A rectilinear channel for muon cooling towards micron scale emittances

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Outline

• Introduction to muon accelerators
• Challenges
• Cooling solution for a muon accelerator
  • Introduction to ionization cooling
  • Lattice design features
  • End-to-End simulation
  • Challenges & feasibility issues
• Summary
### Muon Collider Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Units</th>
<th>Higgs Factory</th>
<th>Top Threshold Options</th>
<th>Multi-TeV Baselines</th>
<th>Accounts for Site Radiation Mitigation</th>
</tr>
</thead>
<tbody>
<tr>
<td>CoM Energy</td>
<td>TeV</td>
<td>0.126</td>
<td>0.35</td>
<td>1.5</td>
<td>6.0</td>
</tr>
<tr>
<td>Avg. Luminosity</td>
<td>$10^{34}$ cm$^{-2}$s$^{-1}$</td>
<td>0.0017</td>
<td>0.07</td>
<td>1.25</td>
<td>12</td>
</tr>
<tr>
<td>Beam Energy Spread</td>
<td>%</td>
<td>0.003</td>
<td>0.01</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>Higgs* or Top* Production/10$^7$sec</td>
<td></td>
<td>3,500*</td>
<td>7,000*</td>
<td>37,500*</td>
<td>820,000*</td>
</tr>
<tr>
<td>Circumference</td>
<td>km</td>
<td>0.3</td>
<td>0.7</td>
<td>2.5</td>
<td>6</td>
</tr>
<tr>
<td>No. of IPs</td>
<td></td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Repetition Rate</td>
<td>Hz</td>
<td>30</td>
<td>15</td>
<td>15</td>
<td>12</td>
</tr>
<tr>
<td>$\beta^*$</td>
<td>cm</td>
<td>3.3</td>
<td>1.5</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>No. muons/bunch</td>
<td>$10^{12}$</td>
<td>2</td>
<td>4</td>
<td>2</td>
<td>2</td>
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<tr>
<td>No. bunches/beam</td>
<td></td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Norm. Trans. Emittance, $\varepsilon_{TN}$</td>
<td>$\pi$ mm-rad</td>
<td>0.4</td>
<td>0.2</td>
<td>0.025</td>
<td>0.025</td>
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<tr>
<td>Norm. Long. Emittance, $\varepsilon_{LN}$</td>
<td>$\pi$ mm-rad</td>
<td>1</td>
<td>1.5</td>
<td>70</td>
<td>70</td>
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<tr>
<td>Bunch Length, $\alpha_s$</td>
<td>cm</td>
<td>5.6</td>
<td>6.3</td>
<td>0.9</td>
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<td>Proton Driver Power</td>
<td>MW</td>
<td>4$^8$</td>
<td>4</td>
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</tbody>
</table>
Dealing with muons: Challenges

- Muon beams are born as tertiary beams
  - Protons → pions → muons
  - Challenge: How to capture & transport muons efficiently?
- Muons are born within a large phase-space
  - To obtain luminosities $O(10^{34})$ cm$^{-2}$s$^{-1}$ need to reduce the initial phase-space by 6 orders of magnitude
  - Challenge: How to cool the muon beam?
- Muons decay fast ( ~ 2 µs at rest)
  - At the same time, deal with high momentum protons, electrons…
  - Challenge: How to accelerate fast the beam? Collider ring design?

This talk
The cooling challenge

- Required for TeV Colliders
- Final 4D Cooling
- 6D Cooling
- Merge 12→1 bunch
- 6D Cooling
- Phase Rotation to 12 bunches

Transverse Emittance (micron)

Required for Higgs Factory

Longitudinal Emittance (mm)

DoE March, 2013
Given the short muon lifetime, ionization cooling is the only practical method that can be realized.

- Energy loss in discrete absorbers
- rf cavities to compensate for lost longitudinal energy
- Multi-tesla magnetic field to confine muon beams
Emittance exchange for 6D cooling

Concept 1: Generate dispersion and cool via emittance exchange in a wedge absorber

Concept 2: Energy loss dependence on path length in a continuous absorber

- Two concepts, same principle
- Dispersion is introduced to spatially separate muons of different momenta
- This study focuses on channels with discrete absorbers only!
Ionization cooling theory

- **Transverse cooling:**
  \[
  \frac{d\varepsilon_\perp}{ds} = -\frac{\varepsilon_\perp}{\beta^2 E} \frac{dE}{ds} + \frac{\beta_\perp (13.6 \text{ MeV})^2}{2\beta^3 E_\mu X_0 E}
  \]
  
  - Cooling term
  
  - Scattering term

- **Minimum emittance:**

- **Emittance evolution:**

- **Emittance can be controlled by:**
  - **Material:** Product of radiation length $X_0$ and energy loss rate $dE/ds$ large. Best choices are LiH & H
  
  - Magnet strength: Transverse beta function $\beta_T$ must be small. Thus, we progressively taper the magnetic field towards higher values

- Minimum emittance:

- Emittance evolution:

  \[
  \varepsilon_\perp(s) = \varepsilon_\perp^{eq} + (\varepsilon_\perp - \varepsilon_\perp^{eq}) \exp\left(-s/s_\perp^c\right)
  \]

  \[
  s_\perp^c = \beta^2 E/(dE/ds)
  \]
• We considered a rectilinear cooling scheme for a MC. Idea first proposed by V. Balbekov (Fermilab).
• Its simple geometry avoids several engineering challenges of previously considered schemes (rings or helix)
Cooling channel: How it works

- Coils are slightly tilted to generate a $B_y$ field component.
- This leads to dispersion, primarily in $x$.
- 6D cooling on wedge absorber.
- Better, if beta is minimum at the absorber.
Tapered lattice design

EARLY STAGE COOLING
- 275 cm long
- Coils far
- 325 MHz
- Axial B ~ 3 T
- Beta ~ 40 cm

LATE STAGE COOLING
- 80 cm long
- Coils near axis
- 650 MHz
- Axial B ~ 12 T
- Beta ~ 3 cm
Numerical study with ICOOL

- End-to-end simulation starting from the muon beam birth place (point 1)
- Notable reduction of 6D-emittance a factor of $10^5$
- Desired emittances for Higgs factory delivered!
Multivariable optimization

- 6 beam & lattice parameters were scanned with the goal to maximize the luminosity using the Nelder-Mead algorithm.

![Graphs showing various parameters and their run numbers](image-url)
Theory and simulation

- Found good agreement between theory and simulation

\[ \varepsilon_{eq} = \left( \frac{dE}{ds} \right)^{-1} \beta \left(13.6 \text{ MeV}\right)^2 \frac{1}{2 \beta E_\mu X_0} \]

\[ s_{eq} = \beta^2 E/(dE/ds) \]

\[ \varepsilon_\perp(s) = \varepsilon_{eq} + (\varepsilon_\perp - \varepsilon_{eq}) \exp(-s/s_{eq}) \]
Influence of space-charge

- At the end of the cooling, the bunch is 2 cm long, has $5 \times 10^{12}$ muons with a peak current of 5 kA at 200 MeV/c!
- We examined the influence of SC fields on cooling.
- SC causes particle loss and longitudinal emittance growth.

Goal

WARP Simulation
Space-charge compensation (2)

- For a Muon Collider to obtain a longitudinal emittance < 1.0 mm the rf gradient of a 805 MHz cavity needs to surpass 32.5 MV/m
- Avoid longitudinal cooling to < 1.3 mm
- Required compensation gradient from theory:

\[ E_{rf,c} = \frac{eN_0g_0c}{4\pi\varepsilon_0 \sqrt{2}\gamma^2 \sigma_\perp^3 (2\pi f n \cos \varphi)} \]
rf operation in B-fields

- Numerical simulations predict that the copper surfaces of a rf cavity may be damaged when $B > 1\, \text{T}$.
Hybrid solution

- Key Idea: Utilize gas filled cavities in a rectilinear channel
- Majority of cooling will be done in LiH and use gas only to protect the cavity from the high-field. Similar idea was used in the past for 4D cooling (Gallardo & Zisman, Nufact09)
Lattice performance

- Essentially, the same performance as the conventional channel with vacuum cavities
- BUT there remains considerable work to do before a hybrid channel can be considered a validated cooling channel option.
Sensitivity to pressure

- Final transverse emittance is correlated to gas pressure
Sensitivity to gas type

![Graph showing emittance vs gas type]

- Helium (100 atm)
- Hydrogen (100 atm)
- Vacuum

Emittance (trans.), $\varepsilon_T$ (mm)

Gas type
Summary

• We have presented a conceptual design of a rectilinear channel that in view its simple geometry may offer several technological advantages (compared to a helix or a ring)
• Showed reduction of 6D by at least 5 orders of magnitude.
• The influence of space-charge fields on the cooling process was thoroughly examined
• $\text{Nb}_3\text{Sn}$ magnets can be used for late stage cooling
• A hybrid solution with gas filled cavities was presented