Exact Neutrino Mixing Angles from Three Subgroups of SU(2)

and the Physics Consequences

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Goal

- Show that the 3 lepton families represent 3 special and related subgroups of SU(2), therefore remaining within the realm of the SM EW gauge group.
- Show that the 2 lepton (and quark) flavor states in each family may not be 'pure' SU(2) basis states
- Mixing makes them behave collectively as SU(2)

SU(2) ≈ unit quaternions

- $q = a + bi + cj + dk$
- $a^2 + b^2 + c^2 + d^2 = 1$
- a, b, c, d $\varepsilon\ \mathbb{R}$
- quaternion rot θ in $\mathbb{R}^3$ is actually rot by θ/2
- e.g. k is a quaternion rot by π in i-j plane

Discrete symmetry subgroups

- The only finite quaternion subgroups are:
- 2T, 2O, 2I, $2D_{2n}$, $2C_n$, $1C_n$ (n odd)
- 2 means binary or double cover [of SO(3)]
- Only 2T, 2O, 2I need include 3-D volume

Assign 2T, 2O, 2I

- 2T ⇨ Electron family (ν_e, e^-)
- 2O ⇨ Muon family (ν_μ, μ^-)
- 2I ⇨ Tau family (ν_τ, τ^-)

Quaternion generators

- Difference in k only

- SU(2): $U_1 = j$ $U_2 = k$ $U_3 = i$
- 2T: $U_1 = j$ $U_2 = ?$ $U_3 = i$
- 2O: $U_1 = j$ $U_2 = ?$ $U_3 = i$
- 2I: $U_1 = j$ $U_2 = ?$ $U_3 = i$

What is U_2?

- $U_2 = -\, i \cos \pi/q \;\; -\, j \cos \pi/p \;\; -\, k \sin \pi/h$
- Alternate names [p,q,2] ⇨
- 2T = [3,3,2]; 2O = [4,3,2]; 2I = [5,3,2]
- h = 4, 6, 10

Want contribution of the 3 U_2's = k by linear superposition

Family	Group	U_2	Factor	Angle	Angle/2
ν_e, e^-	[3,3,2]	$- 1/2\ i - 1/2\ j + 1/\sqrt{2}\ k$	-0.26422	105.3204°	52.660°
ν_μ, μ^-	[4,3,2]	$- 1/2\ i - 1/\sqrt{2}\ j + 1/2\ k$	0.80116	36.7581°	18.379°
ν_τ, τ^-	[5,3,2]	$- 1/2\ i - \phi/2\ j + \phi^{-1}/2\ k$	-0.53695	122.4764°	61.238°

3 equations for 3 unknowns → normalized Factors

$\Phi = (1 + \sqrt{5})/2 = 1.618\ldots$ i.e. Golden Ratio

Angle = arccosine (Factor), the projection angle to the k axis

Neutrino mixing angles

- $\theta_1 = 52.660°$ $\theta_2 = 18.379°$ $\theta_3 = 61.238°$
- $\theta_{12} = 34.281°$ vs. $33.56° \pm 0.77°$
- $\theta_{23} = 42.859°$ vs. $41.6° \pm 1.5°$
- $\theta_{13} = -8.578°$ vs. $8.46° \pm 0.15°$
- Assumed no charged-lepton mixing
- θ_{23} ⇨ normal mass ordering $m_1 < m_2 < m_3$
- NuFit 3.0 (2016)
- As expected: $34.281° = 42.859° - 8.578°$

Major consequences:

- Neutrino mixing occurs because 3 lepton families together act as one SU(2)
- Leptons are 3-D objects representing discrete symmetry properties of subgroups 2T, 2O, 2I
- Total lepton number is conserved, but not each lepton family number separately
- Unitary PMNS matrix: rows/columns $\to$ 1

PMNS matrix

0.8170 (0.822)	0.5570 (0.547)	- - - 0.1491 $e^{-i\delta}$ - (- 0.150 + 0.038i) -
- 0.4129 + 0838 $e^{i\delta}$ - (- 0.356 + 0.020i)	0.6057 +0.0571 $e^{-i\delta}$ (0.704 - 0.013i)	0.6726 (0.614)
0 3831 + 0.0903 $e^{-i\delta}$ (0.442 + 0.025i)	- 0.5620 + 0.0616 $e^{-i\delta}$ - (- 0.452 + 0.017i)	0.7248 (0.774)

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More consequences?

- Phase δ could be 0, -π/2, π/2 ??
- No more lepton families beyond 3
- For two EW basis states in R^3, only 4 d.o.f. → one massive lepton (3 d.o.f.) and one massless lepton (1 d.o.f.)
- For ν to have mass, must "see" 4th dim, where there are 6 d.o.f. → 2 massive

One more great clue?!

- Syzygies from invariant theory, 3 invariant eqs for each group 2T, 2O, 2I   1884 Felix Klein
- Each group related to j-invariant of elliptic modular functions and linear transformations
- Group constants 1,   108,   1728
- Charged leptons: 0.511, 105.66, 1776.82 MeV
- % differences: -48.9%, -2.17%, +2.83%
- Coincidence, Correlation, or Cause and effect?

Anecdote?

- Richard Feynman, in his Caltech office Nov 1987
- The Icosahedron and the solution of equations of the fifth degree (1884) by F. Klein [see Dover edition 1956]

Quarks?

- Same approach works for quark families
- 4 subgroups in R^4 → 4 quark families predicted
- [3,3,3], [4,3,3], [3,4,3], [5,3,3]
- → 4x4 CKM4 matrix → good agreement to CKM 3x3 except for V_{ub}
- 3 lepton families acting as one SU(2) match 4 quark families acting as one SU(2) to cancel triangle anomaly

Possible consequences

- Predicts EW $\theta_W = 30°$ - agrees with latest expts
- No sterile neutrino
- Not Majorana neutrinos
- No neutrinoless double beta decay
- 2 more quarks to be discovered

Thank You!

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- 2I = [5,3,2] ⇨ (ν_τ, τ^-) $\theta_3 = 61.238°$
- 34.281° = 42.859°- 8.578°
- Neutrino mixing occurs because 2T, 2O, 2I act together to make SU(2) for the SM
- See my DISCRETE 2014 Conference writeup in Journal of Physics: Conference Series, Vol 631 (link)