On the Stability of Longitudinal Coupled Bunch Motion Using the Vlasov Equation

Rubén Méndez Rodríguez
Universidad de Puerto Rico, Recinto de Mayagüez, Departamento de Física

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Abstract

Supervisor: Dr. James A. MacLachlan, AD/PS

The longitudinal stability of coupled bunch synchrotron oscillation is studied analytically using the Vlasov equation. The first step is to write the time dependent Vlasov equation for the interacting h bunches. The interaction is made by a coupling impedance that is modeled by a parallel RLC circuit. Because of the complexity of the Vlasov equation we are forced to used a perturbative approach to solve the equation for the distribution. A technique of multiple Fourier transforms was used to significantly reduce the number of differential operator.

1 Introduction

Theories of modern physics try to answer some questions like what is the fundamental constituent of matter and how the fundamental forces in nature interact with each other. One of the theories that describes such phenomena is relativistic quantum field theory. The way that scientists check the theory is to collide particles with each other. By measuring the deflection of the particles one gets information about how particles interact. To make such experiments, the scientific community builds big machines such as the Tevatron at Fermilab to make the particles collide. Accelerator physicists investigate how to guide the particles in the vacuum chamber, how to make the orbit of the beam stable so that it doesn't get lost and to get higher luminosity. Here we investigate the stability of longitudinal coupled bunch motion. We analyze this problem for the interaction of radio frequency (rf) cavity and the beam through a linear potential. We use the equation of motion to write the Vlasov equation. To solve the Vlasov equation we linearized it and treated the coupling as a perturbation.

2 Basics of Synchrotron Accelerator

Particles in a torroidal vacuum chamber are accelerated by a resonant (rf) cavity. These particles in succesive turns have to be in phase with the rf voltage to get the right energy increment. A collection of particles that occupy the same area about a stable phase is a called a bunch. When particles in the bunch are traveling through the pipe they are not all side by side; this means that some particle get more energy and others less when they pass the rf gap. Ultimately this would be disastrous for particles that are not in phase. The persistance of a group of particles near a particular phase of the synchronus rf waveform is called phase stability. Particles that have energy near the energy of an ideal particle that has the right energy and phase will remain nearby, because of a restoring force; the particles will oscillate about this energy and phase. These oscillations are called synchrotron oscillations because they were first studied for the machine now called the synchrotron. The equations that govern the motion of the particle without any interactions are

$$\dot{\phi} = \frac{\eta h \omega_0}{\beta^2 E_0} \epsilon, \tag{1}$$

$$\dot{\epsilon} = \frac{e \omega_0 \hat{V}}{2\pi} \left\{ f\left(\phi\right) - f\left(\phi_s\right) \right\}. \tag{2}$$

We identify the following variables

ϕ_s	Synchronous Phase.
ϕ	Rf phase when particles is at cavity.
E_0	Synchronous energy.
ϵ	Difference between energy of the particle and the synchronous energy E_0.
h	Harmonic number.
ω_0	Angular frequency of beam circulation.
η	Slip Factor $\gamma_T^{-2} - \gamma^{-2}$.
γ	The relativistic energy ratio E/m_0c^2 or $\sqrt{1-\beta^2}$.
γ_T	Transition energy. This value is determined by the type of device studied as well as its design.
β	v/c
$\hat{V}$	Amplitude of rf voltage.
e	Charge of the proton.

The function $f(\phi)$ in equation (2) is periodic with period $2\pi/h$. The quantity h, the harmonic number of the rf is the number of oscillations the rf wave makes during one revolution. In other words it is the maximum number of bunches in the beam pipe. For the booster ring at Fermilab, this number is 84. The slip factor η changes sign when the energy is above transition. Some approximation can be made to equation (2), mainly to expand the function $f(\phi)$ in a Taylor series about ϕ_s and keeping the first order.

$$\dot{\epsilon} = \frac{e\omega_0 \hat{V} \cos\phi_s}{2\pi} (\phi - \phi_s) . \tag{3}$$

The quantity $\phi - \phi_s = h\omega_0\tau$, is the phase difference between a particle and the synchronous particle. The variable τ represent the time of arrival at t_0 ahead of the synchronous particle. Combining equation (2) and (3) we get the equation of a harmonic oscillator. For this linear, noninteracting case the frequency of oscillation is

$$\Omega = \left(\frac{e\omega_0 \eta \hat{V} \cos\phi_s}{2\pi\beta^2 E_0} \right)^{\frac{1}{2}} .$$

When particles go around the beam pipe they produce an electromagnetic field called the wake field, that is produced by the beam current. The wake field is described by the wake function $W_m(\tau)$; formally it can be found by the following equation

$$\int_{-L/2}^{L/2} ds\, \vec{F} = -e\vec{\nabla} I_m r^m W_m(\tau) \cos m\theta, \tag{4}$$

here "L" is the distance from the particle to the wall of the pipe. The letter "m" stands for the multipole moments, arising form the charge and current density in the Maxwell equations. $\vec{F}$ is given by the Lorentz force

$$\vec{F} = e\left(\vec{E} + \vec{v} \times \vec{B}\right). \tag{5}$$

It is more convenient to calculate the wake function $W_m(\tau)$ by

$$W_m(\tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega\, Z(\omega)\, e^{i\omega\beta\tau}, \tag{6}$$

where $Z(\omega)$ is the coupling impedance. It is more convenient to use equation (6) because there are some models that give an analytical result for the impedance like the one for a rf cavity

$$Z(\omega) = \frac{R_s}{1 + iQ\left(\frac{\omega_R}{\omega} - \frac{\omega}{\omega_R}\right)}. \tag{7}$$

The only thing that we have to do is take the Fourier transform. Compared to equation (4), equation (6) is easier to use.

When particles go around the ring their are a lot of interactions that affect the stability of orbits. It may seem surprising that we can do high energy physics because of that. We can because of a stabilizing mechanism called Landau damping. This was first studied by Lev Davidovich Landau in his work on damping of longitudinal space charge waves in plasma. This damping provides a natural stabilizing mechanism against the collective instabilities if particles in the beam have a small spread in their natural synchrotron frequency [3]. This frequency spread for the longitudinal bunched beam case, comes from nonlinearity of the rf focusing voltage. To get frequency spread we have to introduce some nonlinearity in the equation of motion. We accomplish this by expanding $f(\phi)$ in equation (2) in a Taylor series and keeping up to the 3^{rd} order of ϕ

$$\dot{\epsilon} = \frac{e\omega_0 \hat{V}}{2\pi} \left\{ \frac{df}{d\phi}\bigg|_{\phi=\phi_s} (\phi - \phi_s) + \frac{1}{2!}\frac{d^2 f}{d\phi^2}\bigg|_{\phi=\phi_s} (\phi - \phi_s)^2 + \frac{1}{3!}\frac{d^3 f}{d\varphi^3}\bigg|_{\varphi=\phi_s} (\phi - \phi_s)^3 \right\}. \tag{8}$$

In equation (8) we have used a sinusoidal function in equation (2). The equations of motion are

$$\dot{\epsilon} = \frac{h\omega_0^2 e\hat{V}\cos\phi_s}{2\pi}\left\{\tau - \frac{h\omega_0 \tan\phi_s}{2}\tau^2 - \frac{h^2\omega_0^2}{6}\tau^3\right\}, \tag{9}$$

$$\dot{\tau} = \frac{\eta}{\beta^2 E_0}\epsilon. \tag{10}$$

We define the quantity E, with the dimension of time, and ω_s^2 is the frequency of small amplitude synchrotron oscillation.

$$\omega_s^2 = -\frac{eh\omega_0^2\eta\hat{V}\cos\phi_s}{2\pi\beta^2 E_0}, \qquad E = \frac{\eta}{\omega_s\beta^2 E_0}\epsilon$$

And the equations simplify a little as

$$\dot{E} = -\omega_s\left\{\tau - \frac{h\omega_0\tan\phi_s}{2}\tau^2 - \frac{h^2\omega_0^2}{6}\tau^3\right\}, \tag{11}$$

$$\dot{\tau} = \omega_s E. \tag{12}$$

The corresponding Hamiltonian is

$$H = \frac{\omega_s}{2}\left\{E^2 + \tau^2 - \frac{h\omega_0}{3}\tan\phi_s\tau^3 - \frac{h^2\omega_0^2}{12}\tau^4\right\}. \tag{13}$$

3 Longitudinal Coupled Bunch Motion

In a synchrotron, typically there are approximately 10^{10} particles or more. They interact with each other via the wake field that they leave behind. The motion of the particles becomes coupled and the movement of each particle is not independent and has the form

$$z_n'' + \left(\frac{\omega_s}{c}\right)^2 z_n = \frac{Nr_0\eta}{\gamma C}\sum_k\sum_{m=0}^{M-1} W_0'\left\{-kC - \frac{m-n}{M}C + z_n - z_m\left(\frac{t}{v} - kC - \frac{m-n}{M}C\right)\right\}, \qquad n = 0, 1, 2, \ldots, M-1 \quad , \tag{14}$$

n represent the n^{th} bunch and M we identify with the number of equally spaced bunches. The important thing of this equation is the right side. The indexing is such that the $n = 1$ bunch is ahead of the $n = 0$ bunch[1].

4 The Vlasov Equation

One way to describe the beam and find information about the stability is to describe the state of the particles at a given time by a continuous particle density function in phase space. The coordinates that we use in phase space are canonically conjugate to each other and can be derived from a Hamiltonian $H(q, p)$ using the canonical equations of motion

$$\dot{q_j} = \frac{\partial H(q_j, p_j)}{\partial p_j}, \quad \dot{p_j} = -\frac{\partial H(q_j, p_j)}{\partial q_j}$$

The equation that the density function follows is called the Vlasov equation

$$\frac{\partial \psi}{\partial t} + \dot{q}\frac{\partial \psi}{\partial q} + \dot{p}\frac{\partial \psi}{\partial p} = 0. \tag{15}$$

It states the conservation of phase space density seen by a stationary observer. This equation can also be thought of as describing he flow of particles in a similar way to that of a moving liquid. If the motion grows exponentially this will create instability for the beam. For collective instability the Vlasov

[1]For more information about coupled bunch motion using the equation of motion the reader is referred to A. Chao book, *Physics Of Collective Beam Instability In High Energy Accelerators.*

equation can be written formally [8] as

$$\frac{\partial\psi}{\partial t}+\omega_s p\frac{\partial\psi}{\partial q}+\left\{-\omega_s q+N\omega_s\int_q^{\infty}dq'W\left(q-q'\right)\int_{-\infty}^{\infty}dp'\psi\left(q',p',t\right)\right\}\frac{\partial\psi}{\partial p}=0, \tag{16}$$

This equation is for a single bunch. If we want to treat the problem of h bunches we have to modify the equation to incorporate how we describe the position of each bunch in phase space. If we have the case that the interaction is a constant for each bunch, the Vlasov equation is

$$\frac{\partial\psi}{\partial t}+\omega_s\sum_j^h\left\{E_j\frac{\partial\psi}{\partial\tau_j}-\left(1+\xi_j\right)\tau_j\frac{\partial\psi}{\partial E_j}\right\}=0. \tag{17}$$

Now we linearize the Vlasov equation looking for a solution of ψ of the form

$$\psi\left(E_j,\tau_j,t\right) = \psi_0\left(\tau_j\right)+\psi_1\left(E_j,\tau_j,t\right), \tag{18}$$

where ψ_1 describes the contribution of the interaction and the ψ_0 is the unpertubed particle distribution. It will be modeled by a delta function $\psi_0 = eN\delta\left(\tau_j\right)$ is the unpertubed particle distribution. The new Vlasov equation is

$$\frac{\partial\psi_1}{\partial t}+\omega_s\sum_j^h\left\{E_j\frac{\partial\psi_1}{\partial\tau_j}-\zeta_j^2\tau_j\frac{\partial\psi_1}{\partial E_j}\right\}=-eN\omega_s\sum_j^h E_j\frac{d\psi_0\left(\tau_j\right)}{d\tau_j}, \tag{19}$$

where we have define the constant $\zeta_j^2\equiv\xi_j+1$. We now introduce the Dirac delta function for ψ_0 and use the identity

$$x\frac{d\delta\left(x\right)}{dx}=-\delta\left(x\right).$$

The Vlasov equation becomes

$$\frac{\partial\psi_1}{\partial t}+\omega_s\sum_j^h\left\{E_j\frac{\partial\psi_1}{\partial\tau_j}-\zeta_j^2\tau_j\frac{\partial\psi_1}{\partial E_j}\right\}=eN\omega_s\sum_j^h\frac{E_j}{\tau_j}\delta\left(\tau_j\right). \tag{20}$$

5 Technique to Solve the Vlasov Equation

The usual way to solve the time dependent Vlasov equation is to change to polar coordinates and expand ψ_1 as

$$\psi_1=\sum_{l=-\infty}^{\infty}\alpha_l R_l\left(r\right)e^{il\theta},$$

for the case of multiple bunches this is not very adequate, because we have to define h polar transformations to describe the position of the bunches. For the Vlasov equation this means that we will have $2h+1$ differential operators; for the case of Fermilab's booster ring the Vlasov equation will have 169 differential operators.

One way to attack the problem is to take the Fourier transform of the Vlasov equation for the coordinate variables. We have to take $2h$ Fourier transforms on both sides of the equation, but we will reduce the differential operators significantly to only one in time. After that is accomplished we have to solve the following equation and take the inverse Fourier transform of the solution.

$$\left\{\frac{\partial}{\partial t}+\omega_s\sum_j^h\left\{E_j\frac{\partial}{\partial\tau_j}-\zeta_j^2\tau_j\frac{\partial}{\partial E_j}\right\}\right\}\psi_1(E_j,\tau_j,t)=eN\omega_s\sum_j^h\frac{E_j}{\tau_j}\delta\left(\tau_j\right)\qquad(21)$$

The Fourier Transform of the distribution is

$$\psi_1(E_j,\tau_j,t)=\frac{1}{(2\pi)^h}\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}\prod_{j=1}^h d\sigma_j d\rho_j\,\tilde{\psi}_1\left(\sigma_j,\rho_j,t\right)e^{i\sum_j^h(E_j\sigma_j+\tau_j\rho_j)}.\qquad(22)$$

We will write the right side of equation (21) as an arbitrary function, $F(E_j,\tau_j)$, and define it's Fourier transform as

$$F(E_j,\tau_j)=\frac{1}{(2\pi)^h}\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}\prod_{j=1}^h d\sigma_j d\rho_j\;\tilde{F}(\sigma_j,\rho_j)\,e^{i\sum_j^h(E_j\sigma_j+\tau_j\rho_j)},\qquad(23)$$

$$\tilde{F}(\sigma_j,\rho_j)=\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}\prod_{j=1}^h dE_j d\tau_j\;F(E_j,\tau_j)\,e^{-i\sum_j^h(E_j\sigma_j+\tau_j\rho_j)}.\qquad(24)$$

Because $F(E_j,\tau_j)$ is known, we can calculate $\tilde{F}(\sigma_j,\rho_j)$. I will demonstrate $\tilde{F}(\sigma_j,\rho_j)$ for the one particle problem; the generalization is straight forward.

$$\tilde{F}(\sigma,\rho) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} dE\, d\tau F(E,\tau)\, e^{-i(E\sigma+\tau\rho)}$$

For our case the function $\tilde{F}$ is

$$\begin{aligned}
\tilde{F}(\sigma,\rho) &= eN\omega_s \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} dE\, d\tau \frac{E}{\tau}\delta(\tau)\ e^{-i(E\sigma+\tau\rho)} \\
&= -eNi\omega_s \left\{\int_{-\infty}^{\infty} dE\, \frac{d}{d\sigma} e^{-iE\sigma}\right\}\left\{-\int_{-\infty}^{\infty} d\tau \frac{d\delta(\tau)}{d\tau} e^{-i\tau\rho}\right\} \\
&= eNi\omega_s \left\{\frac{d}{d\sigma}\int_{-\infty}^{\infty} dE\, e^{-iE\sigma}\right\}\left\{\int_{-\infty}^{\infty} d\tau \frac{d\delta(\tau)}{d\tau} e^{-i\tau\rho}\right\}.
\end{aligned}$$

The first integral is just the derivative of a Dirac delta function on σ, the next integral we can evaluate by using the property

$$\int dx f(x)\, \frac{d^n\delta(x)}{dx^n} = -\int dx \frac{\partial f}{\partial x}\frac{d^{n-1}\delta(x)}{dx^{n-1}}.$$

$$\tilde{F}(\sigma,\rho) = eN\omega_s\rho\frac{\delta(\sigma)}{\sigma} \tag{25}$$

where we have used the property of the derivative of a delta function. The generalization of (25) is very simple. I this project I did the two particle problem that uses the same technique. Then look at the three particle problem to see the recurrence of the delta functions. The result

$$\tilde{F}\left(\sigma_j,\rho_j\right)=eN\omega_s\sum_j^h\rho_j\frac{\delta(\sigma_i)}{\sigma_j}\prod_{m\neq j}^h\delta(\sigma_m)\,\delta(\rho_m) \tag{26}$$

We now have to solve the following equation and take the inverse Fourier transform to solve the original problem.

$$\frac{\partial\tilde{\psi}_1}{\partial t}+i\omega_s\sum_j^h\left\{E_j\rho_j-\zeta_j^2\tau_j\sigma_j\right\}\tilde{\psi}_1=eN\omega_s\sum_j^h\rho_j\frac{\delta(\sigma_j)}{\sigma_j}\prod_{m\neq j}^h\delta(\sigma_m)\,\delta(\rho_m) \tag{27}$$

For the case that the interaction is not constant with each particle we have to considerer the last term in equation (16) is

$$V(\tau)=\frac{e^2}{C_0}\int_\tau^\infty d\tau' W'\left(\tau-\tau'\right)\int_{-\infty}^\infty dE'\psi\left(\tau',E'\right). \tag{28}$$

Equation (28) makes the time dependent Vlasov equation (17) a nonlinear integrodifferential equation. Equations like this are very difficult to solve, so we use the antsaz that the particle density for the unpertubed part is equal to that of the total particle density. We will considerer a elliptical distribution[2].

$$\psi_0\left(\tau_0,E_0\right)=\frac{3N\left|\eta\right|}{2\pi\beta^2\omega_s E_0\hat{\tau_0}^3}\left\{\hat{\tau_0}^2-E^2-\tau^2\right\}^{1/2} \tag{29}$$

[2]A. Hofmann and F. Pedersen, *Bunches with Local Elliptic Energy Distribution*, 8^{th} Particle Accelerator Conf., 1979.

The corresponding charge distribution ρ_0 is

$$\rho_0(\tau) = \int_{-\infty}^{\infty} dE' \psi_0 \left(\tau', E'\right) = \frac{3N}{4\hat{\tau}_0{}^3} \left\{ \hat{\tau}_0{}^2 - \tau^2 \right\}. \tag{30}$$

We now consider the interaction of the particles and the resonant cavity (7). The wake function is give by [3]

$$W'(\tau) = \begin{cases} 0 & \text{if } \tau > 0 \\ \alpha R_s & \text{if } \tau = 0 \\ 2\alpha R_s \left\{ \cos(\overline{\omega}\xi\tau) + \frac{\alpha}{\overline{\omega}} \sin(\overline{\omega}\xi\tau) \right\} e^{\alpha\xi\tau} & \text{if } \tau < 0 \end{cases},$$

where $\alpha = \omega_r/2Q$ and $\overline{\omega} = \sqrt{\omega_r^2 - \alpha^2}$. The potential that each particle is affected by is

$$V(\tau) = \frac{3Ne^2}{4C_0\hat{\tau}_0} \int_{\tau}^{\infty} d\tau' \left\{ \hat{\tau}_0{}^2 - \tau'^2 \right\} \left\{ \cos\left(\overline{\omega}\xi\left(\tau' - \tau\right)\right) - \frac{\alpha}{\overline{\omega}} \sin\left(\overline{\omega}\xi\left(\tau' - \tau\right)\right) \right\} e^{-\alpha\xi(\tau' - \tau)}. \tag{31}$$

This integral can be done by integration by parts or using some computer program like Mathematica. The result for the last integral is

$$V(\tau) = -\frac{3Ne^2}{2C_0} \left\{ \frac{2\alpha + \left(\alpha^2 + \overline{\omega}^2\right)\xi\tau}{\xi^2\left(\alpha^2 + \overline{\omega}^2\right)^2 \hat{\tau}_0} \right\}. \tag{32}$$

[3] This result can by found in A. Chao book, *Physics Of Collective Beam Instability In High Energy Accelerators*, pag. 73. For a more complete list of expressions for impedance and wake function it can by find on A. Chao, *Handbook of Accelerator Physics and Engineering*, World Scientific, 1999. In the calaculation of the wake function, this was done for the longitudinal coordinate z. Because z is proportional to $v\tau$, where v is the velocity of the particales. The constant of proportionality is ξ.

The time dependent Vlasov equation may be writen as

$$\frac{\partial\psi_1}{\partial t}+\omega_s\sum_{j=1}^{h}E_j\frac{\partial\psi_1}{\partial\tau_j}+\sum_{j=1}^{h}\left\{\frac{e\omega_0\hat{V}}{2\pi}\left[\sin\left(h\omega_0\tau_j+\phi_s\right)-\sin\phi_s\right]-\frac{3Ne^2}{2C_0}\left\{\frac{2\alpha+\left(\alpha^2+\overline{\omega}^2\right)\xi\tau}{\xi^2(\alpha^2+\overline{\omega}^2)^2\hat{\tau}_0}\right\}\right\}\frac{\partial\psi_1}{\partial E_j}=\sum_{j=1}^{h}G_j\left(E_j,\tau_j\right)\quad(33)$$

In this equation we have added the nonlinearity so that we can analyze the stabilizing mechanism of Landau damping. The function $G_j\left(E_j,\tau_j\right)$ contains the terms of the partial derivatives with respect of E_j andτ_j of the unpertubed particle distribution ψ_0.

6 Summary

We have studied the Vlasov equation for the case of coupled bunch motion with the interaction of a resonant cavity. To analyze this problem analytically, we have introduced $2h$ Fourier transforms to reduce the number of differential operators to simplify the equation. In doing this we have transformed to a frequency like domain, because all the dynamical variables have dimensionality of time. Possibly useful information can be interpreted in this domain. Further work has to be done to make the model complete. In the equation we have to take the right unperturbed particle distribution in order to get Landau damping. Also, because this distribution has to obey some boundary condition, it will be useful to make some finite transform such as a finite Hankel transform or the like.

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