Neutrino masses from a minimum principle

Rodrigo Alonso

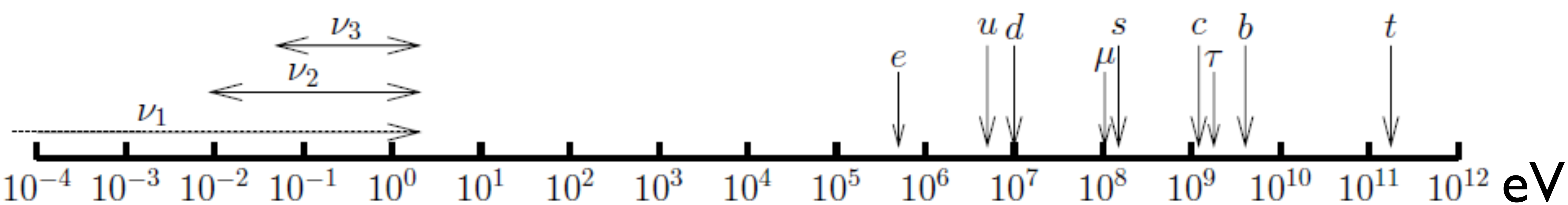
Nu@Fermilab, 07/23, 2015

in collaboration with B. Gavela, L. Merlo, L. Maiani,
D. Hernandez, G. Isidori and S. Rigolin

arXiv:1103.2915, arXiv:1306.5927 & arXiv:1306.5922

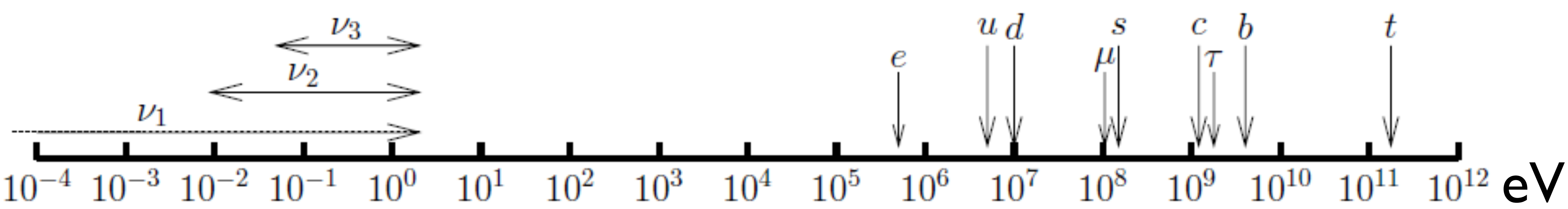
Flavour Puzzle

why are masses so separated from one another?



Flavour Puzzle

why are masses so separated from one another?



lightness of neutrinos
due to a Majorana character?

Flavour Puzzle

why is the mixing pattern so different?

Quarks

$$V_{CKM} = \begin{pmatrix} \sim 1 & \lambda & \lambda^3 \\ \lambda & \sim 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & \sim 1 \end{pmatrix}$$

Leptons

$$U_{PMNS} = \begin{pmatrix} 0.8 & 0.5 & \sim 0.2 \\ -0.4 & 0.5 & -0.7 \\ -0.4 & 0.5 & 0.7 \end{pmatrix}$$

Flavour Puzzle

why is the mixing pattern so different?

Quarks

$$V_{CKM} = \begin{pmatrix} \sim 1 & \lambda & \lambda^3 \\ \lambda & \sim 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & \sim 1 \end{pmatrix}$$

Leptons

$$U_{PMNS} = \begin{pmatrix} 0.8 & 0.5 & \sim 0.2 \\ -0.4 & 0.5 & -0.7 \\ -0.4 & 0.5 & 0.7 \end{pmatrix}$$

Maybe because of the
Majorana Character?

The Standard Theory

Gauge Group

Fermions

	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$
Q_L	3	2	$\frac{1}{6}$
U_R	3	1	$\frac{2}{3}$
D_R	3	1	$-\frac{1}{3}$
ℓ_L	1	2	$-\frac{1}{2}$
E_R	1	1	-1

The Standard Theory

Gauge Group

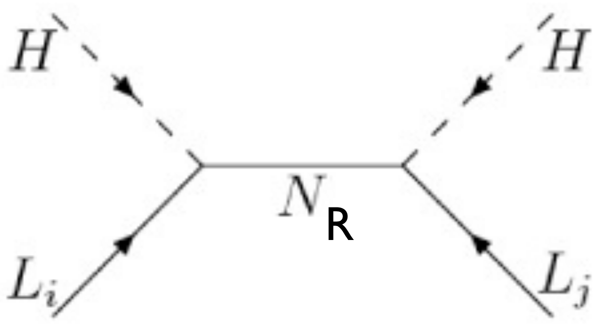
Fermions

	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$
Q_L	3	2	$\frac{1}{6}$
U_R	3	1	$\frac{2}{3}$
D_R	3	1	$-\frac{1}{3}$
ℓ_L	1	2	$-\frac{1}{2}$
E_R	1	1	-1
N_R	1	1	0

Neutrino Masses: type I seesaw

Addition of a gauge singlet

	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$
N_R	1	1	0



with the only "bare" and Majorana mass allowed

$$\mathscr{L}_{seesaw} = -\bar{\ell}_L Y_\nu \tilde{H} N_R - \overline{N}_R^c \frac{M}{2} N_R + h.c.$$

after EWSB and for vY<<M

$$m_\nu = Y_\nu \frac{v^2}{2M} Y_\nu^T$$

$Y_\nu \sim 1 \ M \sim 10^{15}\mathrm{GeV}$

$Y_\nu \sim 10^{-6} \ M \sim 10^3\mathrm{GeV}$

Flavour Symmetry

The largest global symmetry that the free Lagrangian can display:

$$\mathscr{L}_{free} = i \sum_{\psi=Q_L}^{D_R} \overline{\psi} \not{D} \psi$$

that is:

$$\boxed{\begin{array}{c} U(3)_{Q_L} \times U(3)_{U_R} \times U(3)_{D_R} \\ \mathcal{G}_{\mathcal{F}}^{q} \qquad \text{Quarks} \end{array}}$$

[Georgi, Chivukula, 1987
D'Ambrosio, Giudice, Isidori, Strumia, 2002]

Flavour Symmetry

The largest global symmetry that the free Lagrangian can display:

$$\mathscr{L}_{free} = i \sum_{\psi=Q_L}^{D_R} \overline{\psi} \not{D} \psi$$

that is:

$$\boxed{\begin{array}{c} U(3)_{Q_L} \times U(3)_{U_R} \times U(3)_{D_R} \\ \mathcal{G}^q_{\mathcal{F}} \quad \text{Quarks} \end{array}}$$

Flavour Symmetry

The largest global symmetry that the free Lagrangian can display:

$$\mathscr{L}_{free} = i \sum_{\psi=Q_L}^{N_R} \overline{\psi} \not{D} \psi$$

that is:

$$\boxed{\begin{array}{c} U(3)_{Q_L} \times U(3)_{U_R} \times U(3)_{D_R} \\ \mathcal{G}_{\mathcal{F}}^{q} \qquad \text{Quarks} \end{array}}$$

Flavour Symmetry

The largest global symmetry that the free Lagrangian can display:

$$\mathscr{L}_{free} = i \sum_{\psi=Q_L}^{N_R} \overline{\psi} \not{D} \psi - \frac{1}{2} \overline{N}_R^c M N_R \,,$$

that is:

$$\boxed{\begin{array}{c} U(3)_{Q_L} \times U(3)_{U_R} \times U(3)_{D_R} \\[1em] \mathcal{G}_{\mathcal{F}}^{q} \qquad \text{Quarks} \end{array}}$$

Flavour Symmetry

The largest global symmetry that the free Lagrangian can display:

$$\mathscr{L}_{free} = i \sum_{\psi=Q_L}^{N_R} \overline{\psi} \not{D} \psi - \frac{1}{2} \overline{N}_R^c M N_R \,,$$

that is:

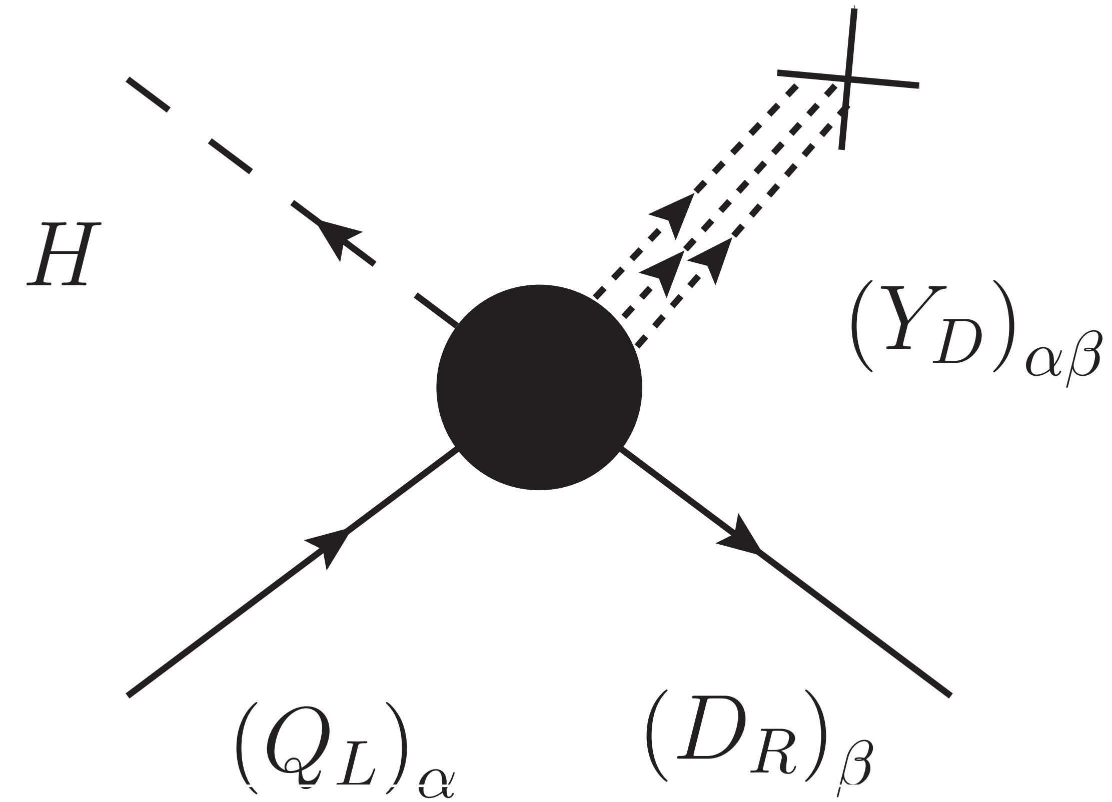
$$\boxed{U(3)_{Q_L} \times U(3)_{U_R} \times U(3)_{D_R}} \times \boxed{U(3)_{\ell_L} \times U(3)_{E_R} \times O(3)_{N_R}}$$

$\mathcal{G}_{\mathcal{F}}^q$ Quarks

$\mathcal{G}_{\mathcal{F}}^l$ Leptons

Flavour Symmetry Breaking

...which we assume spontaneously broken to generate the Yukawa couplings



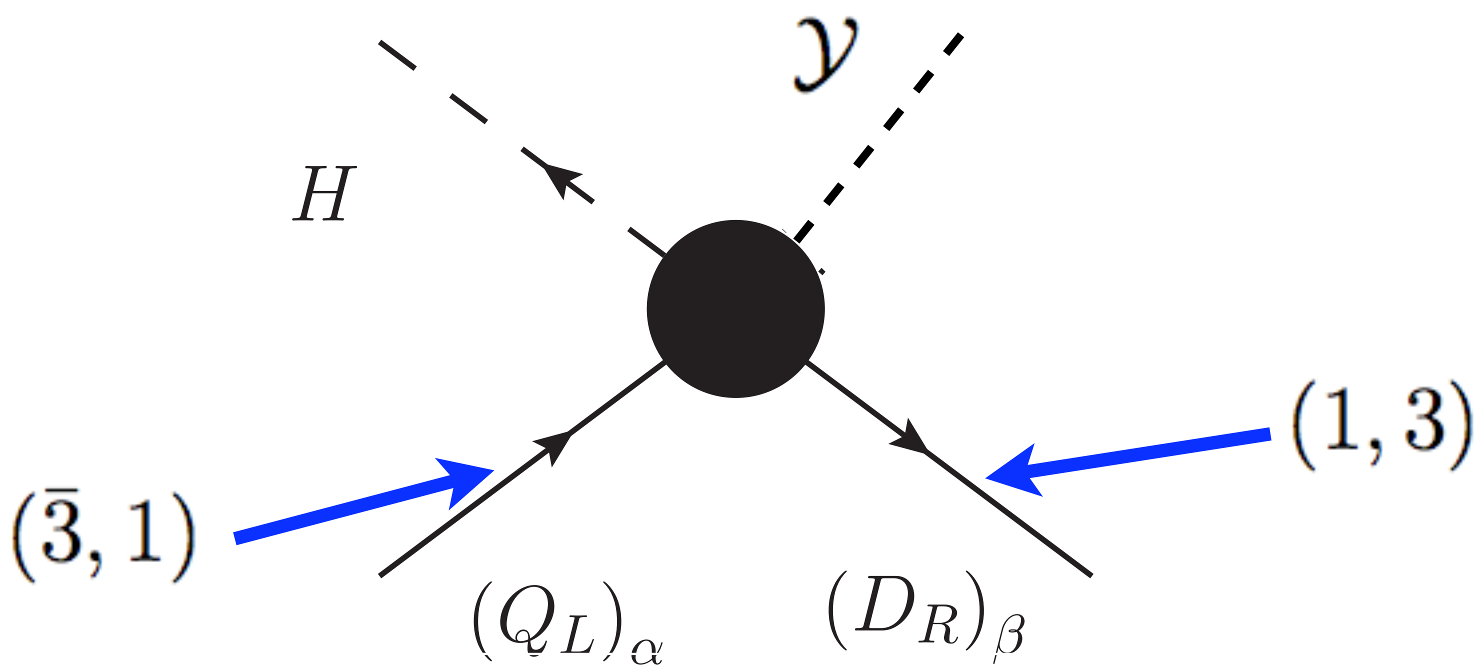
[C. D. Froggat, H. B. Nielsen
Berezhiani, Rosi,
Cabibbo, Maiani
Michel, Radicati ...]

$$Y = \frac{\langle \Phi^n \rangle}{\Lambda_f^n}$$

Flavour Fields

...which we assume spontaneously broken to generate the Yukawa couplings

$$\overline{Q}_L \frac{\mathcal{Y}}{\Lambda_f} U_R \tilde{H}$$



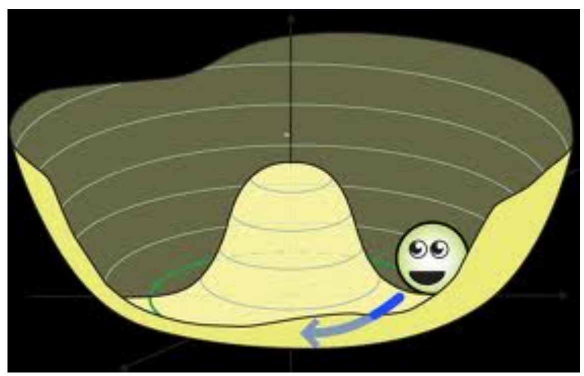
$n = 1$ (d=5)

A single and therefore "bi-fundamental" field

$\mathcal{Y} \sim (3, \bar{3})$

Flavour Symmetry Breaking

To prevent Goldstone Bosons the symmetry can be Gauged



[Grinstein, Redi, Villadoro
Guadagnoli, Mohapatra
Feldman]

Flavour Field's Scalar Potential

The potential shall respect

- *Gauge invariance* $\mathcal{G}$
- *Flavour invariance* $\mathcal{G}_{\mathcal{F}}$

$$V(\Phi) = V(I(\Phi))$$

This means that the potential depends on invariant combinations of the fields: "*I*"

Minimization

a variational principle fixes the vevs of the Fields

$$\delta V = 0$$

$$\sum_j \frac{\partial I_j}{\partial y_i} \frac{\partial V}{\partial I_j} \equiv J_{ij} \frac{\partial V}{\partial I_j} = 0 \, ,$$

This is an homogenous linear equation;

study rank of the Jacobian $J_{ij} = \partial I_j / \partial y_i ,$

Natural Breaking Patterns

$$\det(J) = 0$$ [Cabibbo, Maiani, 1969]

identifies especial solutions
with unbroken symmetry $\mathcal{H}$

$$\mathcal{G} \to \mathcal{H}$$

The "most" natural ones are the maximal subgroups

guaranteed a extremum point in any function [Michel, Radicati, 1969]

e.g. $$SU(5) \to SU(3) \times SU(2) \times U(1)$$ [Glashow, Georgi, 1974]

Quarks

Bi-fundamental Flavour Fields

eigenvalues only

$$I_U = \mathrm{Tr}\left[\mathcal{Y}_U \mathcal{Y}_U^\dagger\right] , \qquad I_D = \mathrm{Tr}\left[\mathcal{Y}_D \mathcal{Y}_D^\dagger\right] ,$$

$$I_{U^2} = \mathrm{Tr}\left[\left(\mathcal{Y}_U \mathcal{Y}_U^\dagger\right)^2\right] , \qquad I_{D^2} = \mathrm{Tr}\left[\left(\mathcal{Y}_D \mathcal{Y}_D^\dagger\right)^2\right] ,$$

$$I_{U^3} = \mathrm{Tr}\left[\left(\mathcal{Y}_U \mathcal{Y}_U^\dagger\right)^3\right] , \qquad I_{D^3} = \mathrm{Tr}\left[\left(\mathcal{Y}_D \mathcal{Y}_D^\dagger\right)^3\right] ,$$

$$I_{U,D} = \mathrm{Tr}\left[\mathcal{Y}_U \mathcal{Y}_U^\dagger \mathcal{Y}_D \mathcal{Y}_D^\dagger\right] , \qquad I_{U,D^2} = \mathrm{Tr}\left[\mathcal{Y}_U \mathcal{Y}_U^\dagger \left(\mathcal{Y}_D \mathcal{Y}_D^\dagger\right)^2\right] ,$$

$$I_{U^2,D} = \mathrm{Tr}\left[\mathcal{Y}_U \mathcal{Y}_U^\dagger \left(\mathcal{Y}_D \mathcal{Y}_D^\dagger\right)^2\right] , \qquad I_{(U,D)^2} = \mathrm{Tr}\left[\left(\mathcal{Y}_U \mathcal{Y}_U^\dagger \mathcal{Y}_D \mathcal{Y}_D^\dagger\right)^2\right] .$$

eigenvalues and mixing

[Feldmann, Jung, Mannel; Jenkins, Manohar]

Jacobian Analysis: [40 years

Breaking of a $SU(3) \times SU(3)$ [Cabibbo, Maiani]

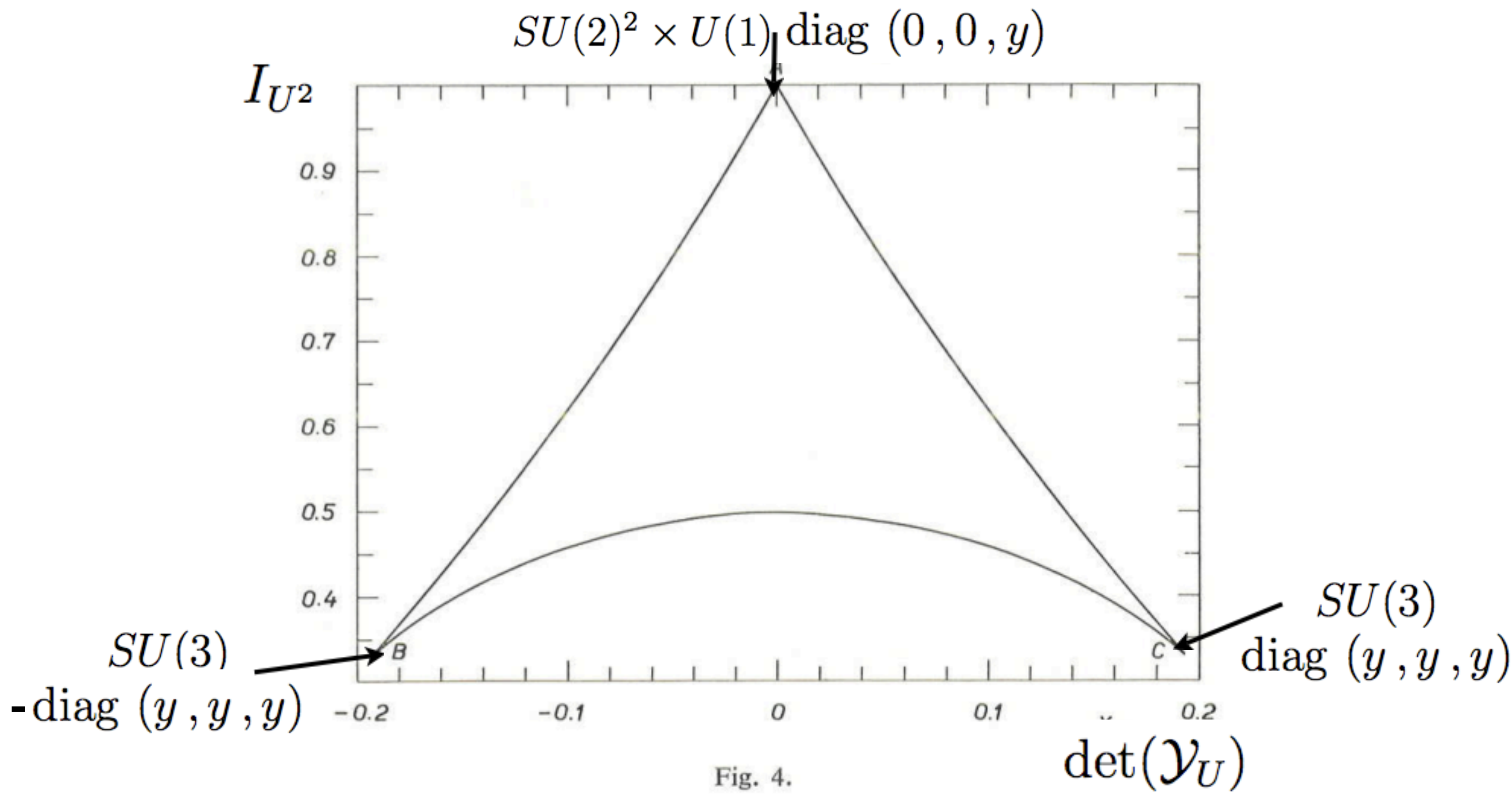


Fig. 4.

Jacobian Analysis: Mixing

$$\det\left(J_{UD}\right) = \left(y_u^2 - y_t^2\right)\left(y_t^2 - y_c^2\right)\left(y_c^2 - y_u^2\right)$$
$$\left(y_d^2 - y_b^2\right)\left(y_b^2 - y_s^2\right)\left(y_s^2 - y_d^2\right)$$
$$\times\ |V_{ud}||V_{us}||V_{cd}||V_{cs}|$$

the rank is reduced the most for:

$\boxed{V_{CKM} = \text{PERMUTATION}}$

no mixing: reordering of states

Quark Natural Flavour Pattern

Summarizing, a possible and natural breaking pattern arises:

$$\not{\mathcal{G}}^q_{\mathcal{F}} \quad : \quad U(3)^3 \to U(2)^3 \times U(1)$$

a hierarchical mass spectrum without mixing

$$\mathcal{Y}_D = \Lambda_f \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_b \end{pmatrix} , \qquad \mathcal{Y}_U = \Lambda_f \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_t \end{pmatrix} ,$$

a good approximation to the observed Yukawas to order $(\lambda_C)^2$

[RA, Gavela, Merlo, Rigolin]

Leptons

Bi-fundamental Flavour Fields

Physical parameters
=Independent Invariants

$$\underset{2\times 18}{\#\,\text{d.o.f. in } \mathcal{Y}_{E,\nu}} \text{ - } \underset{2\times 9+3}{\dim(\mathcal{G}^{\ell}_{\mathcal{F}})} = 15$$

The better suited parametrization is the bi-unitary

$$\mathcal{Y}_\nu = \Lambda_f \mathcal{U}_L \mathbf{y}_\nu \mathcal{U}_R\,, \qquad \mathcal{Y}_E = \Lambda_f \mathbf{y}_E\,;$$

$$\mathcal{U}_L \mathcal{U}_L^\dagger = 1\,, \;\; \mathcal{U}_R \mathcal{U}_R^\dagger = 1\,,$$

the connection with neutrino masses being

$$U_{PMNS}\,\mathbf{m}_\nu\, U_{PMNS}^T = \frac{v^2}{2M} \mathcal{U}_L\, \mathbf{y}_\nu \mathcal{U}_R \mathcal{U}_R^T \mathbf{y}_\nu \mathcal{U}_L^T\,,$$

Leptons

Eigenvalues only

$$I_E = \mathrm{Tr}\left[\mathcal{Y}_E \mathcal{Y}_E^\dagger\right] , \qquad I_\nu = \mathrm{Tr}\left[\mathcal{Y}_\nu \mathcal{Y}_\nu^\dagger\right] ,$$

$$I_{E^2} = \mathrm{Tr}\left[\left(\mathcal{Y}_E \mathcal{Y}_E^\dagger\right)^2\right] , \qquad I_{\nu^2} = \mathrm{Tr}\left[\left(\mathcal{Y}_\nu \mathcal{Y}_\nu^\dagger\right)^2\right] ,$$

$$I_{E^3} = \mathrm{Tr}\left[\left(\mathcal{Y}_E \mathcal{Y}_E^\dagger\right)^3\right] , \qquad I_{\nu^3} = \mathrm{Tr}\left[\left(\mathcal{Y}_\nu \mathcal{Y}_\nu^\dagger\right)^3\right] ,$$

$$I_L = \mathrm{Tr}\left[\mathcal{Y}_\nu \mathcal{Y}_\nu^\dagger \mathcal{Y}_E \mathcal{Y}_E^\dagger\right] ,$$

$$I_{L^2} = \mathrm{Tr}\left[\mathcal{Y}_\nu \mathcal{Y}_\nu^\dagger \left(\mathcal{Y}_E \mathcal{Y}_E^\dagger\right)^2\right] ,$$

$$I_{L^3} = \mathrm{Tr}\left[\mathcal{Y}_E \mathcal{Y}_E^\dagger \left(\mathcal{Y}_\nu \mathcal{Y}_\nu^\dagger\right)^2\right] ,$$

$$I_{L^4} = \mathrm{Tr}\left[\left(\mathcal{Y}_\nu \mathcal{Y}_\nu^\dagger \mathcal{Y}_E \mathcal{Y}_E^\dagger\right)^2\right] ,$$

U_L and eigenvalues

$$I_R = \mathrm{Tr}\left[\mathcal{Y}_\nu^\dagger \mathcal{Y}_\nu \mathcal{Y}_\nu^T \mathcal{Y}_\nu^*\right] ,$$

$$I_{R^2} = \mathrm{Tr}\left[\left(\mathcal{Y}_\nu^\dagger \mathcal{Y}_\nu\right)^2 \mathcal{Y}_\nu^T \mathcal{Y}_\nu^*\right] ,$$

$$I_{R^3} = \mathrm{Tr}\left[\left(\mathcal{Y}_\nu^\dagger \mathcal{Y}_\nu \mathcal{Y}_\nu^T \mathcal{Y}_\nu^*\right)^2\right] ,$$

U_R and eigenvalues

$$I_{LR} = \mathrm{Tr}\left[\mathcal{Y}_\nu \mathcal{Y}_\nu^T \mathcal{Y}_\nu^* \mathcal{Y}_\nu^\dagger \mathcal{Y}_E \mathcal{Y}_E^\dagger\right] , \quad I_{RL} = \mathrm{Tr}\left[\mathcal{Y}_\nu \mathcal{Y}_\nu^T \mathcal{Y}_E^* \mathcal{Y}_E^T \mathcal{Y}_\nu^* \mathcal{Y}_\nu^\dagger \mathcal{Y}_E \mathcal{Y}_E^\dagger\right] ,$$

New Invariants w.r.t. Quarks

Jacobian Analysis: Mixing

$$\det\left(J_{\mathcal{U}_L}\right) = \left(y_{\nu_1}^2 - y_{\nu_2}^2\right)\left(y_{\nu_2}^2 - y_{\nu_3}^2\right)\left(y_{\nu_3}^2 - y_{\nu_1}^2\right)$$
$$\left(y_e^2 - y_\mu^2\right)\left(y_\mu^2 - y_\tau^2\right)\left(y_\tau^2 - y_e^2\right)|\mathcal{U}_L^{e1}||\mathcal{U}_L^{e2}||\mathcal{U}_L^{\mu 1}||\mathcal{U}_L^{\mu 2}|\,.$$

same as for V_{CKM}

O(3) vs U(3)

$$\det J_{\mathcal{U}_R} = \left(y_{\nu_1}^2 - y_{\nu_2}^2\right)^3\left(y_{\nu_2}^2 - y_{\nu_3}^2\right)^3\left(y_{\nu_3}^2 - y_{\nu_1}^2\right)^3$$
$$\times\,|\left(\mathcal{U}_R\mathcal{U}_R^T\right)_{11}||\left(\mathcal{U}_R\mathcal{U}_R^T\right)_{22}||\left(\mathcal{U}_R\mathcal{U}_R^T\right)_{12}|$$

the rank is reduced the most for $\boxed{\mathcal{U}_R\mathcal{U}_R^T \textit{ being a permutation}}$

Jacobian Analysis: Mixing

...which is now **not** trivial mixing...

$$\frac{v^2}{M}\begin{pmatrix} y_{\nu_1}^2 & 0 & 0 \\ 0 & 0 & y_{\nu_2}y_{\nu_3} \\ 0 & y_{\nu_2}y_{\nu_3} & 0 \end{pmatrix} = U_{PMNS}\begin{pmatrix} m_{\nu_1} & 0 & 0 \\ 0 & m_{\nu_2} & 0 \\ 0 & 0 & m_{\nu_2} \end{pmatrix} U_{PMNS}^T\,,$$

...but **maximal** mixing

$$U_{PMNS} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} \\ 0 & -\frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} \end{pmatrix}, \qquad m_{\nu_2} = \frac{v^2}{M} y_{\nu_2}y_{\nu_3}\,, \quad m_{\nu_1} = \frac{v^2}{M} y_{\nu_1}^2\,.$$

+degeneracy and maximal Majorana phase

[RA, Gavela, Hernandez, Merlo, Rigolin
RA, Gavela Isidori, Maiani]

Lepton Natural Flavour Pattern

Summarizing, a possible and natural breaking pattern:

$$\not{\mathcal{G}}^l_{\mathcal{F}} \quad : \quad U(3)^2 \times O(3) \to U(2) \times U(1)$$

brings along hierarchical charged leptons

$$\mathcal{Y}_E = \Lambda_f \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_\tau \end{pmatrix}, \qquad \mathcal{Y}_\nu = \Lambda_f \begin{pmatrix} y_{\nu_1} & 0 & 0 \\ 0 & y_{\nu_2}/\sqrt{2} & -iy_{\nu_2}/\sqrt{2} \\ 0 & y_{\nu_3}/\sqrt{2} & iy_{\nu_3}/\sqrt{2} \end{pmatrix},$$

and (at least) two degenerate neutrinos and maximal angle and Majorana phase

$\theta_{23} = 45°$;

Majorana Phase Pattern (1,1,i)

& Mass degeneracy: $m_{\nu 2} = m_{\nu 3}$

Perturbations can produce a second large angle

if the three neutrinos are quasidegenerate, perturbations:

$$U_{PMNS}\begin{pmatrix} m_0 & 0 & 0 \\ 0 & m_0 & 0 \\ 0 & 0 & m_0 \end{pmatrix} U^T_{PMNS} = \frac{y_\nu v^2}{M}\begin{pmatrix} 1+\delta+\sigma & \epsilon+\eta & \epsilon-\eta \\ \epsilon+\eta & \delta+\kappa & 1 \\ \epsilon-\eta & 1 & \delta-\kappa \end{pmatrix}$$

produce a second large angle and a perturbative one together with mass splittings

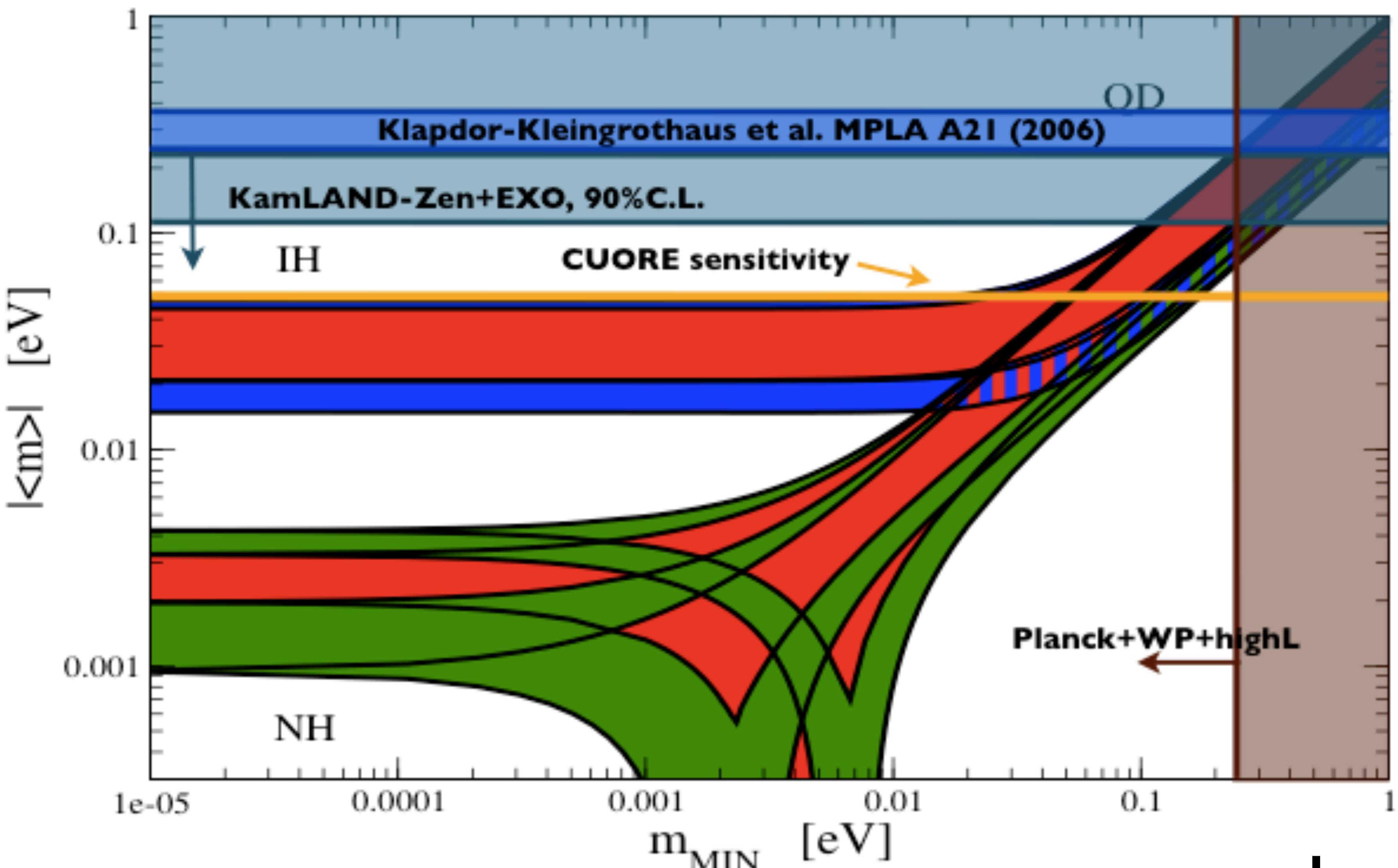
$$\theta_{23} \simeq \pi/4 \ , \ \theta_{12} \text{ large} \quad , \quad \theta_{13} \simeq \epsilon$$

$$\text{Majorana Phases } (\, e^{i\alpha} \, , 1 \, , i)$$

degenerate spetrum

accommodation of angles requires degenerate spectrum

at reach in future neutrinoless double β exps.!

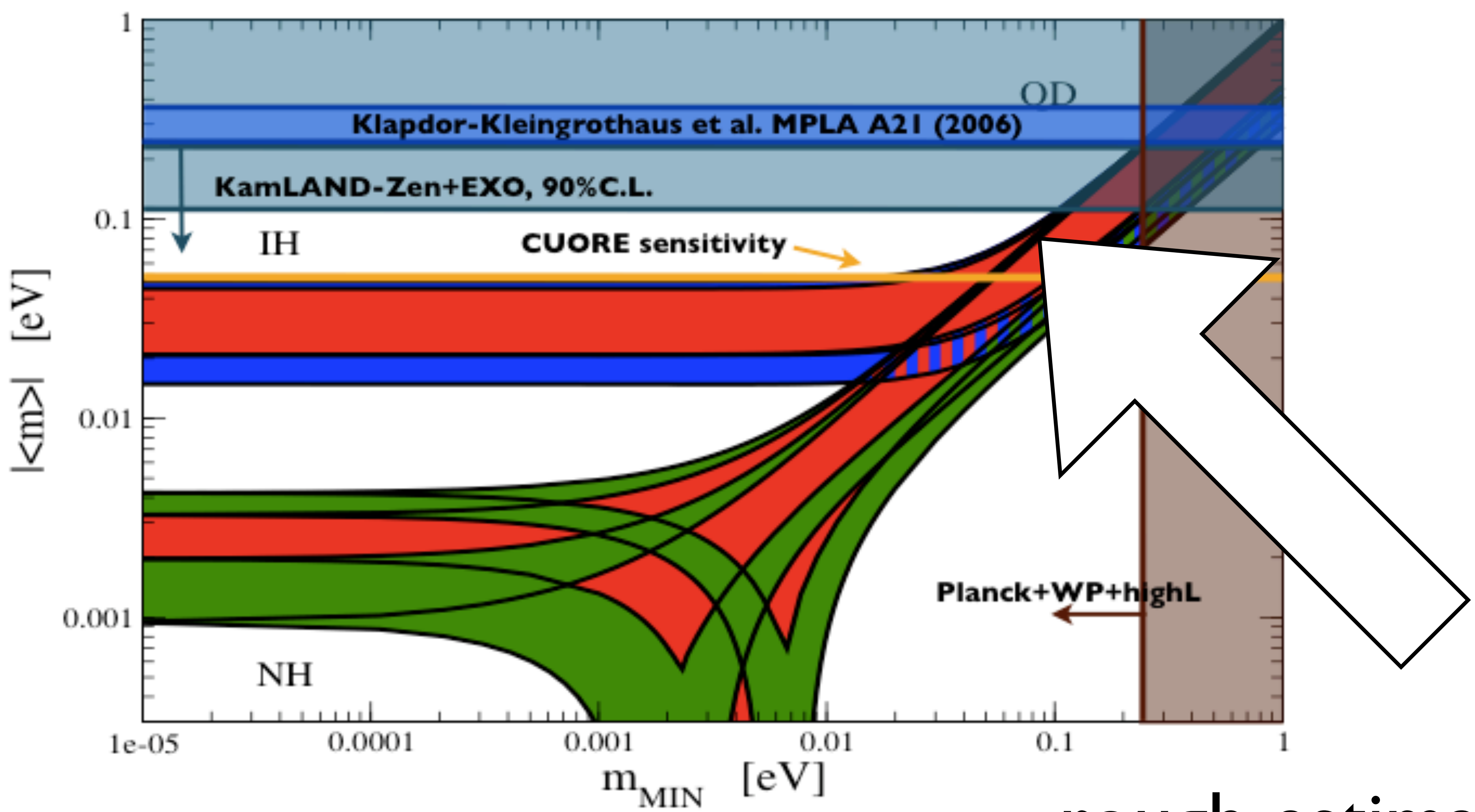


rough estimate

~0.1eV

accommodation of angles requires degenerate spectrum

at reach in future neutrinoless double β exps.!



rough estimate

~0.1eV

Where do the differences in Mixing originate?

From the

MAJORANA vs DIRAC nature of fermions

Where do the differences in Mixing originate?

From the

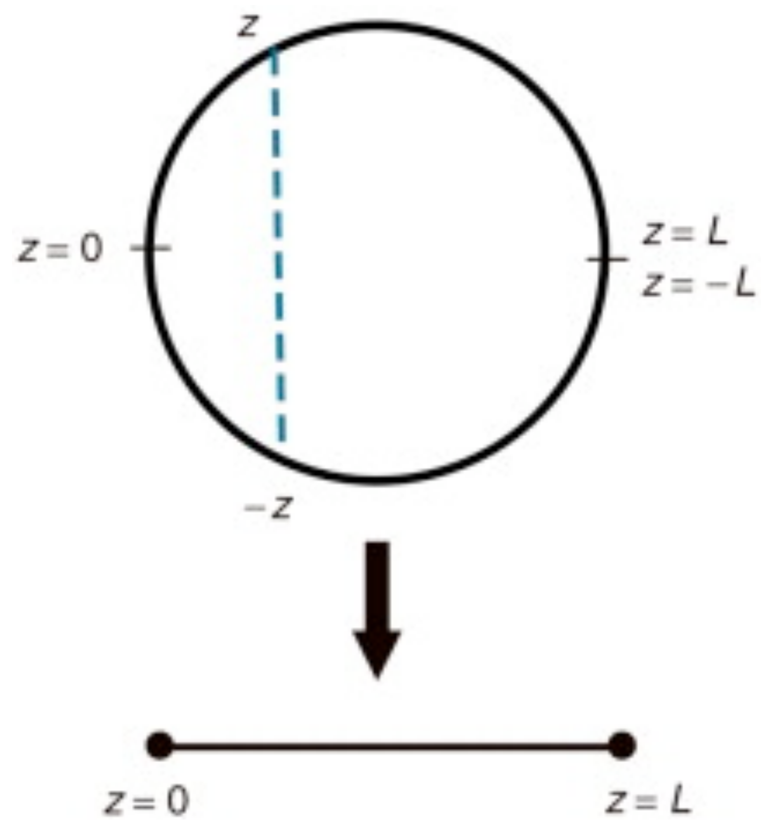
MAJORANA vs DIRAC nature of fermions

THANK YOU

Backup

Boundaries Exhibit Unbroken Symmetry

Extra-Dimensions Example



<u>The smallest boundaries are extremal points of any function</u>

[Michel, Radicati, 1969]

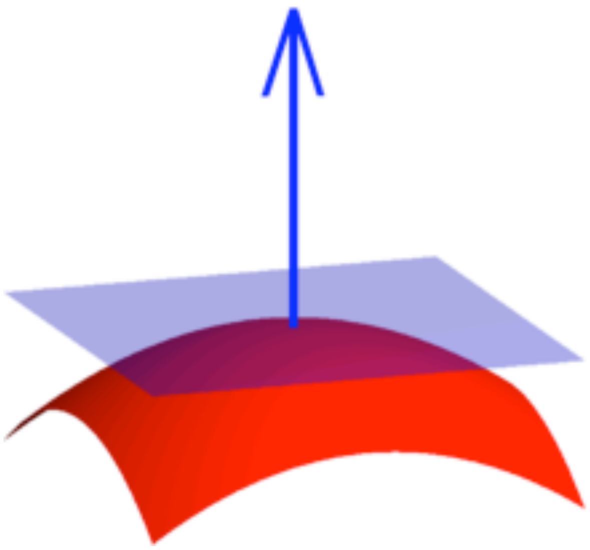
Boundaries

for a reduced rank of the Jacobian,

$$\det(J) = 0$$

there exists (at least) a direction δy_i in which a variation of the field variables does not vary the invariants

$$\delta I_j = \sum_i \frac{\partial I_j}{\partial y_i} \delta y_i = 0$$



that is a Boundary of the *I-manifold*

[Cabibbo, Maiani, 1969]