Deviations of symmetry-based exact neutrino structures using radiative neutrino masses.

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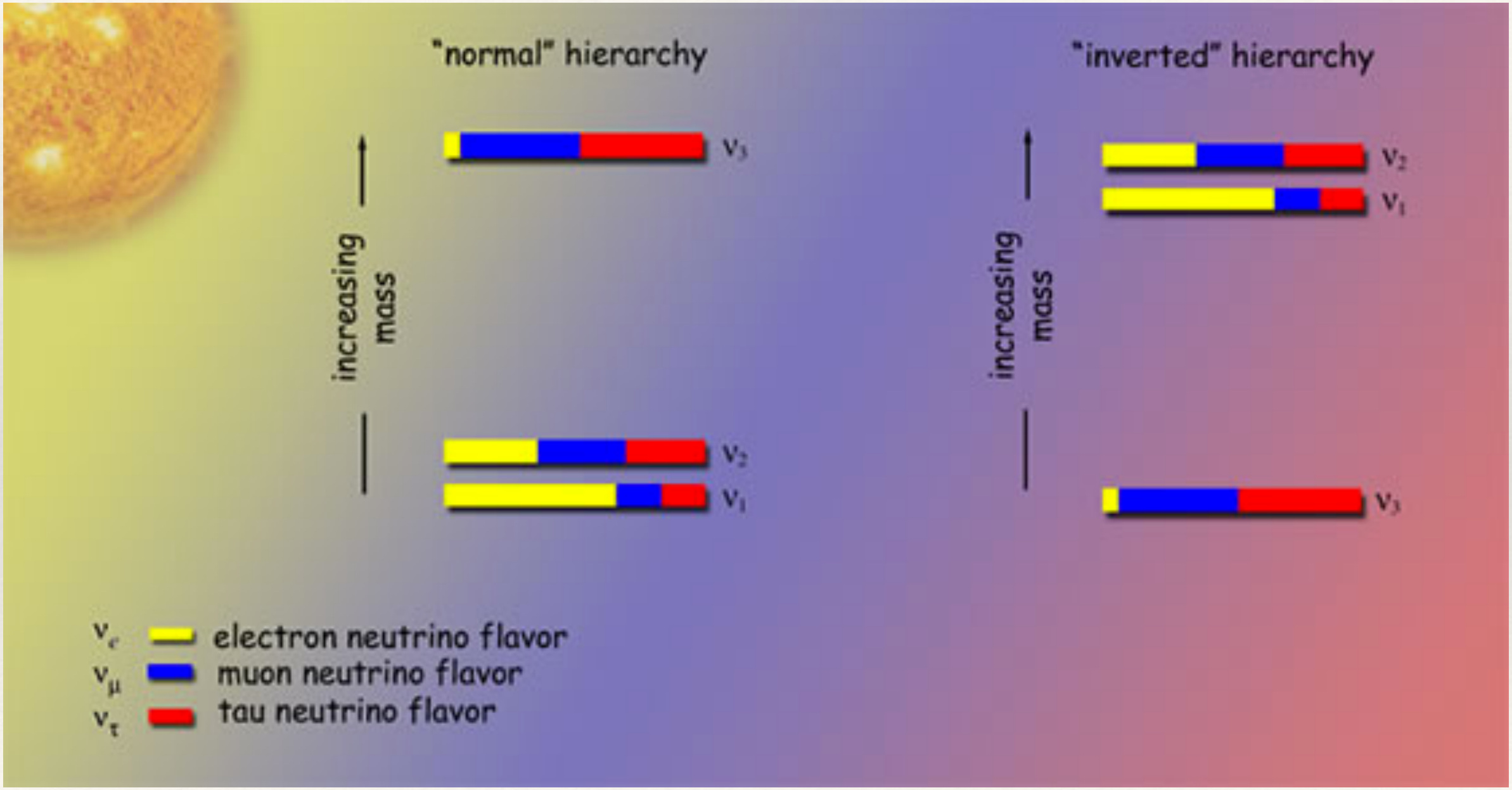
New Physics!

- The baryon asymmetry of the Universe
- The cosmological evidence of Dark Matter
- The existence of neutrino masses

$\theta_{13} \neq 0$



http://www2.lbl.gov/Publications/YOS/Mar/neutrinos-2.html

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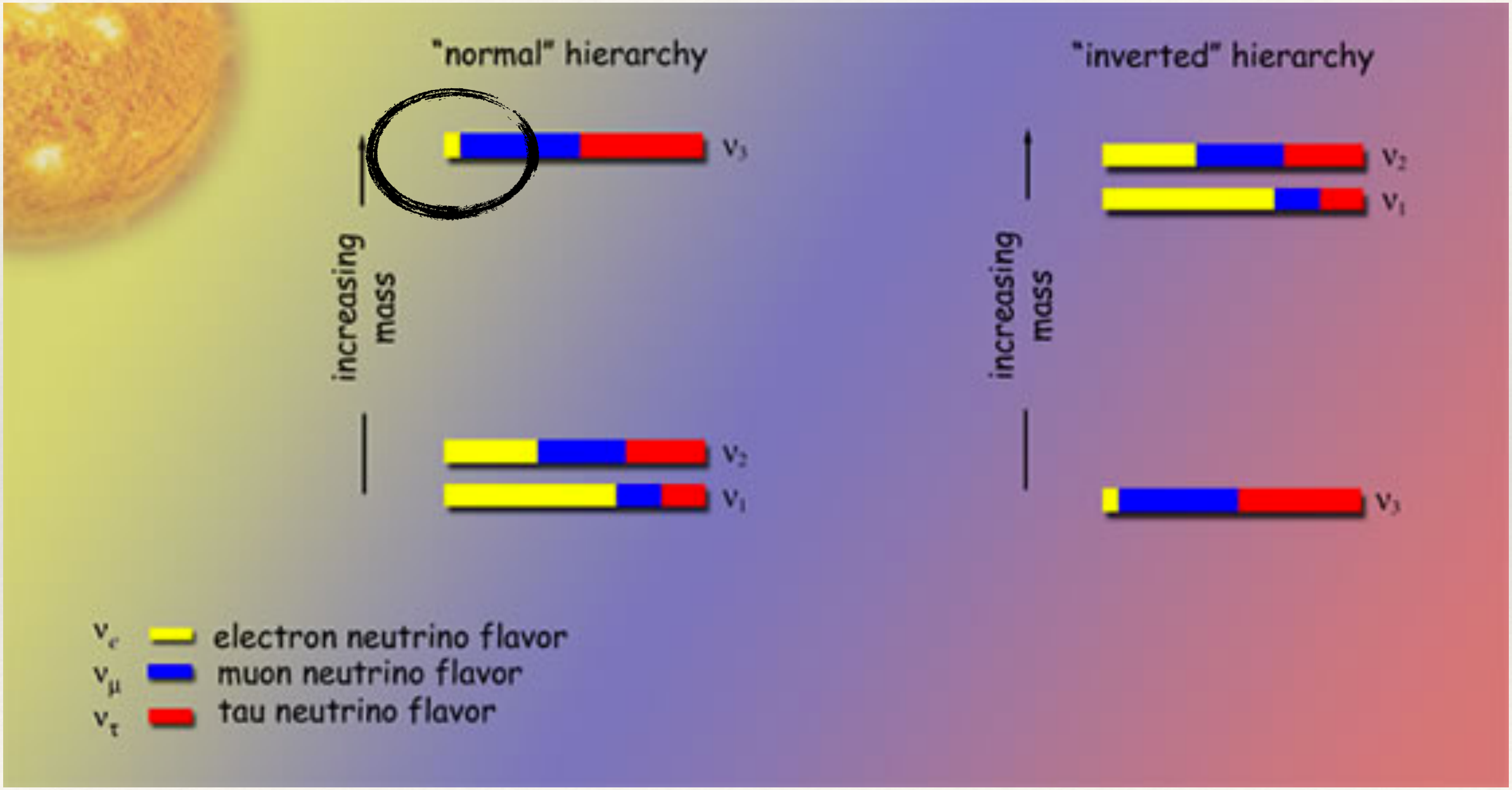

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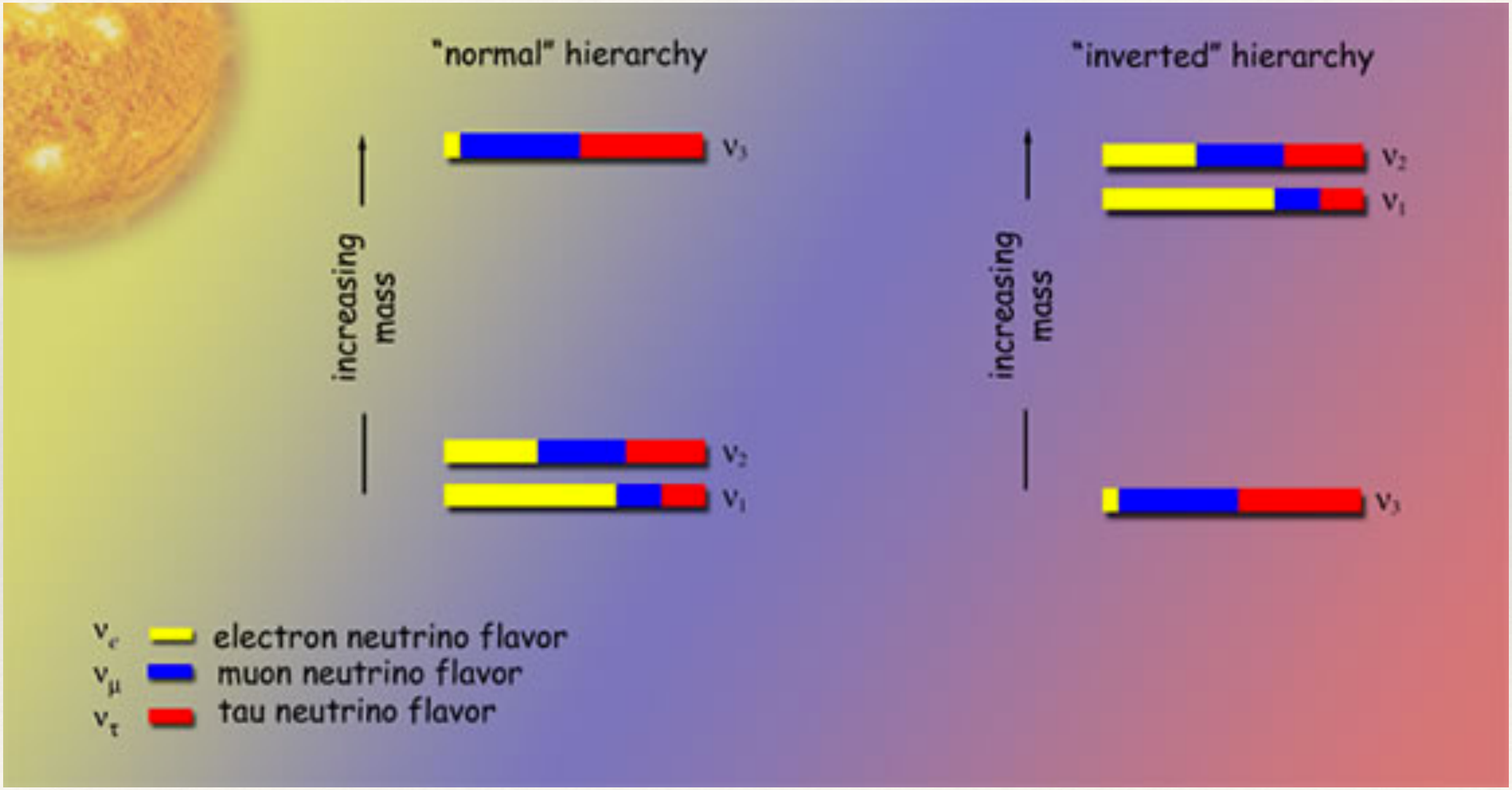

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Neutrino Data

parameter	best fit $\pm$ 1σ	3σ range
Δm^2_{21} $[10^{-5}]\mathrm{eV}^2$	$7.60^{+0.19}_{-0.18}$	7.11–8.18
$\lvert\Delta m^2_{31}\rvert$ $[10^{-3}]\mathrm{eV}^2$ (NH)	$2.48^{+0.05}_{-0.07}$	2.30–2.65
$\lvert\Delta m^2_{31}\rvert$ $[10^{-3}]\mathrm{eV}^2$ (IH)	$2.38^{+0.05}_{-0.06}$	2.20–2.54
$\sin^2\theta_{12}/10^{-1}$	3.23 ± 0.16	2.78–3.75
$\sin^2\theta_{23}/10^{-1}$ (NH)	$5.67^{+0.32}_{-1.28}$	3.92 – 6.43
$\sin^2\theta_{23}/10^{-1}$ (IH)	$5.73^{+0.25}_{-0.43}$	4.03–6.40
$\sin^2\theta_{13}/10^{-2}$ (NH)	2.34 ± 0.20	1.77–2.94
$\sin^2\theta_{13}/10^{-2}$ (IH)	2.40 ± 0.19	1.83–2.97

arXiv:1405.7540 [hep-ph]

Neutrino mixing?

- Why are the values in the neutrino mixing matrix so close to exact values?
- Is it possible to start from an exact symmetry and make deviations?
- What is the explanation to these deviations?

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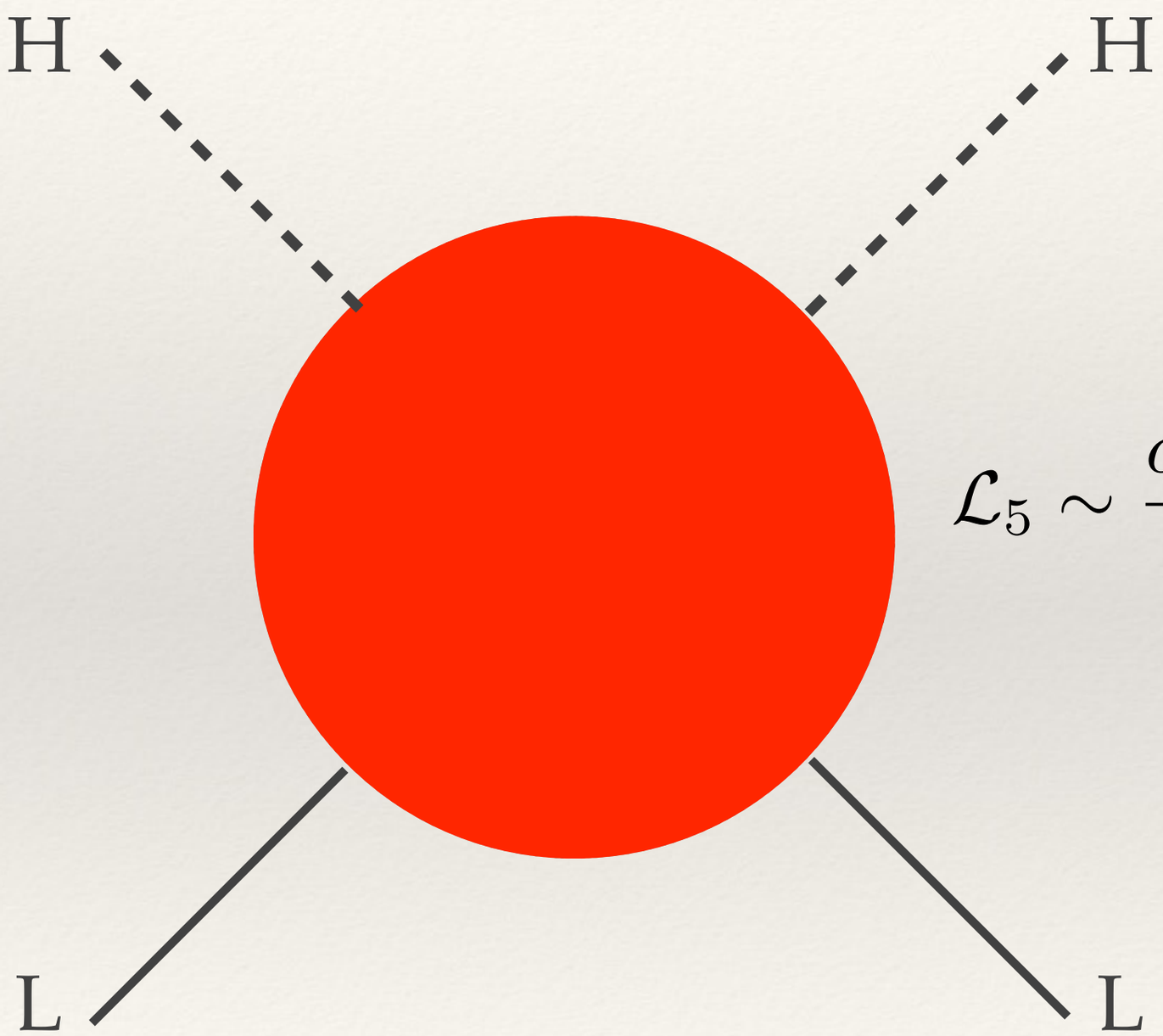
Neutrino mixing!

- An exact mixing matrix (i.e tribimaximal, bimaximal, democratic, etc.) can be accomplished by using the Weinberg operator at one-loop level.
- With the addition of only one more particle to the model, one can generate a two-loop diagram that is ***naturally surpassed*** and explains the deviations.

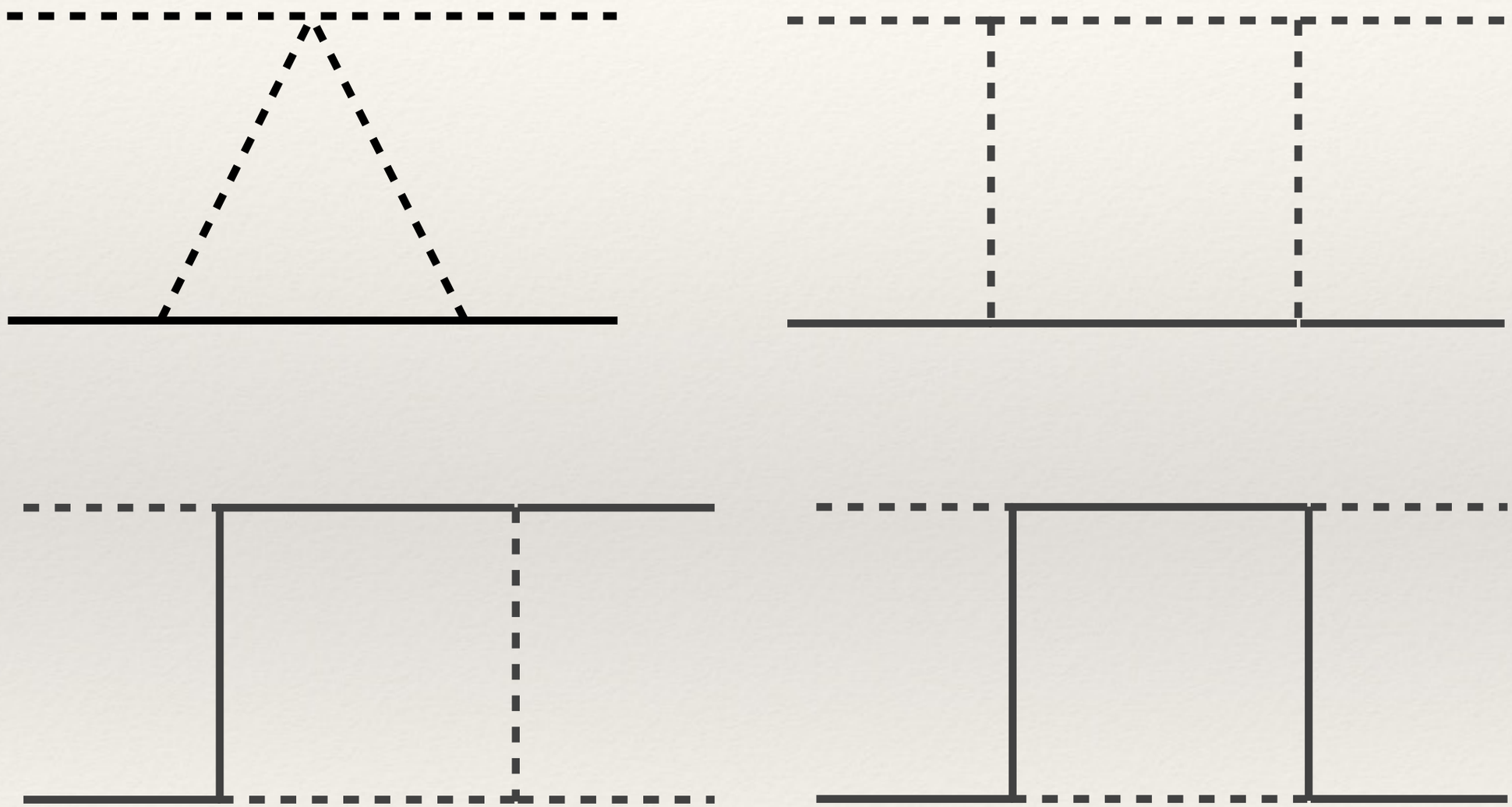
$$m_\nu \propto \left(\frac{1}{16\pi^2}\right)^n \times \frac{v^2}{\Lambda} \times \left(\frac{v}{\Lambda}\right)^{d-5}$$

Weinberg d=5 operator



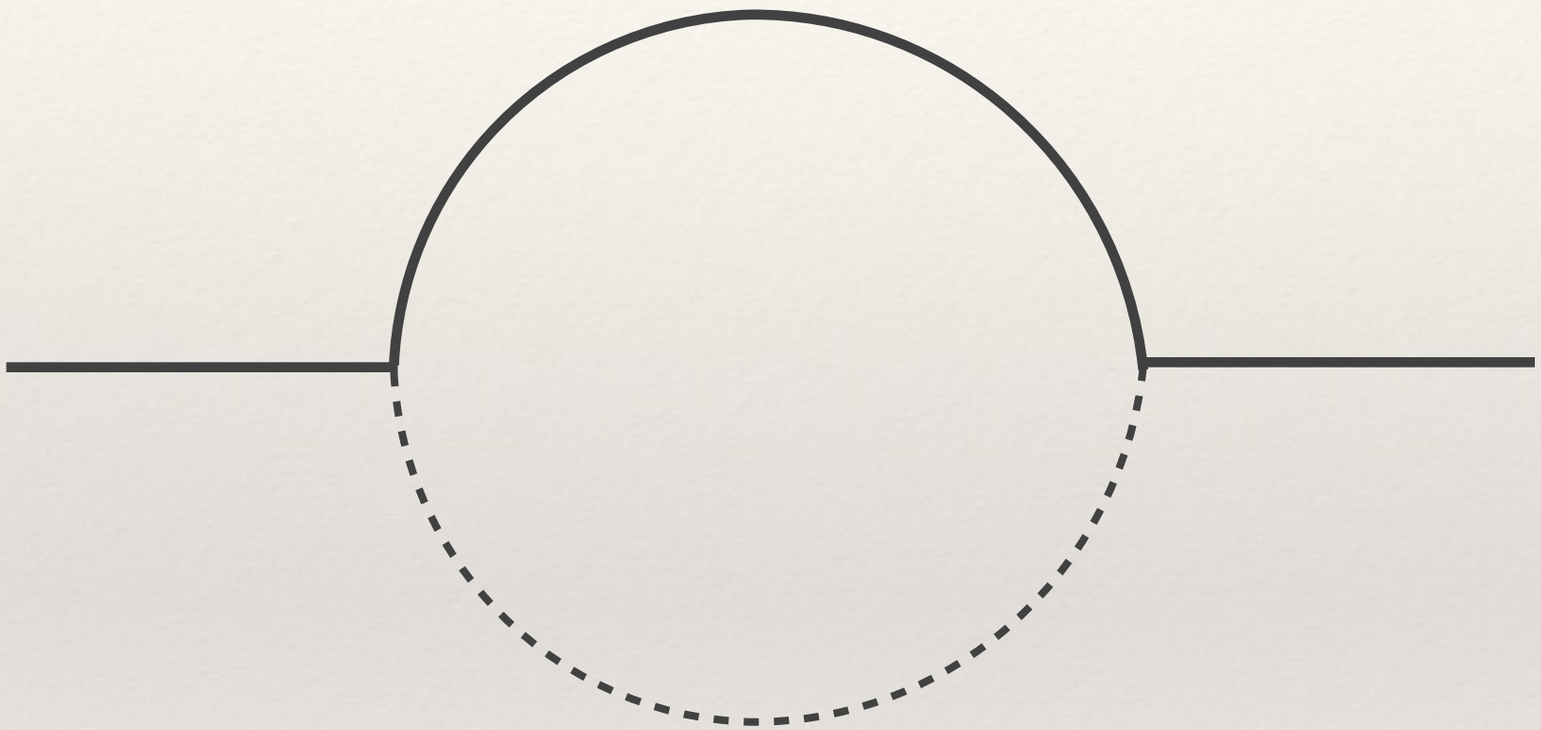
$$\mathcal{L}_5 \sim \frac{c_{\alpha\beta}}{\Lambda}\left(\overline{L^c_\alpha}\tilde{H}^*\,\tilde{H}^\dagger L_\beta\right)$$

One-loop masses



arXiv:1204.5862 [hep-ph]

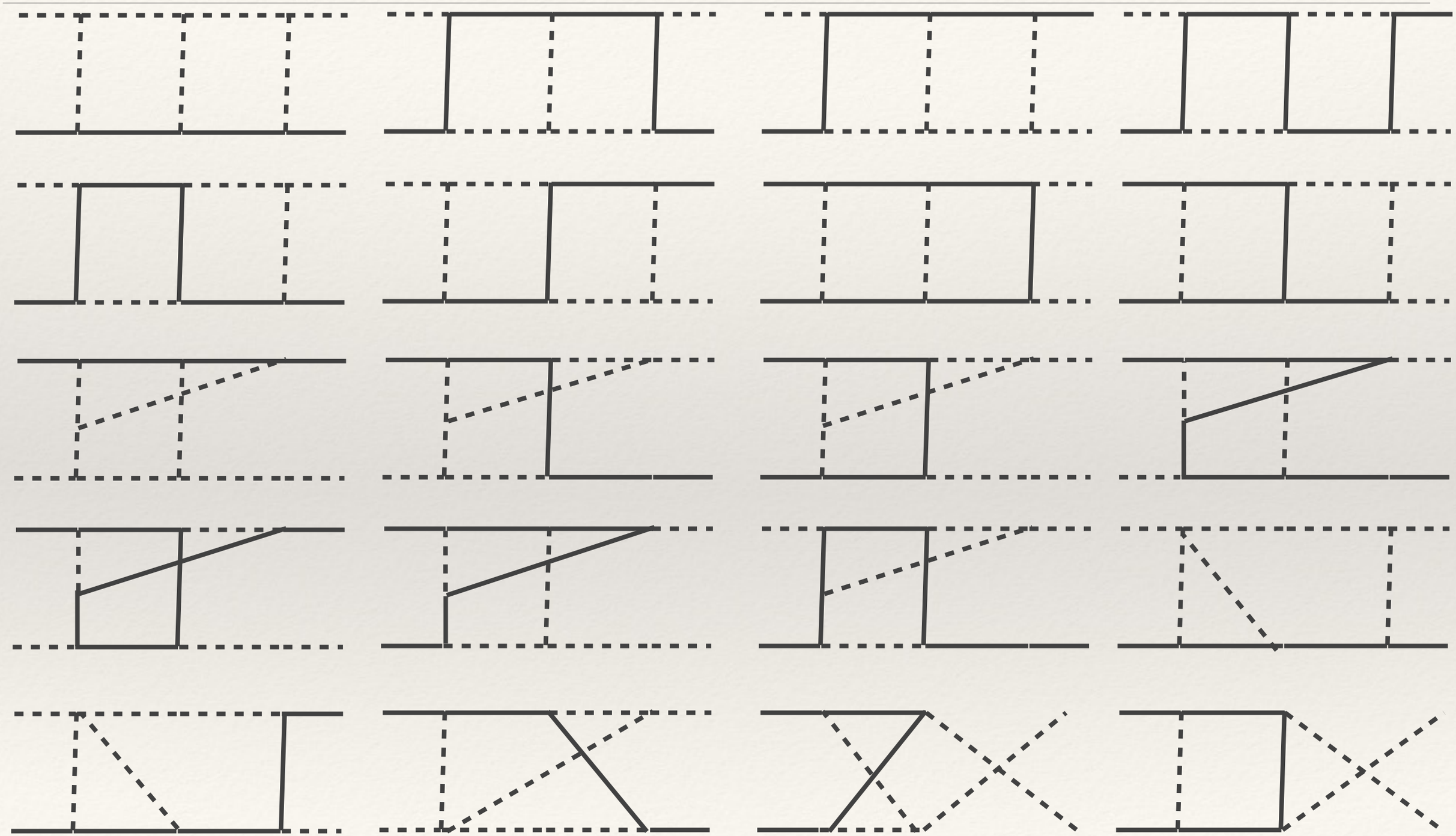
One-loop Integral



$$\equiv \int \frac{\mathrm{d}^d k}{(2\pi)^d \mathrm{i}} \frac{1}{(k^2 - M_A^2)(k^2 - M_B^2)(k^2 - M_C^2)}$$

$$= -\frac{1}{(4\pi)^2} \left[\frac{M_A^2 \ln \frac{M_C^2}{M_A^2}}{(M_A^2 - M_B^2)(M_A^2 - M_C^2)} + \frac{M_B^2 \ln \frac{M_C^2}{M_B^2}}{(M_B^2 - M_A^2)(M_B^2 - M_C^2)} \right]$$

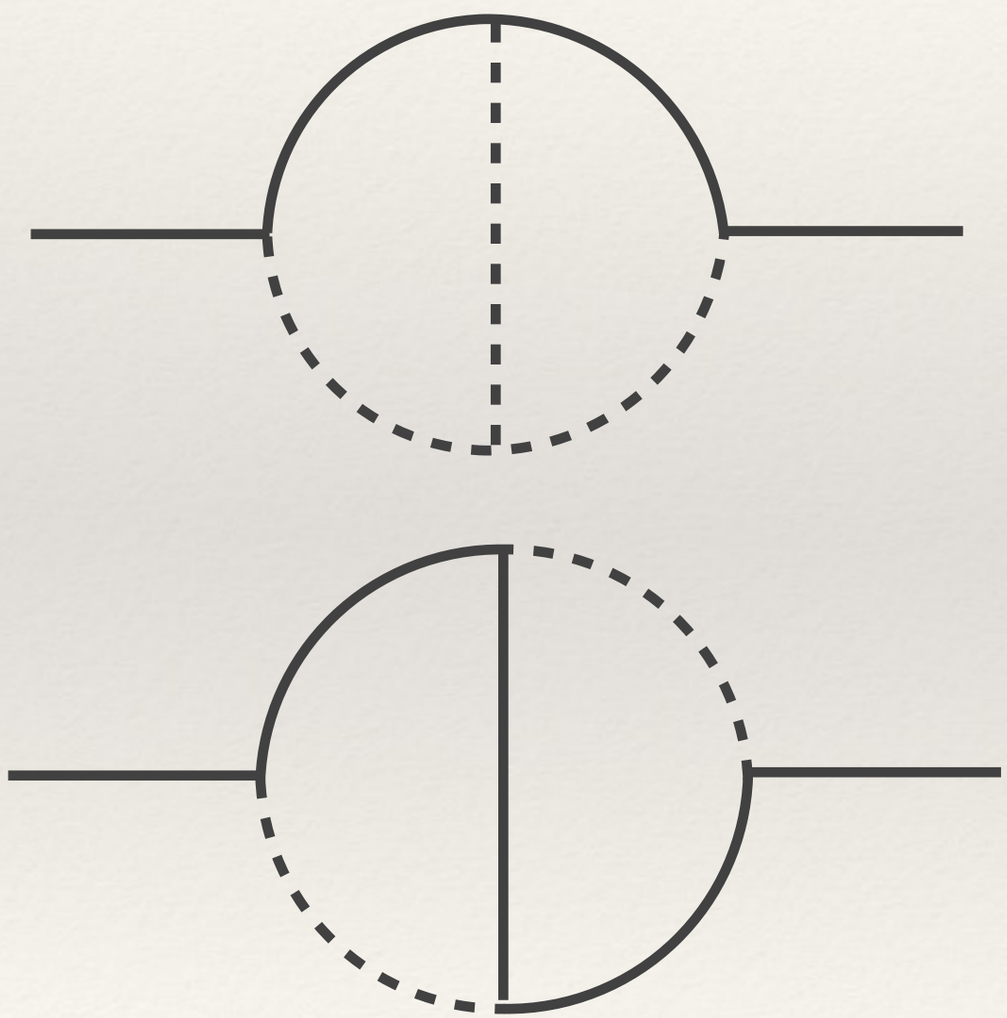
Two-Loops



arXiv:1411.7038 [hep-ph]

Two-loop Integral

$$\hat{\mathcal{I}}_{ab,\alpha\beta}^{\{k^2,q^2,(q+k)^2\}} = \int d^4k \int d^4q \frac{\{k^2, q^2, (q+k)^2\}}{(k^2 - r_a)(k^2 - t_\alpha)(q^2 - r_b)(q^2 - t_\beta)([q+k]^2 - 1)}$$



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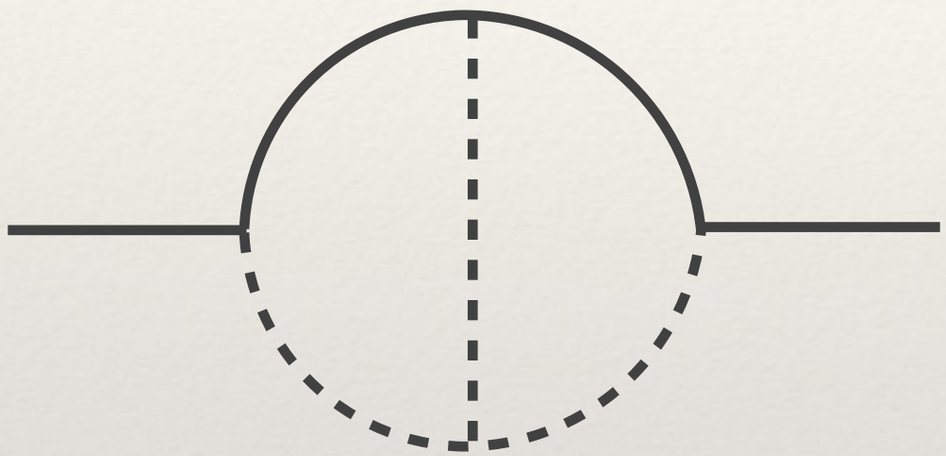
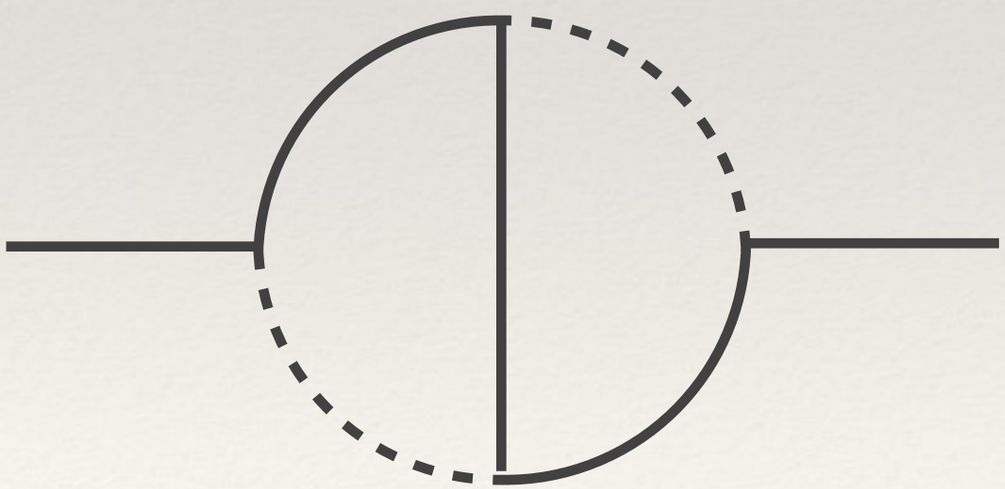


$$\sum_{\alpha,\beta=1}^{2} (-1)^\alpha (-1)^\beta \left(2\, m_5\, m_6\, I_\chi^1 + I_\chi^{k^2} + I_\chi^{q^2} - I_\chi^{(k+q)^2}\right)$$



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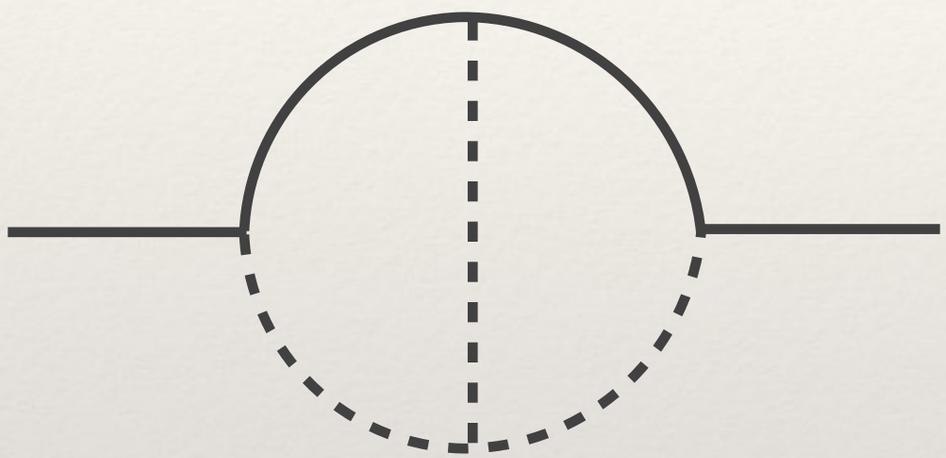
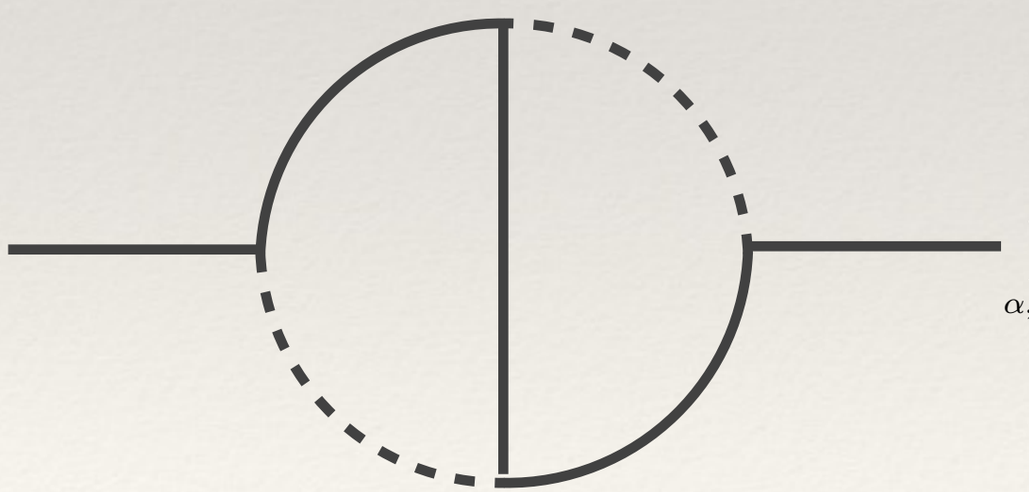


$$\sum_{\alpha,\beta=1}^{2} (-1)^\alpha (-1)^\beta \left(2\, m_5\, m_6\, I_\chi^1 \,+\, I_\chi^{k^2} \,+\, I_\chi^{q^2} \,-\, I_\chi^{(k+q)^2}\right)$$



$$\sum_{\alpha,\,\beta=1}^{2} (-1)^\alpha\, (-1)^\beta \Big(2\, m_2\, m_3\, m_6\, I_\chi^1 \,+\, (m_2 \,-\, m_3 \,-\, m_6)\, I_\chi^{k^2}$$

$$+\, (-m_2 - m_3 + m_6)\, I_\chi^{q^2} \,+\, (m_2 \,+\, m_3 \,+\, m_6)\, I_\chi^{(k+q)^2}\Big)$$

Two Loop solved integral

$$\hat{\mathcal{I}}_{ij,\alpha\beta} = \frac{\pi^4}{(t_\alpha - r_i)(t_\beta - r_j)} \Big\{ r_i[f(t_\beta/r_i, 1/r_i) - f(r_j/r_i, 1/r_i)] \\ + r_j[f(t_\alpha/r_j, 1/r_j) - f(r_i/r_j, 1/r_j)] \\ + t_\alpha[f(r_j/t_\alpha, 1/t_\alpha) - f(t_\beta/t_\alpha, 1/t_\alpha)] \\ + t_\beta[f(r_i/t_\beta, 1/t_\beta) - f(t_\alpha/t_\beta, 1/t_\beta)] \\ + [f(t_\alpha, r_j) - f(r_i, r_j) - f(t_\alpha, t_\beta) + f(r_i, t_\beta)] \Big\}$$

$$f(a,b) = -\frac{1}{2}\ln a \ln b - \frac{1}{2}\Big(\frac{a+b-1}{\kappa}\Big)\Big\{\mathrm{Li}_2(\frac{-x_2}{y_1}) + \mathrm{Li}_2(\frac{-y_2}{x_1}) - \mathrm{Li}_2(\frac{-x_1}{y_2}) - \mathrm{Li}_2(\frac{-y_1}{x_2}) \\ + \mathrm{Li}_2(\frac{b-a}{x_2}) + \mathrm{Li}_2(\frac{a-b}{y_2}) - \mathrm{Li}_2(\frac{b-a}{x_1}) - \mathrm{Li}_2(\frac{a-b}{y_1})\Big\},$$

$$x_{1,2} = \frac{1}{2}(1+b-a\pm\kappa)\ , \quad y_{1,2} = \frac{1}{2}(1+a-b\pm\kappa)\ , \quad \kappa = \sqrt{1-2(a+b)+(a-b)^2}\ .$$

hep-ph/0309270

The mixing matrix originates from the Yukawas, while the mass scale comes from the loop integral.

Casas-Ibarra

$$M = Y.\Lambda.Y^T$$

arXiv:hep-ph/0103065

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$$M_{diag} = U^T.M.U$$

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$$M = Y.\Lambda.Y^T$$

$$M_{diag} = U^T.M.U = U^T.(Y.\Lambda.Y^T).U$$

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$$\xrightarrow{(M_{diag})^{-1/2}} M_{diag} = U^T.M.U = U^T.(Y.\Lambda.Y^T).U \xleftarrow{(M_{diag})^{-1/2}}$$

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R is an 3 x n ortogonal matrix

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$$R^T = (M_{diag})^{-1/2}.U^T.Y.\Lambda^{1/2}$$

$$\boxed{Y = U.(M_{diag})^{1/2}.R^T.\Lambda^{-1/2}}$$

arXiv:hep-ph/0103065

TBM Yukawa

$\alpha_{12}=0,\ \alpha_{23}=0,\ \alpha_{13}=0$

$$Y=\begin{pmatrix} \frac{\sqrt{\frac{2}{3}}\sqrt{\mathrm{mnu1}}}{\sqrt{\Lambda 11}} & \frac{\sqrt{\mathrm{mnu2}}}{\sqrt{3}\sqrt{\Lambda 12}} & 0 \\ -\frac{\sqrt{\mathrm{mnu1}}}{\sqrt{6}\sqrt{\Lambda 11}} & \frac{\sqrt{\mathrm{mnu2}}}{\sqrt{3}\sqrt{\Lambda 12}} & -\frac{\sqrt{\mathrm{mnu3}}}{\sqrt{2}\sqrt{\Lambda 13}} \\ -\frac{\sqrt{\mathrm{mnu1}}}{\sqrt{6}\sqrt{\Lambda 11}} & \frac{\sqrt{\mathrm{mnu2}}}{\sqrt{3}\sqrt{\Lambda 12}} & \frac{\sqrt{\mathrm{mnu3}}}{\sqrt{2}\sqrt{\Lambda 13}} \end{pmatrix}$$

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$$Y=\begin{pmatrix} \sqrt{\frac{2}{3}}K_1 & \frac{1}{\sqrt{3}}K_2 & 0 \\ -\frac{1}{\sqrt{6}}K_1 & \frac{1}{\sqrt{3}}K_2 & -\frac{1}{\sqrt{2}}K_3 \\ -\frac{1}{\sqrt{6}}K_1 & \frac{1}{\sqrt{3}}K_2 & \frac{1}{\sqrt{2}}K_3 \end{pmatrix}$$

TBM mass

$$M = Y.\Lambda.Y^T$$

$$M_{TBM} = \begin{pmatrix} \frac{2A+B}{3} & \frac{-A+B}{3} & \frac{-A+B}{3} \\ \frac{-A+B}{3} & \frac{A+2B+3C}{6} & \frac{A+2B-3C}{6} \\ \frac{-A+B}{3} & \frac{A+2B-3C}{6} & \frac{A+2B+3C}{6} \end{pmatrix}$$

TBM Yukawas examples

$\alpha_{12}=\pi/2,\ \alpha_{23}=\pi/2,\ \alpha_{13}=\pi/2$

$$\begin{pmatrix} 0 & \frac{\sqrt{\text{mnu2}}}{\sqrt{3}\,\sqrt{\Lambda 12}} & \frac{\sqrt{\frac{2}{3}}\,\sqrt{\text{mnu1}}}{\sqrt{\Lambda 13}} \\ \frac{\sqrt{\text{mnu3}}}{\sqrt{2}\,\sqrt{\Lambda 11}} & \frac{\sqrt{\text{mnu2}}}{\sqrt{3}\,\sqrt{\Lambda 12}} & -\frac{\sqrt{\text{mnu1}}}{\sqrt{6}\,\sqrt{\Lambda 13}} \\ -\frac{\sqrt{\text{mnu3}}}{\sqrt{2}\,\sqrt{\Lambda 11}} & \frac{\sqrt{\text{mnu2}}}{\sqrt{3}\,\sqrt{\Lambda 12}} & -\frac{\sqrt{\text{mnu1}}}{\sqrt{6}\,\sqrt{\Lambda 13}} \end{pmatrix}$$

$\alpha_{12}=0,\ \alpha_{23}=\pi/4,\ \alpha_{13}=0$

$$\begin{pmatrix} \frac{\sqrt{\frac{2}{3}}\,\sqrt{\text{mnu1}}}{\sqrt{\Lambda 11}} & \frac{\sqrt{\text{mnu2}}}{\sqrt{6}\,\sqrt{\Lambda 12}} & \frac{\sqrt{\text{mnu2}}}{\sqrt{6}\,\sqrt{\Lambda 13}} \\ -\frac{\sqrt{\text{mnu1}}}{\sqrt{6}\,\sqrt{\Lambda 11}} & \frac{\sqrt{6}\,\sqrt{\text{mnu2}}+3\sqrt{\text{mnu3}}}{6\sqrt{\Lambda 12}} & \frac{\sqrt{6}\,\sqrt{\text{mnu2}}-3\sqrt{\text{mnu3}}}{6\sqrt{\Lambda 13}} \\ -\frac{\sqrt{\text{mnu1}}}{\sqrt{6}\,\sqrt{\Lambda 11}} & \frac{\sqrt{6}\,\sqrt{\text{mnu2}}-3\sqrt{\text{mnu3}}}{6\sqrt{\Lambda 12}} & \frac{\sqrt{6}\,\sqrt{\text{mnu2}}+3\sqrt{\text{mnu3}}}{6\sqrt{\Lambda 13}} \end{pmatrix}$$

$\alpha_{12}=\pi/4,\ \alpha_{23}=\pi/4,\ \alpha_{13}=\pi/4$

$$\begin{pmatrix} \frac{\sqrt{6}\,\sqrt{\text{mnu1}}-\sqrt{3}\,\sqrt{\text{mnu2}}}{6\sqrt{\Lambda 11}} & \frac{2\left(-1+\sqrt{2}\right)\sqrt{\text{mnu1}}+\left(2+\sqrt{2}\right)\sqrt{\text{mnu2}}}{4\sqrt{3}\,\sqrt{\Lambda 12}} & \frac{2\left(1+\sqrt{2}\right)\sqrt{\text{mnu1}}-\left(-2+\sqrt{2}\right)\sqrt{\text{mnu2}}}{4\sqrt{3}\,\sqrt{\Lambda 13}} \\ -\frac{\sqrt{6}\,\sqrt{\text{mnu1}}+2\sqrt{3}\,\sqrt{\text{mnu2}}-6\sqrt{\text{mnu3}}}{12\sqrt{\Lambda 11}} & \frac{\left(\sqrt{3}-\sqrt{6}\right)\sqrt{\text{mnu1}}+\sqrt{3}\left(2+\sqrt{2}\right)\sqrt{\text{mnu2}}+3\sqrt{2}\,\sqrt{\text{mnu3}}}{12\sqrt{\Lambda 12}} & \frac{-\left(\sqrt{3}+\sqrt{6}\right)\sqrt{\text{mnu1}}-\sqrt{3}\left(-2+\sqrt{2}\right)\sqrt{\text{mnu2}}-3\sqrt{2}\,\sqrt{\text{mnu3}}}{12\sqrt{\Lambda 13}} \\ -\frac{\sqrt{6}\,\sqrt{\text{mnu1}}+2\sqrt{3}\,\sqrt{\text{mnu2}}+6\sqrt{\text{mnu3}}}{12\sqrt{\Lambda 11}} & \frac{\left(\sqrt{3}-\sqrt{6}\right)\sqrt{\text{mnu1}}+\sqrt{3}\left(2+\sqrt{2}\right)\sqrt{\text{mnu2}}-3\sqrt{2}\,\sqrt{\text{mnu3}}}{12\sqrt{\Lambda 12}} & \frac{-\left(\sqrt{3}+\sqrt{6}\right)\sqrt{\text{mnu1}}-\sqrt{3}\left(-2+\sqrt{2}\right)\sqrt{\text{mnu2}}+3\sqrt{2}\,\sqrt{\text{mnu3}}}{12\sqrt{\Lambda 13}} \end{pmatrix}$$

TBM Yukawas examples

$\alpha_{12}=\pi/2,\ \alpha_{23}=\pi/2,\ \alpha_{13}=\pi/2$

$$\begin{pmatrix} 0 & \frac{\sqrt{\text{mnu2}}}{\sqrt{3}\,\sqrt{\Lambda 12}} & \frac{\sqrt{\frac{2}{3}}\,\sqrt{\text{mnu1}}}{\sqrt{\Lambda 13}} \\ \frac{\sqrt{\text{mnu3}}}{\sqrt{2}\,\sqrt{\Lambda 11}} & \frac{\sqrt{\text{mnu2}}}{\sqrt{3}\,\sqrt{\Lambda 12}} & -\frac{\sqrt{\text{mnu1}}}{\sqrt{6}\,\sqrt{\Lambda 13}} \\ -\frac{\sqrt{\text{mnu3}}}{\sqrt{2}\,\sqrt{\Lambda 11}} & \frac{\sqrt{\text{mnu2}}}{\sqrt{3}\,\sqrt{\Lambda 12}} & -\frac{\sqrt{\text{mnu1}}}{\sqrt{6}\,\sqrt{\Lambda 13}} \end{pmatrix}$$

3

$\alpha_{12}=0,\ \alpha_{23}=\pi/4,\ \alpha_{13}=0$

$$\begin{pmatrix} \frac{\sqrt{\frac{2}{3}}\,\sqrt{\text{mnu1}}}{\sqrt{\Lambda 11}} & \frac{\sqrt{\text{mnu2}}}{\sqrt{6}\,\sqrt{\Lambda 12}} & \frac{\sqrt{\text{mnu2}}}{\sqrt{6}\,\sqrt{\Lambda 13}} \\ -\frac{\sqrt{\text{mnu1}}}{\sqrt{6}\,\sqrt{\Lambda 11}} & \frac{\sqrt{6}\,\sqrt{\text{mnu2}}+3\sqrt{\text{mnu3}}}{6\sqrt{\Lambda 12}} & \frac{\sqrt{6}\,\sqrt{\text{mnu2}}-3\sqrt{\text{mnu3}}}{6\sqrt{\Lambda 13}} \\ -\frac{\sqrt{\text{mnu1}}}{\sqrt{6}\,\sqrt{\Lambda 11}} & \frac{\sqrt{6}\,\sqrt{\text{mnu2}}-3\sqrt{\text{mnu3}}}{6\sqrt{\Lambda 12}} & \frac{\sqrt{6}\,\sqrt{\text{mnu2}}+3\sqrt{\text{mnu3}}}{6\sqrt{\Lambda 13}} \end{pmatrix}$$

$\alpha_{12}=\pi/4,\ \alpha_{23}=\pi/4,\ \alpha_{13}=\pi/4$

$$\begin{pmatrix} \frac{\sqrt{6}\,\sqrt{\text{mnu1}}-\sqrt{3}\,\sqrt{\text{mnu2}}}{6\sqrt{\Lambda 11}} & \frac{2\left(-1+\sqrt{2}\right)\sqrt{\text{mnu1}}+\left(2+\sqrt{2}\right)\sqrt{\text{mnu2}}}{4\sqrt{3}\,\sqrt{\Lambda 12}} & \frac{2\left(1+\sqrt{2}\right)\sqrt{\text{mnu1}}-\left(-2+\sqrt{2}\right)\sqrt{\text{mnu2}}}{4\sqrt{3}\,\sqrt{\Lambda 13}} \\ -\frac{\sqrt{6}\,\sqrt{\text{mnu1}}+2\sqrt{3}\,\sqrt{\text{mnu2}}-6\sqrt{\text{mnu3}}}{12\sqrt{\Lambda 11}} & \frac{\left(\sqrt{3}-\sqrt{6}\right)\sqrt{\text{mnu1}}+\sqrt{3}\left(2+\sqrt{2}\right)\sqrt{\text{mnu2}}+3\sqrt{2}\,\sqrt{\text{mnu3}}}{12\sqrt{\Lambda 12}} & \frac{-\left(\sqrt{3}+\sqrt{6}\right)\sqrt{\text{mnu1}}-\sqrt{3}\left(-2+\sqrt{2}\right)\sqrt{\text{mnu2}}-3\sqrt{2}\,\sqrt{\text{mnu3}}}{12\sqrt{\Lambda 13}} \\ -\frac{\sqrt{6}\,\sqrt{\text{mnu1}}+2\sqrt{3}\,\sqrt{\text{mnu2}}+6\sqrt{\text{mnu3}}}{12\sqrt{\Lambda 11}} & \frac{\left(\sqrt{3}-\sqrt{6}\right)\sqrt{\text{mnu1}}+\sqrt{3}\left(2+\sqrt{2}\right)\sqrt{\text{mnu2}}-3\sqrt{2}\,\sqrt{\text{mnu3}}}{12\sqrt{\Lambda 12}} & \frac{-\left(\sqrt{3}+\sqrt{6}\right)\sqrt{\text{mnu1}}-\sqrt{3}\left(-2+\sqrt{2}\right)\sqrt{\text{mnu2}}+3\sqrt{2}\,\sqrt{\text{mnu3}}}{12\sqrt{\Lambda 13}} \end{pmatrix}$$

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$$\begin{pmatrix} 0 & \frac{\sqrt{\text{mnu2}}}{\sqrt{3}\,\sqrt{\Lambda 12}} & \frac{\sqrt{\frac{2}{3}}\,\sqrt{\text{mnu1}}}{\sqrt{\Lambda 13}} \\ \frac{\sqrt{\text{mnu3}}}{\sqrt{2}\,\sqrt{\Lambda 11}} & \frac{\sqrt{\text{mnu2}}}{\sqrt{3}\,\sqrt{\Lambda 12}} & -\frac{\sqrt{\text{mnu1}}}{\sqrt{6}\,\sqrt{\Lambda 13}} \\ -\frac{\sqrt{\text{mnu3}}}{\sqrt{2}\,\sqrt{\Lambda 11}} & \frac{\sqrt{\text{mnu2}}}{\sqrt{3}\,\sqrt{\Lambda 12}} & -\frac{\sqrt{\text{mnu1}}}{\sqrt{6}\,\sqrt{\Lambda 13}} \end{pmatrix}$$

3

$\alpha_{12}=0,\ \alpha_{23}=\pi/4,\ \alpha_{13}=0$

$$\begin{pmatrix} \frac{\sqrt{\frac{2}{3}}\,\sqrt{\text{mnu1}}}{\sqrt{\Lambda 11}} & \frac{\sqrt{\text{mnu2}}}{\sqrt{6}\,\sqrt{\Lambda 12}} & \frac{\sqrt{\text{mnu2}}}{\sqrt{6}\,\sqrt{\Lambda 13}} \\ -\frac{\sqrt{\text{mnu1}}}{\sqrt{6}\,\sqrt{\Lambda 11}} & \frac{\sqrt{6}\,\sqrt{\text{mnu2}}+3\sqrt{\text{mnu3}}}{6\sqrt{\Lambda 12}} & \frac{\sqrt{6}\,\sqrt{\text{mnu2}}-3\sqrt{\text{mnu3}}}{6\sqrt{\Lambda 13}} \\ -\frac{\sqrt{\text{mnu1}}}{\sqrt{6}\,\sqrt{\Lambda 11}} & \frac{\sqrt{6}\,\sqrt{\text{mnu2}}-3\sqrt{\text{mnu3}}}{6\sqrt{\Lambda 12}} & \frac{\sqrt{6}\,\sqrt{\text{mnu2}}+3\sqrt{\text{mnu3}}}{6\sqrt{\Lambda 13}} \end{pmatrix}$$

5

$\alpha_{12}=\pi/4,\ \alpha_{23}=\pi/4,\ \alpha_{13}=\pi/4$

$$\begin{pmatrix} \frac{\sqrt{6}\,\sqrt{\text{mnu1}}-\sqrt{3}\,\sqrt{\text{mnu2}}}{6\sqrt{\Lambda 11}} & \frac{2\left(-1+\sqrt{2}\right)\sqrt{\text{mnu1}}+\left(2+\sqrt{2}\right)\sqrt{\text{mnu2}}}{4\sqrt{3}\,\sqrt{\Lambda 12}} & \frac{2\left(1+\sqrt{2}\right)\sqrt{\text{mnu1}}-\left(-2+\sqrt{2}\right)\sqrt{\text{mnu2}}}{4\sqrt{3}\,\sqrt{\Lambda 13}} \\ -\frac{\sqrt{6}\,\sqrt{\text{mnu1}}+2\sqrt{3}\,\sqrt{\text{mnu2}}-6\sqrt{\text{mnu3}}}{12\sqrt{\Lambda 11}} & \frac{\left(\sqrt{3}-\sqrt{6}\right)\sqrt{\text{mnu1}}+\sqrt{3}\left(2+\sqrt{2}\right)\sqrt{\text{mnu2}}+3\sqrt{2}\,\sqrt{\text{mnu3}}}{12\sqrt{\Lambda 12}} & \frac{-\left(\sqrt{3}+\sqrt{6}\right)\sqrt{\text{mnu1}}-\sqrt{3}\left(-2+\sqrt{2}\right)\sqrt{\text{mnu2}}-3\sqrt{2}\,\sqrt{\text{mnu3}}}{12\sqrt{\Lambda 13}} \\ -\frac{\sqrt{6}\,\sqrt{\text{mnu1}}+2\sqrt{3}\,\sqrt{\text{mnu2}}+6\sqrt{\text{mnu3}}}{12\sqrt{\Lambda 11}} & \frac{\left(\sqrt{3}-\sqrt{6}\right)\sqrt{\text{mnu1}}+\sqrt{3}\left(2+\sqrt{2}\right)\sqrt{\text{mnu2}}-3\sqrt{2}\,\sqrt{\text{mnu3}}}{12\sqrt{\Lambda 12}} & \frac{-\left(\sqrt{3}+\sqrt{6}\right)\sqrt{\text{mnu1}}-\sqrt{3}\left(-2+\sqrt{2}\right)\sqrt{\text{mnu2}}+3\sqrt{2}\,\sqrt{\text{mnu3}}}{12\sqrt{\Lambda 13}} \end{pmatrix}$$

TBM Yukawas examples

$\alpha_{12}=\pi/2,\ \alpha_{23}=\pi/2,\ \alpha_{13}=\pi/2$

$$\begin{pmatrix} 0 & \frac{\sqrt{\text{mnu2}}}{\sqrt{3}\,\sqrt{\Lambda 12}} & \frac{\sqrt{\frac{2}{3}}\,\sqrt{\text{mnu1}}}{\sqrt{\Lambda 13}} \\ \frac{\sqrt{\text{mnu3}}}{\sqrt{2}\,\sqrt{\Lambda 11}} & \frac{\sqrt{\text{mnu2}}}{\sqrt{3}\,\sqrt{\Lambda 12}} & -\frac{\sqrt{\text{mnu1}}}{\sqrt{6}\,\sqrt{\Lambda 13}} \\ -\frac{\sqrt{\text{mnu3}}}{\sqrt{2}\,\sqrt{\Lambda 11}} & \frac{\sqrt{\text{mnu2}}}{\sqrt{3}\,\sqrt{\Lambda 12}} & -\frac{\sqrt{\text{mnu1}}}{\sqrt{6}\,\sqrt{\Lambda 13}} \end{pmatrix}$$

3

$\alpha_{12}=0,\ \alpha_{23}=\pi/4,\ \alpha_{13}=0$

$$\begin{pmatrix} \frac{\sqrt{\frac{2}{3}}\,\sqrt{\text{mnu1}}}{\sqrt{\Lambda 11}} & \frac{\sqrt{\text{mnu2}}}{\sqrt{6}\,\sqrt{\Lambda 12}} & \frac{\sqrt{\text{mnu2}}}{\sqrt{6}\,\sqrt{\Lambda 13}} \\ -\frac{\sqrt{\text{mnu1}}}{\sqrt{6}\,\sqrt{\Lambda 11}} & \frac{\sqrt{6}\,\sqrt{\text{mnu2}}+3\sqrt{\text{mnu3}}}{6\sqrt{\Lambda 12}} & \frac{\sqrt{6}\,\sqrt{\text{mnu2}}-3\sqrt{\text{mnu3}}}{6\sqrt{\Lambda 13}} \\ -\frac{\sqrt{\text{mnu1}}}{\sqrt{6}\,\sqrt{\Lambda 11}} & \frac{\sqrt{6}\,\sqrt{\text{mnu2}}-3\sqrt{\text{mnu3}}}{6\sqrt{\Lambda 12}} & \frac{\sqrt{6}\,\sqrt{\text{mnu2}}+3\sqrt{\text{mnu3}}}{6\sqrt{\Lambda 13}} \end{pmatrix}$$

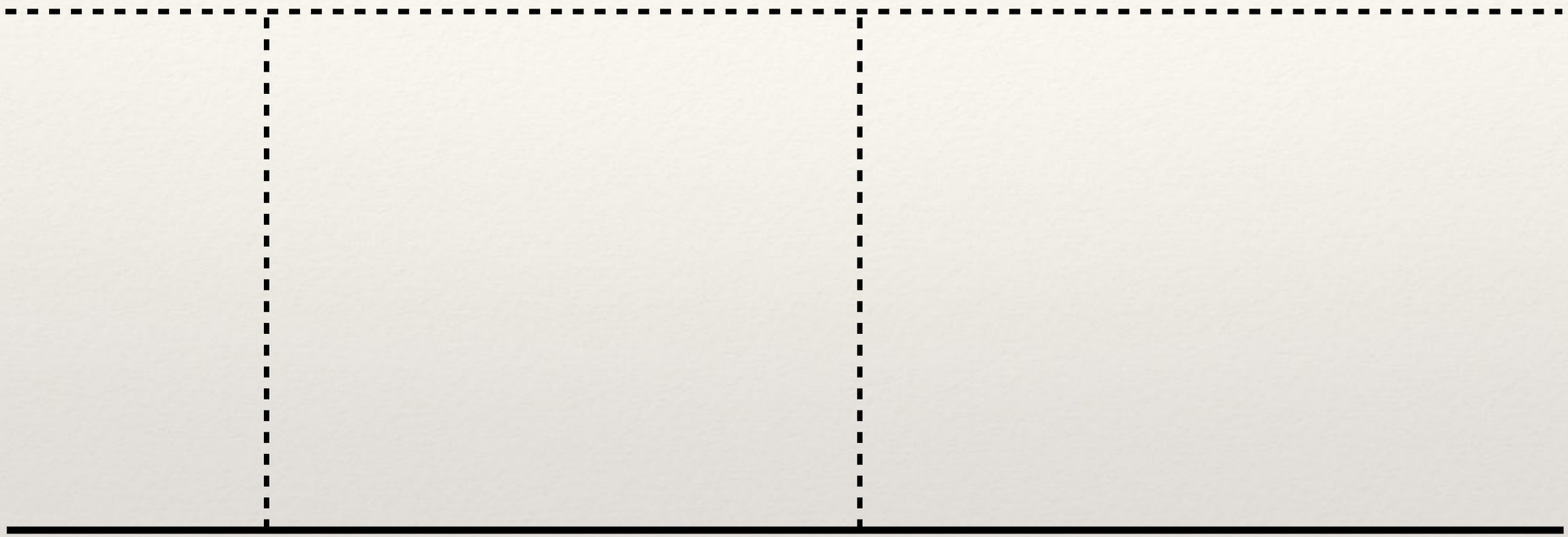
5

$\alpha_{12}=\pi/4,\ \alpha_{23}=\pi/4,\ \alpha_{13}=\pi/4$

$$\begin{pmatrix} \frac{\sqrt{6}\,\sqrt{\text{mnu1}}-\sqrt{3}\,\sqrt{\text{mnu2}}}{6\sqrt{\Lambda 11}} & \frac{2\left(-1+\sqrt{2}\right)\sqrt{\text{mnu1}}+\left(2+\sqrt{2}\right)\sqrt{\text{mnu2}}}{4\sqrt{3}\,\sqrt{\Lambda 12}} & \frac{2\left(1+\sqrt{2}\right)\sqrt{\text{mnu1}}-\left(-2+\sqrt{2}\right)\sqrt{\text{mnu2}}}{4\sqrt{3}\,\sqrt{\Lambda 13}} \\ -\frac{\sqrt{6}\,\sqrt{\text{mnu1}}+2\sqrt{3}\,\sqrt{\text{mnu2}}-6\sqrt{\text{mnu3}}}{12\sqrt{\Lambda 11}} & \frac{\left(\sqrt{3}-\sqrt{6}\right)\sqrt{\text{mnu1}}+\sqrt{3}\left(2+\sqrt{2}\right)\sqrt{\text{mnu2}}+3\sqrt{2}\,\sqrt{\text{mnu3}}}{12\sqrt{\Lambda 12}} & \frac{-\left(\sqrt{3}+\sqrt{6}\right)\sqrt{\text{mnu1}}-\sqrt{3}\left(-2+\sqrt{2}\right)\sqrt{\text{mnu2}}-3\sqrt{2}\,\sqrt{\text{mnu3}}}{12\sqrt{\Lambda 13}} \\ -\frac{\sqrt{6}\,\sqrt{\text{mnu1}}+2\sqrt{3}\,\sqrt{\text{mnu2}}+6\sqrt{\text{mnu3}}}{12\sqrt{\Lambda 11}} & \frac{\left(\sqrt{3}-\sqrt{6}\right)\sqrt{\text{mnu1}}+\sqrt{3}\left(2+\sqrt{2}\right)\sqrt{\text{mnu2}}-3\sqrt{2}\,\sqrt{\text{mnu3}}}{12\sqrt{\Lambda 12}} & \frac{-\left(\sqrt{3}+\sqrt{6}\right)\sqrt{\text{mnu1}}-\sqrt{3}\left(-2+\sqrt{2}\right)\sqrt{\text{mnu2}}+3\sqrt{2}\,\sqrt{\text{mnu3}}}{12\sqrt{\Lambda 13}} \end{pmatrix}$$

9

Example 1

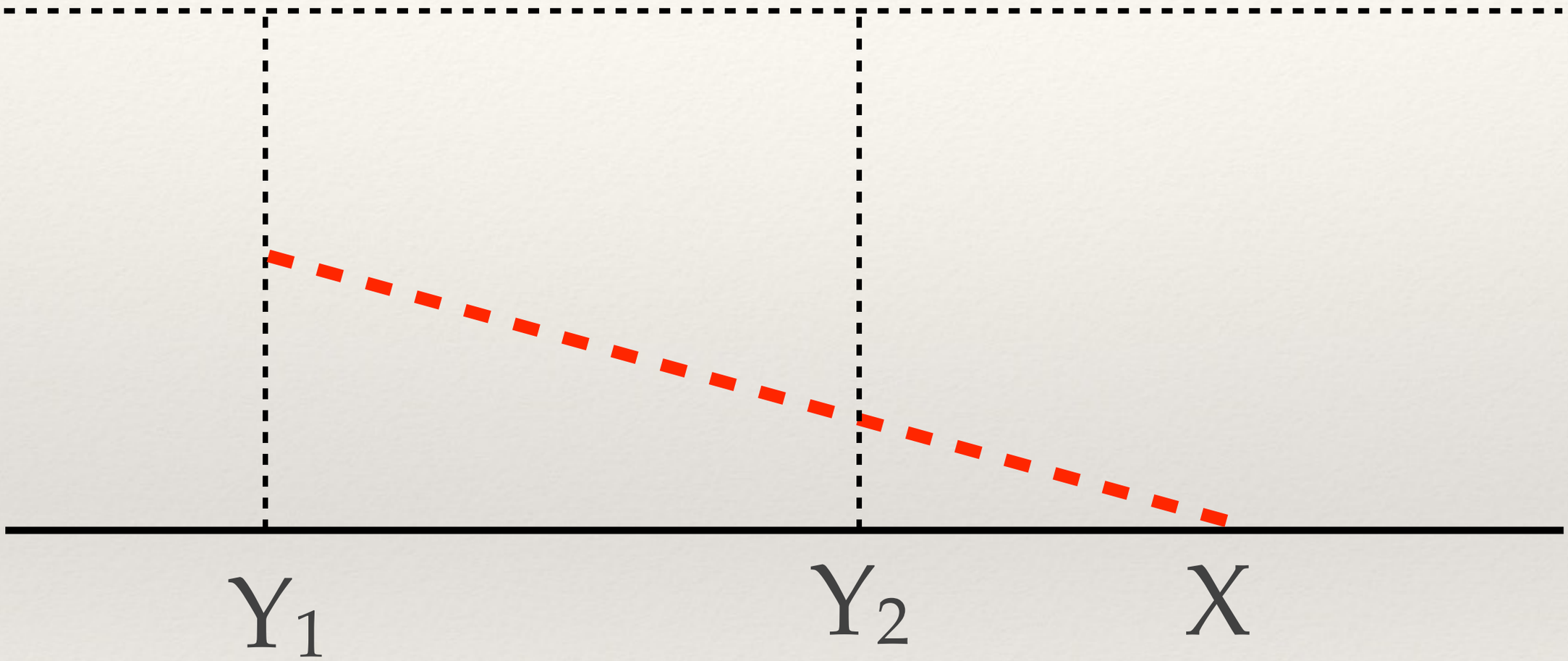


Y_1 Y_2

$\mathcal{L} = Y_2 \bar{L} F_1 S$

$M\nu \sim Y_1.Y_2^T + Y_1.Y_2^T$

Example 1



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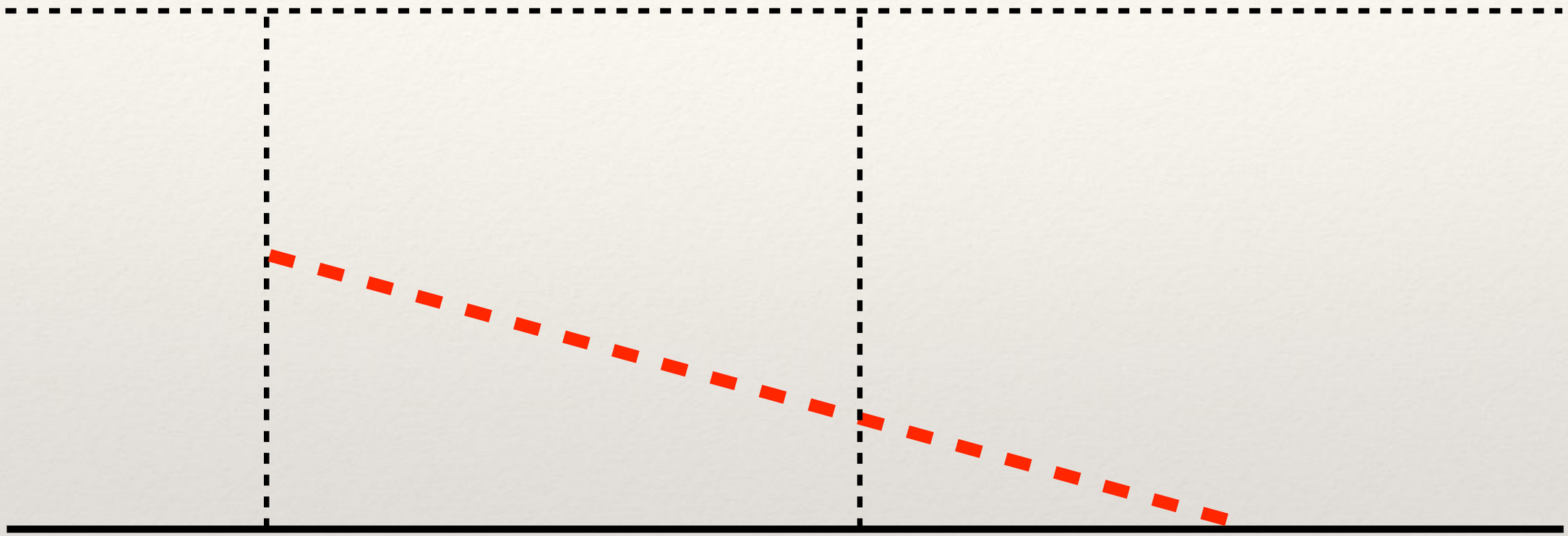
Example 1

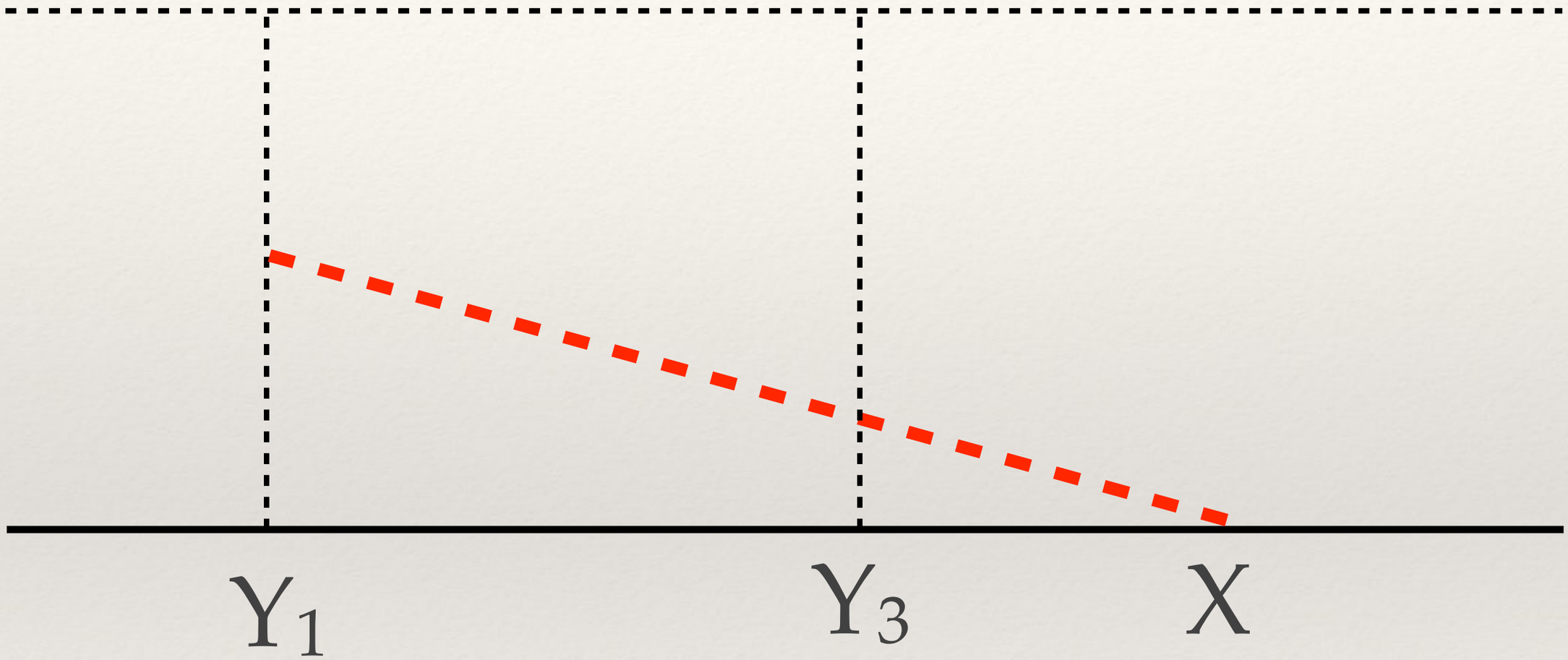


Y_1 Y_3 X

$\mathcal{L} = Y_2 \bar{L} F_1 S$

$M\nu \sim Y_1 . Y_2^T + Y_1 . Y_2^T$

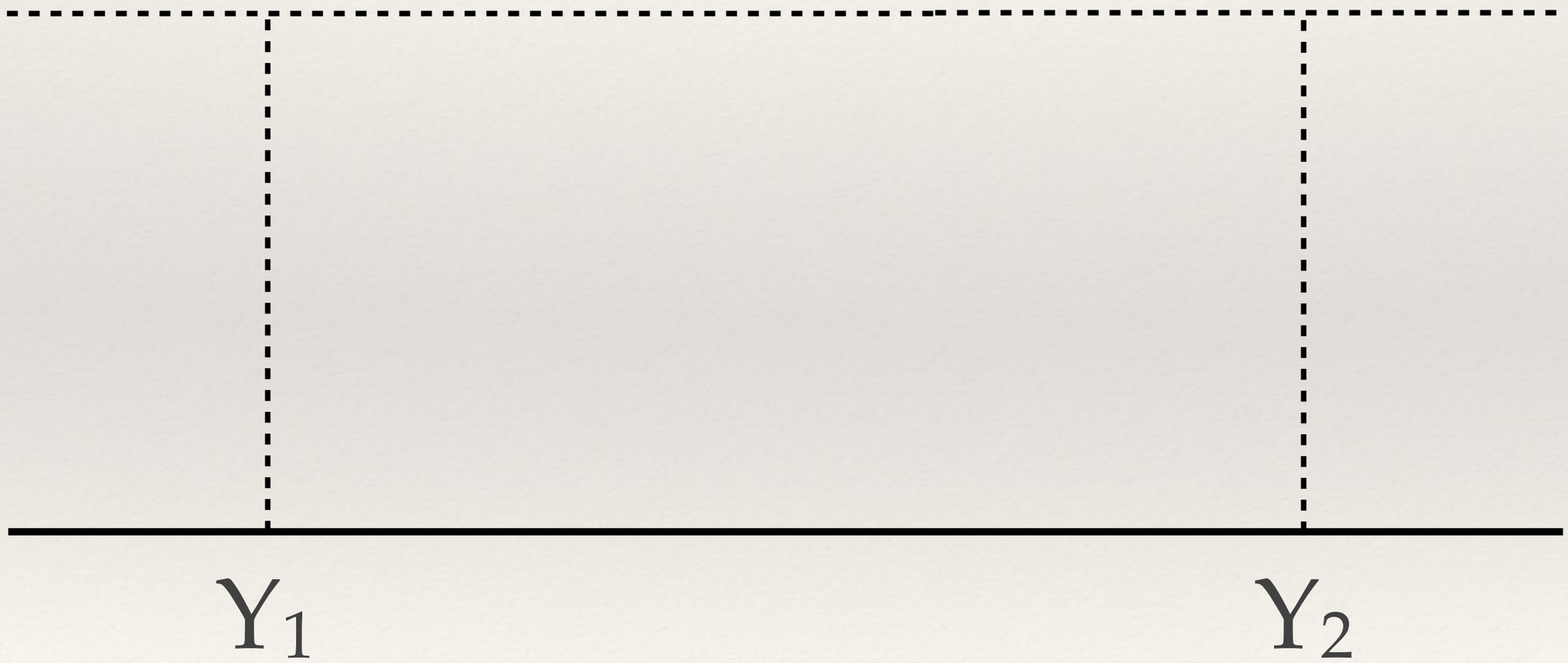
Example 1



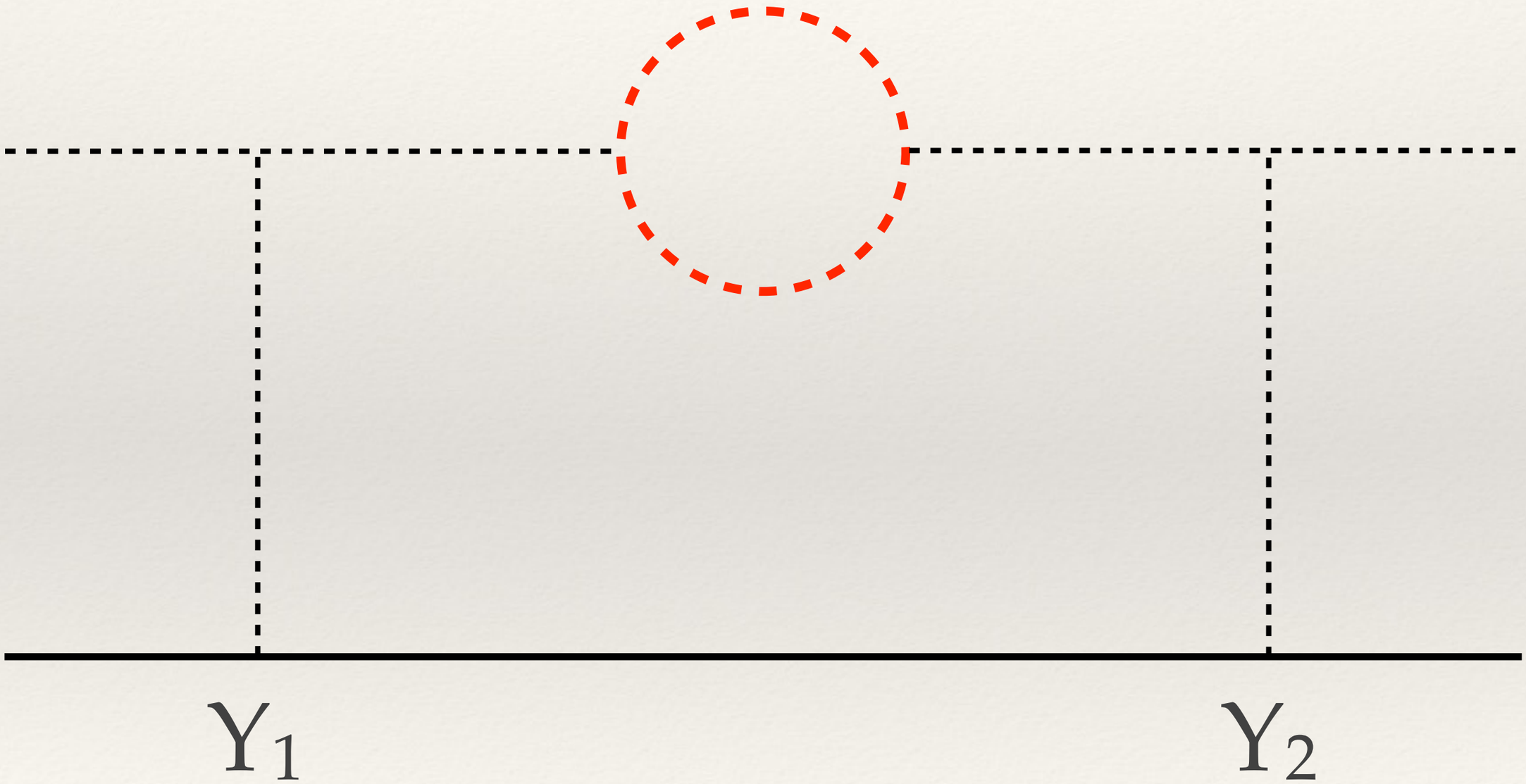
$\mathcal{L} = Y_2 \bar{L} F_1 S$ $\quad M\nu \sim Y_1.Y_2^T + Y_1.Y_2^T$

$\mathcal{L} = Y_3 \bar{F}_2 F_1 S$ $\quad M\nu \sim Y_1.Y_3.X^T + X.Y_3^T.Y_1^T$

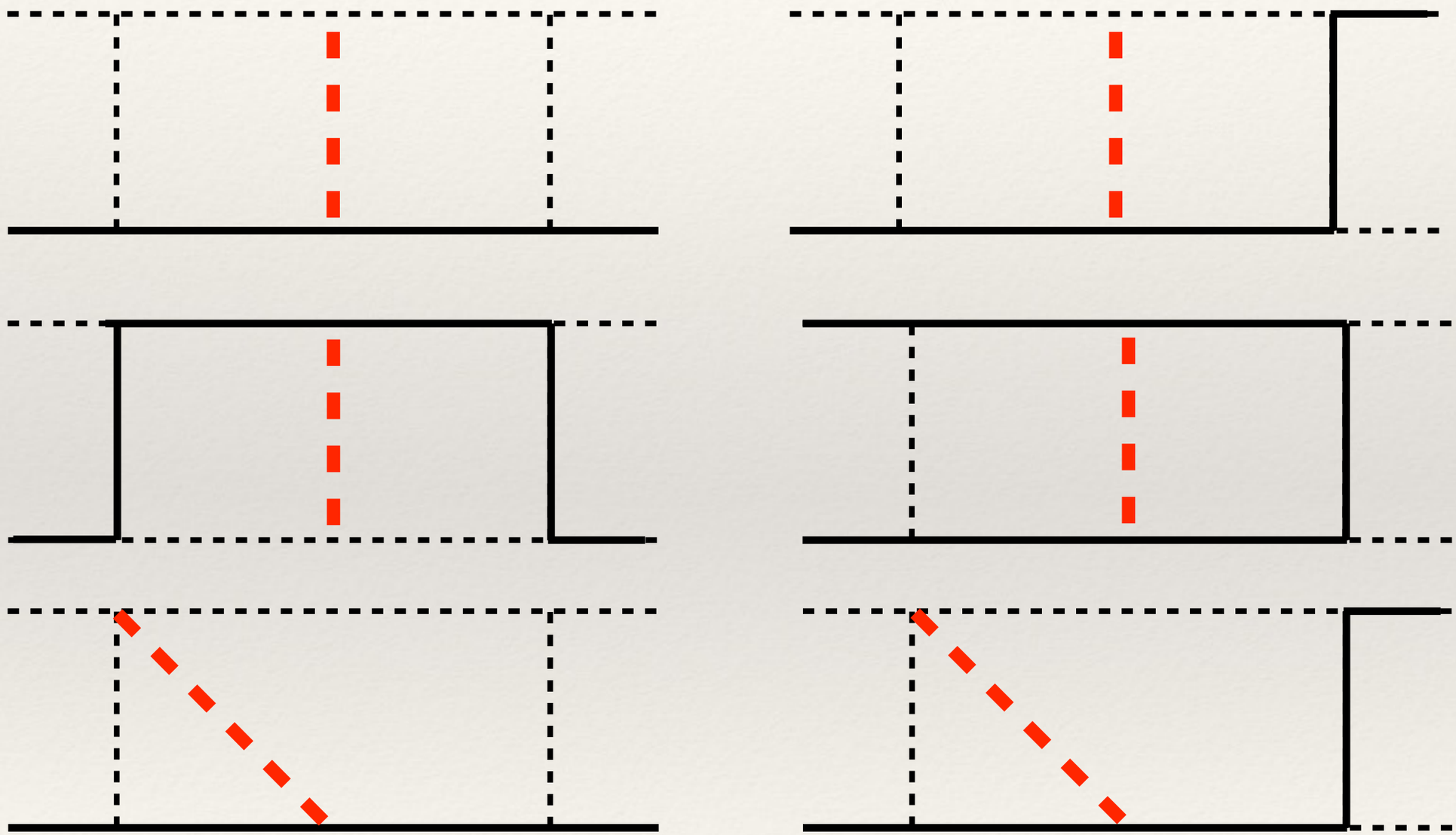
Example 2



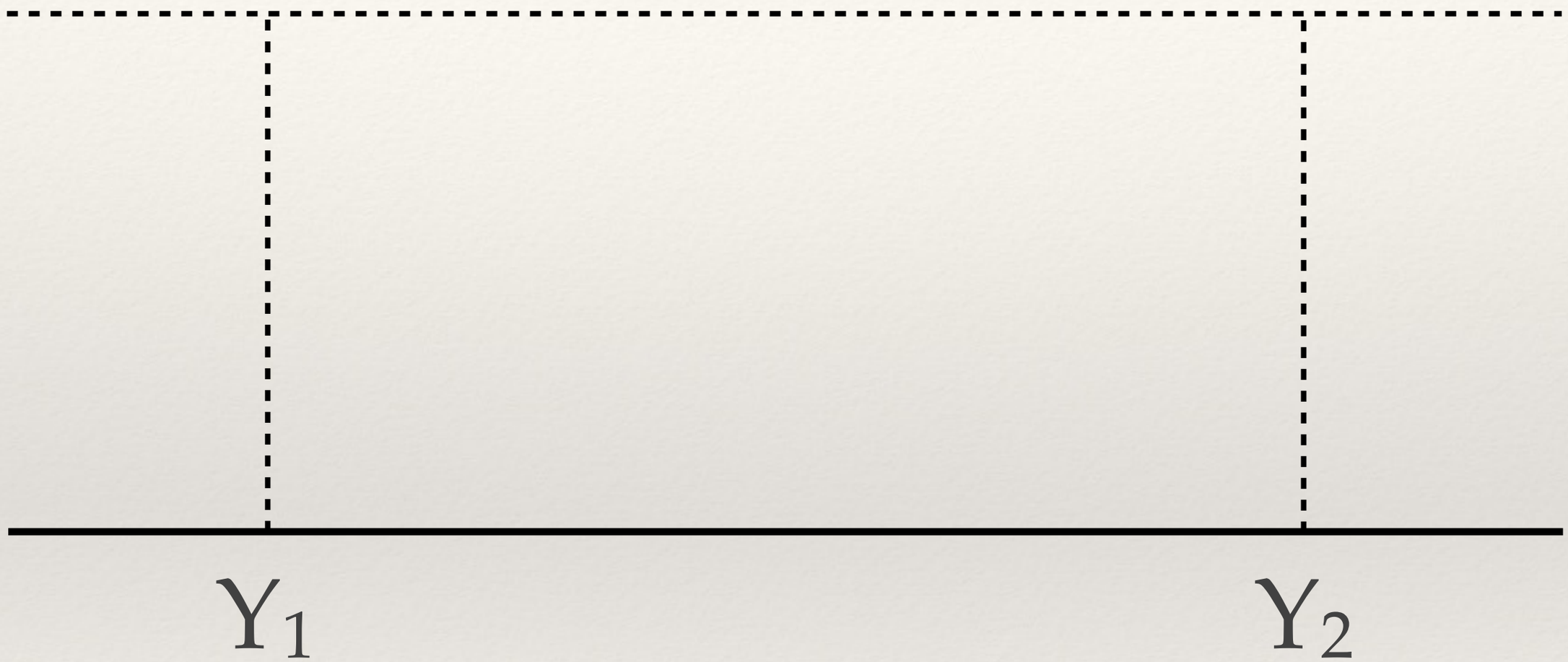
Example 2



Examples that work

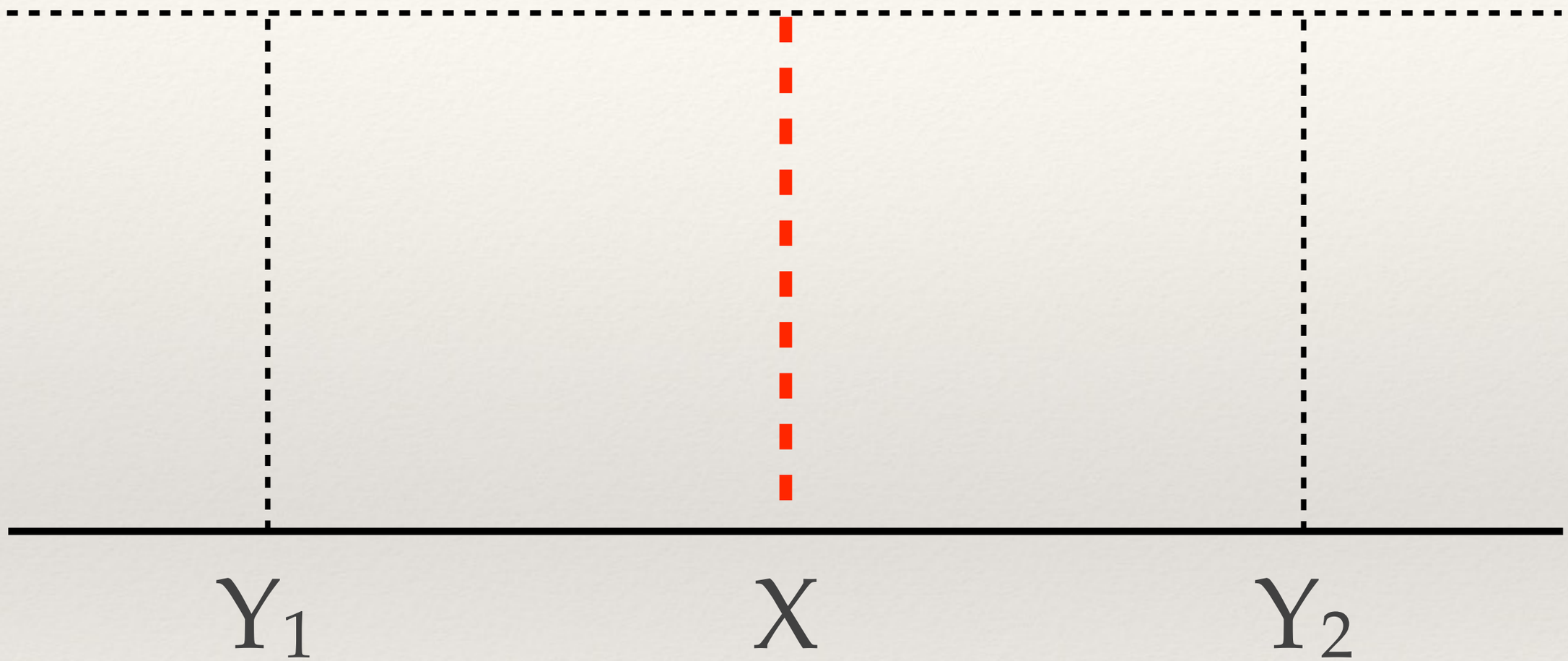


Real Example



$$M\nu_1 \sim Y_1.\Lambda.Y_2^T + Y_1.\Lambda.Y_2^T$$

Real Example



$$M\nu_1 \sim Y_1.\Lambda.Y_2^T + Y_1.\Lambda.Y_2^T$$

$$M\nu_2 \sim Y_1.\Lambda_1.X.\Lambda_2.Y_2^T + Y_2.\Lambda_2.X^T.\Lambda_1.Y_1^T$$

~The model

- SM (H, L)
- One scalar doublet (S_2), Y=1
- One scalar singlet (S_1), Y=0
- Three heavy Fermions (F_i), Y=0
- Z_2 Symmetry
- $Y_1 = Y_2$
- Tribimaximal

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- Three heavy Fermions (F_i), Y=0
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- **One scalar singlet (S_3), Y=0**

Z_2 symmetry

- Avoid FCNC (Scalar doublet needs to have different Z_2 than the Higgs, or terms in the potential (i.e $V = H^\dagger H H^\dagger S_2$ or $V = S_2^\dagger S_2 S_2^\dagger H$) are possible)

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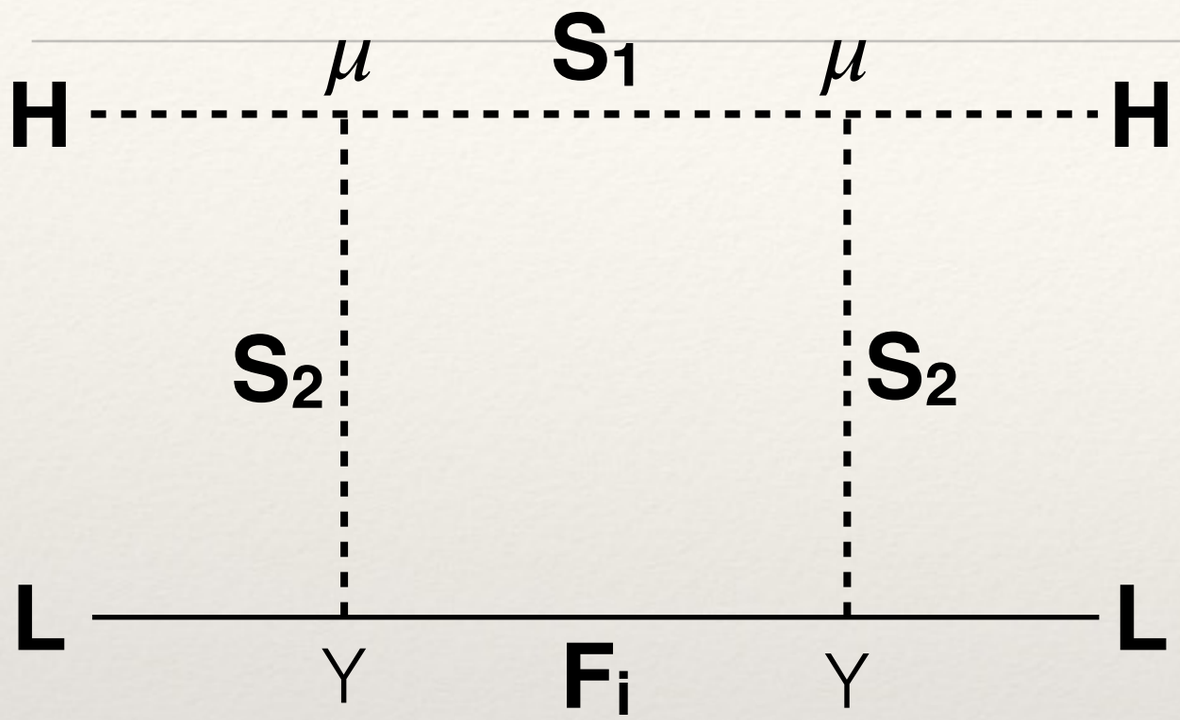
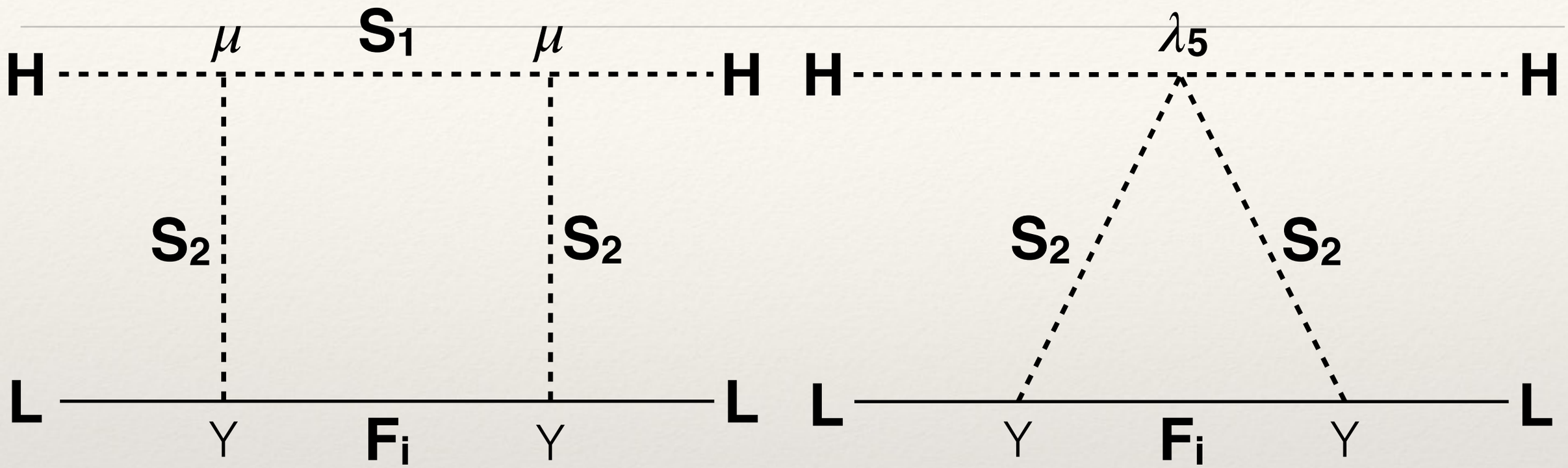
- Forbid Dirac masses (i.e. Seesaw)
- DM candidate

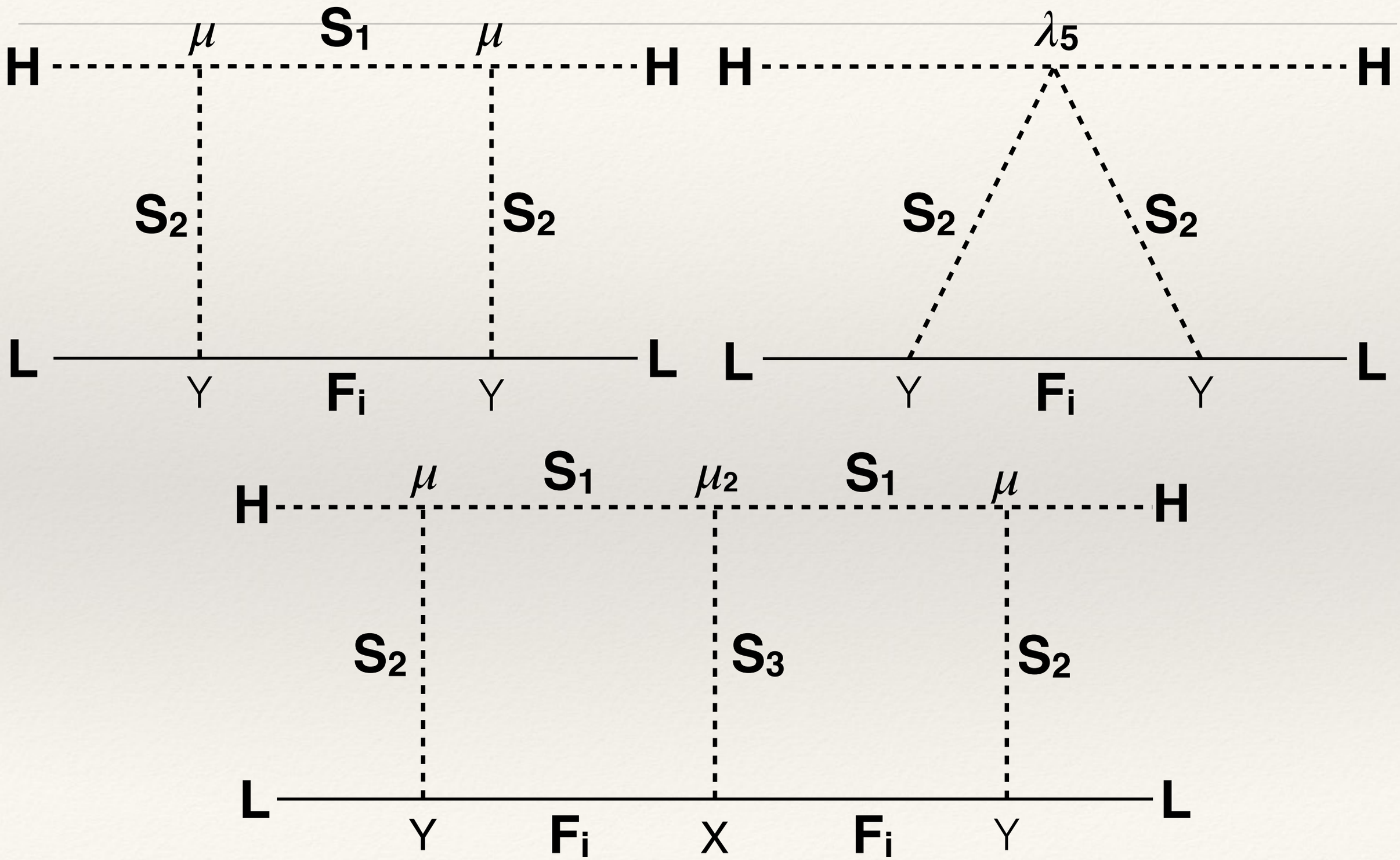
One/two-loops



One/two-loops



One/two-loops



Some math

$$\mathcal{L} = Y^{ia}\,(\overline{L^{\mathtt{c}}}_i\,P_L)F^{\mathtt{c}}{}_a\,S_2 \;+\; X^{ab}\,\overline{F^{\mathtt{c}}}_a\,F^{\mathtt{c}}{}_b\,S_3 \;+\; Y^{bj}\,\overline{F^{\mathtt{c}}}_b\,(P_L\,L_j)\,S_2^\dagger$$
$$+\;\mu\,H\,S_1^\dagger\,S_2 + \mu_2\,S_1\,S_2^\dagger\,S_3^\dagger \;+\; \mu\,H\,S_2\,S_1^\dagger$$

$$(\mathcal{M}_1)_{ij} = \frac{1}{32\pi^2}\sum_{k=1}^{3} Y_{ik} m_{Fk} Ykj\Big((\cos^2\alpha)F(m_{s1},m_{Fk})+(\sin^2\alpha)F(m_{s2},m_{Fk}) - F(m_A,m_{Fk})\Big)$$

$$(\mathcal{M}_2)_{ij} = \frac{\mu_2}{4(2\pi)^8}\sum_{a,b=1}^{3}(Y^{ia}\,X^{ab}\,Y^{bj} + Y^{ib}\,X^{ba}\,Y^{aj})\,\sin^2(2\alpha)$$
$$\times\sum_{\alpha,\beta=1}^{2}(-1)^\alpha(-1)^\beta\left(2\,m_{Fa}\,m_{Fb}\,I_\chi^1 \;+\; I_\chi^{k^2} \;+\; I_\chi^{q^2} \;-\; I_\chi^{(k+q)^2}\right)$$

Approximations

$$M_F << m_{s2} << m_{s1}$$

$$m_{one-loop} = \frac{\lambda_5 v^2}{16\pi^2\, m_s^2} Y \cdot M_F \cdot Y^T$$

$$m_{two-loop} = \frac{\mu}{(96\pi^2)\, m_s^2} \mathcal{F}(m_{s3}/m_{s2}) Y \cdot M_F \cdot X \cdot M_F \cdot Y^T$$

$$\mathcal{F}(m_{s3}/m_{s2}) = \begin{cases} 1 & m_{s2} >> m_{s_3} \\ 1 + \frac{3}{\pi^2}(log^2 \frac{m_{s3}^2}{m_{s2}^2} - 1) & m_{s3} >> m_{s_2} \end{cases}$$

arXiv:hep-ph/0601225
arXiv:hep-ph/0212058

Numerics

$$m_i^0 = \frac{\lambda_5 \, v^2 K_i^2 \, m_{Fi}}{8\pi^2 \, m_s^2}$$

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$$m_{F1} = \pm \frac{\sqrt{\lambda_5^2\, v^4 K_2^4\, m_{F2}^2 - 64\pi^4 m_s^4 \Delta_{21}^2}}{\lambda_5 K_1^2 v^2}$$

$$m_{F3} = \pm \frac{\sqrt{\lambda_5^2\, v^4 K_2^4\, m_{F2}^2 \pm 64\pi^4 m_s^4 \Delta_{32}^2}}{\lambda_5 K_3^2 v^2}$$

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$$+\, \mu\, H\, S_1^\dagger\, S_2 + \mu_2\, S_1\, S_2^\dagger\, S_3^\dagger \,+\, \mu\, H\, S_2\, S_1^\dagger$$

Numerics

$$m_i^0 = \frac{\lambda_5\, v^2 K_i^2\, m_{Fi}}{8\pi^2\, m_s^2} \longrightarrow \begin{aligned} m_{F1} &= \pm\frac{\sqrt{\lambda_5^2\, v^4 K_2^4\, m_{F2}^2 - 64\pi^4 m_s^4 \Delta_{21}^2}}{\lambda_5 K_1^2 v^2} \\ m_{F3} &= \pm\frac{\sqrt{\lambda_5^2\, v^4 K_2^4\, m_{F2}^2 \pm 64\pi^4 m_s^4 \Delta_{32}^2}}{\lambda_5 K_3^2 v^2} \end{aligned}$$

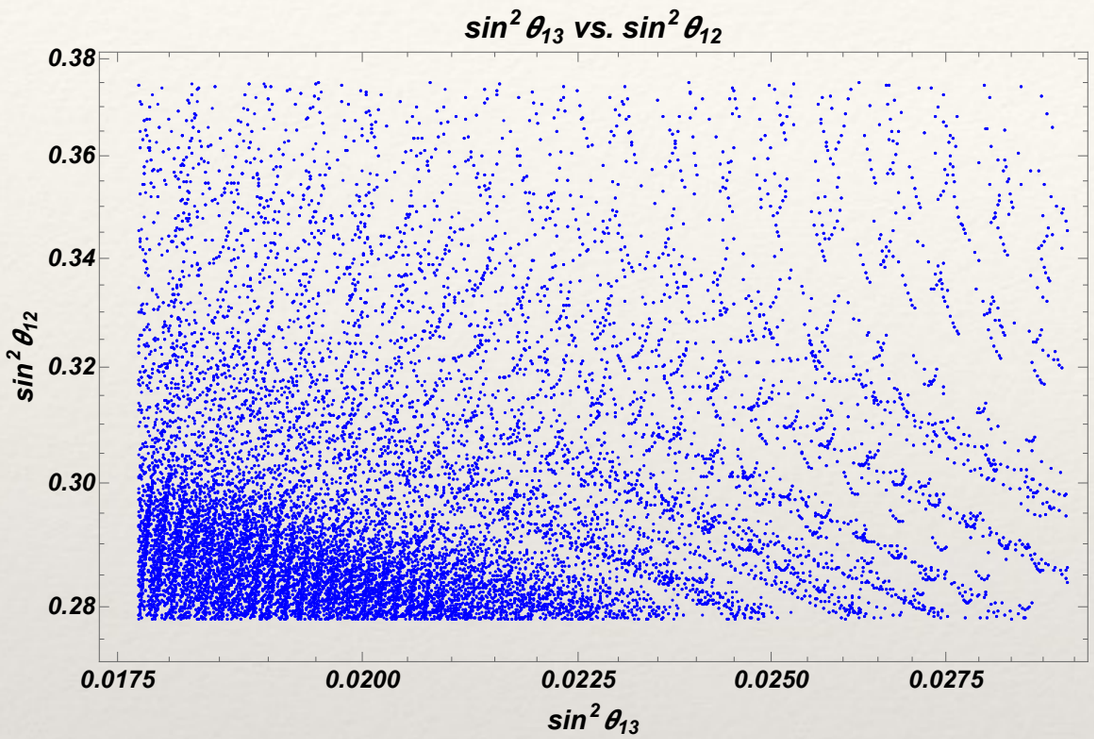
$$\mathcal{L} = Y^{ia}\,(\overline{L^{\mathtt{C}}}_i\, P_L) F^{\mathtt{C}}{}_a\, S_2 + X^{ab}\, \overline{F^{\mathtt{C}}}_a\, F^{\mathtt{C}}{}_b\, S_3 + Y^{bj}\, \overline{F^{\mathtt{C}}}_b\, (P_L\, L_j)\, S_2^\dagger$$
$$+ \mu\, H\, S_1^\dagger\, S_2 + \mu_2\, S_1\, S_2^\dagger\, S_3^\dagger + \mu\, H\, S_2\, S_1^\dagger$$

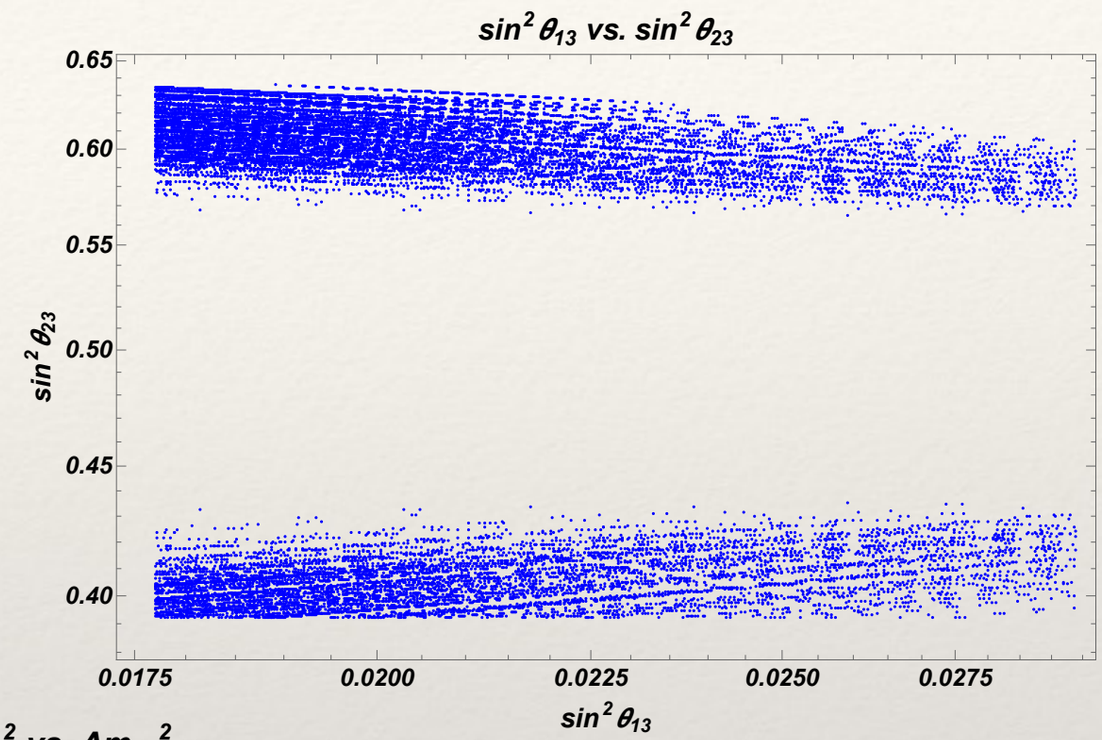
$$X = \begin{pmatrix} X_{11} & X_{12} & X_{13} \\ X_{12} & X_{22} & X_{23} \\ X_{13} & X_{23} & X_{33} \end{pmatrix}$$

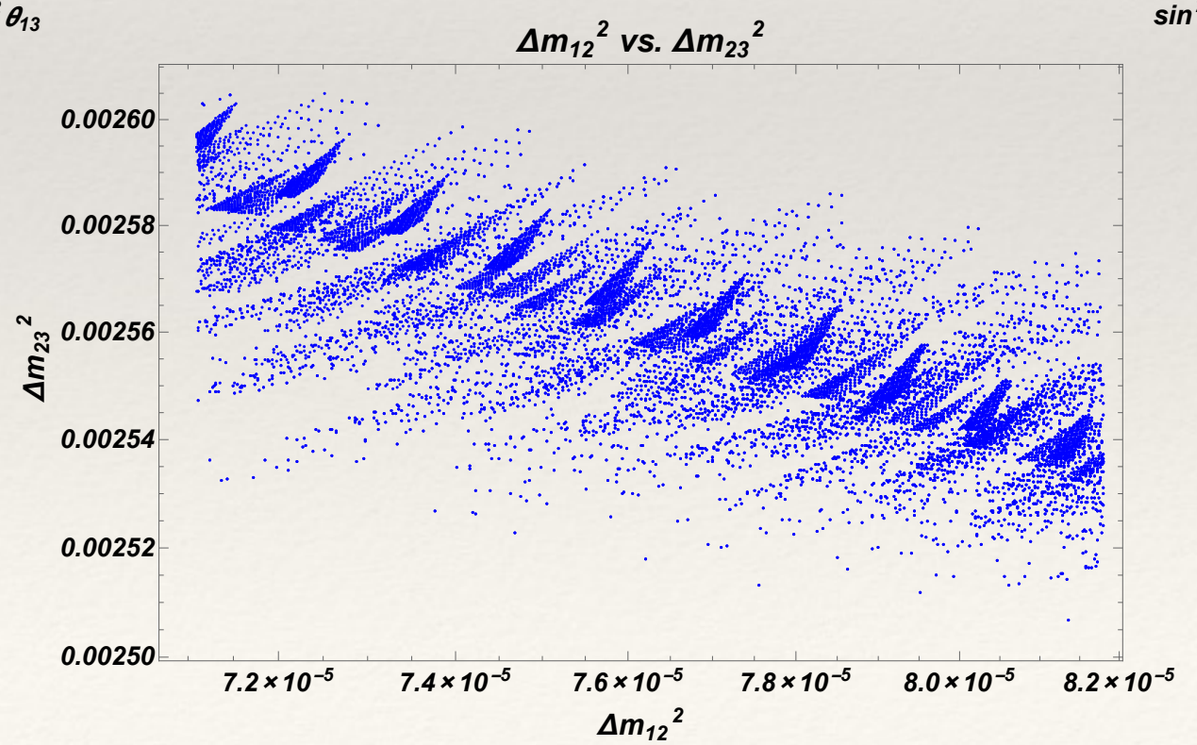
Numerics

K_1, K_2, K_3	0.1
X_{23}	-.08 / .08
X_{13}	-0.1 / 0.1
X_{12}	0 / 0.02
X_{11}, X_{22}, X_{33}	0
M_{f2}	7 / 15×10^{11} eV
m_s	5 / 7×10^{14} eV
μ	1×10^{9} eV
λ_5	0,001
Λ_{21}	10×10^{-3} eV^2
Λ_{32}	2.4×10^{-5} eV^2

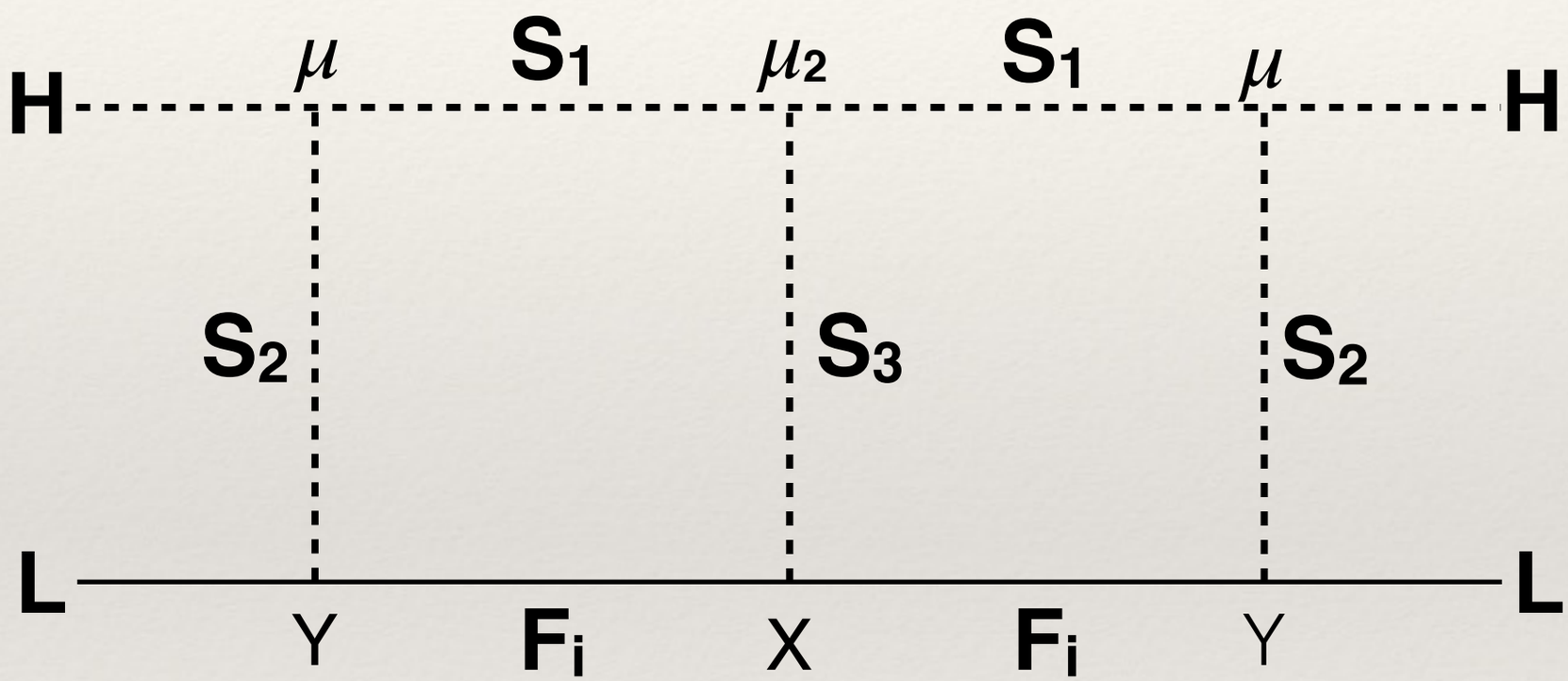
Some plots


$\sin^2\theta_{13}$ vs. $\sin^2\theta_{12}$
0.38
0.36
0.34
0.32
0.30
0.28
$\sin^2\theta_{12}$
0.0175
0.0200
0.0225
0.0250
0.0275
$\sin^2\theta_{13}$

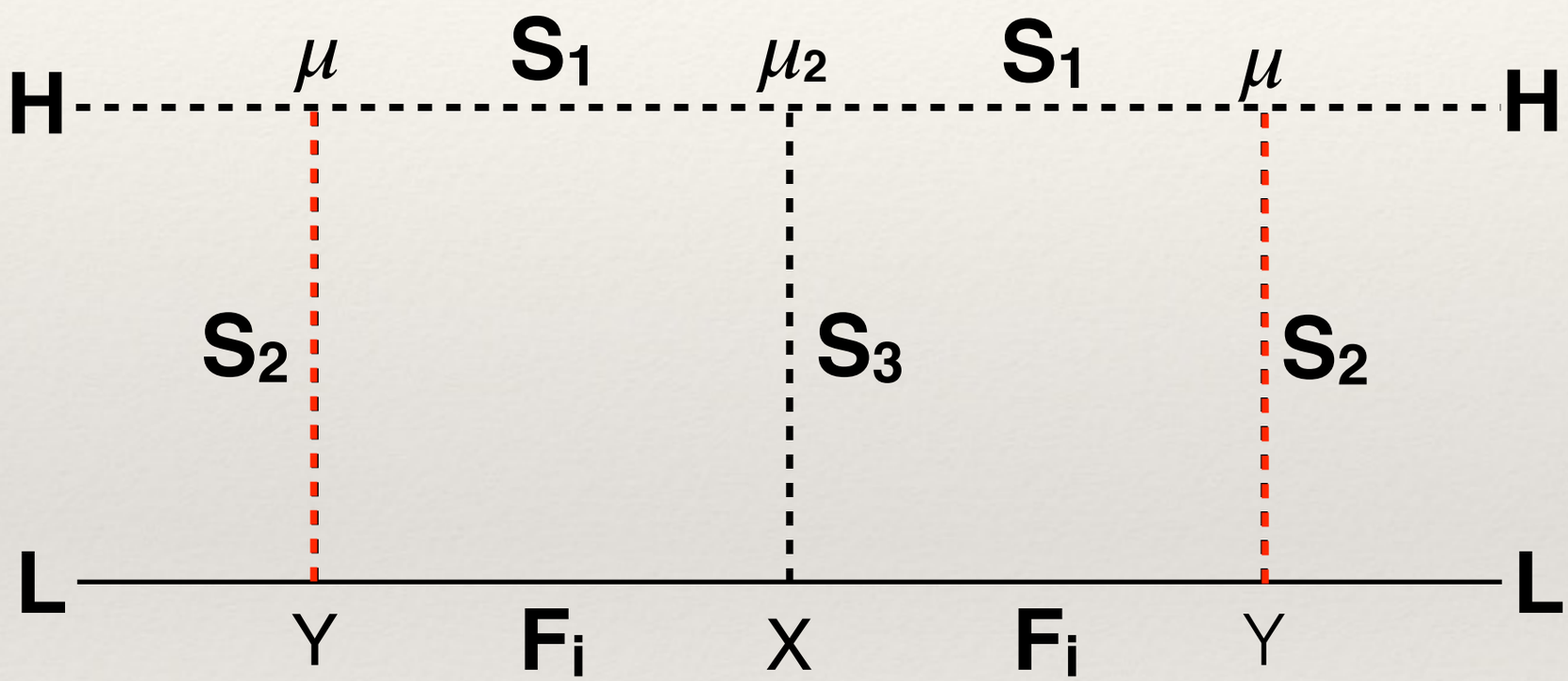

$\sin^2\theta_{13}$ vs. $\sin^2\theta_{23}$
0.65
0.60
0.55
0.50
0.45
0.40
$\sin^2\theta_{23}$
0.0175
0.0200
0.0225
0.0250
0.0275
$\sin^2\theta_{13}$


Δm_{12}^2 vs. Δm_{23}^2
0.00260
0.00258
0.00256
0.00254
0.00252
0.00250
Δm_{23}^2
7.2×10^{-5}
7.4×10^{-5}
7.6×10^{-5}
7.8×10^{-5}
8.0×10^{-5}
8.2×10^{-5}
Δm_{12}^2

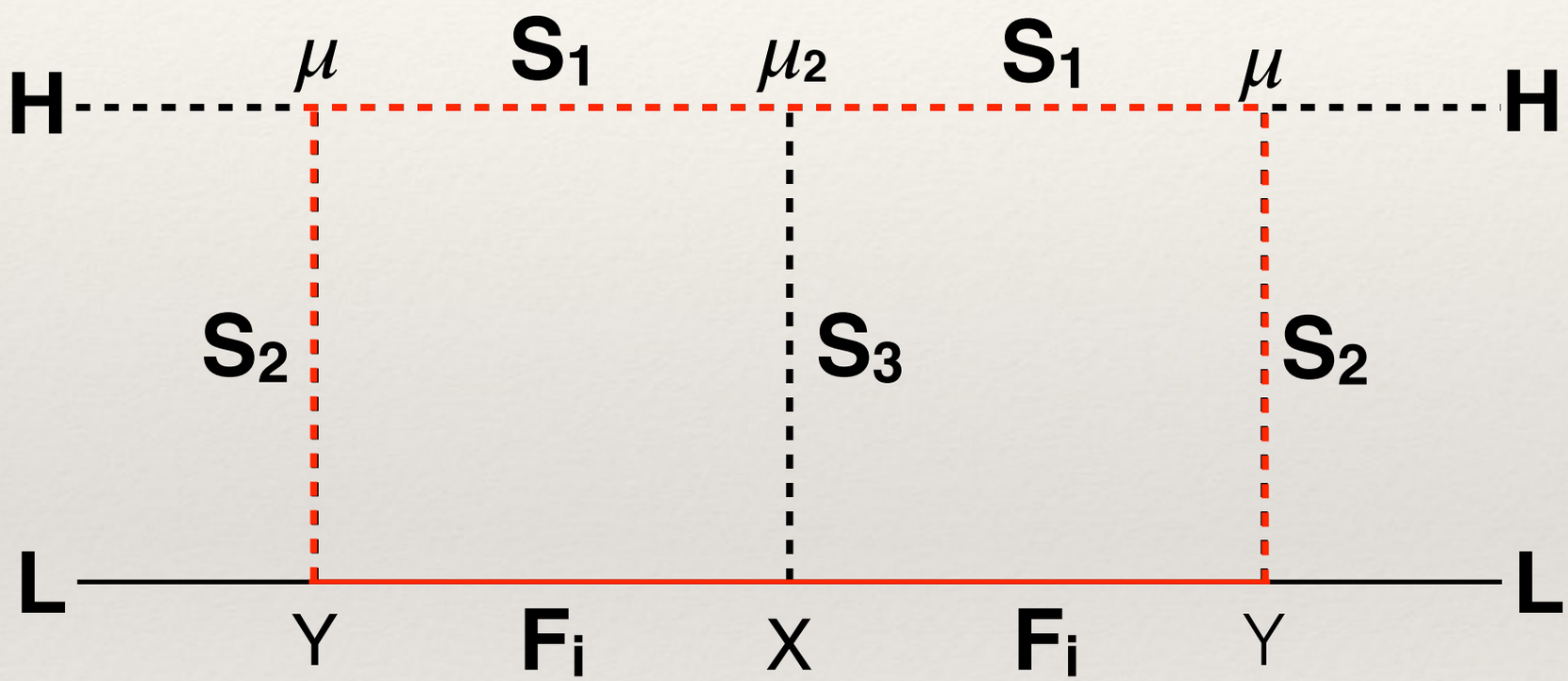
DM candidates?



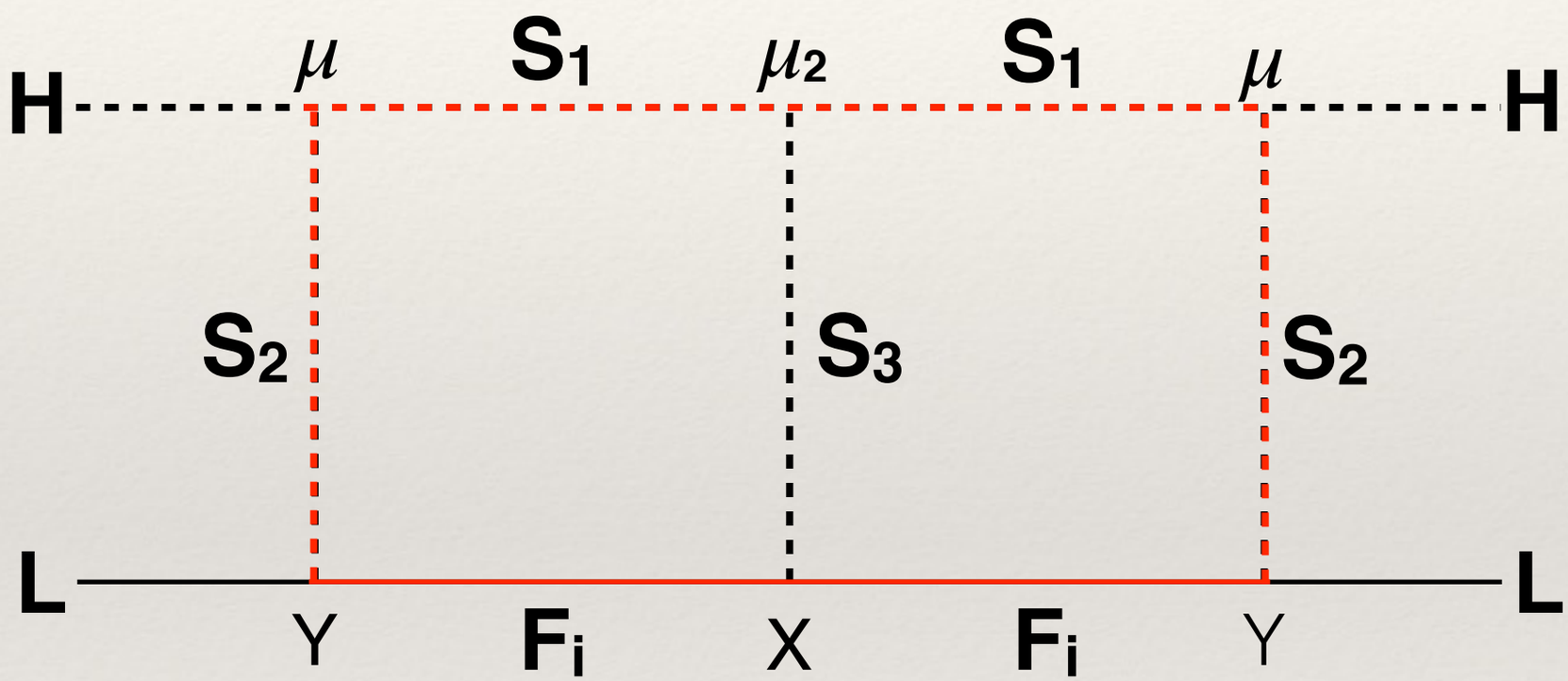
DM candidates?



DM candidates?



DM candidates?



Smallest of F_i can be DM candidate or smallest between S_1 or S_2 can DM candidate

Conclusions

$$M = m_1 + m_2$$

$$m_1 = Y \cdot A \cdot Y^T$$

$$m_2 = Y \cdot B \cdot Y^T$$

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$$M = m_1 + m_2 \ = Y \cdot (A + B) \cdot Y^T$$

$$m_1 = Y \cdot A \cdot Y^T$$

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Conclusions

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$$m_1 = Y \cdot A \cdot Y^T$$

$$m_2 = Y \cdot B \cdot Y^T$$

$$\begin{aligned} M_d &= U_{MNS}^T \cdot M \cdot U_{MNS} \\ m_d &= U_0^T \cdot m_1 \cdot U_0 \\ U_{MNS} &= U_0 \cdot O \end{aligned}$$

Conclusions

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$$m_1 = Y \cdot A \cdot Y^T$$

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$$\begin{aligned} Y &= U_0 \, m_d^{1/2} \, R_1^T \, A^{-1/2} \\ Y &= U_{MNS} \, M_d^{1/2} \, R_2^T \, (A + B)^{-1/2} \\ O &= m_d^{1/2} R_1^T (1 + A^{-1} B)^{1/2} R_2 M_d^{-1/2} \\ O &\simeq m_d^{1/2} R_1^T R_2 M_d^{-1/2} + \frac{1}{2} m_d^{1/2} R_1^T A^{-1} B R_2 M_d^{-1/2} \\ O &\simeq 1 + \frac{1}{2} m_d^{1/2} A^{-1} B m_d^{-1/2} \end{aligned}$$

Conclusions and prospectives

- “Ugly” neutrino mixing, can be explained with radiative masses.

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- DM
- CP violation
- Sterile neutrinos
- Quarks