Lepton flavour violation in a two-Higgs-doublet seesaw model

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Content

- **Description of the model**
- **Numerical results**
- **Conclusions**

Description of the model

- We consider the Standard Model with two Higgs doublets and enlarge the lepton sector by adding to each lepton family a right handed neutrino singlet ν_R.

- We assume that all the Yukawa-coupling matrices are diagonal but the Majorana mass matrix M_R of the right-handed neutrino singlets is an arbitrary symmetric matrix,

- thereby introducing an explicit but soft violation of all lepton numbers.

The Yukawa Lagrangian of the leptons is

$$\mathcal{L}_{\rm Y} = -\sum_{k=1}^{n_H} \sum_{\ell,\ell'=e,\mu,\tau} \left[\left(\varphi_k^-,\ \varphi_k^{0*} \right) \bar{\ell}_R \left(\Gamma_k\right)_{\ell\ell'} + \left(\varphi_k^0,\ -\varphi_k^+ \right) \bar{\nu}_{\ell R} \left(\Delta_k\right)_{\ell\ell'} \right] \begin{pmatrix} \nu_{\ell' L} \\ \ell'_L \end{pmatrix} + \text{H.c.}$$

The mass matrix of the charged leptons and the Dirac neutrino mass matrix are

$$M_\ell = \frac{1}{\sqrt{2}} \sum_k v_k^* \Gamma_k \qquad M_D = \frac{1}{\sqrt{2}} \sum_k v_k \Delta_k$$

right-handed leptons $\ell_R,\ \nu_R$

Yukawa coupling matrices Γ_k are $n_L \times n_L$, Δ_k are $n_R \times n_L$

Description of the model

- **General case**

The left- and right-handed neutrinos are written as linear superpositions of the physical Majorana neutrino fields χ_i

projector operators $\gamma_L = (1-\gamma_5)/2$

$\gamma_R = (1+\gamma_5)/2$

$$\nu_{\ell L} = \sum_i (U_L)_{\ell i}\,\gamma_L \chi_i \quad \text{and} \quad \nu_{\ell R} = \sum_i (U_R)_{\ell i}\,\gamma_R \chi_i$$

diagonalization matrix $\mathcal{U} = \begin{pmatrix} U_L \\ U_R^* \end{pmatrix}$

U is defined in such a way that $\mathcal{U}^T \begin{pmatrix} 0 & M_D^T \\ M_D & M_R \end{pmatrix} \mathcal{U} = \hat{m} = \mathrm{diag}\,(m_1, m_2, \ldots, m_6)$

The charged-current Lagrangian is

$$\mathcal{L}_{\mathrm{cc}} = \frac{g}{\sqrt{2}} \sum_{\ell,i} \left[W_\mu^- \,(U_L)_{\ell i}\, \bar{\ell}\gamma^\mu \gamma_L \chi_i + W_\mu^+ \left(U_L^\dagger\right)_{i\ell} \bar{\chi}_i \gamma^\mu \gamma_L \ell \right]$$

where *g* is the SU(2) gauge coupling

The Yukawa couplings to the charged scalars

$$\mathcal{L}_{\mathrm{Y}}\left(S^\pm\right) = \sum_{a,i,\ell} \left\{ S_a^- \bar{\ell} \left[(R_a)_{\ell i}\,\gamma_R - (L_a)_{\ell i}\,\gamma_L \right] \chi_i + S_a^+ \bar{\chi}_i \left[\left(R_a^\dagger\right)_{i\ell} \gamma_L - \left(L_a^\dagger\right)_{i\ell} \gamma_R \right] \ell \right\}$$

where $R_a = \Delta_a^\dagger U_R$ and $L_a = \Gamma_a U_L$

Description of the model

- **Two Higgs doublets**

In the Higgs basis, the VEVs are given by

$$\langle \varphi_1^0 \rangle_0 = \frac{v}{\sqrt{2}}, \quad \langle \varphi_2^0 \rangle_0 = 0$$

where $v \approx 246$ GeV is real and positive

Higgs doublets writes

$$\phi_1 = \begin{pmatrix} G^+ \\ (v + S_1^0 + iG^0)/\sqrt{2} \end{pmatrix}, \quad \phi_2 = \begin{pmatrix} H^+ \\ (S_2^0 + iS_3^0)/\sqrt{2} \end{pmatrix}$$

where G^+ and G^0 are Goldstone bosons, H^+ is a physical charged scalar with the mass m_C, and $S^0_{1,2,3}$ are physical neutral scalars. The scalar S^0_1 has couplings fully identical to the ones of the SM Higgs boson.

We parametrize the flavour-diagonal Yukawa coupling matrices as

$$\begin{aligned}
\Gamma_1 &= \frac{\sqrt{2}}{v} \operatorname{diag}(m_e, m_\mu, m_\tau) = \frac{\sqrt{2}}{v} M_\ell, \\
\Gamma_2 &= \operatorname{diag}(\gamma_e, \gamma_\mu, \gamma_\tau), \\
\Delta_1 &= \operatorname{diag}(d_e, d_\mu, d_\tau) = \frac{\sqrt{2}}{v} M_D, \\
\Delta_2 &= \operatorname{diag}(\delta_e, \delta_\mu, \delta_\tau).
\end{aligned}$$

$$M_D = \operatorname{diag}(a, b, c)$$

$$\Longrightarrow$$

$$\begin{aligned}
(R_1)_{\ell n} &= d_\ell^* (U_R)_{\ell n}, \\
(R_2)_{\ell n} &= \delta_\ell^* (U_R)_{\ell n}, \\
(L_1)_{\ell n} &= \frac{\sqrt{2} m_\ell}{v} (U_L)_{\ell n}, \\
(L_2)_{\ell n} &= \gamma_\ell (U_L)_{\ell n}.
\end{aligned}$$

Description of the model

- **The amplitude of the process**

We compute one loop amplitude for the process $\tau^-(p_1) \to \mu^-(p_2)\,\gamma(q)$

The amplitude consists from three contributions:

- $W^\pm$ - exchange contribution,
- contribution of the charged Goldstone boson,
- contribution from diagrams in which the photon attaches either to the $W^\pm$ or to the charged scalar in the loop.

The amplitude writes

$$
\begin{aligned}
M^\rho \;=\; e\sum_{n=1}^{6}\overline{u_\mu}\Big\{ & i\sigma^{\rho\lambda}q_\lambda\left(a_{Ln}\gamma_L + a_{Rn}\gamma_R\right) \\
& + b_{Ln}\left[q^\rho\gamma_L - \frac{q^2}{m_\tau^2 - m_\mu^2}\,\gamma^\rho\left(m_\mu\gamma_L + m_\tau\gamma_R\right)\right] \\
& + b_{Rn}\left[q^\rho\gamma_R - \frac{q^2}{m_\tau^2 - m_\mu^2}\,\gamma^\rho\left(m_\mu\gamma_R + m_\tau\gamma_L\right)\right]\Big\}\,u_\tau
\end{aligned}
$$

W. Grimus, L. Lavoura [hep-ph/0204070]

It is finite and respects gauge invariance.

The four-momentum $q^\rho = p_1^\rho - p_2^\rho$ belongs to the outgoing photon; p_1^ρ is the four-momentum of the incoming τ lepton and p_2^ρ is four-momentum of the outgoing μ lepton.

The parameters b are irrelevant when the photon is on mass shell.

Description of the model

- **Coefficients of the amplitude**

The coefficients *a* are given by

$$
\begin{aligned}
a_{Ln} &= (U_L)_{\mu n}\,(U_L^*)_{\tau n}\left\{ m_\mu \left[g^2\left(d_{2nW}+f_{nW}+c_{1nW}\right)+\frac{2m_\tau^2}{v^2}\,k_{1nW}\right.\right.\\
&\quad \left.+\frac{2m_n^2}{v^2}\left(k_{2nW}-k_{3nW}\right)\right]\\
&\quad \left.+m_\tau\gamma_\mu\gamma_\tau^* k_{1nC}-\frac{\sqrt{2}m_n^2\gamma_\mu}{v}\,\frac{\delta_\tau}{d_\tau}\,k_{3nC}+\frac{2m_n^2 m_\mu}{v^2}\,\frac{\delta_\mu^*}{d_\mu^*}\,\frac{\delta_\tau}{d_\tau}\,k_{2nC}\right\}\\
a_{Rn} &= (U_L)_{\mu n}\,(U_L^*)_{\tau n}\left\{ m_\tau \left[g^2\left(d_{1nW}+f_{nW}+c_{2nW}\right)+\frac{2m_\mu^2}{v^2}\,k_{2nW}\right.\right.\\
&\quad \left.+\frac{2m_n^2}{v^2}\left(k_{1nW}-k_{3nW}\right)\right]\\
&\quad \left.+m_\mu\gamma_\mu\gamma_\tau^* k_{2nC}-\frac{\sqrt{2}m_n^2\gamma_\tau^*}{v}\,\frac{\delta_\mu^*}{d_\mu^*}\,k_{3nC}+\frac{2m_n^2 m_\tau}{v^2}\,\frac{\delta_\mu^*}{d_\mu^*}\,\frac{\delta_\tau}{d_\tau}\,k_{1nC}\right\}
\end{aligned}
$$

Where

$$
\begin{aligned}
k_{1nW} &= d_{1nW}+f_{nW}-c_{1nW}\,,\\
k_{2nW} &= d_{2nW}+f_{nW}-c_{2nW}\,,\\
k_{3nW} &= c_{1nW}+c_{2nW}-a_{3nW}\,,
\end{aligned}
$$

and similarly with *W* → *C* in the indices.

Description of the model

- **Feynman integrals**

The coefficient are defined in the following way

$$
\begin{aligned}
& a_{nW} \\
= & \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{1}{\left(k^2 - m_n^2\right)\left[(k-p_1)^2 - m_W^2\right]\left[(k-p_2)^2 - m_W^2\right]}, \\
& c_{1nW} p_1^\theta + c_{2nW} p_2^\theta \\
= & \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{k^\theta}{\left(k^2 - m_n^2\right)\left[(k-p_1)^2 - m_W^2\right]\left[(k-p_2)^2 - m_W^2\right]}, \\
& d_{1nW} p_1^\theta p_1^\psi + d_{2nW} p_2^\theta p_2^\psi + f_{nW}\left(p_1^\theta p_2^\psi + p_2^\theta p_1^\psi\right) + u_{nW} g^{\theta\psi} \\
= & \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{k^\theta k^\psi}{\left(k^2 - m_n^2\right)\left[(k-p_1)^2 - m_W^2\right]\left[(k-p_2)^2 - m_W^2\right]},
\end{aligned}
$$

and similarly with $W \rightarrow C$ in the indices.

The infinities in the amplitude cancel for the:

- unitarity of the diagonalization matrix U,
- flavour-diagonal Yukawa coupling matrices.

More generally, these reasons are responsible for the cancellation of all terms independent of the neutrino masses m_n.

Performing the integrals over k, one obtains

$$
\begin{aligned}
a_{nW} &= \frac{-i}{16\pi^2} \int_0^1 \mathrm{d}x \int_0^x \mathrm{d}y \, \frac{1}{\Delta}, \\
c_{1nW} &= \frac{-i}{16\pi^2} \int_0^1 \mathrm{d}x \int_0^x \mathrm{d}y \, \frac{x-y}{\Delta}, \\
c_{2nW} &= \frac{-i}{16\pi^2} \int_0^1 \mathrm{d}x \int_0^x \mathrm{d}y \, \frac{y}{\Delta}, \\
d_{1nW} &= \frac{-i}{16\pi^2} \int_0^1 \mathrm{d}x \int_0^x \mathrm{d}y \, \frac{(x-y)^2}{\Delta}, \\
d_{2nW} &= \frac{-i}{16\pi^2} \int_0^1 \mathrm{d}x \int_0^x \mathrm{d}y \, \frac{y^2}{\Delta}, \\
f_{nW} &= \frac{-i}{16\pi^2} \int_0^1 \mathrm{d}x \int_0^x \mathrm{d}y \, \frac{y\,(x-y)}{\Delta}.
\end{aligned}
$$

Where

$$
\begin{aligned}
\Delta &= K + p_1^2 (1-x)(y-x) - p_2^2 (1-x)\, y, \\
K &\equiv m_n^2 (1-x) + m_W^2 x.
\end{aligned}
$$

Description of the model

- **BR and M_R calculations**

The partial width

$$\Gamma_{\text{partial}}\left(\tau^- \to \mu^-\gamma\right) = \frac{\alpha\left(m_\tau^2 - m_\mu^2\right)^3}{4m_\tau^3}\left(\left|\sum_{n=1}^{6} \boxed{a_{Ln}}\right|^2 + \left|\sum_{n=1}^{6} \boxed{a_{Rn}}\right|^2\right)$$

L. Lavoura [hep-ph/0302221]

The branching ratio

$$\text{BR}\left(\tau^- \to \mu^-\gamma\right) = \frac{\Gamma_{\text{partial}}\left(\tau^- \to \mu^-\gamma\right)}{\Gamma_{\text{total}}\left(\tau^-\right)}$$

The mass matrix of the light neutrinos is obtained by the seesaw formula

$$\mathcal{M}_\nu = -M_D^T M_R^{-1} M_D = -\frac{v^2}{2}\Delta_1 M_R^{-1}\Delta_1$$

Inverting previous equation, we obtain

$$M_R = -\frac{v^2}{2}\Delta_1 \mathcal{M}_\nu^{-1}\Delta_1$$

The matrix M_ν is diagonalized as

$$V_L^T \mathcal{M}_\nu V_L = \text{diag}\left(m_1, m_2, m_3\right) \equiv \hat{m} \qquad \text{where} \qquad V_L = e^{i\hat{\alpha}}\, U_{\text{PMNS}}\, e^{i\hat{\beta}}$$

using the fact that the matrices $\Delta_1,\ \hat{m},\ e^{i\hat{\alpha}}$ and $e^{i\hat{\beta}}$ are diagonal we obtain

$$M_R = -\frac{v^2}{2} e^{i\hat{\alpha}}\Delta_1\, U_{\text{PMNS}}\left(e^{2i\hat{\beta}}\hat{m}^{-1}\right) U_{\text{PMNS}}^T\, \Delta_1 e^{i\hat{\alpha}}$$

Numerics

- **Assumptions and exp. bounds**

We make some simplifying assumptions:

- all the parameters of the model are real
- U_{PMNS} is real too
- $e^{i\hat{\alpha}} = e^{i\hat{\beta}} = \mathbb{1}$

We allow oscillations parameters to vary in 1σ bounds

	HN	IH
	bfp $\pm 1\sigma$	bfp $\pm 1\sigma$
$\sin^2\theta_{12}$	$0.304^{+0.013}_{-0.012}$	$0.304^{+0.013}_{-0.012}$
$\theta_{12}/^\circ$	$33.48^{+0.78}_{-0.75}$	$33.48^{+0.78}_{-0.75}$
$\sin^2\theta_{23}$	$0.452^{+0.052}_{-0.028}$	$0.579^{+0.025}_{-0.037}$
$\theta_{23}/^\circ$	$42.3^{+3.0}_{-1.6}$	$49.5^{+1.5}_{-2.2}$
$\sin^2\theta_{13}$	$0.0218^{+0.0010}_{-0.0010}$	$0.0219^{+0.0011}_{-0.0010}$
$\theta_{13}/^\circ$	$8.50^{+0.20}_{-0.21}$	$8.51^{+0.20}_{-0.21}$
$\delta_{\text{CP}}/^\circ$	306^{+39}_{-70}	254^{+63}_{-62}
$\frac{\Delta m^2_{21}}{10^{-5}\ \text{eV}^2}$	$7.50^{+0.19}_{-0.17}$	$7.50^{+0.19}_{-0.17}$
$\frac{\Delta m^2_{3\ell}}{10^{-3}\ \text{eV}^2}$	$+2.457^{+0.047}_{-0.047}$	$-2.449^{+0.048}_{-0.047}$

M.C. Gonzalez-Garcia *et all.* [arXiv:1106.0034 [hep-ph]]

Generating m_1 in the range 10^{-6} to 0.1 eV, we obtain for the other two light neutrino masses

$$m_2 = \sqrt{m_1^2 + \Delta m_{21}^2}$$

$$m_3 = \sqrt{m_1^2 + \Delta m_{31}^2}$$

The experimental bounds of the branching ratios

$$\begin{aligned} \text{BR}\,(\mu^+ \to e^+\gamma) &< 4.2 \times 10^{-13} \\ \text{BR}\,(\tau^- \to e^-\gamma) &< 3.3 \times 10^{-8} \\ \text{BR}\,(\tau^- \to \mu^-\gamma) &< 4.4 \times 10^{-8} \end{aligned}$$

A.M Baldini *et all.* [arXiv:1606.05081 [hep-ex]]

C Patrignani *et all.* (PDG), Chin. Phys. C 40 (2016) 100001

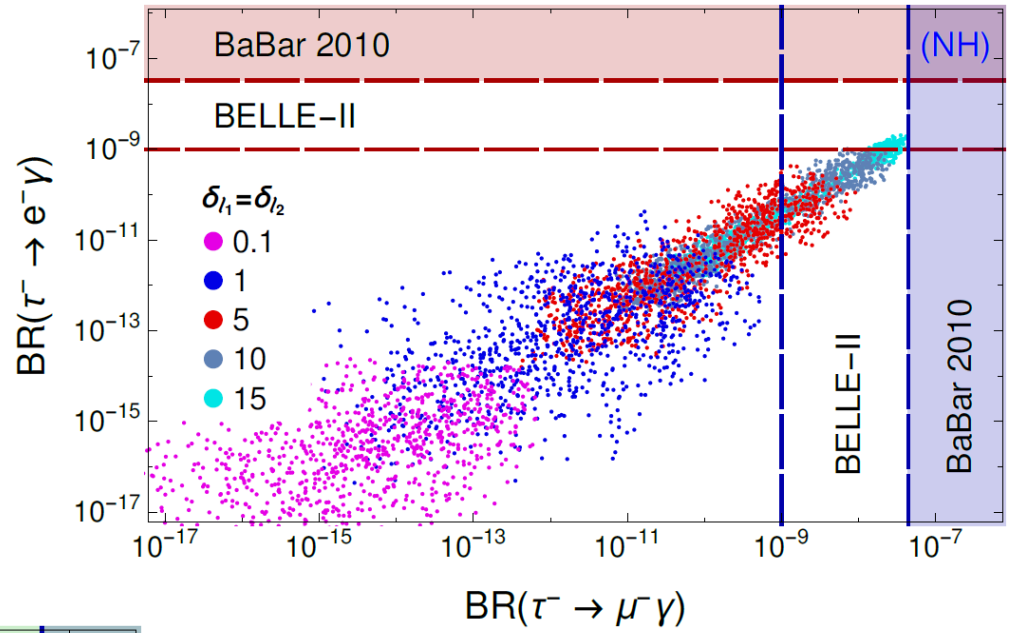
Numerics

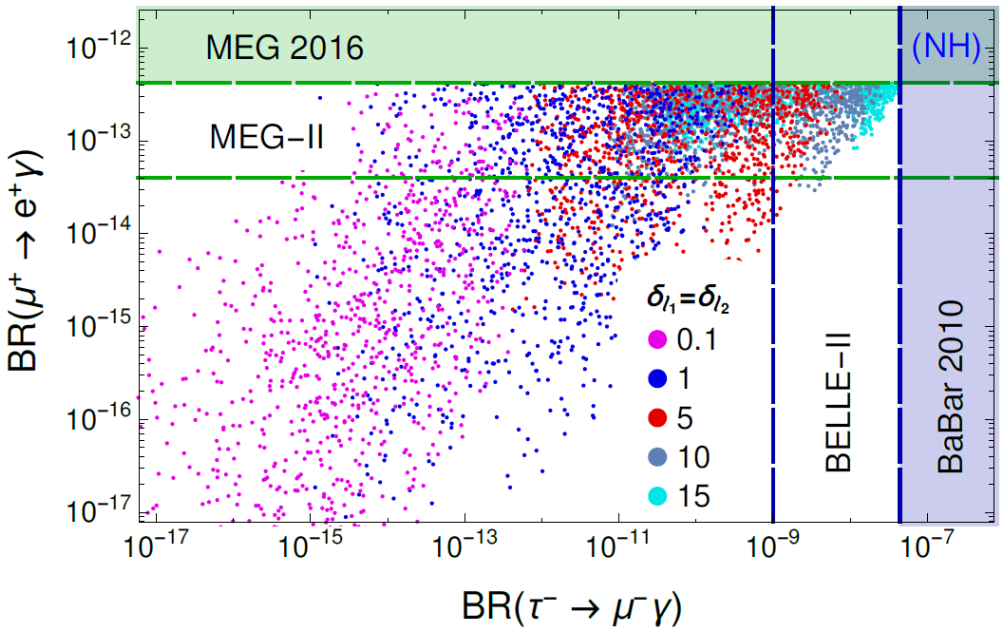
- **The case with NH and masses of the heavy neutrinos 0.1 < m_R < 500 TeV**

We generate impute parameters *a*, *b* and *c* which parametrize Yukawa coupling matrix

$$M_D = \mathrm{diag}\,(a, b, c)$$

and determinate the magnitude of the masses of heavy neutrinos.





We assume that $\gamma_{\ell_1} = \gamma_{\ell_2} = 0$, because the impact of gamma's to BR is not significant.

Calculations were made assuming:

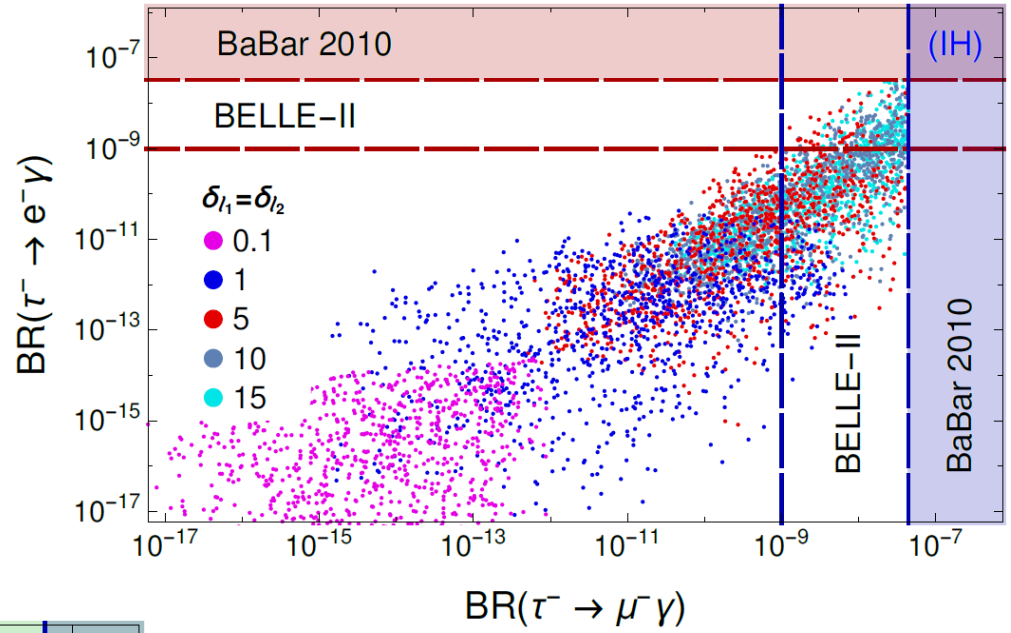
- 0.1 < *a*, *b*, *c* < 3 MeV
- 0.1 < m_R < 500 TeV
- m_C = 500 GeV

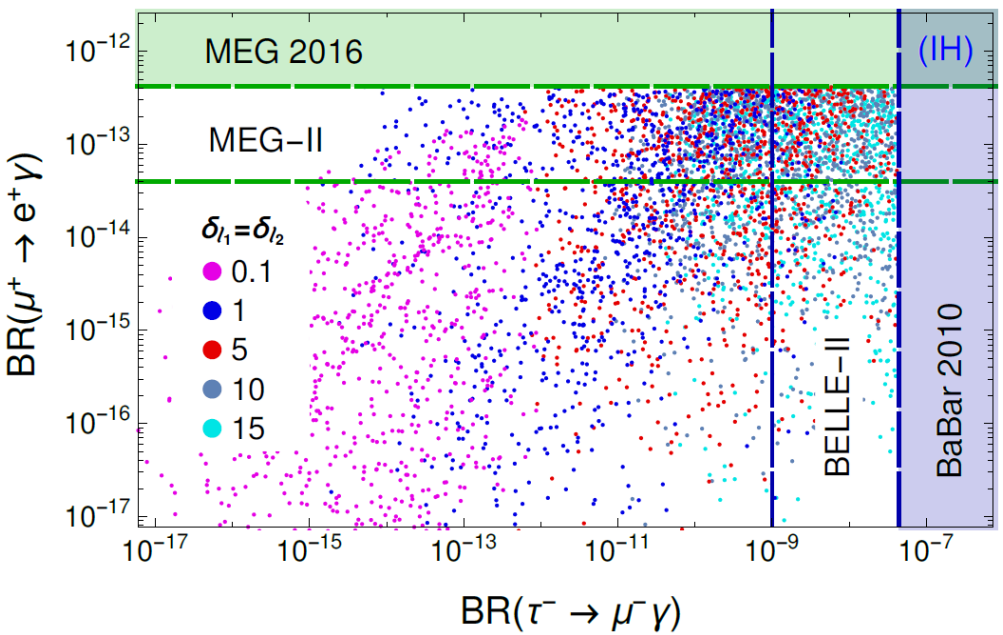
Numerics

- **The case with IH and masses of the heavy neutrinos 0.1 < m_R < 500 TeV**

Calculations were made assuming:

- 0.1 < a, b, c < 3 MeV
- 0.1 < m_R < 500 TeV
- m_C = 500 GeV





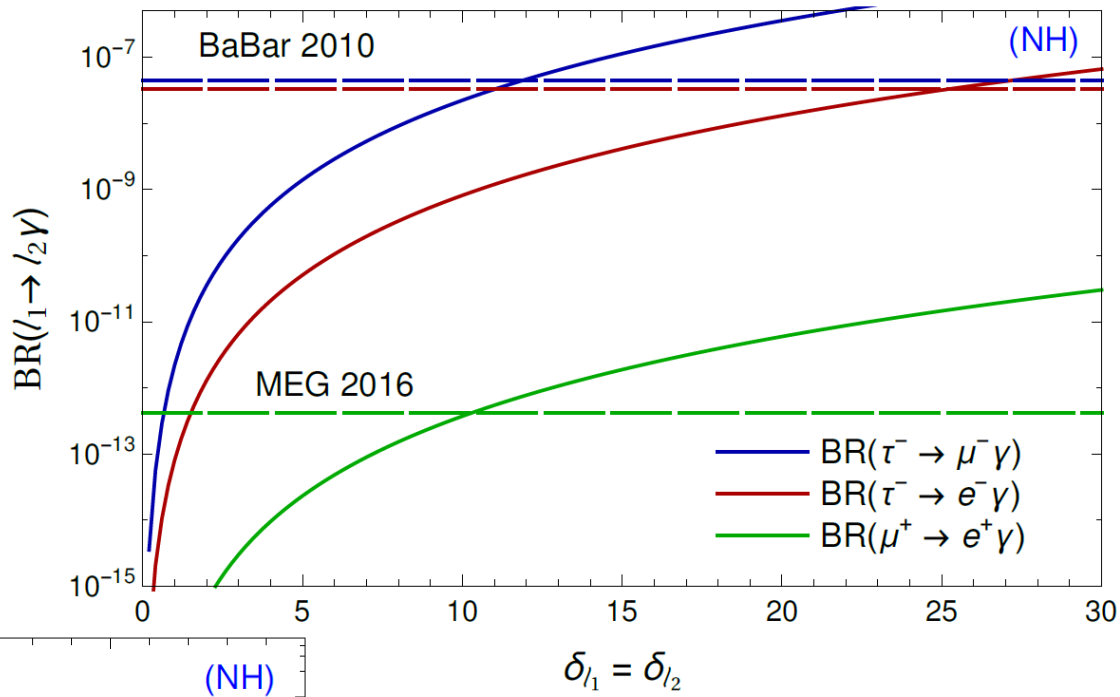
For the case with inverted ordering of light neutrinos branching ratios receive larger values comparing with normal ordering of neutrinos

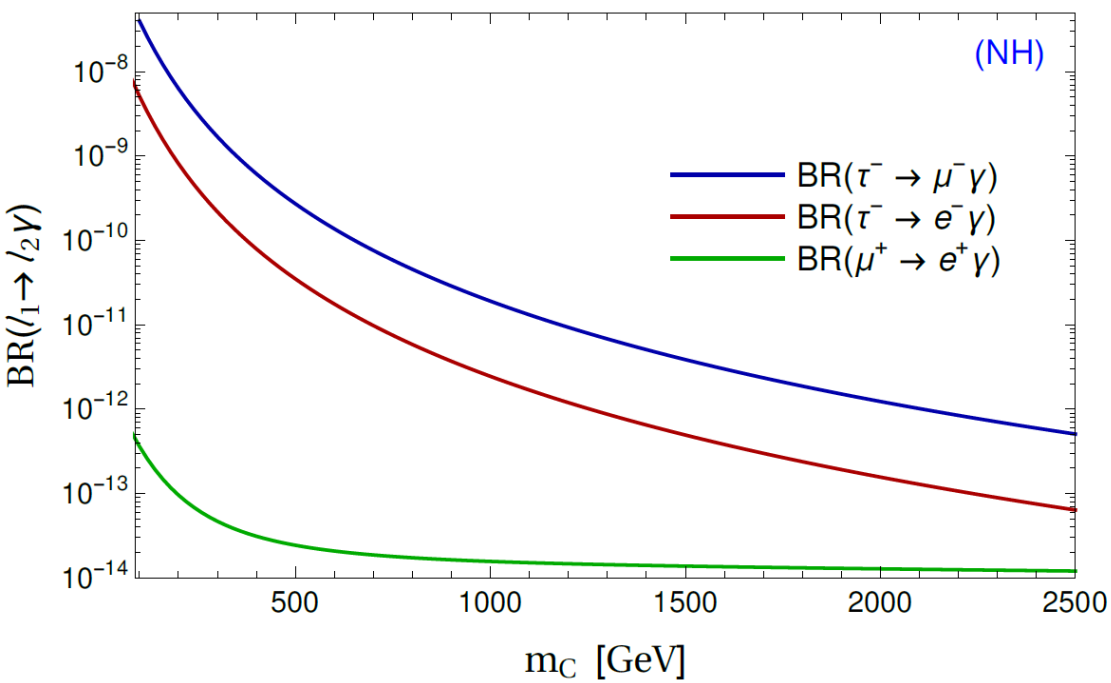
Numerics

- **BR as functions of delta's and mass of charged scalar**

By fixing some point of the previous plots with NH we extrapolate BR as functions of delta's ⇨

assuming that $\gamma_{\ell_1} = \gamma_{\ell_2} = 0$





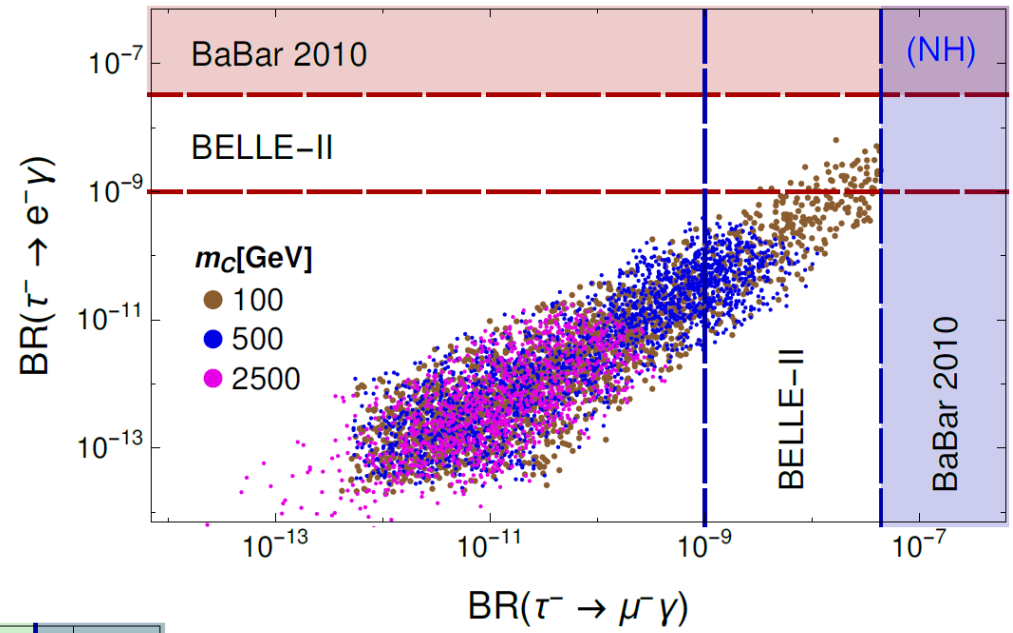
By fixing point with largest BR we extrapolate BR as functions of m_C ⇦

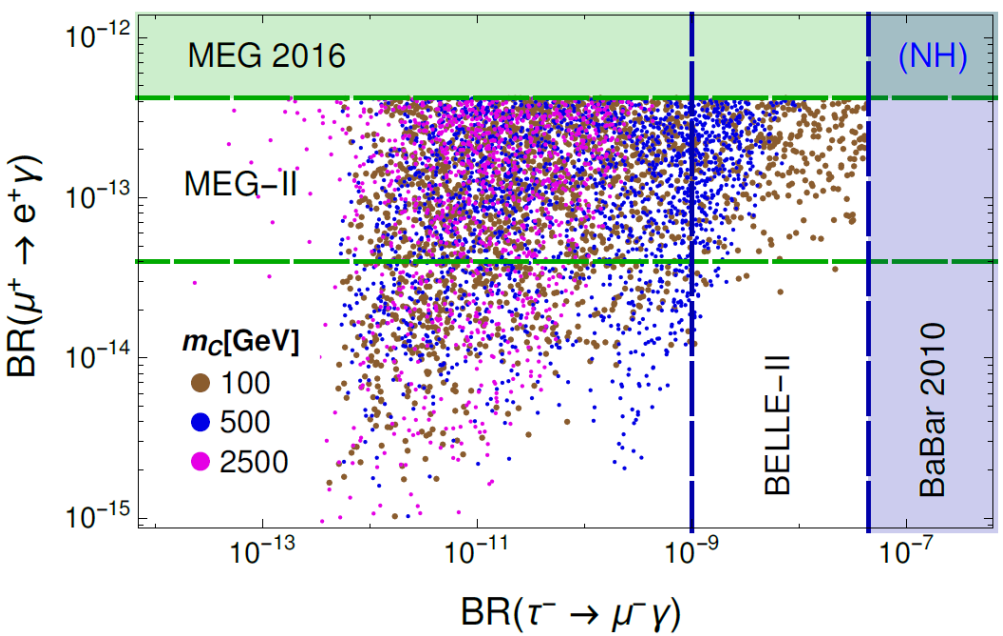
Numerics

- **Scatter plots for different m_C**

Calculations were made assuming:

- $0.1 < a, b, c < 3$ MeV
- $0.1 < m_R < 500$ TeV
- $m_C = 500$ GeV
- $\delta_{l_1} = \delta_{l_2} = 5$





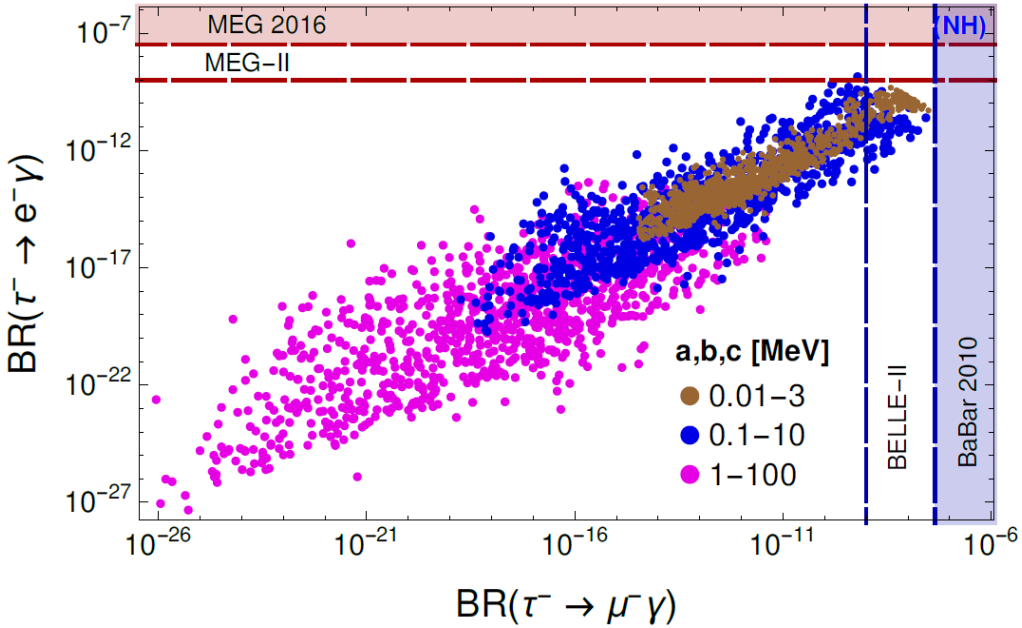
Smaller values of the mass of charged scalar gives larger values of branching ratio

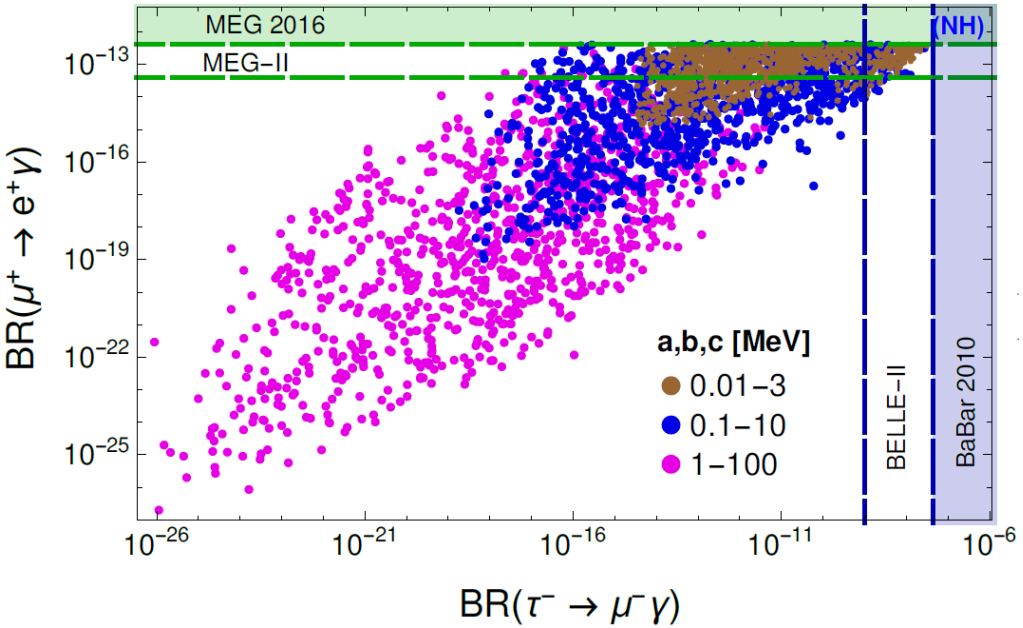
Numerics

- **Scatter plots for different ranges of input parameters *a, b* and *c***

Calculations were made assuming:

- $m_R > 0.1$ TeV
- $m_C = 500$ GeV
- $\delta_{l_1} = \delta_{l_2} = 5$





in this case we do not restricted m_R from above. Comparing with first figures we see wider distributions of branching ratios having smaller values.

Numerics

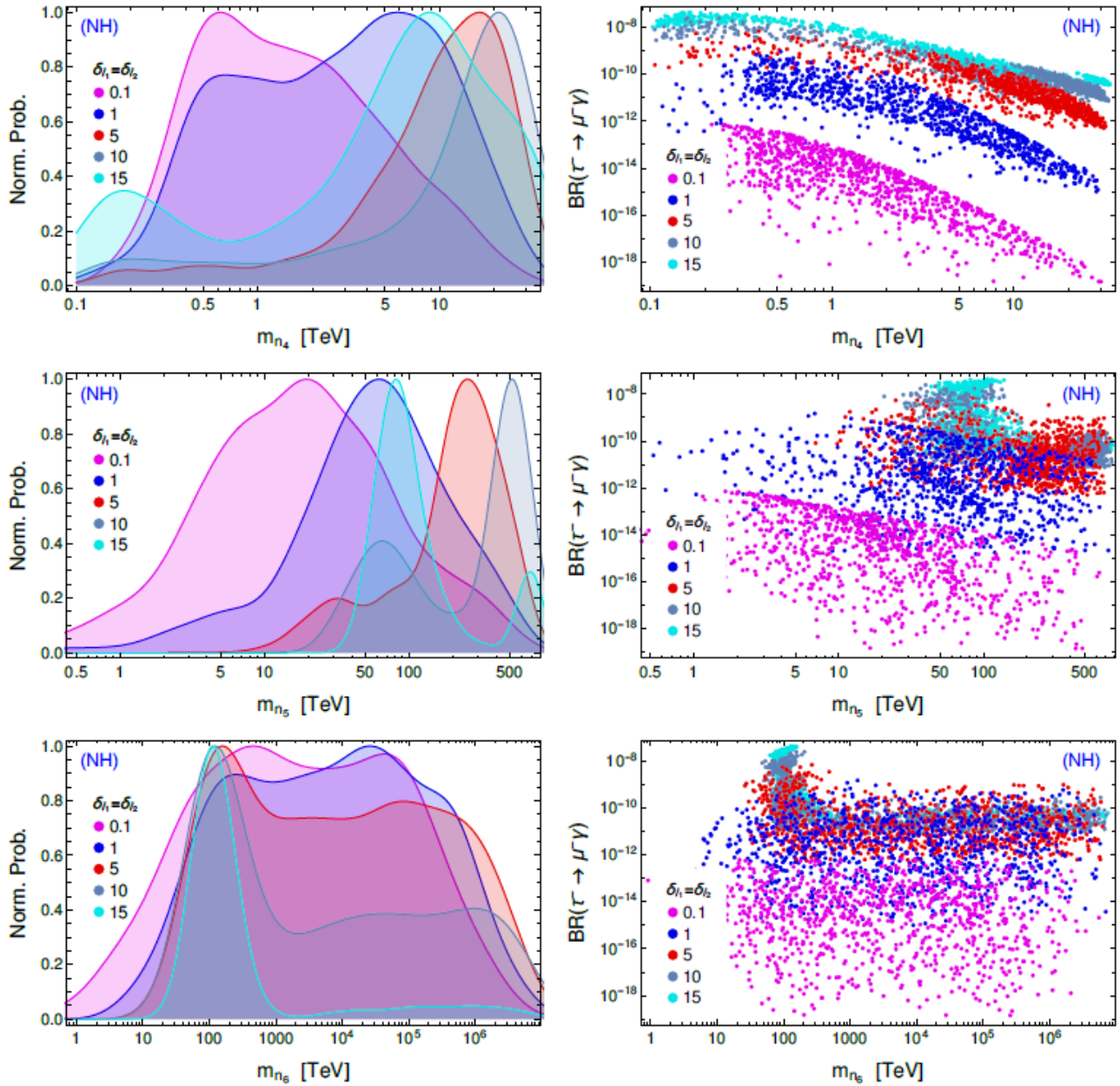
- **Distributions of m_R**

Assumptions:

- $0.1 < a, b, c < 3$ MeV
- $m_R > 0.1$ TeV
- $m_C = 500$ GeV

- figures on the left illustrate distributions of the masses of heavy neutrinos with normalized probability

- figures on the right shows dependence of BR($\tau \to \mu\gamma$) from the masses of heavy neutrinos

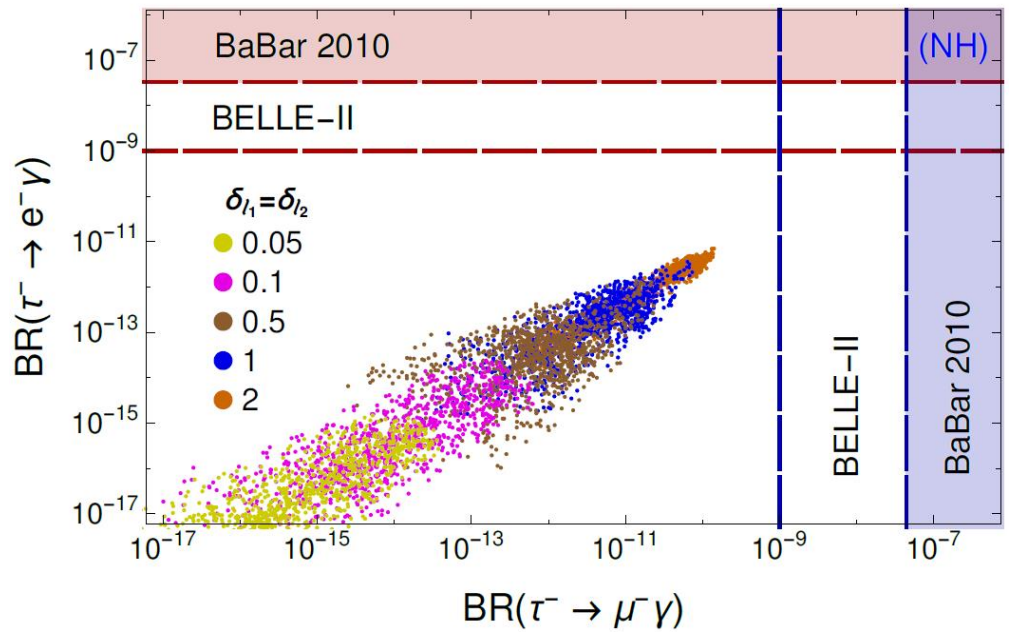


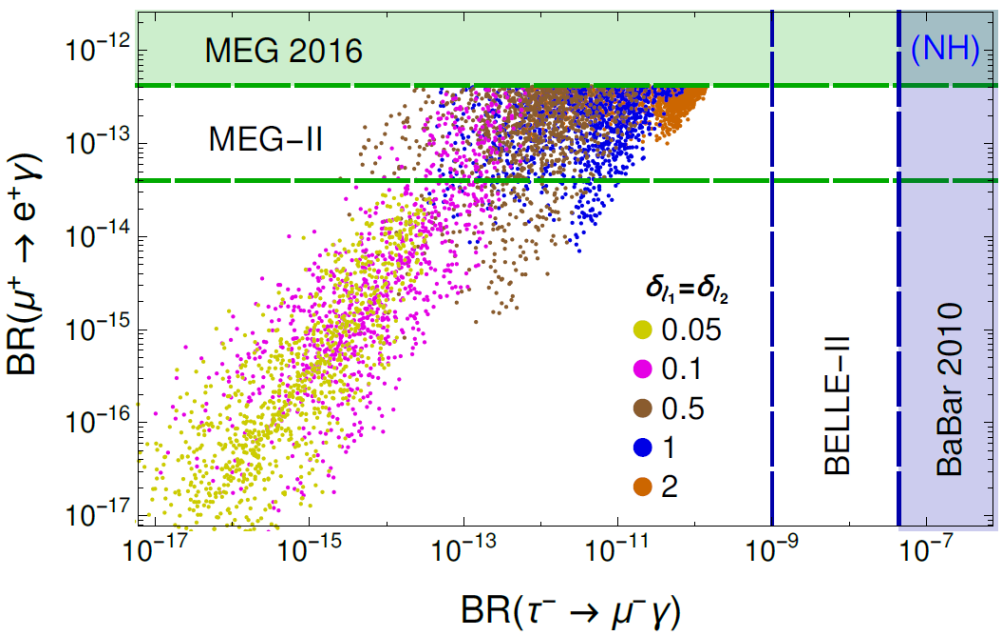
Numerics

- **The case with NH and masses of the heavy neutrinos 0.1 < m_R < 10 TeV**

Calculations were made assuming:

- 0.1 < *a, b, c* < 3 MeV
- 0.1 < m_R < 10 TeV
- m_C = 500 GeV





Restricting all masses of heavy neutrinos < 10 TeV we receive lower values of branching ratios

Conclusions

- **Using Higgs basis (the basis for the Higgs doublets wherein only one of them has nonzero VEV) we simplify model which gives good results.**

- **We have employed several simplifying assumptions in order to reduce the parameter space of the model.**

- **In this model is possible to find the parameter space where all three branching ratios of the decay $l_1 \rightarrow l_2\gamma$ are simultaneously close to their experimental limits.**

- **The idea of the model could be adopted for the processes $Z \rightarrow l_1 l_2$ and $H \rightarrow l_1 l_2$. Work is in progress…**

Thank You…