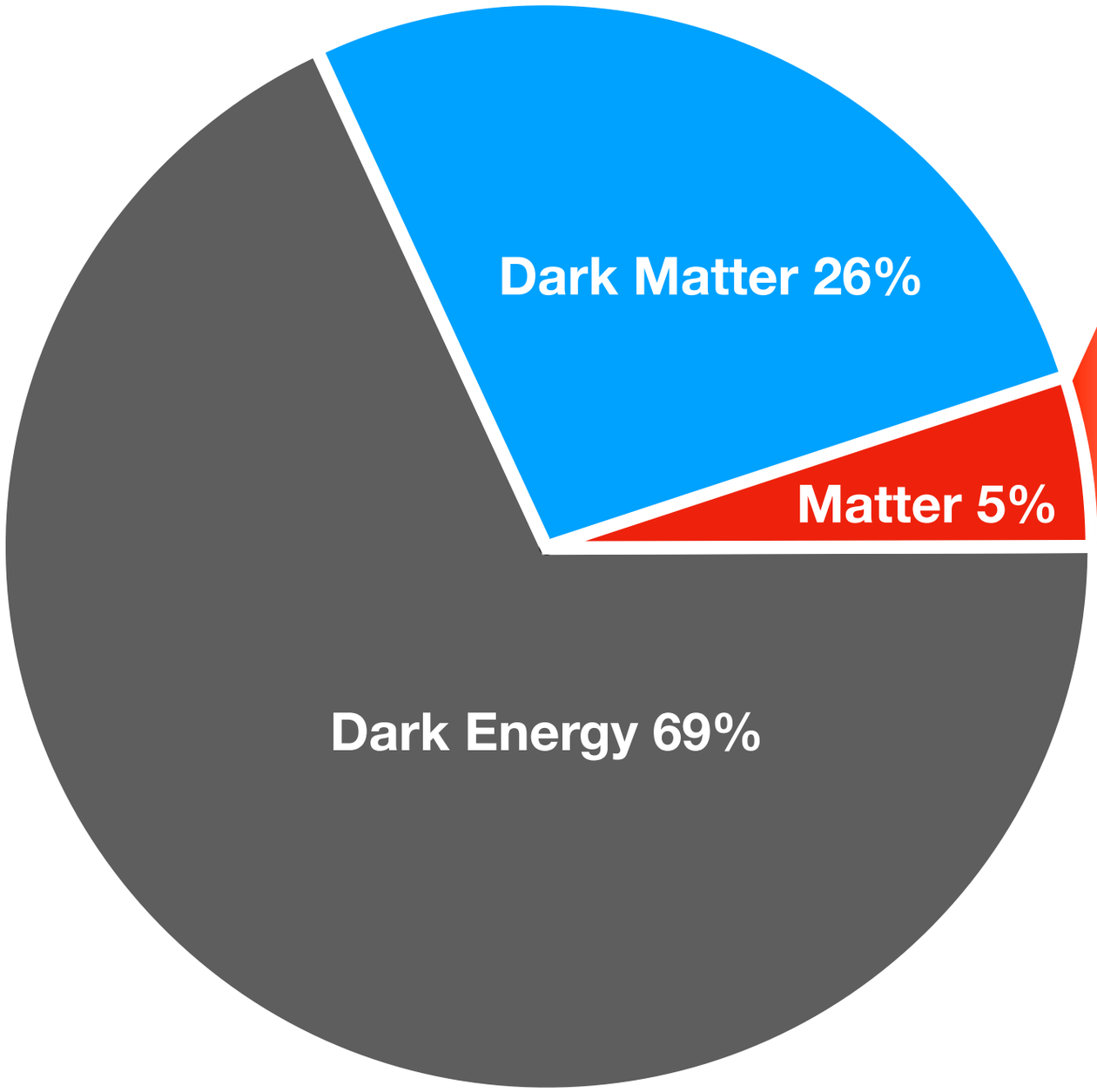
The 26th International Workshop on Weak Interactions and Neutrinos (WIN2017)

Leptogenesis via Weinberg operator

Ye-Ling Zhou, IPPP Durham, 20 June 2017

Baryon-antibaryon asymmetry



Most matter is formed by baryon, not anti-baryon.

$$\eta_B \equiv \frac{n_B - n_{\overline{B}}}{n_\gamma}$$

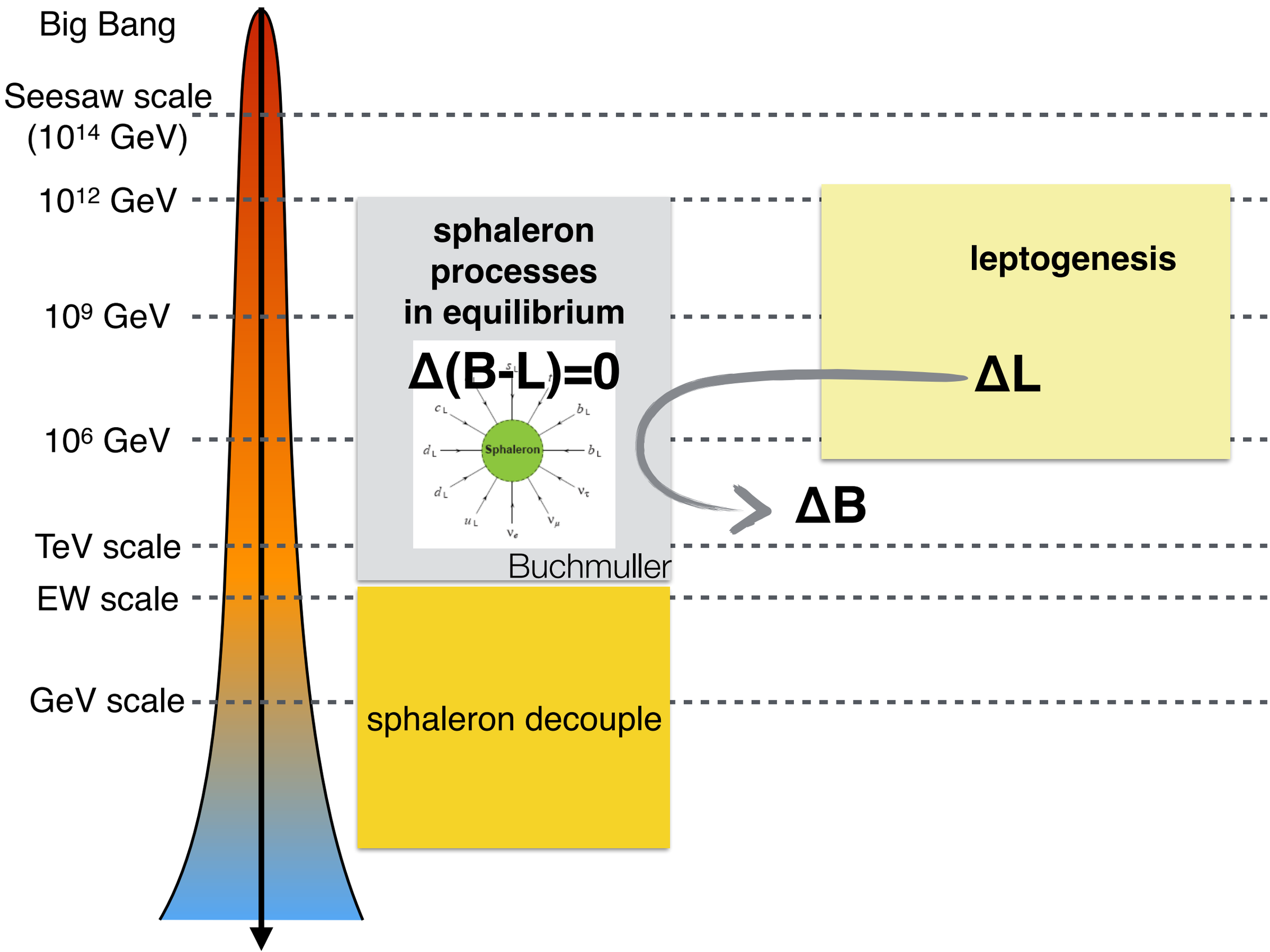
$$= 6.105^{+0.086}_{-0.081} \times 10^{-10}$$

Planck 2015

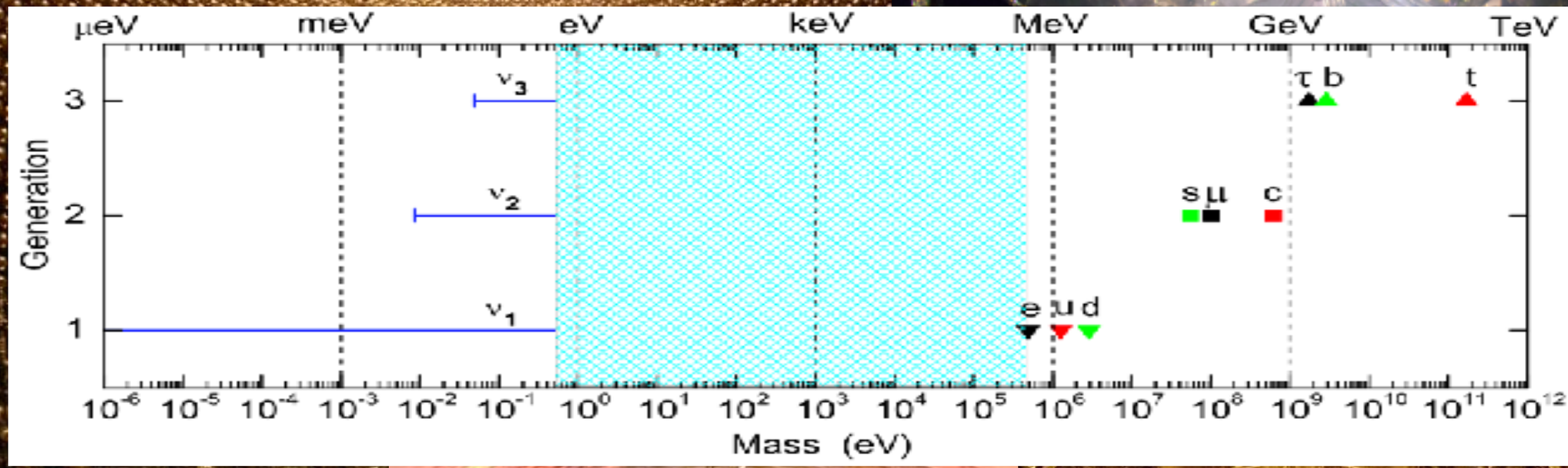
The SM cannot provide strong out-of-equilibrium dynamics and enough CP violation.

[1] Parameter	[2] 2013N(DS)	[3] 2013F(DS)	[4] 2013F(CY)	[5] 2015F(CHM)	[6] 2015F(CHM) (Plik)
$100\theta_{MC}$	1.04131 ± 0.00063	1.04126 ± 0.00047	1.04121 ± 0.00048	1.04094 ± 0.00048	1.04086 ± 0.00048
$\Omega_b h^2$	0.02205 ± 0.00028	0.02234 ± 0.00023	0.02230 ± 0.00023	0.02225 ± 0.00023	0.02222 ± 0.00023
$\Omega_c h^2$	0.1199 ± 0.0027	0.1189 ± 0.0022	0.1188 ± 0.0022	0.1194 ± 0.0022	0.1199 ± 0.0022
H_0	67.3 ± 1.2	67.8 ± 1.0	67.8 ± 1.0	67.48 ± 0.98	67.26 ± 0.98

Baryogenesis via leptogenesis



Leptogenesis and neutrino masses



Why neutrinos have masses and these masses are so tiny?

In the SM without extending particle content, the only way to generate a neutrino mass is using higher dimensional operators.

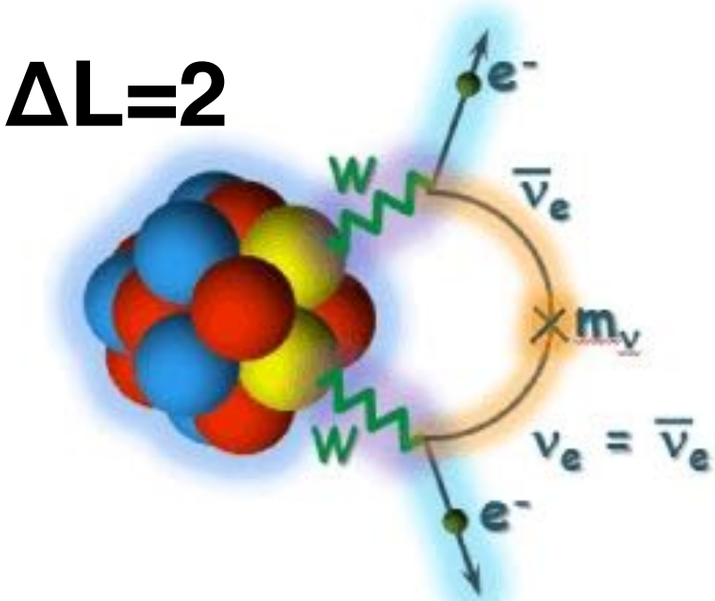
Weinberg operator

$$\mathcal{L}_W = \frac{\lambda_{\alpha\beta}}{\Lambda} \ell_{\alpha L} H C \ell_{\beta L} H + \text{h.c.}$$

$$m_\nu = \lambda \frac{v_H^2}{\Lambda} \qquad \frac{\Lambda}{\lambda} \sim 10^{15}\ \text{GeV}$$

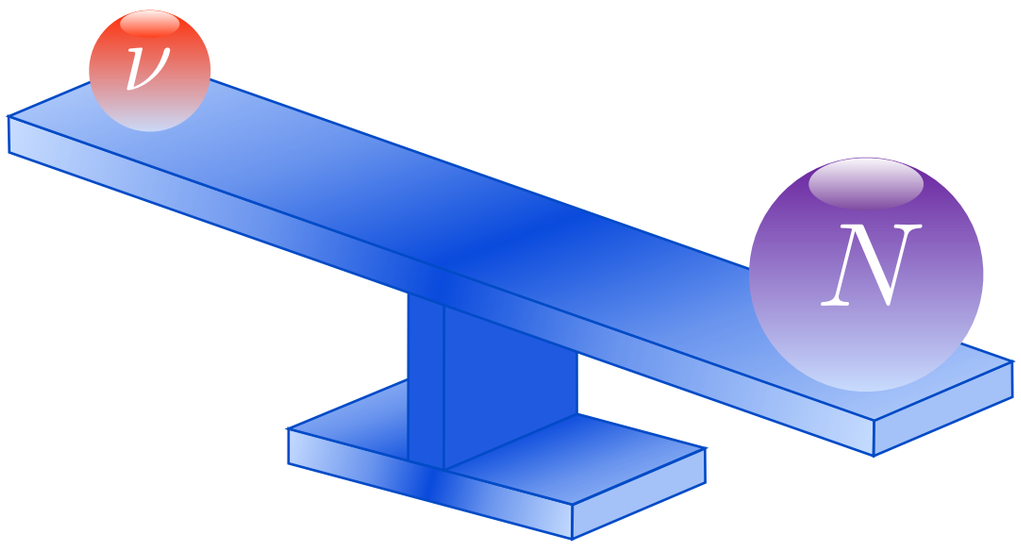
Neutrino masses are Majorana masses

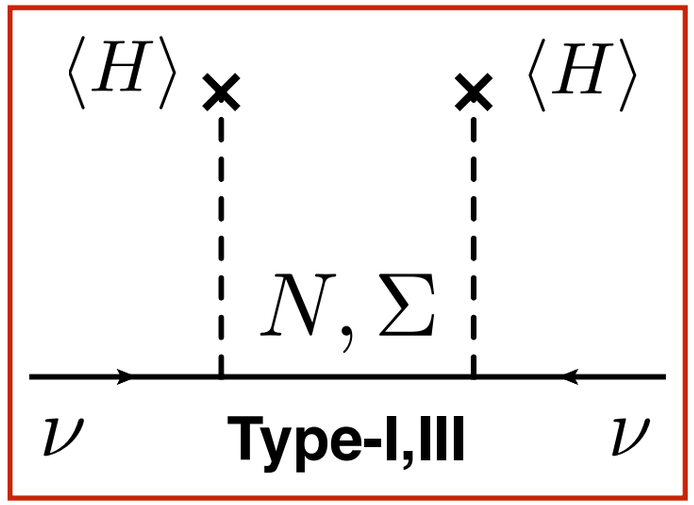
ΔL=2

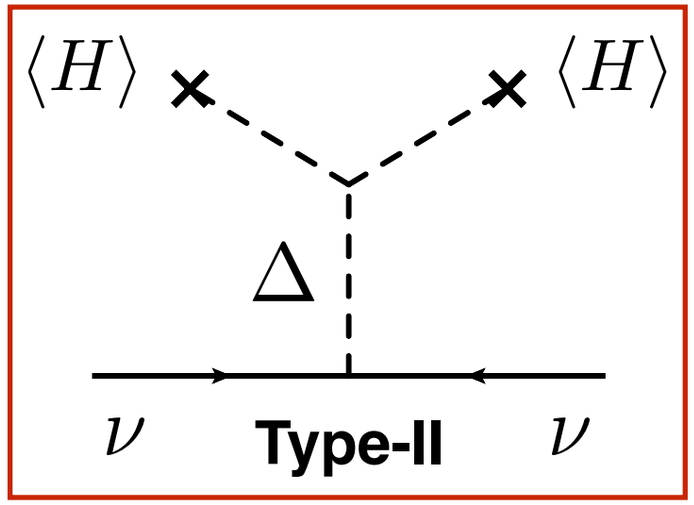


UV completions for Weinberg operator

- **Seesaw mechanism: type-I, II, III**







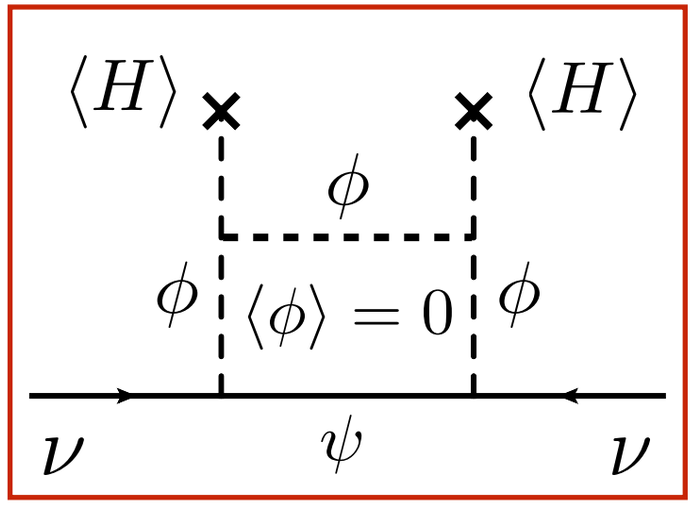
- **Low scale seesaw models**

  inverse seesaw, linear seesaw, multiple seesaw, type-(I+II), seesaw with flavor symmetries, ...

- **Radiative corrections**

  Zee, Zee-Babu models, …

- **SUSY: R-parity violation**



Baryogenesis via leptogenesis

- Sakharov conditions for leptogenesis

SM L/B-L violation

C/CP violation

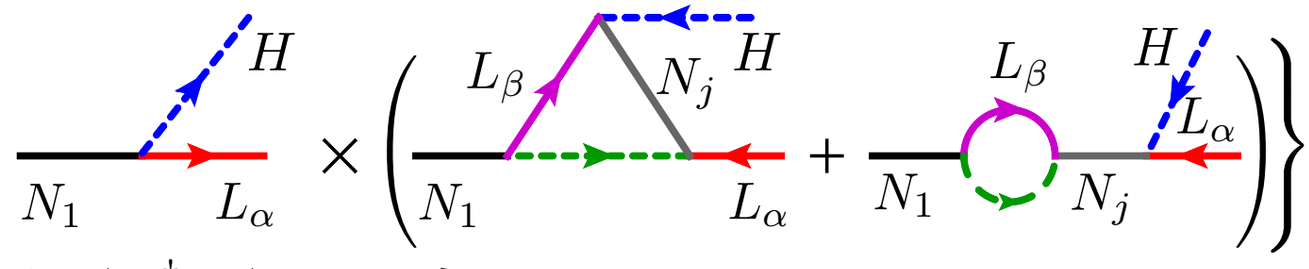
Out of equilibrium dynamics

Leptogenesis in the framework of seesaw

- **Leptogenesis via RH neutrino decays** Fukugita, Yanagida, 1986

RH neutrino *N* **CP violation** **Decay of lightest *N***

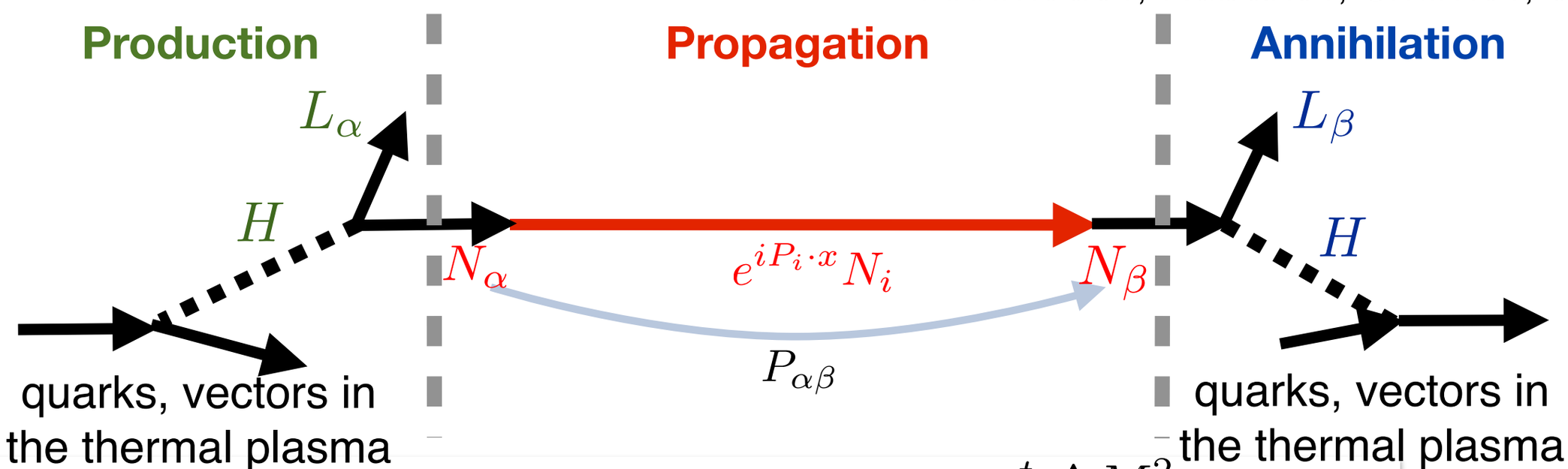
$$\Delta f_{\ell_\alpha} \equiv f_{\ell_\alpha} - f_{\bar{\ell}_\alpha} \propto \text{Im}\left\{ \times \left(+ \right)\right\}$$



$$\propto \text{Im}\{Y^*_{\nu\alpha 1}(Y^\dagger_\nu Y_\nu)_{1j} Y_{\nu\alpha j}\}$$

- **Leptogenesis via RH neutrino oscillations**

Akhmedov, Rubakov, Smirnov, 9803255



$$P(N_\alpha \to N_\beta) - P(\overline{N}_\alpha \to \overline{N}_\beta) \propto \text{Im}\{\exp(-i\int_0^t \frac{\Delta M^2_{ij}}{2E} a(t)dt)\}$$

CP violation $\alpha \neq \beta$ $\times \text{Im}\{Y_{\alpha i} Y^*_{\beta i} Y^*_{\alpha j} Y_{\beta j}\}$ $i \neq j$

The fall of Leptogenesis in the framework of seesaw

However, these mechanisms do not work if ...

- all RH neutrinos have masses above 10^{12} GeV;
- there is no physically imaginary parameter in the Yukawa coupling, thus no CP violation;
- the Majorana neutrino masses are generated by a mechanism different from type-I seesaw?

Leptogenesis via Weinberg operator

Silvia Pascoli, Jessica Turner, **YLZ**, arXiv:1609.07969

Three Sakharov conditions are satisfied as follows:

- The Weinberg operator violates lepton number and leads to LNV processes.

$$H^* H^* \leftrightarrow \ell\ell\,, \quad \bar{\ell} H^* \leftrightarrow \ell H\,, \quad \bar{\ell} H^* H^* \leftrightarrow \ell\,,$$
$$\bar{\ell} \leftrightarrow \ell H H\,, \quad H^* \leftrightarrow \ell\ell H\,, \quad 0 \leftrightarrow \ell\ell H H$$

- The Weinberg operator is very weak and can directly provide out of equilibrium dynamics in the early Universe.

$$\Gamma_{\rm W} \sim \langle \sigma n \rangle \sim \frac{1}{4\pi}\frac{\lambda^2}{\Lambda^2} T^3 \sim \frac{1}{4\pi}\frac{m_\nu^2}{v_H^4} T^3$$

$$T < 10^{12}\,{\rm GeV}$$

$$<$$

$$H_u \sim 10 \frac{T^2}{m_{\rm pl}}$$

No washout if there are no other LNV sources.

- We assume that a phase transition triggers a time-varying Weinberg operator, giving rise to CP violation.

Motivation for varying Weinberg operator

A lot of symmetries have been proposed in the lepton sector.
Their breaking may lead to a time-varying Weinberg operator.

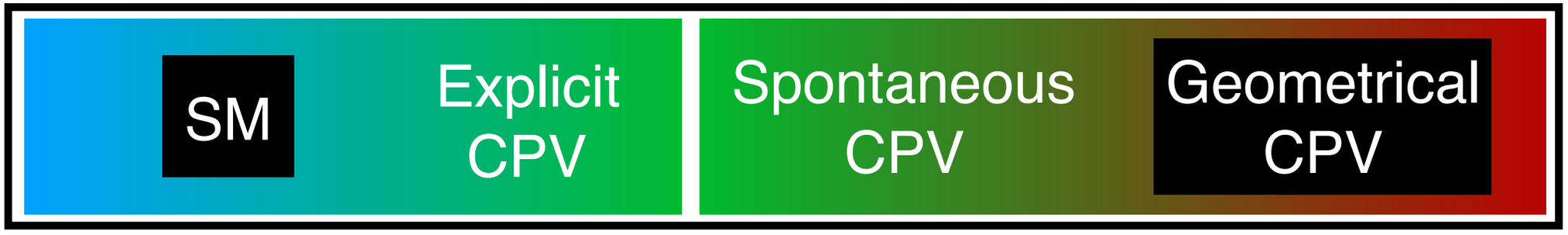
- **B-L symmetry breaking**

  To generate a CP violation, at least two scalars are needed.

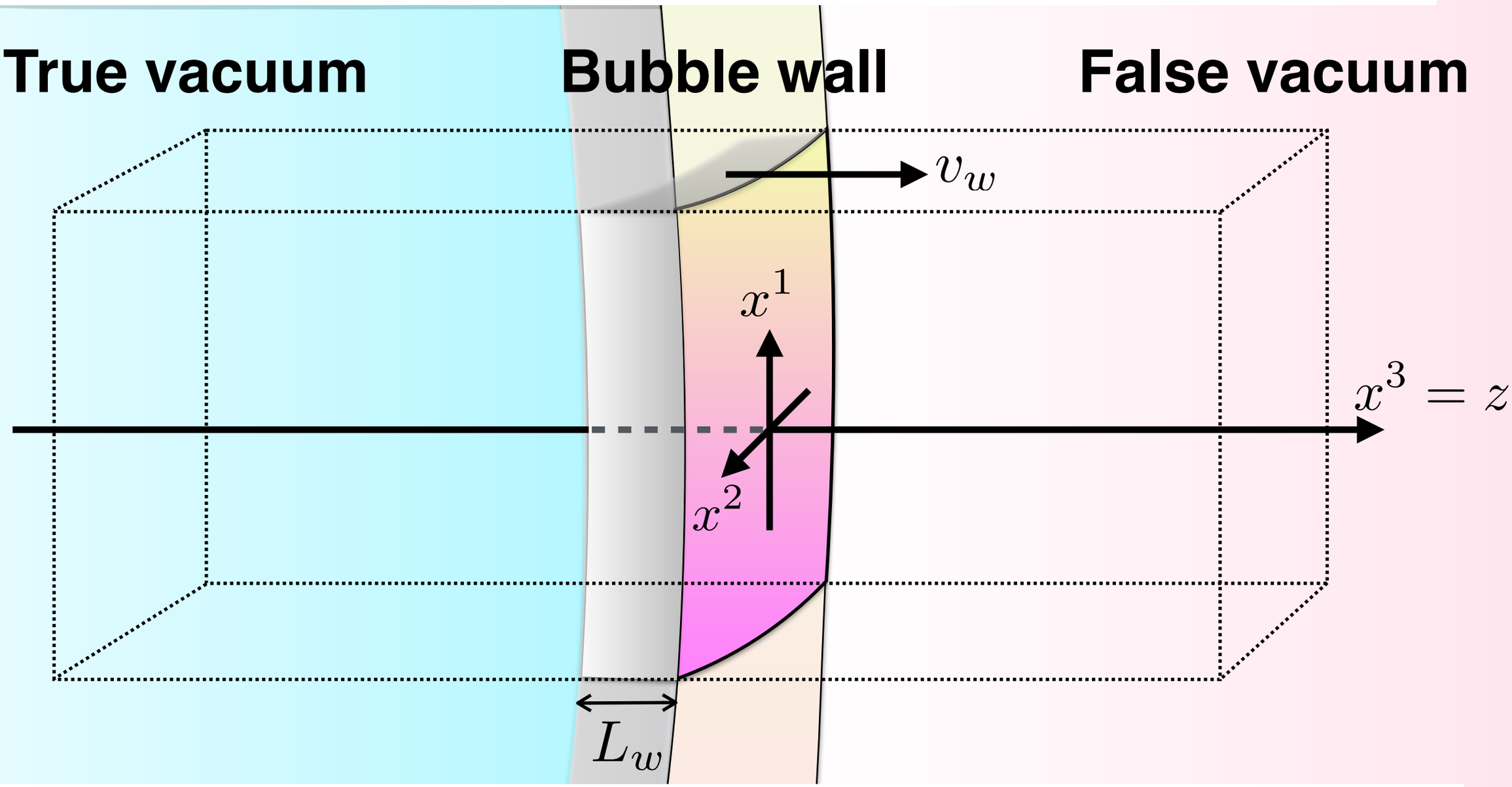
- **Flavour symmetry breaking**

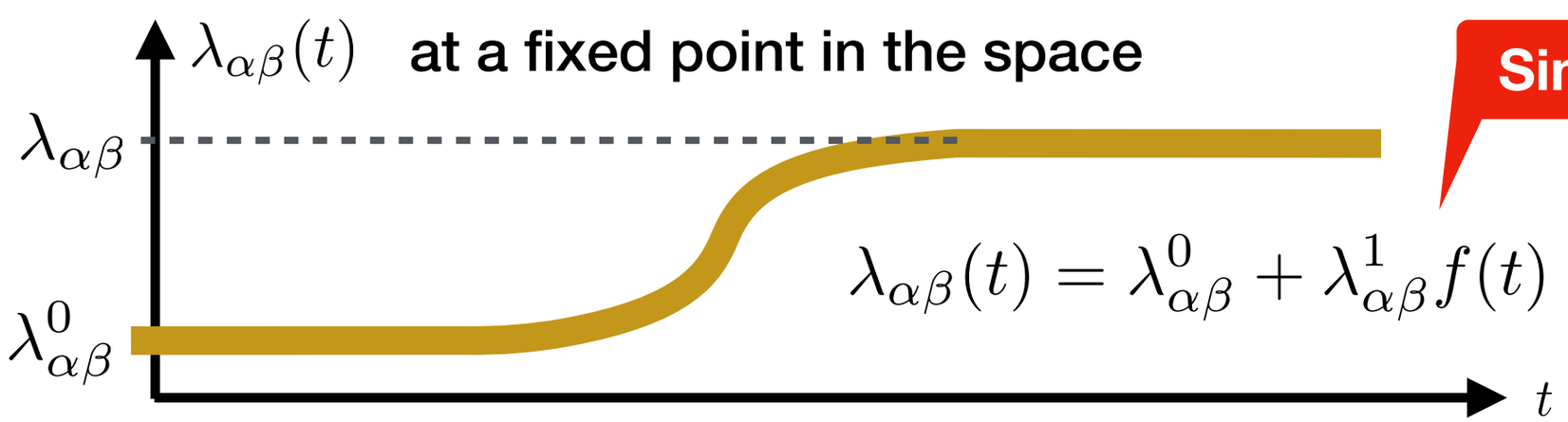
Flavour symmetries	Continuous	Discrete
Abelian	Fraggatt-Nielson, L_{mu}-L_{tau} …	Z_n
Non-Abelian	SU(3), SO(3), …	A_4, S_4, A_5, Δ(48), …

- **CP symmetry breaking**



Assuming first-order phase transition





Single-scalar case

$$\lambda_{\alpha\beta}(t) = \lambda^0_{\alpha\beta} + \lambda^1_{\alpha\beta} f(t)$$

$$f(t \to -\infty) = 0$$

$$f(t \to +\infty) = 1$$

$$\lambda^0_{\alpha\beta} + \lambda^1_{\alpha\beta} \equiv \lambda_{\alpha\beta}$$

CP violation from varying Weinberg operator

- **Example: time-dependent di-lepton production**

$$H^*_{\mathbf{q}} H^*_{\mathbf{q}'} \to \ell_{\mathbf{k}} \ell_{\mathbf{k}'}$$

- **Canonical quantisation**

$$H(x) = \int \frac{d^3\mathbf{q}}{(2\pi)^3\sqrt{2\omega_{\mathbf{q}}}} \left(a_{\mathbf{q}} e^{-i(\omega_{\mathbf{q}} x^0 - \mathbf{q}\cdot\mathbf{x})} + b^\dagger_{\mathbf{q}} e^{i(\omega_{\mathbf{q}} x^0 - \mathbf{q}\cdot\mathbf{x})} \right) ,$$

$$H^*(x) = \int \frac{d^3\mathbf{q}}{(2\pi)^3\sqrt{2\omega_{\mathbf{q}}}} \left(b_{\mathbf{q}} e^{-i(\omega_{\mathbf{q}} x^0 - \mathbf{q}\cdot\mathbf{x})} + a^\dagger_{\mathbf{q}} e^{i(\omega_{\mathbf{q}} x^0 - \mathbf{q}\cdot\mathbf{x})} \right) ;$$

$$\ell_\alpha(x) = \int \frac{d^3\mathbf{k}}{(2\pi)^3\sqrt{2\omega_{\mathbf{k}}}} \sum_s \left(a^s_{\alpha\mathbf{k}} u^s_{\mathbf{k}} e^{-i(\omega_{\mathbf{k}} x^0 - \mathbf{k}\cdot\mathbf{x})} + b^{s\dagger}_{\alpha\mathbf{k}} v^s_{\mathbf{k}} e^{i(\omega_{\mathbf{k}} x^0 - \mathbf{k}\cdot\mathbf{x})} \right)$$

$$\bar{\ell}_\alpha(x) = \int \frac{d^3\mathbf{k}}{(2\pi)^3\sqrt{2\omega_{\mathbf{k}}}} \sum_s \left(b^s_{\alpha\mathbf{k}} \bar{v}^s_{\mathbf{k}} e^{-i(\omega_{\mathbf{k}} x^0 - \mathbf{k}\cdot\mathbf{x})} + a^{s\dagger}_{\alpha\mathbf{k}} \bar{u}^s_{\mathbf{k}} e^{i(\omega_{\mathbf{k}} x^0 - \mathbf{k}\cdot\mathbf{x})} \right)$$

canonical quantisation

$$S = \mathbf{1} + (-i) \int_{-\infty}^{+\infty} dt H_I(t) + \cdots ,$$

$$H_I(t) = \int d^3\mathbf{x} \mathcal{L}_W = \frac{1}{\Lambda} \int d^3\mathbf{x}\, \lambda_{\alpha\beta}(t) \ell_{\alpha L} H C \ell_{\beta L} H + \text{h.c.}$$

- **Magnitude**

Ignoring thermal distribution factors

$$M(H^*_{\mathbf{q}} H^*_{\mathbf{q}'} \to \ell_{\mathbf{k}} \ell_{\mathbf{k}'}) \propto \frac{2}{\Lambda} \int_{-\infty}^{+\infty} dt\, \lambda^*_{\alpha\beta}(t) e^{i\Delta\omega t} e^{-i\Delta\mathbf{k}\cdot\mathbf{x}} ,$$

$$M(H_{\mathbf{q}} H_{\mathbf{q}'} \to \bar{\ell}_{\mathbf{k}} \bar{\ell}_{\mathbf{k}'}) \propto \frac{2}{\Lambda} \int_{-\infty}^{+\infty} dt\, \lambda_{\alpha\beta}(t) e^{i\Delta\omega t} e^{-i\Delta\mathbf{k}\cdot\mathbf{x}} .$$

$$\Delta\mathbf{k} = \mathbf{k} + \mathbf{k}' - \mathbf{q} - \mathbf{q}'.$$

$$\Delta\omega = \omega_{\mathbf{k}} + \omega_{\mathbf{k}'} - \omega_{\mathbf{q}} - \omega_{\mathbf{q}'}$$

$$\lambda_{\alpha\beta}(t) = |\lambda_{\alpha\beta}(t)| e^{i\phi_{\alpha\beta}(t)}$$

$$\Delta_{CP}(H^*_{\mathbf{q}} H^*_{\mathbf{q}'} \to \ell_{\mathbf{k}} \ell_{\mathbf{k}'}) \equiv \frac{|M(H^*_{\mathbf{q}} H^*_{\mathbf{q}'} \to \ell_{\mathbf{k}} \ell_{\mathbf{k}'})|^2 - |M(H_{\mathbf{q}} H_{\mathbf{q}'} \to \bar{\ell}_{\mathbf{k}} \bar{\ell}_{\mathbf{k}'})|^2}{|M(H^*_{\mathbf{q}} H^*_{\mathbf{q}'} \to \ell_{\mathbf{k}} \ell_{\mathbf{k}'})|^2 + |M(H_{\mathbf{q}} H_{\mathbf{q}'} \to \bar{\ell}_{\mathbf{k}} \bar{\ell}_{\mathbf{k}'})|^2}$$

CP violation from varying Weinberg operator

- **CP violation of di-lepton <u>production</u> and <u>annihilation</u>**

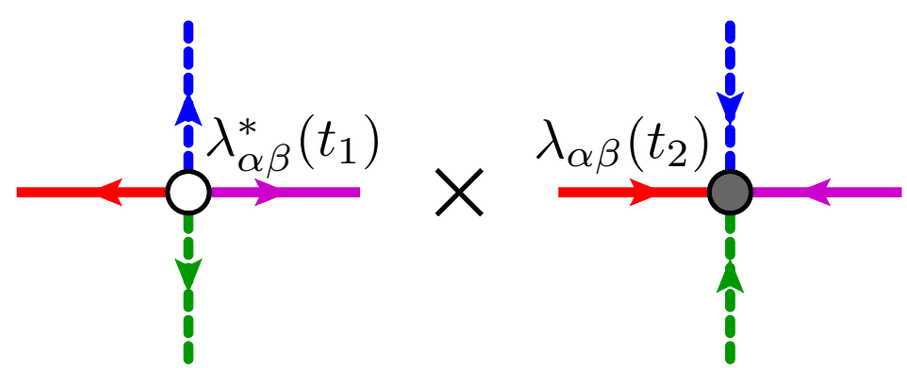
$$\Delta_{CP}(H^*_{\mathbf{q}}H^*_{\mathbf{q}'} \to \ell_{\mathbf{k}}\ell_{\mathbf{k}'}) \propto \int_{-\infty}^{+\infty} dt_1 dt_2 \, \mathrm{Im}\Big\{\mathrm{tr}\,[\lambda^*(t_1)\lambda(t_2)]\Big\}\,\mathrm{Im}\Big\{e^{i\Delta\omega(t_1-t_2)}\Big\}$$

$$\Delta_{CP}(\ell_{\mathbf{k}}\ell_{\mathbf{k}'} \to H^*_{\mathbf{q}}H^*_{\mathbf{q}'}) \propto \int_{-\infty}^{+\infty} dt_1 dt_2 \, \mathrm{Im}\Big\{\mathrm{tr}\,[\lambda(t_1)\lambda^*(t_2)]\Big\}\,\mathrm{Im}\Big\{e^{-i\Delta\omega(t_1-t_2)}\Big\}$$

$$\Delta_{CP}(\ell_{\mathbf{k}}\ell_{\mathbf{k}'} \to H^*_{\mathbf{q}}H^*_{\mathbf{q}'}) = \Delta_{CP}(H^*_{\mathbf{q}}H^*_{\mathbf{q}'} \to \ell_{\mathbf{k}}\ell_{\mathbf{k}'})$$

- **Total lepton asymmetry**

$$\Delta f_\ell \sim 2\Delta_{CP}(H^*H^* \leftrightarrow \ell\ell)\Big[\gamma(H^*H^* \to \ell\ell) - \gamma(\ell\ell \to H^*H^*)\Big]/T \quad + \cdots$$

$\Delta f_{\ell_\alpha} \propto$ Im $\Big\{$ 

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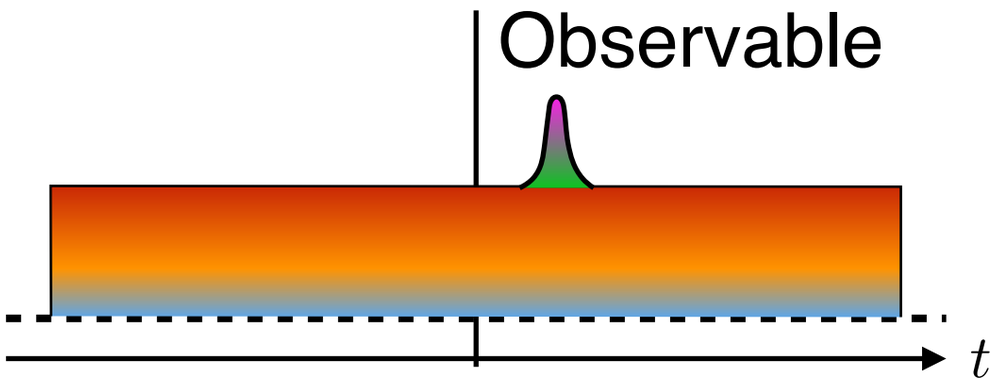
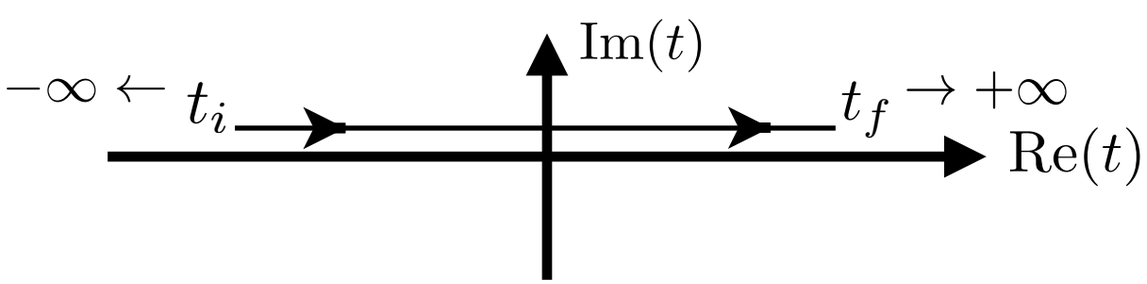
Motivation for closed-time-path (CTP) approach

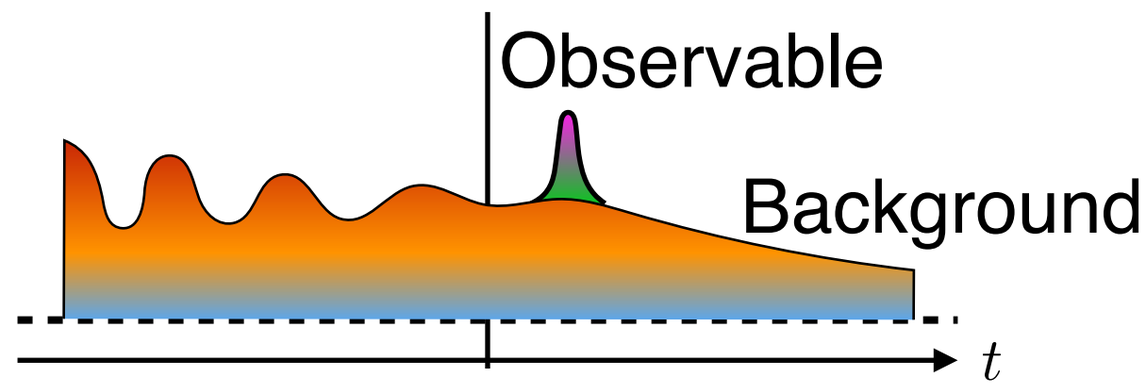
- QFT at zero temperature or in thermal equilibrium



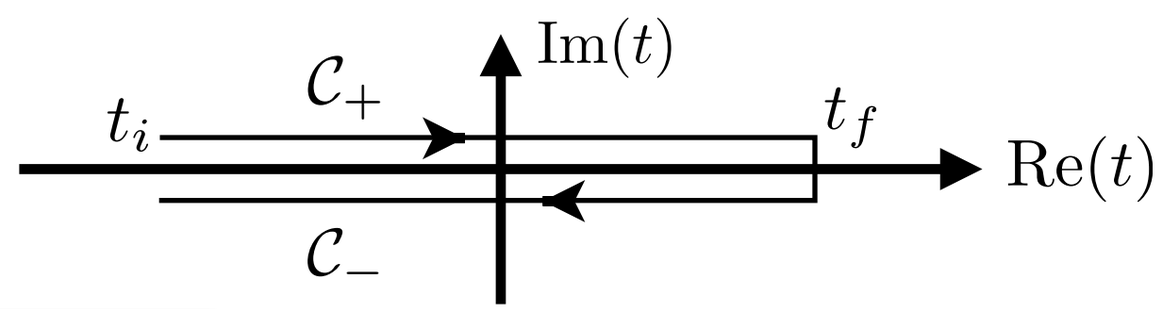
Vacuum/background is in thermal equilibrium, time-dependent



- QFT in non-equilibrium case



Background is time-dependent. We have to specify a time.



$$\mathcal{C} = \mathcal{C}_+ \cup \mathcal{C}_-$$

$$\langle \Omega(t) | \mathcal{O} | \Omega(t) \rangle$$

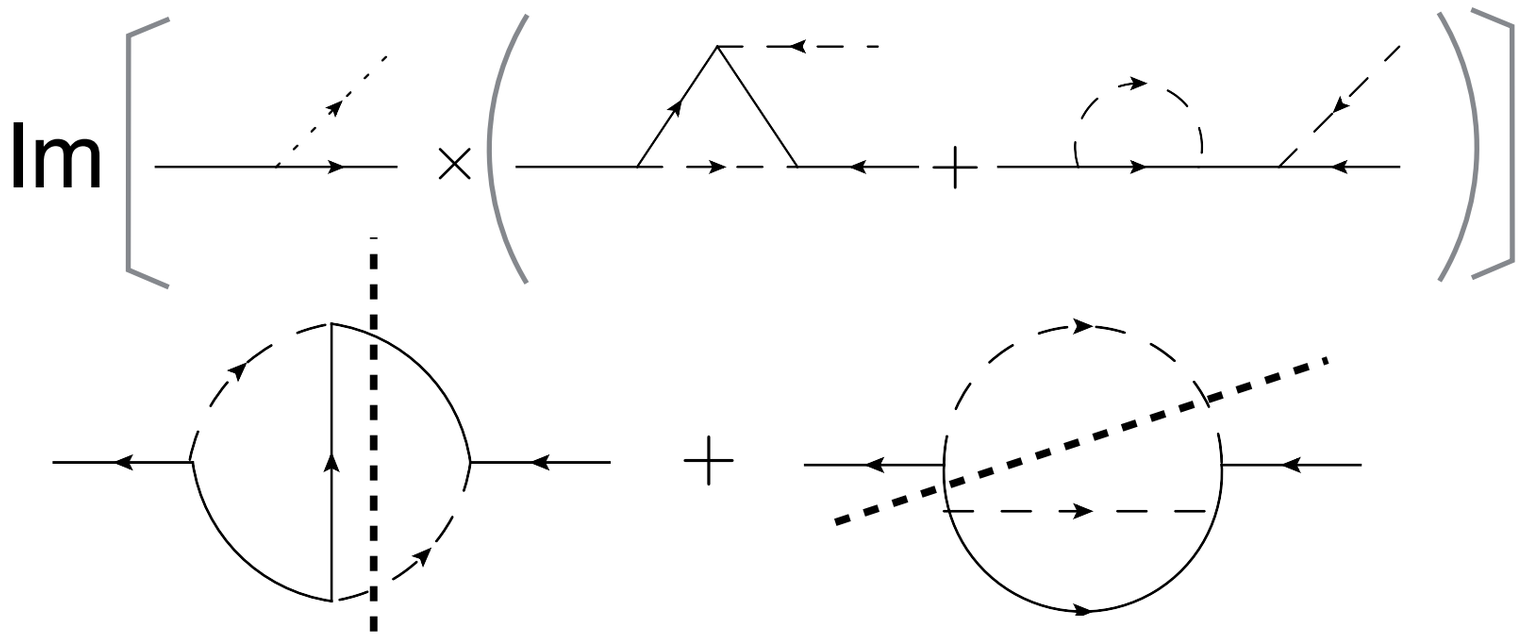
In-out formalism $\langle \Omega(t_f | \mathcal{O} | \Omega(t_i) \rangle$

In-in formalism $\langle \Omega(t_i) | \mathcal{O} | \Omega(t_i) \rangle$

Classical formalism vs CTP formalism

- **Leptogenesis via RH neutrino decay**

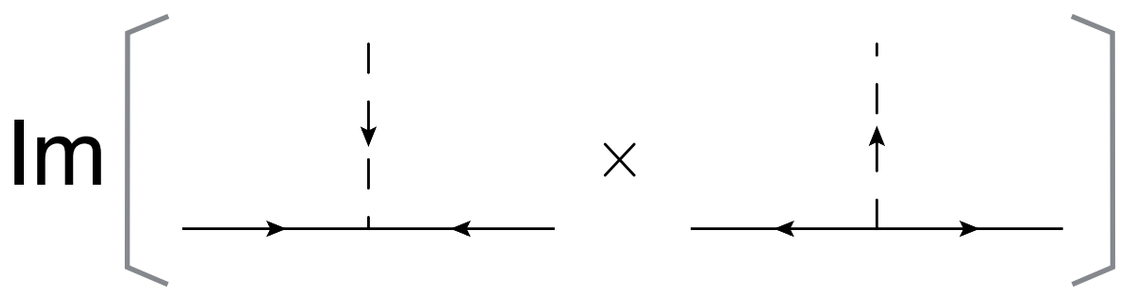
Anisimov, Buchmuller, Drewes, Mendizabal, 1012.5821

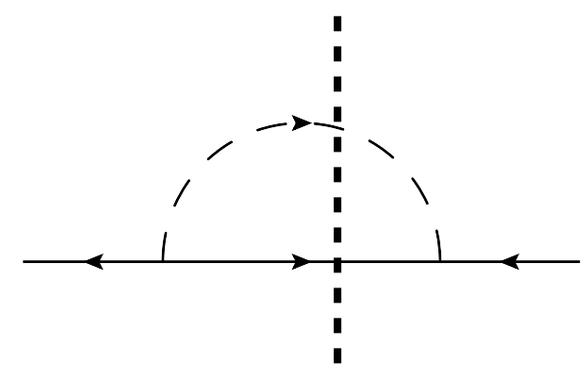


CPV source in classical formalism

Self energies including CPV source in CTP formalism

- **Leptogenesis via RH neutrino oscillation**





CPV source in classical formalism

Self energy including CPV source in CTP formalism

Leptogenesis via Weinberg operator (in CTP approach)

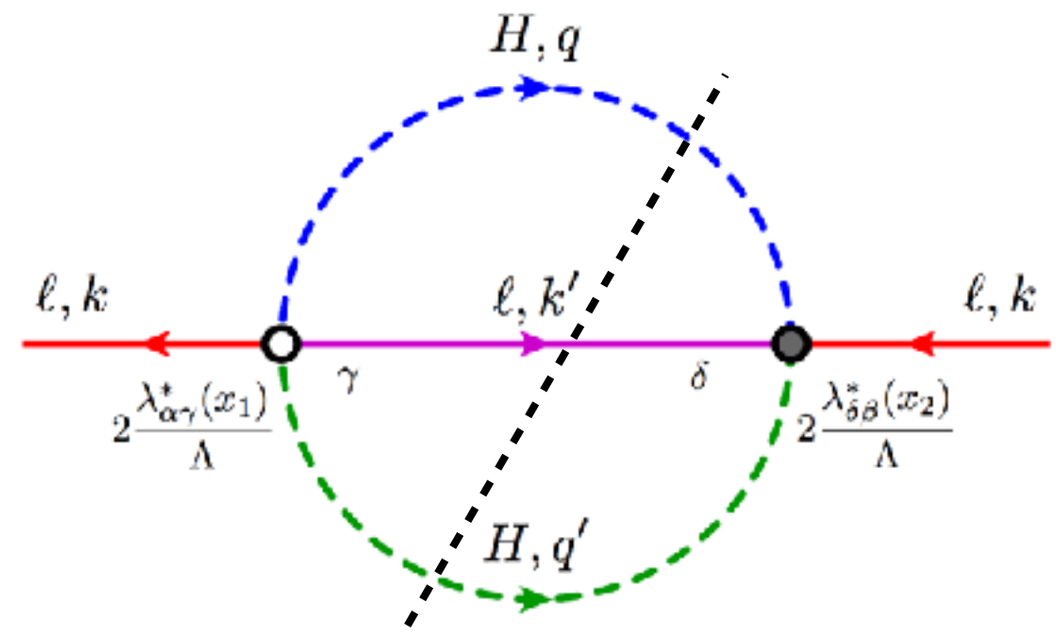
CPV source in classical formalism

$$\Delta f_{\ell_\alpha} \propto \mathrm{Im}\left\{ \lambda^*_{\alpha\beta}(t_1) \times \lambda_{\alpha\beta}(t_2) \right\}$$

Self energies including CPV source in CTP formalism

$$\Sigma^{<,>}_{\alpha\beta}(x_1,x_2) = 3 \times \frac{4}{\Lambda^2} \sum_{\gamma\delta} \lambda^*_{\alpha\gamma}(x_1)\lambda_{\delta\beta}(x_2)$$

$$\times \Delta^{>,<}(x_2,x_1)\Delta^{>,<}(x_2,x_1)S^{>,<}_{\gamma\delta}(x_2,x_1)\,,$$



$$\Delta N_\ell = -\frac{12}{\Lambda^2}\int d^4x d^4r\,(-i)\mathrm{tr}[\lambda^*(x_1)\lambda(x_2)]\mathcal{M}\,.$$

$$\mathcal{M} = i\int \frac{d^4k}{(2\pi)^4}\frac{d^4k'}{(2\pi)^4}\frac{d^4q}{(2\pi)^4}\frac{d^4q'}{(2\pi)^4}e^{iK\cdot(-r)}\left\{\Delta^<_q(x)\Delta^<_{q'}(x)\mathrm{tr}[S^<_k(x)S^<_{k'}(x)] - \Delta^>_q(x)\Delta^>_{q'}(x)\mathrm{tr}[S^>_k(x)S^>_{k'}(x)]\right\}$$

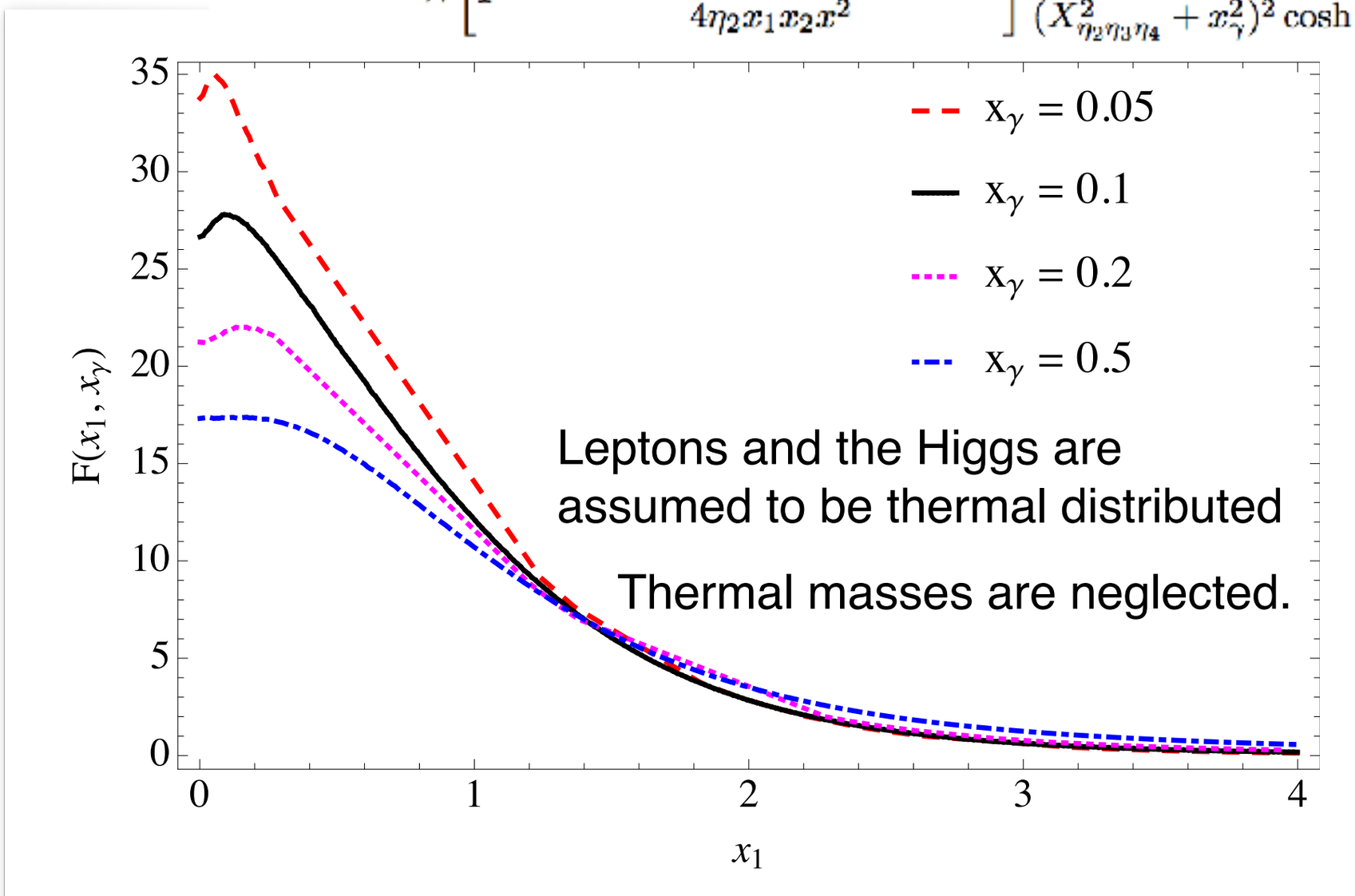
The final lepton asymmetry is determined by **the behaviour of Weinberg operator during the phase transition** and **thermal properties of leptons and the Higgs**.

Leptogenesis via Weinberg operator (in CTP approach)

$$\Delta f_\ell = \frac{3\,\mathrm{Im}\{\mathrm{tr}[m_\nu^0 m_\nu^*]\}T^2}{(2\pi)^4 v_H^4} F(x_1, x_\gamma)$$

$$\Delta n_\ell = \int \frac{d^3\mathbf{k}}{(2\pi)^3} \Delta f_\ell$$

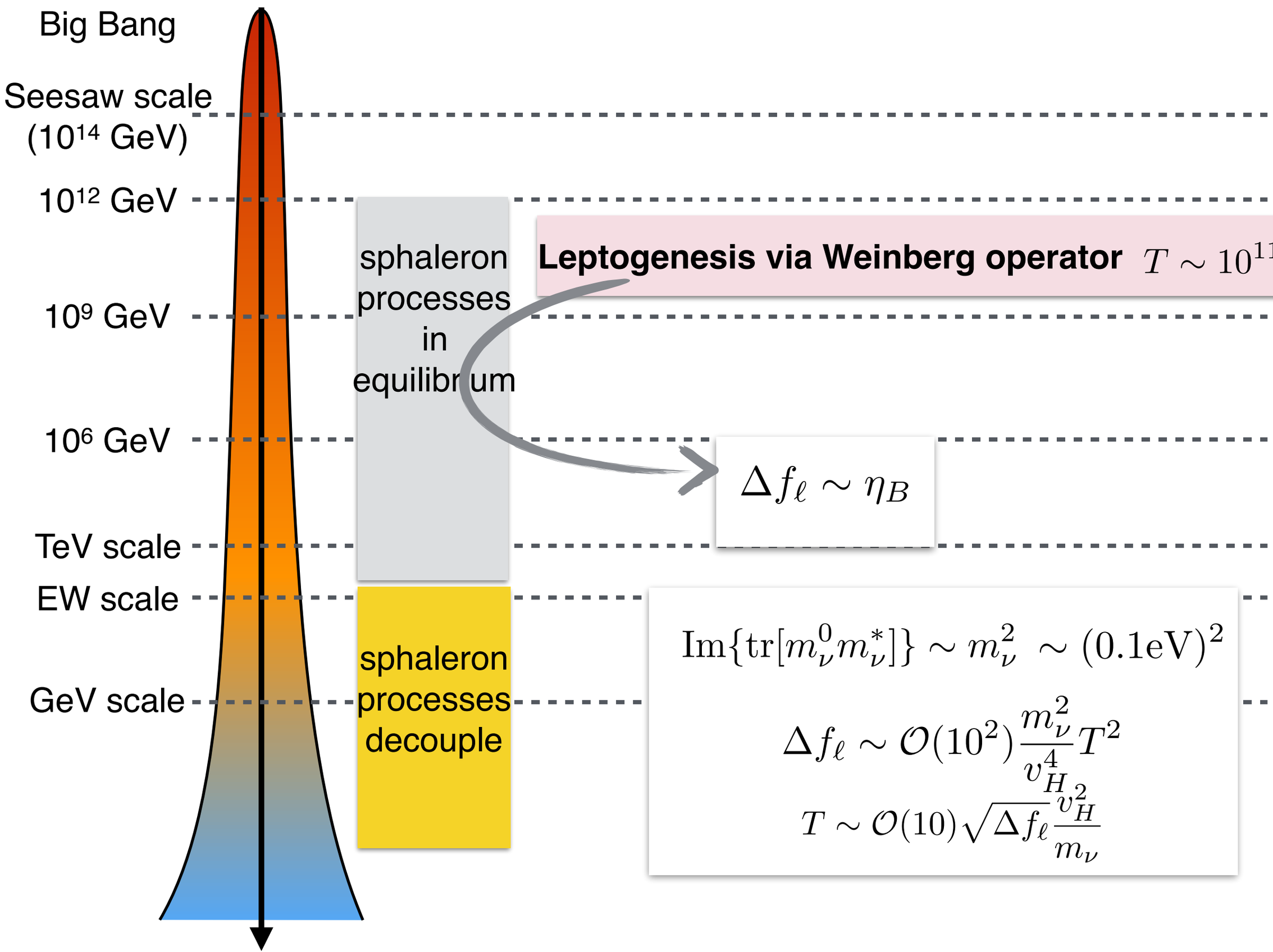
$$F(x_1, x_\gamma) = \frac{1}{x_1}\int_0^{+\infty} dx \int_0^{+\infty} x_2 dx_2 \int_{|x_1-x|}^{x_1+x} dx_3 \int_{|x_2-x|}^{x_2+x} dx_4 \sum_{\eta_2,\eta_3,\eta_4=\pm 1}$$
$$\times \left[1 - \frac{(x_1^2+x^2-x_3^2)(x_2^2+x^2-x_4^2)}{4\eta_2 x_1 x_2 x^2}\right] \frac{X_{\eta_2\eta_3\eta_4} x_\gamma \sinh X_{\eta_2\eta_3\eta_4}}{(X^2_{\eta_2\eta_3\eta_4}+x_\gamma^2)^2 \cosh x_1 \cosh x_2 \sinh x_3 \sinh x_4}$$



$$m_\nu^0 = \lambda^0 \frac{v_H^2}{\Lambda}$$
$$m_\nu = \lambda \frac{v_H^2}{\Lambda}$$

$$x_1 = |\mathbf{k}|\beta/2,$$
$$x_2 = |\mathbf{k}'|\beta/2,$$
$$x_3 = |\mathbf{q}|\beta/2,$$
$$x_4 = |\mathbf{q}'|\beta/2,$$
$$x_\gamma = \Gamma\beta/2,$$
$$\Gamma = 2(\gamma_H + \gamma_\ell)$$
$$\beta \equiv 1/T$$

Leptogenesis via RH neutrino decays



Leptogenesis via ...

in the framework of seesaw

RH neutrino decay

flavour effect

resonant decay

RH neutrino oscillation

Weinberg operator

Thank you very much!

Back up

CTP approach

- **Propagators**

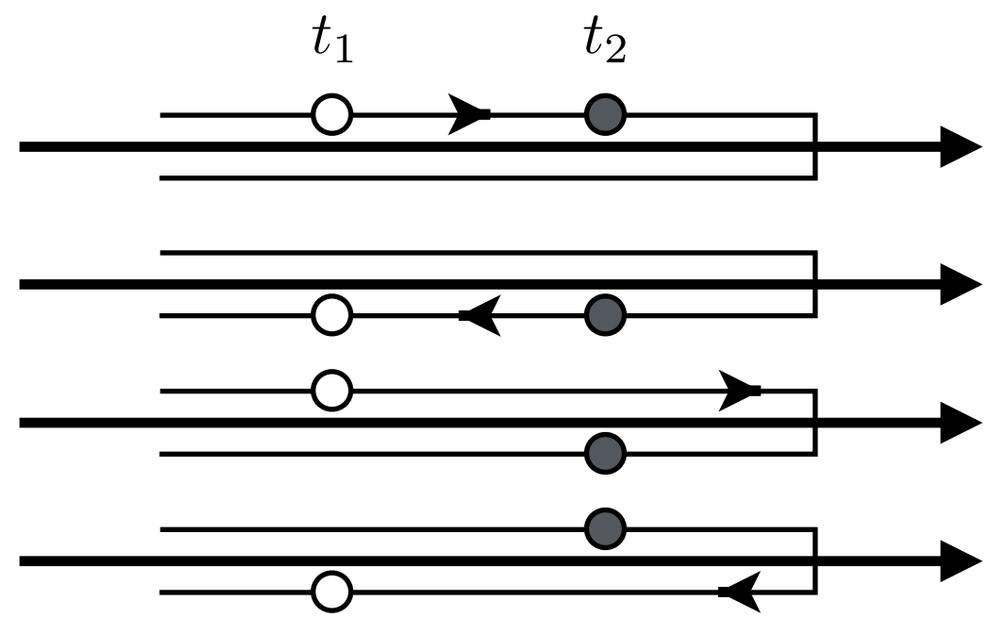
Feynman $S^{T}_{\alpha\beta}(x_1,x_2) = \langle T[\ell_\alpha(x_1)\bar{\ell}_\beta(x_2)]\rangle$

Dyson $S^{\overline{T}}_{\alpha\beta}(x_1,x_2) = \langle \overline{T}[\ell_\alpha(x_1)\bar{\ell}_\beta(x_2)]\rangle$

Wightman

$S^{<}_{\alpha\beta}(x_1,x_2) = -\langle \bar{\ell}_\beta(x_2)\ell_\alpha(x_1)\rangle$

$S^{>}_{\alpha\beta}(x_1,x_2) = \langle \ell_\alpha(x_1)\bar{\ell}_\beta(x_2)\rangle$



$x_1^\mu = (t_1, \vec{x}_1) \qquad x_2^\mu = (t_2, \vec{x}_2)$

- **Kadanoff-Baym equation**

$$i\partial\!\!\!/ S^{<,>} - \Sigma^H \odot S^{<,>} - \Sigma^{<,>} \odot S^H = \frac{1}{2}[\Sigma^> \odot S^< - \Sigma^< \odot S^>]$$

Lepton asymmetry | **Self energy correction** | **Dispersion relations** | **Collision term**

$$\Delta n_{\ell\alpha}(x) = -\frac{1}{2}\mathrm{tr}\left\{\gamma^0\left[S^<_{\alpha\alpha}(x,x) + S^>_{\alpha\alpha}(x,x)\right]\right\}$$

$$\Delta f_{\ell_\alpha}(k) = -\int_{t_i}^{t_f} dt_1 \partial_{t_1} \mathrm{tr}[\gamma_0 S^<_{\vec{k}}(t_1,t_1) + \gamma_0 S^>_{\vec{k}}(t_1,t_1)]$$

$$S^H = S^T - \frac{1}{2}(S^> + S^<)$$

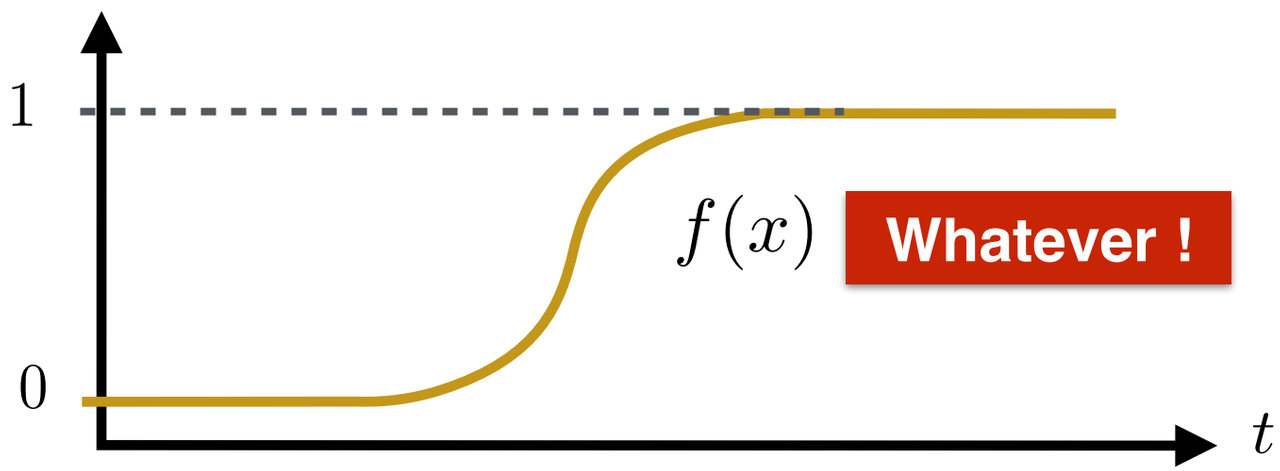
$$\Sigma^H = \Sigma^T - \frac{1}{2}(\Sigma^> + \Sigma^<)$$

CPV source

Influence of phase transition

- **Single-scalar phase transition**

$$\lambda(x) = \lambda^0 + \lambda^1 f(x) \qquad f(x) \equiv \frac{\langle \phi(x) \rangle}{v_\phi}$$



$$m_\nu^0 = \lambda^0 \frac{v_H^2}{\Lambda}$$

$$m_\nu = \lambda \frac{v_H^2}{\Lambda}$$

$$\int d^4x \mathrm{Im}\{\mathrm{tr}[\lambda^*(x_1)\lambda(x_2)]\} = \mathrm{Im}\{\mathrm{tr}[\lambda^0\lambda^*]\}\left(r^0 - \frac{r^3}{v_w}\right)V$$

$$\Delta n_\ell^{\mathrm{I}} = -\frac{12}{\Lambda^2}\mathrm{Im}\{\mathrm{tr}[\lambda^0\lambda^*]\}\boxed{\int d^4r\, r^0\, \mathcal{M}}$$

time-dependent integration

$$\Delta n_\ell^{\mathrm{II}} = \frac{12}{v_w\Lambda^2}\mathrm{Im}\{\mathrm{tr}[\lambda^0\lambda^*]\}\boxed{\int d^4r\, r^3\, \mathcal{M}}$$

space-dependent integration

$$\Delta n_\ell = \Delta n_\ell^{\mathrm{I}} + \Delta n_\ell^{\mathrm{II}}$$

Influence of phase transition

- **Multi-scalar phase transition (in the thick-wall limit)**

e.g., $\lambda(x) = \lambda^0 + \lambda^1 f_1(x) + \lambda^2 f_2(x)$

$$\mathrm{Im}\{\mathrm{tr}[\lambda^*(x_1)\lambda(x_2)]\} = \mathrm{Im}\{\mathrm{tr}[\lambda^0\lambda^{1*}]\}[f_1(x_1) - f_1(x_2)] + \mathrm{Im}\{\mathrm{tr}[\lambda^0\lambda^{2*}]\}[f_2(x_1) - f_2(x_2)]$$
$$\boxed{+\mathrm{Im}\{\mathrm{tr}[\lambda^{1*}\lambda^2]\}[f_1(x_1)f_2(x_2) - f_1(x_2)f_2(x_1)]}$$

Interferences of different scalar VEVs cannot be neglected.

$$\int d^4r \mathrm{Im}\{\mathrm{tr}[\lambda^*(x_1)\lambda(x_2)]\}\mathcal{M} = \int d^4r \mathrm{Im}\{\mathrm{tr}[\lambda^*(x+r/2)\lambda(x-r/2)]\}\mathcal{M}$$
$$\approx \mathrm{Im}\{\mathrm{tr}[\lambda^*(x)\partial_\mu\lambda(x)]\}\int d^4r r^\mu \mathcal{M}\,.$$

$$\Delta n_\ell^{\mathrm{I}} \propto \boxed{\mathrm{Im}\{\mathrm{tr}[\lambda^*(x)\partial_t\lambda(x)]\}}\boxed{\int d^4r\, r^0\, \mathcal{M}}$$ **time-dependent integration**

$$\Delta n_\ell^{\mathrm{II}} \propto \boxed{\mathrm{Im}\{\mathrm{tr}[\lambda^*(x)\partial_z\lambda(x)]\}}\boxed{\int d^4r\, r^3\, \mathcal{M}}$$ **space-dependent integration**

Time derivative/spatial gradient

Silvia Pascoli, Jessica Turner, **YLZ**, in progress

Influence of thermal effects

Thermal effects influence the time- and space-dependent integration.

$$\boxed{\int d^4r\, r^0\, \mathcal{M}} \qquad \boxed{\int d^4r\, r^3\, \mathcal{M}}$$

$$\mathcal{M} = i\int \frac{d^4k}{(2\pi)^4}\frac{d^4k'}{(2\pi)^4}\frac{d^4q}{(2\pi)^4}\frac{d^4q'}{(2\pi)^4} e^{iK\cdot(-r)} \left\{\Delta_q^<(x)\Delta_{q'}^<(x)\mathrm{tr}[S_k^<(x)S_{k'}^<(x)] - \Delta_q^>(x)\Delta_{q'}^>(x)\mathrm{tr}[S_k^>(x)S_{k'}^>(x)]\right\}$$

- **Resummed propagators of the Higgs and leptons**

$$\Delta_q^{<,>} = \frac{-2\varepsilon(q^0)\mathrm{Im}\Pi_q^R}{[q^2+\mathrm{Re}\Pi_q^R]^2+[\mathrm{Im}\Pi_q^R]^2}\left\{\vartheta(\mp q^0) + f_{B,|q^0|}(x)\right\},$$

$$S_k^{<,>} = \frac{-2\varepsilon(k^0)\mathrm{Im}\Sigma_k^{R2}}{[k^2+\mathrm{Re}\Sigma_q^{R2}]^2+[\mathrm{Im}\Sigma_q^{R2}]^2}\left\{\vartheta(\mp k^0) - f_{F,|q^0|}(x)\right\}P_L\not{k}P_R,$$

thermal equilibrium $\quad f_{B,|q^0|} \equiv \frac{1}{e^{\beta|q^0|}-1}, \qquad f_{F,|k^0|} \equiv \frac{1}{e^{\beta|k^0|}+1},$

thermal mass $\quad m_{\mathrm{th},H}^2 = \mathrm{Re}\Pi \qquad m_{\mathrm{th},\ell} = \mathrm{Re}\Sigma$

thermal width $\quad \gamma_H = \frac{\mathrm{Im}\Pi}{2m_{\mathrm{th},H}} \qquad \gamma_\ell = \frac{\mathrm{Im}\Sigma^2}{2m_{\mathrm{th},\ell}}$

By assuming thermal equilibrium in the rest frame of plasma, the space-dependent integration is zero.

$\mathcal{M}$ **is invariant under parity transformation**

$$r \to r^P = (r^0, -\mathbf{r}), \quad k_n \to k_n^P = (k_n^0, -\mathbf{k}_n) \quad \Longrightarrow \quad \boxed{\int d^4r\, r^3\, \mathcal{M} = 0}$$

Influence of thermal effects

- **Performing the time-dependent integration**

From 4D momentum space to 3D momentum space + 1D time

$$\Delta_{\mathbf{q}}^{<,>}(t_1,t_2) = \int \frac{dq^0}{2\pi} e^{-iq^0 y} \Delta_q^{<,>} = \frac{\cos(\omega_{\mathbf{q}} y^{\mp})}{2\omega_{\mathbf{q}} \sinh(\omega_{\mathbf{q}}\beta/2)} e^{-\gamma_{H,\mathbf{q}}|y|} ,$$

$$S_{\mathbf{k}}^{<,>}(t_1,t_2) = \int \frac{dk^0}{2\pi} e^{-ik^0 y} S_k^{<,>} = -P_L \frac{\gamma^0 \cos(\omega_{\mathbf{k}} y^{\mp}) + i\vec{\gamma}\cdot\hat{\mathbf{k}} \sin(\omega_{\mathbf{k}} y^{\mp})}{2\cosh(\omega_{\mathbf{k}}\beta/2)} e^{-\gamma_{\ell,\mathbf{k}}|y|} ,$$

$$y = r^0$$

$$y^- = y - i\beta/2$$

$$\omega_{\mathbf{q}} = \sqrt{m_{H,\mathrm{th}}^2 + \mathbf{q}^2},\ \omega_{\mathbf{k}} = \sqrt{m_{\ell,\mathrm{th}}^2 + \mathbf{k}^2} \text{ and } \hat{\mathbf{k}} \equiv \mathbf{k}/\omega_{\mathbf{k}}$$

Integrating out the time

$$\int d^4r\, y\mathcal{M} = 2\int \frac{d^3\mathbf{k}}{(2\pi)^3}\frac{d^3\mathbf{q}}{(2\pi)^3}\frac{d^3\mathbf{q}'}{(2\pi)^3}\int dy\, yM ;$$

$$\Gamma = 2(\gamma_H + \gamma_\ell)$$

$$\int_{-\infty}^{+\infty} dy y M = 2\int_0^{+\infty} dy y M$$

$$= 2\int_0^{+\infty} dy y \frac{\mathrm{Im}\{\cos(\omega_{\mathbf{q}} y^-)\cos(\omega_{\mathbf{q}'} y^-)[\cos(\omega_{\mathbf{k}} y^-)\cos(\omega_{\mathbf{k}'} y^-) + \hat{\mathbf{k}}\cdot\hat{\mathbf{k}}'\sin(\omega_{\mathbf{k}} y^-)\sin(\omega_{\mathbf{k}'} y^-)]\}}{8\omega_{\mathbf{q}}\omega_{\mathbf{q}'}\sinh(\omega_{\mathbf{q}}\beta/2)\sinh(\omega_{\mathbf{q}'}\beta/2)\cosh(\omega_{\mathbf{k}}\beta/2)\cosh(\omega_{\mathbf{k}'}\beta/2)} e^{-\Gamma y}$$

$$= -\sum_{\eta_2,\eta_3,\eta_4=\pm1} \frac{\Omega_{\eta_2\eta_3\eta_4}\Gamma\sinh(\beta\Omega_{\eta_2\eta_3\eta_4}/2)[1-\eta_2\hat{\mathbf{k}}\cdot\hat{\mathbf{k}}']}{32\omega_{\mathbf{q}}\omega_{\mathbf{q}'}(\Omega_{\eta_2\eta_3\eta_4}^2+\Gamma^2)^2\sinh(\omega_{\mathbf{q}}\beta/2)\sinh(\omega_{\mathbf{q}'}\beta/2)\cosh(\omega_{\mathbf{k}}\beta/2)\cosh(\omega_{\mathbf{k}'}\beta/2)}$$