WIN 2017
Irvine, California 19-24 June 2017

FAST NEUTRINO FLAVOR CONVERSIONS NEAR THE SN CORE

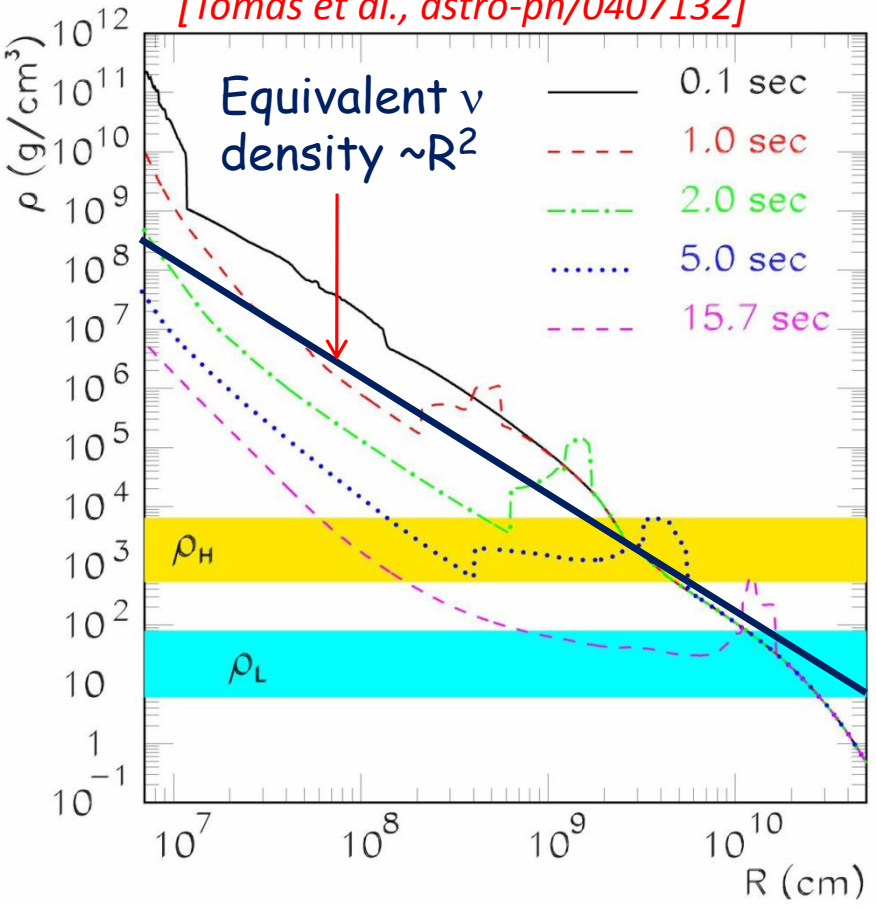
Alessandro MIRIZZI
(BARI Uni. & INFN Bari, Italy)

Based on:

- *Dasgupta, A.M., Sen, arXiV:1609.00528*
- *Capozzi, Dasgupta, Lisi, Marrone, A.M. , arXiV:1706.03360*

SNAPSHOT OF SN DENSITIES

[Tomas et al., astro-ph/0407132]



- Matter bkg potential

$$\lambda = \sqrt{2} G_F N_e \quad \sim R^{-3}$$

- ν–ν interaction

$$\mu = \sqrt{2} G_F n_\nu \quad \sim R^{-2}$$

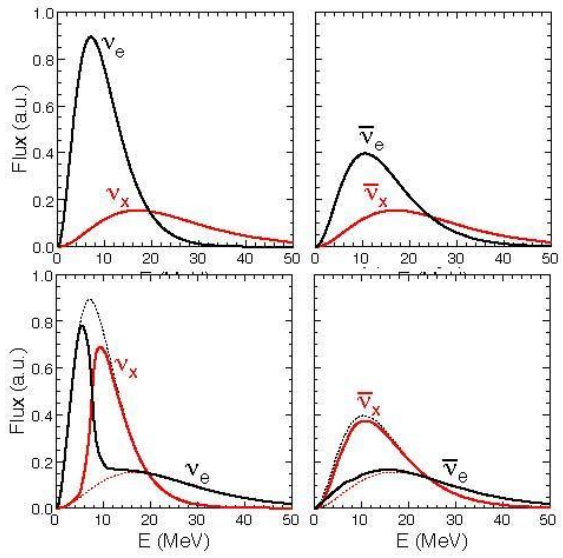
See talk by B. Dasgupta

- Vacuum oscillation frequencies

$$\omega = \frac{\Delta m^2}{2E}$$

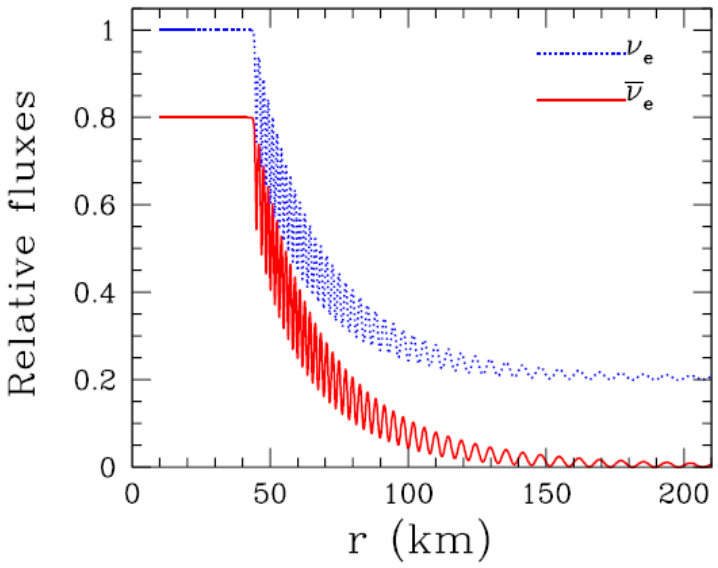
When $\mu >> \lambda$, SN ν oscillations dominated by ν-ν interactions

Collective flavor transitions at low-radii [O (10^2 – 10^3 km)]

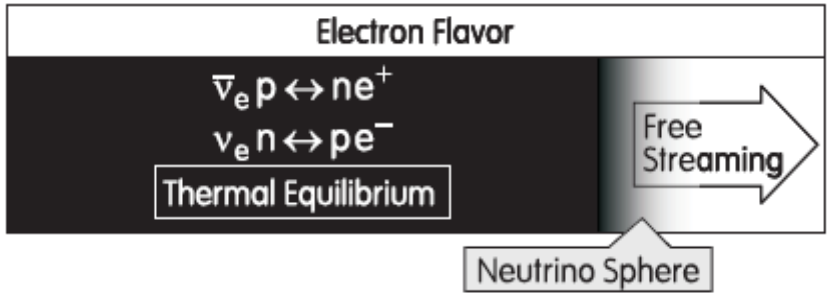


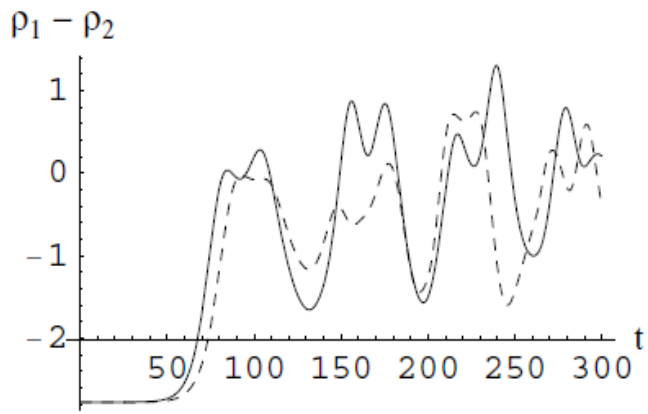
See [A.M., Tamborra, Janka, Saviano, Scholberg et al., arXiv:1508.00785 [astro-ph.HE]] for a review

FLAVOR CONVERSIONS NEAR SN CORE?



- Most of the studies assume no flavor conversion at r < 50 km (only synchronized oscillations). After self-induced conversions develop with a rate ~ $\sqrt{\omega\mu}$ *[see, e.g., Hannestad et al, astro-ph/0608695]*





- However, since more than a decade Ray Sawyer is pointing out that close to nu-sphere nu angular distributions of different species are rather different. This would lead to a new flavor instability (absent assuming equal angular distributions). The outcome would be a possible complete flavor mixing of the outgoing stream just above the nu-sphere. Fast rate ~μ

FAST vs SLOW

SELF-INDUCED FLAVOR CONVERSIONS

FAST FLAVOR CONVERSIONS NEAR SN CORE

PHYSICAL REVIEW D **72,** 045003 (2005)

Speed-up of neutrino transformations in a supernova environment

R. F. Sawyer

Department of Physics, University of California at Santa Barbara, Santa Barbara, California 93106, USA

(Received 8 April 2005; published 5 August 2005)

When the neutral current neutrino-neutrino interaction is treated completely, rather than as an interaction among angle-averaged distributions, or as a set of flavor-diagonal effective potentials, the result can be flavor mixing at a speed orders of magnitude faster than that one would anticipate from the measured neutrino oscillation parameters. It is possible that the energy spectra of the three active species of neutrinos emerging from a supernova are nearly identical.

PHYSICAL REVIEW D **79,** 105003 (2009)

Multiangle instability in dense neutrino systems

R. F. Sawyer

Department of Physics, University of California at Santa Barbara, Santa Barbara, California 93106, USA

(Received 18 April 2008; published 6 May 2009)

We calculate rates of flavor exchange within clouds of neutrinos interacting with each other through the standard model coupling, assuming a conventional mass matrix. For cases in which there is an angular dependence in the relation among intensity, flavor, and spectrum, we find instabilities in the evolution equations and greatly speeded-up flavor exchange. The instabilities are categorized by examining linear perturbations to simple solutions, and their effects are exhibited in complete numerical solutions to the system. The application is to the region just under the neutrino surfaces in the supernova core.

PRL **116,** 081101 (2016) PHYSICAL REVIEW LETTERS week ending 26 FEBRUARY 2016

Neutrino Cloud Instabilities Just above the Neutrino Sphere of a Supernova

R. F. Sawyer

Department of Physics, University of California at Santa Barbara, Santa Barbara, California 93106, USA

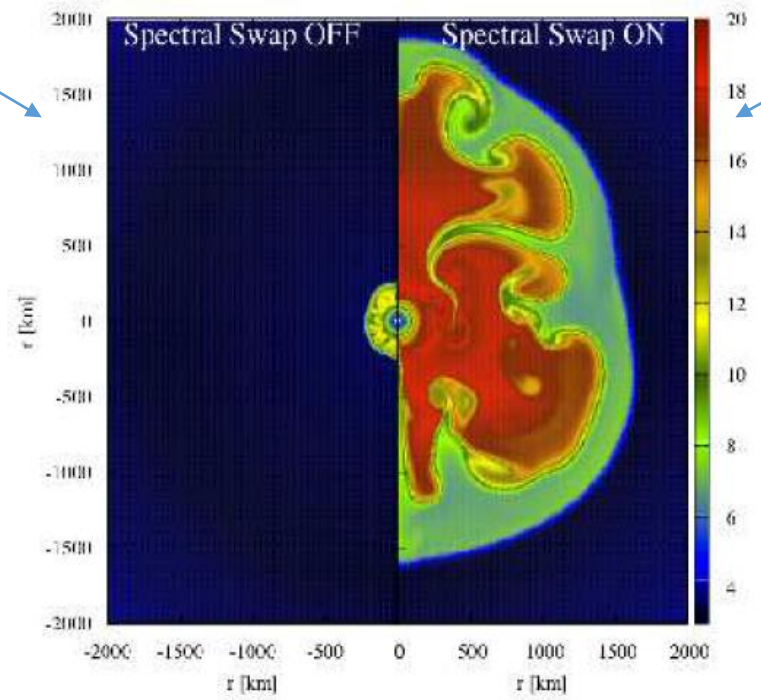
(Received 7 September 2015; revised manuscript received 2 January 2016; published 25 February 2016)

Most treatments of neutrino flavor evolution, above a surface of the last scattering, take identical angular distributions on this surface for the different initial (unmixed) flavors, and for particles and antiparticles. Differences in these distributions must be present, as a result of the species-dependent scattering cross sections lower in the star. These lead to a new set of nonlinear equations, unstable even at the initial surface with respect to perturbations that break all-over spherical symmetry. There could be important consequences for explosion dynamics as well as for the neutrino pulse in the outer regions.

WHY SHOULD WE WORRY ABOUT THESE EFFECTS?

- If flavor changes occur in the deepest SN regions, they would modify the neutrino heating behind the stalled shock wave, possibly helping a SN to explode.

Non-exploding



Exploding. Spectral swap (by hands) behind shock front

[Suwa et al., 1106.5487]

- If flavor equilibrium would occur close to the nu-sphere, all further oscillation effects (self-indued, matter…) would be washed-out. Crucial to predict observable SN nu signal.

SELF-INDUCED FLAVOR CONVERSION

- Flavor conversion (vacuum or MSW) for a neutrino of a given momentum *p*
  - Requires lepton flavor violation by masses and mixing

$$\nu_e \to \nu_\mu$$

$$\frac{\Delta m^2_{atm}}{2E} = 10^{-10}\,\text{eV} = 0.5\ \text{km}^{-1}$$

- Pair-wise flavor exchange by ν–ν refraction (fwd scattering)
  - No net flavor change of pair
  - Requires dense neutrino medium (collective effects of self-interacting neutrinos)
  - Can occur without mass/mixing (and does not depend on $\Delta m^2/2E$)
  - Familiar as neutrino pair processes $O(G^2_F)$. Here as coherent effect $O(G_F)$

$$\nu_e(p) + \bar{\nu}_e(k) \to \nu_\mu(p) + \bar{\nu}_\mu(k)$$

$$\nu_e(p) + \nu_\mu(k) \to \nu_\mu(p) + \nu_e(k)$$

$$\sqrt{2} G_F n_\nu = 10^{-5}\,\text{eV} = 0.5\ \text{cm}^{-1}$$

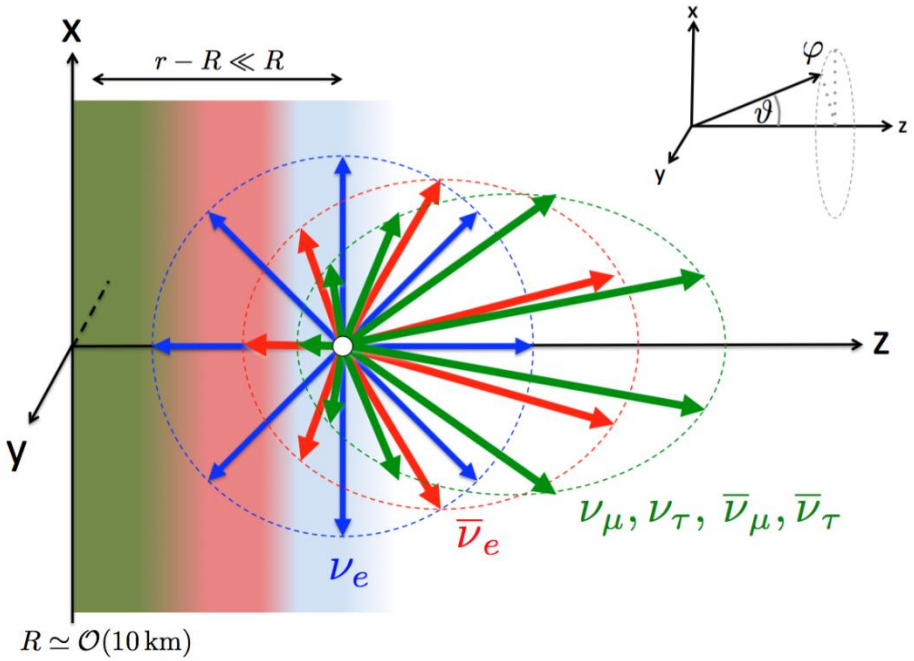
$$E = 12.5\ \text{MeV}$$

$$\text{R} = 80\ \text{km}$$

$$\text{L}_\nu = 40 \times 10^{51}\,\text{erg}$$

NEUTRINO ANGULAR DISTRIBUTIONS AT DECOUPLING

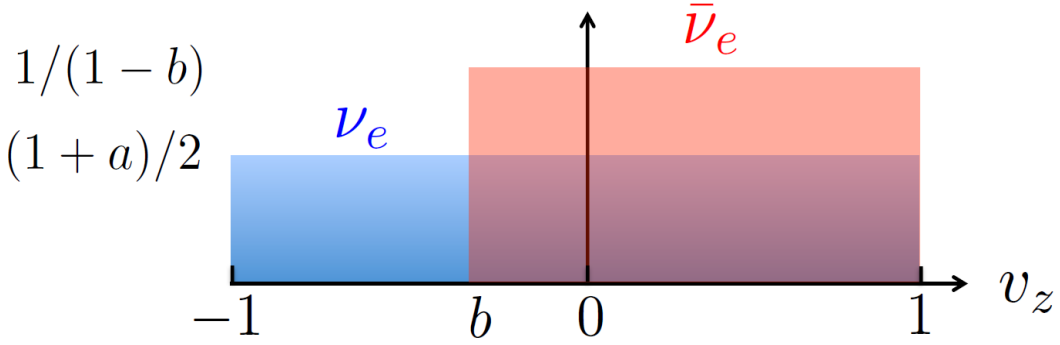
[Dasgupta, A.M., Sen, arXiV:1609.00528]



- Electron flavors remain in equilibrium with matter for a longer period than the non-electron flavors, due to the largest cross-sections of CC interactions
- Non-electron flavors decouple deeper in the star (more fwd-peaked distributions)
- Neutron-richness enhances CC interactions for ν_e keeping them more coupled to matter (more isotropic distribution) than $\overline{\nu}_e$.

TWO-BULB MODEL

[Sawyer, 1509.03323, Chakraborty, Hansen, Izaguirre, Raffelt, 1602.00698 Dasgupta, A.M., Sen, arXiV:1609.00528]

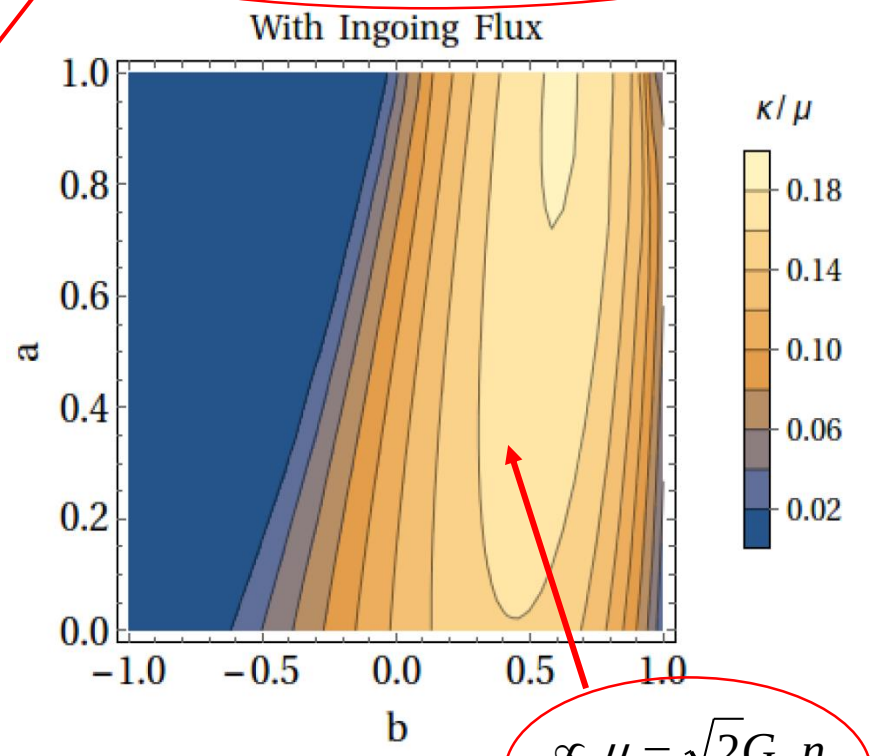


(1+a)/2 ⟶ $\nu_e/\bar{\nu}_e$ flux ratio

b ⟶ bulb-size asymmetry

1/(1-b)> (1+a)/2 ⟶ Crossing in angular spectra

Fast-instability growth rate

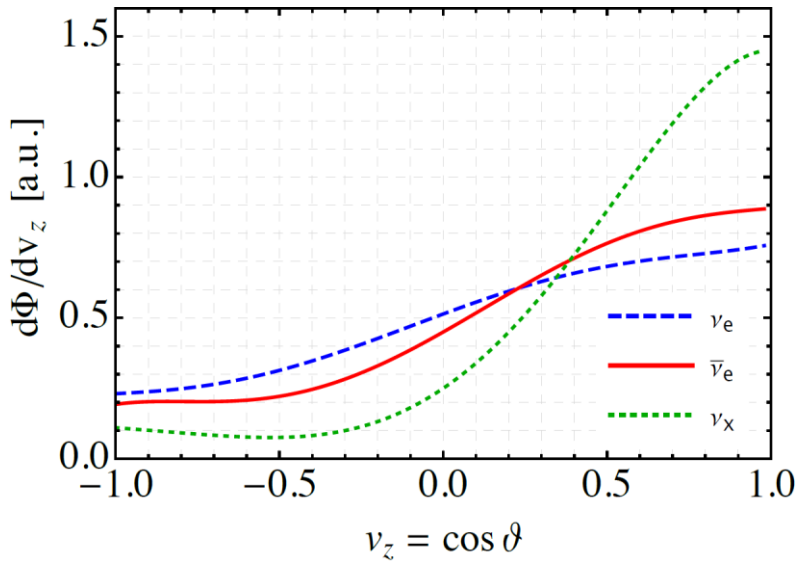


$\propto \mu = \sqrt{2} G_F n_\nu$

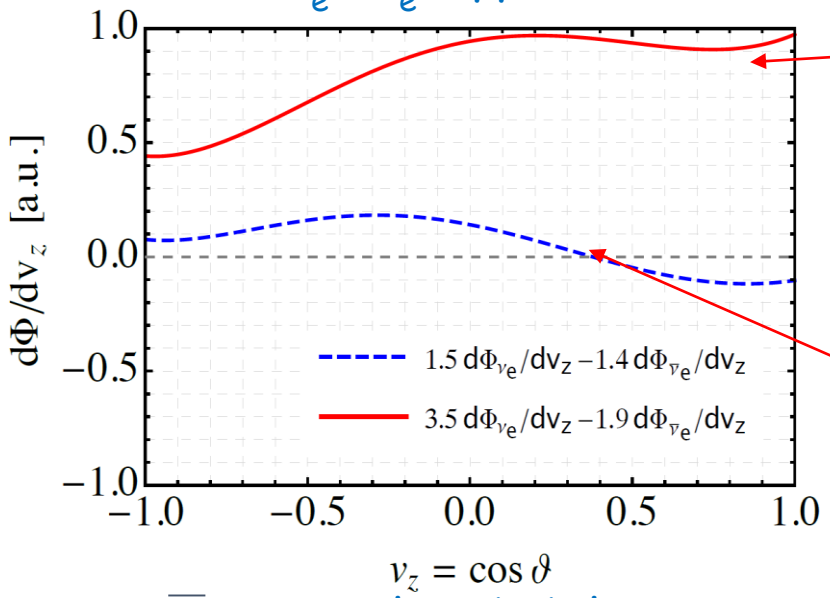
FAST FLAVOR CONVERSIONS IN SN

[Dasgupta, A.M., Sen, arXiV:1609.00528]

Nu angular distributions



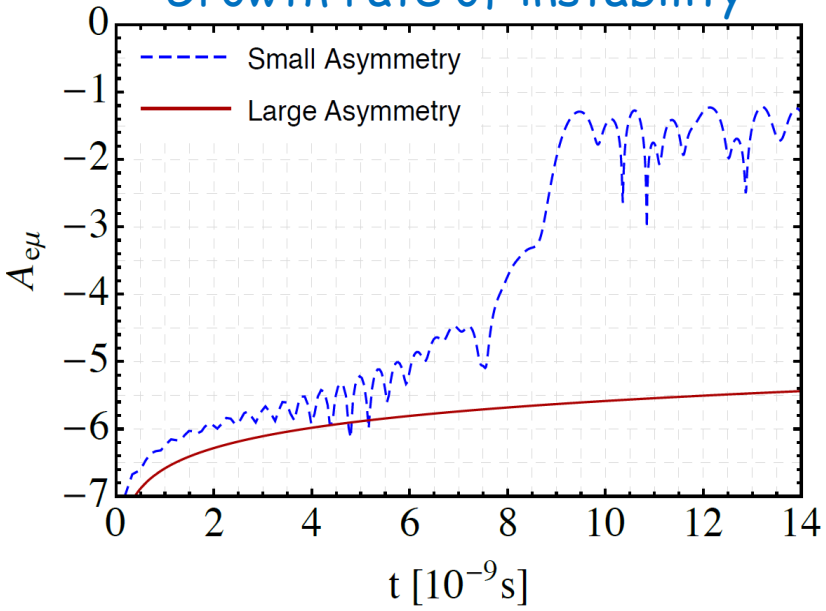
$\nu_e - \bar{\nu}_e$ difference



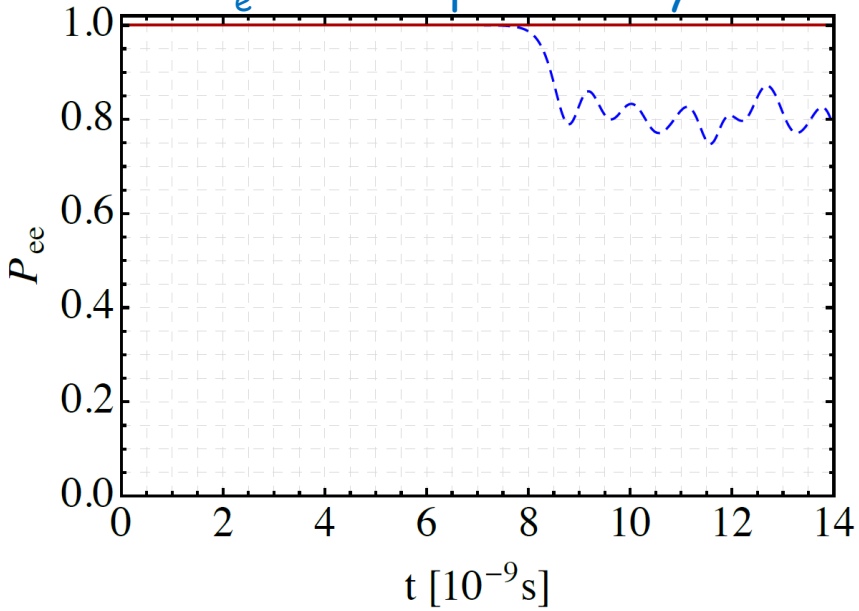
no crossing

crossing

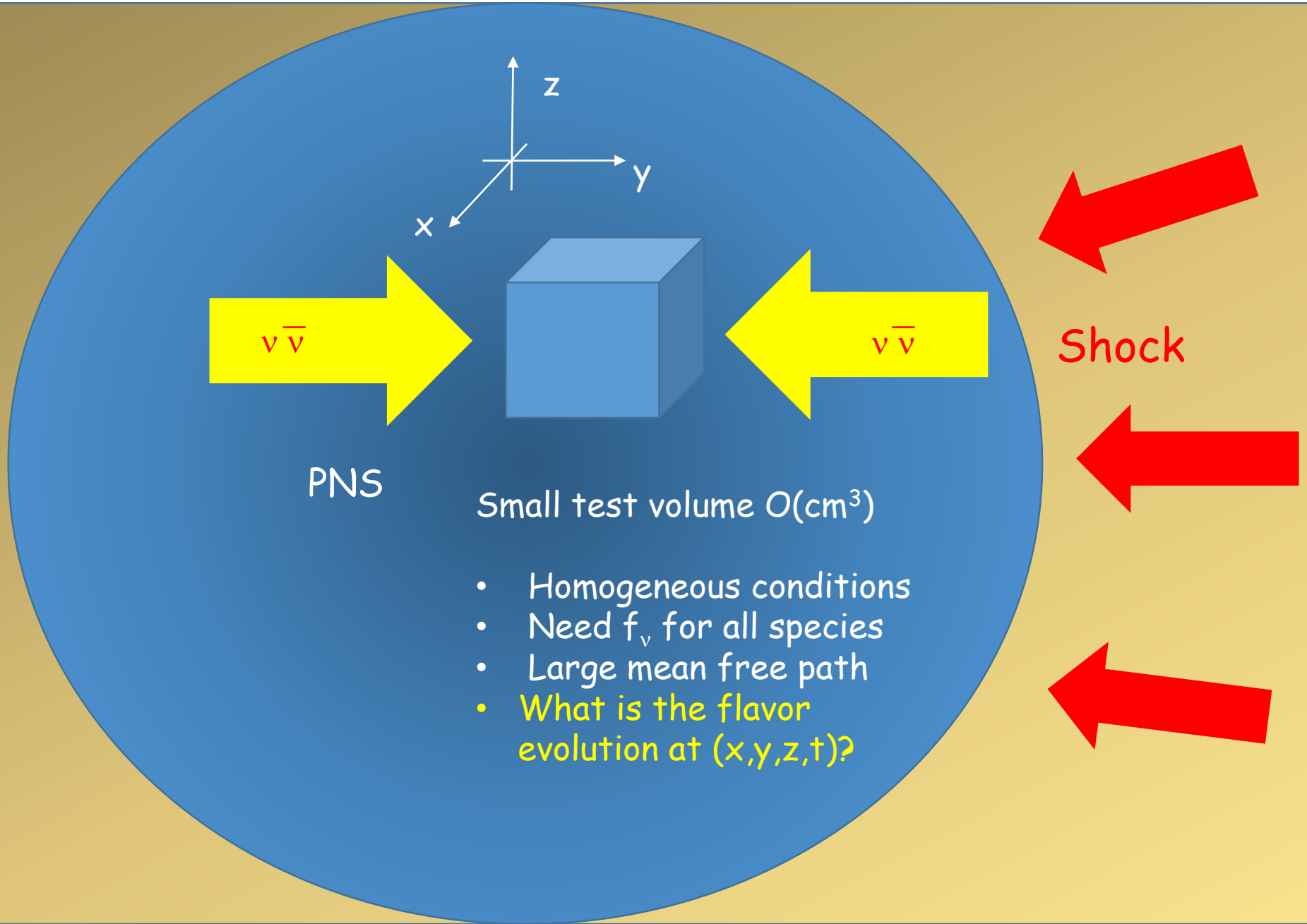
Growth rate of instability



$\bar{\nu}_e$ survival probability



LOCAL STABILITY ANALYSIS



Small test volume O(cm^3)

- Homogeneous conditions
- Need f_ν for all species
- Large mean free path
- What is the flavor evolution at (x,y,z,t)?

EQUATIONS OF MOTION FOR A TWO FLAVOR SYSTEM

[Izaguirre, Raffelt & Tamborra, 1610.01612]

- **Liouville equations for 2x2 density matrices**

$$(\partial_t + \vec{v}\cdot\vec{\nabla})\rho(t,\vec{x},\vec{p}) = -i\left[H(t,\vec{x},\vec{p}),\rho(t,\vec{x},\vec{p})\right] + C\left[\rho(t,\vec{x},\vec{p})\right]$$

(Free-streaming neutrinos Collisions=0)

- **Flavor-dependent phase-space densities**

$$\rho = \begin{pmatrix} f_{\nu_e} & f_{\langle\nu_e|\nu_x\rangle} \\ f_{\langle\nu_x|\nu_e\rangle} & f_{\nu_x} \end{pmatrix}$$

Diagonal elements related to flavor content

Off-diagonal elements responsible for flavor conversions

- **Flavor evolution governed by the "Hamiltonian matrix"**

$$H = \underbrace{\frac{\Delta m^2}{2E}\begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}}_{\text{Vacuum oscillations}} + \underbrace{\sqrt{2}G_F\begin{pmatrix} n_e & 0 \\ 0 & 0 \end{pmatrix}}_{\text{MSW effect}} + \underbrace{\sqrt{2}G_F\int\frac{d^3\vec{p}}{(2\pi)^3}(\rho+\bar{\rho})}_{\text{nu-nu interaction term}}$$

Note the for $\Delta m^2=0$ the nu-nu interaction term could still produce run-away modes (off-diagonal ρ)

LINEAR STABILITY ANALYSIS

- **Flavor-dependent phase-space densities**

$$\rho = \begin{pmatrix} f_{\nu_e} & f_{\langle \nu_e | \nu_x \rangle} \\ f_{\langle \nu_x | \nu_e \rangle} & f_{\nu_x} \end{pmatrix} = \frac{f_{\nu_e} + f_{\nu_x}}{2} + \frac{f_{\nu_e} - f_{\nu_x}}{2} \begin{pmatrix} s & S \\ S^* & -s \end{pmatrix} \qquad s^2 + |S|^2 = 1$$

Complex scalar function S(x,t) contains all information if |S|<<1 and s=1

- **Linearized equations of motion for** $\Delta m^2=0$

$$i(\partial_t + \vec{v} \cdot \vec{\nabla}) S_{\vec{v}}(t,\vec{x}) = (\Lambda_0 + \Phi_0 - \vec{v} \cdot \vec{\Phi}) S_{\vec{v}}(t,\vec{x}) - \int \frac{d^3\vec{v}'}{4\pi} (1 - \vec{v} \cdot \vec{v}') G_{\vec{v}'} S_{\vec{v}'}(t,\vec{x})$$

$\vec{v}$ nu direction of motion (angular variable)

- Matter potential $\Lambda_0 = \sqrt{2} G_F (n_{e-} - n_{e+})$
- Nu potential and current $\Phi_0 = \sqrt{2} G_F (n_{\nu_e} - n_{\bar{\nu}_e})$ $\qquad \vec{\Phi} = \int \frac{d^3\vec{v}}{4\pi} \vec{v} G(\vec{v})$
- Lepton number angle distribution for ν_e $\quad G_{\vec{v}} = \sqrt{2} G_F \int_0^\infty dE \frac{E^2}{2\pi^2} [f_{\nu_e}(E,\vec{v}) - f_{\bar{\nu}_e}(E,\vec{v})]$

PLANE WAVE SOLUTION (FOURIER SPACE)

Look for plane-wave solution of the form $S_{\vec{v}}(t,\vec{r}) = \tilde{S}_{\vec{v}}(\Omega,\vec{K})e^{-i(\Omega t-\vec{K}\cdot\vec{r})}$

Eliminate the differentials $i(\partial_t + \vec{v}\cdot\vec{\nabla})S_{\vec{v}} \rightarrow (-\Omega + \vec{v}\cdot\vec{K})S_{\vec{v}}$

Eigenvalue equation for the plane wave solutions

$$\left[\Omega + \Lambda_0 + \Phi_0 - \vec{v}\cdot(\vec{\Phi} + \vec{K})\right]\tilde{S}_{\vec{v}} = \sqrt{2}G_F \int \frac{d^3\vec{v}}{4\pi}(1-\vec{v}\cdot\vec{v}')G_{\vec{v}'}\tilde{S}_{\vec{v}'}$$

MSW potential from (e^-e^+)

Potential and current from (ν_e-$\bar{\nu}_e$)

Angle distribution for (ν_e-$\bar{\nu}_e$)

DISPERSION RELATION

This equation can be schematically written as a **dispersion relation**

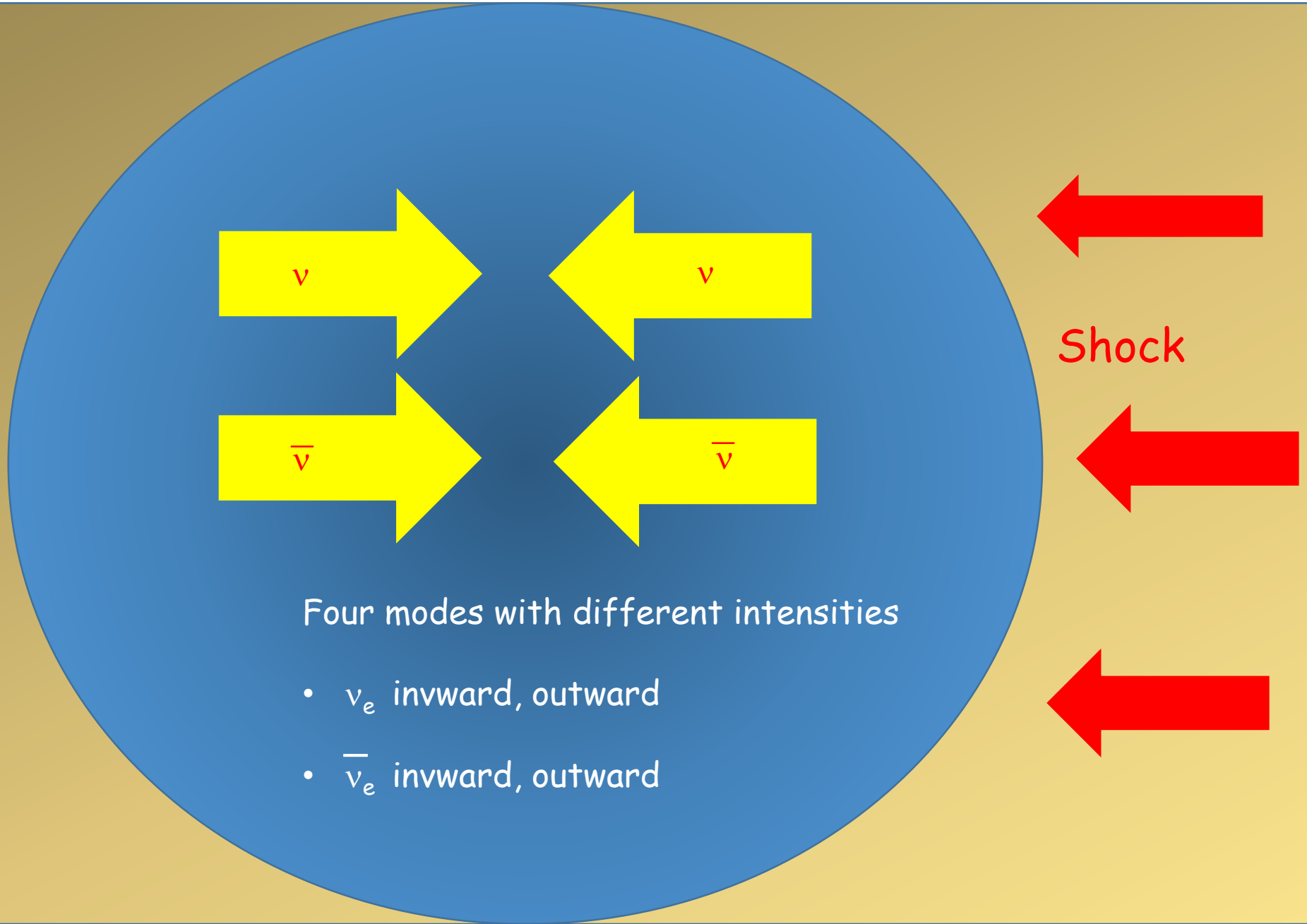
$$D(\Omega,\vec{K})=0$$

Task: Derive solution as

- $\omega = \Omega(\vec{k}) \qquad \omega \in C,\ \vec{k} \in R \longrightarrow$ **Temporal stability analysis**
- $\vec{k} = \vec{K}(\omega) \qquad \vec{k} \in C, \omega \in R \longrightarrow$ **Spatial stability analysis**

Quantitative characterization of the instabilities requires study of time-asymptotic behavior Green's function [see, e.g., *Briggs 1964, Landau & Lifshitz, Physical Kinetics-Volume 10, Chapter VI, Instability Theory*]
See [*Capozzi, Dasgupta, Lisi, Marrone, A.M., arXiV:1706.03360*] for details

TWO-BEAM MODEL



Four modes with different intensities

- ν_e invward, outward
- $\bar{\nu}_e$ invward, outward

DISPERSION RELATION

[Izaguirre, Raffelt & Tamborra, 1610.01612; Capozzi, Dasgupta, Lisi, Marrone, A.M., arXiV:1706.03360]

- Neutrino angular spectrum

$$G_{\mathbf{v}} = 4\pi[G_1\delta(\mathbf{v}-\mathbf{v}_1) + G_2\delta(\mathbf{v}-\mathbf{v}_2)]$$

- Linearized equations of motion

$$i\left(\frac{\partial}{\partial t} + v_1\frac{\partial}{\partial z}\right)S_1(z,t) = -g_2 S_2(z,t) \ ,$$
$$i\left(\frac{\partial}{\partial t} + v_2\frac{\partial}{\partial z}\right)S_2(z,t) = -g_1 S_1(z,t) \ .$$

$$g_1 = (1-\mathbf{v}_1\cdot\mathbf{v}_2)G_1$$
$$g_2 = (1-\mathbf{v}_1\cdot\mathbf{v}_2)G_2 \ .$$

- Dispersion relation

$$(\omega - v_1 k)(\omega - v_2 k) = \varepsilon \quad \Rightarrow \quad \Omega(k) = \frac{1}{2}\left\{(v_1+v_2)k \pm [k^2(v_1-v_2)^2 + 4\varepsilon]^{1/2}\right\}$$

$$\varepsilon = g_1 g_2$$

$\varepsilon > 0 \longrightarrow$ No crossing

$\varepsilon < 0 \longrightarrow$ crossing

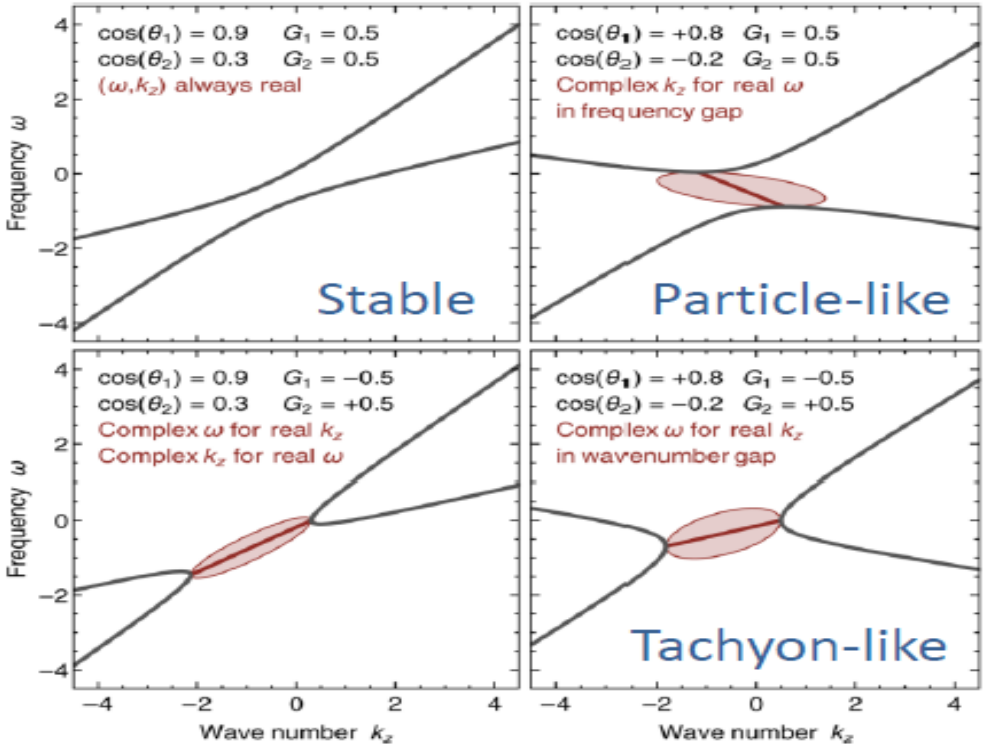
$$K(\omega) = \frac{1}{2v_1v_2}\left\{(v_1+v_2)\omega \pm [\omega^2(v_1-v_2)^2 + 4\varepsilon v_1 v_2]^{1/2}\right\}$$

PRL 118, 021101 (2017) PHYSICAL REVIEW LETTERS week ending 13 JANUARY 2017

Fast Pairwise Conversion of Supernova Neutrinos: A Dispersion Relation Approach

Ignacio Izaguirre,[1] Georg Raffelt,[1] and Irene Tamborra[2]
[1]*Max-Planck-Institut für Physik (Werner-Heisenberg-Institut), Föhringer Ring 6, 80805 München, Germany*
[2]*Niels Bohr International Academy, Niels Bohr Institute, Blegdamsvej 17, 2100 Copenhagen, Denmark*
(Received 10 October 2016; published 10 January 2017)

Classification of instabilities of "flavor waves"



Classification of instabilities of plasma waves

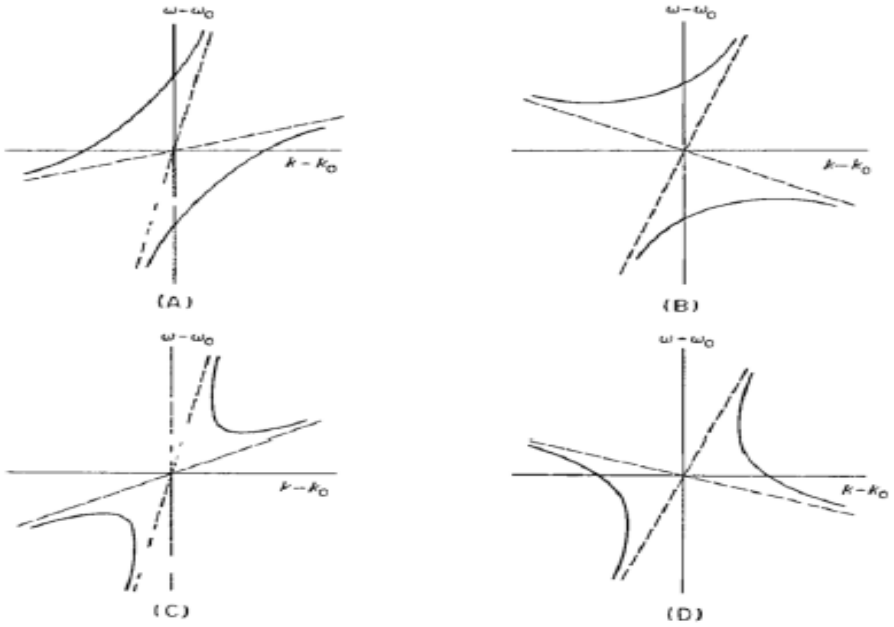
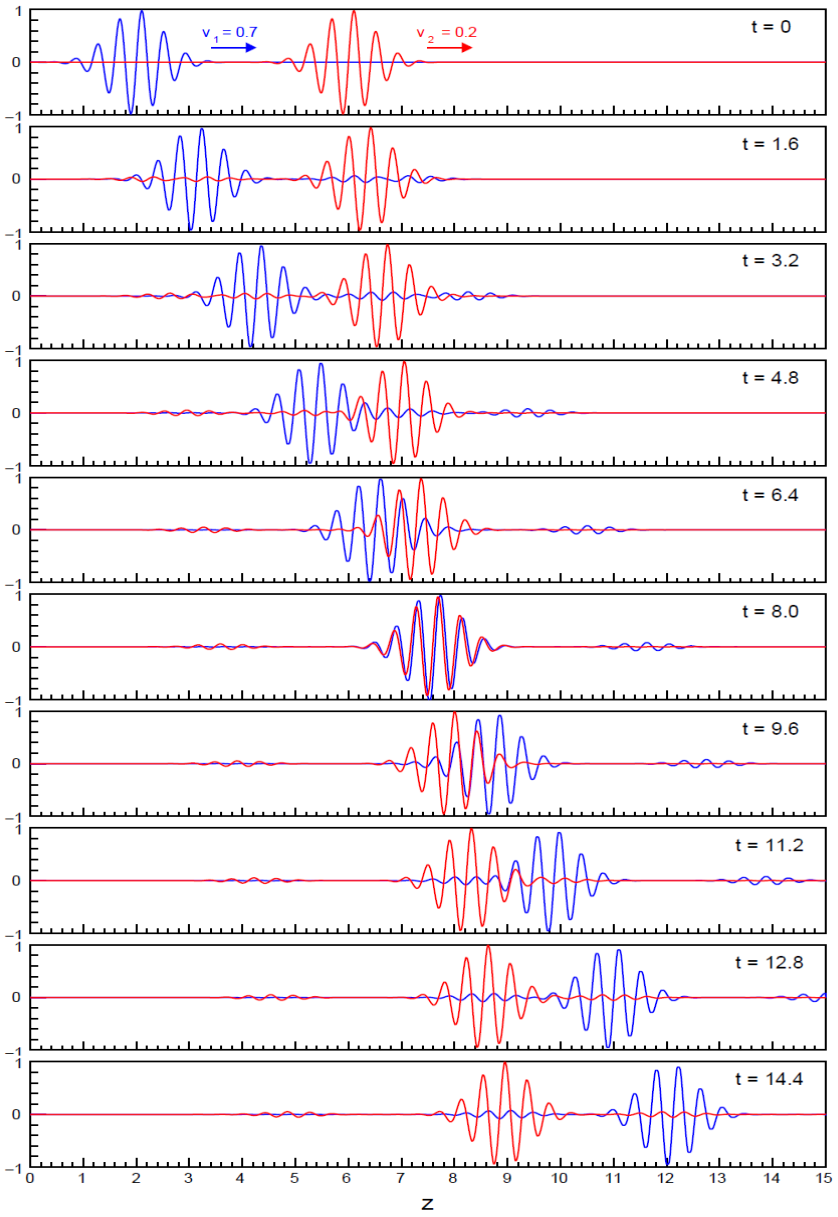


FIG. 23.

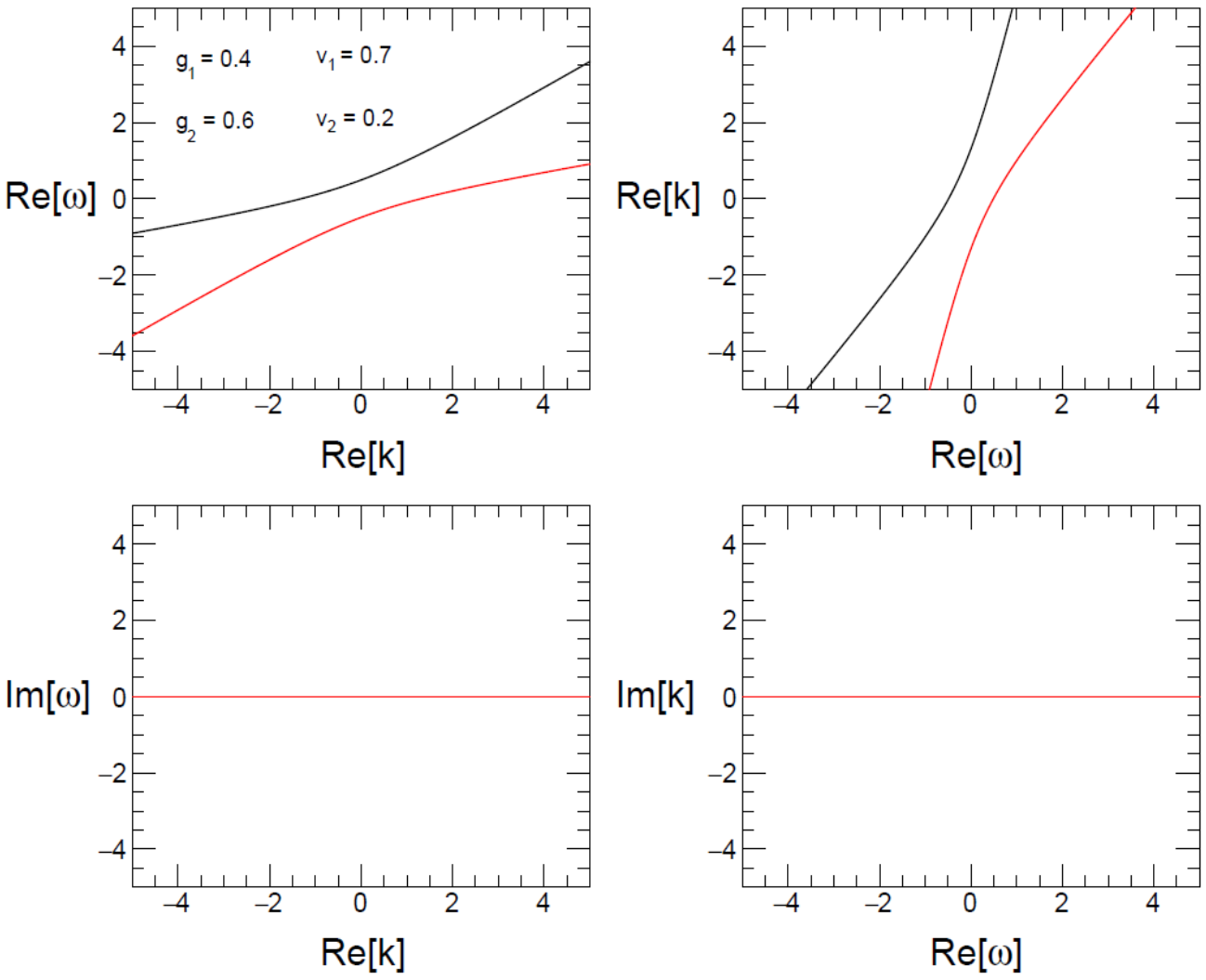
Landau & Lifshitz, Vol.10, Physical Kinetics
Chapter VI, Instability Theory

- **Complete stability** $v_1 v_2 > 0$, $\varepsilon > 0$ → oscillatory behavior, no growing mode



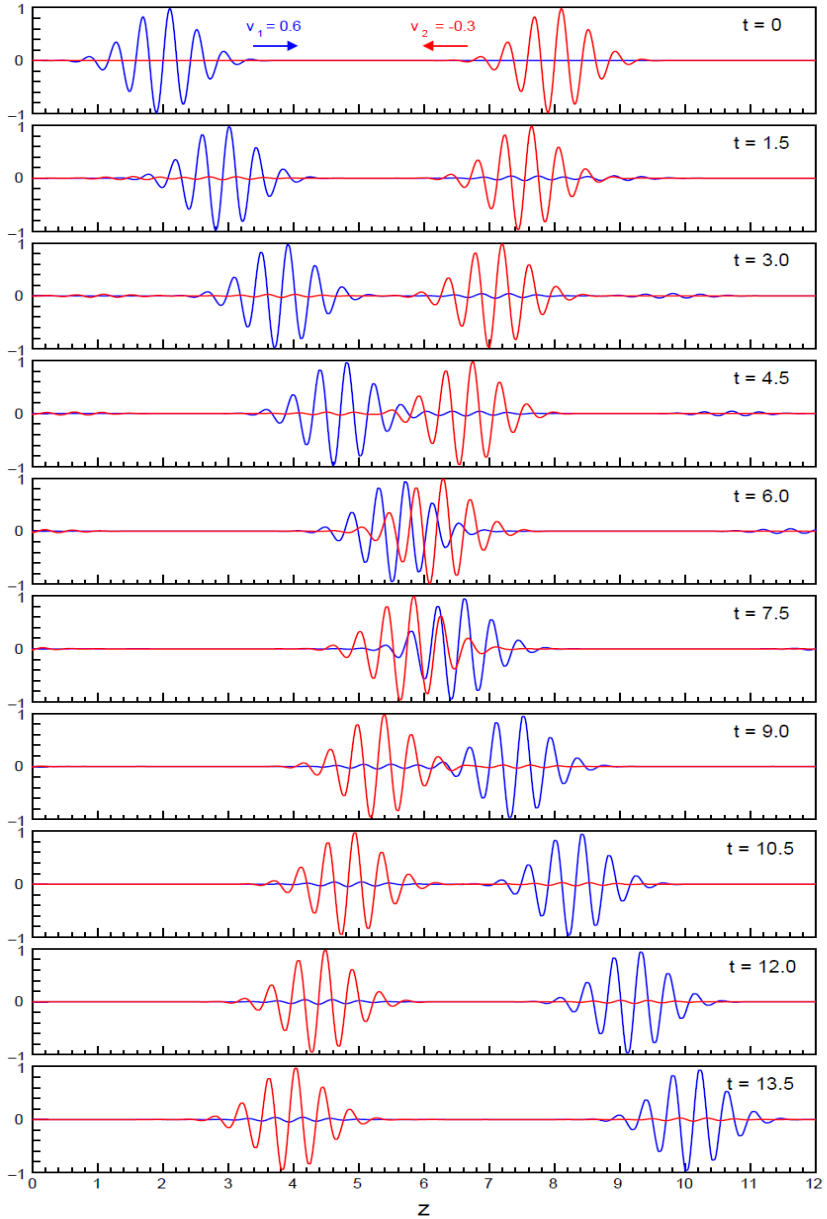
no gap in k or ω

$\omega = \Omega(k)$ $\qquad$ $k = K(\omega)$

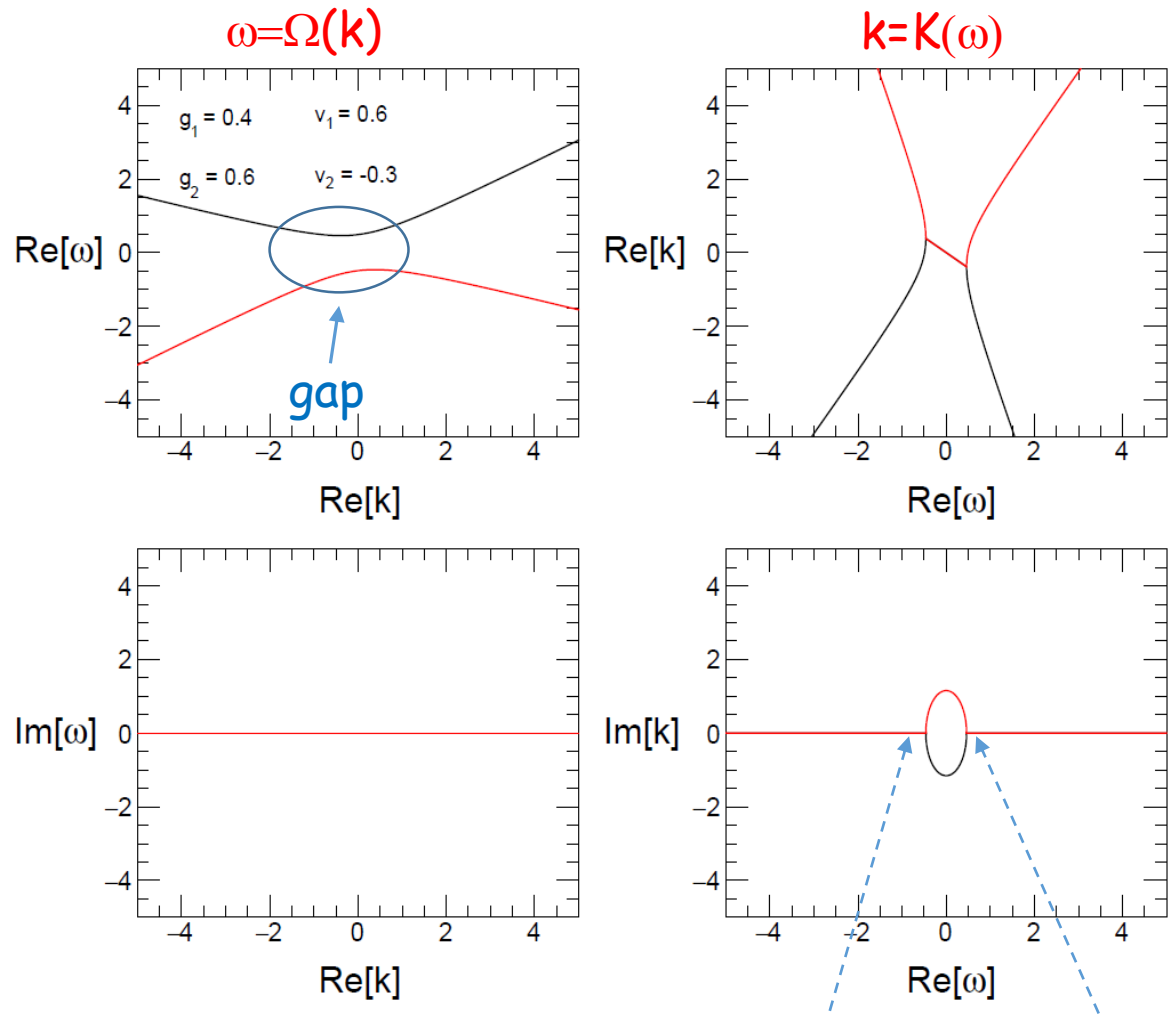


[Capozzi, Dasgupta, Lisi, Marrone, A.M., arXiV:1706.03360]

- **Damping** $v_1 v_2 < 0$, $\varepsilon > 0$ → Evanescent wave. Damped solution

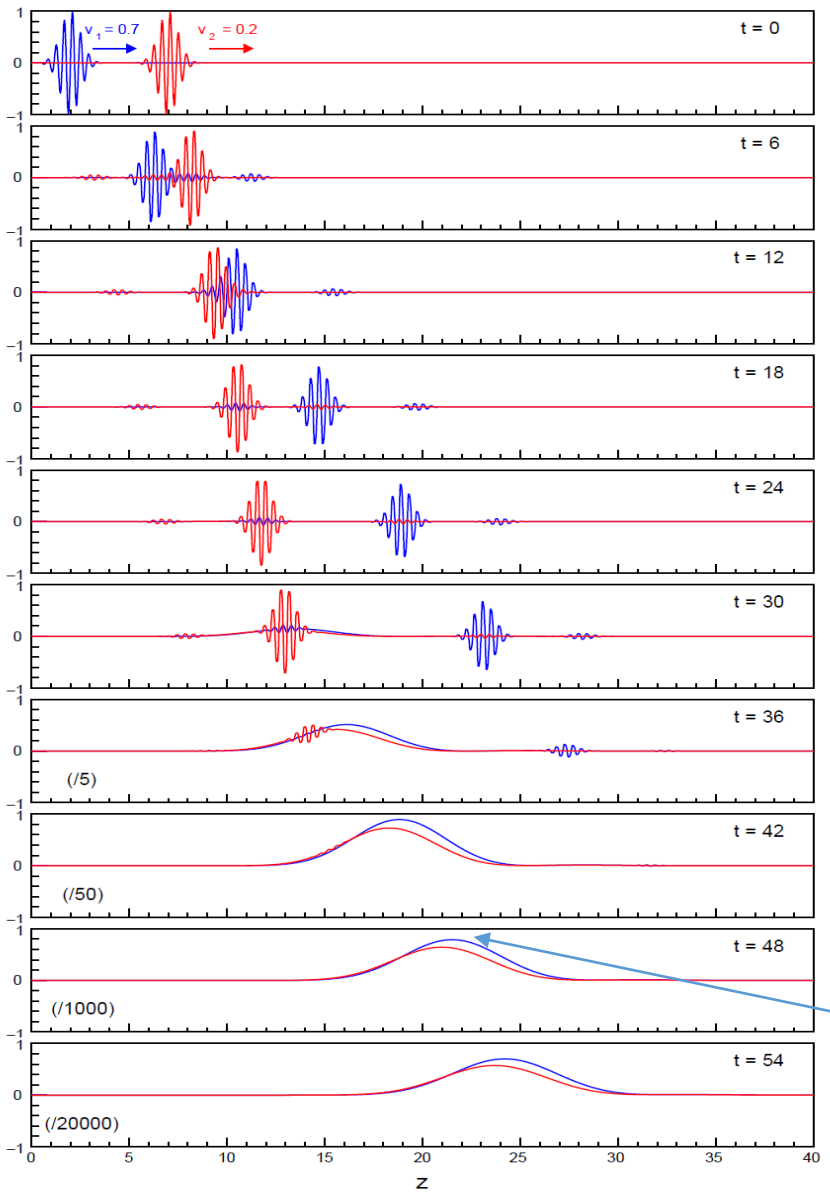


Complex k for real ω in frequency gap

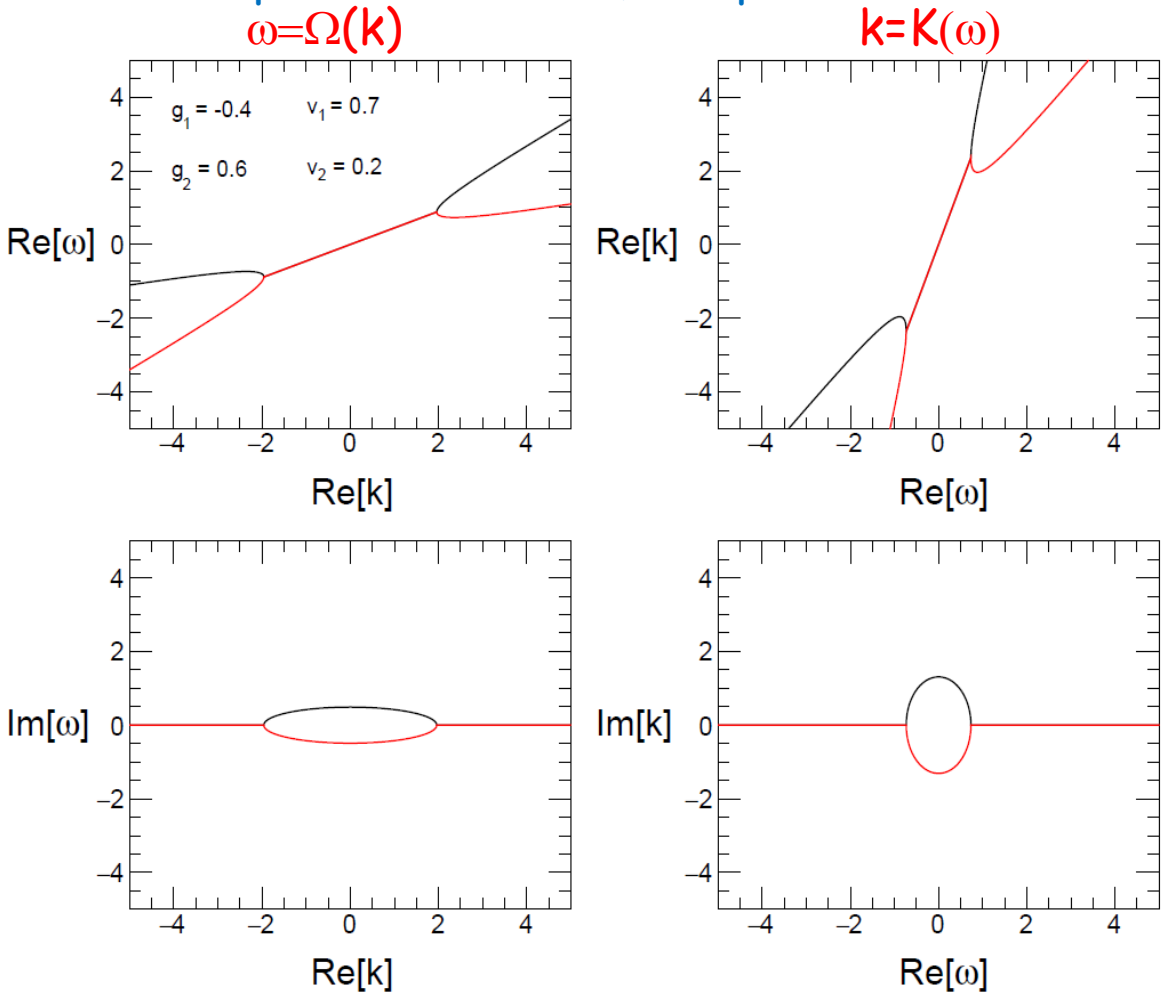


$$\omega^2 < 4\frac{|\varepsilon v_1 v_2|}{(v_1 - v_2)^2}$$

- **Convective instability** $v_1 v_2 > 0$, $\varepsilon < 0$ → The signal grows and is convected away from the original region

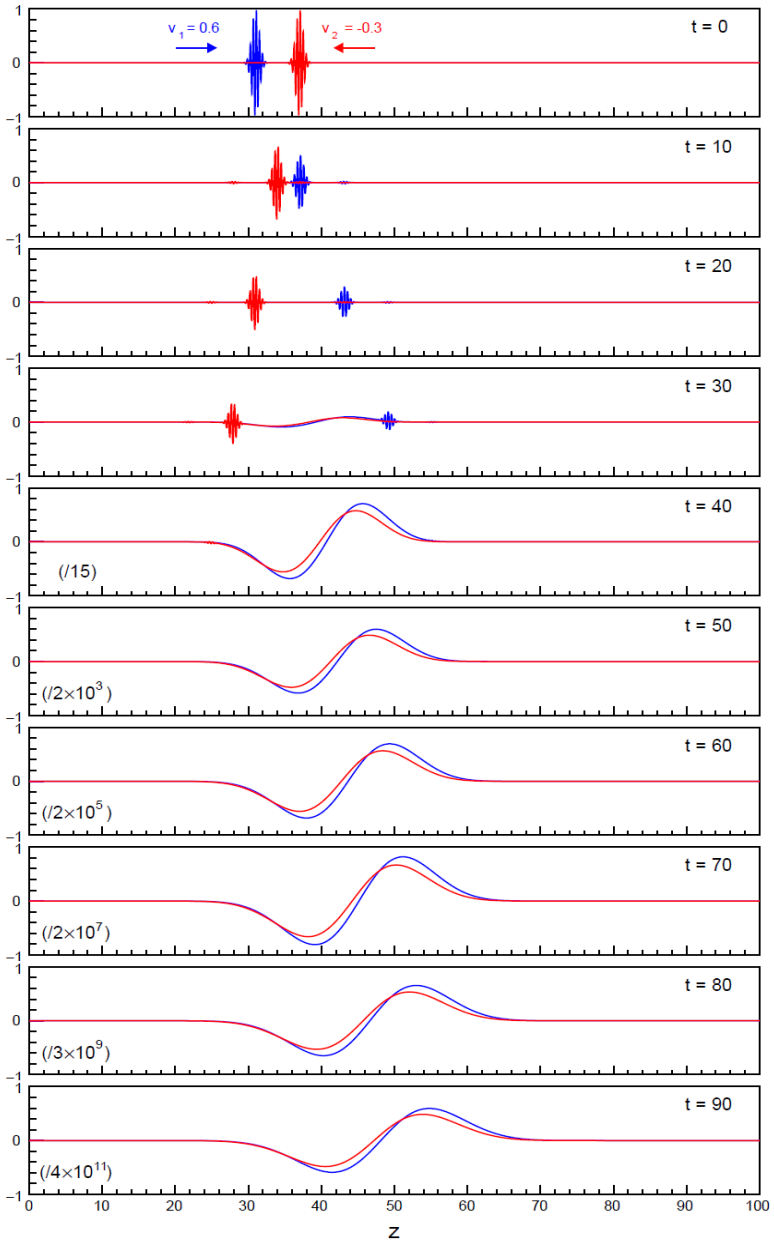


Complex k for real ω, complex ω for real k

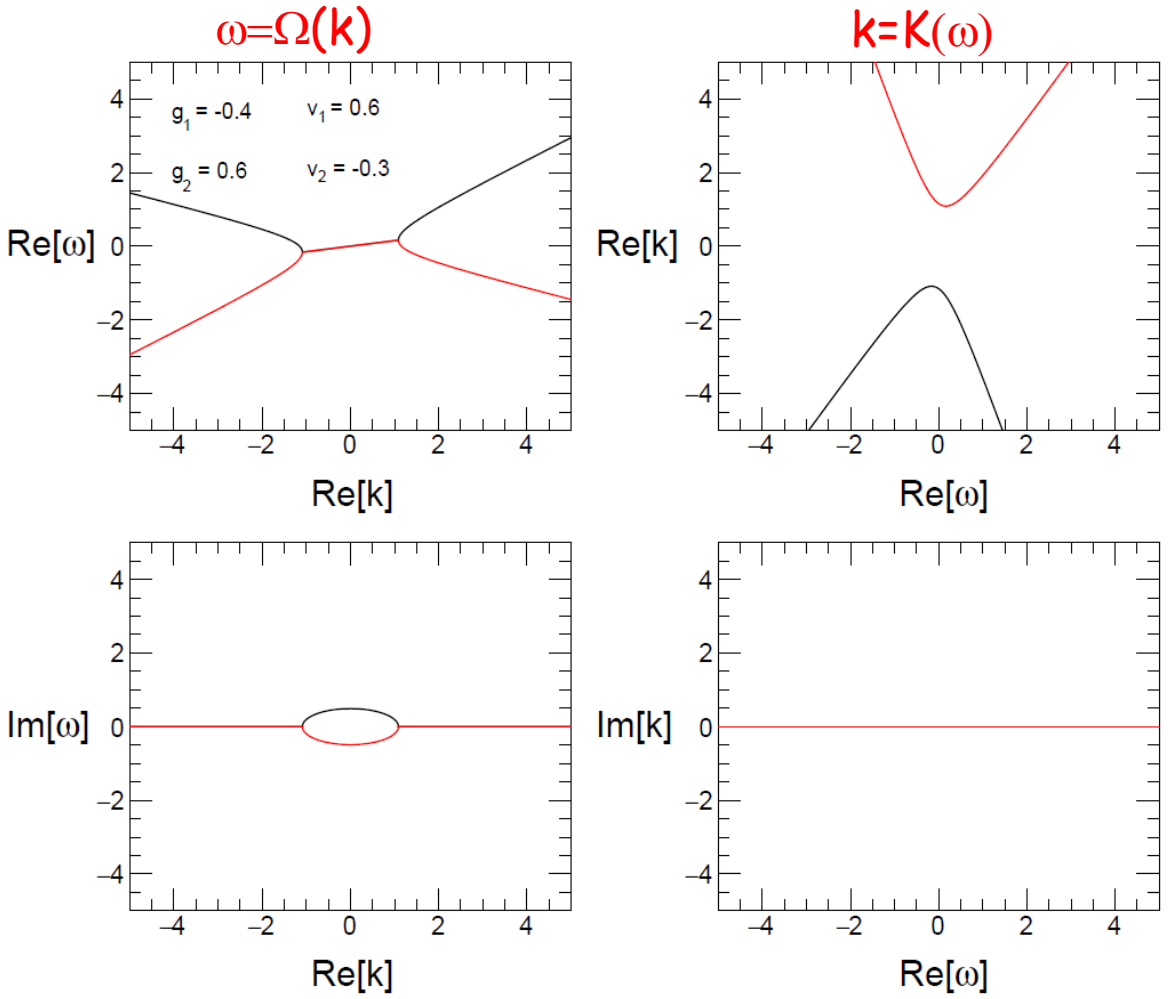


$$V = \frac{1}{2}(v_1 + v_2)$$

- **Absolute instability** $v_1 v_2 < 0$, $\varepsilon < 0$ → growing disturbance that does not propagate



Complex ω for real k, in the wavenumber gap



Growth rate $\omega_c = 2i\frac{\sqrt{(v_1 v_2 \varepsilon)}}{|v_1 - v_2|}$

CONCLUSIONS

- Fast flavor conversions are possible just above a SN core, triggered by an instability induced by the crossing in angular distributions of ν_e and $\overline{\nu}_e$.

- Linear stability analysis provides a valuable tool to classify the possible instabilities through an analysis of the dispersion relation.

- This technique should be applied to more realistic models to understand how generic are fast conversions in SNe.

- Non-linear simulations of the flavor evolution are mandatory to determine if fast conversions lead to flavor equilibrium.

LOT OF WORK STILL ON THE AGENDA!