Fermilab

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Muon Accelerator Program

HFOFO-Snake - Status & Outlook

Yuri Alexahin (FNAL APC)

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Basic Principles

- **Alternating solenoidal field** → ~ equal cooling rates of the transverse normal modes
- **Resonance dispersion generation** → very large dispersion possible:

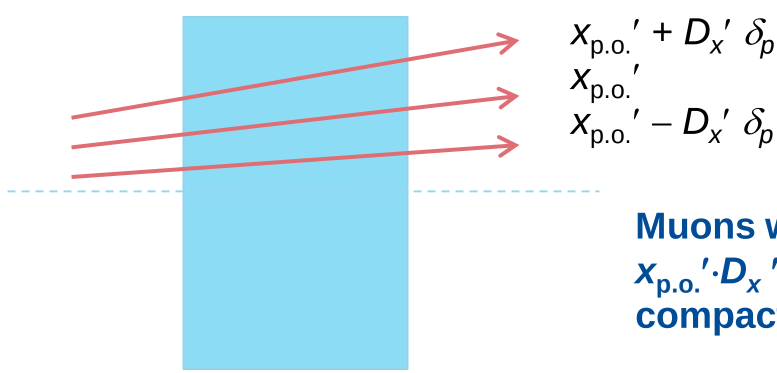
In a periodic structure with alternating transverse field the periodic orbit resonantly depends on the tune: $x_{\text{p.o.}} \sim 1/\sin \pi Q_x$,

if there is also a large chromaticity we obtain dispersion

$$D_x = \frac{dx_{\text{p.o.}}}{d\delta_p} \approx -\pi Q_x' x_{\text{p.o.}} \cot(\pi Q_x)$$

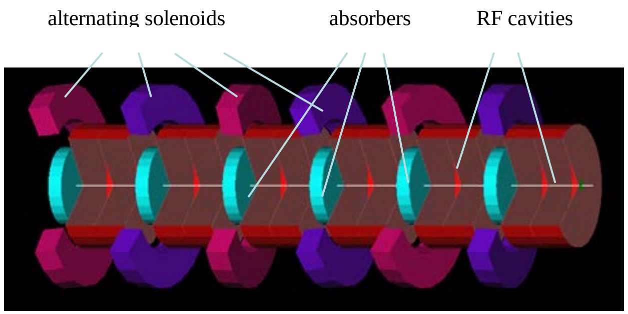
Since naturally the chromaticity is negative, $Q_x'<0$, we can obtain also large positive momentum compaction factor if the tune is just above an integer, $Q_x \geq$ n.

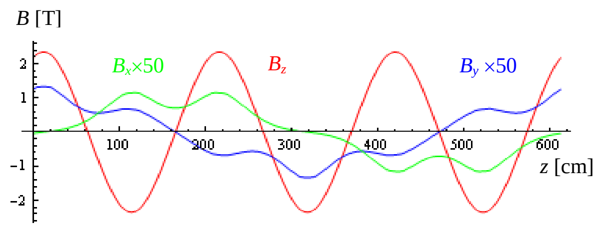
- **Longitudinal cooling in FLAT absorbers due to D′** (only much later it was realized that wedge absorbers also can work for both beams)

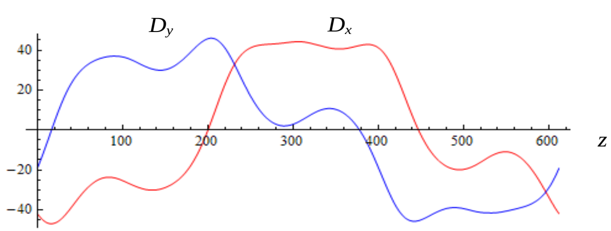


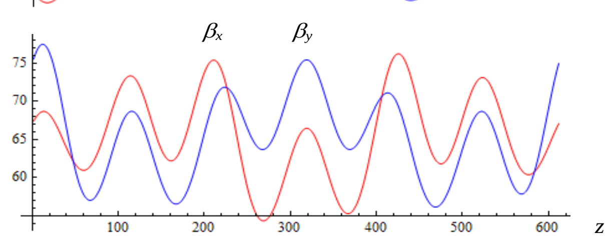
Muons with higher δ_p make a longer path if $x_{\text{p.o.}}' \cdot D_x' > 0$ → positive momentum compaction factor is necessary

201 MHz Helical FOFO Snake Design



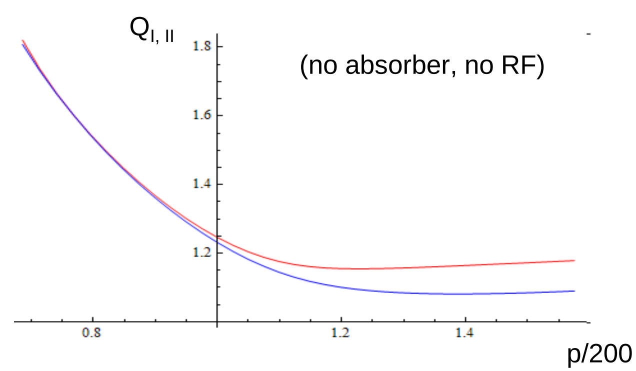






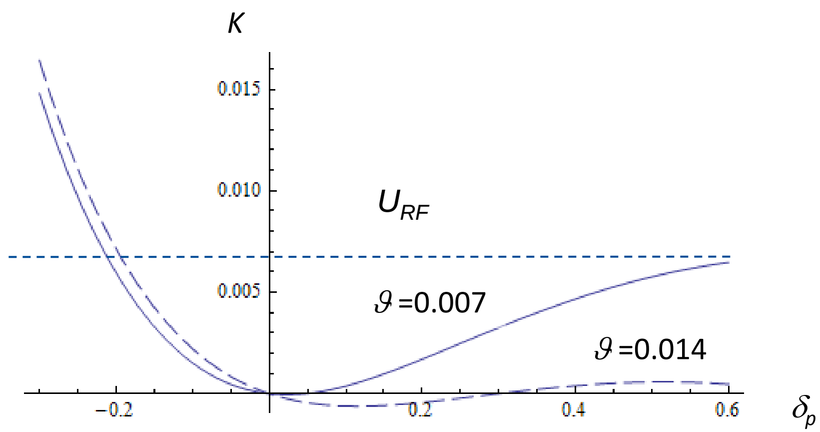
- Rotating ("helical") magnetic field is more efficient, here it is created by solenoid inclination by angle $\vartheta = 0.007$ in rotating plane.
- RF gradient 16 MV/m, 380μm thick Be windows.
- 15 cm wide LH2 absorbers with 100μm thick 2090-T81 walls

- Stable momentum range is unexpectedly wide thanks to higher order chromaticity
- Orbits of both beams are similar – just shifted along the axis by ½ period



Encountered Difficulties

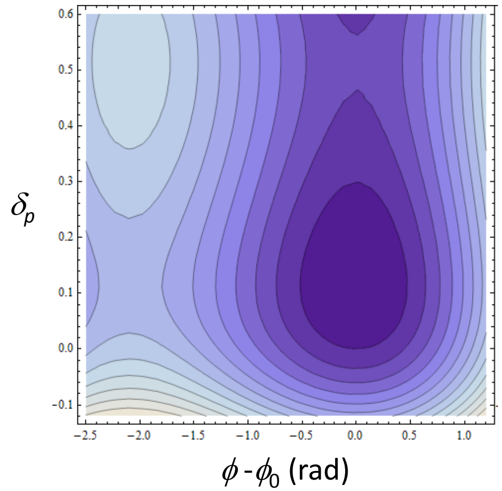
- Momentum acceptance limited from above by the slippage factor sign reversal
- Large difference in the transverse normal modes cooling rates even in the helical snake, was remedied by addition of a quadrupole field
- Large initial transverse amplitude due to the orbit and beta-function mismatch (difficult to match for both beams simultaneously)



Kinetic energy in quasi-static approximation

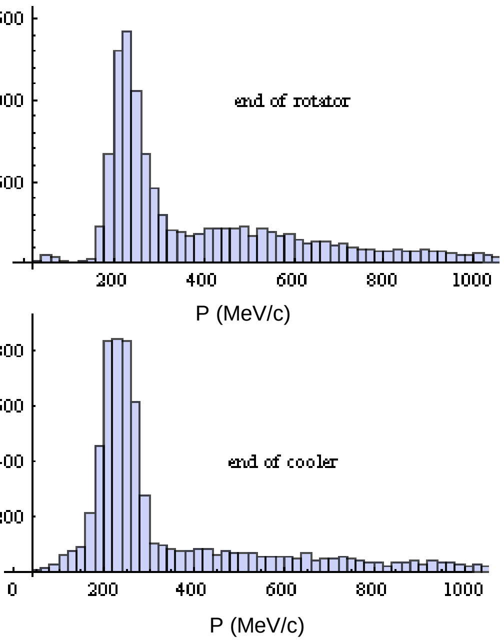
$$K(\delta_p) = \int^{\delta_p} [1-\lambda(\xi)\frac{\beta_0}{\beta(\xi)}]\, d\xi$$

where λ = orbit length/L_0

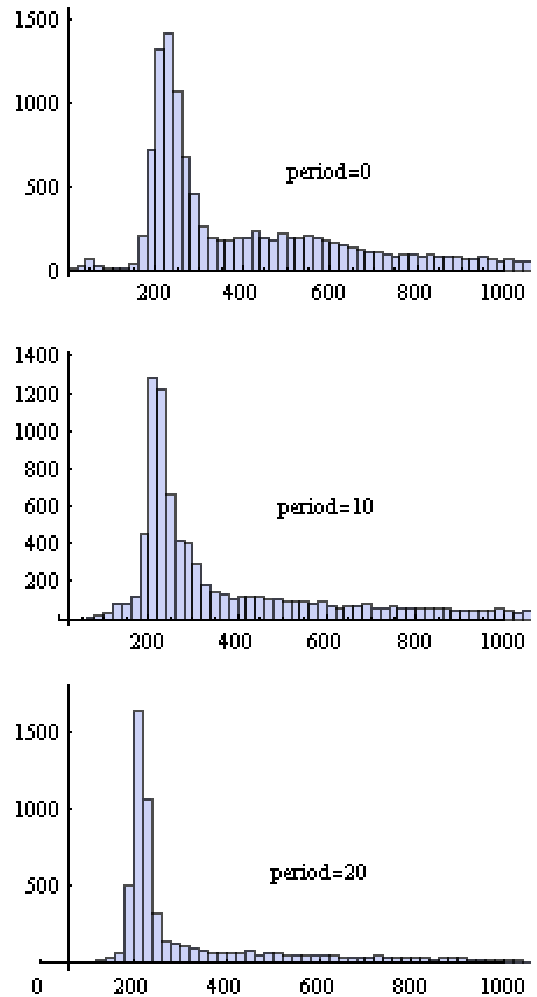


Contour plot of the longitudinal Hamiltonian for ϑ =0.014 – it is the kinematic nonlinearity that limits momentum acceptance, not an insufficient RF bucket depth.

201 MHz Snake Performance



Momentum distribution (w/o weights) before and after 4D cooler (90m long)



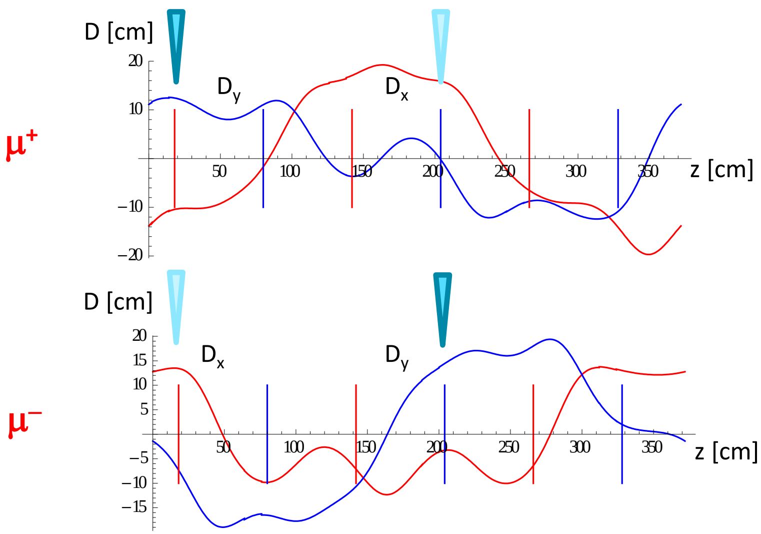
Momentum distribution (w/o weights) before, in the middle and after HFOFO snake (122m)

- This version HFOFO snake had ~10% lower transmission than D. Neuffer's 4D linear cooler : 63% vs 70%, but
- much lower (by a factor of ~5) longitudinal emittance.
- Transverse emittance (Gaussian fit): $\varepsilon_{\perp N}$=8mm vs 6mm

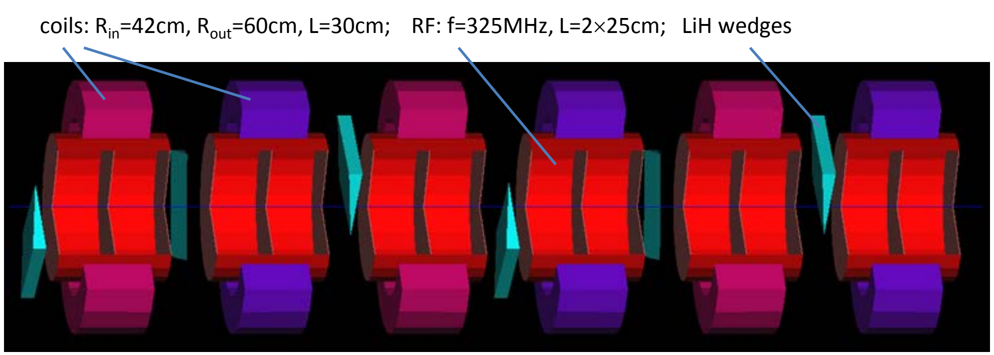
- not bad taking into account insufficient optimization and poor matching

325 MHz HFOFO Snake

- Higher frequency RF is bad for transverse acceptance but it allows for
  - cavities INSIDE solenoids
  - absorbers BETWEEN solenoids at $\beta_\perp$ minima
  - higher Ez in vacuum RF
- It was realized that wedge absorbers in HFOFO can work for both beams simultaneously:
  - LiH wedge + high pressure gaseous H2 = “hybrid channel”

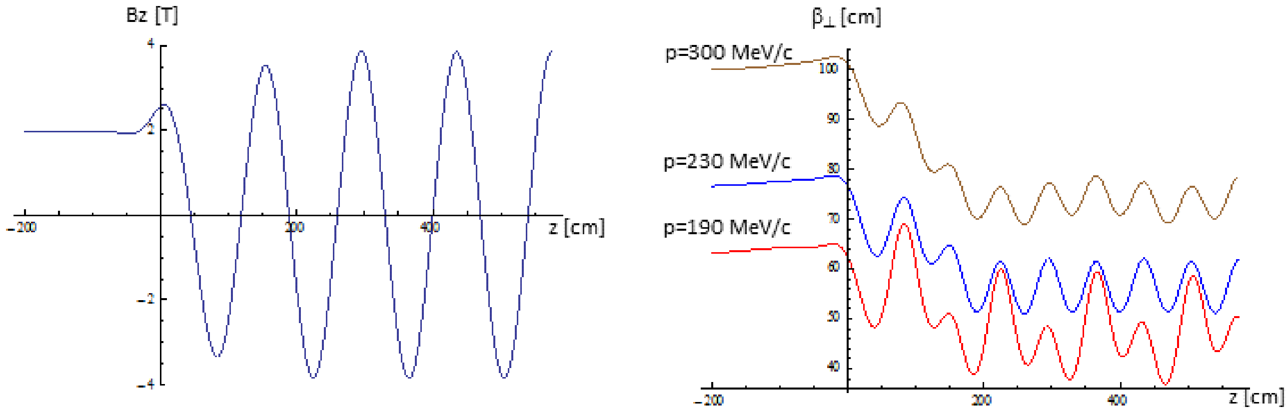


Dispersion and two vertical wedge absorbers: the left works for μ^+ while the right works for μ^-

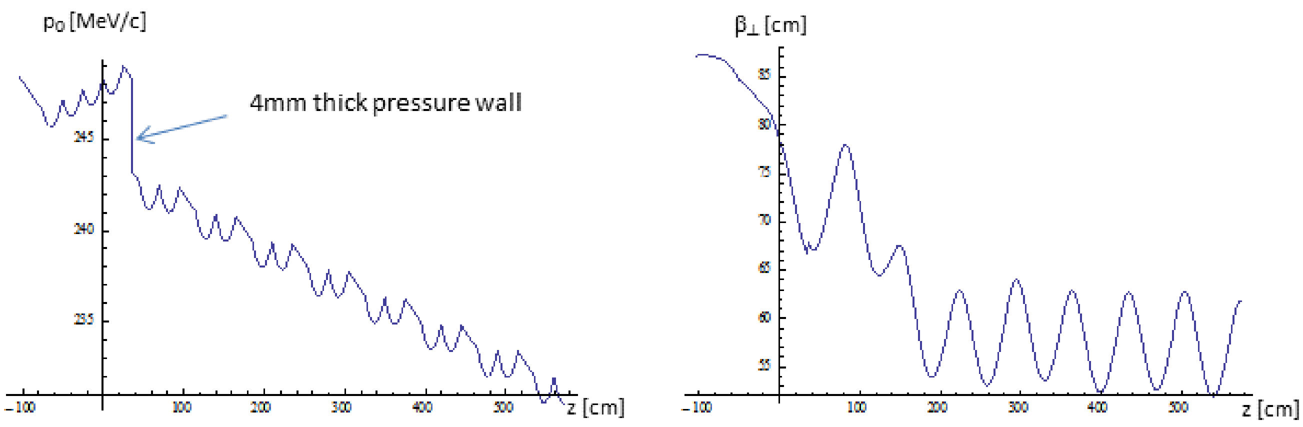


- The channel filled with GH2 with density 1/5 of LH2
- RF gradient 25 MV/m, Be windows with radius and thickness reduced in 3 steps (R=30→20cm, w=120 →70μm)
- LiH absorbers with tapered wedge angle (0.17 →0.2rad)
- Solenoid pitch 2.5mrad, tapered Bz=3.9 → 3.5T
- The channel period =6×0.7m = 4.2m, 30 periods total

Linear Optics Matching

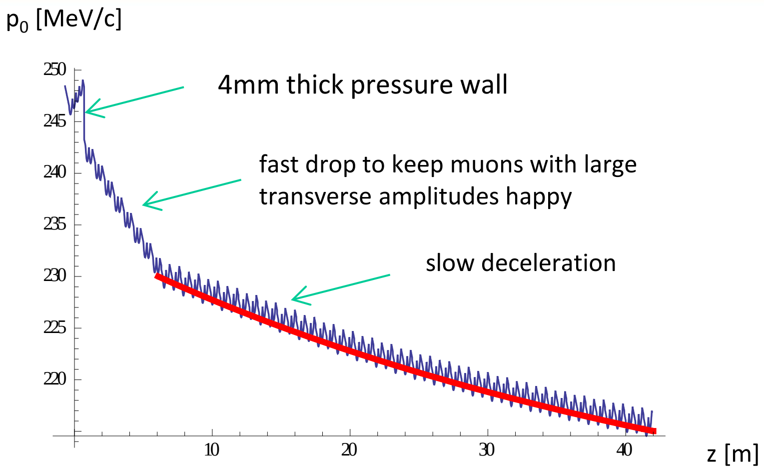


Magnetic field in the transition area (left) and β-function for constant momentum (right)

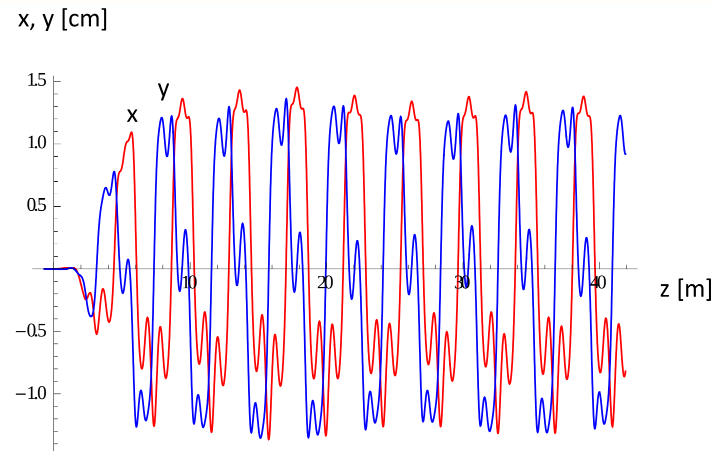


Total momentum of the reference particle in the beginning of the channel (left) and β-function for such momentum dependence (right).

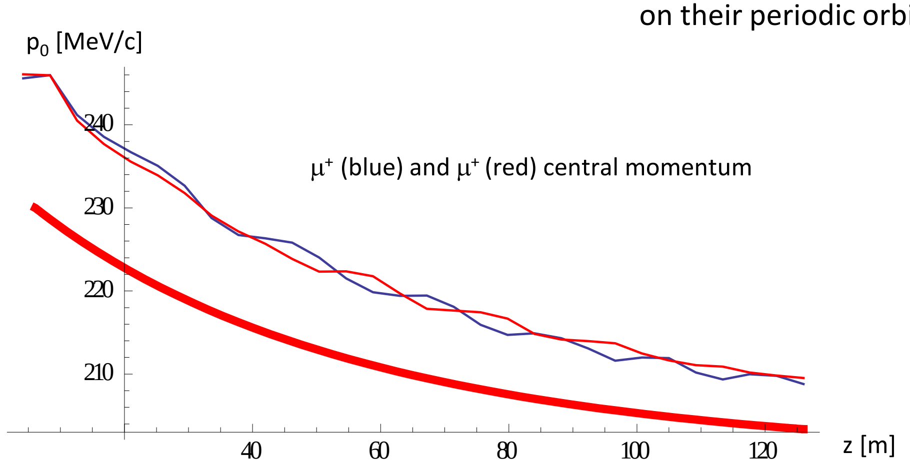
Momentum and orbit matching



Momentum of reference particle (blue) and target value (red).
Adjustment made by RF timing (gradient fixed).

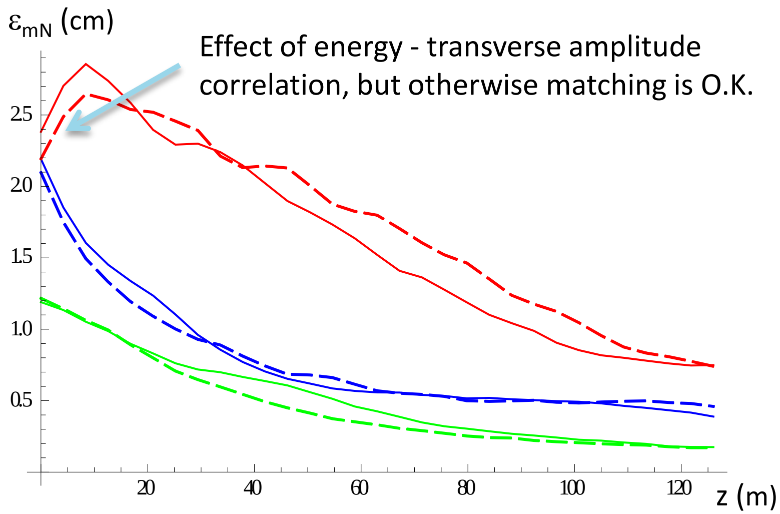


Reference particle trajectory (red – x, blue – y), inclination of solenoids 3-9 was used for placing μ^+ and μ^- on their periodic orbits simultaneously.

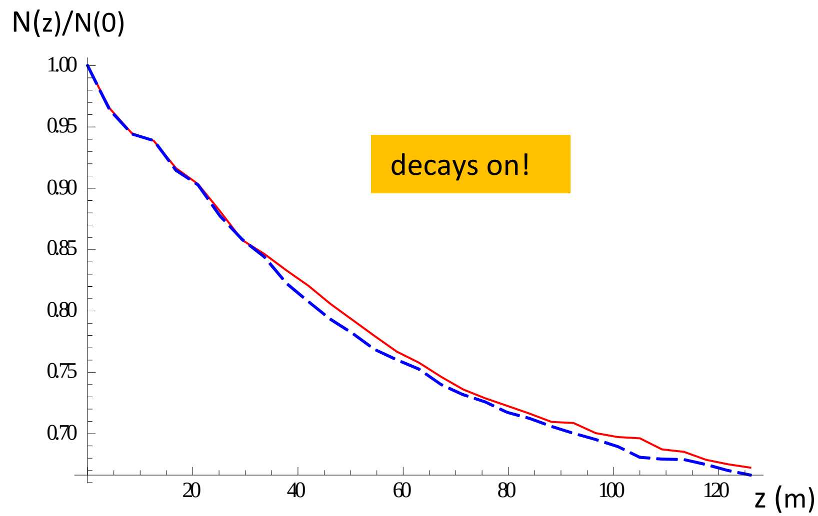


With real beam the RF timing had to be readjusted to compensate for the (residual) effect of momentum-betatron amplitude correlation

Cooling & Transmission (G4BL)



Normalized emittances (cm) from Gaussian fit: μ^+ - solid lines, μ^- - dashed lines.

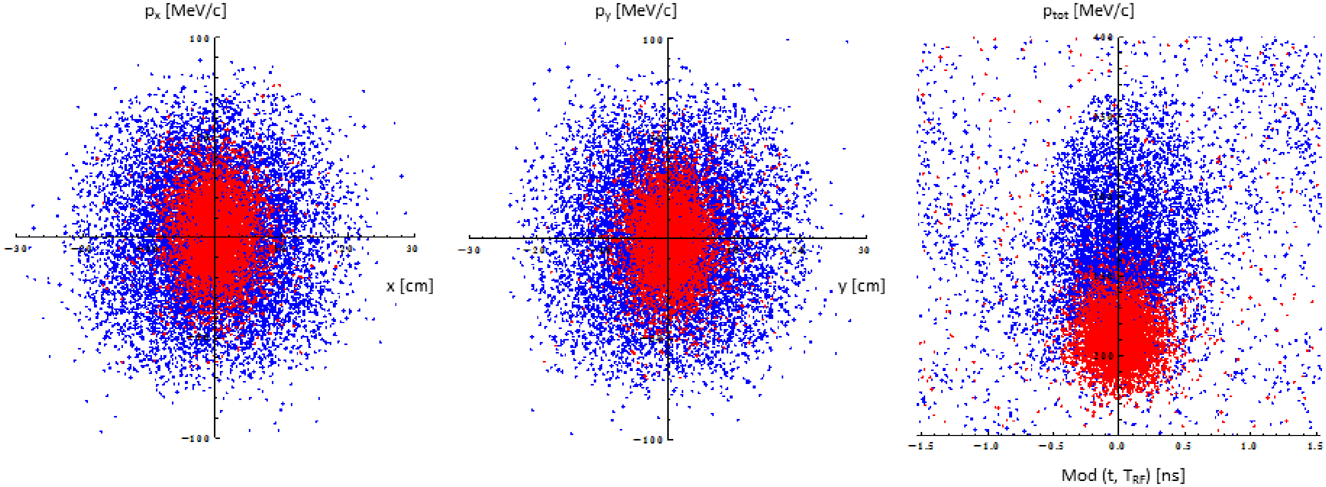


Transmission as a ratio of the number of muons in the Gaussian core: red solid line - μ^+, blue dashed line - μ^-.

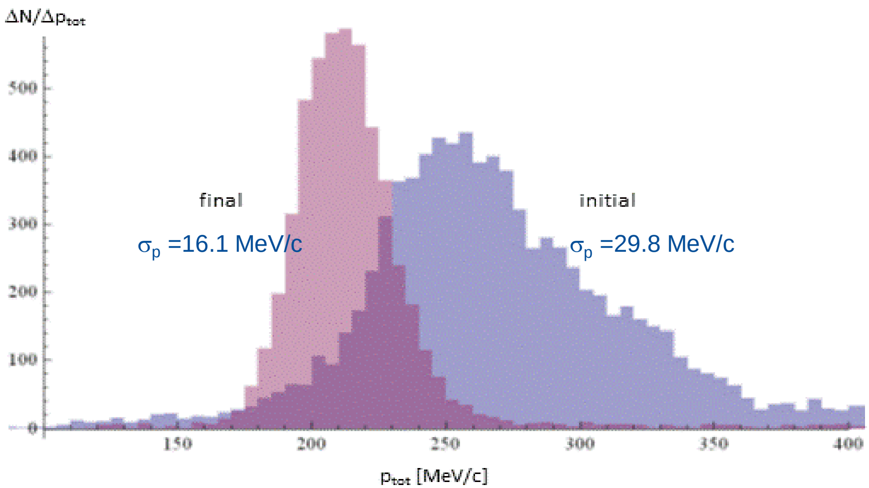
Final/Initial values (Gaussian fit):

	$N^{(total)}$	$N^{(150<p<360)}$	$N^{(core)}$	$p^{(cnt)}$, MeV/c	ε_{mN}, cm			ε_{6D}, cm^3
μ^+	5378/11755	5167/7998	5010/7329	208.2/248.0	0.19/1.19	0.36/2.19	0.76/2.38	0.051/6.22
μ^-	5896/12396	5743/9020	5499/8248	207.7/248.8	0.16/1.22	0.46/2.10	0.72/2.19	0.051/5.59

Initial/Final Phase Space Distributions



Initial μ+ beam (blue) and cooled beam in the 2T exit solenoid (red). All bunches were projected onto the same RF bucket in the right plot. No cuts applied



What is next?

- Transmission can be significantly improved by eliminating initial blowup of longitudinal emittance
- The probable cause of this blowup is the energy - transverse amplitude^2 correlation which is ~$1/\beta_\perp$
- The simplest way is to adiabatically lower $\beta_\perp$ (from present 83cm to ~55cm) in the front end – for that I need the G4BL file for the front end.
- A “stage 2” can be appended with shorter RF cavities (say, 15 cm in length instead of present 25cm)
  $\Rightarrow$ smaller period length $\Rightarrow$ smaller $\beta_\perp$-function
  $\Rightarrow$ 6D emittance reduction by another factor 3 to 5
- Another possibility for “stage 2”: higher tune, $Q_\perp$>2 (see next slides)

To study these possibilities ~ 3 p.-m. is needed

Higher Phase Advance / Period

Channel parameters (6-solenoid period, $Q_{\perp} \geq 2$):

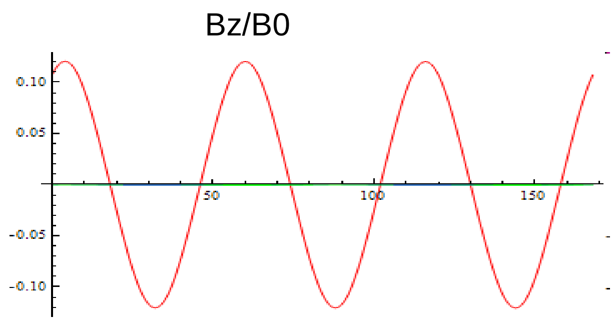
Vacuum RF: 800 MHz pillbox RF 2x8cm, Emax=32MV/m

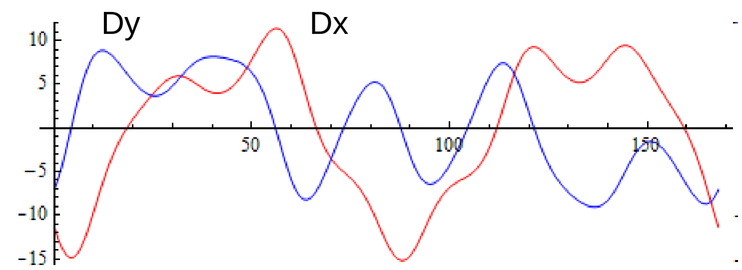
Solenoids: L=8cm, Rin=16cm, Rout=26cm,
Inclination 3mrad in rotated planes: 0, $\pi/3$, $2\pi/3$, π, $4\pi/3$, $5\pi/3$.

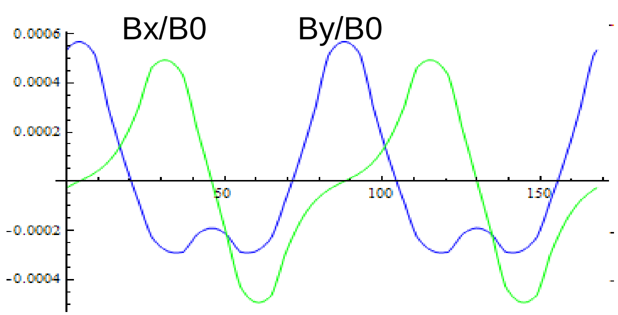
With p0=150MeV/c B0=94.5T $\Rightarrow$ Bzmax=11.5T

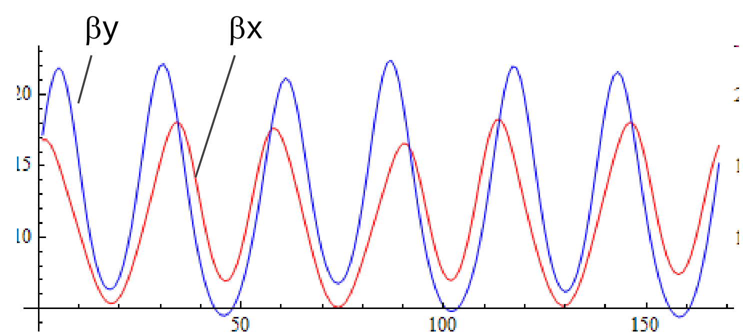
Absorbers: LiH, total width (on-axis) 1.2cm, full wedge angle 0.2rad

Total length of 6-cell period 1.68m



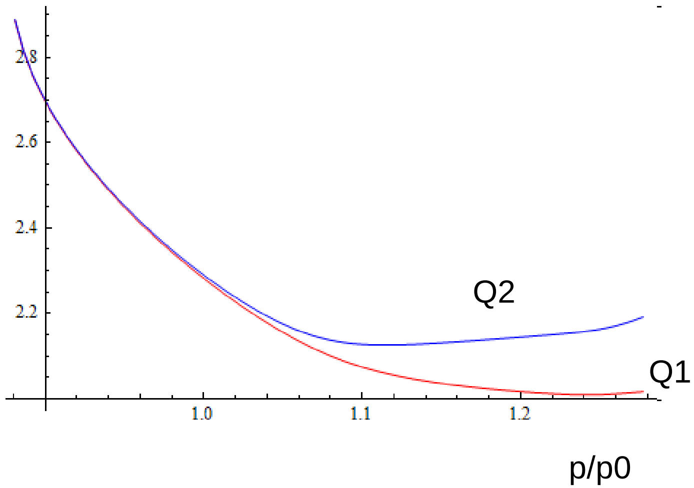




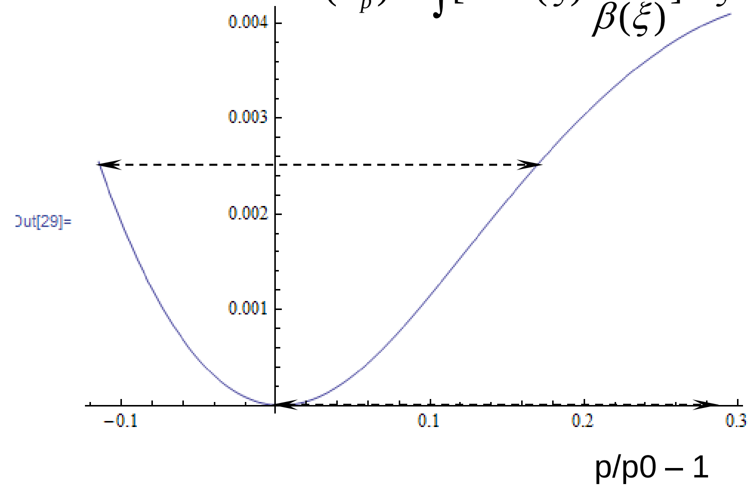


Analytics

p0=150MeV/c



$$K(\delta_p) = \int^{\delta_p} [1 - \lambda(\xi) \frac{\beta_0}{\beta(\xi)}] \, d\xi$$



With constant quadrupole field of just 1.45 T/m and no RF windows

mode	I	II	III
tune	2.272+0.0052i	2.313+0.0053i	0.215+0.0029i
ε_eq (mm)	0.83	0.78	0.80

No tracking simulations performed, would like to try for 325 MHz