Institute for Research in Electronics & Applied Physics

Resonant Excitation of Envelope Modes as an Emittance Diagnostic in High-Intensity Circular Accelerators

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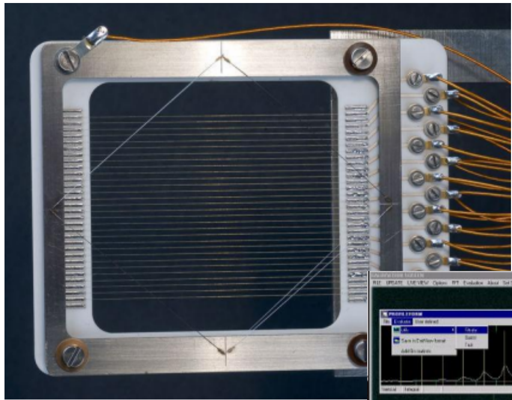
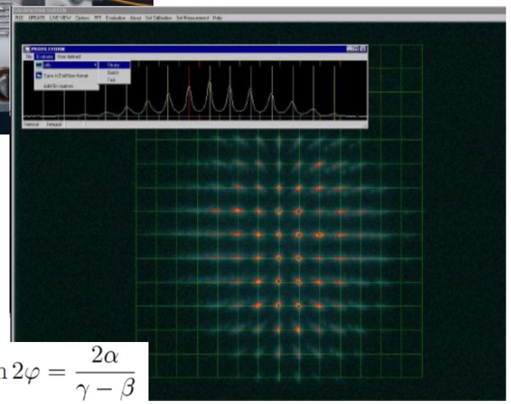
Outline

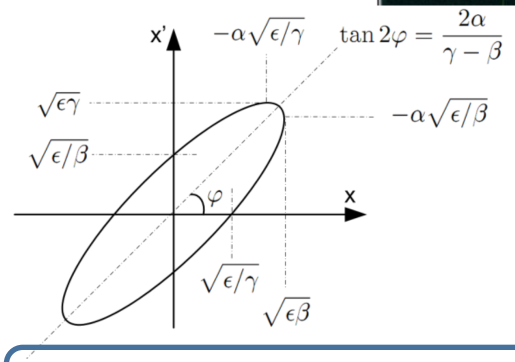
- Some Traditional Methods of Measuring Emittance
- Emittance Dependence on Envelope Mode Frequency
- Experimental Excitation of Envelope Resonances at the University of Maryland Electron Ring (UMER)
- Using Simulations to Infer Emittance from Experimental Measurements
- Application to Other High-Intensity Circular Accelerators

Measuring Emittance

- Wire Scanners
- Pepperpots
- Quad Scans







$$\gamma x^2 + 2\alpha x x' + \beta x'^2 = \epsilon_x$$

Uli Raich. USPAS Lecture Notes, http://uspas.fnal.gov/materials/09UNM/Emittance.pdf

My Idea

- New method of measuring emittance
  - Sensitive
  - Non-invasive
  - Works for high-intensity beams in circular accelerators

- Now: brief introduction to envelope modes

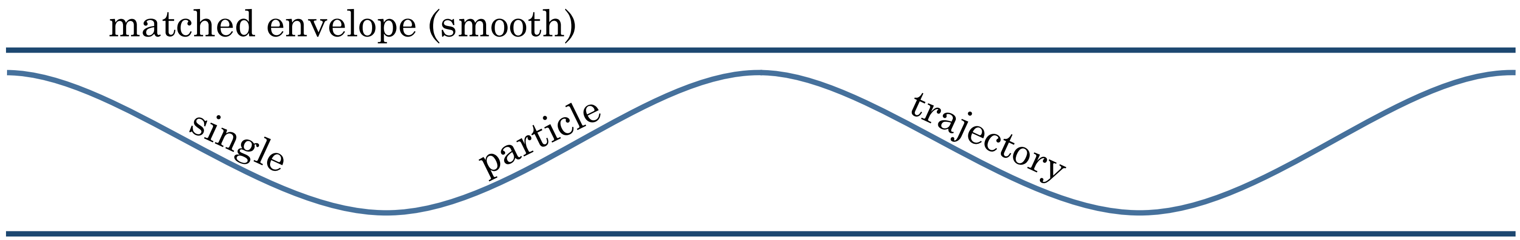
Beam Envelope in the Smooth Approximation

- For simplicity, approximate A-G lattice by an average focusing force

Described by the rms Envelope Equations:

$$X'' + \kappa_x(s)X - \frac{2K}{X+Y} - \frac{\varepsilon_x^2}{X^3} = 0$$

$$Y'' + \kappa_y(s)Y - \frac{2K}{X+Y} - \frac{\varepsilon_y^2}{Y^3} = 0$$



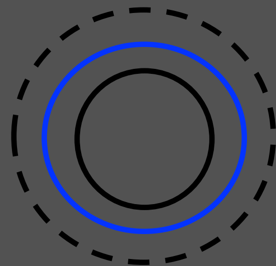
Envelope Modes

- Perturbations to the matched envelope solutions of the rms Envelope Equations drive envelope mode oscillations

Equations of Motion:

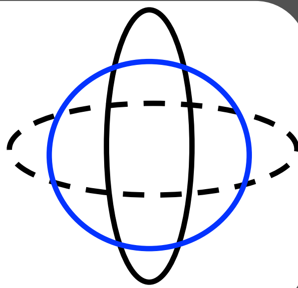
$$R_+'' + k_+^2 R_+ = 0$$

"Breathing"



$$R_-'' + k_-^2 R_- = 0$$

"Quadrupole"



"1-D" Simple Harmonic Motion

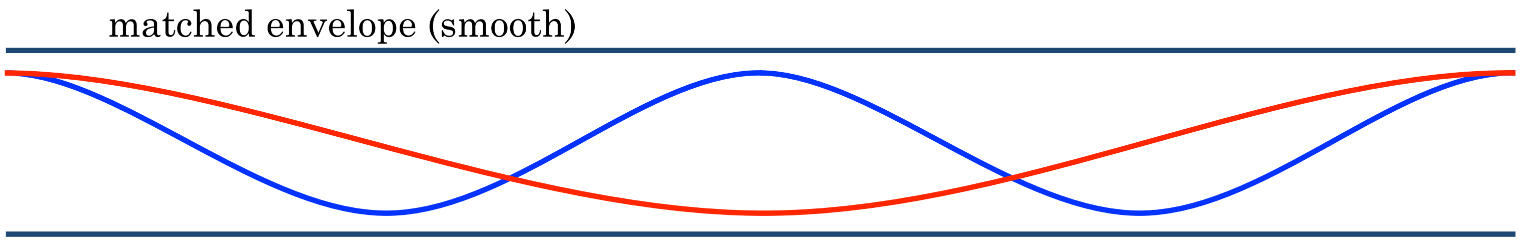
Mode Coordinates:

$$R_+ \equiv \delta X + \delta Y$$

$$R_- \equiv \delta X - \delta Y$$

Space-Charge Effects

- Phase advance can be used as a measure of space-charge intensity

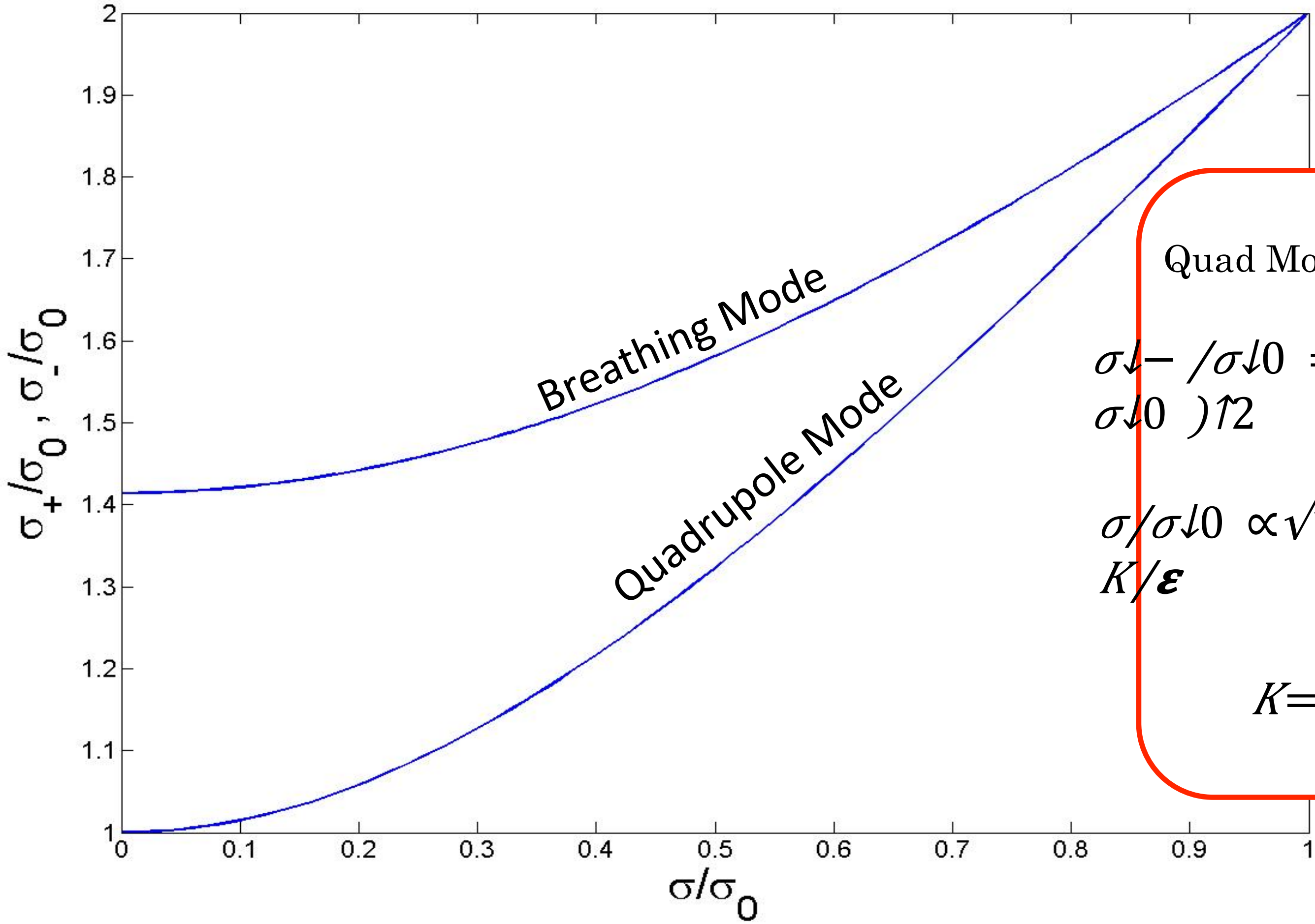


- Undepressed Single Particle Trajectory ~ σ_0
- Space-Charge Depressed Single Particle Trajectory ~ σ

So in this case, normalized phase advance is $\sigma/\sigma_0 = 1/2 = 0.5$

Envelope Modes in the Smooth Approximation

Mode scaling as a function of space-charge (normalized phase advance)



Quad Mode Frequencies

$$\sigma_- /\sigma_0 = \sqrt{1+3(\sigma/\sigma_0)^2}$$

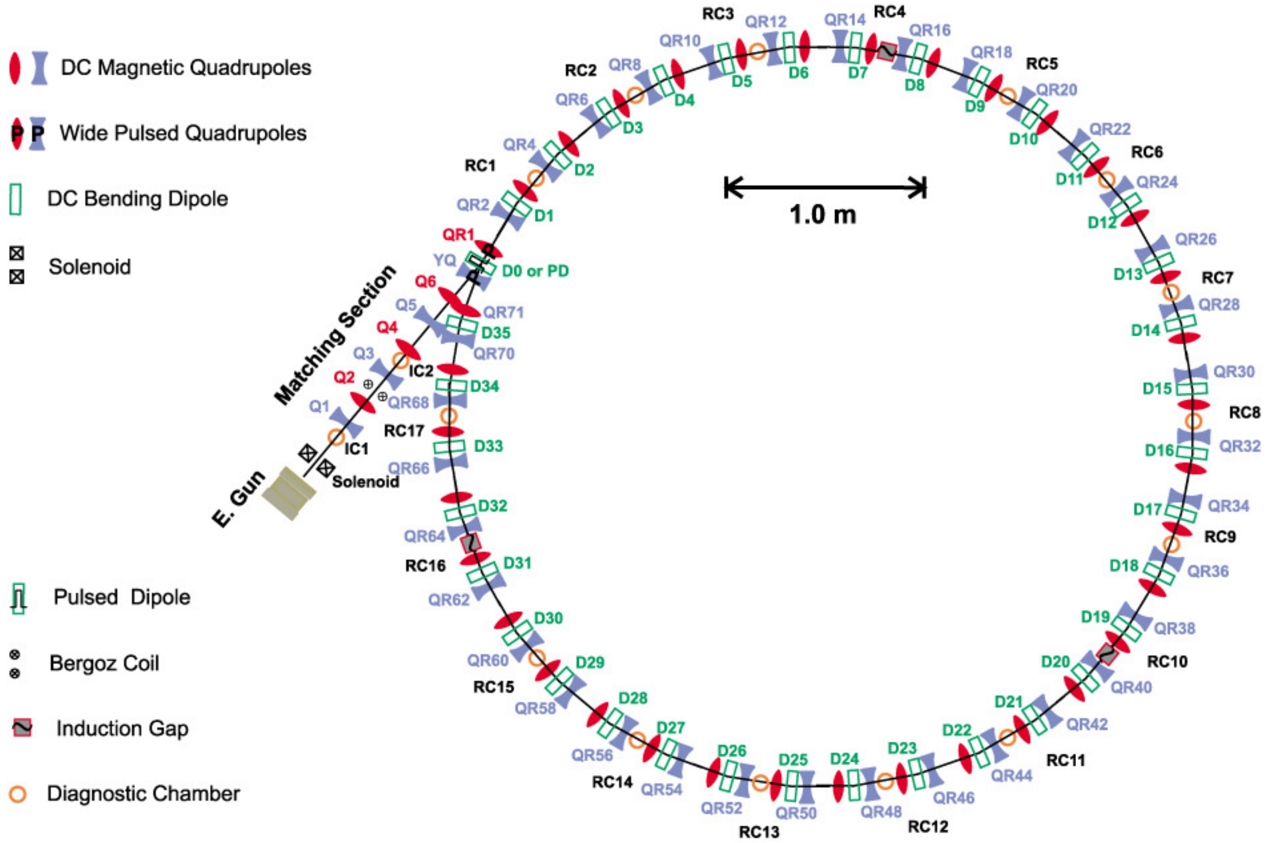
$$\sigma/\sigma_0 \propto \sqrt{1+(K/\boldsymbol{\varepsilon})^2} - K/\boldsymbol{\varepsilon}$$

$$K = I/I_0 \; 2/(\beta\gamma)^3$$

University of Maryland Electron Ring (UMER)

Robust, scalable research facility for intense-beam experiments

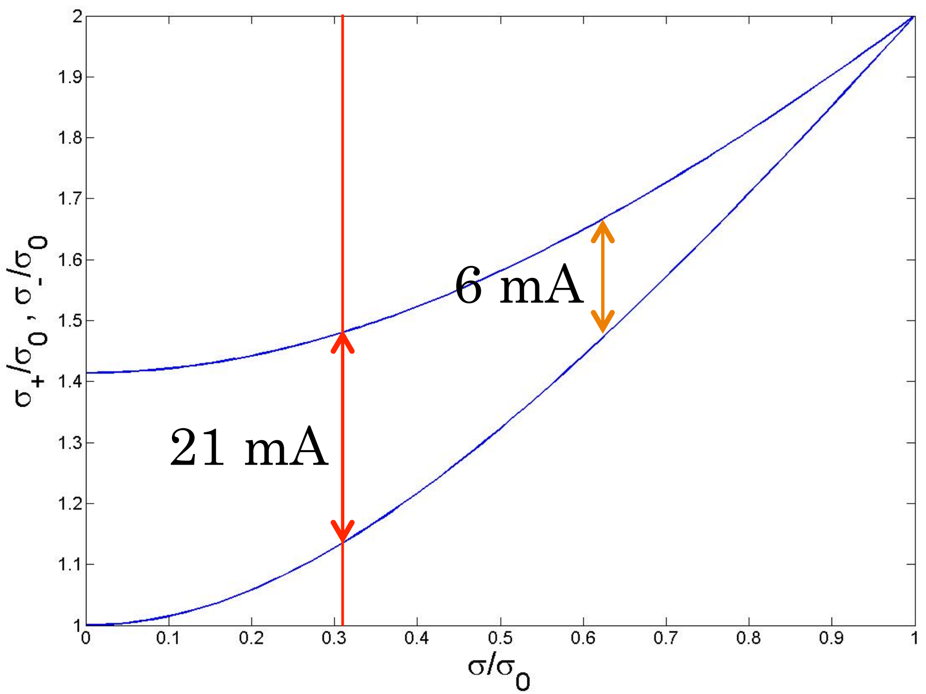
- Beam Energy: 10 keV
  $\Longrightarrow \beta \cong 0.2$
- 11.52 m Circumference
- Circulation Time: 197 ns
- Bunch Length: 100 ns
- 72 Quadrupole Focusing Magnets
- 14 Beam Diagnostic Ring Chambers (RCs)



Tunable UMER

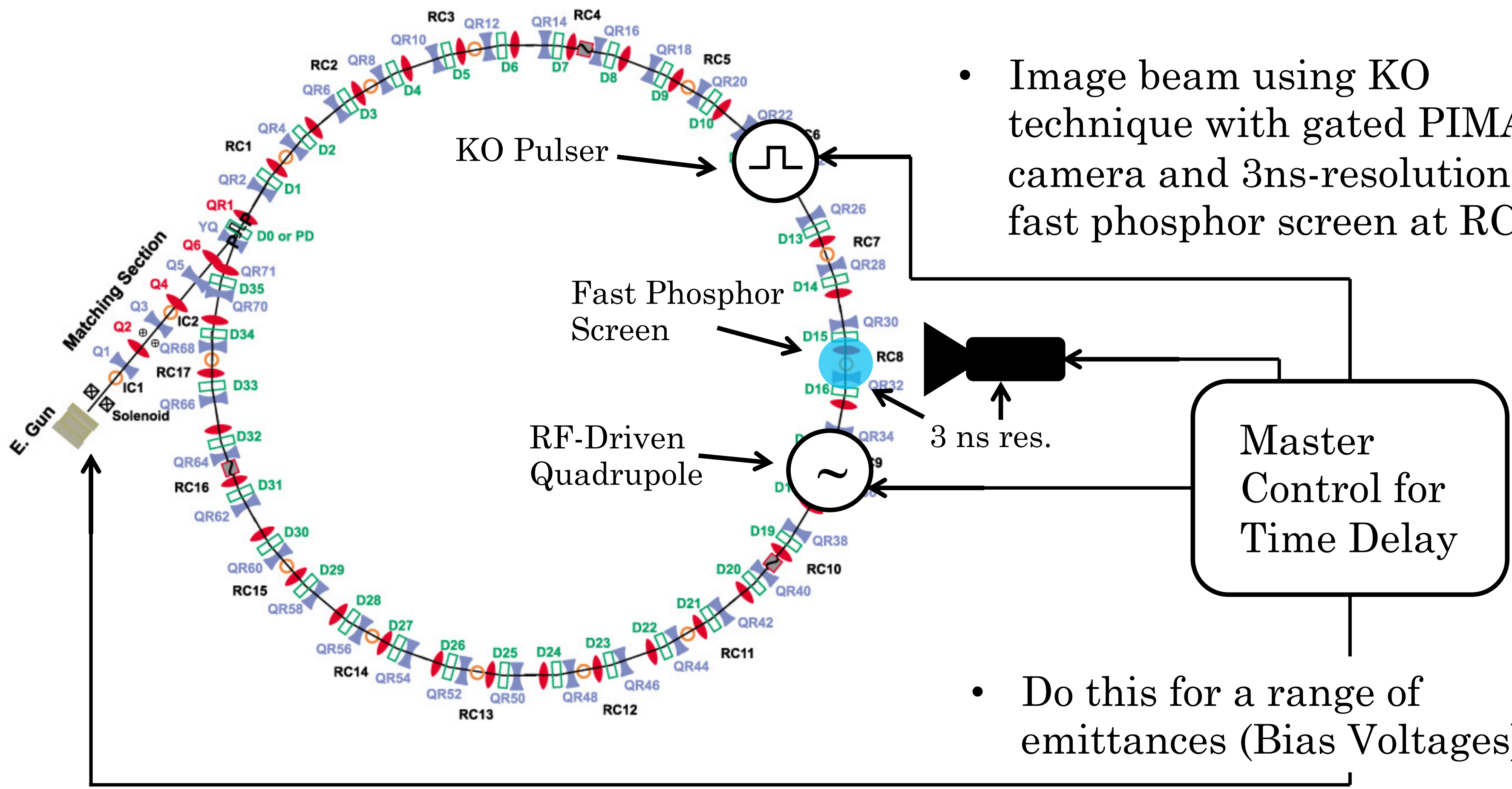
Aperture wheel Tunes Beam Current/ Intensity

Mask Setting	Expected Quad Mode Frequency
0.6 mA	65.5 MHz
6 mA	48.1 MHz
21 mA	36.9 MHz
40 mA	33.7 MHz



Experimental Outline

- Excite quadrupole mode with RF-driven electric quadrupole at RC9

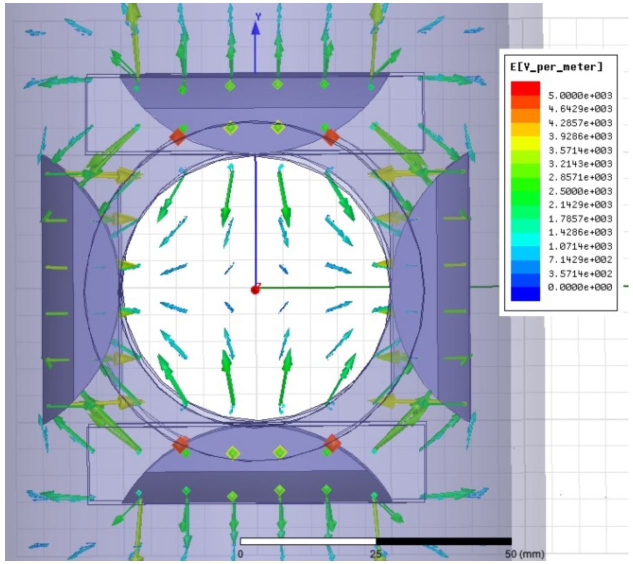


- Image beam using KO technique with gated PIMAX camera and 3ns-resolution fast phosphor screen at RC8

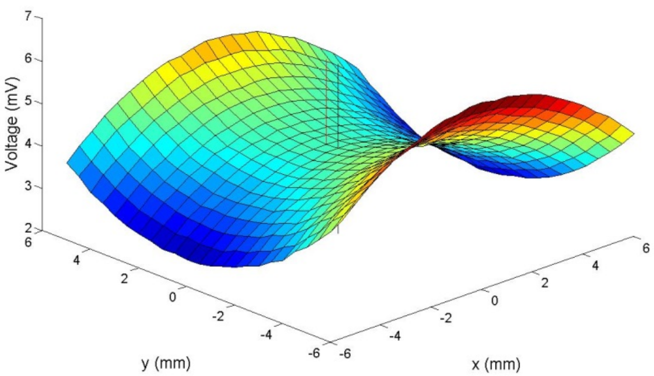
- Do this for a range of emittances (Bias Voltages)

Apparatus – Quadrupole

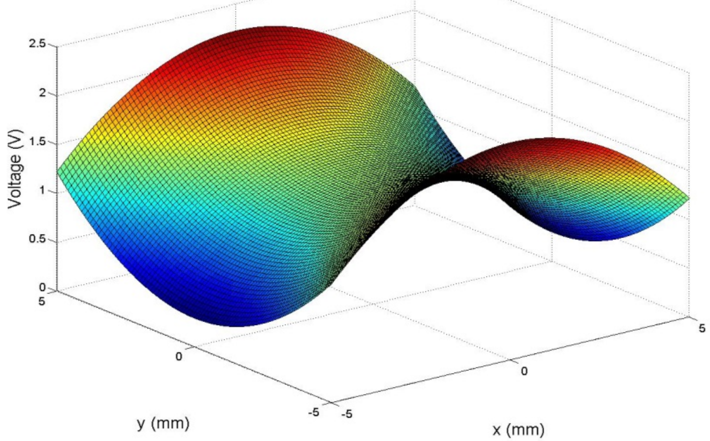
- I designed it in Solidworks
- I built it in the Machine Shop
- I simulated it with Maxwell 3D and Poisson Superfish
- I simulated fringe field particle tracing in Matlab



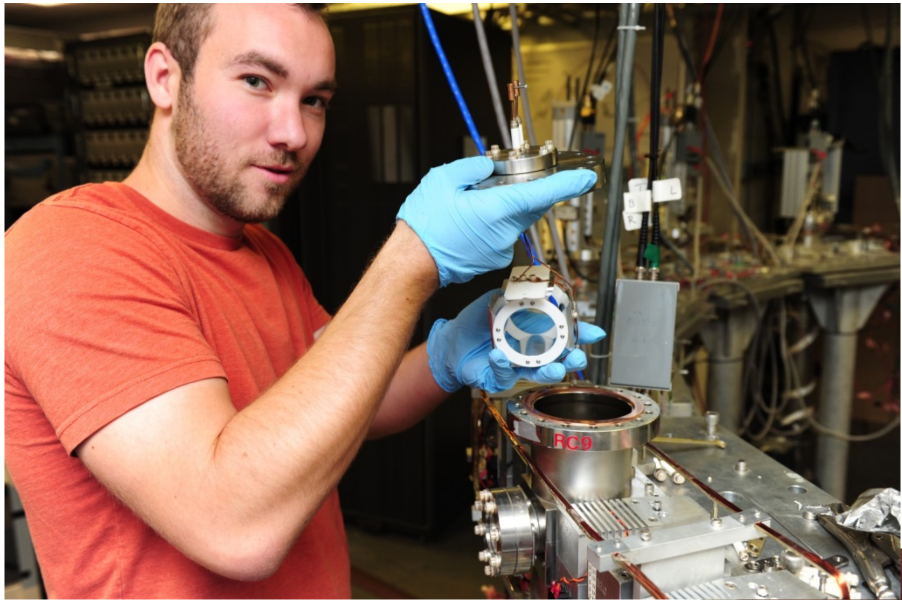
Measurement



Simulation

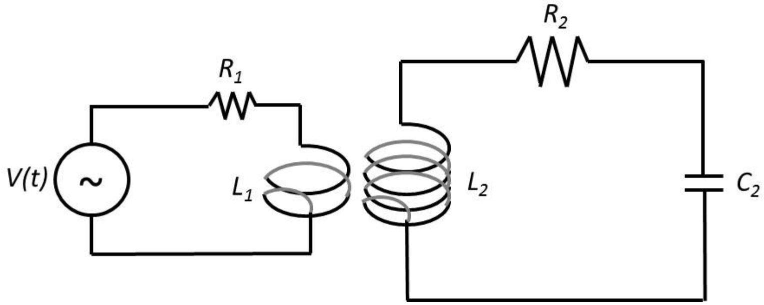


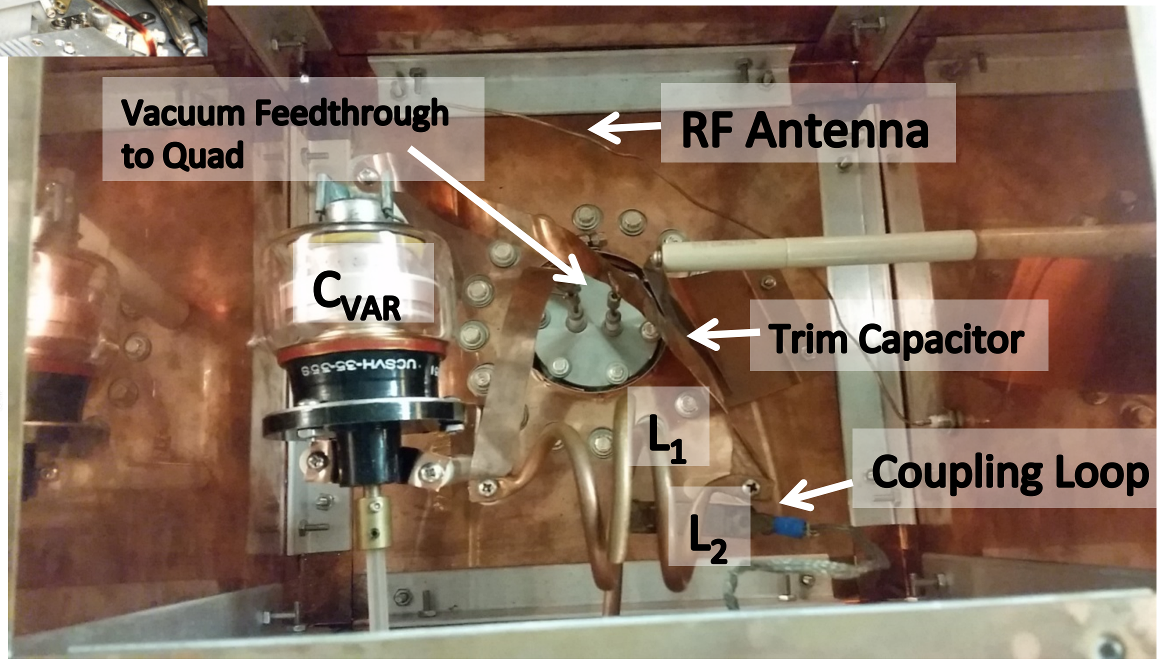
Apparatus – RF Box



- I designed, built, and soldered the RF box
- The quadrupole acts as a capacitor in the RF circuit

Simplified Circuit Diagram





Reminder – Goal of Experiment

- Find the RF driving frequencies at which envelope resonances occur

- Compare results with simulation

- Infer Emittance

Consider a periodically driven 1-D SHO (Reductionist Toy Model)

$$x+\omega_0^{2} x=A_0 \sin(\omega_k t+\varphi)\sum_n \delta(t-nT)$$

- ω_0 is the natural (resonant) frequency of the oscillator (env. mode)
- ω_k is the RF driving frequency of the quadrupole
- A_0 is the amplitude of the rf quadrupole
- n is the number of interactions with the quadrupole (or turn)
- T is the period between interactions (197 ns)

Analytic Solution

$$x(t) = -\frac{A_0}{\omega_0} \sum_{n} \cos(\omega_k nT + \varphi)\, \sin(\omega_0 (t - nT))$$

…Steady State Structure ($n \rightarrow \infty$)…

$$f_{k,1} = \Omega m_1 + f_0$$

$$f_{k,2} = \Omega m_2 - f_0$$

Resonance Conditions

$m_{1,2} = 1,2,3\ldots$

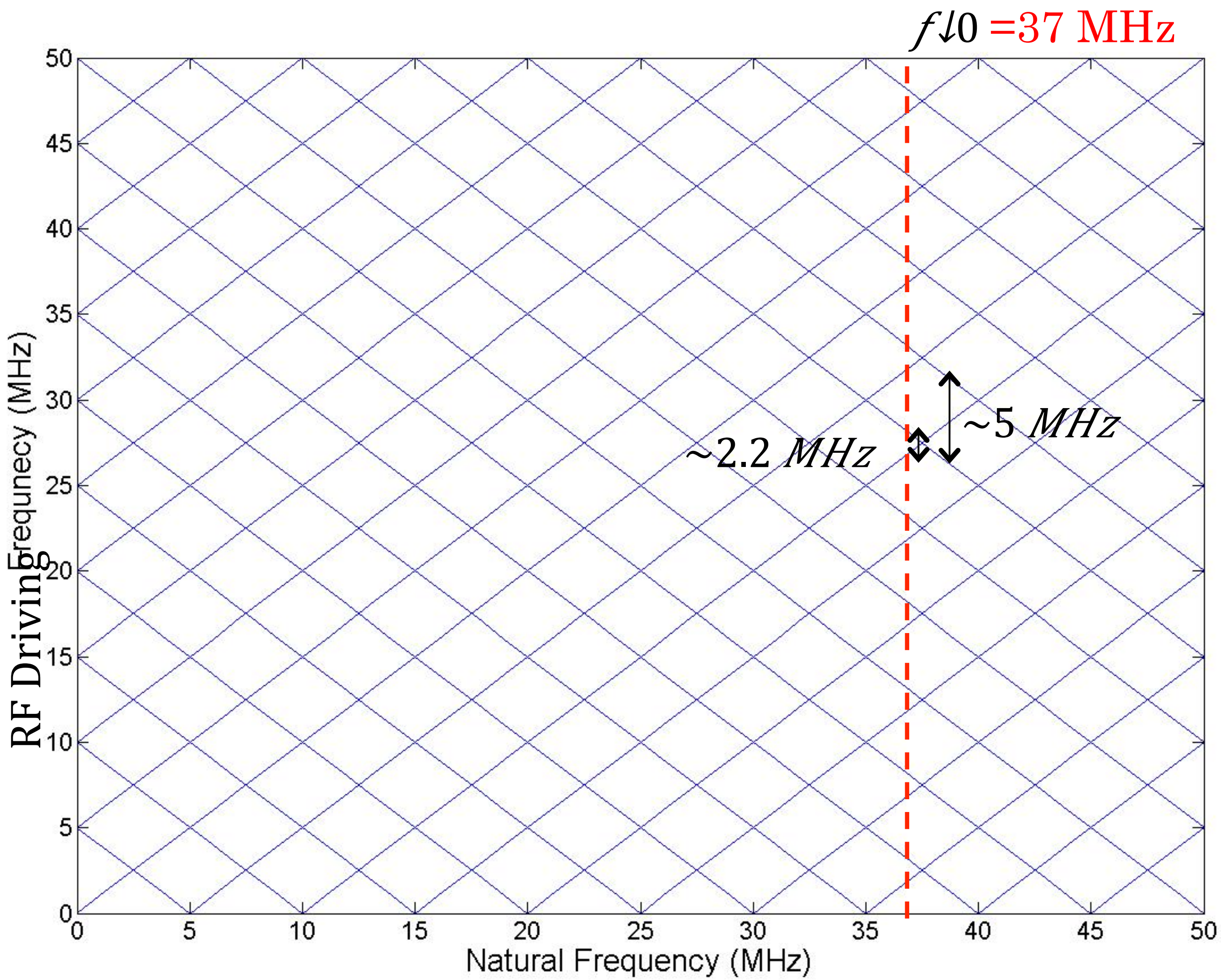
Three Frequency System

$f_0 = \text{"Unknown"} \approx 37\ MHz$

$f_k = \text{Known, Variable}$

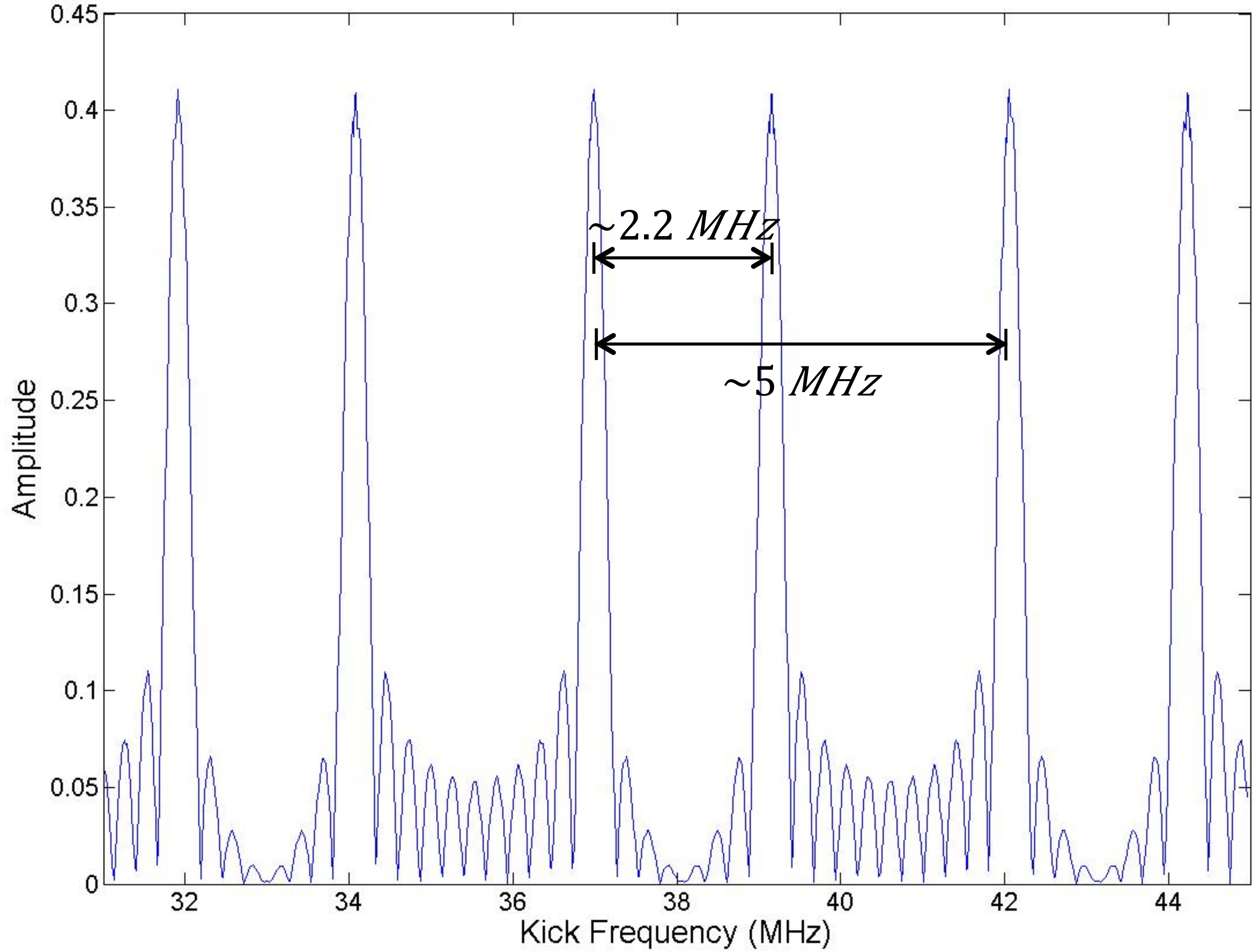
$\Omega \equiv 1/T = 5\ MHz = \text{Known}$

Resonance Lines (Dispersion Relation)



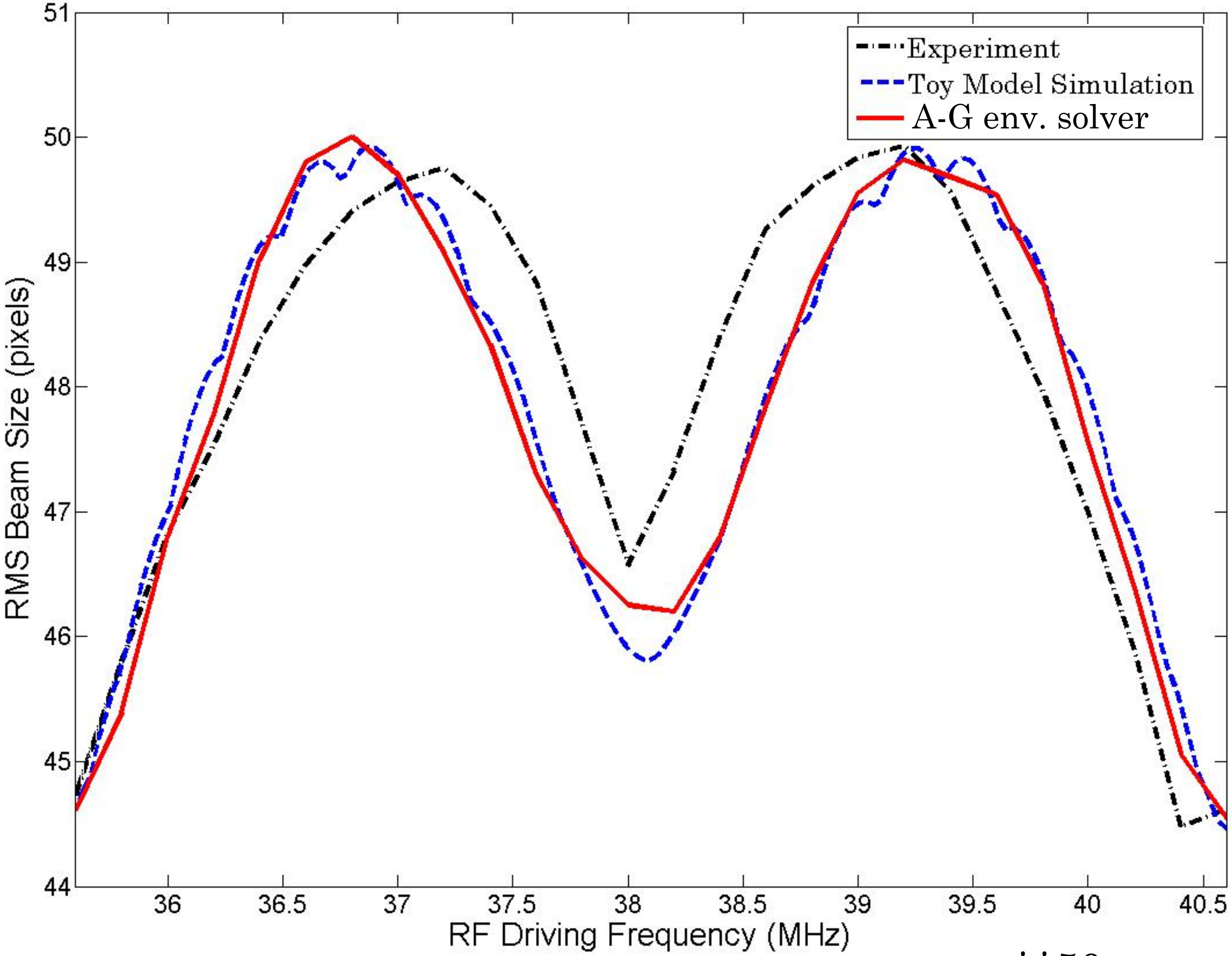
What Frequencies Do Resonances Occur?

$f_0 = 37$ MHz $= \omega_0 / 2\pi$



20^{th} Turn

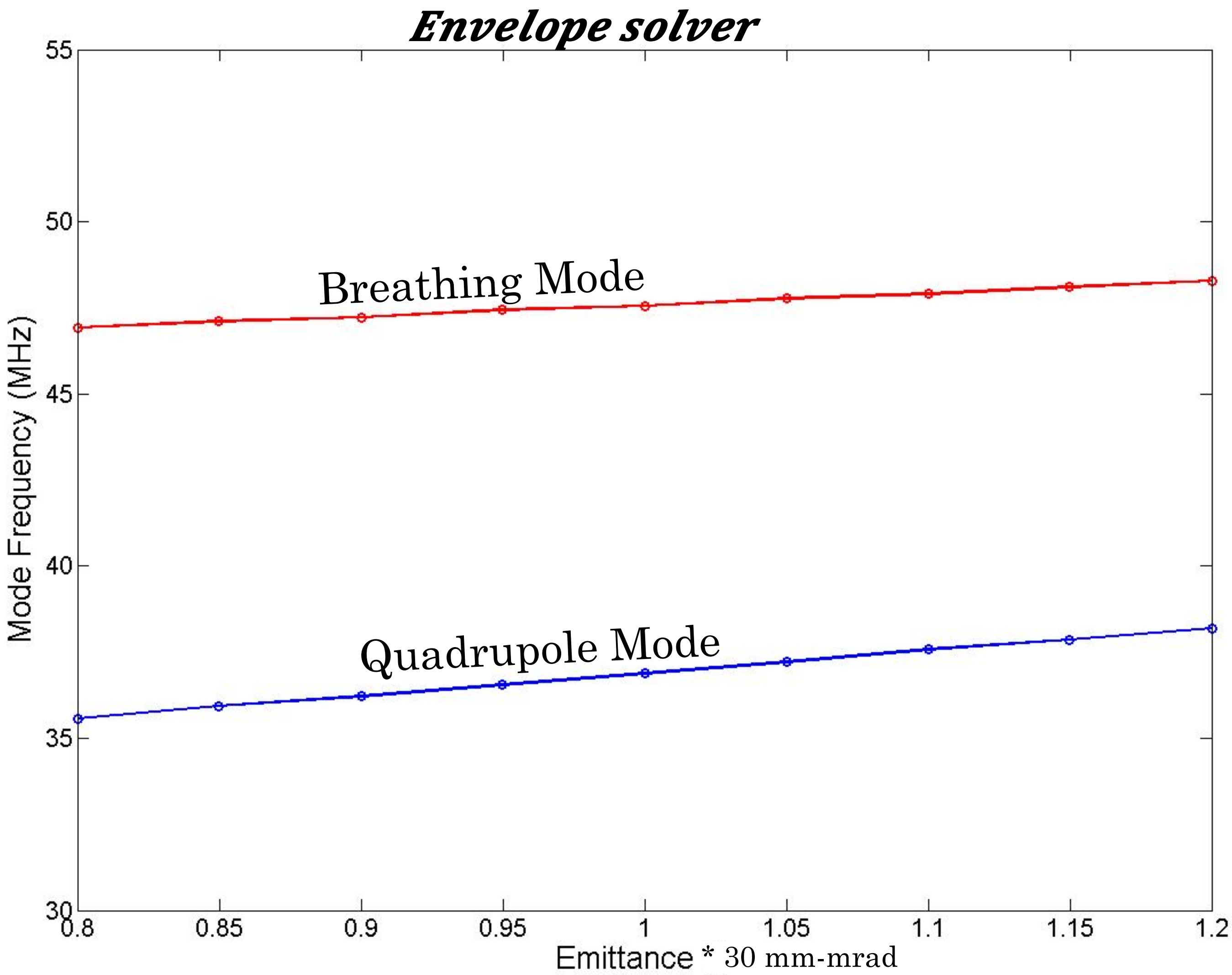
Agreement in Simulation and Experiment



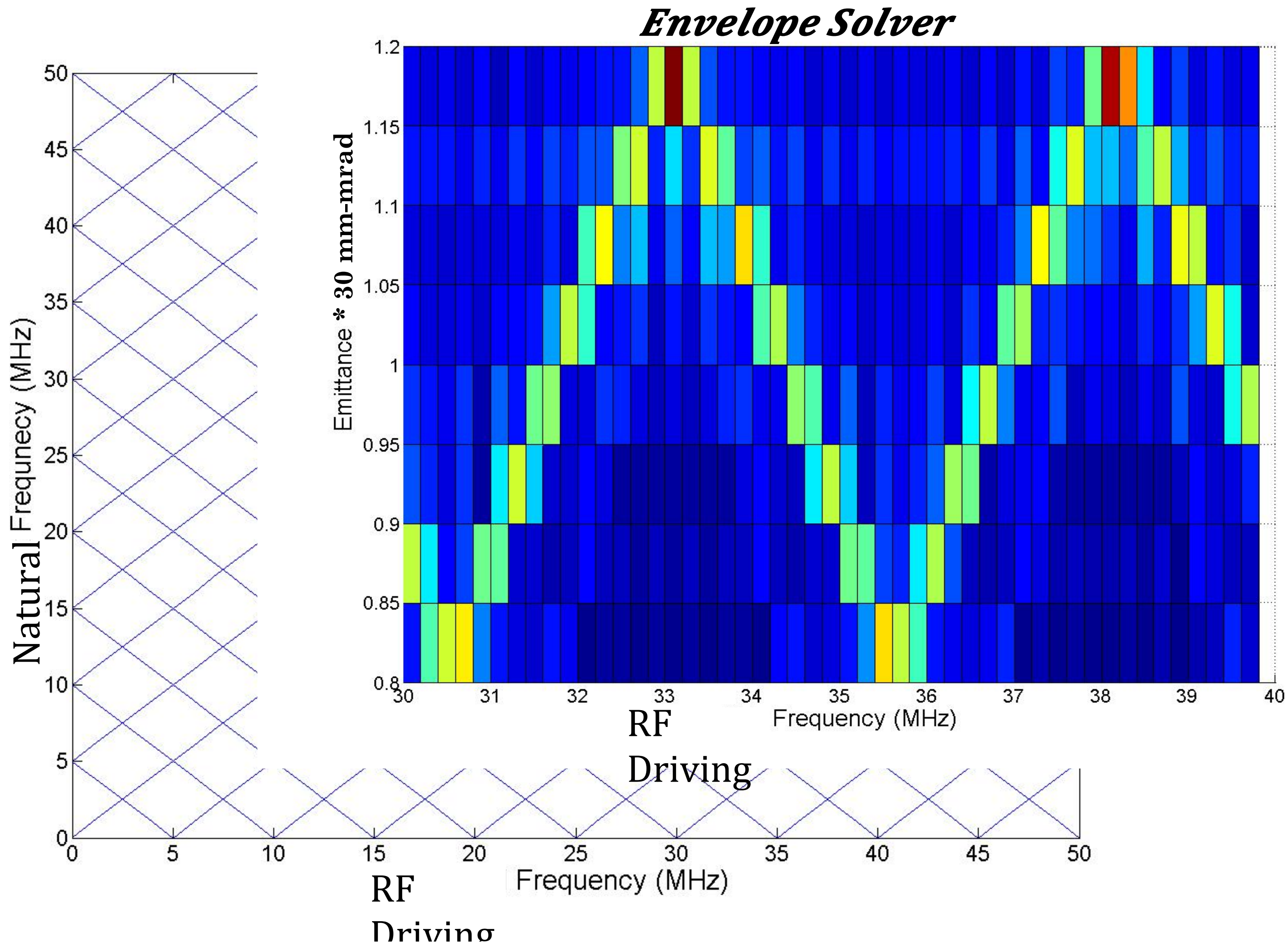
5th Turn

**50 μm per pixel

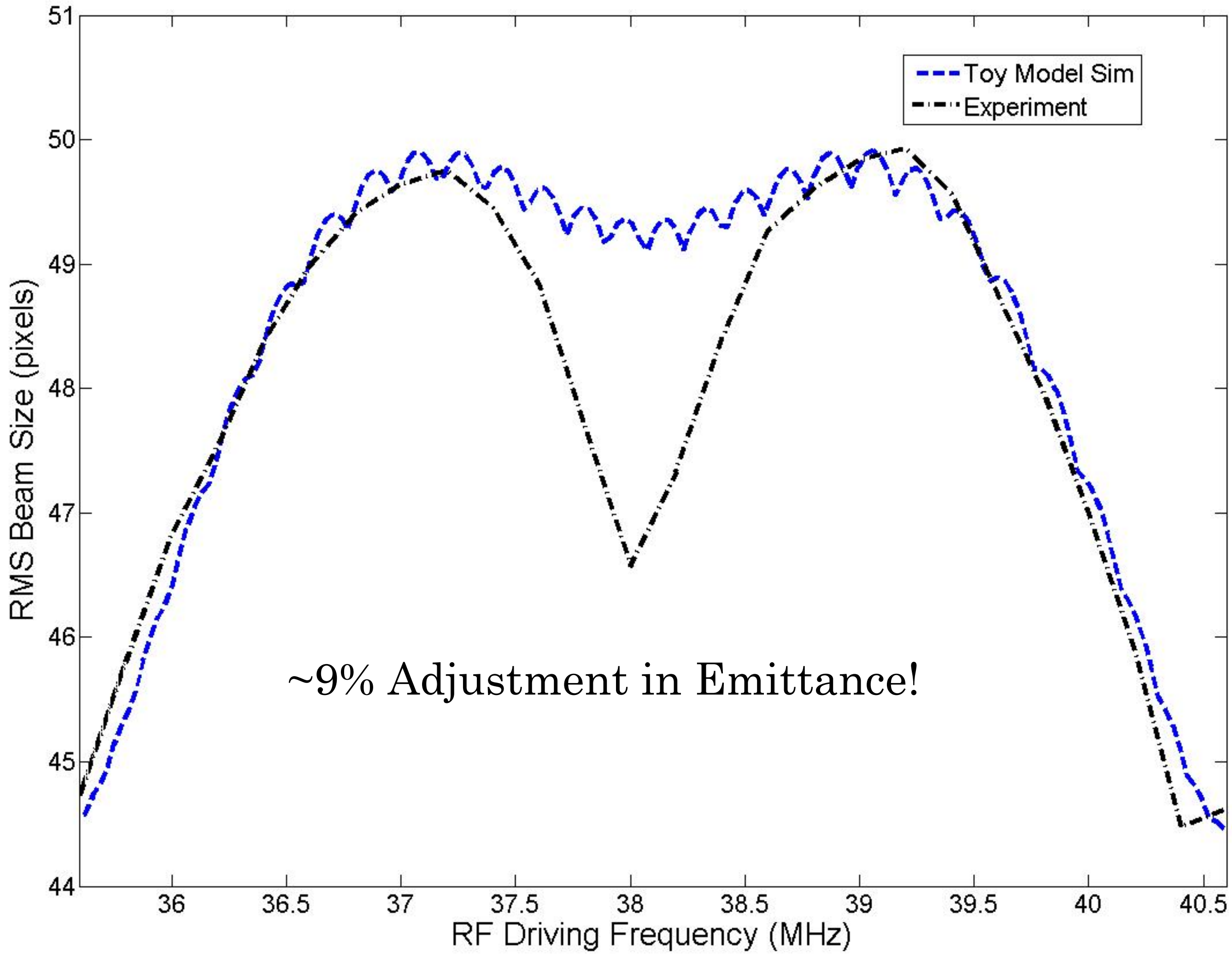
Resonance Frequencies vs Emittance



Resonance Frequencies vs Emittance



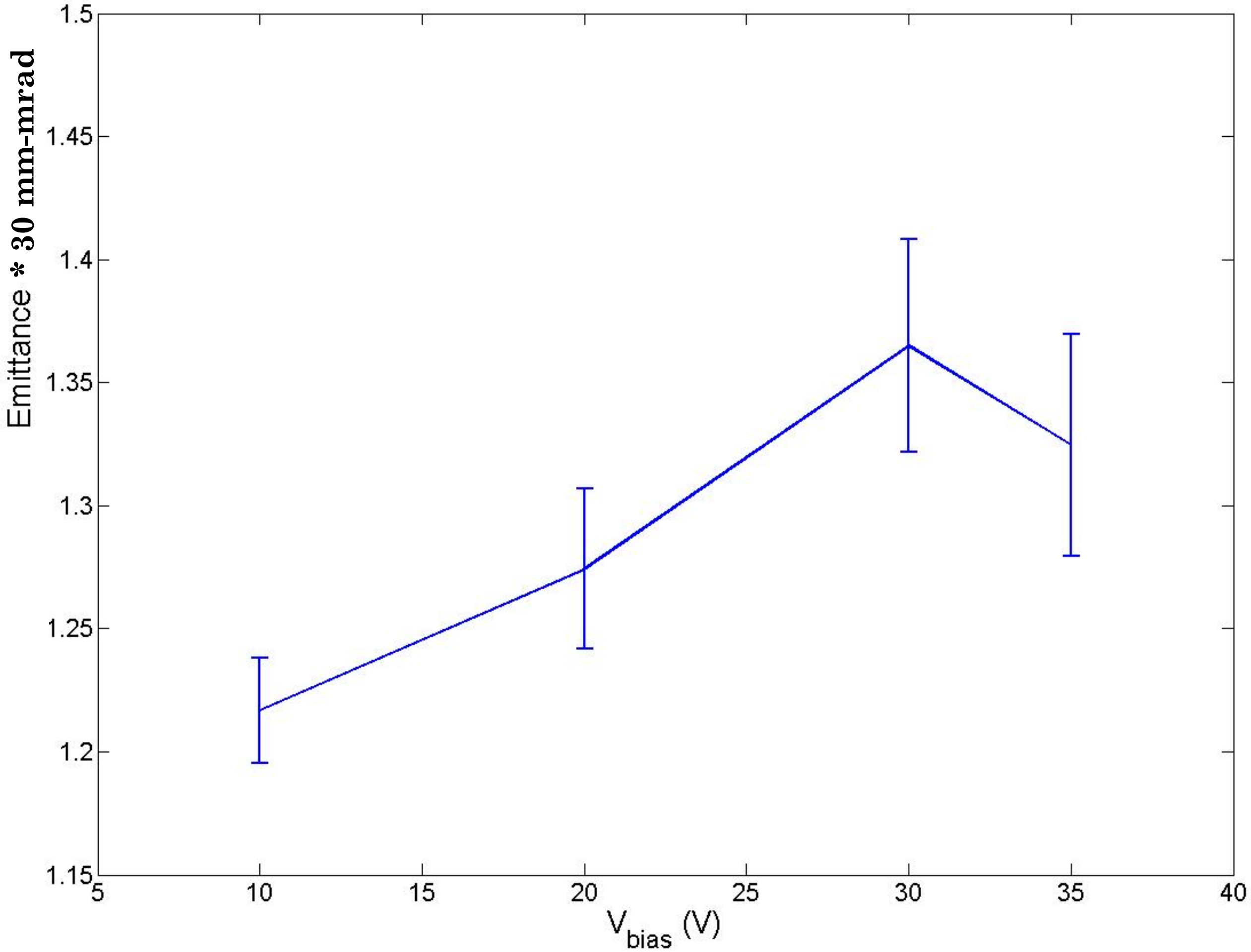
Agreement in Simulation and Experiment



5th Turn

**50 μm per pixel

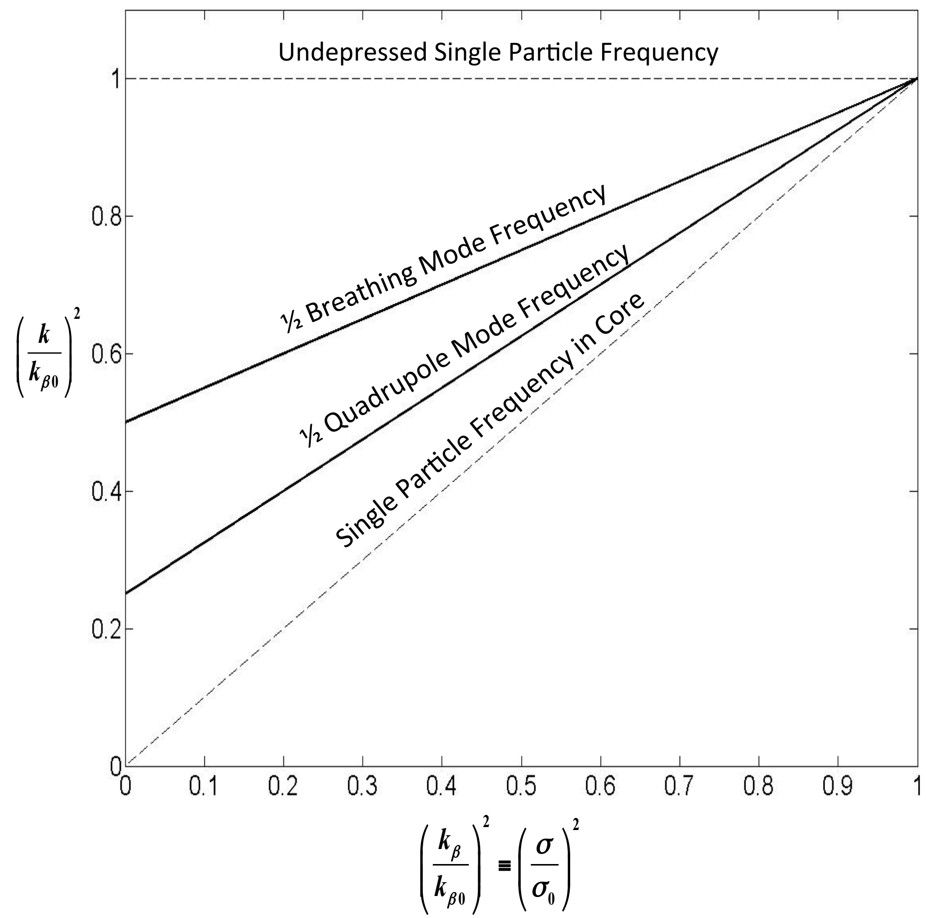
Emittance vs. Bias Voltage

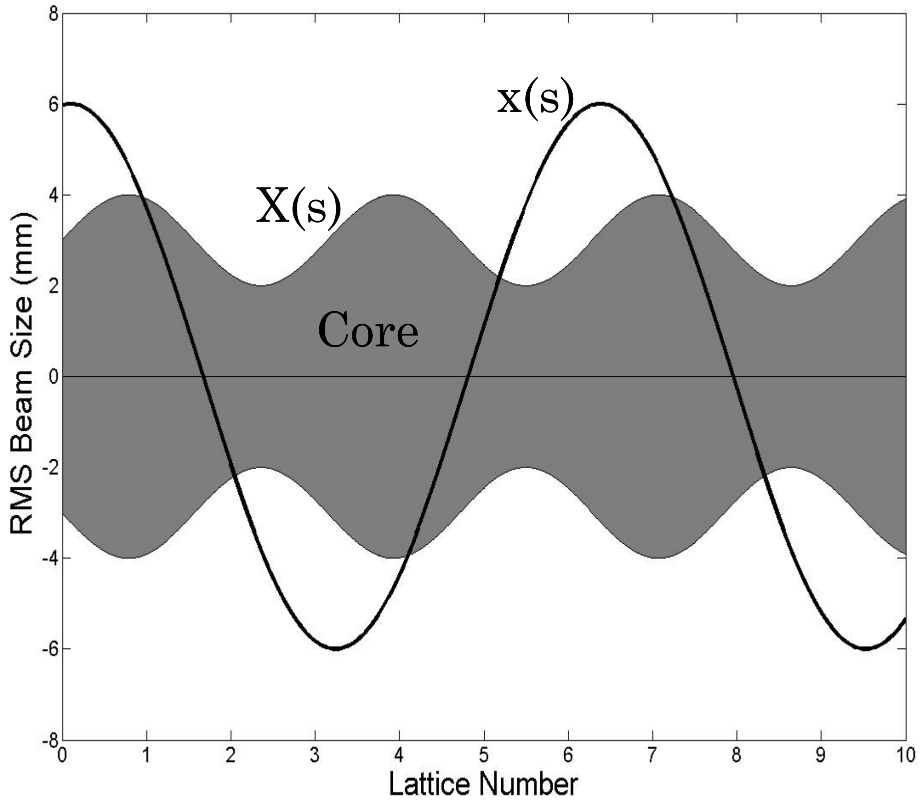


…Working on reducing error!

Measuring Frequency by Beam Halo

Resonance Conditions for Halo Growth





Conclusions

- Envelope mode frequencies can be used as a sensitive, non-invasive emittance diagnostic in high-intensity rings
- Measurements of multi-turn envelope excitations shows good agreement with simulation
- Improvements can be made by applying more kicks before measurement (and before space-charge bunch-end erosion)
- Halo formation can be used as a diagnostic in rings with longer beam lifetime

Acknowledgements

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- ***Misc. Advice and Consultations:*** Steve Lund, Luke Johnson, Aram Vartanyan

References

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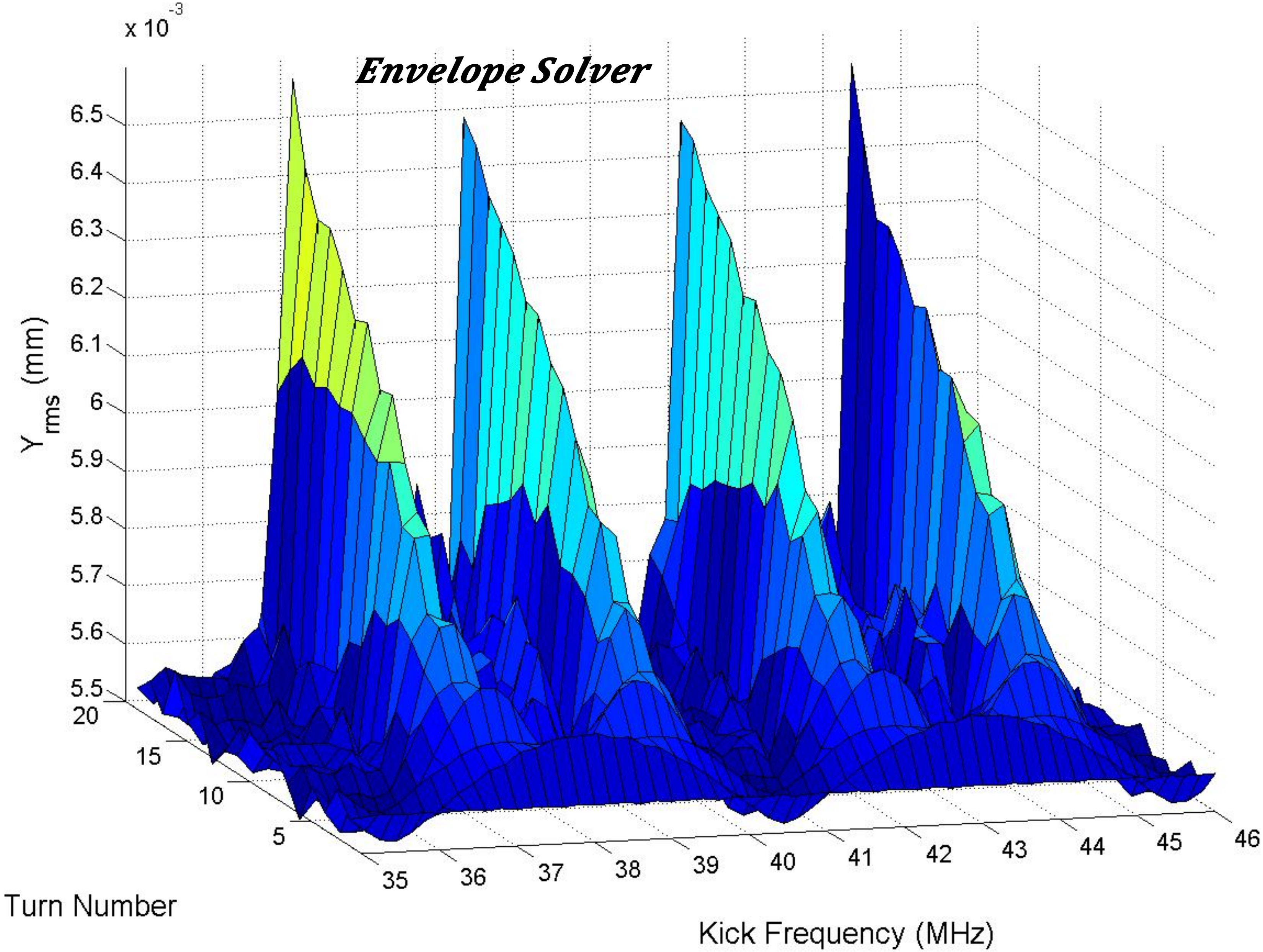
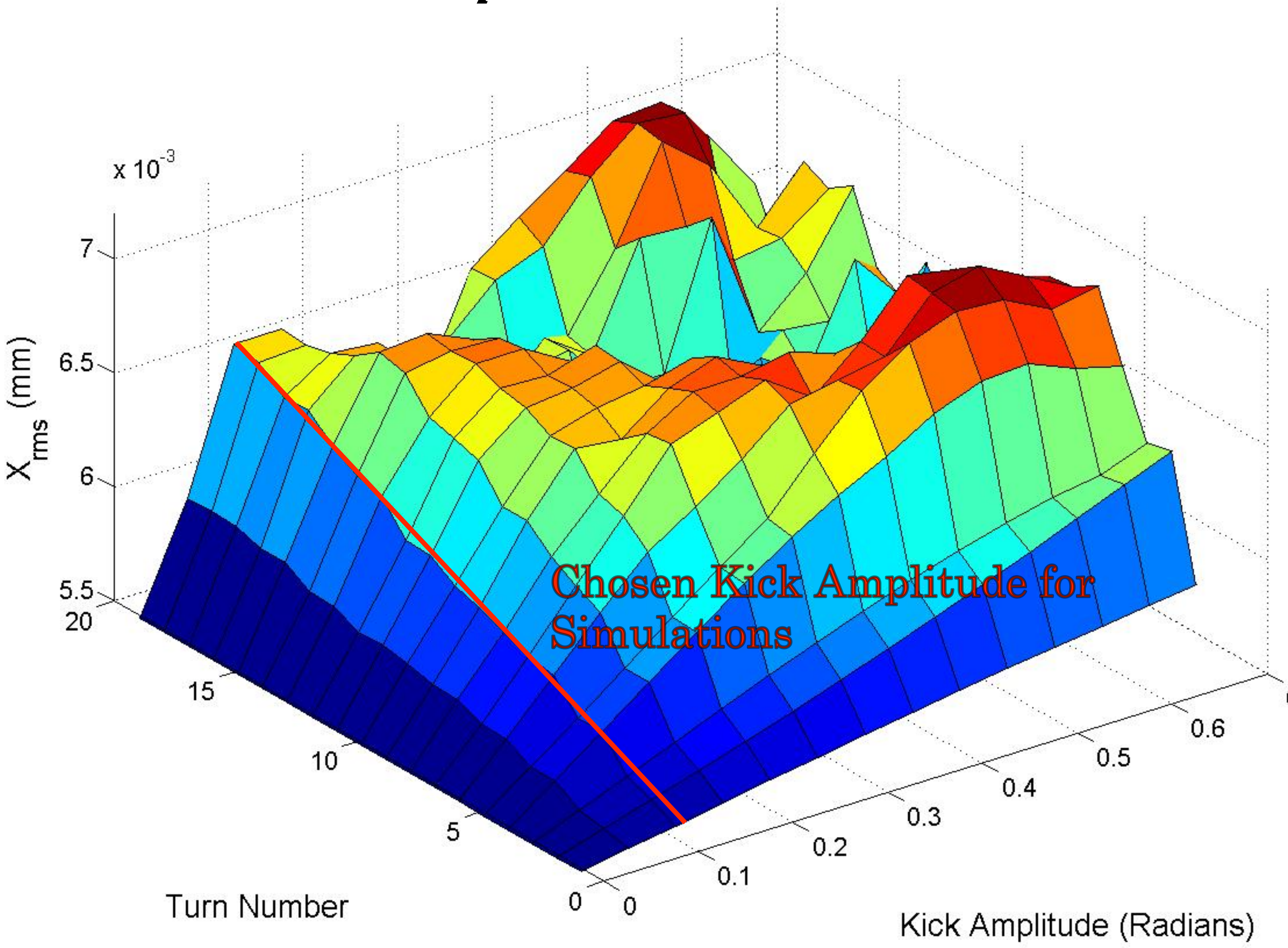
Resonant Growth

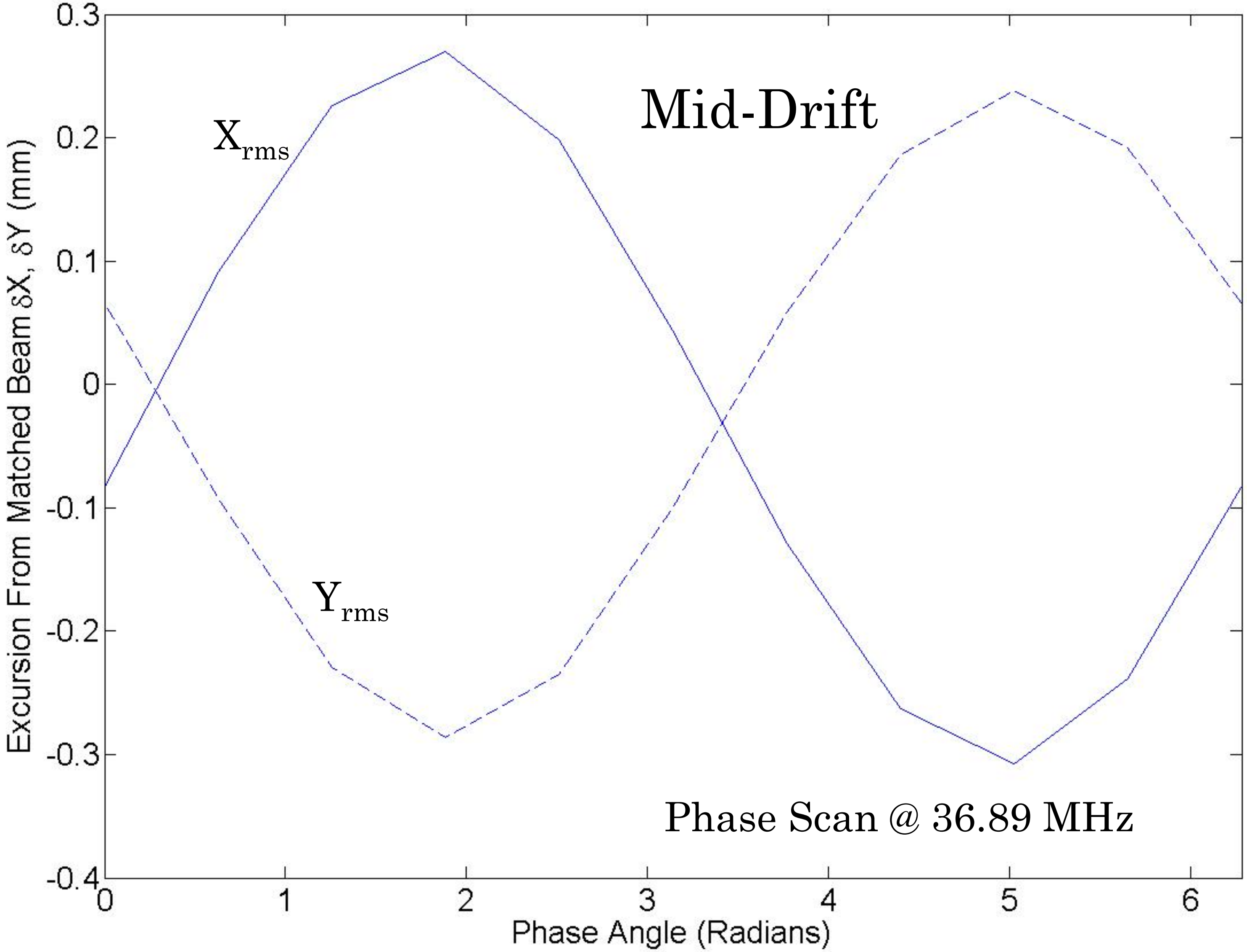


Amplitude Dependence

Envelope Solver

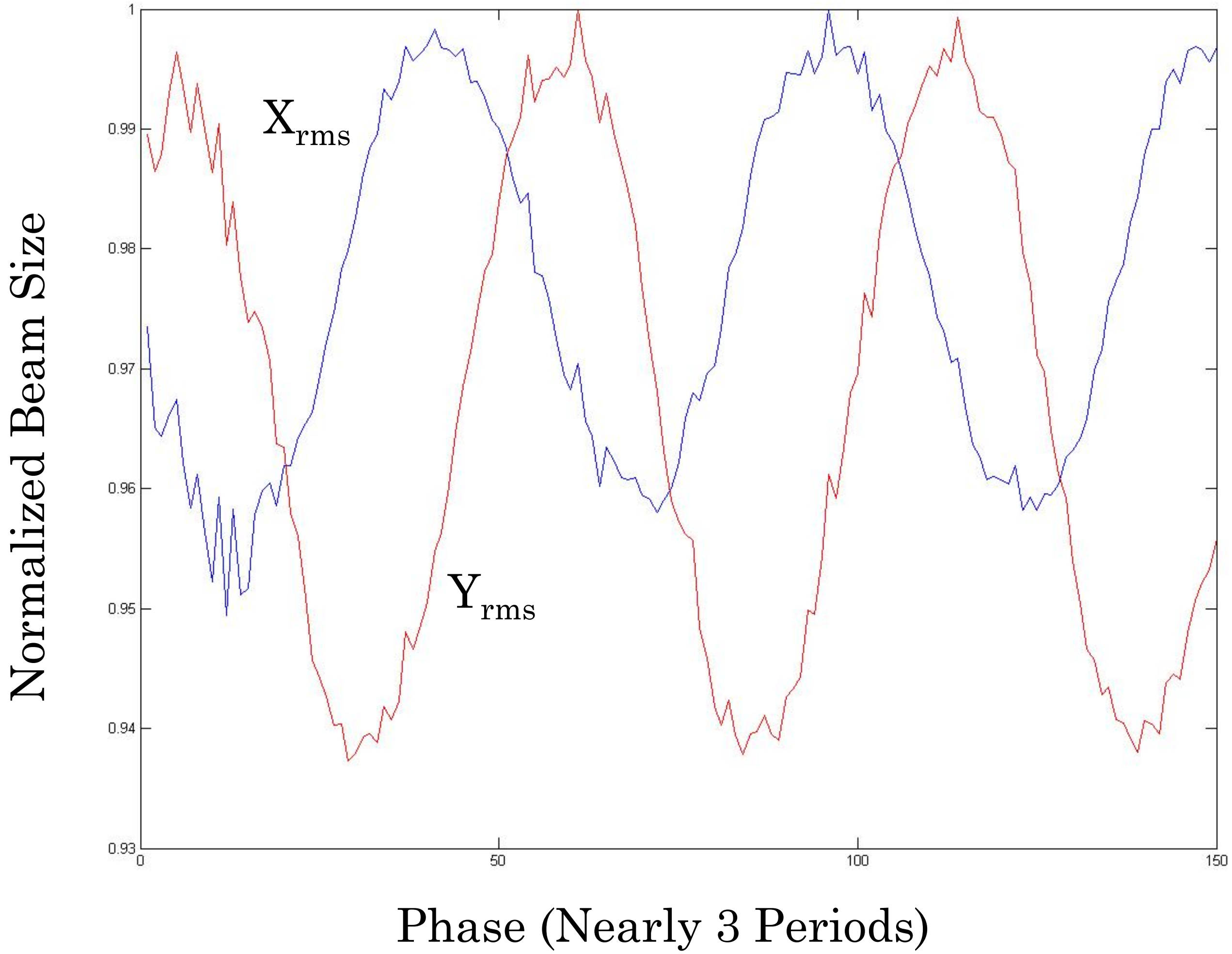


Envelope Simulations



Experimental Phase Scan

Phase Scan @ 37 MHz

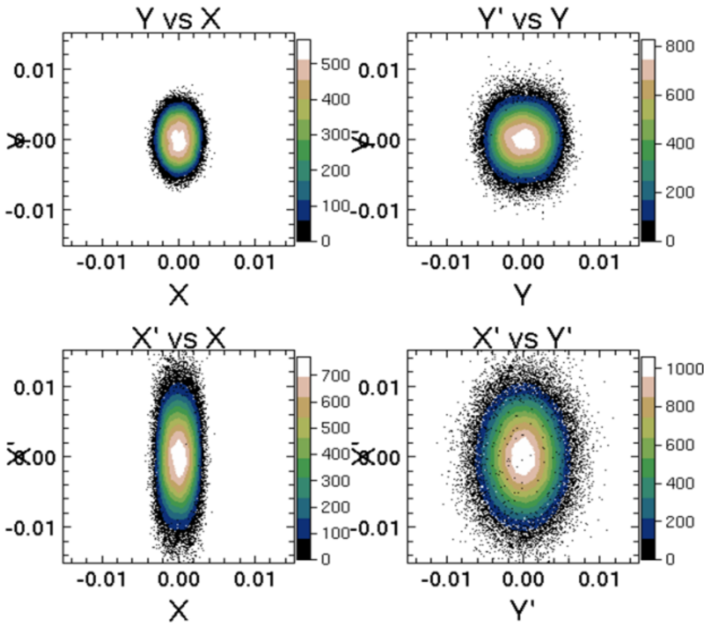


**X,Y not 180 degree out of phase due to skew?

PIC Code Halo

WARP PIC simulations of experiment

Beam with no halo



Beam with halo