The Proton Radius Puzzle

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Introduction: The proton radius puzzle

Form Factors

- Matrix element of EM current between nucleon states give rise to two form factors ($q = p_f - p_i$)

$$\langle N(p_f)| \sum_q e_q \, \bar{q}\gamma^\mu q |N(p_i)\rangle = \bar{u}(p_f) \left[\gamma^\mu F_1(q^2) + \frac{i\sigma_{\mu\nu}}{2m} F_2(q^2) q^\nu \right] u(p_i)$$

- Sachs electric and magnetic form factors

$$G_E(q^2) = F_1(q^2) + \frac{q^2}{4m_p^2} F_2(q^2) \qquad G_M(q^2) = F_1(q^2) + F_2(q^2)$$

$$G_E^p(0) = 1 \qquad G_M^p(0) = \mu_p \approx 2.793$$

- The slope of G_E^p

$$\boxed{\langle r^2 \rangle_E^p = 6 \frac{dG_E^p}{dq^2}\bigg|_{q^2=0}}$$

determines the charge radius $r_E^p \equiv \sqrt{\langle r^2 \rangle_E^p}$

- The proton *magnetic* radius

$$\langle r^2 \rangle_M^p = \frac{6}{G_M^p(0)} \frac{dG_M^p(q^2)}{dq^2}\bigg|_{q^2=0}$$

Charge radius from atomic physics

$$\langle p(p_f)| \sum_q e_q\, \bar{q}\gamma^\mu q |p(p_i)\rangle = \bar{u}(p_f)\left[\gamma^\mu F_1^p(q^2) + \frac{i\sigma_{\mu\nu}}{2m} F_2^p(q^2) q^\nu\right] u(p_i)$$

- For a point particle amplitude for $p + \ell \to p + \ell$

$$\mathcal{M} \propto \frac{1}{q^2} \quad \Rightarrow \quad U(r) = -\frac{Z\alpha}{r}$$

- Including q^2 corrections from proton structure

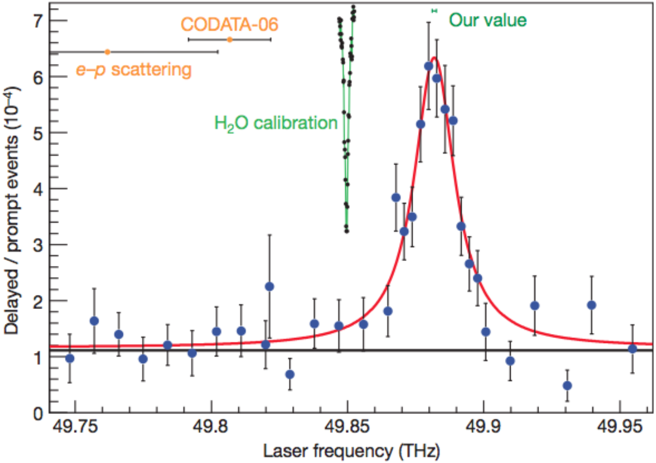
$$\mathcal{M} \propto \frac{1}{q^2} q^2 = 1 \quad \Rightarrow \quad U(r) = \frac{4\pi Z\alpha}{6} \delta^3(r)(r_E^p)^2$$

- Proton structure corrections $\left(m_r = m_\ell m_p/(m_\ell + m_p) \approx m_\ell\right)$

$$\boxed{\Delta E_{r_E^p} = \frac{2(Z\alpha)^4}{3n^3} m_r^3 (r_E^p)^2 \delta_{\ell 0}}$$

- **Muonic hydrogen can give the best measurement of r_E^p!**

Charge radius from Muonic Hydrogen



- CREMA Collaboration measured for the **first time** $2S_{1/2}^{F=1} - 2P_{3/2}^{F=2}$ transition in Muonic Hydrogen [Pohl et al. Nature **466**, 213 (2010)]

Charge radius from atomic physics

- Lamb shift in muonic hydrogen [Pohl et al. Nature **466**, 213 (2010)]
  $r_E^p = 0.84184(67)$ fm
  more recently $r_E^p = 0.84087(39)$ fm [Antognini et al. Science **339**, 417 (2013)]
- CODATA value [Mohr et al. RMP **80**, 633 (2008)]
  $r_E^p = 0.87680(690)$ fm
  more recently $r_E^p = 0.87510(610)$ fm [Mohr et al. RMP **88**, 035009 (2016)]
  extracted mainly from (electronic) hydrogen
- **5σ discrepancy!**

- This is the proton radius puzzle

What could be the reason for the discrepancy?

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1) Problem with the electronic extraction? (Part 1 of this talk)

2) Hadronic Uncertainty? (Part 2 of this talk)

3) New Physics?

- Declaimer: I will focus mainly on published work I am involved in

Outline

- Introduction: The proton radius puzzle
- Part 1: Proton radii from scattering
- Part 2: Hadronic Uncertainty?
- Part 3: Connecting $\mu - p$ scattering and muonic hydrogen
- Conclusions and outlook

Part 1: Proton radii from scattering

What did the PDG 2010 say?

K. Nakamura et al. (Particle Data Group), J. Phys. G 37, 075021 (2010)

p CHARGE RADIUS

This is the rms charge radius, $\sqrt{\langle r^2 \rangle}$.

VALUE (fm)	*DOCUMENT ID*		*TECN*	*COMMENT*
0.8768±0.0069	MOHR	08	RVUE	2006 CODATA value
• • • We do not use the following data for averages, fits, limits, etc. • • •				
0.897 ±0.018	BLUNDEN	05		SICK 03 + 2γ correction
0.8750±0.0068	MOHR	05	RVUE	2002 CODATA value
0.895 ±0.010 ±0.013	SICK	03		$e p \to e p$ reanalysis
0.830 ±0.040 ±0.040	[24] ESCHRICH	01		$e p \to e p$
0.883 ±0.014	MELNIKOV	00		1S Lamb Shift in H
0.880 ±0.015	ROSENFELDR...	00		$e p$ + Coul. corrections
0.847 ±0.008	MERGELL	96		$e p$ + disp. relations
0.877 ±0.024	WONG	94		reanalysis of Mainz $e p$ data
0.865 ±0.020	MCCORD	91		$e p \to e p$
0.862 ±0.012	SIMON	80		$e p \to e p$
0.880 ±0.030	BORKOWSKI	74		$e p \to e p$
0.810 ±0.020	AKIMOV	72		$e p \to e p$
0.800 ±0.025	FREREJACQ...	66		$e p \to e p$ (CH_2 tgt.)
0.805 ±0.011	HAND	63		$e p \to e p$

Citation: K. Nakamura *et al.* (Particle Data Group), JPG **37**, 075021 (2010) (URL: http://pdg.lbl.gov)

[24] ESCHRICH 01 actually gives $\langle r^2 \rangle = (0.69 \pm 0.06 \pm 0.06)$ fm^2.

Proton charge radius from scattering

- The PDG 2010 lists proton radii starting from 1963
- This is more then 50 years of radii extraction.
- You can find almost any value between 0.8-0.9 fm....
- Data sets have changed over the last 50 years
  but even using the same data sets different people get different values
- What is the problem?

Form Factors: What we don't know

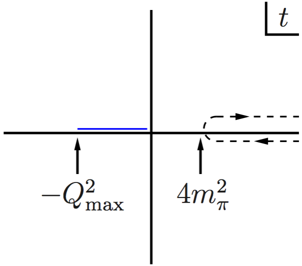
- The form factors are non-perturbative objects.
- **Nobody** knows the *exact* functional form of G_E^p and G_M^p
- They don't have to have a dipole/polynomial/spline
  or any other functional form
- Including such models can bias your extraction of r_E^p and r_M^p

Form Factors: What we do know

- Analytic properties of $G_E^p(t)$ and $G_M^p(t)$ are known
- They are analytic outside a cut $t \in [4m_\pi^2, \infty]$

  [Federbush, Goldberger, Treiman, Phys. Rev. **112**, 642 (1958)]

- $e - p$ scattering data is in $t < 0$ region

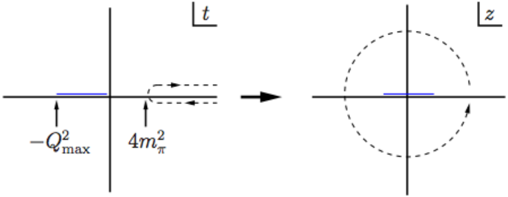


z expansion

- z expansion: map domain of analyticity onto unit circle

$$z(t, t_{\rm cut}, t_0) = \frac{\sqrt{t_{\rm cut} - t} - \sqrt{t_{\rm cut} - t_0}}{\sqrt{t_{\rm cut} - t} + \sqrt{t_{\rm cut} - t_0}}$$

where $t_{\rm cut} = 4m_\pi^2$, $z(t_0, t_{\rm cut}, t_0) = 0$



- Expand $G^p_{E,M}$ in a Taylor series in z: $G^p_{E,M}(q^2) = \sum_{k=0}^{\infty} a_k\, z(q^2)^k$

- **The** method for **meson** form factors
  [Flavor Lattice Averaging Group, EPJ C **74**, 2890 (2014)]

z expansion

- [Zachary Epstein, GP, Joydeep Roy PRD **90**, 074027 (2014)]

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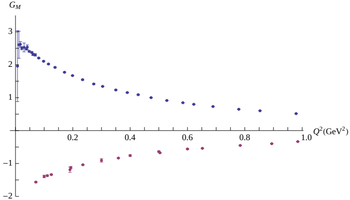
  $G_M(Q^2)$ for proton (blue, above axis) and neutron (red, below axis)



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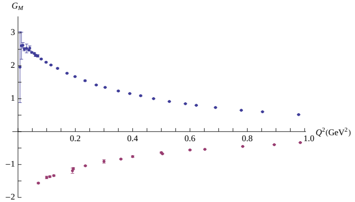

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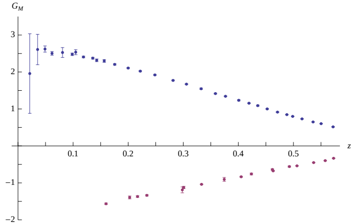

- See also R.J. Hill talk at FPCP 2006 [hep-ph/0606023]

Problem with the electronic extraction?

- Recent development: use of the z expansion
  based on known analytic properties of form factors
- **The** method for **meson** form factors
  [Flavor Lattice Averaging Group, EPJ C **74**, 2890 (2014)]
- Now applied successfully to **baryon** form factors
  to extract r_E^p, r_M^p, r_M^n, m_A...

PDG 2016: r_E^p

Citation: C. Patrignani *et al.* (Particle Data Group), Chin. Phys. C, **40**, 100001 (2016)

p CHARGE RADIUS

This is the rms electric charge radius, $\sqrt{\langle r_E^2 \rangle}$.

VALUE (fm)			*DOCUMENT ID*		*TECN*	*COMMENT*
0.8751	**±0.0061**		MOHR	16	RVUE	2014 CODATA value
0.84087	**±0.00026**	**±0.00029**	ANTOGNINI	13	LASR	μp-atom Lamb shift
• • • We do not use the following data for averages, fits, limits, etc. • • •						
0.895	±0.014	±0.014	[1] LEE	15	SPEC	Just 2010 Mainz data
0.916	±0.024		LEE	15	SPEC	World data, no Mainz
0.8775	±0.0051		MOHR	12	RVUE	2010 CODATA, $e p$ data
0.875	±0.008	±0.006	ZHAN	11	SPEC	Recoil polarimetry
0.879	±0.005	±0.006	BERNAUER	10	SPEC	$e p \to e p$ form factor
0.912	±0.009	±0.007	BORISYUK	10		reanalyzes old $e p$ data
0.871	±0.009	±0.003	HILL	10		z-expansion reanalysis
0.84184	±0.00036	±0.00056	POHL	10	LASR	See ANTOGNINI 13
0.8768	±0.0069		MOHR	08	RVUE	2006 CODATA value
0.844	$^{+0.008}_{-0.004}$		BELUSHKIN	07		Dispersion analysis
0.897	±0.018		BLUNDEN	05		SICK 03 + 2γ correction
0.8750	±0.0068		MOHR	05	RVUE	2002 CODATA value
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[Hill, GP PRD **82** 113005 (2010)]
[Lee, Arrington, Hill, PRD **92**, 013013 (2015)]

PDG 2016: r_M^p

Citation: C. Patrignani *et al.* (Particle Data Group), Chin. Phys. C, **40**, 100001 (2016)

p MAGNETIC RADIUS

This is the rms magnetic radius, $\sqrt{\langle r_M^2 \rangle}$.

VALUE (fm)	*DOCUMENT ID*		*TECN*	*COMMENT*
0.776±0.034±0.017	[1] LEE	15	SPEC	Just 2010 Mainz data
• • • We do not use the following data for averages, fits, limits, etc. • • •				
0.914±0.035	LEE	15	SPEC	World data, no Mainz
0.87 ±0.02	EPSTEIN	14		Using ep, en, $\pi\pi$ data
0.867±0.009±0.018	ZHAN	11	SPEC	Recoil polarimetry
0.777±0.013±0.010	BERNAUER	10	SPEC	$ep \to ep$ form factor
0.876±0.010±0.016	BORISYUK	10		Reanalyzes old $ep \to ep$ data
0.854±0.005	BELUSHKIN	07		Dispersion analysis

[1] Authors also provide values for a combination of all available data.

[Epstein, GP, Roy PRD **90**, 074027 (2014)]
[Lee, Arrington, Hill, PRD **92**, 013013 (2015)]

PDG 2016: r_M^n

Citation: C. Patrignani *et al.* (Particle Data Group), Chin. Phys. C, **40**, 100001 (2016)

n MAGNETIC RADIUS

This is the rms magnetic radius, $\sqrt{\langle r_M^2 \rangle}$.

VALUE (fm)	*DOCUMENT ID*		*COMMENT*
$0.864^{+0.009}_{-0.008}$ OUR AVERAGE			
0.89 ± 0.03	EPSTEIN	14	Using $e p$, $e n$, $\pi\pi$ data
$0.862^{+0.009}_{-0.008}$	BELUSHKIN	07	Dispersion analysis

[Epstein, GP, Roy PRD **90**, 074027 (2014)]

Part 2: Hadronic Uncertainty?

The bottom line

- Scattering:
  - World $e-p$ data [Lee, Arrington, Hill '15]
    $r_E^p = 0.918 \pm 0.024$ fm
  - Mainz $e-p$ data [Lee, Arrington, Hill '15]
    $r_E^p = 0.895 \pm 0.020$ fm
  - Proton, neutron and π data [Hill , GP '10]
    $r_E^p = 0.871 \pm 0.009 \pm 0.002 \pm 0.002\,\mathrm{fm}$

- Muonic hydrogen
  - [Pohl et al. Nature **466**, 213 (2010)]
    $r_E^p = 0.84184(67)$ fm
  - [Antognini et al. Science **339**, 417 (2013)]
    $r_E^p = 0.84087(39)$ fm

- The bottom line:
  using z expansion scattering disfavors muonic hydrogen

- Is there a problem with muonic hydrogen *theory*?

Muonic hydrogen theory

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- Potentially yes!
  [Hill, GP PRL **107** 160402 (2011)]
- Muonic hydrogen measures ΔE and translates it to r_E^p

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  $\Delta E = 206.0573(45) - 5.2262(r_E^p)^2 + 0.0347(r_E^p)^3$ meV

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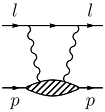
- [Antognini et al. Science **339**, 417 (2013), Ann. of Phy. **331**, 127]
  $\Delta E = 206.0336(15) - 5.2275(10)(r_E^p)^2 + 0.0332(20)$ meV

- Apart from r_E^p we need also the two-photon exchange
- Notice the change in the *theoretical* prediction
- Potential source of hadronic uncertainty

Two photon exchange



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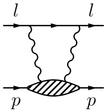
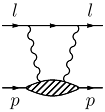


$$
\begin{aligned}
W^{\mu\nu} &= \frac{1}{2}\sum_s i\int d^4x\, e^{iq\cdot x}\langle \boldsymbol{k}, s|\, T\{J^\mu_{\mathrm{e.m.}}(x)J^\nu_{\mathrm{e.m.}}(0)\}|\boldsymbol{k}, s\rangle \\
&= \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right)W_1 + \left(k^\mu - \frac{k\cdot q\, q^\mu}{q^2}\right)\left(k^\nu - \frac{k\cdot q\, q^\nu}{q^2}\right)W_2
\end{aligned}
$$

Two photon exchange



$$W^{\mu\nu} = \frac{1}{2}\sum_s i\int d^4x\, e^{iq\cdot x}\langle \boldsymbol{k}, s|T\{J^\mu_{\rm e.m.}(x)J^\nu_{\rm e.m.}(0)\}|\boldsymbol{k}, s\rangle$$
$$= \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right)W_1 + \left(k^\mu - \frac{k\cdot q\, q^\mu}{q^2}\right)\left(k^\nu - \frac{k\cdot q\, q^\nu}{q^2}\right)W_2$$

Dispersion relations ($\nu = 2k\cdot q,\ Q^2 = -q^2$)

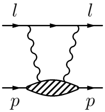
$$W_1(\nu, Q^2) = W_1(0, Q^2) + \frac{\nu^2}{\pi}\int_{\nu_{\rm cut}(Q^2)^2}^{\infty} d\nu'^2\, \frac{{\rm Im}\, W_1(\nu', Q^2)}{\nu'^2(\nu'^2-\nu^2)}$$

$$W_2(\nu, Q^2) = \frac{1}{\pi}\int_{\nu_{\rm cut}(Q^2)^2}^{\infty} d\nu'^2\, \frac{{\rm Im}\, W_2(\nu', Q^2)}{\nu'^2-\nu^2}$$

- W_1 requires subtraction...

Two photon exchange

- Apart from r_E^p we have two-photon exchange



- Imaginary part related to data:
  form factors, structure functions

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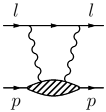

- Imaginary part related to data:
  form factors, structure functions

- *Cannot* reproduce it from its imaginary part:
  Dispersion relation requires subtraction

- Need poorly constrained non-perturbative function $W_1(0, Q^2)$

Two Photon exchange: small Q^2 limit

- *Small* Q^2 limit using NRQED [Hill, GP, PRL **107** 160402 (2011)]
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$$W_1(0,Q^2) = 2a_p(2+a_p) + \frac{Q^2}{m_p^2}\left\{\frac{2m_p^3\bar{\beta}}{\alpha} - a_p - \frac{2}{3}\left[(1+a_p)^2 m_p^2 (r_M^p)^2 - m_p^2 (r_E^p)^2\right]\right\} + \mathcal{O}\left(Q^4\right)$$

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- $a_p = 1.793$, $\bar{\beta} = 2.5(4)\times 10^{-4}$ fm^3
- $r_M = 0.776(34)(17)$ fm,
- $r_E^H = 0.8751(61)$ fm or $r_E^{\mu H} = 0.84087(26)(29)$ fm

$$W_1(0,Q^2) = 13.6 + \frac{Q^2}{m_p^2}\left(-54\pm 7\right) + \mathcal{O}\left(Q^4\right)$$

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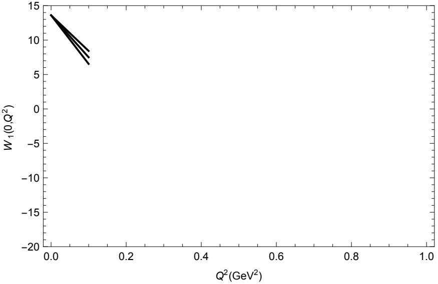
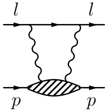


Two Photon Exchange: large Q^2 limit



- Calculable in *large* Q^2 limit using Operator Product Expansion (OPE) [J. C. Collins, NPB **149**, 90 (1979)]

  The photon "sees" the quarks and gluons inside the proton

$$W_1(0, Q^2) = c/Q^2 + \mathcal{O}\left(1/Q^4\right)$$

- Result was used to estimate two photon exchange effects
- c calculated in [J. C. Collins, NPB **149**, 90 (1979)]

RENORMALIZATION OF THE COTTINGHAM FORMULA

John C. COLLINS *
Joseph Henry Laboratories, Princeton University, Princeton, New Jersey 08540, USA

Received 23 October 1978

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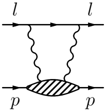

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- Was it?

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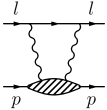


$$W^{\mu\nu} = \frac{1}{2}\sum_s i\int d^4x\, e^{iq\cdot x}\langle \boldsymbol{k}, s|T\{J^{\mu}_{\rm e.m.}(x)J^{\nu}_{\rm e.m.}(0)\}|\boldsymbol{k}, s\rangle$$
$$= \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right)W_1 + \left(k^\mu - \frac{k\cdot q\, q^\mu}{q^2}\right)\left(k^\nu - \frac{k\cdot q\, q^\nu}{q^2}\right)W_2$$

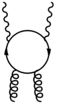
- $W_1(0, Q^2)$ is dimensionless

$$W_1 \sim \frac{\langle \text{Proton}|O|\text{Proton}\rangle}{Q^2} + \mathcal{O}\left(\frac{1}{Q^4}\right)$$

- O is a dimension 4 operator:

- Quarks: Spin 0: $m_q\bar{q}q$ Spin 2: $\bar{q}\left(iD^\mu\gamma^\nu + iD^\nu\gamma^\mu - \frac{1}{4}i\not{D}\, g^{\mu\nu}\right)q$
- Gluons: must be color singlet: $G_a^{\alpha\beta}G_a^{\rho\sigma}$
- What gluon operators can we have?

Gluon operators



- Gluons: must be color singlet $G_a^{\alpha\beta} G_a^{\rho\sigma}$
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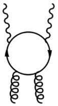
Gluon operators



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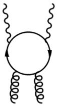

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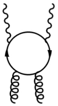

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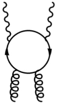
  chromomagnetic stress-energy tensor
- What else?

Gluon operators



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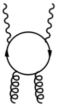
Gluon operators



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$$O^{\mu\alpha\nu\beta} = -\frac{1}{4}\left(\epsilon^{\mu\alpha\rho\sigma}\epsilon^{\nu\beta\kappa\lambda} + \epsilon^{\mu\beta\rho\sigma}\epsilon^{\nu\alpha\kappa\lambda}\right) G_{\rho\kappa} G_{\sigma\lambda} - \text{all possible traces}$$

For example $O^{0123} = G^{01} G^{23} + G^{03} G^{21} = E^1 B^1 - E^3 B^3$

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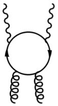

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  For example $O^{0123} = G^{01} G^{23} + G^{03} G^{21} = E^1 B^1 - E^3 B^3$
- For protons: $\langle \text{Proton} | O^{\mu\alpha\nu\beta} | \text{Proton} \rangle = 0$
  What about $\langle \text{Medium} | O^{\mu\alpha\nu\beta} | \text{Medium} \rangle$?
  Solution looking for a problem...

Summary: Possible operators

- In total we have four operators with non-zero proton matrix elements.

- Quarks:
  - Spin 0: $m_q \bar{q} q$
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RENORMALIZATION OF THE COTTINGHAM FORMULA

John C. COLLINS *
Joseph Henry Laboratories, Princeton University, Princeton, New Jersey 08540, USA

Received 23 October 1978

Large Q^2 behavior

- In 1978 Collins calculated EM corrections to the nucleon mass with an emphasis on $m_n - m_p$
  The mass only depends on spin-0 operators (q quark, $G^{\mu\nu}$ gluon)

$$\langle P|m_q\bar{q}q|P\rangle, \qquad \langle P|G^{\mu\nu}G_{\mu\nu}|P\rangle$$

	Quark	Gluon
Spin-0	Collins '78	Collins '78

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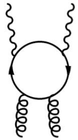
- Need to calculate the spin-2 contribution [Hill, GP arXiv:1611.09917]

	Quark	Gluon
Spin-0	Collins '78	Collins '78
Spin-2	Hill, GP '16	Hill, GP '16

- Collins's result is not enough for muonic hydrogen!

Large Q^2 behavior

- Requires 1-loop calculation



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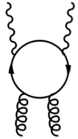

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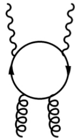

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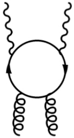

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- For three light quark u, d, s
  Correct result: $\sum_q e_q^2 = (\frac{2}{3})^2 + (\frac{1}{3})^2 + (\frac{1}{3})^2 = \frac{2}{3}$
  Collins: $\sum_q = 3$
  Too large by a factor of 4.5...

Large Q^2 behavior

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Spin-0	Collins '78	~~Collins '78~~ Hill, GP '16
Spin-2	Hill, GP '16	Hill, GP '16

Large Q^2 behavior

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- Even worse, quark spin-0 and gluon spin-0 come with opposite signs
  After correcting the mistake they largely cancel
  $W_1(0, Q^2)$ is **dominated** by spin-2 contribution

- Lesson: It is important to do a full calculation

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- Flip side: You cannot use $m_n - m_p$ to constrain muonic hydrogen

Large Q^2 behavior: Results

[Hill, GP PRD **95**, 094017 (2017), arXiv:1611.09917]

- The *correct* spin 0 result

$$\frac{Q^2}{2m_p^2}W_1^{(\mathrm{spin}-0)}(0,Q^2) = -2\sum_q e_q^2 f_q^{(0)} + \left(\sum_q e_q^2\right)\frac{\alpha_s}{12\pi}\tilde{f}_g^{(0)}$$

$$\langle N(k)|O_q^{(0)}|N(k)\rangle \equiv 2m_N^2 f_{q,N}^{(0)}, \quad \langle N(k)|O_g^{(0)}(\mu)|N(k)\rangle \equiv -2m_N^2 \tilde{f}_{g,N}^{(0)}(\mu)$$

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- The *new* spin 2 result

$$\frac{Q^2}{2m_p^2} W_1^{(\mathrm{spin}-2)}(0, Q^2) = 2\sum_q e_q^2 f_q^{(2)}(\mu) + \left(\sum_q e_q^2\right) \frac{\alpha_s}{4\pi}\left(-\frac{5}{3} + \frac{4}{3}\log\frac{Q^2}{\mu^2}\right) f_g^{(2)}(\mu)$$

  - $\langle N(k)|O_q^{(2)\mu\nu}(\mu)|N(k)\rangle \equiv 2\left(k^\mu k^\nu - \frac{g^{\mu\nu}}{4} m_N^2\right) f_{q,N}^{(2)}(\mu)$

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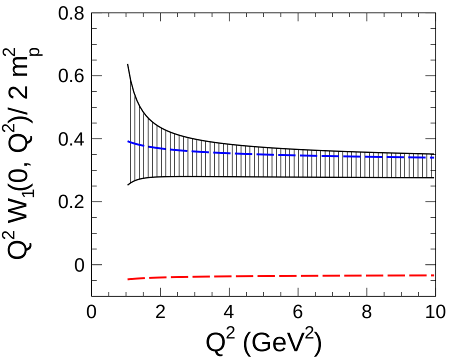
    $\sum_q f_q^{(2)}(\mu) + f_g^{(2)}(\mu) = 1$

Large Q^2 behavior: Total contribution

[Hill, GP PRD **95**, 094017 (2017), arXiv:1611.09917]

- The total contribution



- Dashed red: spin 0
- Dashed blue: spin 2
- Vertical stripes: total contribution with perturbative and hadronic errors added in quadrature

Two Photon exchange: small Q^2 and large Q^2

- Using NRQED we have control over low Q^2

$$W_1(0,Q^2) = 2a_p(2+a_p)+\frac{Q^2}{m_p^2}\left\{\frac{2m_p^3\overline{\beta}}{\alpha}-a_p-\frac{2}{3}\left[(1+a_p)^2m_p^2(r_M^p)^2-m_p^2(r_E^p)^2\right]\right\}+\mathcal{O}\left(Q^4\right)$$

- Using OPE we *now* have control over the high Q^2

$$\frac{Q^2}{2m_p^2}W_1^{(\mathrm{spin}-0)}(0,Q^2) = -2\sum_q e_q^2 f_q^{(0)} + \left(\sum_q e_q^2\right)\frac{\alpha_s}{12\pi}\tilde{f}_g^{(0)}$$

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- The problem, like the joke, is how to make a whole fish from a head and a tail...

- Before this work we had only the low Q^2
  knowing the large Q^2 allows to connect the dots

Two Photon Exchange: Modeling

- "Aggressive" modeling: use OPE for $Q^2 \geq 1$ GeV^2
- Model unknown Q^4: add $\Delta_L(Q^2) = \pm Q^2/\Lambda_L^2$ with $\Lambda_L \approx 500$ MeV
- Model unknown $1/Q^4$: add $\Delta_H(Q^2) = \pm \Lambda_H^2/Q^2$ with $\Lambda_H \approx 500$ MeV

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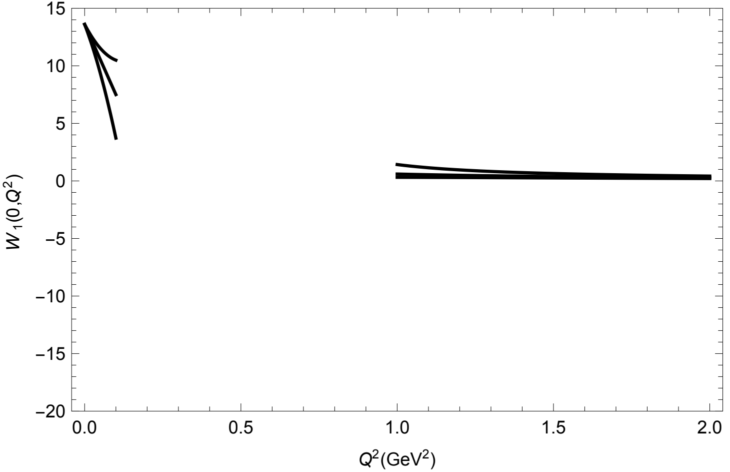
- How to connect the curves?



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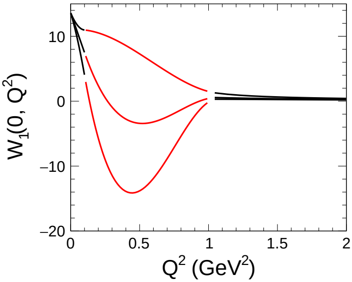

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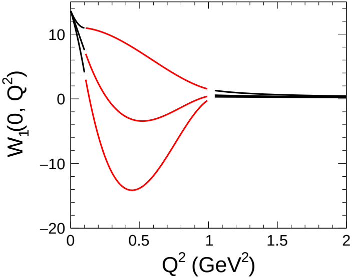
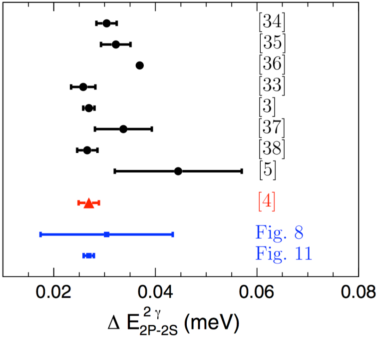


- Energy contribution: $\delta E(2\mathrm{S})^{W_1(0,Q^2)} \in [-0.046\,\mathrm{meV},\ -0.021\,\mathrm{meV}]$
  To explain the puzzle need this to be ~ -0.3 meV
- Caveats: OPE might be only valid for larger Q^2
  $W_1(0, Q^2)$ might be different than the interpolated lines

Two Photon Exchange: Other approaches

- Similar results found by other groups



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[Fig. 8] Hill, GP PRD 95, 094017 (2017).

Experimental test

- How to test?
- New experiment: $\mu - p$ scattering
  MUSE (MUon proton Scattering Experiment) at PSI
  [R. Gilman et al. (MUSE Collaboration), arXiv:1303.2160]

- Need to connect muon-proton scattering and muonic hydrogen can use a new effective field theory: QED-NRQED
  [Hill, Lee, GP, Mikhail P. Solon, PRD **87** 053017 (2013)]
  [Steven P. Dye, Matthew Gonderinger, GP, PRD **94** 013006 (2016)]

Part 3: Connecting muon-proton scattering and muonic hydrogen

MUSE

- Muonic hydrogen:

  Muon momentum $\sim m_\mu c\alpha \sim 1$ MeV

  Both proton and muon non-relativistic

MUSE

- Muonic hydrogen:

  Muon momentum $\sim m_\mu c\alpha \sim 1$ MeV

  Both proton and muon non-relativistic

- MUSE:

  Muon momentum $\sim m_\mu \sim 100$ MeV

  Muon is relativistic, proton is still non-relativistic

MUSE

- Muonic hydrogen:

  Muon momentum $\sim m_\mu c\alpha \sim 1$ MeV

  Both proton and muon non-relativistic

- MUSE:

  Muon momentum $\sim m_\mu \sim 100$ MeV

  Muon is relativistic, proton is still non-relativistic

- QED-NRQED effective theory:

- Use QED for muon
- Use NRQED for proton

  $m_\mu/m_p \sim 0.1$ as expansion parameter

- A *new* effective field theory suggested in

  [Hill, Lee, GP, Mikhail P. Solon, PRD **87** 053017 (2013)]

QED-NRQED Effective Theory

- Example: TPE at the lowest order in $1/m_p$
  [Steven P. Dye, Matthew Gonderinger, GP, PRD **94** 013006 (2016)]

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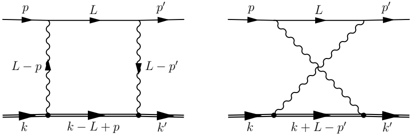
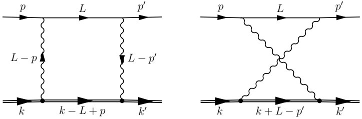


$$\frac{1}{p^0 - L^0 + i\epsilon} + \frac{1}{L^0 - p^0 + i\epsilon} \Rightarrow \delta(L^0 - p^0)$$

- In total

$$\delta(p^0 - p'^0)\,\delta(L^0 - p^0) = \delta(L^0 - p^0)\,\delta(L^0 - p'^0)$$

QED-NRQED Effective Theory



- The amplitude

$$
\begin{aligned}
i\mathcal{M}(2\pi)^4\delta^4(k+p-k'-p') &= Z^2e^4\int\frac{d^4L}{(2\pi)^4}\frac{1}{(L-p)^2L-p')^2}\\
\times \quad & \bar{u}(p')\gamma^0\frac{i}{\not{L}-m}\gamma^0u(p)\chi^\dagger\chi(2\pi)\delta(L^0-p^0)\,(2\pi)\delta(L^0-p'^0)\\
\times \quad & (2\pi)^3\delta^4(\vec{p}-\vec{p}'-\vec{k}')
\end{aligned}
$$

- The cross section

$$
\frac{d\sigma}{d\Omega}=\frac{Z^2\alpha^2 4E^2\left(1-v^2\sin^2\frac{\theta}{2}\right)}{\vec{q}^{\,4}}\left[1+\frac{Z\alpha\pi v\sin\frac{\theta}{2}\left(1-\sin\frac{\theta}{2}\right)}{1-v^2\sin^2\theta}\right]
$$

$Z=1$, $E=$ muon energy, $v=|\vec{p}|/E$, $q=p'-p,\theta$ scattering angle

QED-NRQED Effective Theory

- QED-NRQED result

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- *Same result* as scattering relativistic lepton off static $1/r$ potential
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- *Same result* as $m_p \to \infty$ of "point particle proton" QED scattering (For $m_p \to \infty$ only proton charge is relevant)

QED-NRQED Effective Theory beyond $m_p \to \infty$ limit

- QED-NRQED allows to calculate $1/m_p$ corrections

- Example: one photon exchange $\mu + p \to \mu + p$:
  QED-NRQED = $1/m_p$ expansion of form factors
  [Steven P. Dye, Matthew Gonderinger, GP, PRD **94** 013006 (2016)]

Connecting muon-proton scattering to muonic hydrogen

- Matching

QED, QCD $\qquad$ $G_{E,M}$, Structure func., $W_1(0, Q^2)$

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NRQED-NRQED: *muonic H*	r_E^p, $\psi_\mu^\dagger\psi_\mu\psi_p^\dagger\psi_p$

- Need to match QED-NRQED contact interaction, e.g. $\bar{\mu}\gamma^0\mu\psi_p^\dagger\psi_p$ to NRQED-NRQED contact interaction, e.g. $\psi_\mu^\dagger\psi_\mu\psi_p^\dagger\psi_p$
  [Dye, Gonderinger, GP *in progress*]

Connecting muon-proton scattering to muonic hydrogen

- To do list:

1) Relate QED-NRQED contact interactions
   to NRQED contact interactions and $W_1(0, Q^2)$

2) Calculate $d\sigma(\mu + p \to \mu + p)$ and asymmetry in terms of r_E^p and d_2

3) *Direct* relation between μ–p scattering and muonic H

Conclusions

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- Proton radius puzzle: $> 5\sigma$ discrepancy between
  - r_E^p from muonic hydrogen
  - r_E^p from hydrogen and $e - p$ scattering

- Recent muonic deuterium results find similar discrepancies
  [Pohl et al. Science **353**, 669 (2016)]

- After more than 6 years the origin is still not clear
  1) Is it a problem with the electronic extraction?
  2) Is it a hadronic uncertainty?
  3) is it new physics?

- Motivates a reevaluation of our understanding of the proton

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- Much more work to do!

- Thank you