LEPTOGENESIS IN LOW SCALE SEESAW MODELS

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In collaboration with:

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Non-zero neutrino masses

Baryon asymmetry

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Bariogenesis via leptogenesis

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Standard Leptogenesis scenario

Out of equilibrium decay of heavy states associated to neutrino masses (typically require large scale hard to test)

Fukugita, Yanagida, 1986;
...many works

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Standard Leptogenesis scenario

Out of equilibrium decay of heavy states associated to neutrino masses (typically require large scale hard to test)

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...many works

Leptogenesis via neutrino oscillations
Out of equilibrium in production of sterile
neutrinos (natural at low-scale: testable?)
Akhmedov, Rubakov, Smirnov, 1998;
Asaka, Shaposhnikov, 2005; ...

Minimal extension to SM- adding $N \geq 2$ right handed neutrinos

$$\begin{aligned} & \text{Minimal extension to SM- adding } N = \underset{3}{3} & \text{right handed neutrinos} \\ & \mathcal{L} = \mathcal{L}_{SM} - \sum_{\alpha,i} \bar{L}^{\alpha} Y^{\alpha i} \tilde{\Phi} \nu_R^i - \sum_{i,j=1}^3 \frac{1}{2} \bar{\nu}_R^{ic} M_N^{ij} \nu_R^j + h.c. \end{aligned}$$

Minimal extension to SM- adding $\,N=3\,\,$ right handed neutrinos

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In Majorana mass basis

$$Y \equiv V^{\dagger} Diag(y_1, y_2, y_3) W$$

3x3 Majorana mass matrix

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6 CP phases, 2 of them Majorana phases

3x3 Majorana mass matrix

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3. C and CP violation processes

CP- odd phases (W, V matrices) + CP- even phases (oscillations)

CP asymmetries generated in the different flavours with

$$oldsymbol{\Sigma}_{
m active} oldsymbol{\Delta} oldsymbol{
m L}_{
m active} + oldsymbol{\Sigma}_{
m sterile} oldsymbol{\Delta} oldsymbol{N}_{
m sterile} = oldsymbol{0}$$

PREVIOUS WORK

Akhmedov-Rubakov-Smirnov (ARS)

- estimated the asymmetry only in the sterile sector (N=3 needed)
- concluded that the right asymmetry could be generated without degeneracies

• Shaposhnikov, Asaka and collaborators (u MSM):

- included the transfer to the leptons
- reduced to N=2 (different CP phases than ARS)
- concluded that degeneracies were necessary

Drewes et al; and Shuve et al

 N=3 degeneracies can be lifted (proved for some points of phase space)

OUR GOAL

- Explore systematically the N=3 case (N=2 is a subclass):
 - identify the CP invariants that are relevant
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For all this having precise analytical predictions is a must!

SETUP

- Mass range 0.1-100 GeV (decay before BBN) talk by J. Lopez-Pavon
- \bullet The Majorana nature is irrelevant since $\,M/T\ll 1\,$
- The sterile neutrino production out of equilibrium
- ullet The yukawa couplings are small and $\,y_3\ll y_1,y_2\,$
- Other particles in kinetic equilibrium $ho_x = e^{\mu_x/T}
 ho_{eq}$
- Include only chemical potential of the lepton doublet
- Lepton asymmetry $\stackrel{\rm sphalerons}{\longleftarrow}$ Baryons $T=T_{EW}$

CP INVARIANTS

Asymmetry – CP odd

Proportional to weak basis rephasing invariants:

CP INVARIANTS

Asymmetry – CP odd ————

Proportional to weak basis rephasing invariants:

$$\begin{split} J_W &= -\mathrm{Im}[\mathbf{W}_{23}^{\star}\mathbf{W}_{32}^{\star}\mathbf{W}_{33}\mathbf{W}_{22}] \simeq \theta_{12}\theta_{13}\theta_{23}\sin\delta \quad \text{Relevant for ASR} \\ I_1^{(2)} &= -\mathrm{Im}[\mathbf{W}_{22}\mathbf{V}_{21}^{\star}\mathbf{V}_{11}\mathbf{W}_{12}^{\star}] \simeq \theta_{12}\bar{\theta}_{12}\sin\phi_1 \\ I_1^{(3)} &= -\mathrm{Im}[W_{11}V_{13}^{*}V_{23}W_{21}^{*}] \simeq \theta_{12}\bar{\theta}_{13}\bar{\theta}_{23}\sin(\bar{\delta}+\phi_1) \\ I_2^{(3)} &= \mathrm{Im}[\mathbf{W}_{23}\mathbf{V}_{22}^{\star}\mathbf{V}_{12}\mathbf{W}_{13}^{\star}] \simeq \bar{\theta}_{12}\theta_{13}\theta_{23}\sin(\delta-\phi_1) \end{split}$$
 Relevant for ASR

CP INVARIANTS

A generic expectation for the CP-asymmetry relevant for leptogenesis is

$$\Delta_{CP} = \sum_{\alpha,k} |Y_{\alpha k}|^2 \sum_{i,j} \operatorname{Im}[Y_{\alpha i} Y_{\alpha j}^* (Y^{\dagger} Y)_{ij}] f(M_i, M_j)$$

• In the limit of vanishing y_3

$$\Delta_{CP} = y_1^2 y_2^2 (y_2^2 - y_1^2) \sum_{i,j} \operatorname{Im}[W_{1i}^* W_{1j} W_{2j}^* W_{2i}] f(M_i, M_j)$$

$$+ y_1 y_2 (y_2^2 - y_1^2) \left\{ \left[(y_1^2 - y_2^2) I_1^{(2)} + y_2^2 I_1^{(3)} \right] \left[f(M_1, M_2) - f(M_2, M_1) \right] - I_2^{(3)} [g(M_3) - g(M_2)] \right\}$$

$$\begin{split} \dot{\rho}_{+} &= -i[H_{re}, \rho_{+}] + [H_{im}, \rho_{-}] - \frac{\gamma_{N}^{a} + \gamma_{N}^{b}}{2} \{Y^{\dagger}Y, \rho_{+} - \rho_{\mathrm{FD}}\} \\ &+ i\gamma_{N}^{b} \mathrm{Im}[Y^{\dagger}\mu Y] \rho_{\mathrm{FD}} + \mathrm{i} \frac{\gamma_{N}^{a}}{2} \{ \mathrm{Im}[Y^{\dagger}\mu Y], \rho_{+}\}, \\ \dot{\rho}_{-} &= -i[H_{re}, \rho_{-}] + [H_{im}, \rho_{+}] - \frac{\gamma_{N}^{a} + \gamma_{N}^{b}}{2} \{Y^{\dagger}Y, \rho_{-}\} \\ &+ \gamma_{N}^{b} \mathrm{Re}[Y^{\dagger}\mu Y] \rho_{\mathrm{FD}} + \frac{\gamma_{N}^{a}}{2} \{ \mathrm{Re}[Y^{\dagger}\mu Y], \rho_{-}\}, \\ \dot{\mu}_{\alpha} &= -\mu_{\alpha} (\gamma_{\nu}^{b} \mathrm{Tr}[YY^{\dagger}I_{\alpha}] + \frac{\gamma_{\nu}^{a}}{\rho_{\mathrm{FD}}} \mathrm{Tr}[\mathrm{Re}[Y^{\dagger}I_{\alpha}Y], \rho_{+}]) \\ &+ \frac{\gamma_{\nu}^{a} + \gamma_{\nu}^{b}}{\rho_{\mathrm{FD}}} \mathrm{Tr}[\mathrm{Re}[YI_{\alpha}Y]\rho_{-} + \mathrm{i} \mathrm{Im}[Y^{\dagger}I_{\alpha}Y]\rho_{+}] \end{split}$$

$$\dot{\rho}_{+} = -i[H_{re}, \\
+i\gamma_{N}^{b} \text{Im} \qquad \rho_{\pm} = \frac{\rho_{N} \pm \bar{\rho}_{N}}{2} \\
\dot{\rho}_{-} = -i[H_{re}, \\
+\gamma_{N}^{b} \text{Re}[Y^{\dagger}\mu Y]\rho_{FD} + \frac{\gamma_{N}}{2} \{\text{Re}[Y^{\dagger}\mu Y], \rho_{-}\}, \\
\dot{\mu}_{\alpha} = -\mu_{\alpha}(\gamma_{\nu}^{b} \text{Tr}[YY^{\dagger}I_{\alpha}] + \frac{\gamma_{\nu}^{a}}{\rho_{FD}} \text{Tr}[\text{Re}[Y^{\dagger}I_{\alpha}Y], \rho_{+}]) \\
+\frac{\gamma_{\nu}^{a} + \gamma_{\nu}^{b}}{\rho_{FD}} \text{Tr}[\text{Re}[YI_{\alpha}Y]\rho_{-} + i \text{Im}[Y^{\dagger}I_{\alpha}Y]\rho_{+}]$$

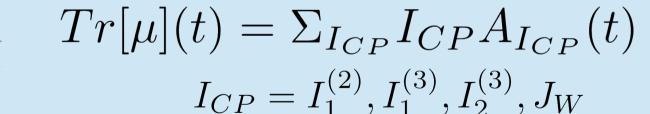
$$\begin{split} \dot{\rho}_{+} &= -i[H_{re},\rho_{+}] + [H_{im},\rho_{-}] - \frac{\gamma_{N}^{a} + \gamma_{N}^{b}}{2} \{Y^{\dagger}Y,\rho_{+} - \rho_{\mathrm{FD}}\} \\ &+ i\gamma_{N}^{b} \mathrm{Im}[Y^{\dagger}\mu Y] \rho_{\mathrm{FD}} + \mathrm{i} \frac{\gamma_{N}^{a}}{2} \{\mathrm{Im}[Y^{\dagger}\mu Y],\rho_{+}\}, \\ \dot{\rho}_{-} &= -i[H_{re},\rho_{-}] + [H_{im},\rho_{+}] - \frac{\gamma_{N}^{a} + \gamma_{N}^{b}}{2} \{Y^{\dagger}Y,\rho_{-}\} \\ &+ \gamma_{N}^{b} \mathrm{Re} \\ \hline \dot{\mu_{\alpha}} &= -\mu_{\alpha}(\gamma_{l}^{l}) \\ &+ \frac{\gamma_{\nu}^{a} + \gamma_{N}^{a}}{\rho_{\mathrm{FD}}} \end{split}$$
 Chemical potential, diagonal in the charged leptons basis
$$Y[,\rho_{+}]) \\ &+ \frac{\gamma_{\nu}^{a} + \gamma_{N}^{a}}{\rho_{\mathrm{FD}}} \end{split}$$

$$\dot{\rho}_{+} = -i[H_{re}, \rho_{+}] + [H_{im}, \rho_{-}] - \underbrace{\gamma_{N}^{a} + \gamma_{N}^{b}}_{Q_{L}} \{Y^{\dagger}Y, \rho_{+} - \rho_{FD}\} + i\gamma_{N}^{b} Im$$

$$\dot{\rho}_{-} = -i[H_{re}, \lambda_{L_{\alpha}}] + \underbrace{\gamma_{N}^{b} Re}_{L_{\alpha}} + \underbrace{\gamma_{N}^{b} Re}_{L_{\alpha}} + \underbrace{\gamma_{N}^{b} Re}_{Q_{L}} + \underbrace{\gamma_{N}^{b} Re}_{Q_{N}Q} = 2\gamma_{N,Q}^{a} = 2\gamma_{N,Q}^{b} = 4\gamma_{\nu,Q}^{a} = \frac{3}{16\pi^{3}} \frac{y_{t}^{2}T^{2}}{k_{0}} + \underbrace{\gamma_{\nu}^{a} + \sum_{\nu} Tr[Re[YI_{\alpha}Y]\rho_{-} + i Im[Y^{\dagger}I_{\alpha}Y]\rho_{+}]}_{PFD}$$

- There is no analytical solution
- ullet We try to solve equations perturbing in the mixing of V and W matrices
- Usual assumptions:
 - Neglecting non-linear effects
 - Average momentum approximation

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ANALYTICAL SOLUTION

$$A_{I_{1}^{(2)}}(t) = y_{1}y_{2}(y_{2}^{2} - y_{1}^{2}) \left(1 - \frac{\gamma_{N}}{\bar{\gamma}_{N}}\right) \gamma_{N}^{2} G_{1}(t),$$

$$A_{I_{1}^{(3)}}(t) = -y_{1}y_{2}(y_{2}^{2} - y_{1}^{2}) \left(1 - \frac{\gamma_{N}}{\bar{\gamma}_{N}}\right) \gamma_{N}^{2} G_{2}(t),$$

$$A_{I_{2}^{(3)}}(t) = y_{1}y_{2} \left(1 - \frac{\gamma_{N}}{\bar{\gamma}_{N}}\right) \gamma_{N} G_{3}(t),$$

$$A_{J_{W}}(t) = \gamma_{1}\gamma_{2} \left(1 - \frac{\gamma_{N}}{\bar{\gamma}_{N}}\right) G_{41}(t) - \frac{\gamma_{N}}{2\bar{\gamma}_{N}} G_{42}(t).$$

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- the right yukawa dependence!
- valid in the fast collision regime $\, {f t} > \gamma_{f i}^{-1} \,$ provided that $\, {f t} < \gamma_{f i}^{-1} \theta_{f i3}^2 \,$

ANALYTICAL SOLUTION

$$A_{I_1^{(2)}}(t) = y_1 y_2 (y_2^2 - y_1^2) \left(1 - \frac{\gamma_N}{\bar{\gamma}_N} \right) \gamma_N^2 G_1(t),$$

$$(t) \equiv \left(e^{-\bar{\gamma}_2 t} - e^{-\bar{\gamma}_1 t} \right) \operatorname{Re} \left[i J_{20}(\Delta_{12}, -\Delta_{12}, t) + 2 \Delta_v J_{201}(\Delta_{12}, -\Delta_{12}, t) \right]$$

$$G_{1}(t) \equiv \left(e^{-\bar{\gamma}_{2}t} - e^{-\bar{\gamma}_{1}t}\right) \operatorname{Re}\left[iJ_{20}(\Delta_{12}, -\Delta_{12}, t) + 2\Delta_{v}J_{201}(\Delta_{12}, -\Delta_{12}, t)\right] + \frac{1}{2}\sum_{j=0}^{2}(-1)^{k}e^{-\bar{\gamma}_{k}t}\operatorname{Re}\left[J_{210}(\Delta_{12}, -\Delta_{12}, t)\left(-2\Delta_{v} + i(2\bar{\gamma}_{k} - \gamma_{1} - \gamma_{2})\right)\right]$$

$$J_{20}(\alpha_1, \alpha_2, t) \equiv \int_0^t dx_1 \ e^{i\frac{\alpha_1 x_1^3}{3}} \int_0^{x_1} dx_2 x_2 \ e^{i\frac{\alpha_2 x_2^3}{3}}$$

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$$J_{201}(\alpha_1, \alpha_2, t) \equiv \int_0^t dx_1 x_1 \ e^{i\frac{\alpha_1 x_1^3}{3}} \int_0^{x_1} dx_2 \ e^{i\frac{\alpha_2 x_2^3}{3}}$$

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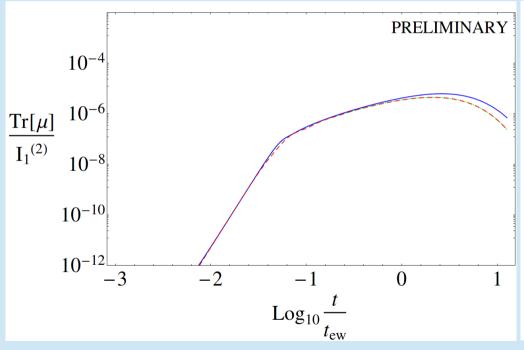
NUMERICAL CHECK

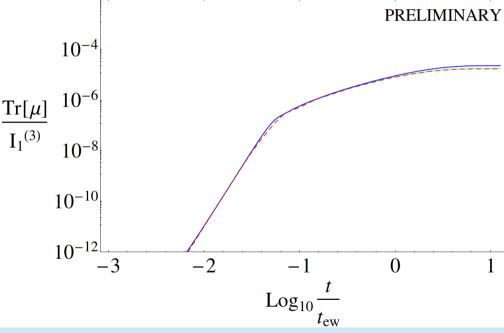
We solve equations numerically and compare with our analytic solution

NUMERICAL CHECK

•
$$A_{I_1^{(2)}}(t)$$

•
$$A_{I_1^{(3)}}(t)$$

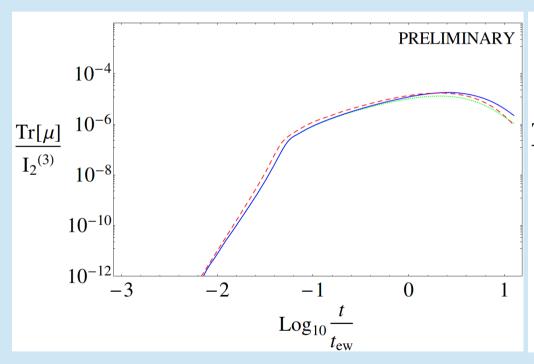


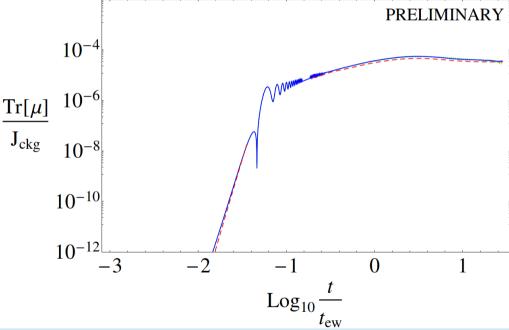


NUMERICAL CHECK

$$\bullet \ A_{I_3^{(2)}}(t)$$

 $\bullet A_{J_W}(t)$





BARYON ASYMMETRY

Measured baryon asymmetry

$$Y_B^{exp} = \frac{n_B - \overline{n}_B}{s} \simeq 8.6(1) \times 10^{-11}$$

Transfer of lepton asymmetry

$$Y_B = -\frac{28}{79}Y_L; \quad Y_L = \frac{90}{\pi^4 g_{\star}} Tr[\mu]; \quad g_* = 106.75$$

$$Y_B \simeq 3 \times 10^{-3} Tr[\mu(t_{\rm EW})]$$

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• Simple exercise: Naive seesaw scaling in νMSM model:

$$y_1^2 = 2\frac{\sqrt{\Delta_{sol}}M_1}{v^2}, \quad y_2^2 = 2\frac{\sqrt{\Delta_{atm}}M_2}{v^2}$$

$$Tr[\mu](t_{EW}) \simeq 7 \times 10^{-10} \frac{I_1^{(2)} - 2I_1^{(3)}}{|\Delta M_{12}^2(\text{GeV}^2)|^{2/3}}$$

PARAMETER SCAN

We have used Casas-Ibarra parameters

$$Y = -iU_{PMNS}^{\star} \sqrt{m_{light}} R(z_{ij})^{T} \sqrt{M} \frac{\sqrt{2}}{v}$$

We allow order 10 theoretical uncertainty

PARAMETER SCAN $\nu { m MSM}$

ullet First we consider the case of $u{
m MSM}$;where M_3 is effectively decoupled

$$m_{3(1)}=0, \ \ z_{i3}=0, \ \ R \to R(z_{ij})(P)$$

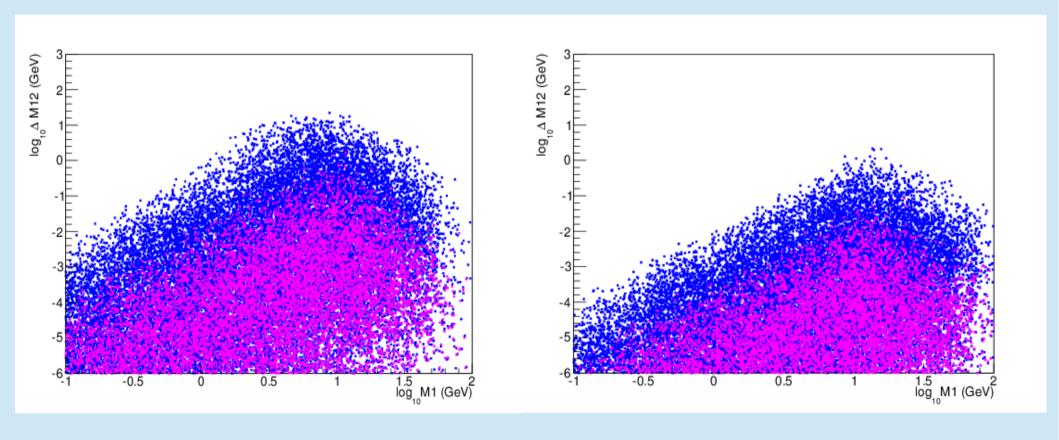
$$P \ \ {\rm is} \ \ 123 \to 312 \ \ {\rm permutation} \ \ {\rm matrix}$$
 for NH only

7 free parameters:

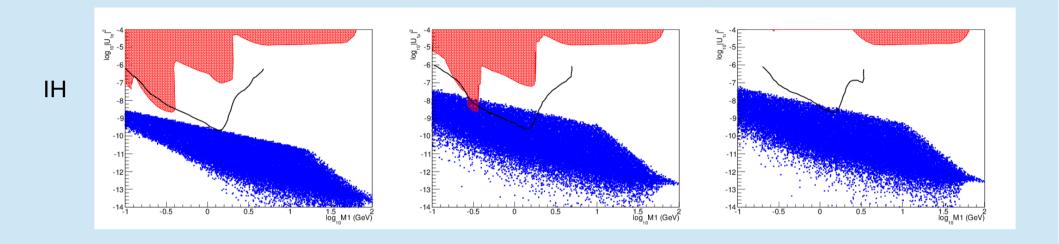
3 heavy masses, one complex angle and 3 phases from PMNS matrix

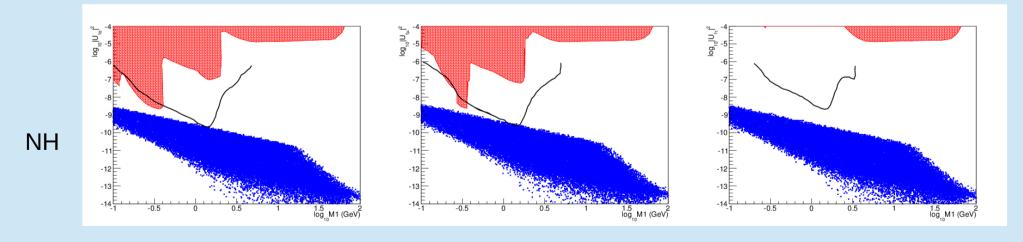
PARAMETER SCAN $\nu { m MSM}$

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PARAMETER SCAN $\nu { m MSM}$





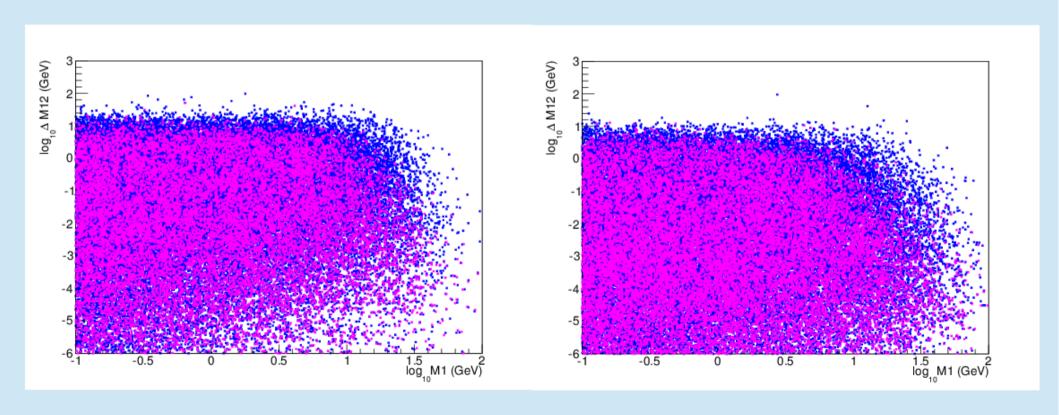
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PARAMETER SCAN FULL PARAMETER SPACE

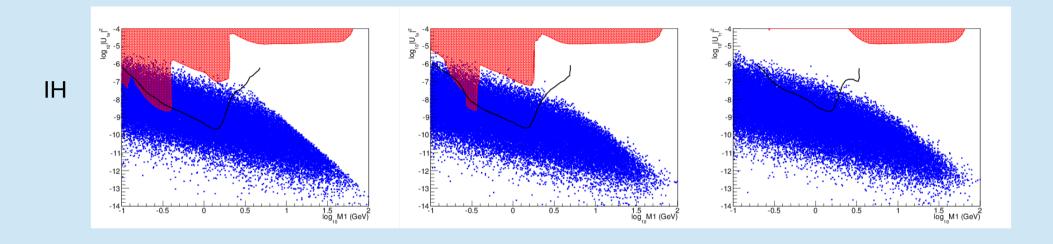
• In the 3 sterile neutrino case there are 5 more free parameters: the lightest neutrino mass and 2 complex angles.

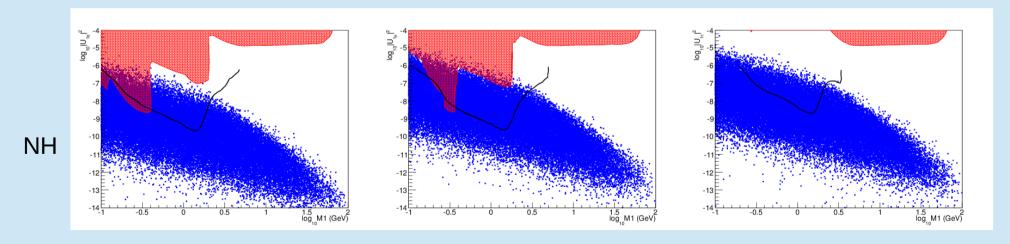
PARAMETER SCAN FULL PARAMETER SPACE

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PARAMETER SCAN FULL PARAMETER SPACE





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PARAMETER SCAN

 We do not find direct correlations between baryon asymmetry with delta, neutrinoless double beta decay amplitude, or, in the case of ASR, with the lightest neutrino mass.

CONCLUSION

- We have studied the mechanism of leptogenesis in a low-scale seesaw model.
- We have developed an analytical approximation to the quantum kinetic equations which works both in the weak and strong washout regimes (provided mixings are small).
- We have used this analytical solution to scan the full parameter space.
- No direct correlations are found between Y_B and delta or the neutrinoless double beta decay rate, nor the lightest neutrino mass.
- We are still trying to clarify the degeneracy requirement.

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- We have studied the mechanism of leptogenesis in a low-scale seesaw model.
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THANK YOU

BACKUP SLIDES

$$\rho_{\pm} = \frac{\rho_N \pm \bar{\rho}_N}{2}$$

$$\gamma_N^{a,b} = \frac{1}{2k_0} \sum_i \int_{\mathbf{p_1}, \mathbf{p_2}, \mathbf{p_3}} \rho_{eq} |M_{N(\nu),i}|^2 (2\pi)^4 \delta(k + p_1 - p_2 - p_3)|$$

$$r_{\pm} = \frac{\sum_{i} \int_{\mathbf{p_1}, \mathbf{p_2}, \mathbf{p_3}} \rho_{\pm}(\mathbf{p_1}) |M_i^{(a)}|^2 (2\pi)^4 \delta(k + p_1 - p_2 - p_3)|}{\sum_{i} \int_{\mathbf{p_1}, \mathbf{p_2}, \mathbf{p_3}} \rho_{eq}(\mathbf{p_1}) |M_i^{(a)}|^2 (2\pi)^4 \delta(k + p_1 - p_2 - p_3)|}$$

In the case of dominant top quark scattering:

$$\gamma_{N,Q}^b = 2\gamma_{N,Q}^a = 2\gamma_{\nu,Q}^b = 4\gamma_{\nu,Q}^a = \frac{3}{16\pi^3} \frac{y_t^2 T^2}{k_0}$$

