^2\frac{\mu^2}{Q^2t^2}+\frac{7}{2}+3\ln 2-\frac{5\pi^2}{12}\Big]
\end{aligned}
$$

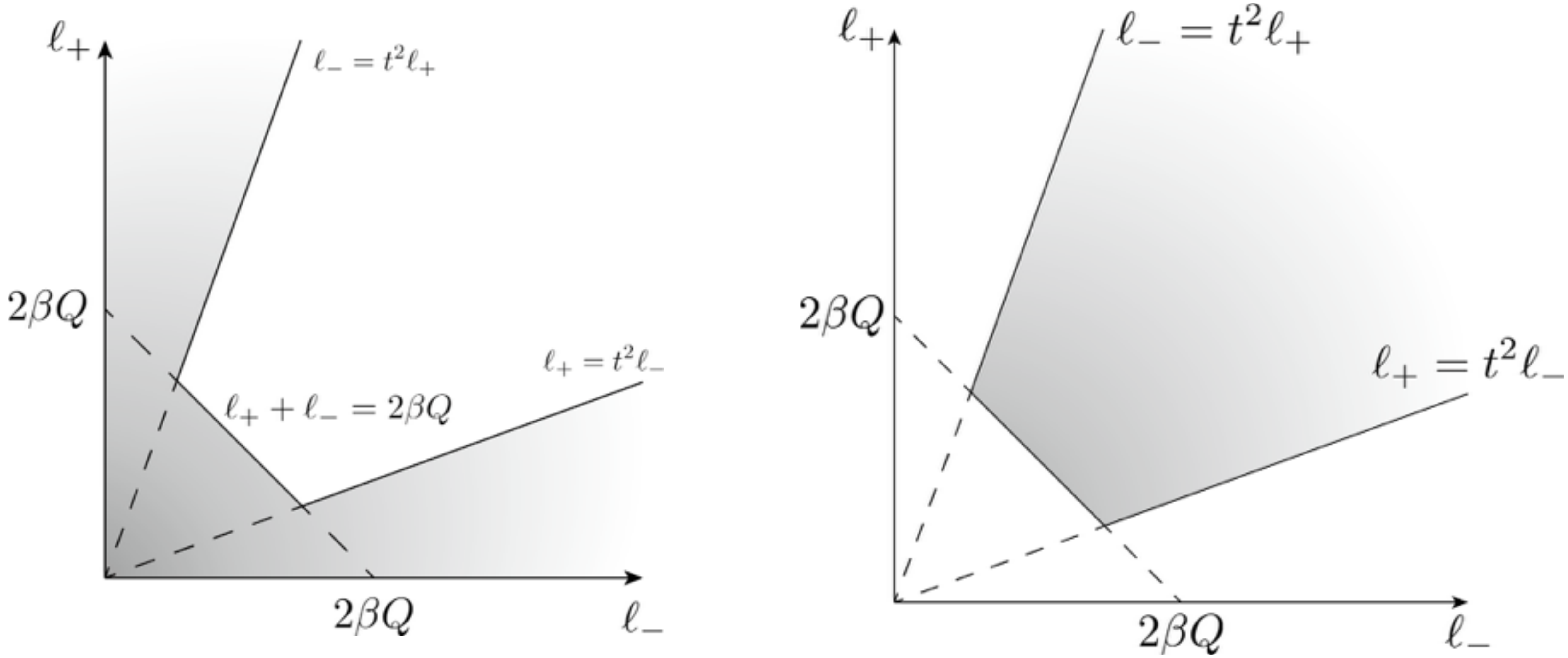
Conclusion

- The (integrated, unintegrated) jet functions are IR finite.
- So is the soft function.
- $J_{\rm int}(tQ,\mu) = \int dp^2 J_{\rm unint}(tQ,\mu;p^2)$
- Other jet algorithms in progress.

$$
\Theta_{\text{soft}} = \begin{cases} \Theta\Big(t^2 > \dfrac{l_+}{l_-}\Big), & (n \text{ jet}), \\ \Theta\Big(t^2 > \dfrac{l_-}{l_+}\Big), & (\overline{n} \text{ jet}), \\ \Theta(l_+ + l_- < 2\beta Q), & (\text{outside the jet}). \end{cases}
$$

$$
\beta \sim \lambda^2
$$




ℓ_+
$\ell_- = t^2\ell_+$
$2\beta Q$
$\ell_+ = t^2\ell_-$
$\ell_+ + \ell_- = 2\beta Q$
$2\beta Q$
ℓ_-
ℓ_+
$\ell_- = t^2\ell_+$
$2\beta Q$
$\ell_+ = t^2\ell_-$
$2\beta Q$
ℓ_-