Optical Stochastic Cooling Experiment at IOTA

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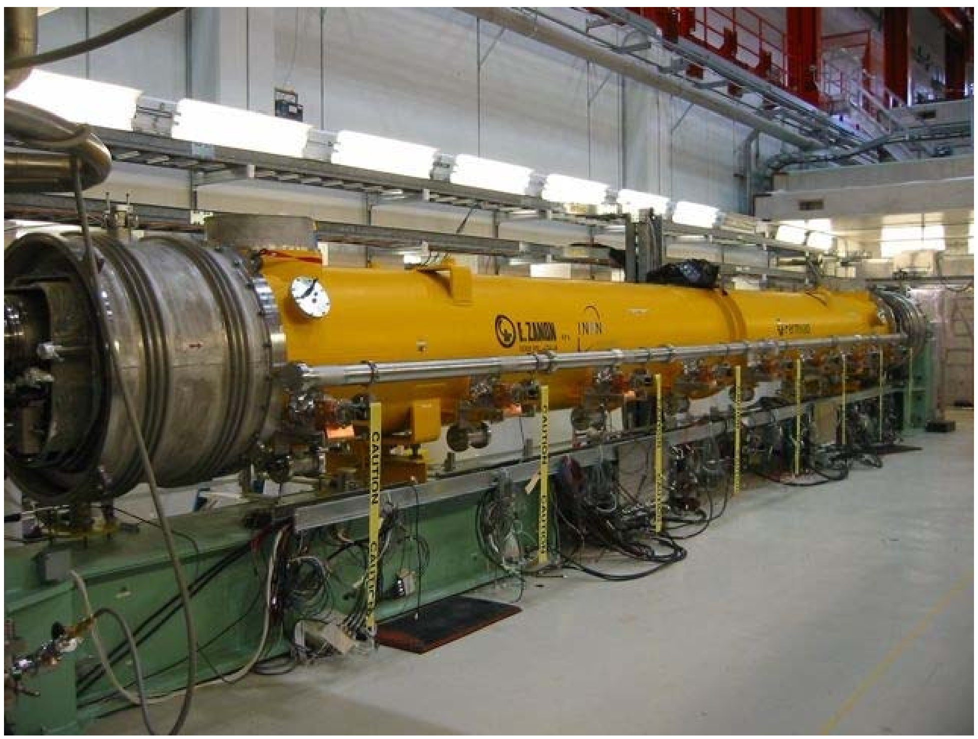
Fermilab

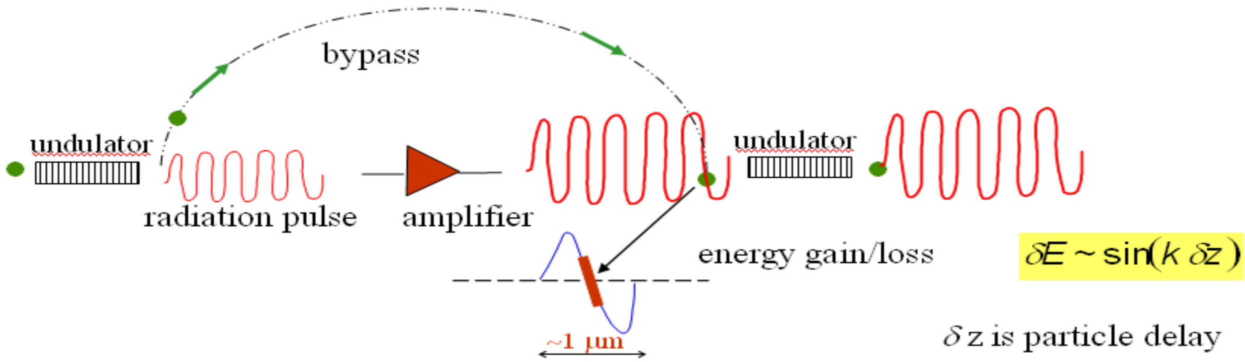


Contents

- Introduction to Optical Stochastic Cooling
- Basics of Optical Stochastic Cooling
- Optical Stochastic Cooling at IOTA ring
- Conclusions

Principles of Optical Stochastic Cooling

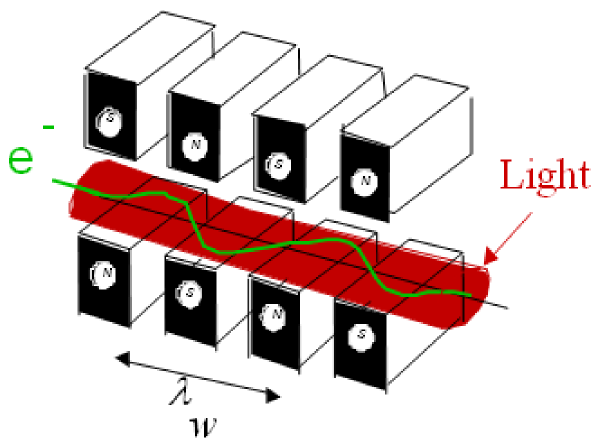
- OSC - suggested by Zolotorev, Zholents and Mikhailichenko (1994)
- OSC obeys the same principles as the microwave stochastic cooling, but exploits the superior bandwidth of optical amplifiers ~ 10^{14} Hz



- At optimum the cooling rates of stochastic cooling are

Dimensionless damping rate: $$\lambda f_0 \approx \frac{W}{N} \quad \Leftrightarrow \quad \lambda \approx \frac{1}{N_{sample}}$$

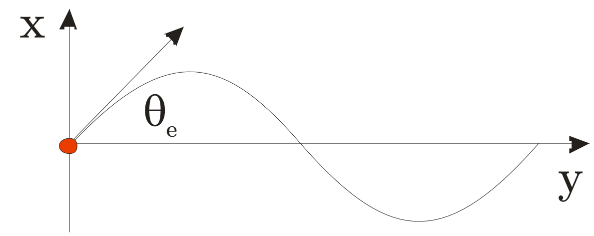
  - Potential gain in damping rates: $10^3 \div 10^4$
- Pickup and kicker must operate at the optical frequencies (same band as an opt. amplifier)
  - Undulators suggested for pickups & kickers
- Slow particles do not radiate at optical frequencies
  - OSC can operate only with ultra-relativistic particles



Principles of Optical Stochastic Cooling (continue)

- Radiation wave length

$$\lambda = \frac{\lambda_{wgl}}{2\gamma^2}\begin{cases}\left(1+\gamma^2\left(\theta_e^{\,2}+\theta^2\right)\right) & -\ helical\ undulator\\ \left(1+\gamma^2\left(\frac{1}{2}\theta_e^{\,2}+\theta^2\right)\right) & -\ flat\ undulator\end{cases}$$



Undulator parameter: $K=\gamma\theta_e$ => $\lambda\big|_{\theta=0}=\lambda_{wgl}\left(1+K^2/2\right)/\left(2\gamma^2\right)\quad -\quad flat\ undulator$

- Correction signal is proportional to longitudinal position change on the travel from pickup to kicker
- Only longitudinal kicks are effective for ultra-relativistic beam
  - *s-x* coupling for long. cooling
  - *x-y* coupling for vertical cooling
- Introduce partial slip factor: describes a long. particle displacement on the way from pickup to kicker with Δp/p≠0 & no betatron motion

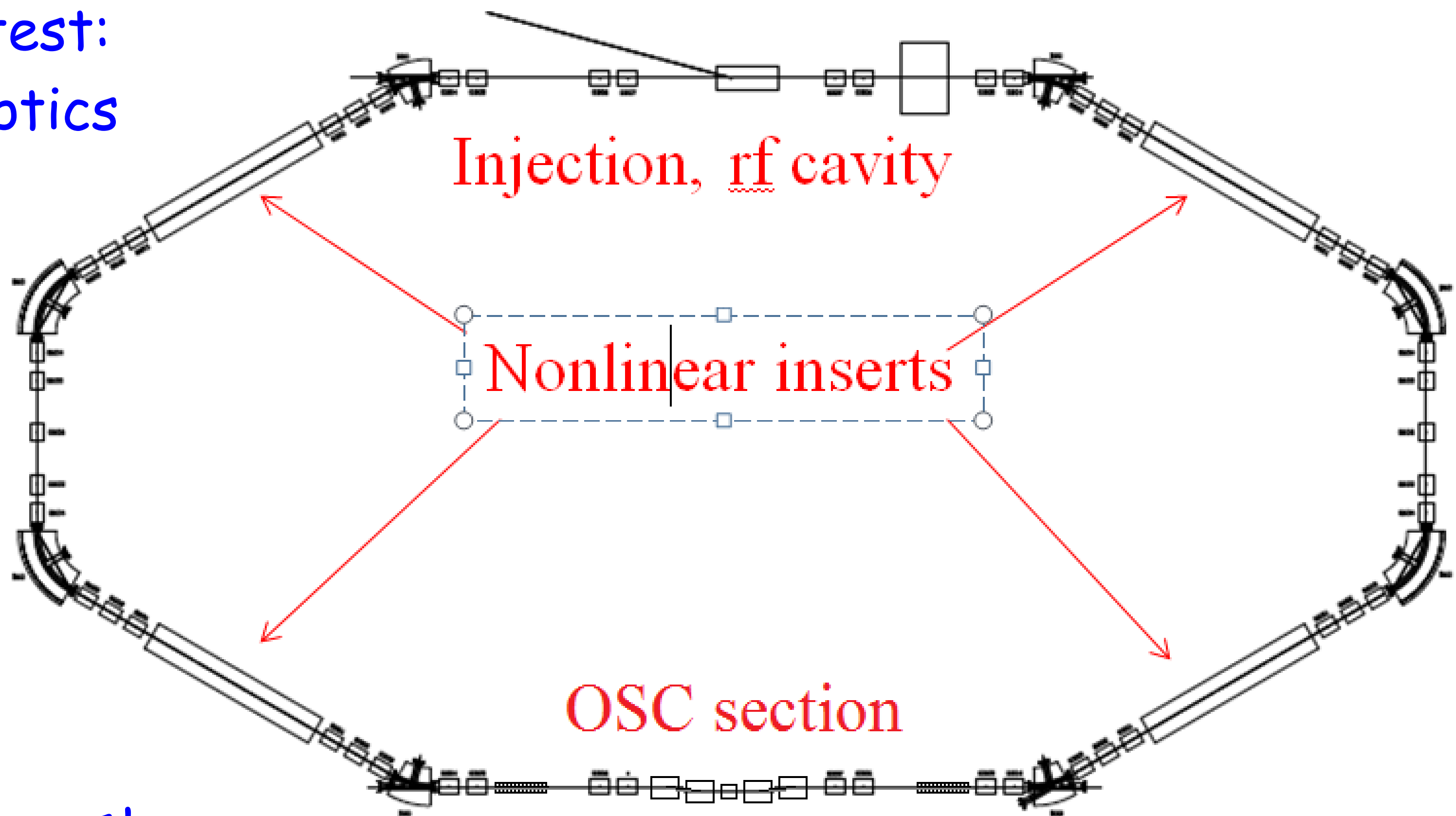
$$\tilde{M}_{56}=M_{51}D_1+M_{52}D_1'+M_{56}\qquad\Leftrightarrow\qquad\Delta s=\tilde{M}_{56}\left(\Delta p/\mathrm{p}\right)$$

- Cooling rates:

$$\boxed{\begin{aligned}\lambda_x&=\frac{k\xi_0}{2}\left(M_{56}-\tilde{M}_{56}\right)\\ \lambda_s&=\frac{k\xi_0}{2}\tilde{M}_{56}\end{aligned}}\qquad\Leftrightarrow\qquad\boxed{\lambda_x+\lambda_s=\frac{k\xi_0}{2}M_{56}^{pk}}$$

Test of OSC in Fermilab

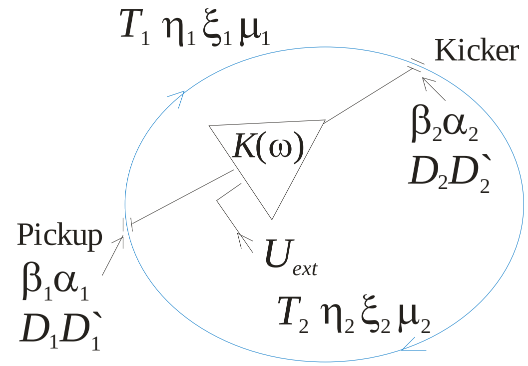
- First attempt to test the OSC in BATES, ~2007
  - Existing electron synchrotron
  - Did not get sufficient support
- Presently Fermilab is constructing a dual purpose small electron ring called IOTA to test:
  - Integrable optics
  - OSC
- Part of ASTA program
  - Full energy injection from SC linac
- Test in a small electron ring is a cost effective way to test the OSC



Basics of OSC: Damping Rates

- Pickup-to-Kicker Transfer Matrix
  - Vertical plane is uncoupled and we omit it

$$\mathbf{M}^{pk} = \begin{bmatrix} M_{11} & M_{12} & 0 & M_{16} \\ M_{21} & M_{22} & 0 & M_{26} \\ M_{51} & M_{52} & 1 & M_{56} \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} x \\ \theta_x \\ s \\ \Delta p / p \end{bmatrix}$$



$\mathbf{M}^{pk}$ - pickup-to-kicker matrix

$\mathbf{M}^{kp}$ - kicker-to-pickup matrix

$\mathbf{M} = \mathbf{M}^{pk}\mathbf{M}^{kp}$ – ring matrix

- Partial slip factor (pickup-to-kicker) describes a longitudinal particle displacement in the course of synchrotron motion

$$\tilde{M}_{56} = M_{51} D_1 + M_{52} D_1' + M_{56}$$

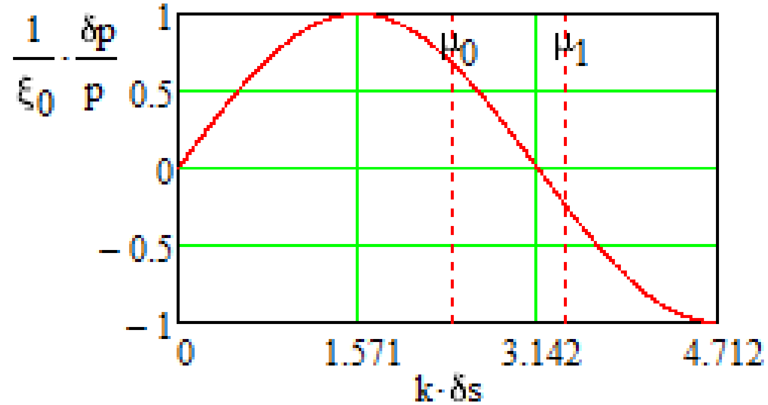
- Linearized longitudinal kick in pickup wiggler

$$\frac{\delta p}{p} = k\xi_0\, \Delta s = k\xi_0 \left(M_{51} x_1 + M_{52} \theta_{x1} + M_{56} \frac{\Delta p}{p} \right)$$

- Cooling rates (per turn)

$$\boxed{\begin{aligned} \lambda_x &= \frac{k\xi_0}{2}\left(M_{56} - \tilde{M}_{56} \right) \\ \lambda_s &= \frac{k\xi_0}{2} \tilde{M}_{56} \end{aligned}} \quad \Leftrightarrow \quad \boxed{\lambda_x + \lambda_s = \frac{k\xi_0}{2} M_{56}^{pk}}$$

Basics of OSC: Cooling Range



- Cooling force depends on Δs nonlinearly

$$\frac{\delta p}{p} = k\xi_0 \Delta s \quad \Rightarrow \quad \frac{\delta p}{p} = \xi_0 \sin\left(k\delta s\right)$$

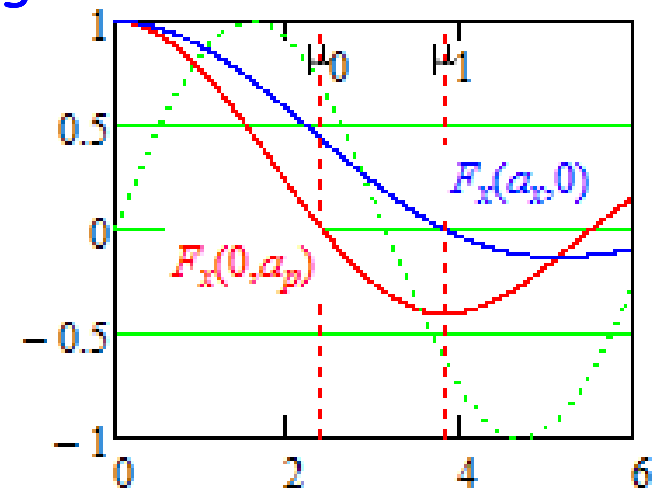
where $k\delta s = a_x \sin(\psi_x) + a_p \sin(\psi_p)$

and a_x & a_p are the amplitudes of longitudinal displacements in cooling chicane due to ⊥ and L motions measured in units of laser phase

$$\boxed{\begin{aligned} a_x &= k\sqrt{\varepsilon\left(\beta_p M_{51}{}^2 - 2\alpha_p M_{51} M_{52} + \gamma_p M_{52}{}^2\right)}, \quad \text{where } \varepsilon = \beta_p \theta^2 - 2\alpha_p x\theta + \gamma_p x^2 \\ a_p &= k\tilde{M}_{56}\left(\Delta p / p\right) \end{aligned}}$$

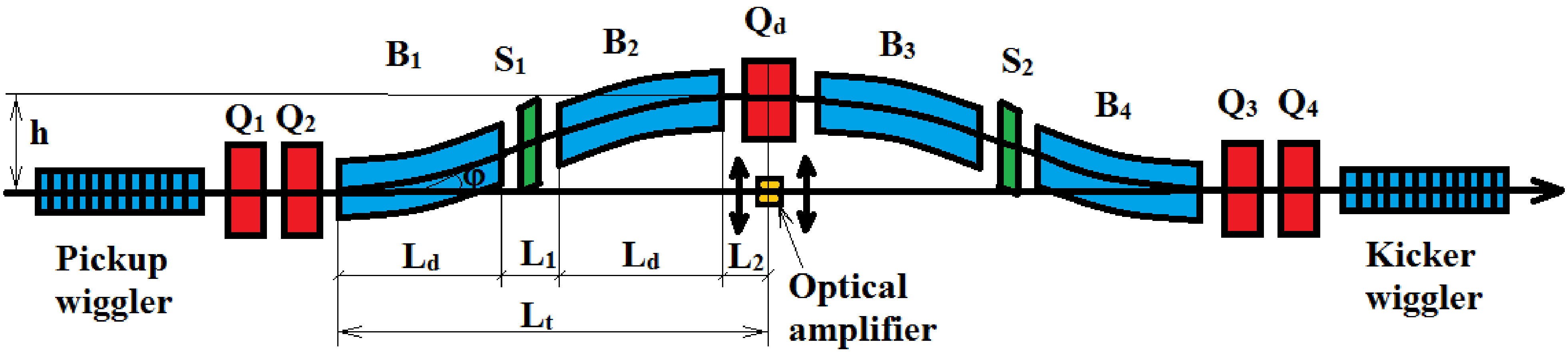
- Averaging yields the form-factors for damping rates

$$\boxed{\begin{aligned} \lambda_{s,x}(a_x, a_p) &= F_{s,x}(a_x, a_p)\lambda_{s,x} \\ F_x(a_x, a_p) &= \frac{2}{a_x} J_0(a_p) J_1(a_x) \\ F_p(a_x, a_p) &= \frac{2}{a_p} J_0(a_x) J_1(a_p) \end{aligned}}$$



- Damping requires both lengthening amplitudes (a_x and a_p) to be smaller than $\mu_0 \approx 2.405$

Transfer Matrix for OSC Chicane



Chicane displaces the beam closer to its center

$$M_{ta}=\begin{pmatrix}1 & L_d & 0 & \frac{L_d\cdot\varphi}{2}\\ 0 & 1 & 0 & \varphi\\ -\varphi & -\frac{L_d\cdot\varphi}{2} & 1 & -\frac{L_d\cdot\varphi^2}{6}\\ 0 & 0 & 0 & 1\end{pmatrix}\cdot\begin{pmatrix}1 & L_1 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{pmatrix}\cdot\begin{pmatrix}1 & L_d & 0 & -\frac{L_d\cdot\varphi}{2}\\ 0 & 1 & 0 & -\varphi\\ \varphi & \frac{L_d\cdot\varphi}{2} & 1 & -\frac{L_d\cdot\varphi^2}{6}\\ 0 & 0 & 0 & 1\end{pmatrix}\cdot\begin{pmatrix}1 & L_2 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{pmatrix}\cdot\begin{pmatrix}1 & 0 & 0 & 0\\ \Phi & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{pmatrix}\cdot\begin{pmatrix}1 & L_2 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{pmatrix}\cdot\begin{pmatrix}1 & L_d & 0 & -\frac{L_d\cdot\varphi}{2}\\ 0 & 1 & 0 & -\varphi\\ \varphi & \frac{L_d\cdot\varphi}{2} & 1 & -\frac{L_d\cdot\varphi^2}{6}\\ 0 & 0 & 0 & 1\end{pmatrix}\cdot\begin{pmatrix}1 & L_1 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{pmatrix}\cdot\begin{pmatrix}1 & L_d & 0 & \frac{L_d\cdot\varphi}{2}\\ 0 & 1 & 0 & \varphi\\ -\varphi & -\frac{L_d\cdot\varphi}{2} & 1 & -\frac{L_d\cdot\varphi^2}{6}\\ 0 & 0 & 0 & 1\end{pmatrix}$$

Leaving only major terms we obtain

$$M_{ta}=\begin{bmatrix}L_t\cdot\Phi+1 & L_t\cdot(L_t\cdot\Phi+2) & 0 & \Phi\cdot h\cdot L_t\\ \Phi & L_t\cdot\Phi+1 & 0 & \Phi\cdot h\\ -\Phi\cdot h & -\Phi\cdot h\cdot L_t & 1 & 2\cdot\Delta s\\ 0 & 0 & 0 & 1\end{bmatrix}$$

$$\Delta s=\varphi^2\cdot\left(L_1+\frac{2}{3}\cdot L_d\right)$$

$$h=\varphi\cdot(L_1+L_d)$$

Matrix comparison:

Exact (M_t) versus

approximate (M_{ta})

$$M_t=\begin{pmatrix}1.093 & 90.635 & 0 & 0.268\\ 2.152\times10^{-3} & 1.093 & 0 & 6.18\times10^{-3}\\ -6.18\times10^{-3} & -0.268 & 1 & 0.591\\ 0 & 0 & 0 & 1\end{pmatrix}\qquad M_{ta}=\begin{pmatrix}1.092 & 89.971 & 0 & 0.262\\ 2.148\times10^{-3} & 1.092 & 0 & 6.093\times10^{-3}\\ -6.093\times10^{-3} & -0.262 & 1 & 0.601\\ 0 & 0 & 0 & 1\end{pmatrix}$$

OSC Chicane and Limitations on IOTA Optics

Dispersion in the chicane center

- In the first approximation
  the orbit offset in the chicane (h),
  the path lengthening (Δs),
  the defocusing strength of Q_d (Φ)
  and dispersion in the chicane center (D^*)
  determine the entire cooling dynamics
- Δs is set by delay in the amplifier => M_{56}
  (Δs = 3 mm is chosen, includes delay in lenses)
- Choose $(dD/ds)^* = 0$ => $D\big|_{s=\pm L_t} \approx D^*$
- $\Phi D^* h$ determines the ratio of decrements
  - Choose: $\lambda_x = 2\lambda_s$ => $\Phi D^* h \approx 4\Delta s/3$

$$M_{56} \approx 2\Delta s\ ,$$

$$\tilde{M}_{56} \approx 2\Delta s - \Phi D^* h\ ,$$

$$\frac{\lambda_x}{\lambda_s} = \frac{\tilde{M}_{56}}{M_{56} - \tilde{M}_{56}} \approx \frac{\Phi D^* h}{2\Delta s - \Phi D^* h}\ ,$$

$$k\sigma_p \left(\frac{\Delta p}{p}\right)_{\max} \tilde{M}_{56} < \mu_0$$

$$\xrightarrow{n_{\sigma p}\sigma_p = \left(\frac{\Delta p}{p}\right)_{\max}}$$

$$n_{\sigma p} \approx \frac{\mu_0}{\left(2\Delta s - \Phi D^* h\right) k\sigma_p}\ ,$$

- For the wave length of λ=2.2 μm and momentum spread of $\sigma_p = 1.2\cdot 10^{-4}$
  ⇨ Cooling acceptance for longitudinal degree of freedom ($n_{\sigma p}$=3.6)
- Thus D* determines the ratio of cooling rates and cooling acceptance in momentum

  This is the first limitation which sets the wave length to be $\geq$ 2 μm

OSC Chicane and Limitations on IOTA Optics (2)

Beta-function in the chicane center

- Behavior of the horizontal β-function determines the cooling range for horizontal degree of freedom
  - At optimum $\alpha^* = 0$
  - ⇨ Cooling acceptance:

$$\varepsilon_{\max} = \frac{\mu_0^{\,2}}{k^2\left(\beta_p M_{51}^{\,2} - 2\alpha_p M_{51} M_{52} + \gamma_p M_{52}^{\,2}\right)} \xrightarrow[\alpha_p \approx \frac{L_t}{\beta^*}]{\beta_p \approx \frac{L_t^{\,2}}{\beta^*}} \approx \frac{\mu_0^{\,2}}{k^2 \Phi^2 h^2 \beta^*}$$

- For known rms emittance, ε, we can rewrite it as following

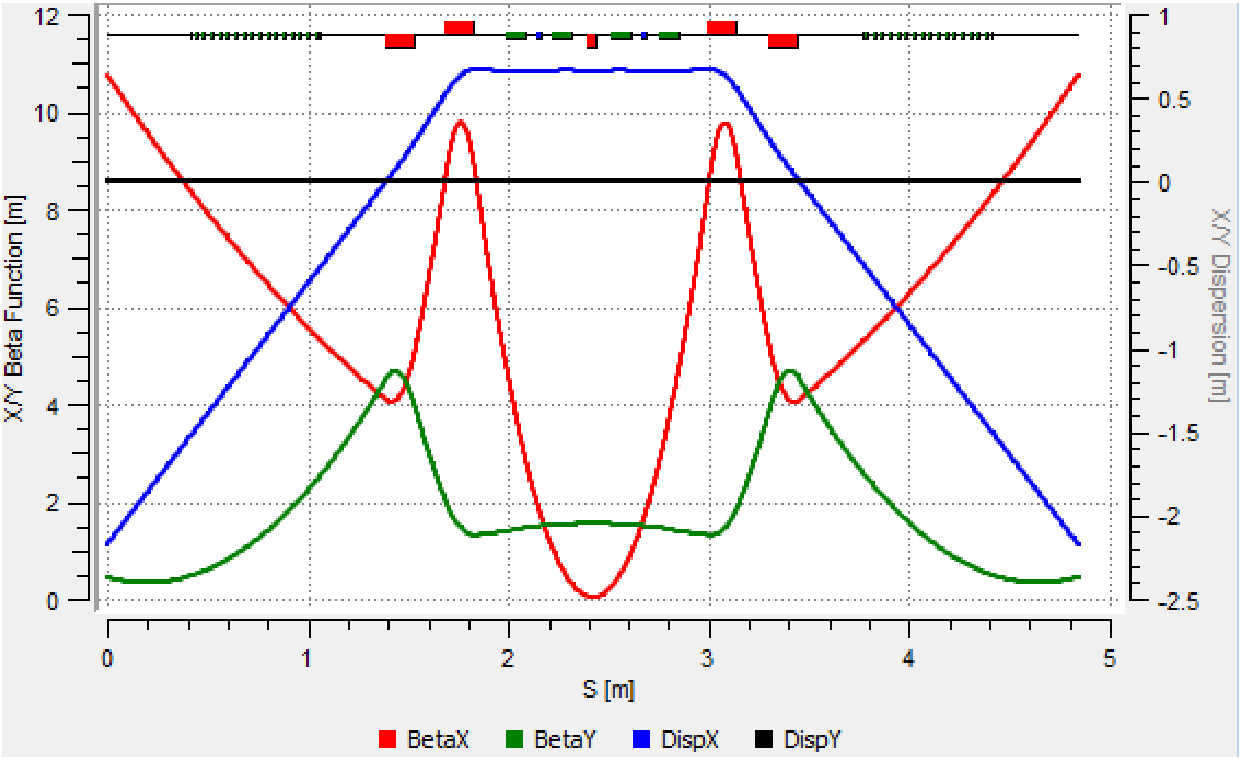
$$n_{\sigma x} \equiv \sqrt{\frac{\varepsilon_{\max}}{\varepsilon}} \approx \frac{\mu_0}{k\Phi h \sqrt{\varepsilon \beta^*}} \xrightarrow{\Phi D^* h = 2\Delta s \frac{\lambda_x}{\lambda_s + \lambda_x}} \boxed{n_{\sigma x} = \frac{\mu_0}{2k\Delta s}\left(1 + \frac{\lambda_s}{\lambda_x}\right)\sqrt{\frac{A_x^{\;*}}{\varepsilon}}} \qquad A_x^{\;*} = \frac{D^{*2}}{\beta^*}$$

- Thus the cooling range, $n_{\sigma x}$, determines the dispersion invariant A_x^*
- Average value of A_x in dipoles determines the equilibrium emittance.
  - A_x^* is large and A_x needs to be reduced fast to get an acceptable value of the equilibrium emittance (ε)
- Getting sufficiently large cooling acceptance requires long wave length of the radiation: another reason for $\lambda \geq 2$ μm

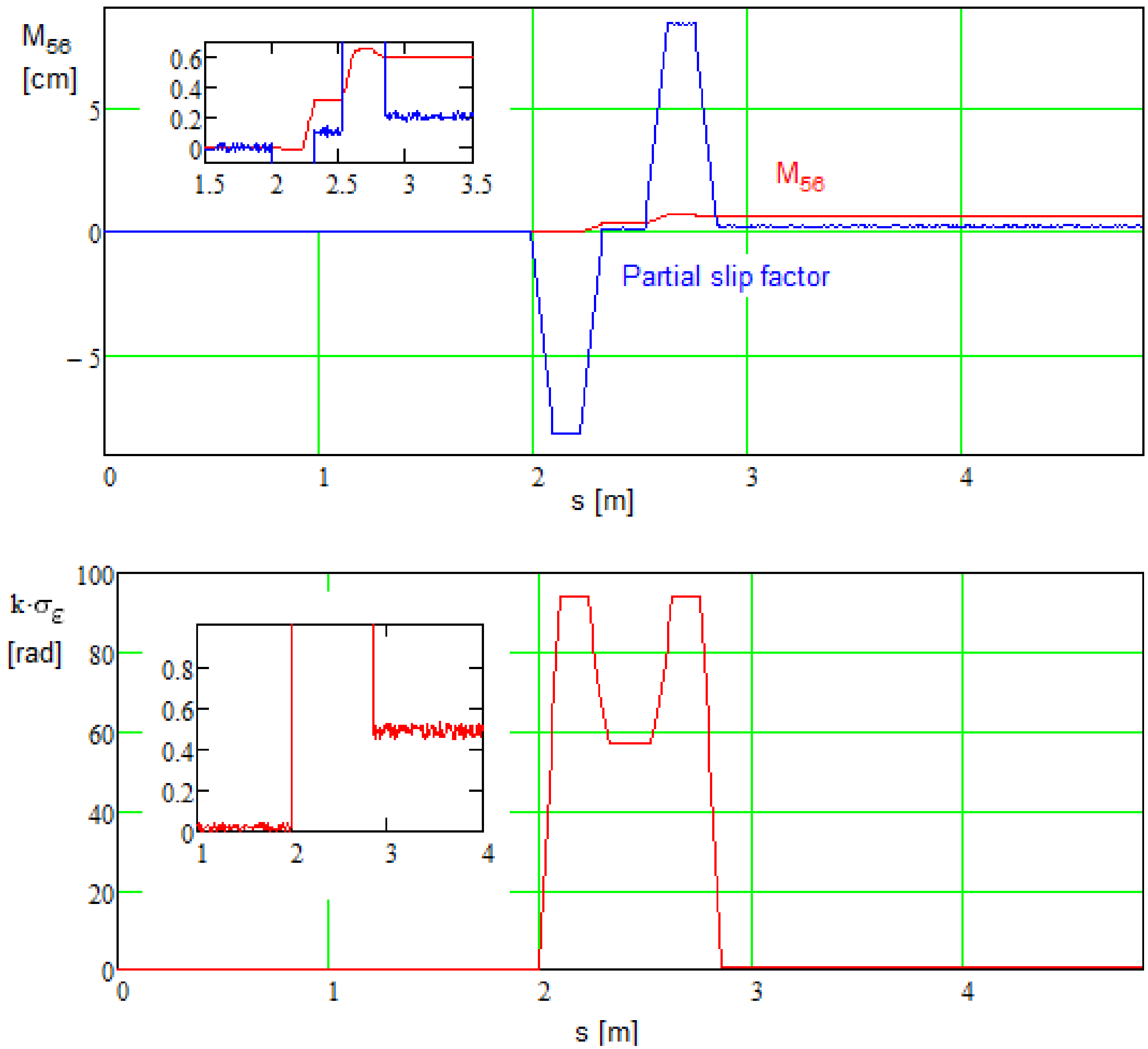
Linear Beam Optics for Cooling Chicane

Major parameters of cooling chicane

Beam energy	100 MeV
Dipole type	Rbend
B of dipole	4.14 kG
L of dipole	10 cm
Orbit offset, h	28.4 mm
Delay, Δs	3 mm
GdL of Qd quad	720 Gs
β_x*	4 cm
D_x^*	66 cm
Damping rates ratio, λ_x/λ_s	1.86
Basic wave length, λ	2.2 μm
Cooling range, $(\Delta p/p)_{max}$	$\pm 1.2 \cdot 10^{-3}$
Cooling acceptance, ε_{max}	0.46 μm



Sample Lengthening on the Travel through Chicane



Sample lengthening due to momentum spread (top) and due to betatron motion (bottom)

- Very large sample lengthening on the travel through chicane
- High accuracy of dipole field is required to prevent uncontrolled lengthening, $\Delta(BL)/(BL)_{dipole} < 10^{-3}$

Non-liner Sample Lengthening

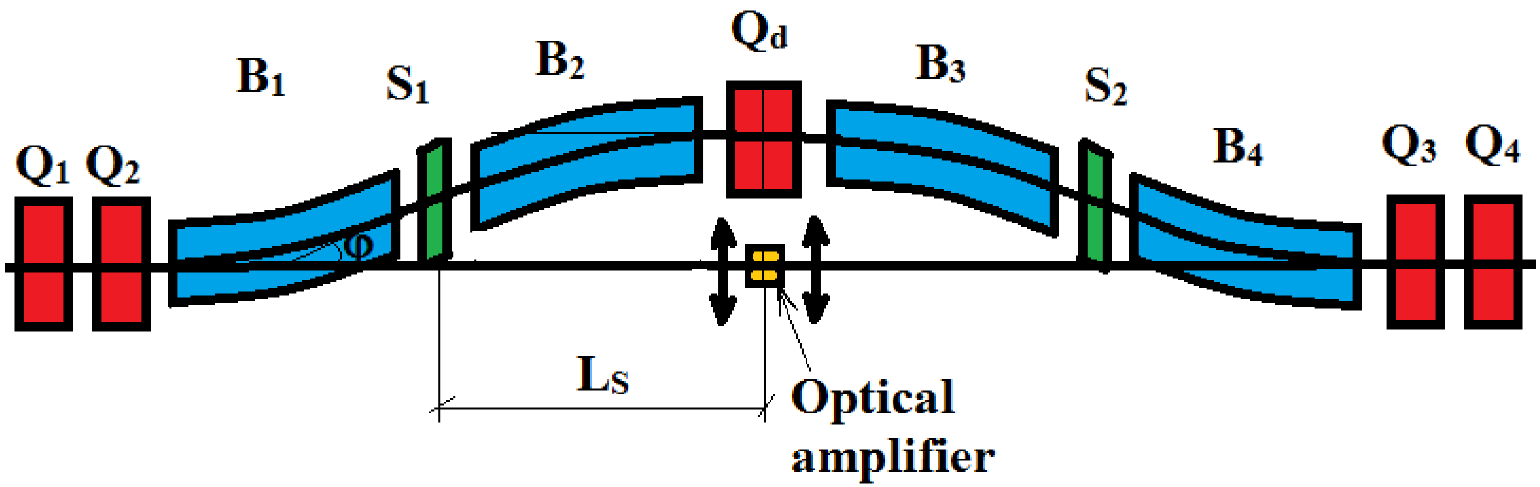
- Major contribution to the 2nd order lengthening comes from particle angle: $$\Delta s_2 = \int_{-L_t/2}^{L_t/2} \frac{\theta(s)^2}{2} ds$$
- Expressing it through particle phase in the chicane center (μ_0), particle Courant-Snyder invariant (ε) and Twiss parameters =>

$$\Delta s_2 = \frac{\varepsilon}{2}\left(I_1 - I_2 \cos(2\mu_0)\right), \qquad I_1 = \int_0^{\mu_t/2} \left(1+\alpha^2(\mu)\right) d\mu\,, \quad I_2 = \int_0^{\mu_t/2} \left(\left(1-\alpha^2(\mu)\right)\cos(2\mu) - 2\alpha(\mu)\sin(2\mu)\right) d\mu$$

⇨ maximum lengthening: $\Delta s_2 = \frac{\varepsilon}{2}\left(I_1 + |I_2|\right)$

- For IOTA cooling chicane we have: $k\Delta s_{2x} \approx 26\text{ rad}$, $k\Delta s_{2x} \approx 5\text{ rad}$ for the boundary of cooling acceptance (ε_{max}=0.46 μm)
- Cooling is weakly affected if $k\Delta s_2 \le 1.5$
  - Thus, in the absence of compensation we lose a factor of 4 in cooling range ($\sqrt{26/1.5}$)
  - Effect of vertical motion is at the boundary of acceptable
  - It is the main reason why $\lambda \ge 2$ μm

Compensation of Nonlinear Sample Lengthening



- Non-linear sample lengthening due to H. betatron motion can be compensated by 2 sextupoles in the chicane
  - Lengthening due to angle:

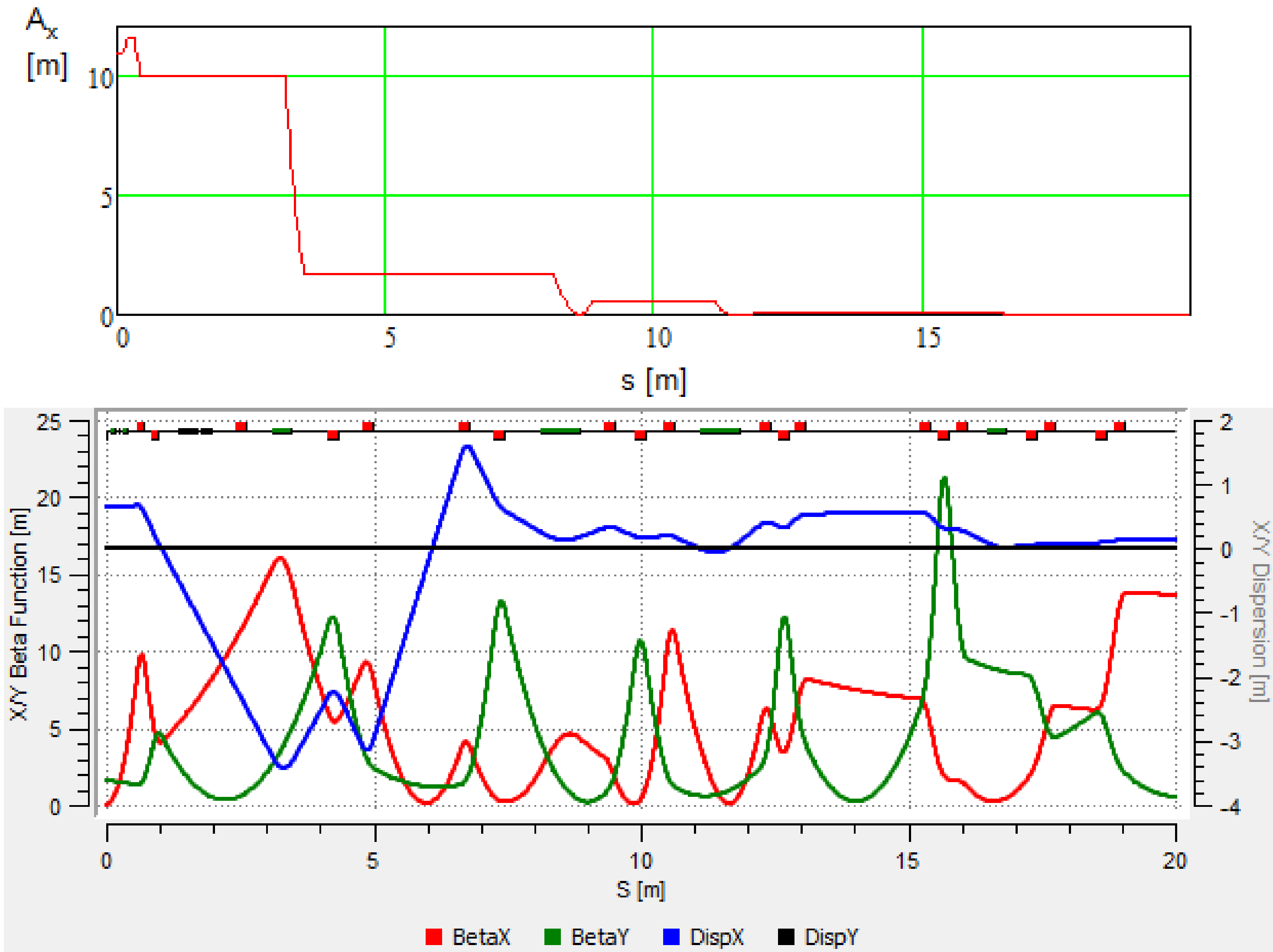
$$\Delta s_2 = -L_Q \theta^2 \,, \quad L_Q = \frac{\beta^*}{2}\left(I_1 + \left|I_2\right|\right) \approx 77 \text{ cm}$$

  - Shortening due to sextupoles ($\delta\theta_S = x^2 / \left(2 x_{os}{}^2\right)$ for defocusing sext) :

$$\Delta s_2 = M_{S_{5,2}} \delta\theta_S \xrightarrow[M_{S_{5,2}} = 2\varphi L_S]{\delta\theta_S = \frac{x^2}{2 x_{os}{}^2} = \frac{(L_S\theta)^2}{2 x_{os}{}^2}} = \frac{L_S{}^3 \varphi}{x_{os}{}^2} \theta^2 \,, \quad \begin{cases} L_S \approx 26.5 \text{ cm} \\ \varphi \approx 0.124 \text{rad} \end{cases}$$

  - Comparing, we obtain the sextupole strength: SdL≈ -11 kG/cm (defocus.)
- Vertical compensation is questionable
- Next round of simulations will follow

IOTA Optics for OSC



Optics functions and dispersion invariant for IOTA half ring

- Doublet focusing is adjusted to greatly reduce A_x at the first ring dipole
- Tunes are adjusted to be near half-integer
- Geometric acceptances: ε_x=20 μm, ε_x=16 μm, $\Delta p/p$=±0.005

IOTA Optics

Main Parameters of IOTA storage ring for OSC

Circumference	40 m
Nominal beam energy	100 MeV
Bending field of main dipoles	4.8 kG
Tunes, Q_x / Q_y	5.464/4.454
Natural chromaticities, ξ_x / ξ_y	-19 / -23
Chromaticities with OSC sextupoles	-81 / 29♥
$\perp$ emittance, $\varepsilon_{SR}/2 = \varepsilon_x = \varepsilon_y$, rms	8.6 nm
Rms momentum spread, σ_p	$1.29 \cdot 10^{-4}$
SR damping times (ampl.), τ_s / ($\tau_x = \tau_y$)	1.5 / 1.4 s
Cooling ranges* (before OSC), $n_{\sigma x}$ / $n_{\sigma s}$	6.9 / 3.4

* For hor. plane it is defined as $\sqrt{\varepsilon_{max}/\varepsilon_x}$. The 2nd order lengthening is neglected. Expected that in the hor. plane it will not be a problem after compensation with sextupoles.
The 2nd order lengthening limits the vertical cooling range to $n_{\sigma y} \approx 4$

♥ Chromaticities need to be compensated to be $|\xi| \le 20$

- Energy is reduced 150→100 MeV to reduce ε, σ_p and undulator period and length
- Operation on coupling resonance reduces horizontal emittance and introduces vertical damping
- Tunes are chosen to maximize dynamic aperture limitation by OSC sextupoles

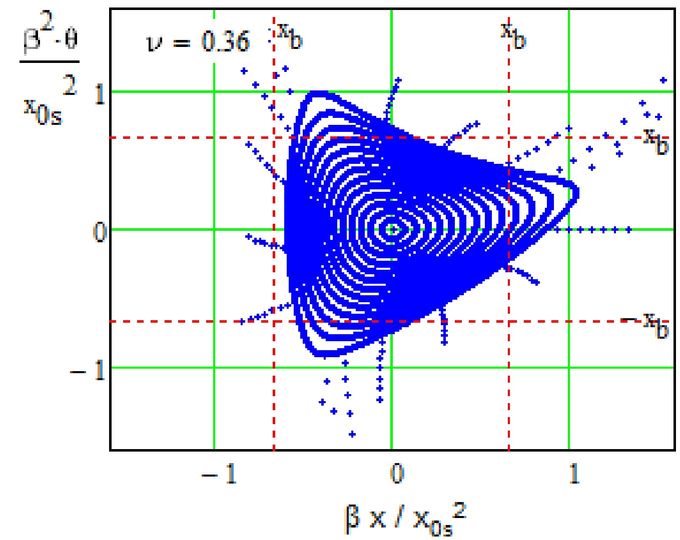
Dynamic Aperture Limitation by Sextupoles of OSC Insert

- Introduce dimensionless variables

$$\tilde{\theta} = \beta^2 \frac{\theta + \alpha x / \beta}{{x_{0S}}^2}, \quad \tilde{x} = \frac{\beta x}{{x_{0S}}^2} \quad \text{where} \quad {x_{0S}}^2 = \frac{pc}{e(SL)}$$

- Then the following transforms drive particle motion

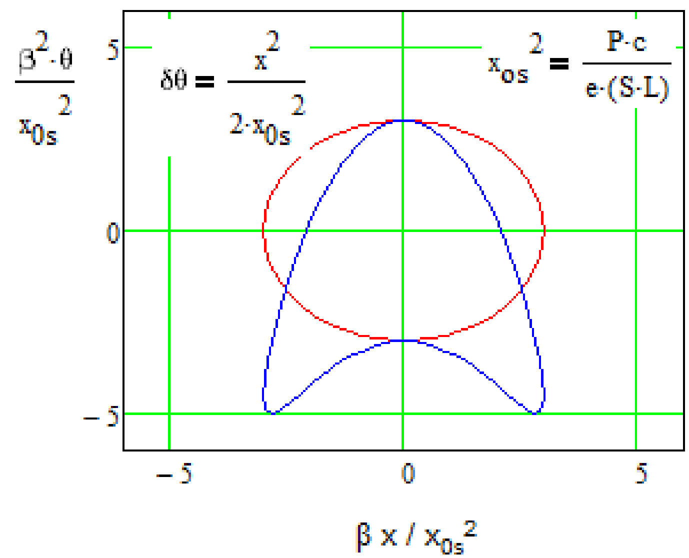
$$\begin{bmatrix} \tilde{x} \\ \tilde{\theta} \end{bmatrix}' = \begin{bmatrix} \cos\mu & \sin\mu \\ -\sin\mu & \cos\mu \end{bmatrix} \begin{bmatrix} \tilde{x} \\ \tilde{\theta} \end{bmatrix}, \quad \tilde{\theta}' = \tilde{\theta} + \frac{\tilde{x}^2}{{x_{0S}}^2}$$



- In vicinity of 3rd order resonance:

$$\tilde{x}_b \approx 25\left|[\nu] - \frac{1}{3}\right| \Rightarrow \varepsilon_b \approx \frac{625 {x_{0S}}^4}{\beta^3}\left([\nu] - \frac{1}{3}\right)^2$$

Sengle sextupole transformation



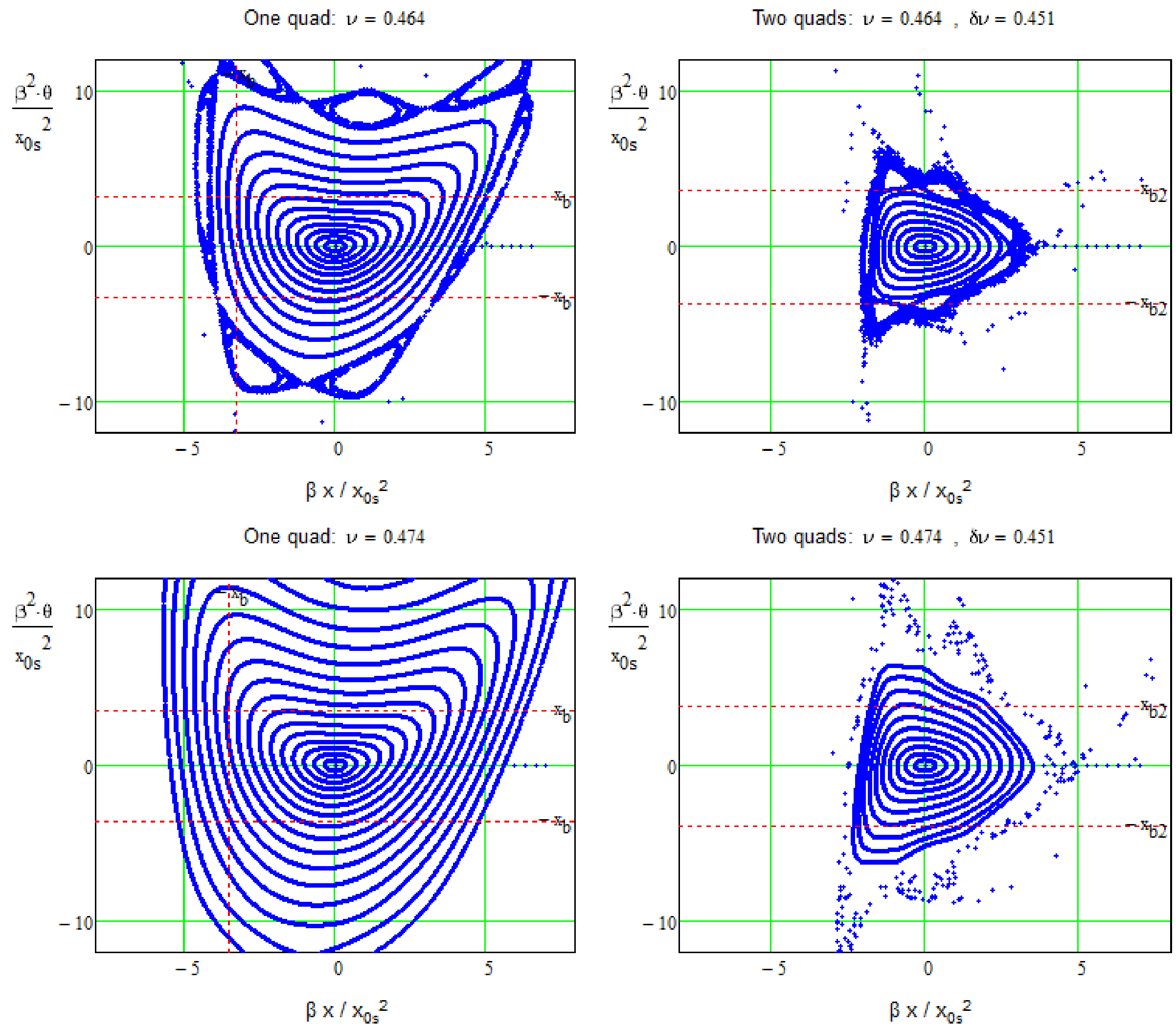
- Far from the resonance the stability boundary can be estimated from the phase space distortion =>

$$\tilde{x}_b \approx 3 \Rightarrow \varepsilon_b \approx \frac{9 {x_{0S}}^4}{\beta^3}$$

- Transition happens at detuning $\Delta\nu \approx 0.1$

Dynamic Aperture Limitation by Sextupoles of OSC Insert(2)

- Phase advance between OSC sextupoles $\Delta Qx=0.451$
  - Although it is close to half integer it does not help with cancellation of sextupole effect



Phase space immediately upstream of first sextupole; top – nominal tune, bottom closer to half integer

- Operation closer to half-integer resonance improves dynamic aperture
- For estimate we use

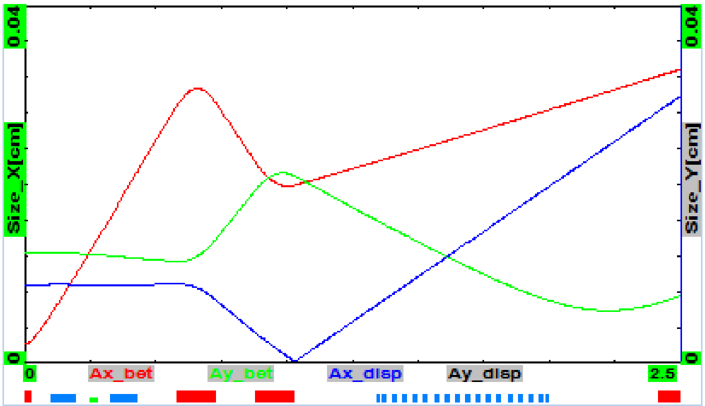
$$\varepsilon_{bx} \approx \frac{9 x_{0S}^{\ 4}}{\beta_x^{\ 3}}, \quad x_{0S}^{\ 2} = \frac{pc}{e(SL)}$$

⇨ ε_{bx}=14 μm, $n_{\sigma x}$≈40
(compare to ε_{x_geom}=20 μm)

- Looks like aperture limitation by OSC sextupoles looks OK
- Orbit stability within sextupoles <100 μm
- Detailed simulations are required

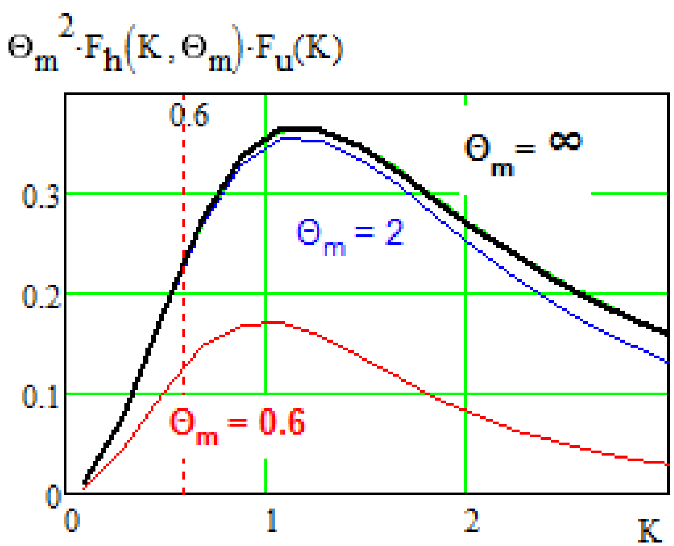
Undulators

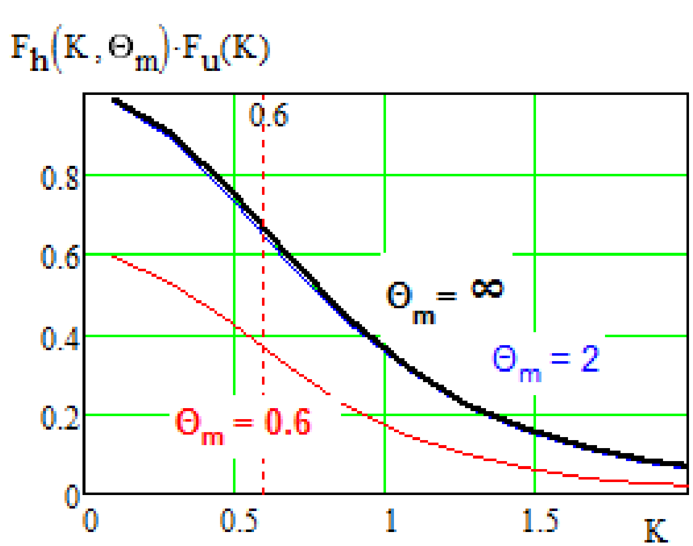
- Undulator period was chosen so that $\lambda|_{\theta=0}$=2.2 μm



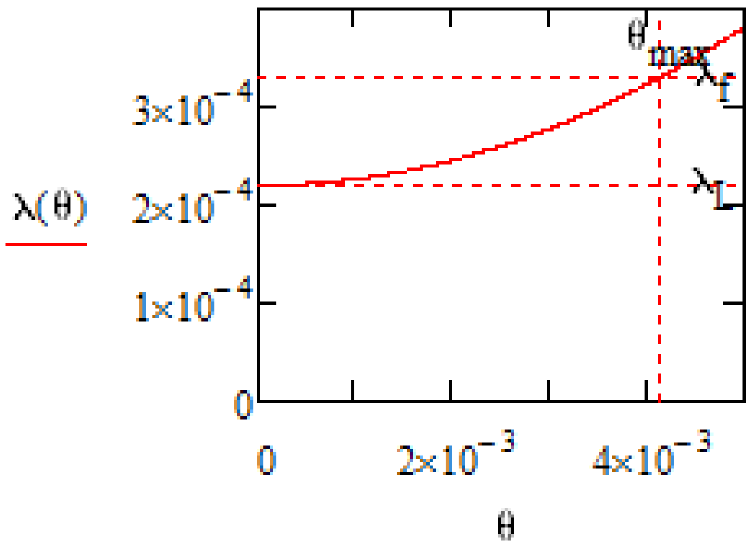
Rms beam sizes in absence of OSC,
σ_x=0.25 mm - in undulators

Radiation wavelength at zero angle	2.2 μm
Undulator parameter, K	0.8
Undulator period	12.9 cm
Number of periods, *m*	6
Total undulator length, L_w	0.77 m
Peak magnetic field	664 G
Distance between centers of undulators	3.3 m
Energy loss per undulator per pass	22 meV
Average power per undulator for $N_e=10^6$	26 nW
Radiation size in 2-nd undulator, HWHM	0.35 mm





Cooling Rates



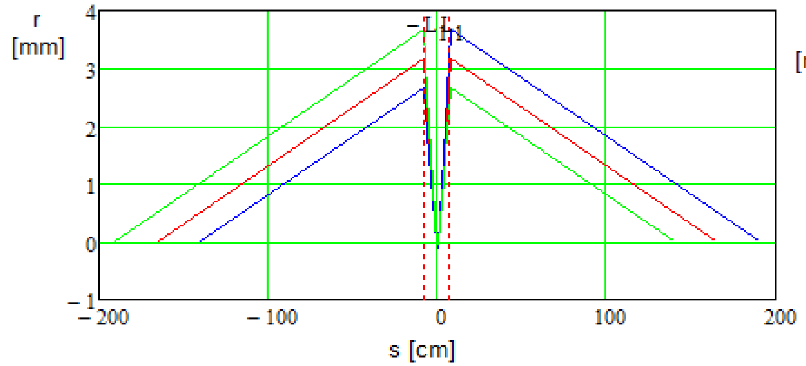
Main parameters of passive OSC

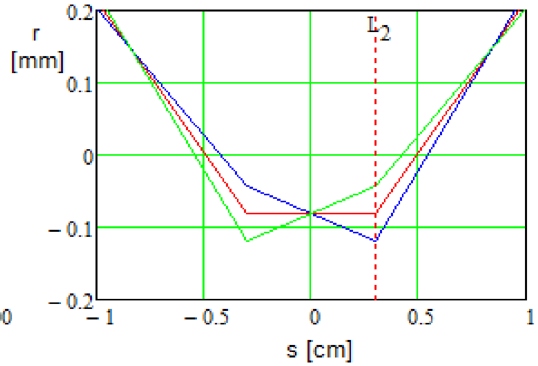
Band	2.2 – 3.3 μm
Angular acceptance	4.1 mrad
Optics system radius	7 mm
Damp. rates (*x*=*y*/*s*)	6.5/7.6 s^{-1}

- Passive OSC increases the SR damping rates by about one order of magnitude
- Optical amplifier with 20 dB gain could increase the damping rate by factor of ~3
  - Factor of 3 will be lost due to smaller bandwidth
  - Detailed design is pending

Focusing of Beam Radiation to OA and Kicker

- Two possibilities
  - For passive OSC: four lens system with complete suppression of depth of field
  - Two lens system (F=8 cm, radius – 3.5 mm)
    - Reasonable compromise between 4 major requirements
    - The spot size in OA to be sufficiently small: r<60 μm
      - ⇒ diffraction limited size in OA: HWHM=12 μm or total size r≈30 μm
      - ⇒ size due to beam convergence/divergence at OA input/exit ≈50 μm





Other Limitations

- Touschek lifetime and multiple IBS limit the number of particles in the bunch, $N_e \sim 10^6$
- Scattering on the residual gas results in short lifetime in the conditions of small cooling acceptance
- Quantum effects play little role in the OSC cooling

Quantum Mechanical Treatment of Transit-Time Optical Stochastic Cooling of Muons

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Quantum theory of Optical Stochastic Cooling *

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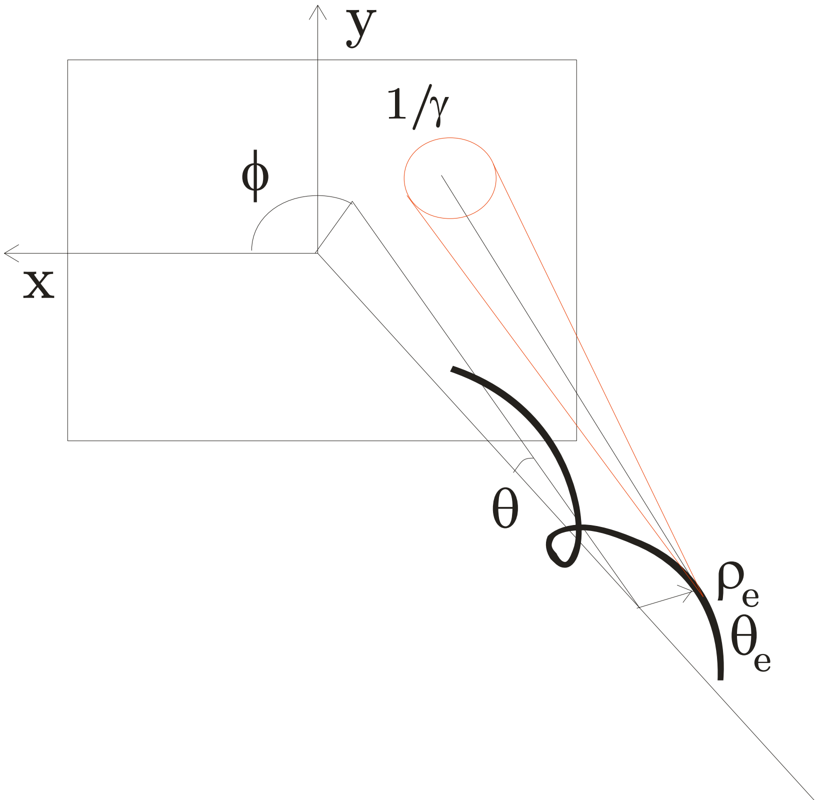
Conclusions

- Optical stochastic cooling looks as a promising technique for future hadron colliders
- Experimental study of OSC in Fermilab is in its initial phase
  - It is aimed to validate cooling principles and to demonstrate cooling with and without optical amplifier
    - Even in the absence of amplification (passive system, $G = 1$) the OSC damping exceeds SR damping by about an order of magnitude
- The beam intensity ranges from a single electron to the bunch population limited by operation at the optimum gain (10^8-10^9)
  - Single electron cooling - localization of electron wave function and essence of quantum mechanics
    - Quantum noise for passive cooling
  - Cooling at the optimal gain (ultimate cooling) gets us to otherwise hidden details of OSC, in particular, to signal suppression

Backup Slides

Basics of OSC – Radiation from Undulator



- Liénard-Wiechert potentials and E-field of moving charge in wave zone

$$\begin{cases} \varphi(\mathbf{r},t) = \left.\dfrac{e}{(R-\boldsymbol{\beta}\cdot\mathbf{R})}\right|_{t-R/c} \\ \mathbf{A}(\mathbf{r},t) = \left.\dfrac{e\mathbf{v}}{(R-\boldsymbol{\beta}\cdot\mathbf{R})}\right|_{t-R/c} \end{cases} \Rightarrow$$

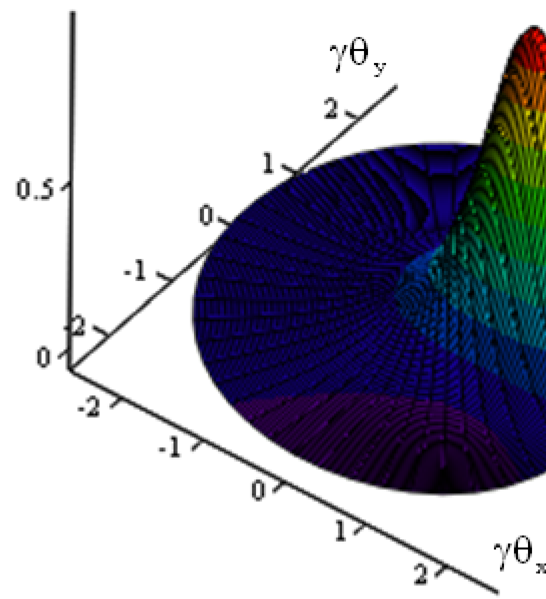
$$\mathbf{E}(\mathbf{r},t) = \frac{e}{c^2}\left.\frac{(\mathbf{R}-\boldsymbol{\beta}\cdot R)(\mathbf{a}\cdot\mathbf{R}) - \mathbf{a}R(R-\boldsymbol{\beta}\cdot\mathbf{R})}{(R-\boldsymbol{\beta}\cdot\mathbf{R})^3}\right|_{t-R/c}$$

where $\mathbf{a} \equiv \dfrac{d\mathbf{v}}{dt}$

- Radiation of ultra-relativistic particle is concentrated in $1/\gamma$ angle
- Undulator parameter:

$$K \equiv \gamma\theta_e = \frac{\lambda_{wgl}}{2\pi}\frac{eB_0}{mc^2}$$

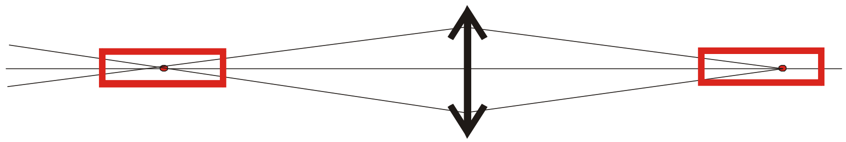
- For K ≥ 1 the radiation is mainly radiated into higher harmonics



E_x for K=1

Basics of OSC – Radiation Focusing to Kicker Undulator

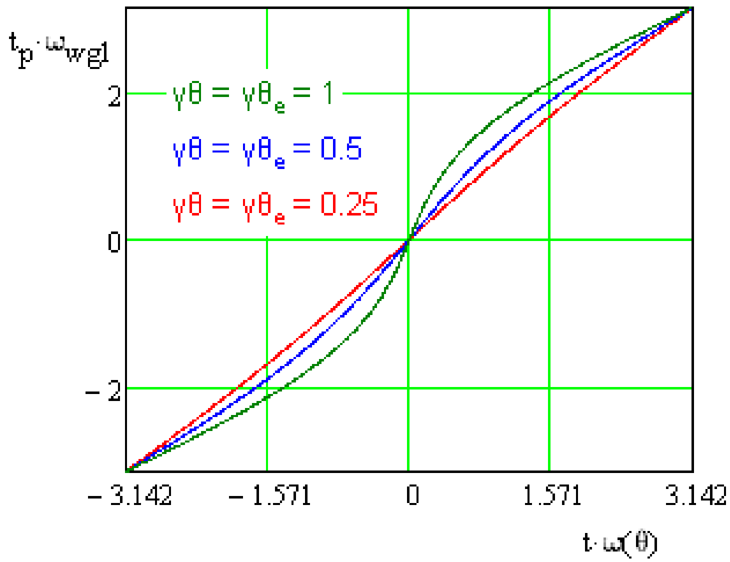
- Modified Kirchhoff formula

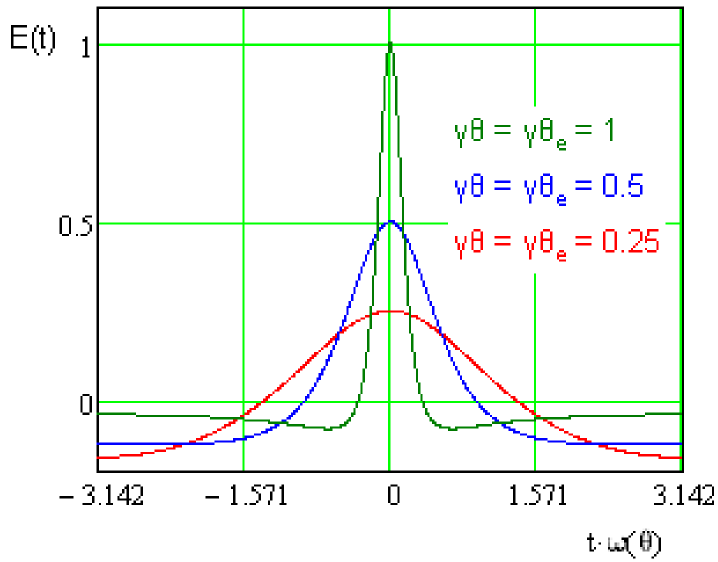


$$E(r) = \frac{\omega}{2\pi i c}\int_S \frac{E(r')}{|r-r'|}\mathrm{e}^{i\omega|r-r'|}\,ds'$$

=> $$E(r) = \frac{1}{2\pi i c}\int_S \frac{\omega(r')E(r')}{|r-r'|}\mathrm{e}^{i\omega|r-r'|}\,ds'$$

- Effect of higher harmonics
  - Higher harmonics are normally located outside window of optical lens transparency and are absorbed in the lens material



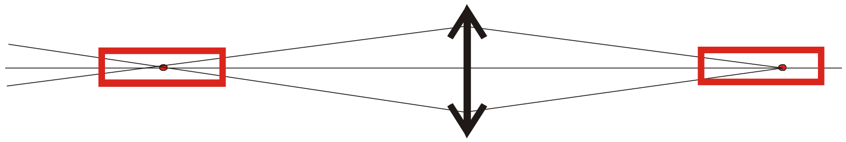


Dependences of retarded time (t_p) and E_x on time for helical undulator

- Only first harmonic is retained in the calculations presented below

Basics of OSC – Longitudinal Kick for K<<1

- For $K << 1$ refocused radiation of pickup undulator has the same structure as radiation from kicker undulator. They are added coherently:

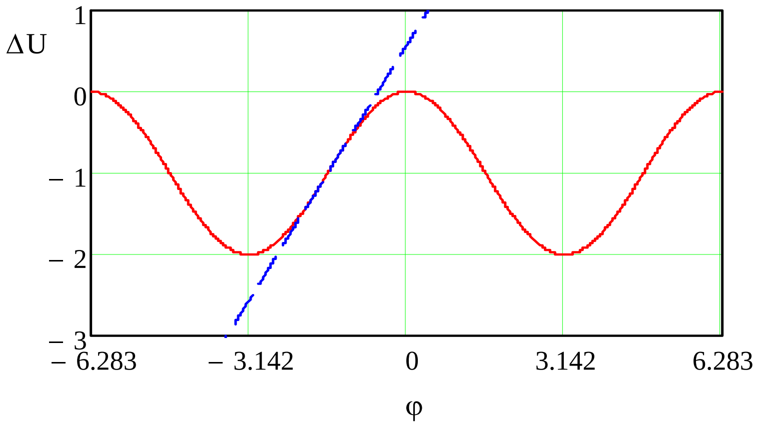


$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 e^{i\phi} \xrightarrow{\mathbf{E}_1 = \mathbf{E}_2} 2\cos\left(\phi / 2\right) \mathbf{E}_1 e^{i\phi/2}$$

⇨ Energy loss after passing 2 undulators

$$\Delta U \propto \left| E^2 \right| = 4\cos\left(\phi/2\right)^2 \left| \mathbf{E}_1{}^2 \right| = 2\left(1+\cos\phi\right)\left| \mathbf{E}_1{}^2 \right| = 2\left(1+\cos\left(kM_{56}\frac{\Delta p}{p}\right)\right)\left| \mathbf{E}_1{}^2 \right|$$

- Large derivative of energy loss on momentum amplifies damping rates and creates a possibility to achieve damping without optical amplifier



  - SR damping: $\lambda_{\|\_SR} \approx \dfrac{2\Delta U_{SR}}{pc} f_0$
  - OSC: $\lambda_{\|\_OSC} \approx f_0 \dfrac{2\Delta U_{wgl}}{pc}\left(GkM_{56}\right) \xrightarrow{kM_{56}(\Delta p/p)_{\max}=\pi} f_0 \dfrac{2\Delta U_{wgl}}{pc}\left(\dfrac{G}{\left(\Delta p/p\right)_{\max}}\right)$

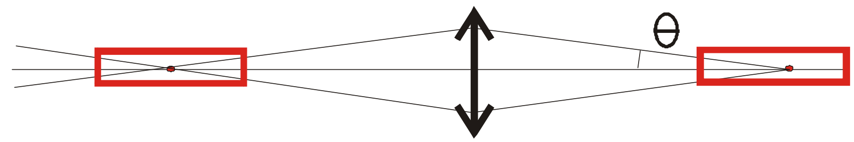
where G - optical amplifier gain, $(\Delta p/p)_{max}$ - cooling system acceptance

⇨ $\lambda_{\|\_OSC} \propto B^2 L \propto K^2 L$ - but cooling efficiency drops with K increase above ~1

Basics of OSC – Longitudinal Kick for K<<1(continue)

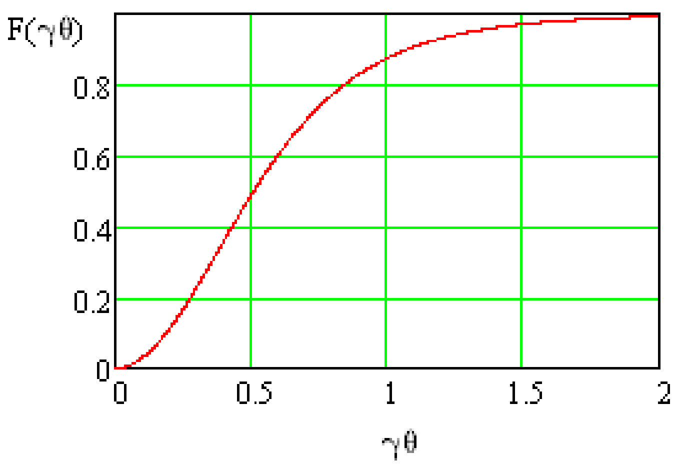
- Radiation wavelength depends on θ as

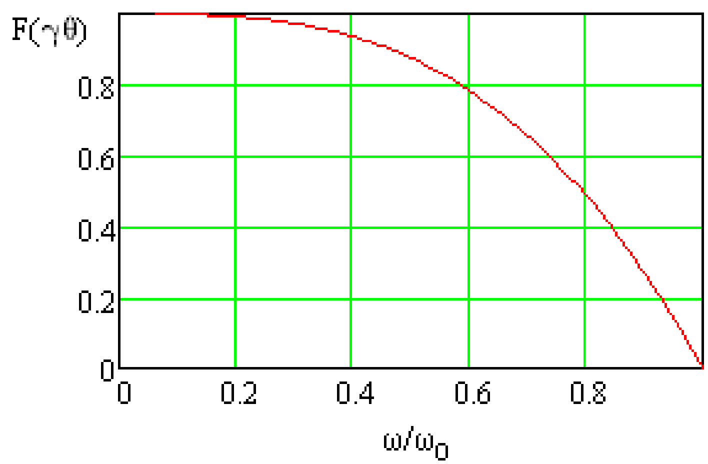
$$\lambda = \frac{\lambda}{2\gamma^2}\left(1+\gamma^2\theta^2\right)$$



Limitation of system bandwidth by (1) optical amplifier band or (2) subtended angle reduce damping rate

$$\lambda_{\|\_SR} = \lambda_{\|\_SR0} F\left(\gamma\theta_{\mathrm{m}}\right), \qquad F\left(x\right) = 1 - \frac{1}{\left(1+x^2\right)^3}$$





- For narrow band: $\Delta U_{wgl} = \Delta U_{wgl0}\left(\frac{3\Delta\omega}{\omega}\right), \quad \frac{3\Delta\omega}{\omega} << 1$

where $\Delta U_{wgl0} = \frac{e^4 B^2 \gamma^2 L}{3m^2c^4}\begin{cases} 1, & \text{Flat wiggler} \\ 2, & \text{Helical wiggler}\end{cases}$ the energy radiated in one undulator

Basics of OSC – Radiation from Flat Undulator

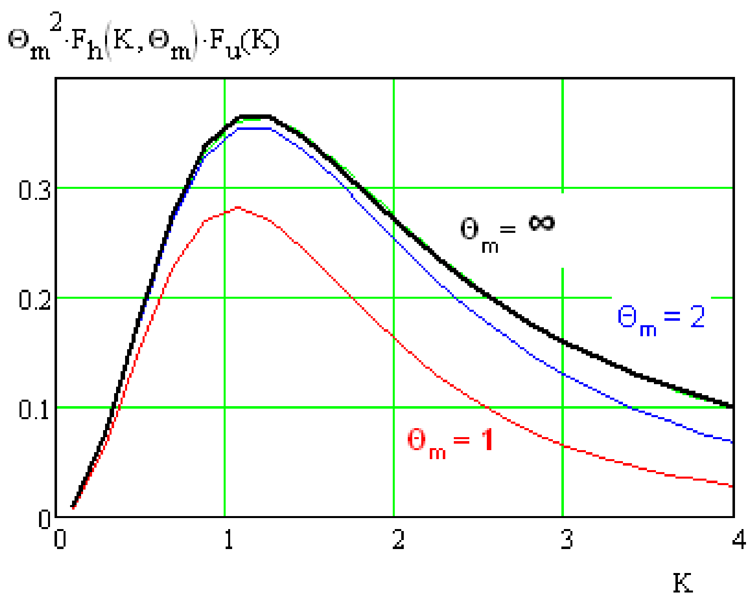
- For arbitrary undulator parameter we have

$$\Delta U_{OSC\_F} = \frac{1}{2}\frac{4e^4 B_0^{\,2}\gamma^2 L}{3m^2c^4} G F_f\left(K,\gamma\theta_{\max}\right) F_u\left(\kappa_u\right)$$

$$F_u\left(\kappa_u\right) = \mathrm{J}_0\left(\kappa_u\right) - \mathrm{J}_1\left(\kappa_u\right), \quad \kappa_u = K^2 / \left(4\left(1+K^2/2\right)\right)$$

Fitting results of numerical integration yields:

$$F_h\left(K,\infty\right) \approx \frac{1}{1+1.07K^2+0.11K^3+0.36K^4}, \quad K \equiv \gamma\theta_e \le 4$$



- Dependence of wave length on θ:

$$\lambda \approx \frac{\lambda_{wgl}}{2\gamma^2}\left(1+\gamma^2\left(\theta^2+\frac{\theta_e^{\,2}}{2}\right)\right)$$

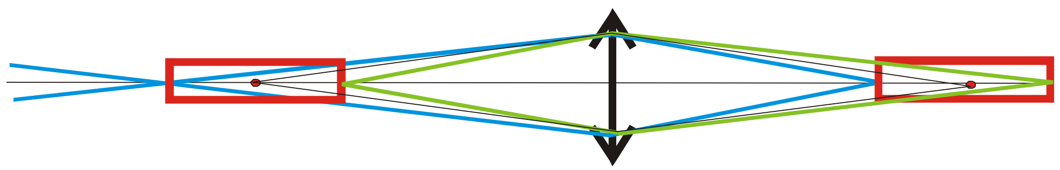
$$K \equiv \gamma\theta_e$$

- Flat undulator is "more effective" than the helical one
- For the same K and λ_{wgl} flat undulator generates shorter wave lengths

- For both cases of the flat and helical undulators and for fixed *B* a decrease of λ_{wgl} and, consequently, λ yields kick increase
  - ♦ but wavelength is limited by both beam optics and light focusing

Basics of OSC – Correction of the Depth of Field

- It was implied above that the radiation coming out of the pickup undulator is focused on the particle during its trip through the kicker undulator



  - It can be achieved with lens located at infinity

$$\frac{1}{2F+\Delta s}+\frac{1}{2F-\Delta s}=\frac{1}{F} \rightarrow \frac{1}{F-\Delta s^2/4F}=\frac{1}{F} \xrightarrow{F\to\infty} \frac{1}{F}=\frac{1}{F}$$

  - but this arrangement cannot be used in practice

- A 3-lens telescope can address the problem within limited space

$$\begin{bmatrix}1 & L\\0 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\-F_1^{-1} & 1\end{bmatrix}\begin{bmatrix}1 & L_1\\0 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\-F_2^{-1} & 1\end{bmatrix}\begin{bmatrix}1 & L_1\\0 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\-F_1^{-1} & 1\end{bmatrix}\begin{bmatrix}1 & L\\0 & 1\end{bmatrix}=\begin{bmatrix}-1 & 0\\0 & -1\end{bmatrix}$$