Radiative corrections in a general Yukawa model

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Main question

How do radiative corrections to masses and mixing angles behave in ν-mass models?

Renormalization and radiative corrections to masses in a general Yukawa model

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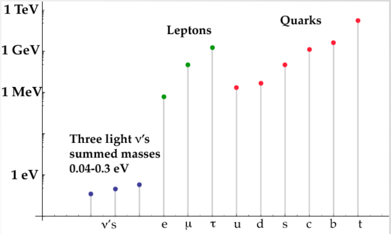
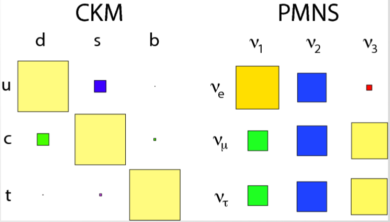
M. Fox, W. Grimus and M. Löschner,
Renormalization and radiative corrections to masses in a general Yukawa model,
arXiv:1705.09589 [hep-ph].

Motivation

Open problems:

1. Smallness of ν-masses
2. Mild hierarchy in ν-mass spectrum vs. strong hierarchy in spectra of charged leptons and quarks
3. Relation between mixing angles and masses
4. Majorana nature of neutrinos





Taken from [1]

Motivation

Flavour symmetries

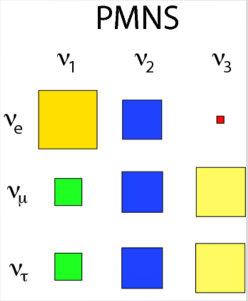
- Attempt to describe/explain structure of U_{PMNS} via symmetries of the mass matrix
- Use (combination of) discrete symmetries to approximate U_{PMNS} , e.g. μ-τ symmetry [3]

$$S\mathcal{M}_\nu S = \mathcal{M}_\nu^*$$

$$\text{with } S = \begin{array}{c} \nu_e \\ \nu_\mu \\ \nu_\tau \end{array} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\Rightarrow |U_{\mu i}| = |U_{\tau i}| \quad \forall i$$

$$\Rightarrow \theta_{23} = 45^\circ,\ \delta = \pm\frac{\pi}{2}.$$



Taken from [1]

- Models of this kind often show *proliferation of scalars*
- Need suitable treatment of potentially large number of scalars

Radiative Corrections in ν-mass models

Setup of general Yukawa toy model:

- n_χ Majorana fermions
- n_φ real scalars
- $\mathbb{Z}_4$-Symmetry: no tree-level mass terms and (tri-)linear terms in scalar potential:

$$\mathcal{S}: \quad \chi_L \to i\chi_L \quad \varphi \to -\varphi$$

Toy model Lagrangian

$$\begin{aligned}\mathcal{L} = i\bar{\chi}_{iL}\not{\partial}_\mu\chi_{iL} + \frac{1}{2}\left(\partial_\mu\varphi_a\right)\left(\partial^\mu\varphi_a\right) \\ + \left(\frac{1}{2}\left(Y_a\right)_{ij}\,\chi_{iL}{}^T C^{-1}\chi_{jL}\varphi_a + \mathrm{H.c.}\right) \\ - \frac{1}{2}(\mu^2)_{ab}\varphi_a\varphi_b - \frac{1}{4}\lambda_{abcd}\,\varphi_a\varphi_b\varphi_c\varphi_d\end{aligned}$$

Radiative Corrections in ν-mass models

Generation of masses

- Masses are generated via *spontaneous symmetry breaking* (SSB)

$$\varphi_a = \mathcal{M}^{-\varepsilon/2}\bar{v}_a + h_a$$

  - Majorana masses $m_0 = \sum_{a=1}^{n_\varphi} v_a Y_a$
  - Scalar masses $\left(M_0^2\right)_{ab} = \mu_{ab}^2 + 3\lambda_{abcd} v_c v_d$

- Masses fully determined via model parameters, also in terms of renormalization
- Note: number of masses $\ll$ number of model parameters

  $\Rightarrow$ Renormalization of masses not sufficient to make theory finite

Radiative Corrections in ν-mass models

Renormalization

Want to achieve:

- Clear and straightforward approach
- Non-specific on peculiarities of the investigated models
- Take nature of masses as functions of parameters of the model serious

Radiative Corrections in ν-mass models

Renormalization

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Programme:

1. $\overline{\text{MS}}$ renormalization for all parameters of the model and the field strength renormalization constants (using dim. reg.)
2. Finite shifts Δv_a of vacuum expectation values such that scalar one-point functions vanish
3. Finite field strength renormalization to get on-shell self-energies

Radiative Corrections in ν-mass models

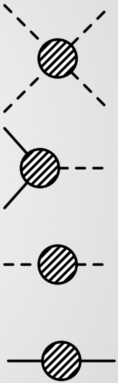
Determination of Counterterms

(a) $\delta\hat{\lambda}_{abcd}$ determined from the quartic scalar coupling

(b) $\delta\hat{Y}_a$ obtained from Yukawa vertex

(c) $\delta\hat{\mu}^2_{ab}$ removes UV-divergent terms from p^2-independent part of scalar selfenergy

(d) $\hat{\delta}^{(\chi)}$ and $\hat{\delta}^{(\varphi)}$ determined from momentum-dependent parts of respective selfenergies.



Results

One-point functions and VEV shifts

- Unobscured treatment of tadpole contributions

$0 =$ 

$$= \mathcal{M}^{-\varepsilon/2} \frac{i}{-M_{0a}^2} \times (-i) \left(\hat{t}_a + T_a + \Delta\hat{t}_a + \delta\hat{\mu}^2_{ab}\hat{v}_b + \delta\hat{\lambda}_{abcd}\, \hat{v}_b\hat{v}_c\hat{v}_d \right)$$

- At one loop order:

$$\Delta\hat{t}_a = \hat{\mu}^2_{ab}\Delta\hat{v}_b + 3\hat{\lambda}_{abcd}\hat{v}_c\hat{v}_d\Delta\hat{v}_b = \left(\hat{M}_0^2\right)_{ab} \Delta\hat{v}_b,$$

- Finite tadpole contributions are absorbed in VEV shifts

$$\Delta\hat{v}_a = -\left(\hat{M}_0^2\right)^{-1}_{ab} (T_{\text{fin}})_b \,.$$

- Note: $\hat{t}_a = 0$ equivalent to finding stationary points of potential

$$\hat{t}_a = \hat{\mu}^2_{ab}\hat{v}_b + \hat{\lambda}_{abcd}\hat{v}_b\hat{v}_c\hat{v}_d = \left.\frac{\partial V}{\partial \varphi_a}\right|_{\varphi_i = \hat{v}_i} \stackrel{!}{=} 0$$

Results

Renormalized self-energies

Majorana fermions

$$\begin{aligned}\Sigma(p) &= \Sigma^{\text{1-loop}}(p) - \not{p}\left[\hat{\delta}^{(\chi)}\gamma_L + \left(\hat{\delta}^{(\chi)}\right)^*\gamma_R\right] \\ &\quad + \hat{v}_a\left[\delta\hat{Y}_a\gamma_L + (\delta\hat{Y}_a)^*\gamma_R\right] + \Delta\hat{v}_a\left[\hat{Y}_a\gamma_L + \hat{Y}_a^*\gamma_R\right],\end{aligned}$$

Real scalars

$$\Pi_{ab}(p^2) = \Pi_{ab}^{\text{1-loop}}(p^2) - \hat{\delta}_{ab}^{(\varphi)}p^2 + \delta\hat{\mu}_{ab}^2 + 3\delta\hat{\lambda}_{abcd}\hat{v}_c\hat{v}_d + 6\hat{\lambda}_{abcd}\hat{v}_c\Delta\hat{v}_d$$

*explicit results for self-energies and counterterms in [2]

Results

One-loop masses

Majorana fermions:

$$m_i = m_{0i} + m_{0i}\left(\Sigma_L^{(A)}\right)_{ii}(m_{0i}^2) + \mathrm{Re}\left(\Sigma_L^{(B)}\right)_{ii}(m_{0i}^2),$$

Dirac fermions:

$$\begin{aligned} m_i &= m_{0i} + \frac{1}{2}m_{0i}\left[\left(\Sigma_L^{(A)}\right)_{ii}(m_{0i}^2) + \left(\Sigma_R^{(A)}\right)_{ii}(m_{0i}^2)\right] \\ &\quad + \mathrm{Re}\left(\Sigma_L^{(B)}\right)_{ii}(m_{0i}^2) \end{aligned}$$

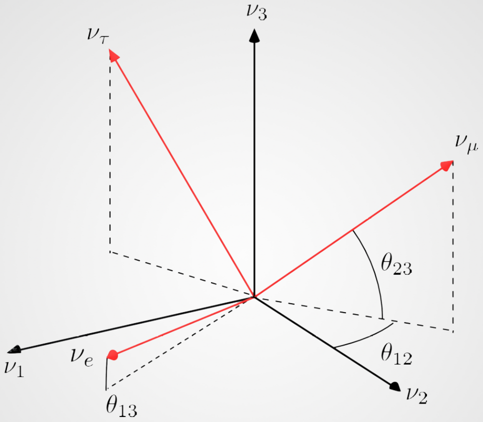
Real scalars:

$$M_a^2 = M_{0a}^2 + \Pi_{aa}(M_{0a}^2)$$

- Finite mass shifts via radiative corrections to couplings

Conclusion/Outlook

- **General idea**: build a tool for studying radiative corrections to masses and mixing angles in neutrino mass models with flavor symmetries
- Contribute to recent discussions on tadpole renormalization
  - see e. g. [4, 5, 6]
- **Next steps**:
  - Need gauge interactions to see effect of mixing angle renormalization
  - Elevate toy model to gauge theory
  - Relate approach to N-Higgs doublet models (NHDM)
  - Determine radiative corrections to masses and mixing angles in showcase ν-mass models known from the literature



Thanks!

[1] S. Stone, *New physics from flavour*, PoS ICHEP **2012** (2013) 033 [arXiv:1212.6374 [hep-ph]].

[2] M. Fox, W. Grimus and M. Löschner, *Renormalization and radiative corrections to masses in a general Yukawa model*, arXiv:1705.09589 [hep-ph].

[3] W. Grimus and L. Lavoura, *A Nonstandard CP transformation leading to maximal atmospheric neutrino mixing*, Phys. Lett. B **579** (2004) 113 doi:10.1016/j.physletb.2003.10.075 [hep-ph/0305309].

[4] A. Denner, L. Jenniches, J. N. Lang and C. Sturm, *Gauge-independent $\overline{MS}$ renormalization in the 2HDM*, JHEP **1609** (2016) 115 [arXiv:1607.07352 [hep-ph]].

[5] M. Krause, R. Lorenz, M. Mühlleitner, R. Santos and H. Ziesche, *Gauge-independent renormalization of the 2-Higgs-doublet model*, JHEP **1609** (2016) 143 [arXiv:1605.04853 [hep-ph]].

[6] L. Altenkamp, S. Dittmaier and H. Rzehak, *Renormalization schemes for the two-Higgs-doublet model and applications to $h \to WW/ZZ \to 4$ fermions*, arXiv:1704.02645 [hep-ph].