

Discrete Flavor Symmetries and Origin of CP Violation

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Work done in collaboration with

Maximilian Fallbacher, K.T. Mahanthappa, Michael Ratz, Andreas Trautner, Nucl. Phys. B883 (2014) 267
K.T. Mahanthappa, Phys. Lett. B681, 444 (2009)

CP Violation in Nature

- CP violation: required to explain matter-antimatter asymmetry
- So far observed only in flavor sector
 - SM: CKM matrix for the quark sector
 - experimentally established δ_{CKM} as major source of CP violation
 - not sufficient for observed cosmological matter-antimatter asymmetry
 - Search for new source of CP violation:
 - CP violation in neutrino sector
 - if found \Rightarrow phase in PMNS matrix
- Discrete family symmetries:
 - suggested by large neutrino mixing angles
 - neutrino mixing angles from group theoretical CG coefficients

Discrete (family) symmetries \Leftrightarrow Physical CP violation

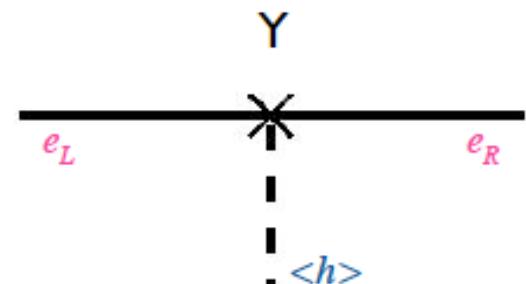
Origin of CP Violation

- CP violation \Leftrightarrow complex mass matrices

$$\overline{U}_{R,i}(M_u)_{ij}Q_{L,j} + \overline{Q}_{L,j}(M_u^\dagger)_{ji}U_{R,i} \xrightarrow{\mathcal{CP}} \overline{Q}_{L,j}(M_u)_{ij}U_{R,i} + \overline{U}_{R,i}(M_u)^*_{ij}Q_{L,j}$$

- Conventionally, CPV arises in two ways:

- Explicit CP violation: complex Yukawa coupling constants Y
- Spontaneous CP violation: complex scalar VEVs $\langle h \rangle$

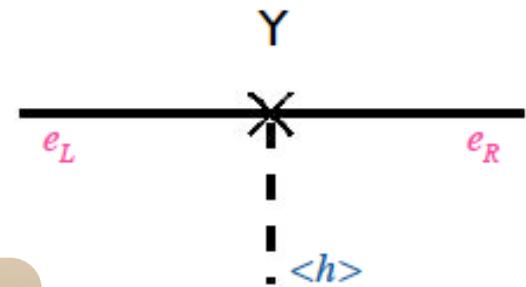


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Fermion mass and hierarchy problem \rightarrow
Many free parameters in the Yukawa sector

A Novel Origin of CP Violation

M.-C.C., K.T. Mahanthappa
Phys. Lett. B681, 444 (2009)

- **Reduce the number of parameters \Leftrightarrow non-Abelian discrete family symmetry**
 - e.g. A_4 family symmetry \Leftrightarrow TBM mixing from CG coefficients
- **Complex CG coefficients in certain discrete groups \Rightarrow explicit CP violation**
 - real Yukawa couplings, real Higgs VEV
 - CPV in quark and lepton sectors purely from complex CG coefficients
 - No additional parameters needed \Rightarrow extremely predictive model!

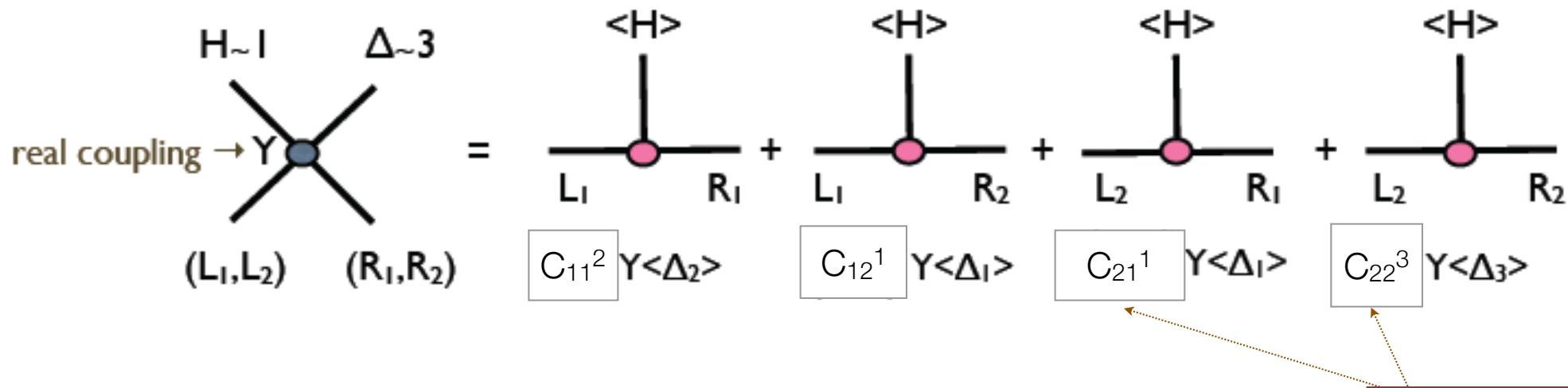
CG coefficients in non-Abelian discrete symmetries
 \Leftrightarrow relative strengths and phases in entries of Yukawa matrices
 \Leftrightarrow mixing angles and phases (and mass hierarchy)

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Basic idea

Discrete symmetry \mathbf{G}

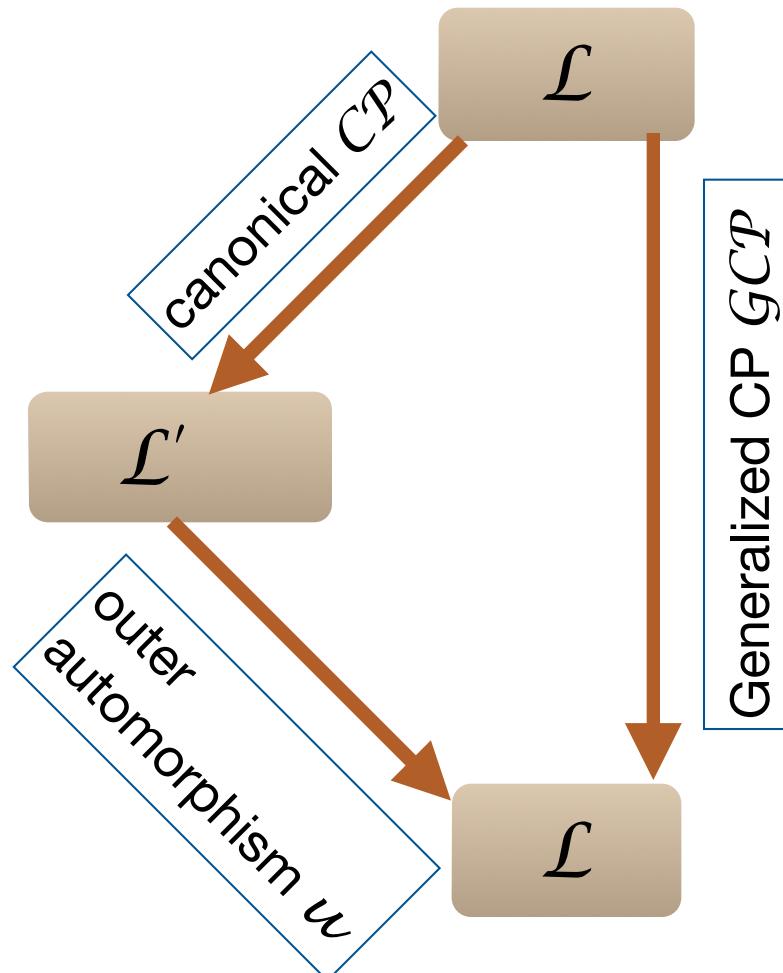


- Scalar potential: if Z_3 symmetric $\Rightarrow \langle \Delta_1 \rangle = \langle \Delta_2 \rangle = \langle \Delta_3 \rangle \equiv \langle \Delta \rangle$ real
- Complex effective mass matrix: **phases determined by group theory**

$$M = \begin{pmatrix} L_1 & L_2 \\ C_{12}^1 & C_{22}^3 \end{pmatrix} Y \langle \Delta \rangle \begin{pmatrix} R_1 \\ R_2 \end{pmatrix}$$

Physical CP vs. Generalized CP Transformations

complex CGs $\Leftrightarrow G$ and physical CP transformations do not commute



Generalized CP transformation:

$$\Phi(x) \xrightarrow{\widetilde{CP}} U_{CP} \Phi^*(\mathcal{P} x)$$

contains all
reps in model

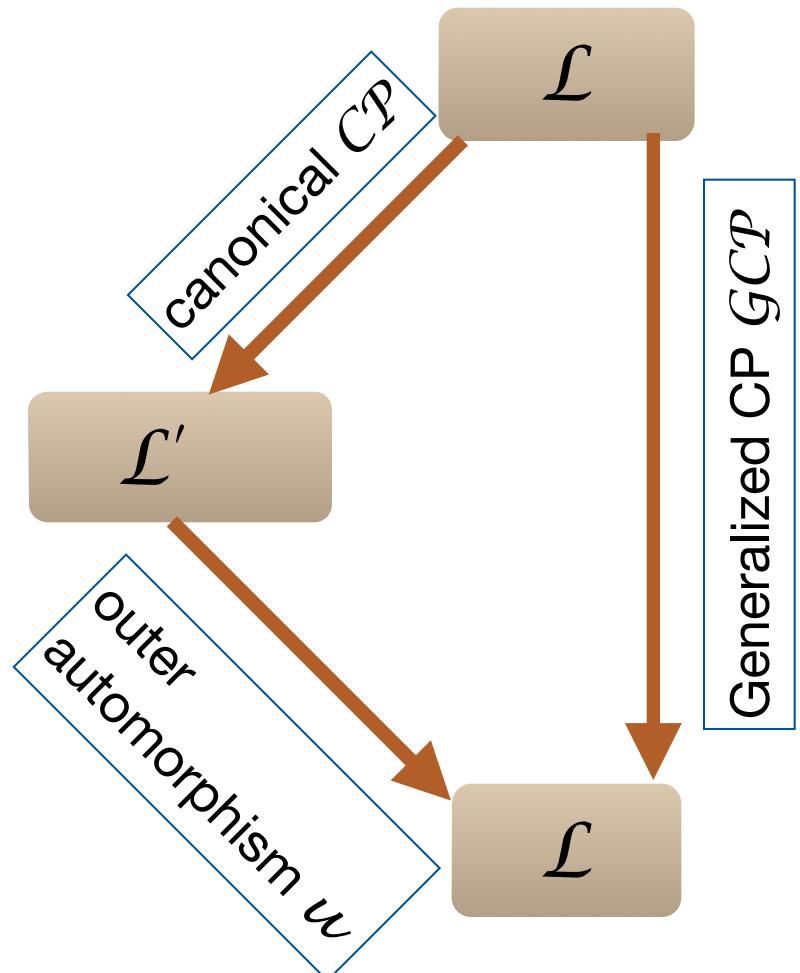
Necessary Consistency condition:

Holthausen, Lindner, Schmidt (2013)

$$\rho(u(g)) = U_{CP} \rho(g)^* U_{CP}^\dagger \quad \forall g \in G$$

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However, GCP may not correspond to physical CP transformation
 \Leftrightarrow for GCP = physical CP:
more stringent consistency condition

Physical CP vs. Generalized CP Transformations

- generalized CP transformation
- Necessary consistency condition

$$\Phi(x) \xrightarrow{\widetilde{CP}} U_{\text{CP}} \Phi^*(\mathcal{P} x)$$

$$\rho(u(g)) = U_{\text{CP}} \rho(g)^* U_{\text{CP}}^\dagger \quad \forall g \in G$$

Holthausen, Lindner, Schmidt (2013)

- **Necessary and sufficient consistency condition**

M.-C.C., M. Fallbacher, K.T. Mahanthappa, M. Ratz,
A. Trautner (2014)

$$\rho_{\mathbf{r}_i}(u(g)) = U_{\mathbf{r}_i} \rho_{\mathbf{r}_i}(g)^* U_{\mathbf{r}_i}^\dagger \quad \forall g \in G \text{ and } \forall i$$



physical CP

Physical CP vs. Generalized CP Transformations

- generalized CP transformation
- Necessary consistency condition

$$\Phi(x) \xrightarrow{\widetilde{CP}} U_{\text{CP}} \Phi^*(\mathcal{P} x)$$

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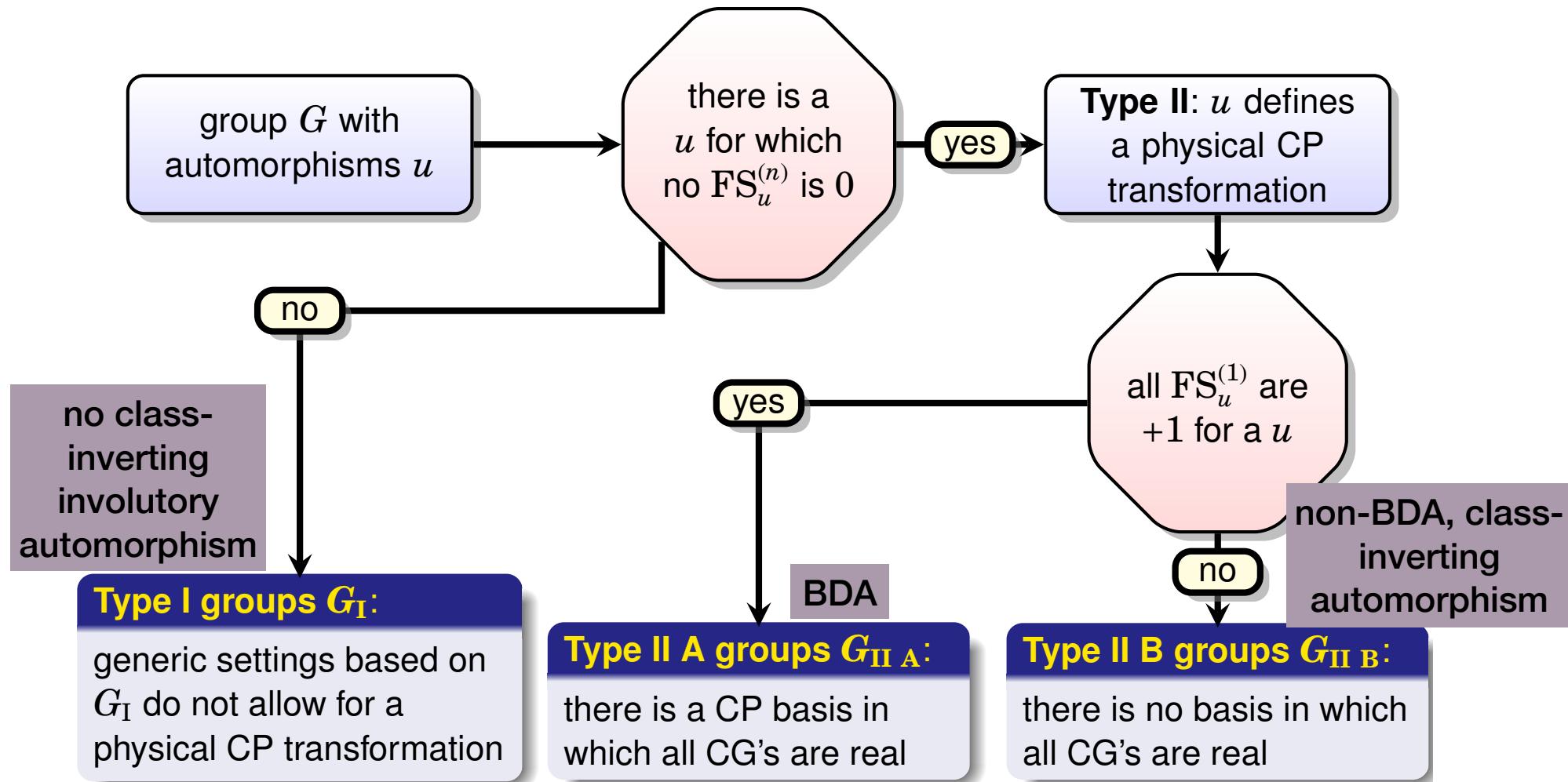


physical CP

u has to be a **class-inverting, involuntary automorphism of G**
⇒ non-existence of such automorphism in certain groups
⇒ explicit physical CP violation

Three Types of Finite Groups

M.-C.C., M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner (2014)

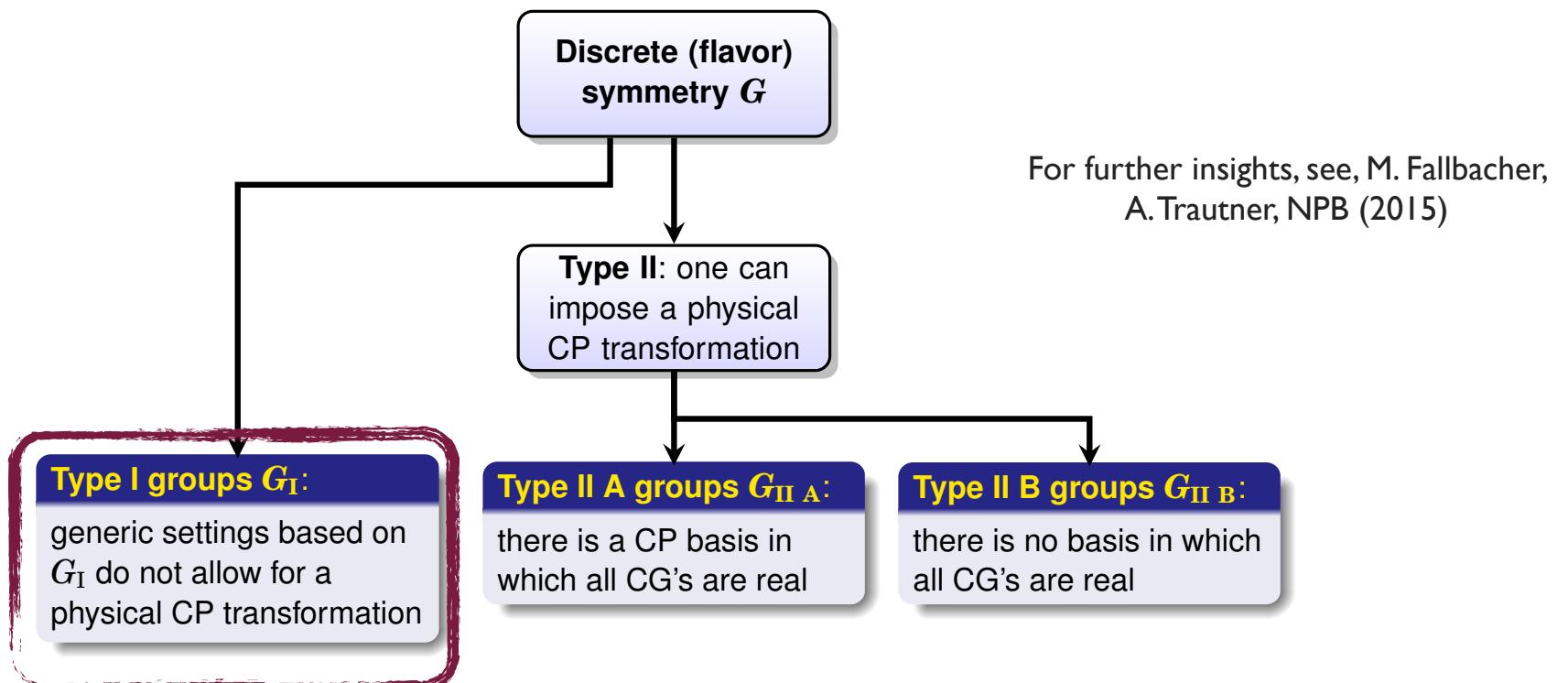


A Novel Origin of CP Violation

M.-C.C, M. Fallbacher, K.T. Mahanthappa,
M. Ratz, A.Trautner, NPB (2014)

- For discrete groups that do not have class-inverting, involutory automorphism, CP is generically broken by complex CG coefficients (**Type I Group**)
- Non-existence of such automorphism \Leftrightarrow Physical CP violation

CP Violation from Group Theory!



Examples

M.-C.C., M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner (2014)

- Type I: all odd order non-Abelian groups

| | | | | |
|-------|-------------------------------------|--------|--------------|-------------------------------------|
| group | $\mathbb{Z}_5 \rtimes \mathbb{Z}_4$ | T_7 | $\Delta(27)$ | $\mathbb{Z}_9 \rtimes \mathbb{Z}_3$ |
| SG | (20,3) | (21,1) | (27,3) | (27,4) |

- Type IIA: dihedral and all Abelian groups

| | | | | | | | |
|-------|-------|-------|--------|-------------------------------------|--------|---------|--------|
| group | S_3 | Q_8 | A_4 | $\mathbb{Z}_3 \rtimes \mathbb{Z}_8$ | T' | S_4 | A_5 |
| SG | (6,1) | (8,4) | (12,3) | (24,1) | (24,3) | (24,12) | (60,5) |

- Type IIB

| | | |
|-------|--------------|----------------------------------------------------------------------------------|
| group | $\Sigma(72)$ | $((\mathbb{Z}_3 \times \mathbb{Z}_3) \rtimes \mathbb{Z}_4) \rtimes \mathbb{Z}_4$ |
| SG | (72,41) | (144,120) |

Example for a type I group:

$$\Delta(27)$$

- decay asymmetry in a toy model
- prediction of CP violating phase from group theory



Toy Model based on $\Delta(27)$

M.-C.C., M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner (2014)

- Field content

| field | S | X | Y | Ψ | Σ |
|--------------|---------------------|---------------------|----------------|--------------|--------------|
| $\Delta(27)$ | $\mathbf{1}_0$ | $\mathbf{1}_1$ | $\mathbf{1}_3$ | $\mathbf{3}$ | $\mathbf{3}$ |
| $U(1)$ | $q_\Psi - q_\Sigma$ | $q_\Psi - q_\Sigma$ | 0 | q_Ψ | q_Σ |

fermions

- Interactions

$q_\Psi - q_\Sigma \neq 0$

$$\mathcal{L}_{\text{toy}} = F^{ij} S \bar{\Psi}_i \Sigma_j + G^{ij} X \bar{\Psi}_i \Sigma_j + H_\Psi^{ij} Y \bar{\Psi}_i \Psi_j + H_\Sigma^{ij} Y \bar{\Sigma}_i \Sigma_j + \text{h.c.}$$

$$F = f \mathbb{1}_3$$

$$G = g \begin{pmatrix} 0 & q_\Psi - q_\Sigma & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

$$H_{\Psi/\Sigma} = h_{\Psi/\Sigma} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix}$$

with $\omega := e^{2\pi i/3}$

“flavor” structures determined by
(complex) CG coefficients

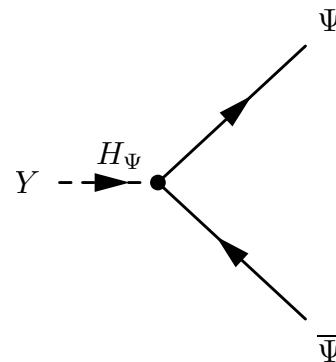
arbitrary coupling constants:
 f, g, h_Ψ, h_Σ

Toy Model based on $\Delta(27)$

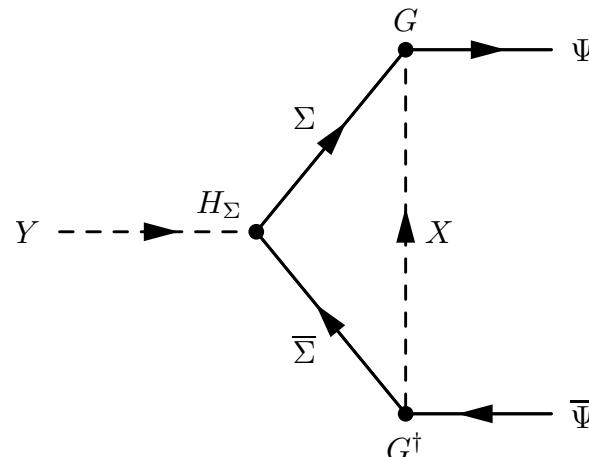
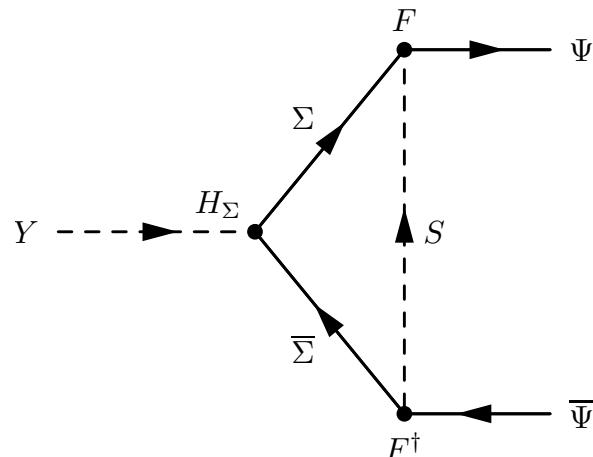
M.-C.C., M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner (2014)

- Particle decay $Y \rightarrow \bar{\Psi}\Psi$

interference of



with



Decay Asymmetry

M.-C.C., M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner (2014)

- Decay asymmetry

$$\begin{aligned}\varepsilon_{Y \rightarrow \bar{\Psi} \Psi} &= \frac{\Gamma(Y \rightarrow \bar{\Psi} \Psi) - \Gamma(Y^* \rightarrow \bar{\Psi} \Psi)}{\Gamma(Y \rightarrow \bar{\Psi} \Psi) + \Gamma(Y^* \rightarrow \bar{\Psi} \Psi)} \\ &\propto \text{Im}[I_S] \text{ Im} \left[\text{tr} \left(F^\dagger H_\Psi F H_\Sigma^\dagger \right) \right] + \text{Im}[I_X] \text{ Im} \left[\text{tr} \left(G^\dagger H_\Psi G H_\Sigma^\dagger \right) \right] \\ &= |f|^2 \text{ Im}[I_S] \text{ Im}[h_\Psi h_\Sigma^*] + |g|^2 \text{ Im}[I_X] \text{ Im}[\omega h_\Psi h_\Sigma^*] .\end{aligned}$$

one-loop integral
 $I_S = I(M_S, M_Y)$

one-loop integral
 $I_X = I(M_X, M_Y)$

- properties of ε

- invariant under rephasing of fields
- independent of phases of f and g
- basis independent

Decay Asymmetry

M.-C.C., M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner (2014)

- Decay asymmetry

$$\varepsilon_{Y \rightarrow \bar{\Psi}\Psi} = |\textcolor{green}{f}|^2 \operatorname{Im} [I_S] \operatorname{Im} [\textcolor{red}{h}_\Psi h_\Sigma^*] + |\textcolor{green}{g}|^2 \operatorname{Im} [I_X] \operatorname{Im} [\omega h_\Psi h_\Sigma^*]$$

- cancellation requires delicate adjustment of relative phase $\varphi := \arg(\textcolor{red}{h}_\Psi h_\Sigma^*)$
- for non-degenerate M_S and M_X : $\operatorname{Im} [I_S] \neq \operatorname{Im} [I_X]$
 - phase φ unstable under quantum corrections
- for $\operatorname{Im} [I_S] = \operatorname{Im} [I_X]$ & $|\textcolor{green}{f}| = |\textcolor{green}{g}|$
 - phase φ stable under quantum corrections
 - relations cannot be ensured by an outer automorphism (i.e. GCP) of $\Delta(27)$
 - require symmetry larger than $\Delta(27)$

model based on $\Delta(27)$ violates CP!

Spontaneous CP Violation with Calculable CP Phase

M.-C.C., M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner (2014)

| field | X | Y | Z | Ψ | Σ | ϕ |
|--------------|----------------|----------------|----------------|--------------|--------------|----------------|
| $\Delta(27)$ | $\mathbf{1}_1$ | $\mathbf{1}_3$ | $\mathbf{1}_8$ | $\mathbf{3}$ | $\mathbf{3}$ | $\mathbf{1}_0$ |
| $U(1)$ | $2q_\Psi$ | 0 | $2q_\Psi$ | q_Ψ | $-q_\Psi$ | 0 |

$$\Delta(27) \subset SG(54, 5): \begin{cases} (X, Z) & : \text{doublet} \\ (\Psi, \Sigma^C) & : \text{hexaplet} \\ \phi & : \text{non-trivial 1-dim. representation} \end{cases}$$

☞ non-trivial $\langle \phi \rangle$ breaks $SG(54, 5) \rightarrow \Delta(27)$

☞ allowed coupling leads to mass splitting $\mathcal{L}_{\text{toy}}^\phi \supset M^2 (|X|^2 + |Z|^2) + \left[\frac{\mu}{\sqrt{2}} \langle \phi \rangle (|X|^2 - |Z|^2) + \text{h.c.} \right]$

→ CP asymmetry with calculable phases

$$\varepsilon_{Y \rightarrow \bar{\Psi}\Psi} \propto |g|^2 |h_\Psi|^2 \text{Im} [\omega] (\text{Im} [I_X] - \text{Im} [I_Z])$$

phase predicted by group theory

CG coefficient of $SG(54, 5)$

Group theoretical origin
of CP violation!

M.-C.C., K.T. Mahanthappa (2009)

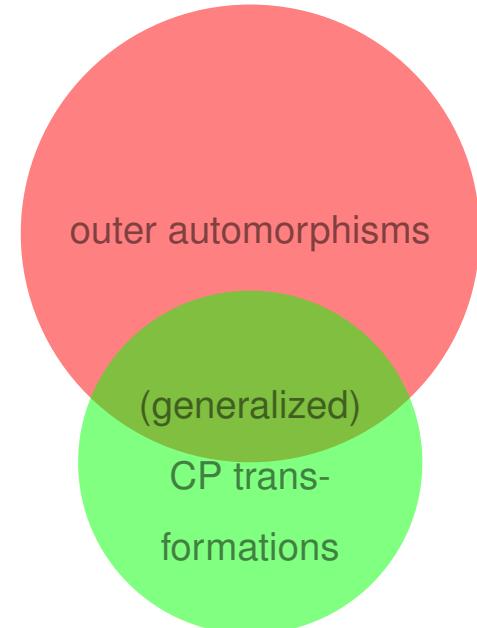
CP-Like Symmetries

- ➡ outer automorphism u_5

$$X \rightarrow X^*, \quad Z \rightarrow Z^*, \quad Y \rightarrow Y^*, \quad \Psi \rightarrow U_{u_5} \Sigma \quad \& \quad \Sigma \rightarrow U_{u_5} \Psi$$

$$U_{u_5} = \begin{pmatrix} 0 & 0 & \omega^2 \\ 0 & 1 & 0 \\ \omega & 0 & 0 \end{pmatrix}$$

- ➡ does **not** lead to a vanishing decay asymmetry
- ➡ in general, imposing an outer automorphism as a symmetry does not lead to physical CP conservation!
- ➡ CP-like symmetry



Summary

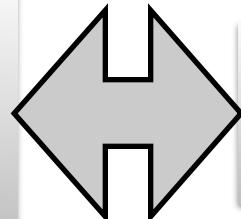
Summary

- NOT all outer automorphisms correspond to physical CP transformations
- Condition on automorphism for *physical* CP transformation

$$\rho_{\mathbf{r}_i}(u(g)) = U_{\mathbf{r}_i} \rho_{\mathbf{r}_i}(g)^* U_{\mathbf{r}_i}^\dagger \quad \forall g \in G \text{ and } \forall i$$

M.-C.C, M. Fallbacher, K.T. Mahanthappa, M. Ratz, A.Trautner, NPB (2014)

class inverting,
involutory
automorphisms

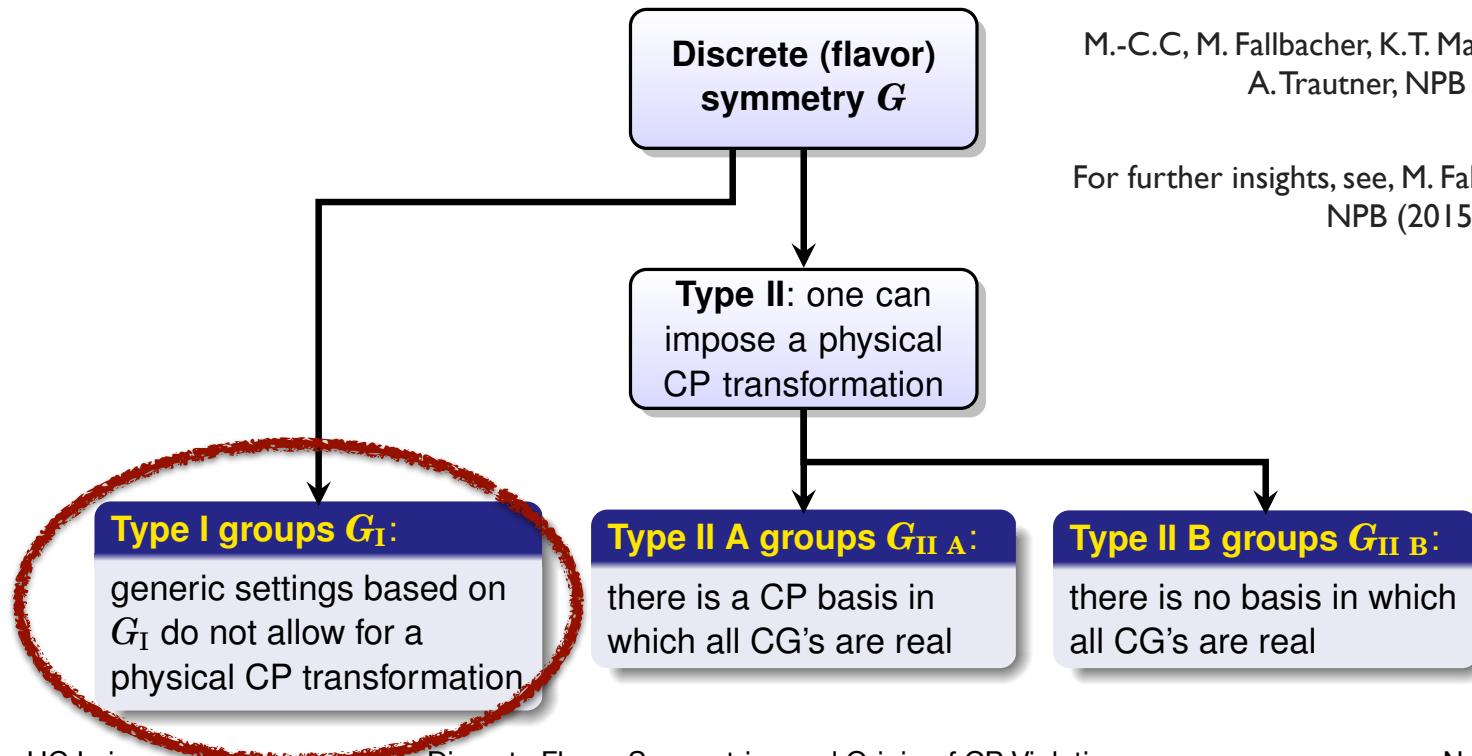


physical CP
transformations

Summary

- For discrete groups that do not have class-inverting, involutory automorphism, CP is generically broken by complex CG coefficients (**Type I Group**)
- Non-existence of such automorphism \Leftrightarrow physical CP violation

CP Violation from Group Theory!



Backup Slides

CP Transformation

- Canonical CP transformation

$$\phi(x) \xrightarrow{C\bar{P}} \eta_{C\bar{P}} \phi^*(\bar{P}x)$$

freedom of re-phasing fields

- Generalized CP transformation

Ecker, Grimus, Konetschny (1981); Ecker, Grimus, Neufeld (1987);
Grimus, Rebelo (1995)

$$\Phi(x) \xrightarrow{\widetilde{C\bar{P}}} U_{CP} \Phi^*(\bar{P}x)$$

unitary matrix

Generalized CP Transformation

- (setting w/ discrete symmetry G)

G and CP transformations do not commute

- generalized CP transformation

Feruglio, Hagedorn, Ziegler (2013); Holthausen, Lindner, Schmidt (2013)

- invariant contraction/coupling in A_4 or T'

$$[\phi_{\mathbf{1}_2} \otimes (x_{\mathbf{3}} \otimes y_{\mathbf{3}})_{\mathbf{1}_1}]_{\mathbf{1}_0} \propto \phi (x_1 y_1 + \omega^2 x_2 y_2 + \omega x_3 y_3)$$

$$\omega = e^{2\pi i/3}$$

- canonical CP transformation maps A_4/T' invariant contraction to something non-invariant

- need generalized CP transformation \widetilde{CP} : $\phi \xrightarrow{\widetilde{CP}} \phi^*$ as usual but

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \xrightarrow{\widetilde{CP}} \begin{pmatrix} x_1^* \\ x_3^* \\ x_2^* \end{pmatrix} \quad \& \quad \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} \xrightarrow{\widetilde{CP}} \begin{pmatrix} y_1^* \\ y_3^* \\ y_2^* \end{pmatrix}$$

The Bickerstaff-Damhus automorphism (BDA)

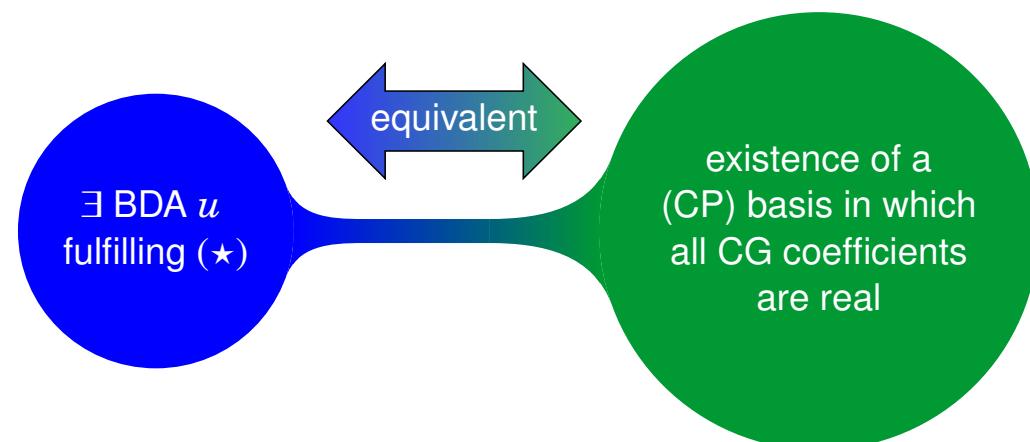
- Bickerstaff-Damhus automorphism (BDA) \mathbf{u}

Bickerstaff, Damhus (1985)

$$\rho_{\mathbf{r}_i}(\mathbf{u}(g)) = U_{\mathbf{r}_i} \rho_{\mathbf{r}_i}(g)^* U_{\mathbf{r}_i}^\dagger \quad \forall g \in G \text{ and } \forall i \quad (\star)$$

unitary & symmetric

- BDA vs. Clebsch-Gordan (CG) coefficients



Constraints on generalized CP transformations

☞ generalized CP transformation

$$\Phi(x) \xrightarrow{\widetilde{CP}} U_{\text{CP}} \Phi^*(\mathcal{P} x)$$

☞ consistency condition

Holthausen, Lindner, and Schmidt (2013)

$$\rho(u(g)) = U_{\text{CP}} \rho(g)^* U_{\text{CP}}^\dagger \quad \forall g \in G$$

☞ further properties:

M.-C.C., M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner (2014)

$$\rho_{\mathbf{r}_i}(u(g)) = U_{\mathbf{r}_i} \rho_{\mathbf{r}_i}(g)^* U_{\mathbf{r}_i}^\dagger \quad \forall g \in G \text{ and } \forall i$$



physical CP
transformations

- u has to be class-inverting
- in all known cases, u is equivalent to an automorphism of order two

bottom-line:

u has to be a class-inverting (involutory) automorphism of G

Twisted Frobenius-Schur Indicator

- How can one tell whether or not a given automorphism is a BDA?
- Frobenius-Schur indicator:

$$\text{FS}(\mathbf{r}_i) := \frac{1}{|G|} \sum_{g \in G} \chi_{\mathbf{r}_i}(g^2) = \frac{1}{|G|} \sum_{g \in G} \text{tr} [\rho_{\mathbf{r}_i}(g)^2]$$

$$\text{FS}(\mathbf{r}_i) = \begin{cases} +1, & \text{if } \mathbf{r}_i \text{ is a real representation,} \\ 0, & \text{if } \mathbf{r}_i \text{ is a complex representation,} \\ -1, & \text{if } \mathbf{r}_i \text{ is a pseudo-real representation.} \end{cases}$$

- Twisted Frobenius-Schur indicator

Bickerstaff, Damhus (1985); Kawanaka, Matsuyama (1990)

$$\text{FS}_u(\mathbf{r}_i) = \frac{1}{|G|} \sum_{g \in G} [\rho_{\mathbf{r}_i}(g)]_{\alpha\beta} [\rho_{\mathbf{r}_i}(\mathbf{u}(g))]_{\beta\alpha}$$

$$\text{FS}_u(\mathbf{r}_i) = \begin{cases} +1 \quad \forall i, & \text{if } \mathbf{u} \text{ is a BDA,} \\ +1 \text{ or } -1 \quad \forall i, & \text{if } \mathbf{u} \text{ is class-inverting and involutory,} \\ \text{different from } \pm 1, & \text{otherwise.} \end{cases}$$

CP Conservation vs Symmetry Enhancement

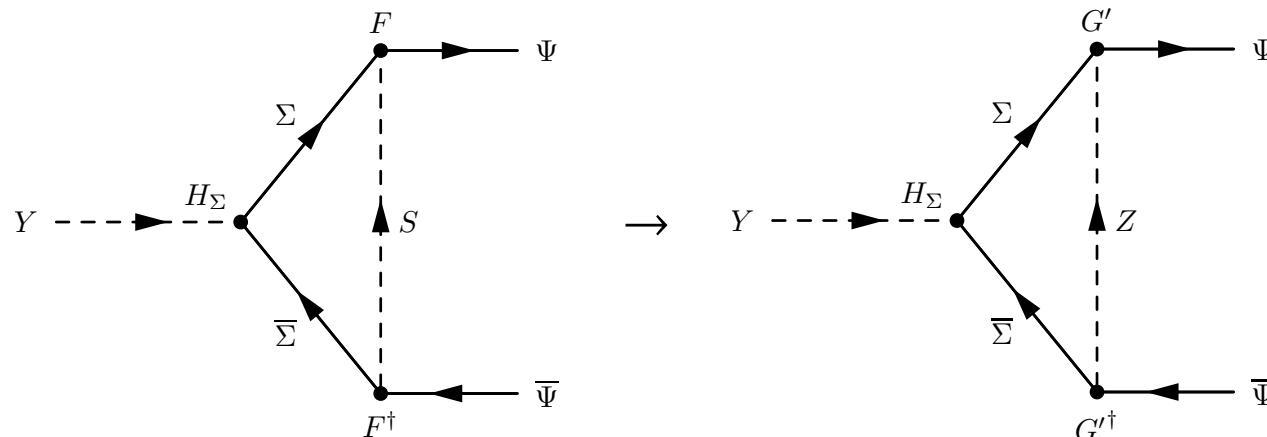
M.-C.C., M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner (2014)

👉 replace $\mathbf{S} \sim \mathbf{1}_0$ by $\mathbf{Z} \sim \mathbf{1}_8 \curvearrowright$ interaction

$$\mathcal{L}_{\text{toy}}^{\mathbf{Z}} = g' \left[\mathbf{Z}_{\mathbf{1}_8} \otimes (\bar{\Psi} \Sigma)_{\mathbf{1}_4} \right]_{\mathbf{1}_0} + \text{h.c.} = (G')^{ij} \mathbf{Z} \bar{\Psi}_i \Sigma_j + \text{h.c.}$$

$$G' = g' \begin{pmatrix} 0 & 0 & \omega^2 \\ 1 & 0 & 0 \\ 0 & \omega & 0 \end{pmatrix}$$

and leads to new interference diagram



Some Outer Automorphisms of $\Delta(27)$

- sample outer automorphisms of $\Delta(27)$

$$\begin{aligned}
 u_1 &: \mathbf{1}_1 \leftrightarrow \mathbf{1}_2, \mathbf{1}_4 \leftrightarrow \mathbf{1}_5, \mathbf{1}_7 \leftrightarrow \mathbf{1}_8, \mathbf{3} \rightarrow U_{u_1} \mathbf{3}^* \\
 u_2 &: \mathbf{1}_1 \leftrightarrow \mathbf{1}_4, \mathbf{1}_2 \leftrightarrow \mathbf{1}_8, \mathbf{1}_3 \leftrightarrow \mathbf{1}_6, \mathbf{3} \rightarrow U_{u_2} \mathbf{3}^* \\
 u_3 &: \mathbf{1}_1 \leftrightarrow \mathbf{1}_8, \mathbf{1}_2 \leftrightarrow \mathbf{1}_4, \mathbf{1}_5 \leftrightarrow \mathbf{1}_7, \mathbf{3} \rightarrow U_{u_3} \mathbf{3}^* \\
 u_4 &: \mathbf{1}_1 \leftrightarrow \mathbf{1}_7, \mathbf{1}_2 \leftrightarrow \mathbf{1}_5, \mathbf{1}_3 \leftrightarrow \mathbf{1}_6, \mathbf{3} \rightarrow U_{u_4} \mathbf{3}^* \\
 u_5 &: \mathbf{1}_i \leftrightarrow \mathbf{1}_i^*, \mathbf{3} \rightarrow U_{u_5} \mathbf{3}
 \end{aligned}$$

- twisted Frobenius-Schur indicators

| R | $\mathbf{1}_0$ | $\mathbf{1}_1$ | $\mathbf{1}_2$ | $\mathbf{1}_3$ | $\mathbf{1}_4$ | $\mathbf{1}_5$ | $\mathbf{1}_6$ | $\mathbf{1}_7$ | $\mathbf{1}_8$ | $\mathbf{3}$ | $\bar{\mathbf{3}}$ |
|----------------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|--------------|--------------------|
| $\text{FS}_{u_1}(R)$ | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 |
| $\text{FS}_{u_2}(R)$ | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 1 |
| $\text{FS}_{u_3}(R)$ | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 1 |
| $\text{FS}_{u_4}(R)$ | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 1 |
| $\text{FS}_{u_5}(R)$ | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |

- none of the u_i maps all representations to their conjugates
- however, it is possible to impose CP in (non-generic) models, where only a subset of representations are present, e.g. $\{\mathbf{r}_i\} \subset \{\mathbf{1}_0, \mathbf{1}_5, \mathbf{1}_7, \mathbf{3}, \bar{\mathbf{3}}\}$
- CP conservation possible in non-generic models
 - e.g. some well-known multiple Higgs model Branco, Gerard, and Grimus (1984)

CP Conservation vs Symmetry Enhancement

M.-C.C., M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner (2014)

- 👉 replace $\textcolor{violet}{S} \sim \mathbf{1}_0$ by $\textcolor{violet}{Z} \sim \mathbf{1}_8 \curvearrowright$ interaction

$$\mathcal{L}_{\text{toy}}^Z = g' \left[Z_{\mathbf{1}_8} \otimes (\bar{\Psi} \Sigma)_{\mathbf{1}_4} \right]_{\mathbf{1}_0} + \text{h.c.} = (G')^{ij} Z \bar{\Psi}_i \Sigma_j + \text{h.c.}$$

- ➡ different contribution to decay asymmetry: $\varepsilon_{Y \rightarrow \bar{\Psi}\Psi}^S \rightarrow \varepsilon_{Y \rightarrow \bar{\Psi}\Psi}^Z$

- 👉 total CP asymmetry of the Y decay vanishes if $\begin{cases} \text{(i)} & M_Z = M_X \\ \text{(ii)} & |g| = |g'| \\ \text{(iii)} & \varphi = 0 \end{cases}$

- 👉 relations (i)–(iii) can be due to an **outer automorphism**

$$X \xleftrightarrow{u_3} Z, \quad Y \xrightarrow{u_3} Y, \quad \Psi \xrightarrow{u_3} U_{u_3} \Sigma^C \quad \& \quad \Sigma \xrightarrow{u_3} U_{u_3} \Psi^C$$

requires $q_\Sigma = -q_\Psi$

$$U_{u_3} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega^2 \end{pmatrix}$$

... BUT this enlarges $\Delta(27) \rightarrow \text{SG}(54, 5) \simeq \Delta(27) \rtimes \mathbb{Z}_2^{u_3}$

$\text{SG}(54, 5)$: group name from GAP library

Example for a type II A group:

T'

- CP basis and its complications
- generalized CP transformation

(Generalized) CP Transformation for T'

- ☞ unique outer automorphism

$$\textcolor{red}{u} : (S, T) \rightarrow (S^3, T^2) \quad \sim \quad \begin{cases} \mathbf{1}_i & \rightarrow U_{\mathbf{1}_i} \mathbf{1}_i^* \\ \mathbf{2}_i & \rightarrow U_{\mathbf{2}_i} \mathbf{2}_i^* \\ \mathbf{3} & \rightarrow U_{\mathbf{3}} \mathbf{3}^* \end{cases}$$

- ☞ twisted Frobenius–Schur indicators

| \mathbf{R} | $\mathbf{1}_0$ | $\mathbf{1}_1$ | $\mathbf{1}_2$ | $\mathbf{2}_0$ | $\mathbf{2}_1$ | $\mathbf{2}_2$ | $\mathbf{3}$ |
|---------------------------|----------------|----------------|----------------|----------------|----------------|----------------|--------------|
| $\text{FS}_u(\mathbf{R})$ | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

- u is a Bickerstaff–Damhus automorphism
- there is a basis in which all Clebsch–Gordan coefficients are real

basis can be found e.g. in Ishimori, Kobayashi, Ohki, Shimizu, Okada, et al. (2010)

- ☞ u defines a physical CP transformation
- ☞ invariance of \mathcal{L} under u restricts the phases of the coupling coefficients

Issues with the CP basis and other bases

M.-C.C., M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner (2014)

- ☞ **3** of T' is a **real representation**
- ☞ however, in many T' bases (including the CP basis), **3** transforms with **complex matrices**
- ☞ need to describe a **real 3–plet** by **complex field(s)** and impose ‘Majorana–like condition’ $\phi^* = U\phi$

with e.g. $U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$ in the ‘Feruglio basis’ “Fefo basis” (2001)

with the ‘Feruglio basis’ defined in Appendix A of Feruglio, Hagedorn, Lin, and Merlo (2007)

- ☞ problems do not appear in the T' extension of the ‘Ma basis’ for A_4

A_4 basis can be found in Ma and Rajasekaran (2001)

- ☞ proper CP transformation

$$\mathbf{1}_i \xrightarrow{\widetilde{CP}} \mathbf{1}_i^*, \quad \mathbf{2}_i \xrightarrow{\widetilde{CP}} \mathbf{2}_i^*, \quad \mathbf{3} \xrightarrow{\widetilde{CP}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \mathbf{3}^*$$

Example for a type II B group:

$$\Sigma(72)$$

- absence of CP basis but generalized CP transformation ensures physical CP conservation
- CP forbids couplings

Example of a type IIB group: $\Sigma(72)$

☞ presentation of $\Sigma(72)$

$$M^4 = N^4 = P^3 = (M^2 P^{-1})^2 = \mathbb{1}, \quad M^2 = N^2, \quad M^{-1}N = NM$$

$$PMPN^{-1}MP^{-1}N = \mathbb{1}, \quad NPM^{-1}P = MPN$$

☞ 6 inequivalent irreducible representations: **1₀₋₃**, **2** and **8**

☞ character table

| | C_{1a} | C_{3a} | C_{2a} | C_{4a} | C_{4b} | C_{4c} |
|----------------------|--------------|----------|----------|----------|----------|----------|
| $\Sigma(72)$ | $\mathbb{1}$ | P | M^2 | MN | N | M |
| 1₀ | 1 | 1 | 1 | 1 | 1 | 1 |
| 1₁ | 1 | 1 | 1 | 1 | -1 | -1 |
| 1₂ | 1 | 1 | 1 | -1 | 1 | -1 |
| 1₃ | 1 | 1 | 1 | -1 | -1 | 1 |
| 2 | 2 | 2 | -2 | 0 | 0 | 0 |
| 8 | 8 | -1 | 0 | 0 | 0 | 0 |

Example of a type IIB group: $\Sigma(72)$

M.-C.C., M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner (2014)

- ☞ $\Sigma(72)$ is ambivalent, i.e. each conjugacy class contains with an element g also its inverse element g^{-1}
- ☞ identity is already class-inverting (and involutory)
- ☞ twisted Frobenius–Schur indicators of identity

| R | $\mathbf{1}_0$ | $\mathbf{1}_1$ | $\mathbf{1}_2$ | $\mathbf{1}_3$ | $\mathbf{2}$ | $\mathbf{8}$ |
|----------------------------|----------------|----------------|----------------|----------------|--------------|--------------|
| $\text{FS}_{\text{id}}(R)$ | 1 | 1 | 1 | 1 | -1 | 1 |

☞ there is no CP basis

no BDA

☞ generalized CP transformation

$$\mathbf{1}_i \xrightarrow{\widetilde{CP}} \mathbf{1}_i^*, \quad \mathbf{2} \xrightarrow{\widetilde{CP}} U_2 \mathbf{2}^*, \quad \mathbf{8} \xrightarrow{\widetilde{CP}} \mathbf{8}^*$$

$$U_2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

Example of a type IIB group: $\Sigma(72)$

- ☞ generalized CP transformation

$$\mathbf{1}_i \xrightarrow{\widetilde{CP}} \mathbf{1}_i^*, \quad \mathbf{2} \xrightarrow{\widetilde{CP}} U_2 \mathbf{2}^*, \quad \mathbf{8} \xrightarrow{\widetilde{CP}} \mathbf{8}^*$$

- ☞ imposing this CP transformation as a symmetry enlarges the flavor group by an additional \mathbb{Z}_2 factor to $\Sigma(72) \times \mathbb{Z}_2$
- ☞ additional symmetry generator acts trivially on all representations except for the $\mathbf{2}$ on which it acts as $V_2 = U_2 U_2^* = -\mathbb{1}$
- ☞ this additional \mathbb{Z}_2 forbids all terms which contain an odd number of fields in the representation $\mathbf{2}$ such as

$$\mathcal{L} \supset c (\mathbf{2} \otimes (\mathbf{8} \otimes \mathbf{8})_2)_{\mathbf{1}_0}$$

forbidden by additional \mathbb{Z}_2

unusual feature of type II B groups:

CP may forbid couplings rather than restricting the phases!

Summary

Three examples:

M.-C.C, M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner, NPB (2014)

☞ Type I group: $\Delta(27)$

- generic settings based on $\Delta(27)$ violate CP!
- spontaneous breaking of type II A group $SG(54, 5) \rightarrow \Delta(27)$
 \curvearrowright prediction of CP violating phase from group theory!

☞ Type II A group: T'

- CP basis exists but has certain shortcomings
- advantageous to work in a different basis & impose generalized CP transformation
- CP constrains phases of coupling coefficients

☞ Type II B group: $\Sigma(72)$

- absence of CP basis but generalized CP transformation ensures physical CP conservation
- CP forbids couplings