IOTA Working Geroup 1 Discussion Start-up paper on ND and SC

Experimental Demonstration of Integrable Optics Lattices at IOTA

Introduction

What prevents us from building super-high intensity accelerators? The answer is case-specific, but it often points to one of the following phenomena: machine resonances, various tune shifts (and spreads), and instabilities. These three phenomena are interdependent in all present machines, which are built to have “linear” focusing optics (also called lattice). A path towards alleviating these phenomena can be opened by making accelerators nonlinear. This idea is not totally new: Orlov [1] and McMillan [2] have proposed initial ideas on nonlinear focusing systems for accelerators. However, practical implementations of such ideas proved elusive, until recently when Danilov and Nagaitsev proposed a solution for nonlinear integrable accelerator lattice that can be implemented with special magnets [3]. In this document we propose a proof-of-principle experiment for demonstration of the concept, and describe the design of a machine for this demonstration—the Integrable Optics Test Accelerator (IOTA).

The ASTA facility will offer a unique opportunity to carry out the proposed research toward demonstration of the feasibility of the integrable optics technique. That research requires construction and operation of a dedicated storage ring (IOTA). It cannot be carried out anywhere else (e.g., at the existing storage rings) as it involves very special insertions (highly nonlinear magnets) which extend over a significant fraction of the ring circumference, special arrangements of the optics lattice and precise control of the elements (strength, positions, etc.).

Concept of Nonlinear Integrable Optics

Nonlinear Integrable Optics and Potential

The lattice design of all present accelerators incorporates dipole magnets to bend particle trajectory and quadrupoles to keep particles stable around the reference orbit. These are “linear” elements because the transverse force is proportional to the particle displacement, x and y. This linearity results (after the action-phase variable transformation) in a Hamiltonian of the following type:

$$H(J_1, J_2) = \nu_x J_1 + \nu_y J_2 , \tag{1}$$

where ν_x and ν_y are betatron tunes and J_1 and J_2 are actions. This is an integrable Hamiltonian. The drawback of this Hamiltonian is that the betatron tunes are constant for all particles regardless of their action values. It has been known since early 1960-s that the spread of betatron tunes is extremely beneficial for beam stability due to the so-called Landau damping. However, because the Hamiltonian (1) is linear, any attempt to add non-linear elements (sextupoles, octupoles) to the accelerator generally results in a reduction of its dynamic aperture, resonant behavior and particle loss. A breakthrough in understanding of stability of Hamiltonian systems, close to integrable, was made by Nekhoroshev [4]. He considered a perturbed Hamiltonian system:

$$H = h(J_1, J_2) + \varepsilon q(J_1, J_2, \theta_1, \theta_2), \tag{2}$$

where h and q are analytic functions and ε is a small perturbation parameter. He proved that under certain conditions on the function h, the perturbed system (2) remains stable for an exponentially long time. Functions h satisfying such conditions are called *steep* functions with quasi-convex and convex being the steepest. In general, the determination of steepness is quite complex. One example of a non-steep function is a linear Hamiltonian Eq. (1).

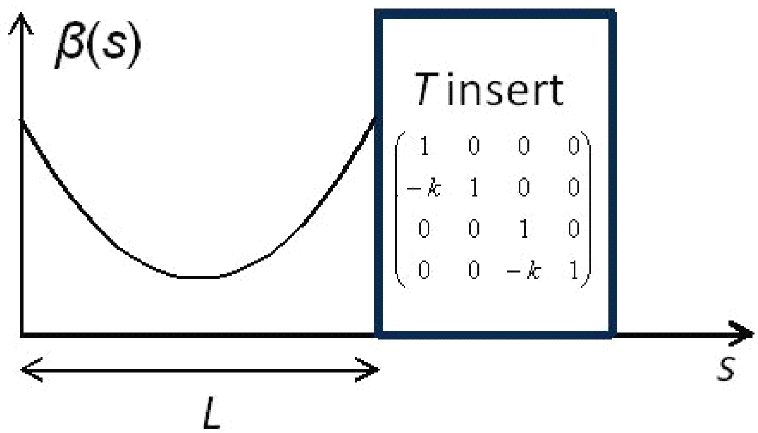

Figure 1: An element of periodicity: a drift space with equal beta-functions followed by a *T*-insert.

In Ref. [1] three examples of nonlinear accelerator lattices were proposed. Here we will concentrate on one of the lattices, which results in a steep (convex) Hamiltonian.

Consider an element of lattice periodicity consisting of two parts: (1) a drift space, *L*, with exactly equal horizontal and vertical beta-functions, followed by (2) an optics insert, *T*, which is comprised of linear elements and has the transfer matrix of a thin axially symmetric lens; see Figure 1.

Let us now introduce additional transverse magnetic field along the drift space *L.* The potential, $V(x, y, s)$, associated with this field satisfies the Laplace equation, $\Delta V = 0$.

Now we will make a normalized-variable substitution [3] to obtain the following Hamiltonian for a particle moving in the drift space *L* with an additional potential *V*:

$$H_N = \frac{p_{xN}^2 + p_{yN}^2}{2} + \frac{x_N^2 + y_N^2}{2} + U(x_N, y_N, \psi), \tag{3}$$

Where

$$U\left(x_N, y_N, \psi\right) = \beta(\psi) V\left(x_N \sqrt{\beta(\psi)}, y_N \sqrt{\beta(\psi)}, s(\psi)\right) \quad , \tag{4}$$

and ψ is the "new time" variable defined as the betatron phase,

$$\psi' = \frac{1}{\beta(s)} . \tag{5}$$

The potential U in equation (3) can be chosen such that it is time-independent [1]. This results in a time-independent Hamiltonian (3). We will now choose a potential such that the Hamiltonian (3) possesses the second integral of motion. We will omit the subscript N from now on.

Consider potentials [5] that can be presented in elliptic coordinates in the following way

$$U(x, y) = \frac{f(\xi) + g(\eta)}{\xi^2 - \eta^2} , \tag{6}$$

where f and g are arbitrary functions,

$$\xi = \frac{\sqrt{(x+c)^2 + y^2} + \sqrt{(x-c)^2 + y^2}}{2c} \qquad \eta = \frac{\sqrt{(x+c)^2 + y^2} - \sqrt{(x-c)^2 + y^2}}{2c} \tag{7}$$

are elliptic variables and c is an arbitrary constant.

The second integral of motion yields

$$I\left(x, y, p_x, p_y\right) = \left(xp_y - yp_x\right)^2 + c^2 p_x^2 + 2c^2 \frac{f(\xi)\eta^2 + g(\eta)\xi^2}{\xi^2 - \eta^2} \tag{8}$$

First, we notice that the harmonic oscillator potential ($x^2 + y^2$) can be presented in the form of Eq. (6) with $f_1(\xi) = c^2\xi^2\left(\xi^2 - 1\right)$ and $g_1(\eta) = c^2\eta^2\left(1 - \eta^2\right)$. Second, we find the following family of potentials that satisfy the Laplace equation and, at the same time, can be presented in the form of Eq. (6):

$$f_2(\xi) = \xi\sqrt{\xi^2 - 1}\left(d + t\,\mathrm{acosh}(\xi)\right) \quad g_2(\eta) = \eta\sqrt{1 - \eta^2}\left(q + t\,\mathrm{acos}(\eta)\right) \quad , \tag{9}$$

where d, q, and t are arbitrary constants. Thus, the total potential energy in Hamiltonian (3) is given by

$$U(x, y) = \frac{x^2}{2} + \frac{y^2}{2} + \frac{f_2(\xi) + g_2(\eta)}{\xi^2 - \eta^2} . \tag{10}$$

Of a particular interest is the potential with $d = 0$ and $q = \frac{\pi}{2}t$, because its lowest multipole expansion term is a quadrupole. Figure 2 presents a contour plot of the potential energy Eq. (10) for $c = 1$ and $t = 0.4$.

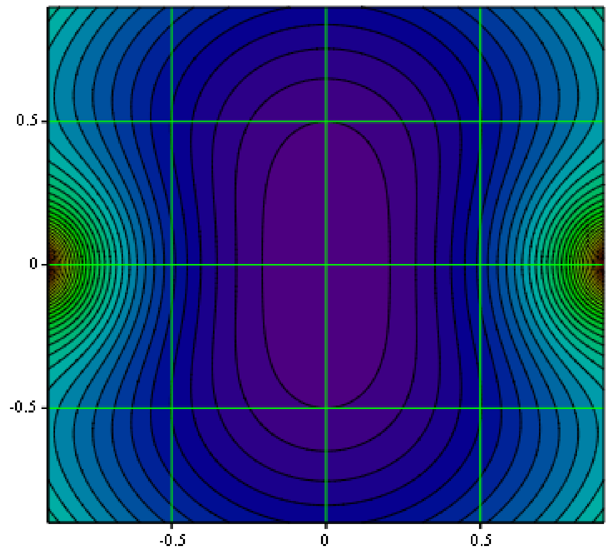

Figure 2: Contour plot of the potential energy Eq. (10) with *c* = 1 and *t* = 0.4. The repulsive singularities are located at *x* = ±*c* and *y* = 0.

The multipole expansion of this potential for *c* = 1 is as follows:

$$U(x,y) \approx \frac{x^2}{2}+\frac{y^2}{2}+t\,\mathrm{Re}\left((x+iy)^2+\frac{2}{3}(x+iy)^4+\frac{8}{15}(x+iy)^6+\frac{16}{35}(x+iy)^8+\ldots\right) \tag{11}$$

where *t* is the magnitude of the nonlinear potential.

Since the 2D Hamiltonian with this potential has two analytic integrals of motion, it is integrable and thus can be expressed as an analytic function of actions:

$$H = h(J_1, J_2)\,, \tag{12}$$

where

$$J_1 = \frac{1}{2\pi}\oint p_\eta d\eta \quad J_2 = \frac{1}{2\pi}\oint p_\xi d\xi \tag{13}$$

Maximum Nonlinear Tune Shift

The potential (10) provides additional focusing in *x* for *t* > 0 and defocusing in *y*. Thus, for a small-amplitude motion to be stable, one needs 0 < *t* < 0.5. This corresponds to the following small-amplitude betatron frequencies,

$$\nu_1 = \nu_0\sqrt{1+2t} \quad \nu_2 = \nu_0\sqrt{1-2t} \quad , \tag{14}$$

where ν_0 is the unperturbed linear-motion betatron frequency. For arbitrary amplitudes the frequencies are obtained by

$$\nu_1(J_1,J_2) = \frac{\partial h}{\partial J_1} \quad \nu_2(J_1,J_2) = \frac{\partial h}{\partial J_2} \quad . \tag{15}$$

Figure 3 presents frequencies $\nu_1(J_1, 0)$ and $\nu_2(0, J_2)$, normalized by ν_0 for t = 0.4.

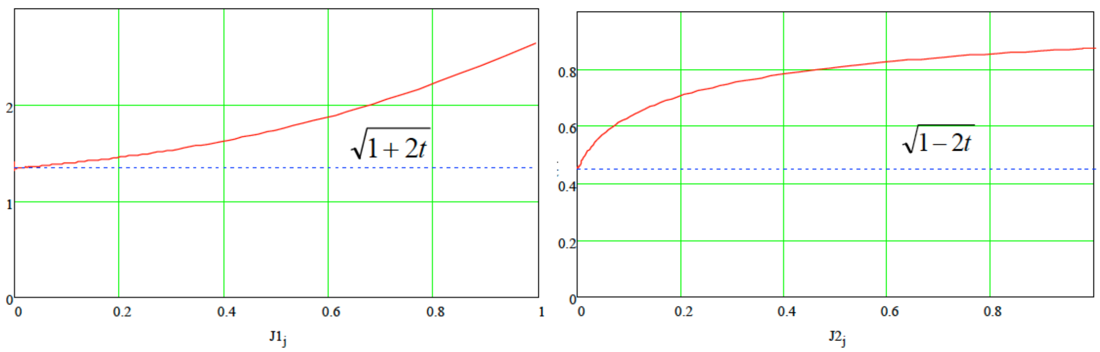

Figure 3: Betatron oscillation frequencies of the two modes - 1 (left) and 2 (right) normalized by ν_0 as functions of actions. Nonlinearity strength parameter t = 0.4.

The unperturbed linear motion tune ν_0 - the betatron phase advance over the drift space L, is limited to 0.5 (in units of 2π). The phase advance in the T-insert must be a multiple of 0.5. This makes the full tune of one element of periodicity 0.5+0.5n. Thus, the theoretical maximum attainable nonlinear tune shift per cell is ~0.5 for mode 1 and ~0.25 for mode 2. Expressed in terms of the full betatron tune per cell, this tune shift can reach 50% (0.5/(0.5+0.5)).

Numerical simulations with single and multi-particle tracking codes were carried out in order to determine the tune spread that can be achieved in a machine built according to the above recipe [6-8]. Various imperfections were taken into account, such as the perturbations of the T-insert lattice, synchrotron oscillations, and other machine nonlinearities. In Figure 4 a result of one of the simulations is presented. The tune footprint obtained with Frequency Map Analysis [9] demonstrates that vertical tune spread exceeding 1 can be achieved and very little resonances are caused by imperfections. No dynamic aperture was observed in the system.

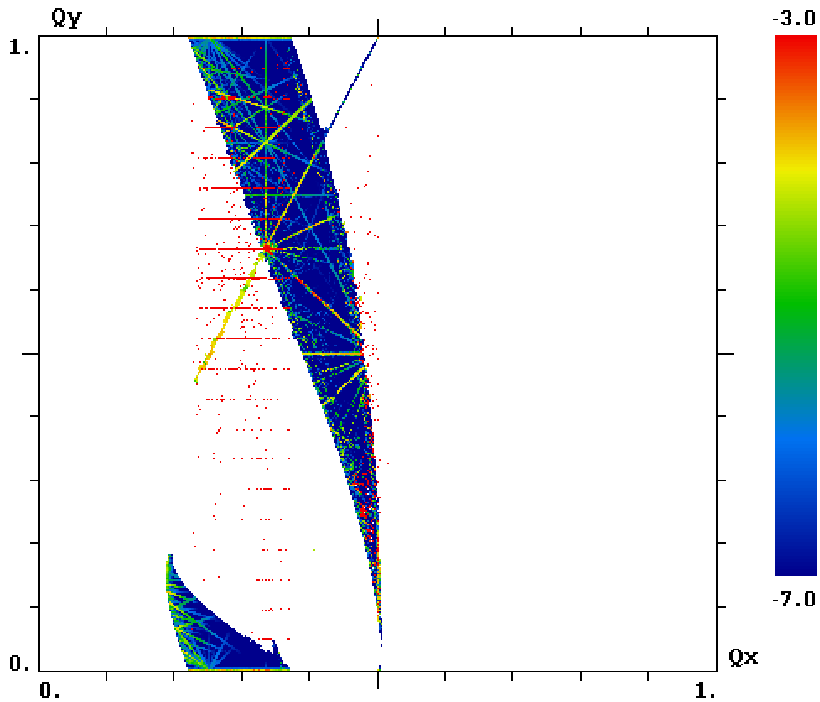

Figure 4: Beam tune footprint for $\nu_0 = 0.3$, four elements of periodicity, $t = 0.4$. Simulation with Lifetrac Frequency Map Analysis.

IOTA Goals and Scope

In Section 8.2.1.2 we demonstrated that using conventional and special nonlinear focusing magnets it is possible to construct an accelerator lattice, in which the betatron motion is strongly nonlinear yet stable. The strong nonlinearity of betatron motion would result in a significant spread of betatron tunes of particles within the bunch (up to 50% of the nominal tune), thus providing strong Landau damping of coherent instabilities.

The superconducting RF linac test at Fermilab's ASTA will provide electron beam with energies up to 800 MeV. The high energy Experimental Area-3 at the end of the linac will be located in a 20×15 m hall, which is large enough to house a small electron storage ring.

The proposed Integrable Optics Test Accelerator (IOTA) will get beam from ASTA's 1.3 GHz SRF linac and will be used for the *demonstration of the possibility to achieve very large nonlinear tune shifts in a realistic accelerator design*. This proof-of principle experiment will *initially concentrate on the single-particle motion stability in the nonlinear integrable system*, leaving the studies of collective effects and attainment of high beam current to future research.

Research at IOTA will include experiments on the following topics:

- Attainment of large nonlinear tune shift/spread without degradation of dynamic aperture
- Suppression of strong lattice resonances (e.g. by crossing the integer resonance by part of the beam without intensity loss)
- Stability of the nonlinear system to perturbations: chromatic effects, effect of synchrotron oscillations, lattice distortions
- Studies of different variants of nonlinear magnet design

In addition to the primary goal, the ring can accommodate other Advanced Accelerator R&D experiments and/or users. This is possible because only a portion of the ring circumference will be occupied with nonlinear magnets and otherwise the machine is a conventional low-energy storage ring. One of the AARD experiments incorporated in the current design of the ring is the Optical Stochastic Cooling; see Section 8.3.2.

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Space Charge Compensation in High Intensity Circular Accelerators

We propose to explore a novel scheme of space-charge compensation that could lead to a significant increase of the beam intensity for future accelerator-based high-energy physics experiments and other sciences.

Through its past success in electron cooling of high-energy antiprotons [1], beam-beam compensation using the electron lenses [2], and controlled halo removal by hollow electron beams [3], Fermilab has gained extensive experience and resources in manipulating high-energy particle beams by means of well-controlled electrons. As the mission of US high energy physics program is pushing the Intensity Frontier, it is of great technical and scientific merit for the community if this remarkable tradition of Fermilab can be applied to overcome the beam intensity limit in the present accelerator technology. Hence, we propose to investigate a novel method of space-charge compensation to achieve very intense and stable beams in circular accelerators through trapping and controlling of the electrons generated from beam-induced residual gas ionization. The method has a great potential to improve performance of leading high-current proton accelerator facilities and experiments, such as LBNE with Project X intensities, Mu2e and “g-2” after the intensity upgrades, compressor and accumulation rings envisioned in the Neutrino Factory and Muon Collider projects. The method may also offer a transformational technology for the next generation high-intensity proton sources, e.g., such as those needed for the Accelerator Driven Systems.

The main idea of this compensation method is based on the long-known fact that the negative effect of Coulomb repulsion can be mitigated if beams are made to pass through a plasma column of opposite charge. This idea has been successfully applied to transport high-current low-energy proton and H^- beams into the RFQ in many linacs. In circular machines, partial neutralization by ionized electrons was attempted with notable improvements in beam intensity, namely one order of magnitude higher than the space-charge limit. However, the beam-plasma system was subject to strong transverse electron-proton (e-p) instability. In principle, this difficulty can be overcome if protons and electrons are immersed in a longitudinal magnetic field which is a) strong enough to freeze the electron density distribution; b) strong enough to suppress the e-p instability; c) weak enough to allow positive ions to escape transversely, in addition to longitudinal draining; and d) uniform enough to avoid beta-beat excitations. In addition, we note that significant improvements have been made on the physics of non-neutral plasmas and on the stability of beam-plasma systems in the plasma physics community over the past decade, some of which could be readily adopted for the present project.

The scope of this proposal will be based on the resources and facilities available at Fermilab within the five year timeline. The existing ion source (proton and H^-), LEBT system, and RFQ of High Intensity Neutrino Source (HINS) program will be reused as an injector for the ring with currents up to 20 mA and energy of 2.5 MeV. The Integrable Optics Test Accelerator (IOTA) ring, which is now under construction at Fermilab’s ASTA with completion expected in 2015-16, will be used to accumulate protons through charge-exchange injection. The Tevatron electron lens system, a nonlinear element to be installed in IOTA ring, can be used to trap electrons for the initial space-charge compensation experiments. The scientific program will consist of both extensive theoretical modeling, and installation and operation of the test accelerator, outlined as follows:

- Studies of the physics of electron column [4] formation and the stability of beam-plasma system
- Measurements of electron accumulation and beam-plasma stability at HINS beamline
- Design and construction of charge-exchange injection system for IOTA ring
- Installation of HINS front-end (with H^- source) to ASTA hall
- Measurements of electron accumulation and beam-plasma stability at IOTA ring using the electron lens system [5]
- Upgrade of the electron lens system with dedicated diagnostic and control equipment.

The present proposal [6] perfectly fits the main thrusts of the Fermilab's accelerator R&D plan, and it will create lots of synergies with other programs as well. For example, once the IOTA ring stores low energy proton beams, combined effects of space-charge compensation and nonlinear integral optics could be readily studied.

The ASTA facility will offer unique opportunities to carry out the proposed research toward demonstration of the feasibility of the space-charge compensation methods, such as with electron columns, electron lenses, or in combination of the two with the elements of the integrable optics technique. That research requires a dedicated storage ring (IOTA) and its operation with protons. It cannot be carried out anywhere else as there are no existing proton storage rings or synchrotrons which can afford the installation of special insertions (columns, lenses, etc.), and offer special arrangements of the optics lattice and precise control of the insertion devices and the ring elements.

1.1.1.1 References

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