Flavour scenarios from 5D SO(10): order and anarchy interplay

Denise Vicino,
University of Padova

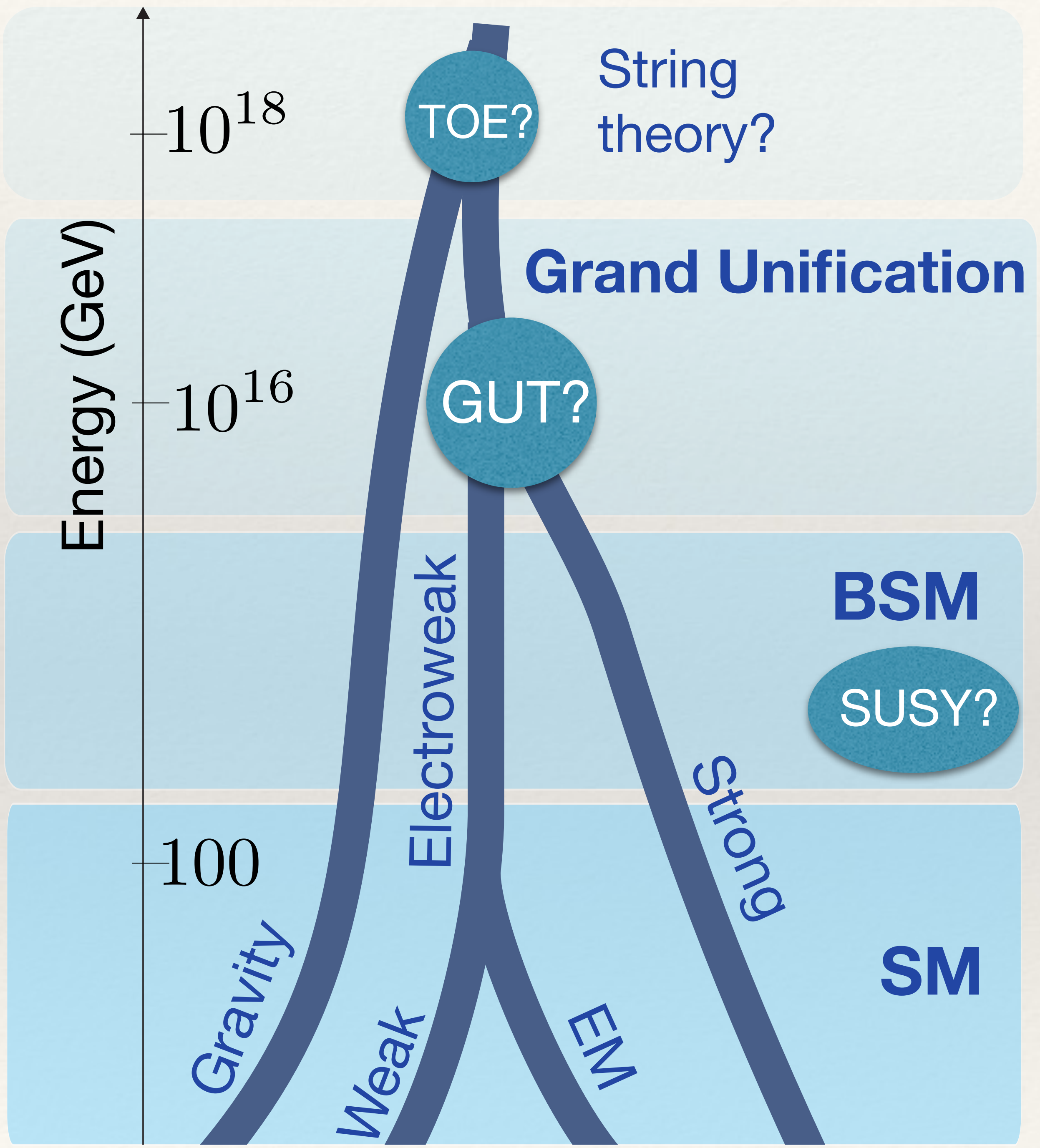
in collaboration with:
F. Feruglio and **K. Patel**

Based on:
arXiv: 1507.00669
and **JHEP 1409(2014)095** - **arXiv: 1407.2913**

Nu@Fermilab 2015 • 21-25 Jul 2015

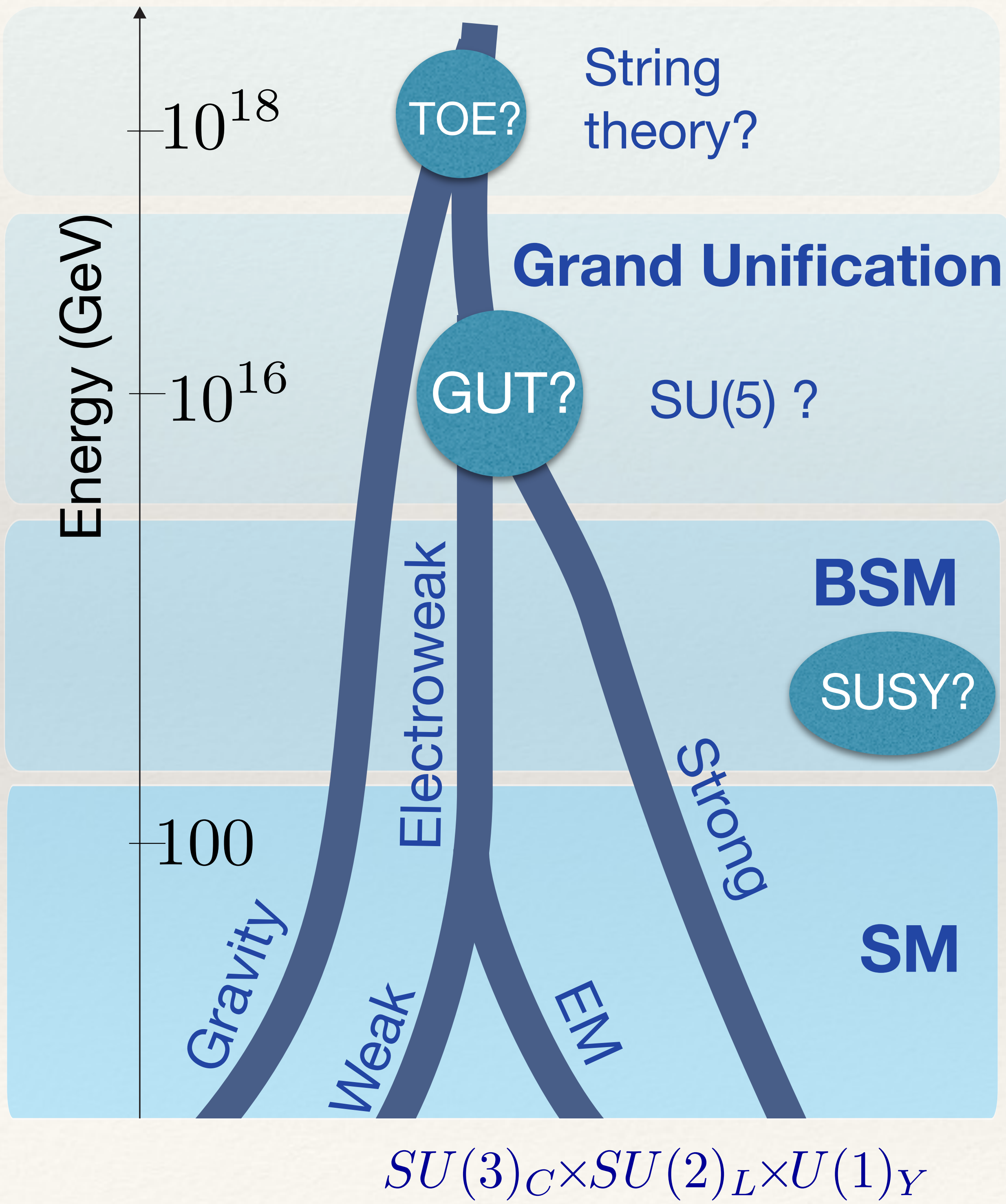
Grand Unification and the Flavour puzzle

Unification of Forces:

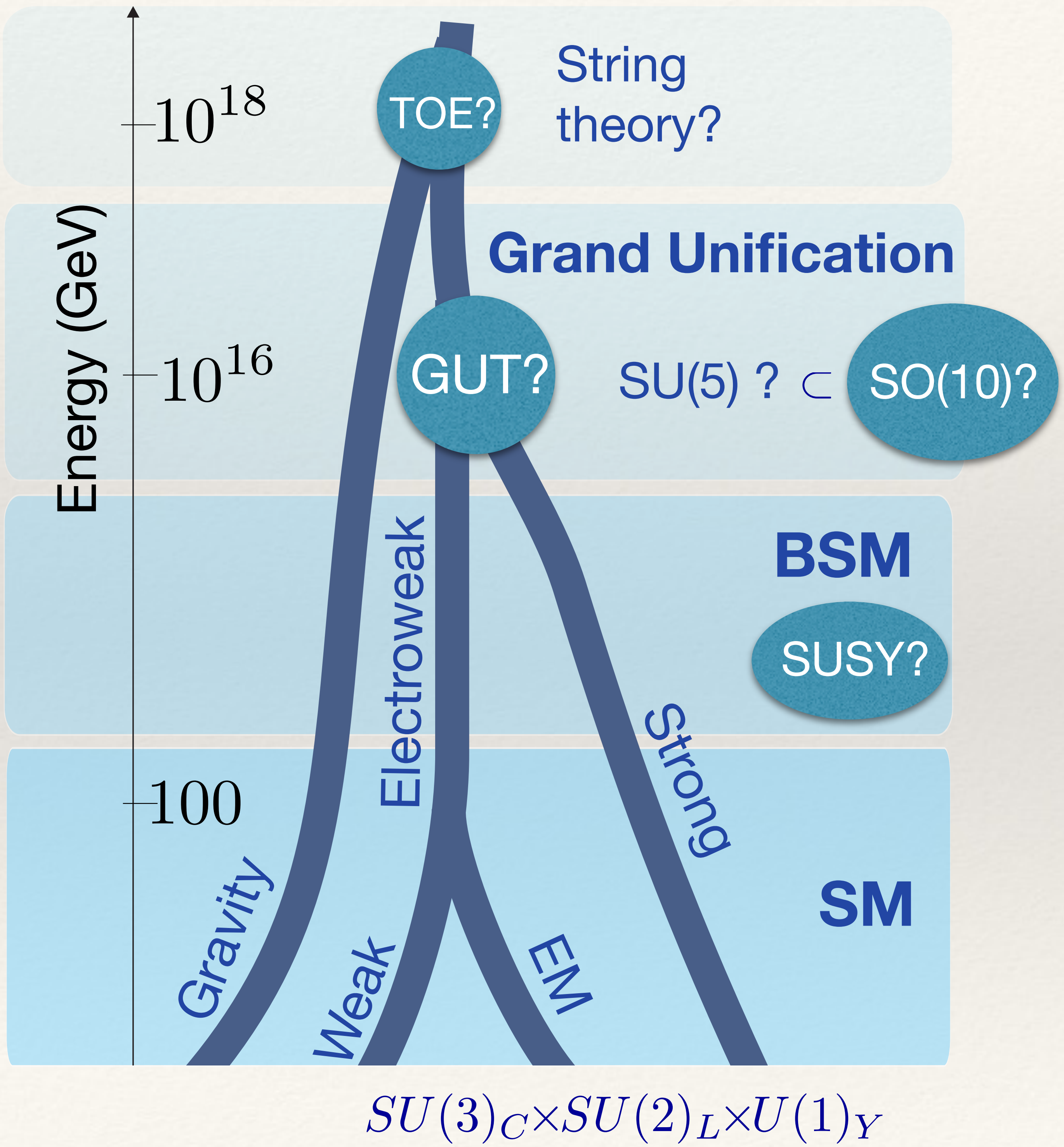


$SU(3)_C \times SU(2)_L \times U(1)_Y$

Unification of Forces:

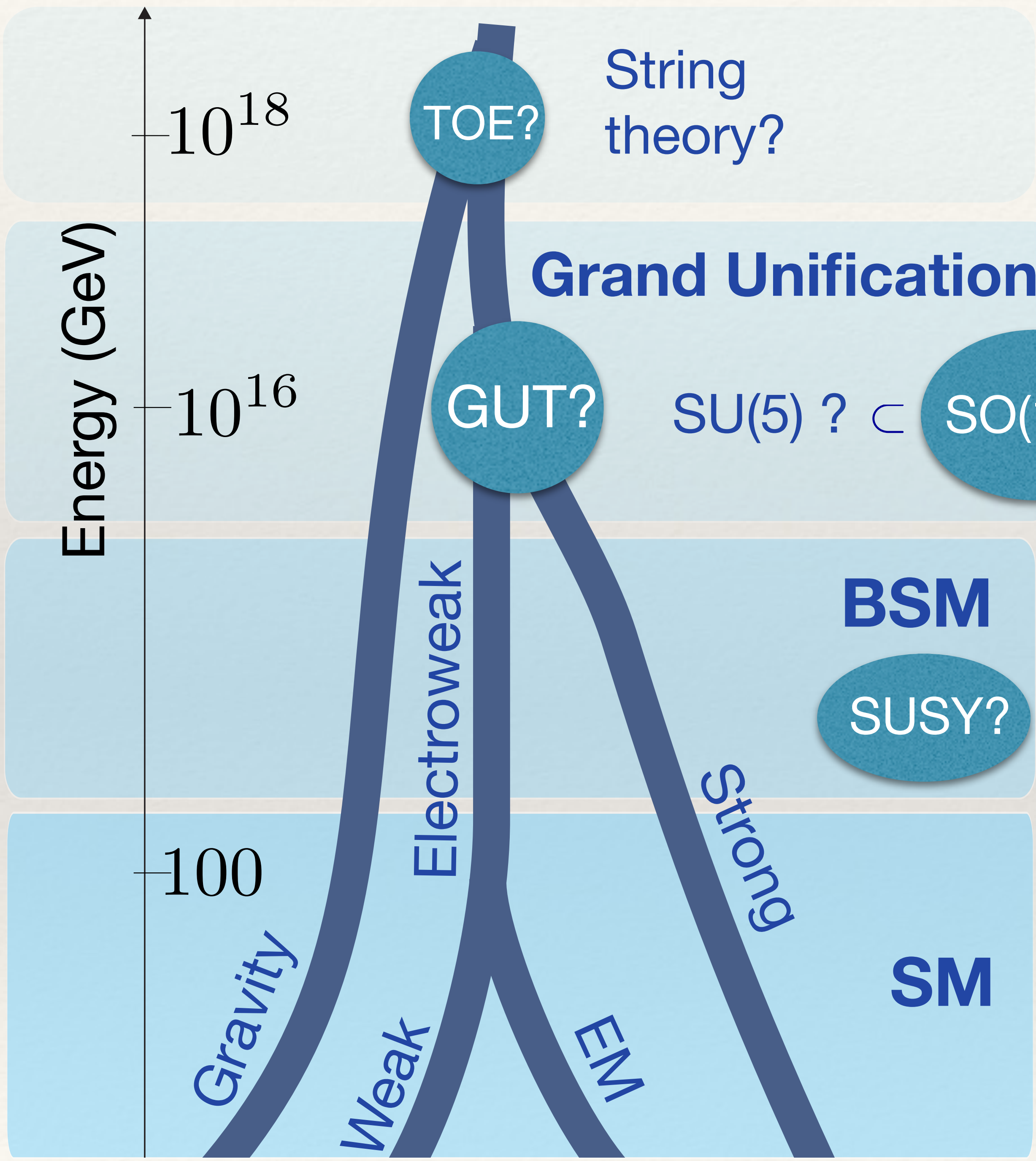


Unification of Forces:



Grand Unification and the Flavour puzzle

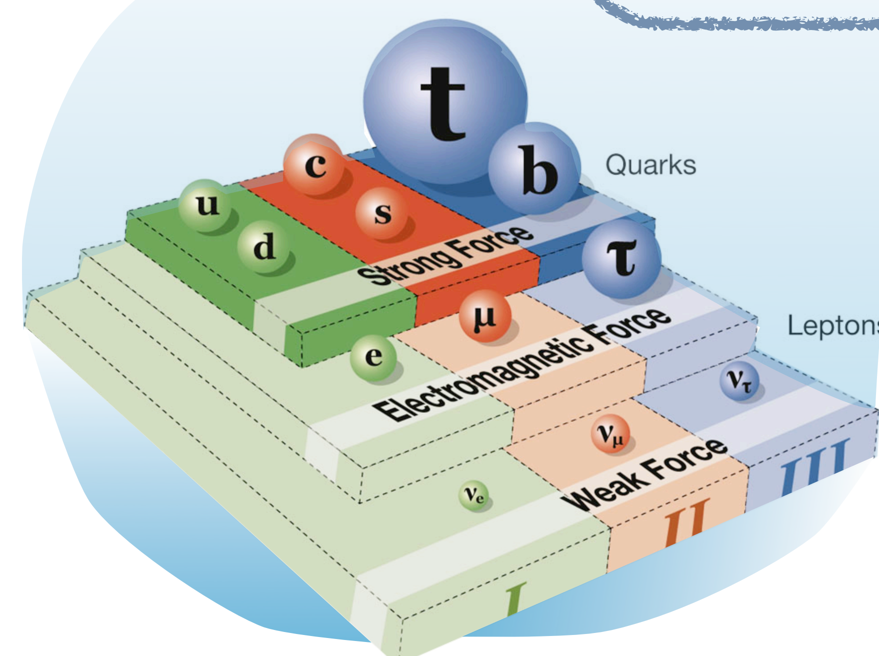
Unification of Forces:



$SU(3)_C \times SU(2)_L \times U(1)_Y$

- Explain the origin of SM gauge structure: but which symmetry breaking down to the SM?
- Unified description of SM fermions:

SM fermions + RH neutrinos → a single SO(10) representation: $16_{i=1,2,3}$



Grand Unification and the Flavour puzzle

Why this peculiar structure of the Yukawa couplings?

Masses

- **Charged Fermions**

$$m_u : m_c : m_t \approx \lambda^8 : \lambda^4 : 1$$
$$m_d : m_s : m_b \approx \lambda^5 : \lambda^3 : 1$$
$$m_e : m_\mu : m_\tau \approx \lambda^6 : \lambda^2 : 1$$

- **Neutrinos**

$$m_\nu \leq \mathcal{O}(\mathrm{eV}) \qquad \frac{\Delta_S}{\Delta_A} \approx \lambda^2$$
$$\Delta_S \equiv m_{\nu 2}^2 - m_{\nu 1}^2$$
$$\Delta_A \equiv \left| m_{\nu 3}^2 - m_{\nu 2}^2 \right|$$

Mixing

- **Quark sector**

$$|V_{\mathrm{CKM}}| \approx \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}$$

- **Lepton sector**

$$|U_{\mathrm{PMNS}}| \approx \begin{pmatrix} 0.8 & 0.5 & 0.2 \\ 0.5 & 0.6 & 0.6 \\ 0.3 & 0.6 & 0.7 \end{pmatrix}$$

SO(10) GUT: which advantages?

- RH neutrinos, natural implementation of (type I)

[Minkowski (1977), Yanagida (1979), Gell-Mann,Ramond, Slansky (1979), Mohapatra and Senjanovic (1980)]

- Embedding SU(5) ⊂ SO(10): explain similar hierarchy in down quarks and charged leptons [Georgi-Glashow (1974)]

Grand Unification and the Flavour puzzle

Why this peculiar structure of the Yukawa couplings?

Masses

- **Charged Fermions**

$$m_u : m_c : m_t \approx \lambda^8 : \lambda^4 : 1$$
$$m_d : m_s : m_b \approx \lambda^5 : \lambda^3 : 1$$
$$m_e : m_\mu : m_\tau \approx \lambda^6 : \lambda^2 : 1$$

- **Neutrinos**

$$m_\nu \leq \mathcal{O}(\mathrm{eV}) \qquad \frac{\Delta_S}{\Delta_A} \approx \lambda^2$$
$$\Delta_S \equiv m_{\nu 2}^2 - m_{\nu 1}^2$$
$$\Delta_A \equiv \left| m_{\nu 3}^2 - m_{\nu 2}^2 \right|$$

Mixing

- **Quark sector**

$$|V_{\mathrm{CKM}}| \approx \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}$$

- **Lepton sector**

$$|U_{\mathrm{PMNS}}| \approx \begin{pmatrix} 0.8 & 0.5 & 0.2 \\ 0.5 & 0.6 & 0.6 \\ 0.3 & 0.6 & 0.7 \end{pmatrix}$$

SO(10) GUT: which advantages?

- RH neutrinos, natural implementation of (type I) **See-Saw mechanism**

  [Minkowski (1977), Yanagida (1979), Gell-Mann,Ramond, Slansky (1979), Mohapatra and Senjanovic (1980)]

- Embedding SU(5) $\subset$ SO(10): explain similar hierarchy in down quarks and charged leptons [Georgi-Glashow (1974)]

Grand Unification and the Flavour puzzle

Why this peculiar structure of the Yukawa couplings?

Masses

- **Charged Fermions**

$$m_u : m_c : m_t \approx \lambda^8 : \lambda^4 : 1$$
$$m_d : m_s : m_b \approx \lambda^5 : \lambda^3 : 1$$
$$m_e : m_\mu : m_\tau \approx \lambda^6 : \lambda^2 : 1$$

- **Neutrinos**

$$m_\nu \leq \mathcal{O}(\mathrm{eV}) \qquad \frac{\Delta_S}{\Delta_A} \approx \lambda^2$$
$$\Delta_S \equiv m^2_{\nu 2} - m^2_{\nu 1}$$
$$\Delta_A \equiv \left| m^2_{\nu 3} - m^2_{\nu 2} \right|$$

Mixing

- **Quark sector**

$$|V_{\mathrm{CKM}}| \approx \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}$$

- **Lepton sector**

$$|U_{\mathrm{PMNS}}| \approx \begin{pmatrix} 0.8 & 0.5 & 0.2 \\ 0.5 & 0.6 & 0.6 \\ 0.3 & 0.6 & 0.7 \end{pmatrix}$$

SO(10) GUT: which advantages?

- RH neutrinos, natural implementation of (type I) **See-Saw mechanism**

  [Minkowski (1977), Yanagida (1979), Gell-Mann, Ramond, Slansky (1979), Mohapatra and Senjanovic (1980)]

- Embedding SU(5) $\subset$ SO(10): explain similar hierarchy in down quarks and charged leptons [Georgi-Glashow (1974)]

SO(10) GUT: which disadvantages?

Structure of the Yukawa couplings: $\mathcal{Y}_{10}^{ij} 16_i 16_j 10_H + \ldots$

$\boxed{\mathbf{16 \times 16 = 10 + 120 + 126}}$ **3 possible Higgs representations**

- **No minimal coupling** quarks and leptons);
- Large representations;
- Lots of parameters,

SO(10) GUT: which disadvantages?

Structure of the Yukawa couplings: $\mathcal{Y}_{10}^{ij} 16_i 16_j 10_H + \ldots$

$\boxed{\mathbf{16 \times 16 = 10 + 120 + 126}}$ **3 possible Higgs representations**

- **No minimal coupling** is possible (more then one Higgs, necessary to distinguish quarks and leptons);
- Large representations;
- Lots of parameters,

SO(10) GUT: which disadvantages?

Structure of the Yukawa couplings: $\mathcal{Y}_{10}^{ij} 16_i 16_j 10_H + \ldots$

$$\mathbf{16} \times \mathbf{16} = \mathbf{10} + \mathbf{120} + \mathbf{126}$$

3 possible Higgs representations

- **No minimal coupling** is possible (more then one Higgs, necessary to distinguish quarks and leptons);
- Large representations; **Doublet-Triplet** splitting problem;
- Lots of parameters,

Grand Unification and the Flavour puzzle

SO(10) GUT: which disadvantages?

Structure of the Yukawa couplings: $\mathcal{Y}_{10}^{ij} 16_i 16_j 10_H + \ldots$

$\mathbf{16 \times 16 = 10 + 120 + 126}$ **3 possible Higgs representations**

- **No minimal coupling** is possible (more then one Higgs, necessary to distinguish quarks and leptons);
- Large representations; **Doublet-Triplet** splitting problem;
- Lots of parameters, **hierarchical** and **fine-tuned** (as much as in the SM)

SO(10) GUT: which disadvantages?

Structure of the Yukawa couplings: $\mathcal{Y}_{10}^{ij} 16_i 16_j 10_H + \ldots$

$\boxed{\mathbf{16 \times 16 = 10 + 120 + 126}}$ **3 possible Higgs representations**

- **No minimal coupling** is possible (more then one Higgs, necessary to distinguish quarks and leptons);
- Large representations; **Doublet-Triplet** splitting problem;
- Lots of parameters, **hierarchical** and **fine-tuned** (as much as in the SM)

Grand Unification and the Flavour puzzle

SO(10) GUT: which disadvantages?

Structure of the Yukawa couplings: $\mathcal{Y}_{10}^{ij} 16_i 16_j 10_H + \ldots$

$$\boxed{\mathbf{16 \times 16 = 10 + 120 + 126}}$$

3 possible Higgs representations

- **No minimal coupling** is possible (more then one Higgs, necessary to distinguish quarks and leptons);
- Large representations; **Doublet-Triplet** splitting problem;
- Lots of parameters, **hierarchical** and **fine-tuned** (as much as in the SM)

Are <u>ANARCHICAL</u> $\mathcal{O}(1)$ Yukawas allowed?

Can any mechanism <u>ORDER</u> the parameters and create the hierarchies?

Is this compatible with unified description of fermions in SO(10)?

Anarchy and order interplay: the basic idea

Hierarchical **Anarchical** $\mathcal{O}(1)$

$$\left.\begin{aligned} \mathcal{Y}_u &= F_Q Y_u F_{u^c} \\ \mathcal{Y}_d &= F_Q Y_d F_{d^c} \end{aligned}\right\} \quad \begin{aligned} &F_{Q1} \ll F_{Q2} \ll F_{Q3} \\ &F_{u1} < F_{u2} < F_{u3} \\ &F_{d1} \lesssim F_{d2} \lesssim F_{d3} \end{aligned}$$

$$\mathcal{Y}_e = F_L Y_e F_{e^c} \quad \left\} \begin{aligned} &F_{L1} \approx F_{L2} \approx F_{L3} \\ &F_{e1} \ll F_{e2} \ll F_{e3} \end{aligned}\right.$$

[Kawamura, (2001

Anarchy and order interplay: the basic idea

Hierarchical **Anarchical** $\mathcal{O}(1)$ **“Ordering”**

$$\left.\begin{aligned}\mathcal{Y}_u &= F_Q Y_u F_{u^c}\\ \mathcal{Y}_d &= F_Q Y_d F_{d^c}\end{aligned}\right\}\qquad \begin{aligned}&F_{Q1} \ll F_{Q2} \ll F_{Q3}\\ &F_{u1} < F_{u2} < F_{u3}\\ &F_{d1} \lesssim F_{d2} \lesssim F_{d3}\end{aligned}$$

$$\left.\mathcal{Y}_e = F_L Y_e F_{e^c}\right\}\qquad \begin{aligned}&F_{L1} \approx F_{L2} \approx F_{L3}\\ &F_{e1} \ll F_{e2} \ll F_{e3}\end{aligned}$$

Which mechanism can generate these hierarchies ?

[Kawamura, (2001

Anarchy and order interplay: the basic idea

Hierarchical **Anarchical** $\mathcal{O}(1)$

$$\mathcal{Y}_u = F_Q Y_u F_{u^c}$$
$$\mathcal{Y}_d = F_Q Y_d F_{d^c}$$
$$\mathcal{Y}_e = F_L Y_e F_{e^c}$$

"Ordering"

$$F_{Q1} \ll F_{Q2} \ll F_{Q3}$$
$$F_{u1} < F_{u2} < F_{u3}$$
$$F_{d1} \lesssim F_{d2} \lesssim F_{d3}$$

$$F_{L1} \approx F_{L2} \approx F_{L3}$$
$$F_{e1} \ll F_{e2} \ll F_{e3}$$

Which mechanism can generate these hierarchies ?

Froggatt-Nielsen charges: $G_f = U(1)_{FN}$

[Kawamura, (2001

Anarchy and order interplay: the basic idea

Hierarchical **Anarchical** $\mathcal{O}(1)$

$$\left.\begin{array}{l}\mathcal{Y}_u = F_Q Y_u F_{u^c} \\ \mathcal{Y}_d = F_Q Y_d F_{d^c}\end{array}\right\}$$

$$\mathcal{Y}_e = F_L Y_e F_{e^c} \Big\}$$

"Ordering"

$$F_{Q1} \ll F_{Q2} \ll F_{Q3}$$
$$F_{u1} < F_{u2} < F_{u3}$$
$$F_{d1} \lesssim F_{d2} \lesssim F_{d3}$$

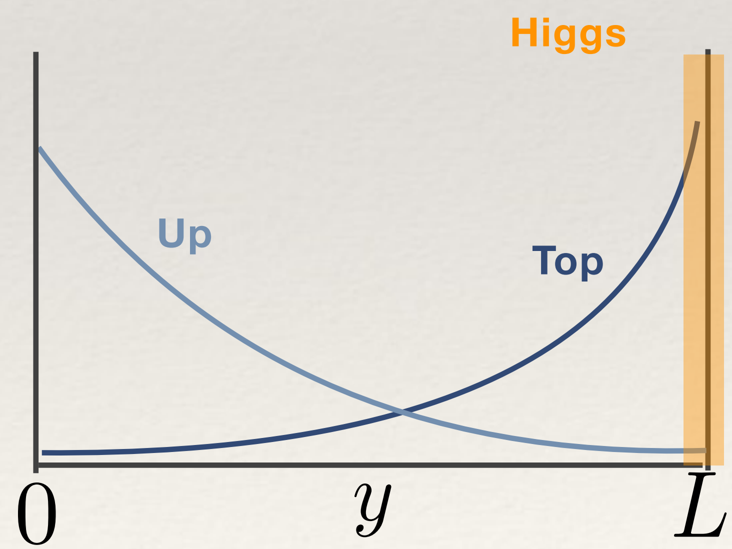
$$F_{L1} \approx F_{L2} \approx F_{L3}$$
$$F_{e1} \ll F_{e2} \ll F_{e3}$$

Which mechanism can generate these hierarchies ?

Froggatt-Nielsen charges: $G_f = U(1)_{FN}$

Extra dimension (ED):

different localisation of fermions



[Kawamura, (2001

Anarchy and order interplay: the basic idea

Hierarchical **Anarchical** $\mathcal{O}(1)$ **"Ordering"**

$$\left.\begin{aligned}\mathcal{Y}_u &= F_Q Y_u F_{u^c} \\ \mathcal{Y}_d &= F_Q Y_d F_{d^c}\end{aligned}\right\}$$

$$\left.\mathcal{Y}_e = F_L Y_e F_{e^c}\right\}$$

$$F_{Q1} \ll F_{Q2} \ll F_{Q3}$$
$$F_{u1} < F_{u2} < F_{u3}$$
$$F_{d1} \lesssim F_{d2} \lesssim F_{d3}$$

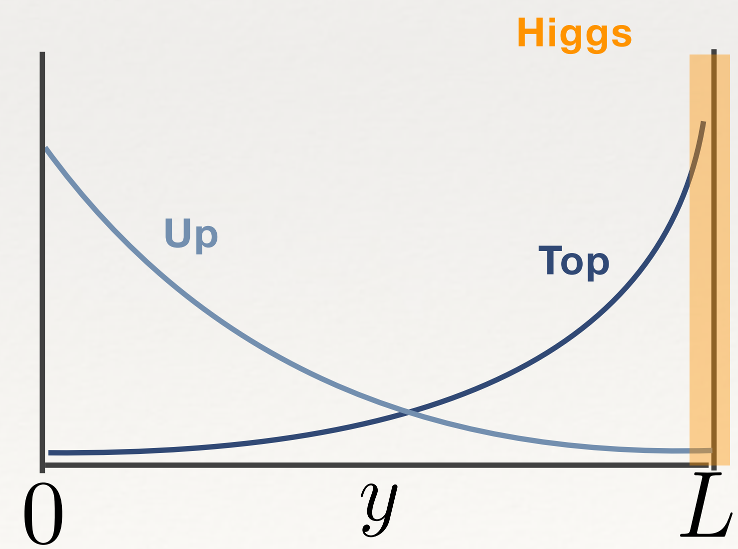
$$F_{L1} \approx F_{L2} \approx F_{L3}$$
$$F_{e1} \ll F_{e2} \ll F_{e3}$$

Which mechanism can generate these hierarchies ?

Froggatt-Nielsen charges: $G_f = U(1)_{FN}$

Extra dimension (ED):

different localisation of fermions



Combined with
SO(10), N=1 SUSY

New mechanisms of symmetry breaking

Solution to Doublet-Triplet splitting problem
[Kawamura, (2001

More predictive model
[Kitano, Li (2004)] [Feruglio, Patel, DV (2014)]

Anarchy and order interplay: the basic idea

Hierarchical **Anarchical** $\mathcal{O}(1)$ **"Ordering"**

$$\mathcal{Y}_u = F_Q Y_u F_{u^c}$$
$$\mathcal{Y}_d = F_Q Y_d F_{d^c}$$
$$\mathcal{Y}_e = F_L Y_e F_{e^c}$$

$$F_{Q1} \ll F_{Q2} \ll F_{Q3}$$
$$F_{u1} < F_{u2} < F_{u3}$$
$$F_{d1} \lesssim F_{d2} \lesssim F_{d3}$$

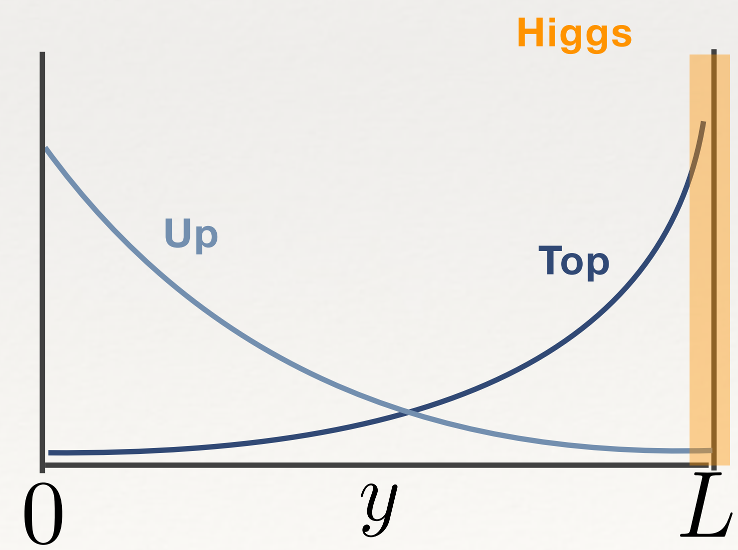
$$F_{L1} \approx F_{L2} \approx F_{L3}$$
$$F_{e1} \ll F_{e2} \ll F_{e3}$$

Which mechanism can generate these hierarchies ?

Froggatt-Nielsen charges: $G_f = U(1)_{FN}$

Extra dimension (ED):

different localisation of fermions



Combined with **SO(10), N=1 SUSY**

New mechanisms of symmetry breaking

Solution to Doublet-Triplet splitting problem

[Kawamura, (2001

More predictive model

[Kitano, Li (2004)] [Feruglio, Patel, DV (2014)]

Anarchy and order interplay: the basic idea

Hierarchical **Anarchical** $\mathcal{O}(1)$

$$\left.\begin{aligned}\mathcal{Y}_u &= F_Q Y_u F_{u^c}\\ \mathcal{Y}_d &= F_Q Y_d F_{d^c}\end{aligned}\right\}$$

$$\left.\mathcal{Y}_e = F_L Y_e F_{e^c}\right\}$$

"Ordering"

$$F_{Q1} \ll F_{Q2} \ll F_{Q3}$$
$$F_{u1} < F_{u2} < F_{u3}$$
$$F_{d1} \lesssim F_{d2} \lesssim F_{d3}$$

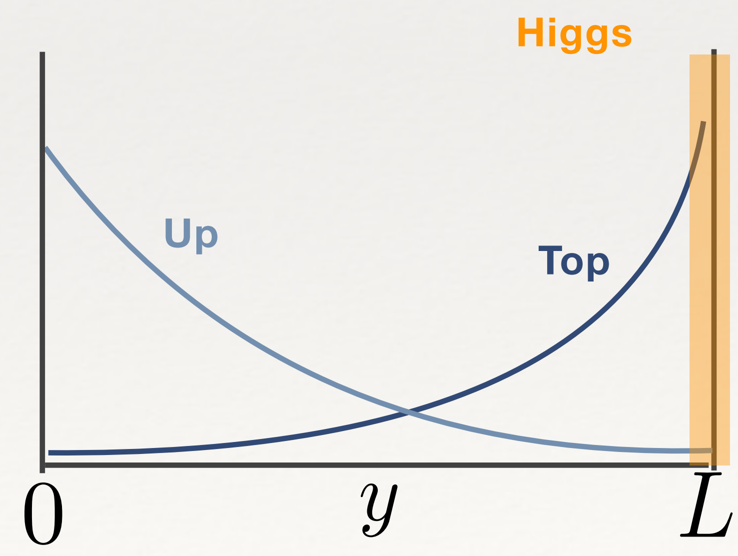
$$F_{L1} \approx F_{L2} \approx F_{L3}$$
$$F_{e1} \ll F_{e2} \ll F_{e3}$$

Which mechanism can generate these hierarchies ?

Froggatt-Nielsen charges: $G_f = U(1)_{FN}$

Extra dimension (ED):

different localisation of fermions



Combined with **SO(10), N=1 SUSY**

New mechanisms of symmetry breaking

Solution to Doublet-Triplet splitting problem
[Kawamura, (2001)]

More predictive model

[Kitano, Li (2004)] [Feruglio, Patel, DV (2014)]

Anarchy and order interplay: the basic idea

Hierarchical **Anarchical** $\mathcal{O}(1)$

$$\left.\begin{aligned} \mathcal{Y}_u &= F_Q Y_u F_{u^c} \\ \mathcal{Y}_d &= F_Q Y_d F_{d^c} \end{aligned}\right\}$$

$$\left.\mathcal{Y}_e = F_L Y_e F_{e^c}\right\}$$

"Ordering"

$$F_{Q1} \ll F_{Q2} \ll F_{Q3}$$
$$F_{u1} < F_{u2} < F_{u3}$$
$$F_{d1} \lesssim F_{d2} \lesssim F_{d3}$$

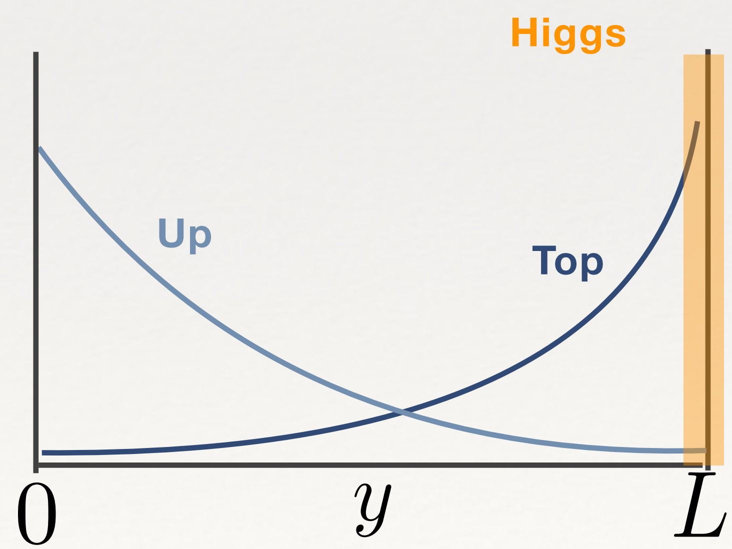
$$F_{L1} \approx F_{L2} \approx F_{L3}$$
$$F_{e1} \ll F_{e2} \ll F_{e3}$$

Which mechanism can generate these hierarchies ?

Froggatt-Nielsen charges: $G_f = U(1)_{FN}$

Extra dimension (ED):

different localisation of fermions



Combined with **SO(10), N=1 SUSY**

New mechanisms of symmetry breaking

Solution to Doublet-Triplet splitting problem
[Kawamura, (2001)]

More predictive model
[Kitano, Li (2004)] [Feruglio, Patel, DV (2014)]

A 5D SO(10) SUSY model: compactification

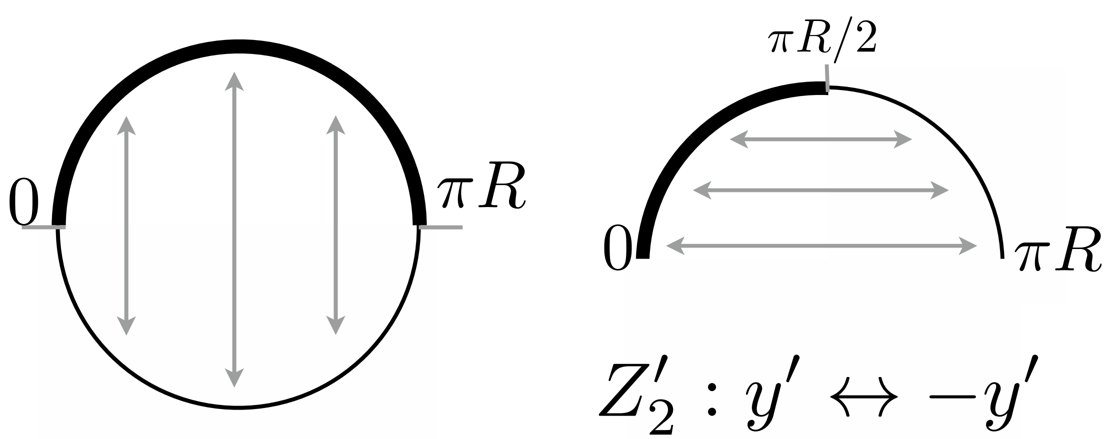
- **Kaluza-Klein expansion:** for each field propagating in the ED

$$H(x_\mu, y) = \sum_n H_n(x_\mu) f_n(y)$$

Profile in the extra dimension

n=0 mode describes the massless particle (MSSM field)

- **Extra dimension compactified on Orbifold:** $S^1/(Z_2 \times Z_2')$ with flat metric

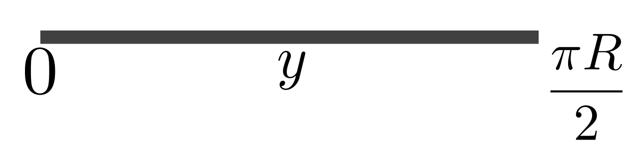


$Z_2 : y \leftrightarrow -y$

$Z_2' : y' \leftrightarrow -y'$

$y' \equiv y - \pi R/2$

All the fields in ED are defined in the fundamental interval:



$\frac{1}{R} \gtrsim M_{GUT}$

$\approx 10^{16}\,\mathrm{GeV}$

with assigned parities (*P*, *P'*) under $Z_2 \times Z_2'$

[Pomarol, Quiros (1998), Arkani-Hamed et al. (2002)]

[Pati Salam (1974), Kawamura, (2001)]

A 5D SO(10) SUSY model: compactification

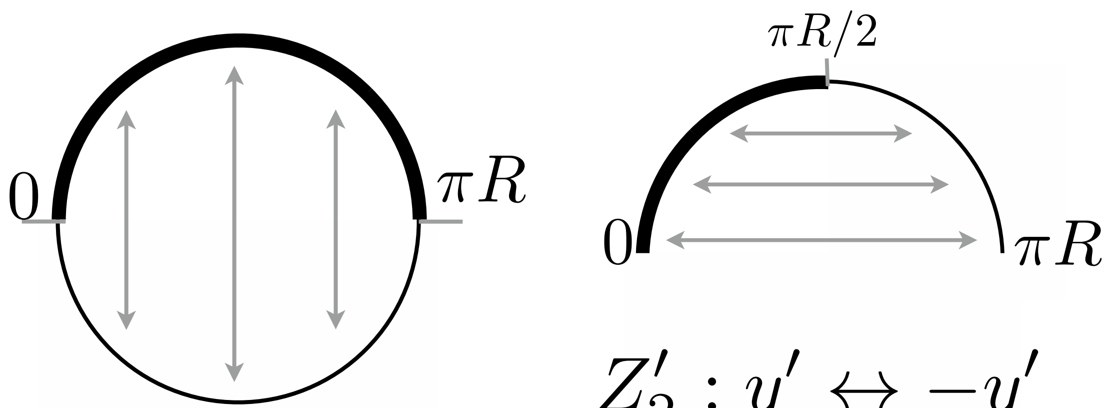
- **Kaluza-Klein expansion:** for each field propagating in the ED

$$H(x_\mu, y) = \sum_n H_n(x_\mu) f_n(y)$$

Profile in the extra dimension

n=0 mode describes the massless particle (MSSM field)

- **Extra dimension compactified on Orbifold:** $S^1/(Z_2 \times Z_2')$ with flat metric

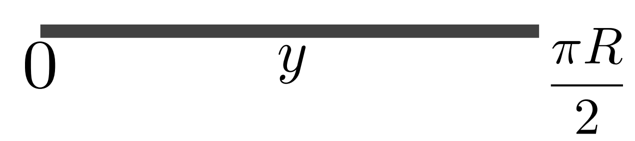


$Z_2 : y \leftrightarrow -y$

$Z_2' : y' \leftrightarrow -y'$

$y' \equiv y - \pi R/2$

All the fields in ED are defined in the fundamental interval:



$\frac{1}{R} \gtrsim M_{GUT}$

$\approx 10^{16}$ GeV

with assigned parities (*P, P'*) under $Z_2 \times Z_2'$

Vanishing of some profiles:
in the bulk or in one of the two branes

[Pomarol, Quiros (1998), Arkani-Hamed et al. (2002)]

[Pati Salam (1974), Kawamura, (2001)]

A 5D SO(10) SUSY model: compactification

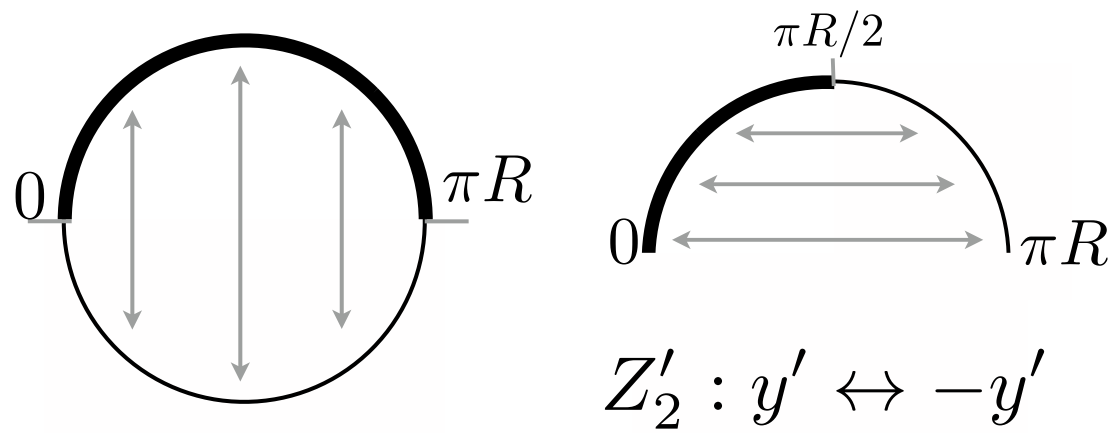
- **Kaluza-Klein expansion:** for each field propagating in the ED

$$H(x_\mu, y) = \sum_n H_n(x_\mu) f_n(y)$$

Profile in the extra dimension

n=0 mode describes the massless particle (MSSM field)

- **Extra dimension compactified on Orbifold:** $S^1/(Z_2 \times Z_2')$ with flat metric

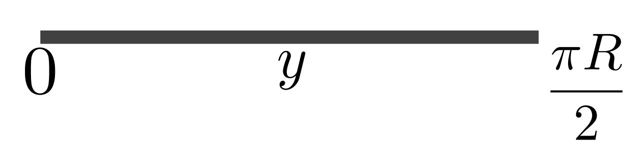


$Z_2 : y \leftrightarrow -y$

$Z_2' : y' \leftrightarrow -y'$

$y' \equiv y - \pi R/2$

All the fields in ED are defined in the fundamental interval:



$\frac{1}{R} \gtrsim M_{GUT}$
$\approx 10^{16}$ GeV

with assigned parities (*P, P'*) under $Z_2 \times Z_2'$

Vanishing of some profiles:
in the bulk or in one of the two branes

SYMMETRY BREAKING !

[Pomarol, Quiros (1998), Arkani-Hamed et al. (2002)]

[Pati Salam (1974), Kawamura, (2001)]

A 5D SO(10) SUSY model: compactification

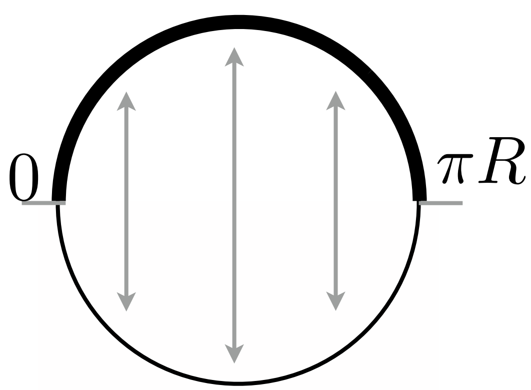
- **Kaluza-Klein expansion:** for each field propagating in the ED

$$H(x_\mu, y) = \sum_n H_n(x_\mu) f_n(y)$$

Profile in the extra dimension

n=0 mode describes the massless particle (MSSM field)

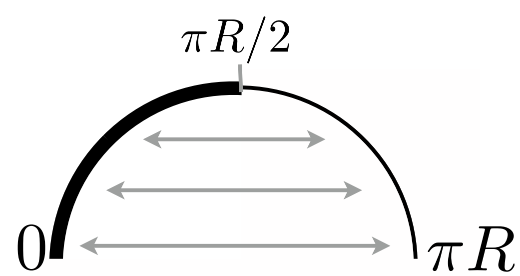
- **Extra dimension compactified on Orbifold:** $S^1/(Z_2 \times Z_2')$ with flat metric



$Z_2 : y \leftrightarrow -y$

Breaks
5D N=1 SUSY
(4D N=2 SUSY)
↓
4D N=1 SUSY

[Pomarol, Quiros (1998), Arkani-Hamed et al. (2002)]

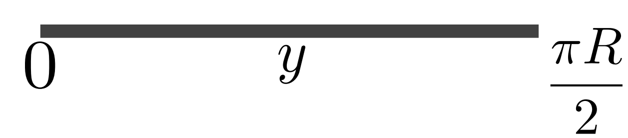


$Z_2' : y' \leftrightarrow -y'$

$y' \equiv y - \pi R/2$

[Pati Salam (1974), Kawamura, (2001)]

All the fields in ED are defined in the fundamental interval:



$\frac{1}{R} \gtrsim M_{GUT} \approx 10^{16}\,\mathrm{GeV}$

with assigned parities *(P, P')* under $Z_2 \times Z_2'$

Vanishing of some profiles:
in the bulk or in one of the two branes

SYMMETRY BREAKING !

A 5D SO(10) SUSY model: compactification

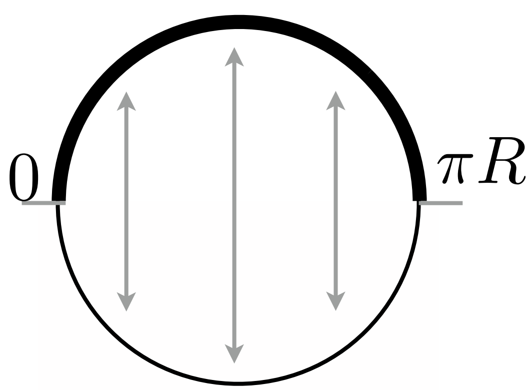
- **Kaluza-Klein expansion:** for each field propagating in the ED

$$H(x_\mu, y) = \sum_n H_n(x_\mu) f_n(y)$$

Profile in the extra dimension

n=0 mode describes the massless particle (MSSM field)

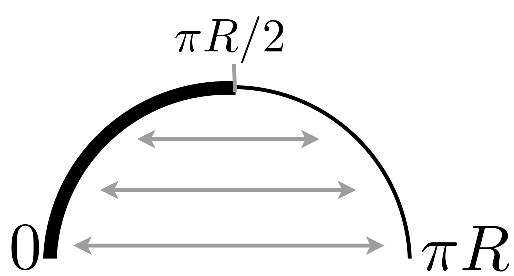
- **Extra dimension compactified on Orbifold:** $S^1/(Z_2 \times Z_2')$ with flat metric



$$Z_2 : y \leftrightarrow -y$$

Breaks
5D N=1 SUSY
(4D N=2 SUSY)
↓
4D N=1 SUSY

[Pomarol, Quiros (1998), Arkani-Hamed et al. (2002)]

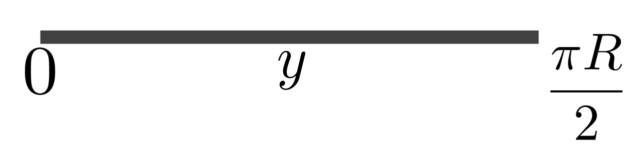


$$Z_2' : y' \leftrightarrow -y'$$
$$y' \equiv y - \pi R/2$$

Breaks
SO(10)
↓
Pati Salam group:
$SU(4) \times SU(2)_L \times SU(2)_R$

[Pati Salam (1974), Kawamura, (2001)]

All the fields in ED are defined in the fundamental interval:



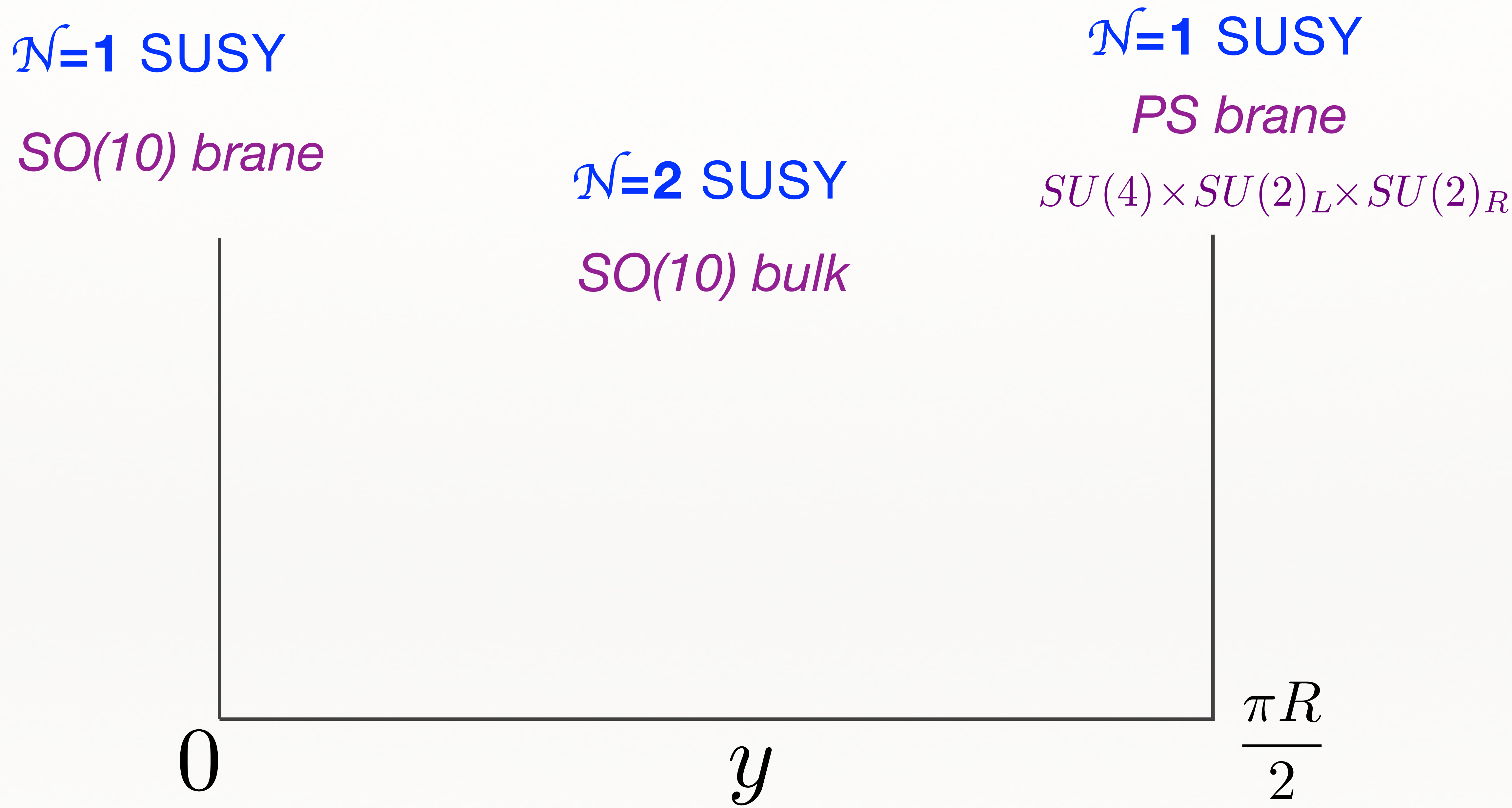
$$\frac{1}{R} \gtrsim M_{GUT} \approx 10^{16}\,\mathrm{GeV}$$

with assigned parities (*P, P'*) under $Z_2 \times Z_2'$

Vanishing of some profiles:
in the bulk or in one of the two branes

SYMMETRY BREAKING !

The model: symmetries



The model: symmetries

$\mathcal{N}$=**1** SUSY

SO(10) brane

$\mathbf{45_V}$

$\mathcal{N}$=**2** SUSY

SO(10) bulk

$\mathbf{45_V}$

$\mathcal{N}$=**1** SUSY

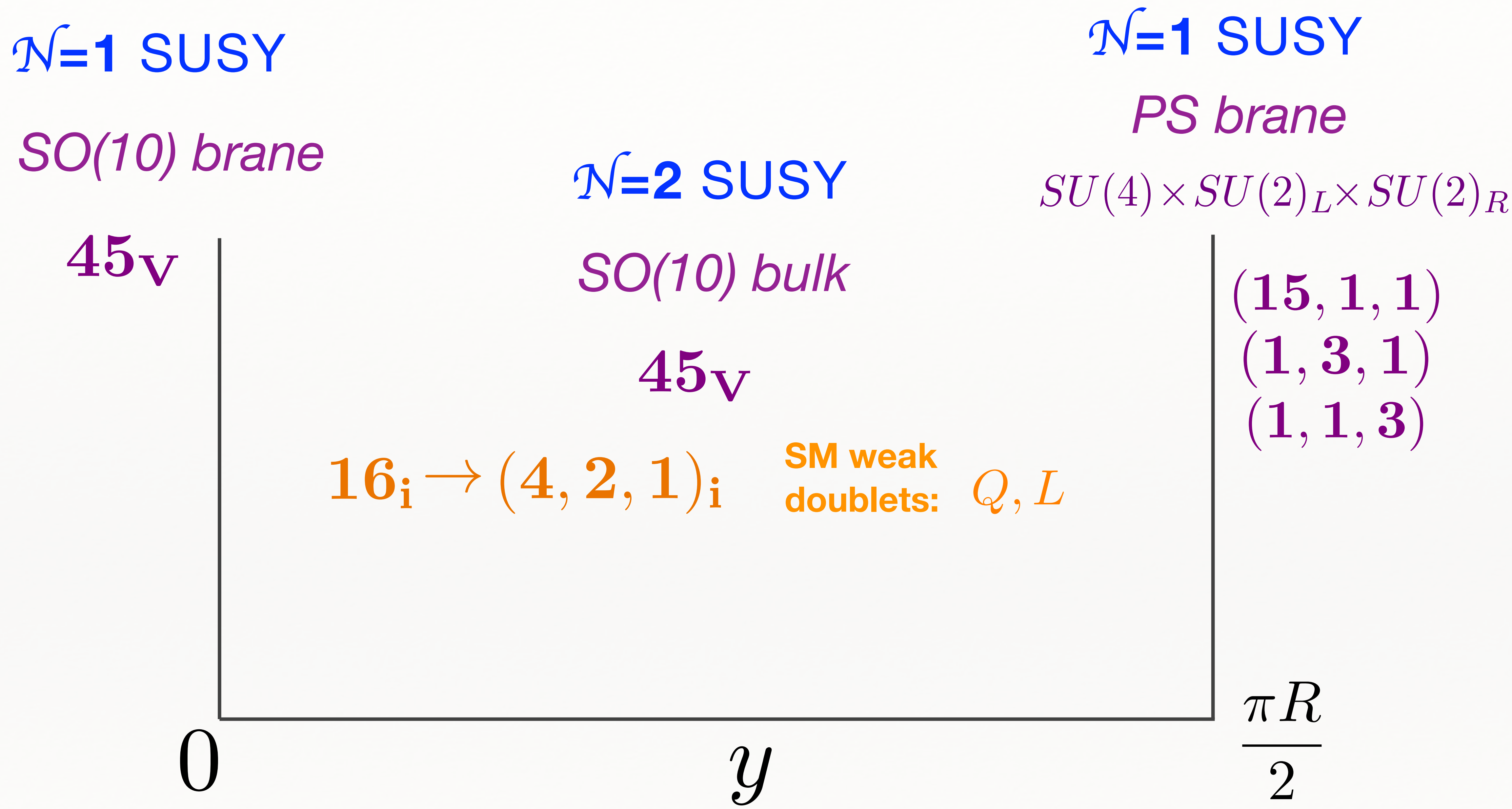
PS brane

$SU(4)\times SU(2)_L\times SU(2)_R$

$(\mathbf{15},\mathbf{1},\mathbf{1})$
$(\mathbf{1},\mathbf{3},\mathbf{1})$
$(\mathbf{1},\mathbf{1},\mathbf{3})$

0 $\qquad y \qquad$ $\dfrac{\pi R}{2}$

The model: symmetries



The model: symmetries

$\mathcal{N}$=1 SUSY

SO(10) brane

$\mathbf{45_V}$

$\mathcal{N}$=2 SUSY

SO(10) bulk

$\mathbf{45_V}$

$\mathbf{16_i} \rightarrow (\mathbf{4}, \mathbf{2}, \mathbf{1})_\mathbf{i}$ **SM weak doublets:** Q, L

$\mathbf{16'_i} \rightarrow (\bar{\mathbf{4}}, \mathbf{1}, \mathbf{2})_\mathbf{i}$ **SM weak singlets + RH neutrinos:** $u^c_L, d^c_L, e^c_L, \nu^c_L$

$\mathcal{N}$=1 SUSY

PS brane

$SU(4)\times SU(2)_L \times SU(2)_R$

$(\mathbf{15}, \mathbf{1}, \mathbf{1})$
$(\mathbf{1}, \mathbf{3}, \mathbf{1})$
$(\mathbf{1}, \mathbf{1}, \mathbf{3})$

0 $\qquad y \qquad$ $\frac{\pi R}{2}$

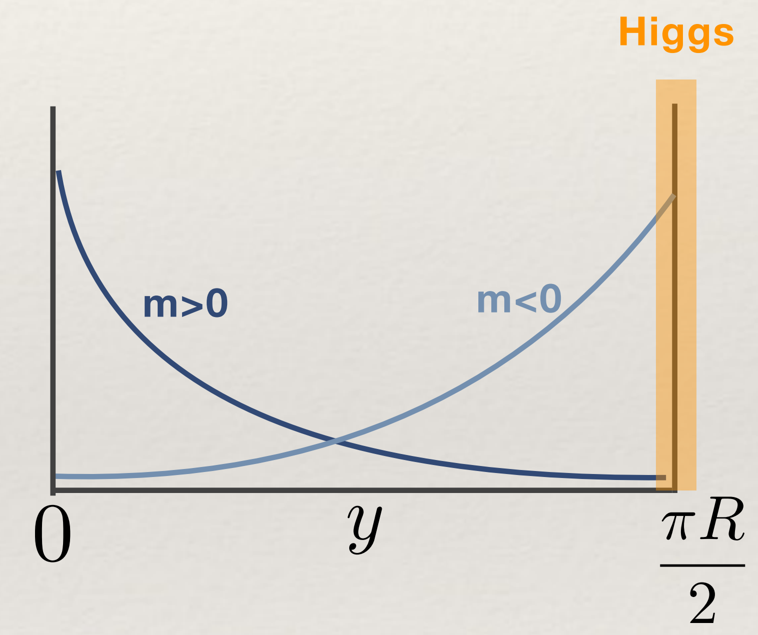
The model: fermions profiles

- **Superpotential in the bulk:**

$$\mathcal{W}_{\text{bulk}} = \mathbf{16}_i^c \left[\hat{m}_i + \partial_y - \sqrt{2} g_5\, \mathbf{45}_\Phi\right] \mathbf{16}_i + \mathbf{16}_i'^c \left[\hat{m}_i' + \partial_y - \sqrt{2} g_5\, \mathbf{45}_\Phi\right] \mathbf{16}_i'$$

- **0-mode profiles:**

$$f_{16_i}(m_i, y) = \sqrt{\frac{2m_i}{1 - e^{-m_i \pi R}}}\, e^{-m_i y}\,; \quad f_{16'_i}(y, m_i')$$



exponentials modulated by **bulk mass** parameters:

- distinguish between doublets and singlets → **still leptons + quark unified**

- bulk masses can be corrected by VEV $\langle \mathbf{45}_\Phi \rangle$

The model: fermions profiles

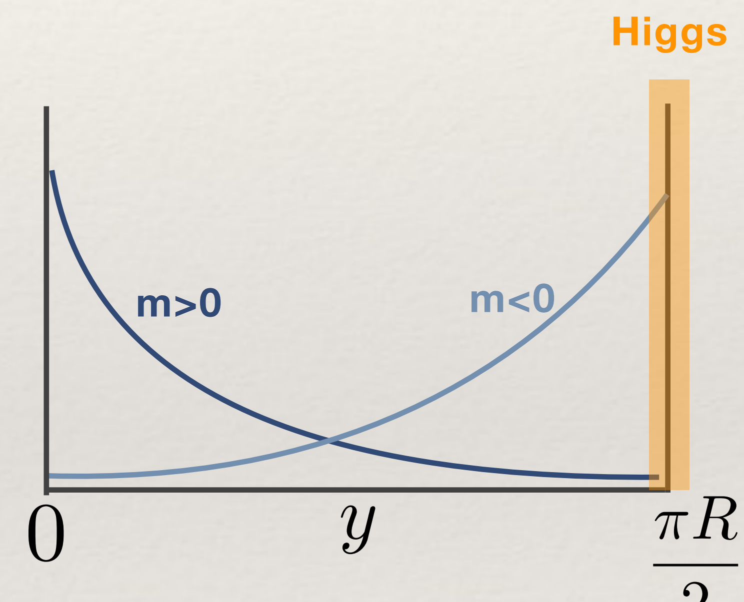
- **Superpotential in the bulk:**

$$\mathcal{W}_{\text{bulk}} = \mathbf{16}_i^c \left[\hat{m}_i + \partial_y - \sqrt{2} g_5\, \mathbf{45}_\Phi\right] \mathbf{16}_i + \mathbf{16}_i'^c \left[\hat{m}_i' + \partial_y - \sqrt{2} g_5\, \mathbf{45}_\Phi\right] \mathbf{16}_i'$$

bulk masses

- **0-mode profiles:**

$$f_{16_i}(m_i, y) = \sqrt{\frac{2m_i}{1 - e^{-m_i \pi R}}}\, e^{-m_i y}\,; \quad f_{16'_i}(y, m_i')$$



exponentials modulated by **bulk mass** parameters:

- distinguish between doublets and singlets → **still leptons + quark unified**

- bulk masses can be corrected by VEV $\langle \mathbf{45}_\Phi \rangle$

The model: fermions profiles

- **Superpotential in the bulk:**

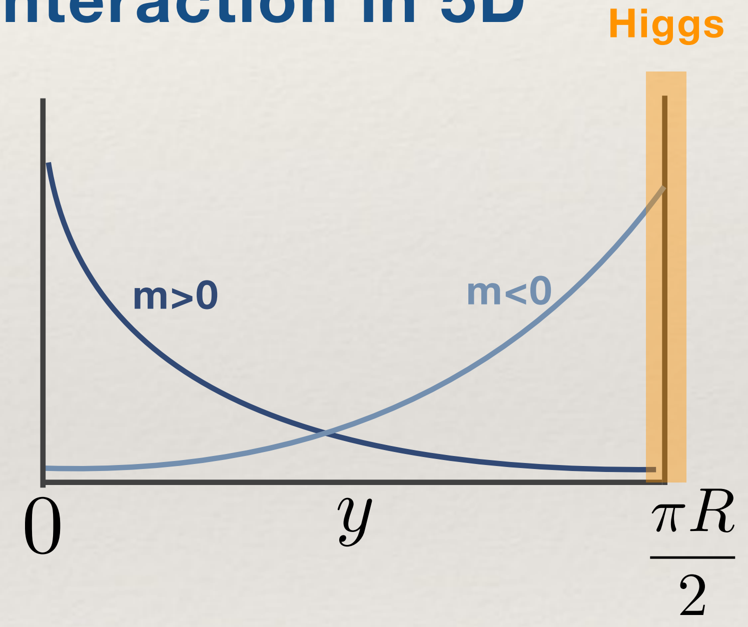
$$\mathcal{W}_{\text{bulk}} = \mathbf{16}_i^c \left[\hat{m}_i + \partial_y - \sqrt{2} g_5 \mathbf{45}_\Phi\right] \mathbf{16}_i + \mathbf{16}_i'^c \left[\hat{m}_i' + \partial_y - \sqrt{2} g_5 \mathbf{45}_\Phi\right] \mathbf{16}_i'$$

bulk masses

from gauge interaction in 5D

- **0-mode profiles:**

$$f_{16_i}(m_i, y) = \sqrt{\frac{2m_i}{1 - e^{-m_i \pi R}}} e^{-m_i y}; \quad f_{16'_i}(y, m_i')$$



exponentials modulated by **bulk mass** parameters:

- distinguish between doublets and singlets → still leptons + quark unified

- bulk masses can be corrected by VEV $\langle \mathbf{45}_\Phi \rangle$

The model: fermions profiles

- **Superpotential in the bulk:**

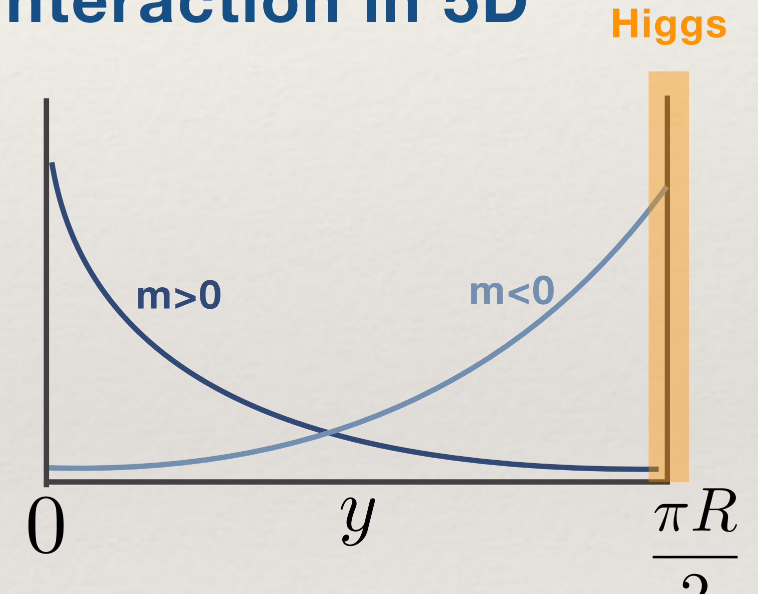
$$\mathcal{W}_{\text{bulk}} = \mathbf{16}_i^c \left[\hat{m}_i + \partial_y - \sqrt{2} g_5 \mathbf{45}_\Phi\right] \mathbf{16}_i + \mathbf{16}_i'^c \left[\hat{m}_i' + \partial_y - \sqrt{2} g_5 \mathbf{45}_\Phi\right] \mathbf{16}_i'$$

bulk masses

from gauge interaction in 5D

- **0-mode profiles:**

$$\underbrace{f_{16_i}}_{Q,L}(m_i, y) = \sqrt{\frac{2m_i}{1 - e^{-m_i \pi R}}}\, e^{-m_i y}\ ; \quad f_{16'_i}(y, m_i')$$



exponentials modulated by **bulk mass** parameters:

- distinguish between doublets and singlets → still leptons + quark unified

- bulk masses can be corrected by VEV $\langle \mathbf{45}_\Phi \rangle$

The model: fermions profiles

- **Superpotential in the bulk:**

$$\mathcal{W}_{\rm bulk} = \mathbf{16}_i^c \left[\hat{m}_i + \partial_y - \sqrt{2} g_5 \mathbf{45}_\Phi \right] \mathbf{16}_i + \mathbf{16}_i'^c \left[\hat{m}_i' + \partial_y - \sqrt{2} g_5 \mathbf{45}_\Phi \right] \mathbf{16}_i'$$

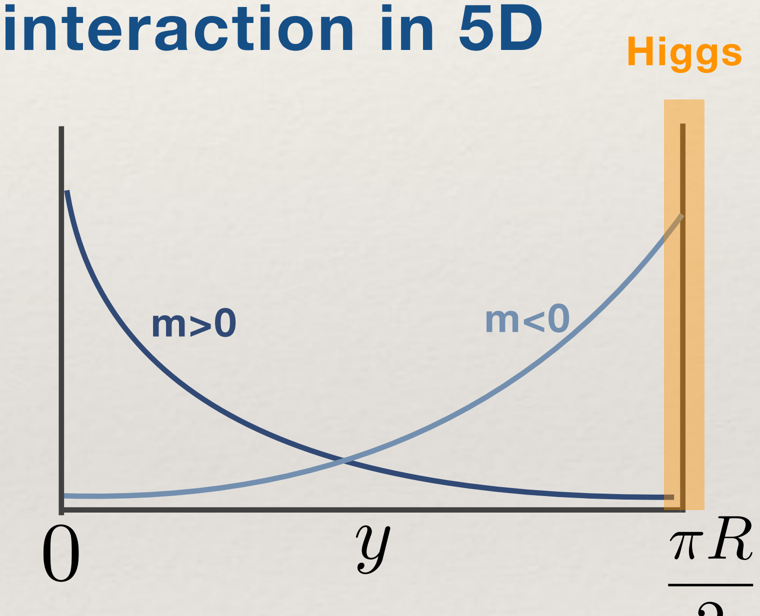
bulk masses

from gauge interaction in 5D

- **0-mode profiles:**

$$f_{16_i}(m_i, y) = \sqrt{\frac{2 m_i}{1 - e^{-m_i \pi R}}} e^{-m_i y} \; ; \; f_{16'_i}(y, m_i')$$

Q, L ; $u_L^c, d_L^c, e_L^c, \nu_L^c$



exponentials modulated by **bulk mass** parameters:

- distinguish between doublets and singlets → **still leptons + quark unified**
- bulk masses can be corrected by VEV $\langle \mathbf{45}_\Phi \rangle$

The model: fermions profiles

- **Superpotential in the bulk:**

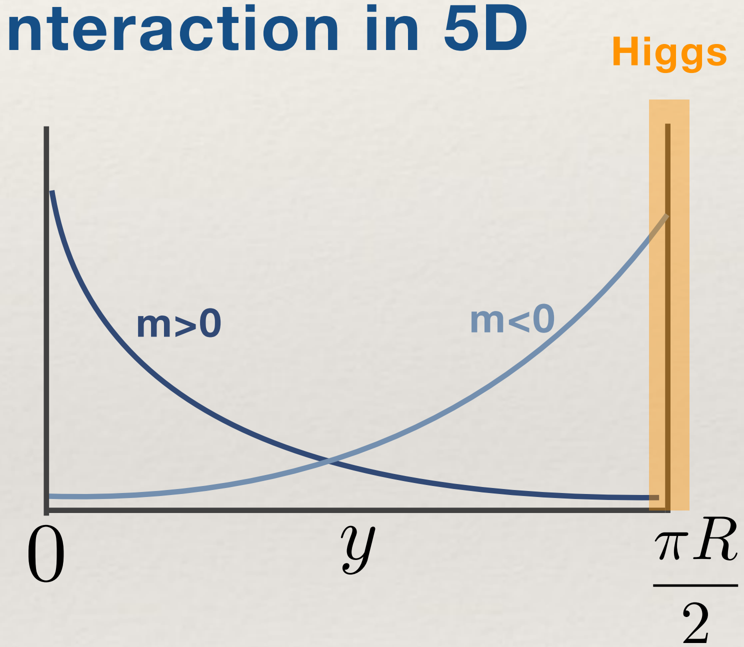
$$\mathcal{W}_{\text{bulk}} = \mathbf{16}_i^c \left[\hat{m}_i + \partial_y - \sqrt{2} g_5 \mathbf{45}_\Phi\right] \mathbf{16}_i + \mathbf{16}_i'^c \left[\hat{m}_i' + \partial_y - \sqrt{2} g_5 \mathbf{45}_\Phi\right] \mathbf{16}_i'$$

bulk masses **from gauge interaction in 5D**

- **0-mode profiles:**

$$f_{16_i}(m_i, y) = \sqrt{\frac{2m_i}{1 - e^{-m_i \pi R}}}\, e^{-m_i y} \; ; \; f_{16'_i}(y, m_i')$$

Q, L $\qquad$ $u_L^c, d_L^c, e_L^c, \nu_L^c$



exponentials modulated by **bulk mass** parameters:

- distinguish between doublets and singlets → **still leptons + quark unified**

- bulk masses can be corrected by VEV $\langle \mathbf{45}_\Phi \rangle$

The model: fermions profiles

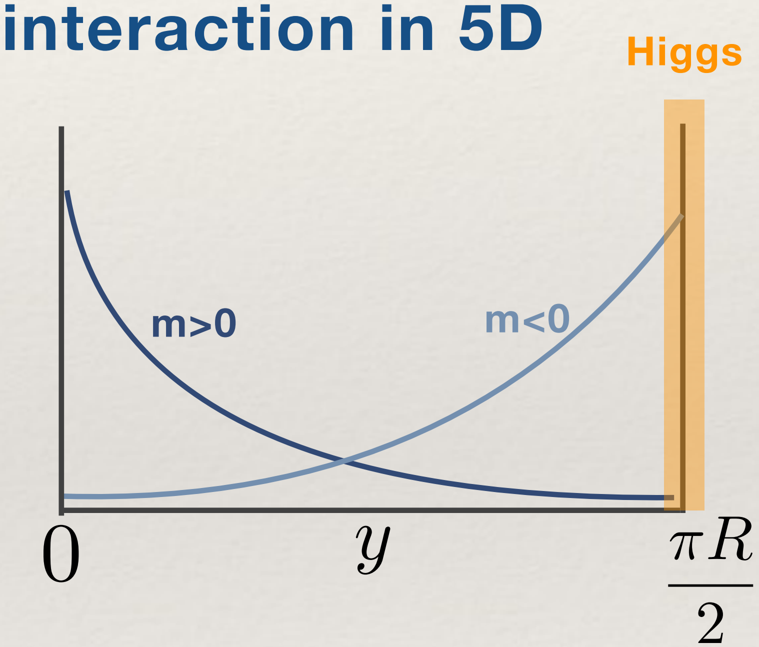
- **Superpotential in the bulk:**

$$\mathcal{W}_{\text{bulk}} = \mathbf{16}_i^c \left[\hat{m}_i + \partial_y - \sqrt{2} g_5 \, \mathbf{45}_\Phi \right] \mathbf{16}_i + \mathbf{16}_i^{\prime c} \left[\hat{m}_i^\prime + \partial_y - \sqrt{2} g_5 \, \mathbf{45}_\Phi \right] \mathbf{16}_i^\prime$$

- **0-mode profiles:**

bulk masses **from gauge interaction in 5D**

$$\underbrace{f_{16_i}}_{Q,\,L}(m_i, y) = \sqrt{\frac{2 m_i}{1 - e^{-m_i \pi R}}} e^{-m_i y} \; ; \; \underbrace{f_{16'_i}}_{u_L^c,\, d_L^c,\, e_L^c,\, \nu_L^c}(y, m_i^\prime)$$



exponentials modulated by **bulk mass** parameters:

- distinguish between doublets and singlets → **still leptons + quark unified**

- bulk masses can be corrected by VEV $\langle \mathbf{45}_\Phi \rangle$

The model: quarks-leptons splitting

- **Splitting from spontaneous symmetry breaking:**

  - in the bulk: $SO(10) \xrightarrow{\langle 45_\Phi \rangle} SU(5) \times U(1)_X$ [Kitano, Li (2004)]

  - decomposition under $SU(5) \times U(1)_X$: $\mathbf{16} = 10_{-1} + \bar{5}_3 + 1_{-5}$

    (Q, u^c, e^c) (d^c, L) (N^c)

  - bulk mass correction $\propto U(1)_X$ charges:

$$m_i \longrightarrow m_i^r = m_i - \sqrt{2} g_5 Q_X^r \langle \mathbf{45}_\Phi \rangle$$

$$f_{16} \longrightarrow \{f_{10},\ f_{\bar{5}},\ f_1\}$$

flavour universal
due to gauge interaction
(SUSY constraint)

  - the same is happening for m_i'

The model: fermions profiles splitting

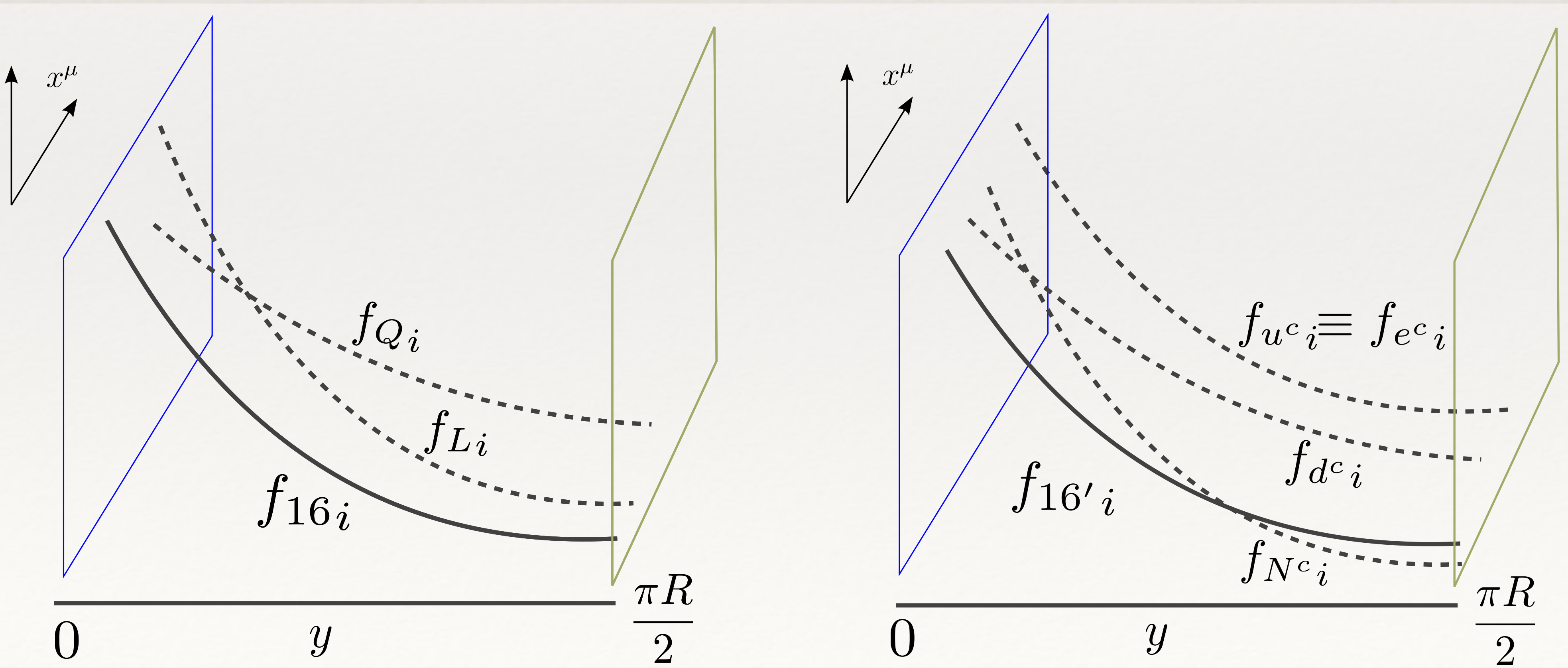
- Combining $SO(10) \xrightarrow{Z_2'}$ PS with $SO(10) \xrightarrow{\langle 45_\Phi \rangle} SU(5) \times U(1)_X$:

$Q_X^{10} = -1$ $\qquad$ $Q_X^{\bar{5}} = 3$

$$a_i^Q = m_i + \sqrt{2} g_5 \langle \mathbf{45}_\Phi \rangle \; ; \qquad a_i^L = m_i - 3\sqrt{2} g_5 \langle \mathbf{45}_\Phi \rangle$$
$$a_i^{u^c} = m_i' + \sqrt{2} g_5 \langle \mathbf{45}_\Phi \rangle \; ; \qquad a_i^{d^c} = m_i' - 3\sqrt{2} g_5 \langle \mathbf{45}_\Phi \rangle$$
$$a_i^{e^c} = m_i' + \sqrt{2} g_5 \langle \mathbf{45}_\Phi \rangle \; ; \qquad a_i^{N^c} = m_i' + 5\sqrt{2} g_5 \langle \mathbf{45}_\Phi \rangle$$

$Q_X^1 = -5$

- Globally 3+3+1=7 parameters create 15 different profiles



The Higgs sector on the brane

- **Yukawa couplings on the PS brane:**
  - lower dimensional representations with respect to SO(10) brane: less number of 4D fields
  - For the Higgs we can select only doublets: no DT splitting problem

$$H, H' \sim (1,2,2)$$

$$\mathcal{W}_{\text{brane}} = \delta\left(y - \frac{\pi R}{2}\right)\frac{1}{\Lambda}\left[Y_{ij}\mathbf{16}_i\mathbf{16}'_j H + Y'_{ij}\mathbf{16}_i\mathbf{16}'_j H' + \frac{1}{2}Y^R_{ij}\mathbf{16}'_i\mathbf{16}'_j\frac{\overline{\Sigma\Sigma}}{\Lambda} + \ldots\right] + \ldots$$

  - Majorana mass term: $\Sigma, \overline{\Sigma} \sim (\overline{4},1,2),(4,1,2)$

- **Superpotential on the branes:**

$$\cdots + \delta\left(y - \frac{\pi R}{2}\right) w_\pi(H, H', \Sigma, \overline{\Sigma}, T) \;\; + \delta(y)\, w_0(\mathbf{16}_H, \overline{\mathbf{16}}_H)$$

The Higgs sector on the brane

- **Yukawa couplings on the PS brane:**
  - lower dimensional representations with respect to SO(10) brane: less number of 4D fields
  - For the Higgs we can select only doublets: no DT splitting problem

$$\boxed{H, H' \sim (1,2,2)}$$

$$\mathcal{W}_{\mathrm{brane}} = \delta\left(y - \frac{\pi R}{2}\right)\frac{1}{\Lambda}\left[Y_{ij}\mathbf{16}_i\mathbf{16}'_j \boxed{H} + Y'_{ij}\mathbf{16}_i\mathbf{16}'_j \boxed{H'} + \frac{1}{2}Y^R_{ij}\mathbf{16}'_i\mathbf{16}'_j\frac{\overline{\Sigma\Sigma}}{\Lambda} + \ldots\right] + \ldots$$

  - Majorana mass term: $\Sigma, \overline{\Sigma} \sim (\overline{4},1,2),(4,1,2)$

- **Superpotential on the branes:**

$$\cdots + \delta\left(y - \frac{\pi R}{2}\right) w_\pi(H, H', \Sigma, \overline{\Sigma}, T) \;\; + \delta(y)\, w_0(\mathbf{16}_H, \overline{\mathbf{16}}_H)$$

The Higgs sector on the brane

- **Yukawa couplings on the PS brane:**
  - lower dimensional representations with respect to SO(10) brane: less number of 4D fields
  - For the Higgs we can select only doublets: no DT splitting problem

$$H, H' \sim (1,2,2)$$

$$\mathcal{W}_{\text{brane}} = \delta\left(y - \frac{\pi R}{2}\right)\frac{1}{\Lambda}\left[Y_{ij}\mathbf{16}_i\mathbf{16}'_j H + Y'_{ij}\mathbf{16}_i\mathbf{16}'_j H' + \frac{1}{2}Y^R_{ij}\mathbf{16}'_i\mathbf{16}'_j\frac{\overline{\Sigma\Sigma}}{\Lambda} + \ldots\right] + \ldots$$

  - Majorana mass term: $\Sigma, \overline{\Sigma} \sim (\overline{4},1,2),(4,1,2)$

- **Superpotential on the branes:**

$$\ldots + \delta\left(y - \frac{\pi R}{2}\right) w_\pi(H, H', \Sigma, \overline{\Sigma}, T) \; + \delta(y)\, w_0(\mathbf{16}_H, \overline{\mathbf{16}}_H)$$

The Higgs sector on the brane

- **Yukawa couplings on the PS brane:**
  - lower dimensional representations with respect to SO(10) brane: less number of 4D fields
  - For the Higgs we can select only doublets: no DT splitting problem

$$H, H' \sim (1,2,2)$$

$$\mathcal{W}_{\text{brane}} = \delta\left(y - \frac{\pi R}{2}\right)\frac{1}{\Lambda}\left[Y_{ij}\mathbf{16}_i\mathbf{16}'_j H + Y'_{ij}\mathbf{16}_i\mathbf{16}'_j H' + \frac{1}{2}Y^R_{ij}\mathbf{16}'_i\mathbf{16}'_j\frac{\overline{\Sigma}\Sigma}{\Lambda} + \ldots\right] + \ldots$$

  - Majorana mass term: $\Sigma, \overline{\Sigma} \sim (\overline{4},1,2),(4,1,2)$

- **Superpotential on the branes:**

$$\cdots + \delta\left(y - \frac{\pi R}{2}\right) w_\pi(H, H', \Sigma, \overline{\Sigma}, T) + \delta(y)\, w_0(\mathbf{16}_H, \overline{\mathbf{16}}_H)$$

Needed to generate non trivial CKM mixing $T \sim (1,1,3)$

The Higgs sector on the brane

- **Yukawa couplings on the PS brane:**
  - lower dimensional representations with respect to SO(10) brane: less number of 4D fields
  - For the Higgs we can select only doublets: no DT splitting problem

$$H, H' \sim (1,2,2)$$

$$\mathcal{W}_{\text{brane}} = \delta\left(y - \frac{\pi R}{2}\right)\frac{1}{\Lambda}\left[Y_{ij}\mathbf{16}_i\mathbf{16}'_j H + Y'_{ij}\mathbf{16}_i\mathbf{16}'_j H' + \frac{1}{2}Y^R_{ij}\mathbf{16}'_i\mathbf{16}'_j\frac{\overline{\Sigma}\Sigma}{\Lambda} + \ldots\right] + \ldots$$

  - Majorana mass term: $\Sigma, \overline{\Sigma} \sim (\overline{4},1,2),(4,1,2)$

- **Superpotential on the branes:**

$$\cdots + \delta\left(y - \frac{\pi R}{2}\right) w_\pi(H, H', \Sigma, \overline{\Sigma}, T) + \delta(y)\, w_0(\mathbf{16}_H, \overline{\mathbf{16}}_H)$$

Needed to generate non trivial CKM mixing

$T \sim (1,1,3)$

Needed to preserve SUSY on the branes

Effective Yukawas

- **Effective Yukawa couplings**

$$F_r \equiv \begin{pmatrix} f_{r1}(\frac{\pi R}{2}) & 0 & 0 \\ 0 & f_{r2}(\frac{\pi R}{2}) & 0 \\ 0 & 0 & f_{r3}(\frac{\pi R}{2}) \end{pmatrix}$$

$$\mathcal{Y}_u = F_Q \; Y_u \; F_{u^c}$$
$$\mathcal{Y}_d = F_Q \; Y_d \; F_{d^c}$$
$$\mathcal{Y}_e = F_L \; Y_d \; F_{e^c}$$
$$\mathcal{Y}_\nu = F_L \; Y_u \; F_{N^c}$$

$$M_R \equiv \frac{\langle\overline{\Sigma}\rangle^2}{\Lambda} F_{N^c} Y_R F_{N^c}$$

$$M_\nu \equiv -\frac{\Lambda v^2 \sin^2\beta}{\langle\overline{\Sigma}\rangle^2} F_L \; (Y_u Y_R^{-1} Y_u^T) \; F_L$$

- **Parameters counting:**

Profiles		Higgs Mixing		Yukawas
μ_1, μ_2, μ_3, k_X μ'_1, μ'_2, μ'_3	+	θ_u, θ_d	+	Y, Y', Y_R $0.5 \le \lvert Y_{ij}\rvert \le 1.5$
7 free bulk mass parameters		2 free angles		44 parameters constrained $\approx \mathcal{O}(1)$

→ fitting **17 observables** (masses and mixing angles of quarks and leptons)

Numerical fit

- **Agreement is not so trivial:** only large *tanβ* allowed (unification of the third generation)

[tanβ=50]

from global χ^2 minimization (including Yukawas)

Observable	Normal ordering Fitted value	Normal ordering Pull	Inverted ordering Fitted value	Inverted ordering Pull
y_t	0.51	0	0.52	0.33
y_b	0.37	0	0.38	0.50
y_τ	0.51	0	0.51	0
m_u/m_c	0.0027	0	0.0028	0.17
m_d/m_s	0.051	0	0.052	0.14
m_e/m_μ	0.0048	0	0.0048	0
m_c/m_t	0.0023	0	0.0023	0
m_s/m_b	0.016	0	0.017	0.50
m_μ/m_τ	0.050	0	0.050	0
$\lvert V_{us}\rvert$	0.227	0	0.227	0
$\lvert V_{cb}\rvert$	0.037	0	0.037	0
$\lvert V_{ub}\rvert$	0.0033	0	0.0030	-0.50
J_{CP}	0.000023	0	0.000023	0
Δ_S/Δ_A	0.0305	0	0.0305	0
$\sin^2\theta_{12}$	0.304	0	0.304	0
$\sin^2\theta_{23}$	0.452	0	0.442	-0.20
$\sin^2\theta_{13}$	0.0218	0	0.0218	-0.10
$\chi^2_{\min}$		≈ 0		≈ 0.96

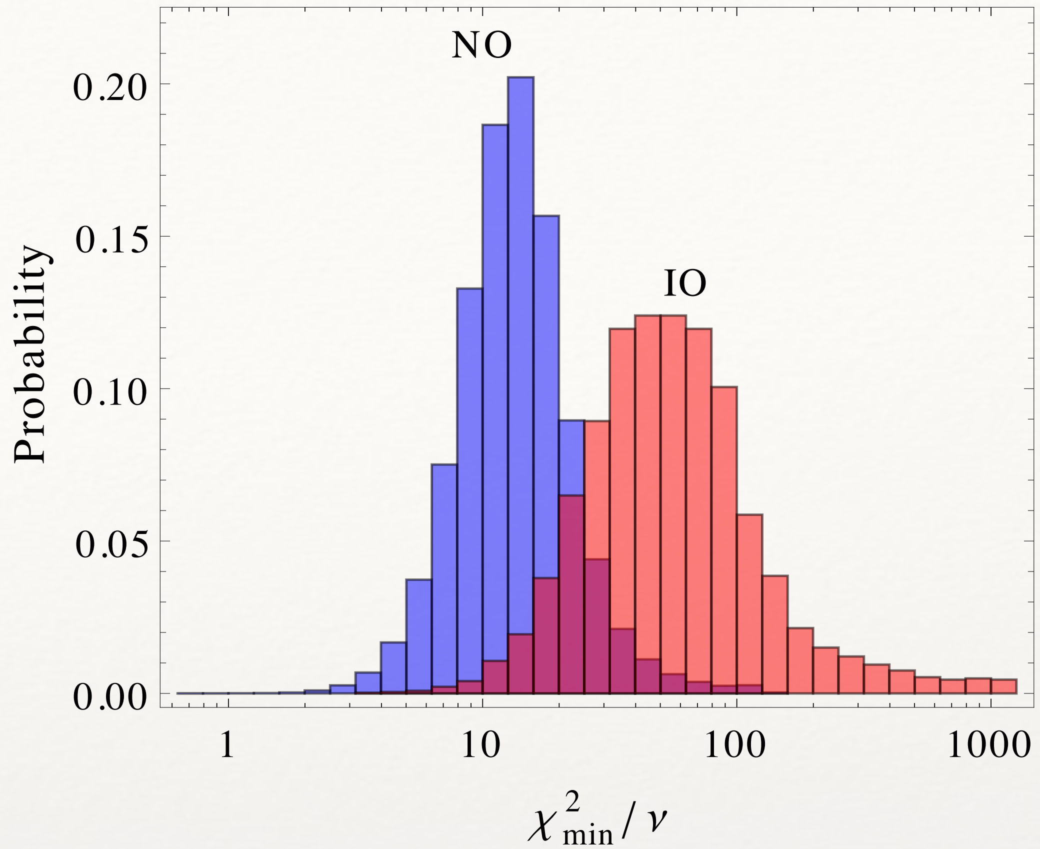
Naturalness test

Random $\mathcal{O}(1)$ Yukawas

- Uniform variation of the parameters:

$$|Y_{ij}| \in [0.5,\ 1.5]$$
$$arg(Y_{ij}) \in [0,\ 2\pi]$$

- Fitting 17 observables with 9 free parameters (8 d.o.f)



Naturalness test

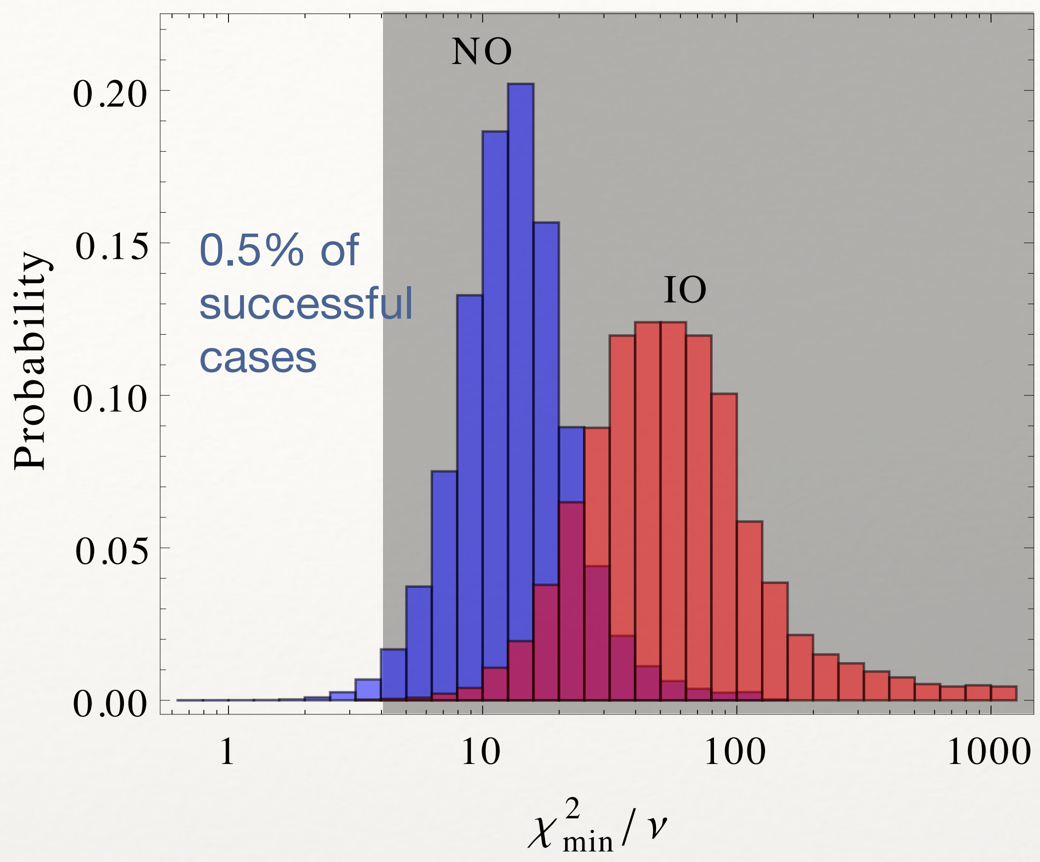
$$\text{Random } \mathcal{O}(1) \text{ Yukawas}$$

- Uniform variation of the parameters:

$$|Y_{ij}| \in [0.5,\ 1.5]$$

$$arg(Y_{ij}) \in [0,\ 2\pi]$$

- Fitting 17 observables with 9 free parameters (8 d.o.f)



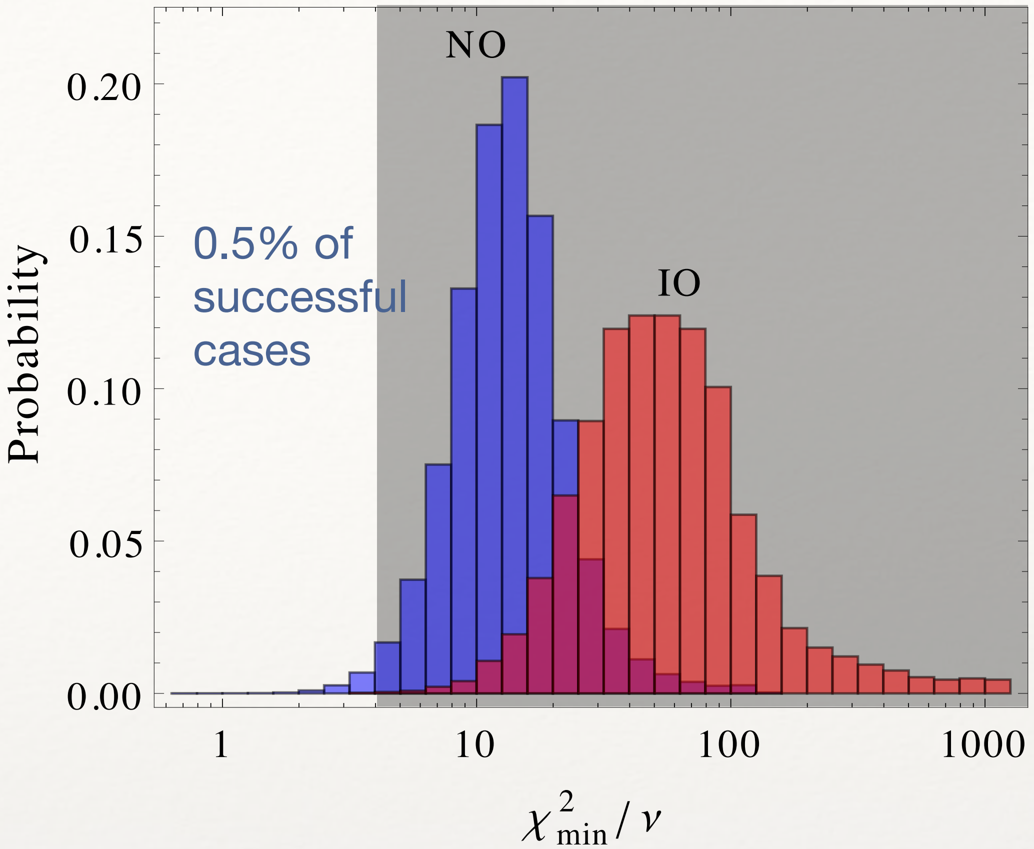
Naturalness test

Random $\mathcal{O}(1)$ Yukawas

- Uniform variation of the parameters:

$$|Y_{ij}| \in [0.5,\ 1.5]$$
$$arg(Y_{ij}) \in [0,\ 2\pi]$$

- Fitting 17 observables
  with 9 free parameters (8 d.o.f)



- **If the Higgs sector was on SO(10) brane…**

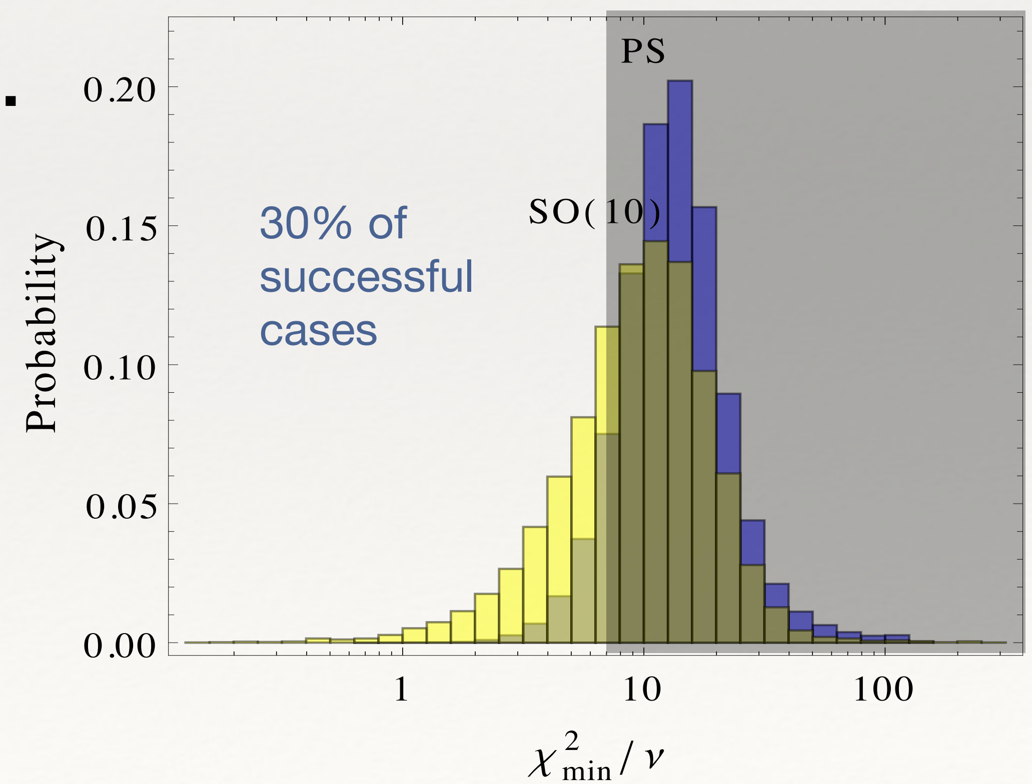
  - Minimal Higgs content:

$$\underset{\text{light}}{\mathbf{10}_H, \mathbf{120}_H} \qquad \underset{\text{heavy}}{\overline{\mathbf{126}}_H}$$

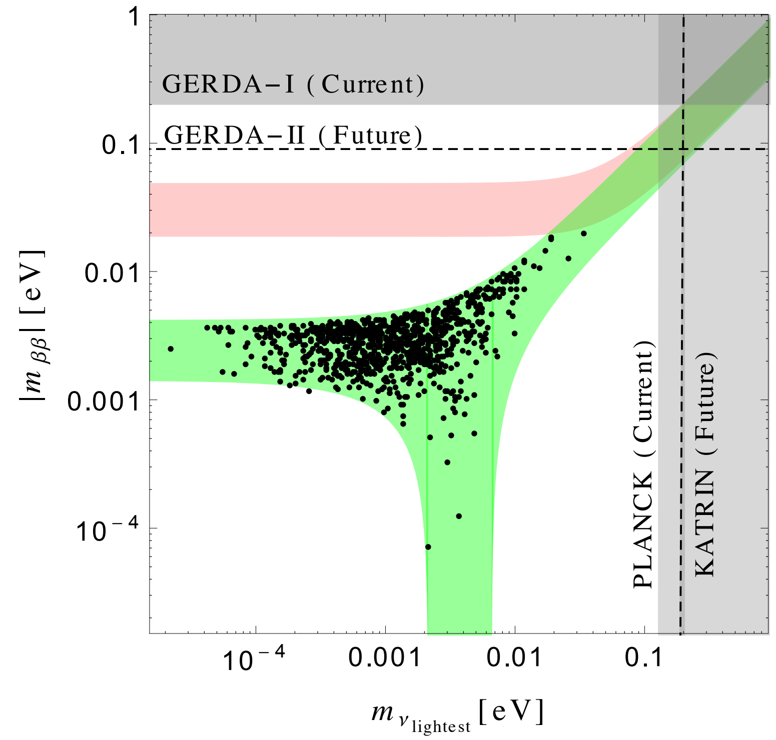
  - 8 Higgs mixing parameters

[Feruglio, Patel, DV (2014)]

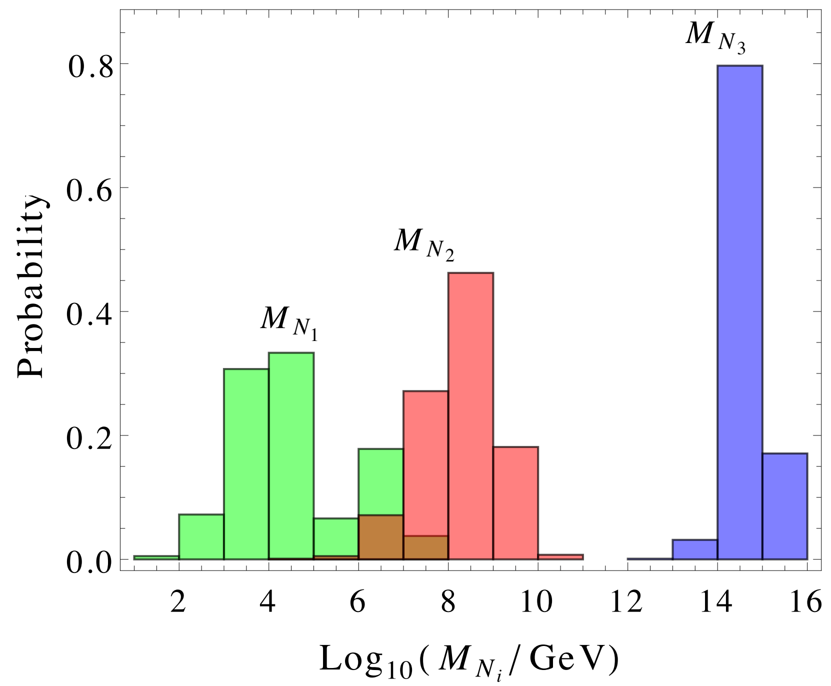
- Fitting 17 observables
  with 15 free parameters (2 d.o.f)



- Effective Majorana neutrino mass and lightest neutrino mass:

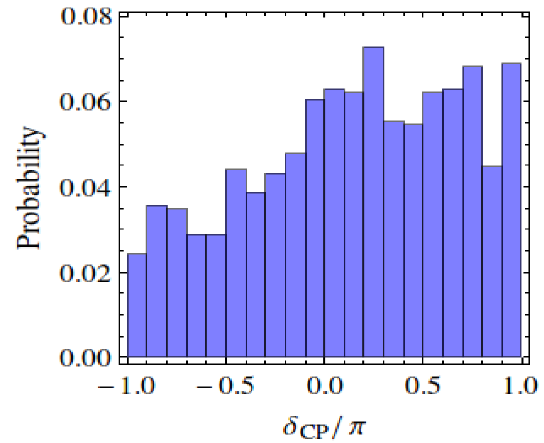


- RH neutrinos mass spectrum: very hierarchical



- Predictions result **quite stable** with respect to the **Higgs dynamics on the branes**, they depend almost entirely on the mechanism of lepton-quarks distinction.

- δ_{CP} and Majorana phases: no preferred value



Conclusions

- O(1) *Anarchical* Yukawa matrices for both quarks and leptons can be nicely reconciled with the observed fermion masses and mixing angles in the framework of extra-dimension, where the hierarchies are created by different localisation of the fermions;
- This scenario can be combined with the unification of one fermion generation implied by the SO(10) GUT, exploiting a dynamical mechanism for splitting the profiles of quarks and leptons.
- Tendency to unify the third generations makes the model compatible only with large $\tan\beta$.
- Both NO and IO are allowed, but NO is more natural with respect to the random variation of Yukawas.
- Different models can be realised, changing the dynamics on the branes, but the predictions depend almost entirely on the mechanism of lepton-quarks distinction. More free parameters on the branes improve the success rate,
- Drawbacks: currently no experimental test can confirm the model.

Conclusions

- O(1) *Anarchical* Yukawa matrices for both quarks and leptons can be nicely reconciled with the observed fermion masses and mixing angles in the framework of extra-dimension, where the hierarchies are created by different localisation of the fermions;
- This scenario can be combined with the unification of one fermion generation implied by the SO(10) GUT, exploiting a dynamical mechanism for splitting the profiles of quarks and leptons.
- Tendency to unify the third generations makes the model compatible only with large *tanβ.*
- Both NO and IO are allowed, but NO is more natural with respect to the random variation of Yukawas.
- Different models can be realised, changing the dynamics on the branes, but the predictions depend almost entirely on the mechanism of lepton-quarks distinction. More free parameters on the branes improve the success rate,
- Drawbacks: currently no experimental test can confirm the model.

Thank you!

Backup slides

The whole action in abelian theory

$$S_5 = \int d^5x [\frac{1}{g^2} \int d^4\theta \left(\partial_5 V - \frac{1}{\sqrt{2}} (\Phi + \bar{\Phi}) \right)^2$$

$$+ \frac{1}{4g^2} \int (d^2\theta \; W^\alpha W_\alpha + \text{h.c.})$$

$$+ \int d^4\theta \; \left(\bar{H} e^{2qQV} H + \bar{H}_c e^{-2gQV} H_c \right)$$

$$+ \left(\int d^2\theta \; H_c \left(m + \partial_5 - \sqrt{2} g Q \Phi \right) H + \text{h.c.} \right)$$

Bulk fields content:

Z_2 Z_2'

	5D N=1	4D N=1	4D N=1 in PS		(P, P')	
Gauge fields	SO(10) Adjoint $\mathbf{45}_{\mathcal{V}}$ Vector multiplet	$\mathbf{45}_V$ Vector multiplet	$(15,1,1)+(1,3,1)+(1,1,3)$		$(+,+)$	Imposed
			$(6,2,2)$	PS Adjoint	$(+,-)$	
		$\mathbf{45}_\Phi$ Chiral multiplet	$(15,1,1)+(1,3,1)+(1,1,3)$		$(-,-)$	
			$(6,2,2)$		$(-,+)$	
Matter fields	$\mathbf{16}_{\mathcal{H}}$ Hypermultiplet	$\mathbf{16}$ Chiral multiplet	$(4,2,1)$	SM weak doublets:	$(+,+)$	Consequence or invariance
			$(\bar{4},1,2)$		$(+,-)$	
		$\mathbf{16}^c$ Chiral multiplet	$(4,1,2)$	Q, L	$(-,+)$	
			$(\bar{4},2,1)$		$(-,-)$	
	$\mathbf{16}'_{\mathcal{H}}$ Hypermultiplet	$\mathbf{16}'$ Chiral multiplet	$(4,2,1)$	SM weak singlets + RH neutrinos:	$(+,-)$	
			$(\bar{4},1,2)$		$(+,+)$	
		$\mathbf{16}'^c$ Chiral multiplet	$(4,1,2)$	$u^c_L, d^c_L, e^c_L, \nu^c_L$	$(-,-)$	
			$(\bar{4},2,1)$		$(-,+)$	

Higgs mass splitting and mixing angles

$$w_\pi = \frac{M_H}{2} H^2 + \frac{M_{H'}}{2} H'^2 + mHH' + \lambda THH' + T(\lambda_H H^2 + \lambda_{H'} H'^2) + \ldots$$

$$\begin{pmatrix} H_u & H'_u \end{pmatrix} \mathcal{M} \begin{pmatrix} H_d \\ H'_d \end{pmatrix} , \quad \text{with } \mathcal{M} = \begin{pmatrix} M_H & m - \lambda\langle T\rangle \\ m + \lambda\langle T\rangle & M_{H'} \end{pmatrix} .$$

$$h_{u,d} = \cos\theta_{u,d} H_{u,d} + \sin\theta_{u,d} H'_{u,d}$$

$$\theta_{u,d} = \frac{1}{2}\tan^{-1}\left(\frac{2M_{H'}(m \mp \lambda\langle T\rangle)}{M_{H'}^2 - (m \mp \lambda\langle T\rangle)^2}\right)$$

Backup slides

- **Profiles parameters distributions (NO)**