Flavor Physics With Higgs and Leptons

Xiao-Gang He

SJTU/NTU

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Standard Model and Flavor Physics

Standard Model is based on $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge interaction.

In SM mis-match of weak and mass eigen-bases, leads to flavor mixing and CP violation, the core of the flavor physics.

When going beyond SM, more possibilities!
Number of SM generations

In the SM, only 3 generations of quarks and leptons are allowed.

\[ gg \rightarrow \text{Higgs} \sim (\text{number of heavy quarks})^2, \] if fourth generation exist, their mass should be large, 9 times bigger production of Higgs. LHC data ruled out more than 3 generations of quarks.

LEP already ruled out more than 3 neutrinos with mass less than \( m_Z/2 \).

Cosmology and astrophysics, number of light neutrinos also less than 4.

SM, triangle anomaly cancellation: equal number of quarks and leptons!

There are only three generations of sequential quarks and leptons!

Why 3 generations? How do they mix with each other?
Quark and Lepton mixing patterns

The mis-match of weak and mass eigen-state bases lead quark and lepton mix within generations.

Quark mixing  the Cabibbo-Kobayashi-Maskawa (CKM) matrix $V_{CKM}$,

lepton mixing the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix $U_{PMNS}$

$$ L = -\frac{g}{\sqrt{2}} \bar{U}_L \gamma^\mu V_{CKM} D_L W^\mu + \frac{g}{\sqrt{2}} E_L \gamma^\mu U_{PMNS} N_L W^- + H.C., $$

$U_L = (u_L, c_L, t_L, ...)^T$, $D_L = (d_L, s_L, b_L, ...)^T$, $E_L = (e_L, \mu_L, \tau_L, ...)^T$, and $N_L = (\nu_1, \nu_2, \nu_3, ...)^T$

For $n$-generations, $V = V_{CKM}$ or $U_{PMNS}$ is an $n \times n$ unitary matrix.

A commonly used form of mixing matrix for three generations of fermions is given by

$$ V = \begin{pmatrix}
    c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\
    -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\
    s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13}
\end{pmatrix}, $$

where $s_{ii} = \sin \theta_{ii}$ and $c_{ii} = \cos \theta_{ii}$ are the mixing angles and $\delta$ is the CP violating phase.

If neutrinos are of Majorana type, for the PMNS matrix one should include an additional diagonal matrix with two Majorana phases $\text{diag}(e^{i\alpha_1/2}, e^{i\alpha_2/2}, 1)$ multiplied to the matrix from right in the above.
Status of Quark and Lepton

Quark Mixing

\[
\begin{pmatrix}
1 - \lambda^2/2 & \lambda A\lambda^3(\rho - i\eta) \\
-\lambda & 1 - \lambda^2/2 A\lambda^2 \\
A\lambda^3(1 - \rho - i\eta) & -A\lambda^2
\end{pmatrix} + O(\lambda^4)
\]

Why they mix the pattern shown above? Some understanding.

Neutrino Mixing

\[\Delta m^2 = m_3^2 - (m_2^2 + m_1^2)/2. \text{ Thus, } \Delta m^2 = \Delta m_{31}^2 = \Delta m_{21}^2/2 > 0, \text{ if } m_1 < m_2 < m_3 \]
\[\text{and } \Delta m^2 = \Delta m_{32}^2 + \Delta m_{21}^2/2 < 0 \text{ for } m_3 < m_1 < m_2.\]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>best-fit</th>
<th>3σ</th>
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<tbody>
<tr>
<td>$\Delta m_{21}^2$ [$10^{-5}$ eV $^2$]</td>
<td>7.37</td>
<td>6.93 − 7.97</td>
</tr>
<tr>
<td>$</td>
<td>\Delta m^2</td>
<td>$ [$10^{-3}$ eV $^2$]</td>
</tr>
<tr>
<td>$\sin^2 \theta_{12}$</td>
<td>0.297</td>
<td>0.250 − 0.354</td>
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<tr>
<td>$\sin^2 \theta_{23}$, $\Delta m^2 &gt; 0$</td>
<td>0.437</td>
<td>0.379 − 0.616</td>
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<tr>
<td>$\sin^2 \theta_{23}$, $\Delta m^2 &lt; 0$</td>
<td>0.569</td>
<td>0.383 − 0.637</td>
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<td>0.0185 − 0.0246</td>
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<tr>
<td>$\sin^2 \theta_{13}$, $\Delta m^2 &lt; 0$</td>
<td>0.0218</td>
<td>0.0186 − 0.0248</td>
</tr>
<tr>
<td>$\delta/\pi$</td>
<td>1.35 (1.32)</td>
<td>(0.92 − 1.99)</td>
</tr>
<tr>
<td>&amp;</td>
<td>(0.83 − 1.99)</td>
<td></td>
</tr>
</tbody>
</table>

$\lambda = 0.22537 \pm 0.00061$, $\ A = 0.814^{+0.023}_{-0.024}$, $\ \bar{\rho} = 0.117 \pm 0.021$, $\ \bar{\eta} = 0.353 \pm 0.013$. 

NuFIT 3.0 (2016)
The 125 GeV Higgs is consistent with SM one!
1. Flavor Physics with Leptons
(g-2)_μ, B and lepton anomalies, ⁸Be^* \rightarrow ⁸Be e e and a 17 MeV X-boson

2. Flavor Physics with Higgs
General Yukawa coupling, precision test from B_s \rightarrow \mu \mu. CP violation in h \rightarrow \tau \tau

3. Flavor Physics with Neutrino Mixing
Model for \delta_{CP} = -\pi/2 and \theta_{23} = \pi/4? Grand Unification prediction?
1. Flavor Physics with Leptons

The longstanding problem of $(g-2)_\mu$

\[ a_{\mu}^{\text{exp}} = 11659209.1(5.4)(3.3) \times 10^{-10}, \]
\[ a_{\mu}^{\text{SM}} = 11659180.3(0.1)(4.2)(2.6) \times 10^{-10}. \]

\[ \Delta a_{\mu} = a_{\mu}^{\text{exp}} - a_{\mu}^{\text{SM}} = 288(63)(49) \times 10^{-11} \]

Abstract

I present a status report of the hadronic vacuum polarization effects for the muon $g-2$, to be considered as an update of [1]. The update concerns recent new inclusive $R$ measurements from KEDR in the energy range 1.84 to 3.72 GeV. For the leading order contributions I find $a_{\mu}^{\text{had}(1)} = (688.07 \pm 4.14)(688.77 \pm 3.38) \times 10^{-10}$ based on $e^+e^- \text{ data [incl. } \tau \text{ data], } a_{\mu}^{\text{had}(2)} = (-9.93 \pm 0.07) \times 10^{-10}$ (NLO) and $a_{\mu}^{\text{had}(3)} = (1.22 \pm 0.01) \times 10^{-10}$ (NNLO). Collecting recent progress in the hadronic light-by-light scattering I adopt $\pi^0, \eta, \eta' [95 \pm 12]$ + axial–vector $[8 \pm 3]$ + scalar $[-6 \pm 1] + \pi, K \text{ loops } [-20 \pm 5] + \text{quark loops } [22 \pm 4] + \text{tensor } [1 \pm 0] + \text{NLO } [3 \pm 2]$ which yields $a_{\mu}^{\text{NLO}}(l_{\mu}, \text{had}) = (103 \pm 29) \times 10^{-11}$. With these updates I find $a_{\mu}^{\text{exp}} - a_{\mu}^{\text{the}} = (31.3 \pm 7.7) \times 10^{-10}$ a 4.1 $\sigma$ deviation. Recent lattice QCD results and future prospects to improve hadronic contributions are discussed.
B and lepton physics anomalies

The $R_{K^*}$ Anomaly

S. Bifani, CERN Seminar, 18th April, 2017

All these processes are induced by $b \rightarrow s \ell\ell$ interaction. Consistently lower than SM predictions.

Combined effects are now about $4\sigma$!

The $R_{D(*)}$ anomalies

$R_{D(*)} = B(B \rightarrow D(*)\tau\nu)/B(B \rightarrow D(*)\ell\nu)$

Again, a $4\sigma$ effects!
Models for $R(D^{(*)})$ and $b\to s\mu^+\mu^-$ anomalies

A lot of model building activities trying to provide solutions to $R(D^{(*)})$ and $b\to s\mu^+\mu^-$ induced anomalies.

Making $b\to s\mu^+\mu^-$ smaller or $b\to s\,e^+e^-$ larger than SM predictions.

$Z'$ and $W'$ models, Multi-Higgs models, leptoquark models, Susy, R-parity violating models, ...

Hundreds of papers written on related subjects!

Since the new LHCb $R_{K^*}$ data, just to the end of April 17 papers:

arXiv: 1703.09189, 1703.09627, 1704.05340, 1704.05435, 1704.05438, 1704.05444, 1704.05446, 1704.05447, 1704.05672, 1704.05835, 1704.06005, 1704.06188, 1704.06200, 1704.06240, 1704.07347, 1704.07397, 1704.08168, ...
Model independent analysis for flavor physics

Too many ways to build theoretical models. Effective operator analysis provides a model independent method.

Operator analysis, more complicated than our collider friends, they usually just take flavor conserving interactions.

For flavor physics, this is obviously not enough! The flavor changing interaction is at the core of the study. More complicated.

Some guideline to parameterize flavour changing interactions needed!

Minimal Flavor violation provides a very economic and efficient way for guide flavor physics when using effective operators:

All flavor and CP violations have their origin in the known structure of Yukawa couplings.
The MFV framework for quarks
D'Ambrosio, Giudice, Isidori, Strumia, arXiv: 0207036

$L_K$ and $L_m$ are formally invariant under a global group
$U(3)_Q \times U(3)_U \times U(3)_D = G_q \times U(1)_Q \times U(1)_U \times U(1)_D.$

with $G_q = SU(3)_Q \times SU(3)_U \times SU(3)_D.$

$Q_{i,L}, U_{i,R},$ and $D_{i,R}$ as fundamental representations of $SU(3)_{Q,U,D}.$

The Yukawa couplings $(Y_{u,d})_{ij}$ as spurions which transform as

$Q_L \rightarrow V_Q Q_L,$ \hspace{1em} $U_R \rightarrow V_U U_R,$ \hspace{1em} $D_R \rightarrow V_D D_R,$

$Y_u \rightarrow V_Q Y_u V_U^\dagger,$ \hspace{1em} $Y_d \rightarrow V_Q Y_d V_D^\dagger,$ \hspace{1em} $V_{Q,U,D} \in SU(3).$

$Y_u \sim (3, \bar{3}, 1)$ and $Y_d \sim (3, 1, \bar{3})$

Construct $\Delta_{q,l}$ so that $O_{1,2}^6$ are invariant of the flavor global groups. Building blocks:

$O_1^6 = \overline{Q} \gamma_\eta \Delta_q P_L Q \overline{L} \gamma_\eta \Delta_\ell P_L L,$

$O_2^6 = \overline{Q} \gamma_\eta \Delta_q P_L \tau_a Q \overline{L} \gamma_\eta \Delta_\ell P_L \tau_a L,$

$A_q = Y_u Y_u^\dagger,$ \hspace{1em} $B_q = Y_d Y_d^\dagger,$

$A_\ell = Y_\nu Y_\nu^\dagger,$ \hspace{1em} $B_\ell = Y_e Y_e^\dagger.$

Similarly for quarks
Practically for B anomalies, the first two terms are sufficient.

$$\Delta_f = \sum_{ijk...} \xi_{ijk...} A^i B^j A^k ... \text{ infinite!}$$

Resume into 17 terms

$$\Delta_f = \xi_1 I + \xi_2 A + \xi_3 B + \xi_4 A^2 + \xi_5 B^2 + \xi_6 AB + \xi_7 BA + \xi_8 ABA + \xi_9 B^2 + \xi_{10} BAB$$

$$+ \xi_{11} AB^2 + \xi_{12} ABA^2 + \xi_{13} A^2 B + \xi_{14} B^2 A + \xi_{15} B^2 AB + \xi_{16} AB^2 A + \xi_{17} B^2 A^2 B$$

The anomalies discussed earlier can be carried out in this framework.

(Lee, Tandean, arXiv:1505.04692)

More restrictive way of generating $O_i$, by $Z'$

$$\mathcal{L}_{Z'} = - (\bar{Q} \gamma^\eta \Delta_q P_L Q + \bar{L} \gamma^\eta \Delta_\ell P_L L) Z'_\eta,$$

Able to solve $b \rightarrow s \mu\mu$ anomalies?

C-W. Cheng, He, Tandean, Yuan, arXiv: 1706.02696
\[ \mathcal{L}_{\text{eff}} \supset \frac{\sqrt{2} \alpha_e \lambda_{sb} G_F}{\pi} C_{\ell\ell'} \bar{s} \gamma^n P_L b \bar{\ell} \gamma_\eta P_L \ell', \]

\[ c_{\ell_j \ell_k} = \frac{\pi}{\sqrt{2} \alpha_e G_F m^2_{Z'}} (\zeta_1 y_t^2 + \zeta_2 y_t^4) (\Delta_{\ell})_{jk}, \]

\[ \approx -25.3 \text{ TeV}^2 \frac{(\zeta_1 y_t^2 + \zeta_2 y_t^4) (\Delta_{\ell})_{jk}}{m^2_{Z'}}, \]

\[ \lambda_{sb} = V^*_{ts} V_{tb}, \quad C_{\ell\ell'} = \delta_{\ell\ell'} C_9^{\text{SM}} + c_{\ell\ell'}, \quad |\zeta_1 y_t^2 + \zeta_2 y_t^4| \frac{m^2_{Z'}}{m^2_{Z'}} \leq \frac{0.13}{\text{TeV}}. \]

\[ 0.60 \leq \frac{\Delta M^\text{exp}_{d}}{\Delta M^\text{SM}_{d}} \leq 1.16, \quad 0.71 \leq \frac{\Delta M^\text{exp}_{s}}{\Delta M^\text{SM}_{s}} \leq 1.19. \]
Constraints on $\Delta l$

$$\mathcal{A}_\ell = \frac{2}{\nu^2} \mathcal{M} U_{PMNS} \hat{m}_\nu^{1/2} O O^\dagger \hat{m}_\nu^{1/2} U_{PMNS}^\dagger,$$

$$OO^\dagger = e^{2iR}, \quad R = \begin{pmatrix} 0 & r_1 & r_2 \\ -r_1 & 0 & r_3 \\ -r_2 & -r_3 & 0 \end{pmatrix}.$$

Real $O$ disfavored. Complex $O$ can do the job!

<table>
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<tr>
<th>Parameter</th>
<th>NH</th>
<th>IH</th>
</tr>
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<tbody>
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<td>$\sin^2 \theta_{12}$</td>
<td>$0.306 \pm 0.012$</td>
<td>$0.306 \pm 0.012$</td>
</tr>
<tr>
<td>$\sin^2 \theta_{23}$</td>
<td>$0.441^{+0.027}_{-0.021}$</td>
<td>$0.587^{+0.020}_{-0.024}$</td>
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<tr>
<td>$\sin^2 \theta_{13}$</td>
<td>$0.02166 \pm 0.00075$</td>
<td>$0.02179 \pm 0.00076$</td>
</tr>
<tr>
<td>$\delta / ^\circ$</td>
<td>$261^{+51}_{-59}$</td>
<td>$277^{+40}_{-46}$</td>
</tr>
<tr>
<td>$\Delta m^2_{21} = m^2_2 - m^2_1$</td>
<td>$(7.50^{+0.19}_{-0.17}) \times 10^{-5}$ eV$^2$</td>
<td>$(7.50^{+0.19}_{-0.17}) \times 10^{-5}$ eV$^2$</td>
</tr>
<tr>
<td>$\Delta m^2_{3\ell}$</td>
<td>$m^2_3 - m^2_\ell = (2.524^{+0.039}_{-0.040}) \times 10^{-3}$ eV$^2$</td>
<td>$m^2_3 - m^2_\ell = (-2.514^{+0.038}_{-0.041}) \times 10^{-3}$ eV$^2$</td>
</tr>
</tbody>
</table>

Wrong sign to solve $a_\mu$ anomaly

$$B(\tau \to e\nu\nu')_{exp} / B(\tau \to e\nu\nu')_{SM} = 1.002 \pm 0.006,$$

$$B(\tau \to e\mu\bar{\nu})_{exp} < 1.5 \times 10^{-8}, \quad B(\tau \to \mu\mu\bar{e})_{exp} < 1.7 \times 10^{-8},$$

$$B(\tau \to e\mu\bar{\nu})_{exp} < 2.7 \times 10^{-8}, \quad B(\tau \to 3\mu)_{exp} < 2.1 \times 10^{-8},$$

$$B(\tau \to \mu\mu\bar{e})_{exp} < 1.8 \times 10^{-8}, \quad B(\mu \to e\gamma)_{exp} < 4.2 \times 10^{-13},$$

$$B(\tau \to e\gamma)_{exp} < 3.3 \times 10^{-8}, \quad B(\tau \to \mu\gamma)_{exp} < 4.4 \times 10^{-8},$$

15
Complex O

IO

NO
<table>
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<tr>
<th>Decay mode</th>
<th>Measured upper limit at 90% CL [39, 56]</th>
<th>Prediction ranges</th>
</tr>
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<tr>
<td></td>
<td></td>
<td>NO</td>
</tr>
<tr>
<td>$B \to Ke^+\mu^+$</td>
<td>$3.8 \times 10^{-8}$</td>
<td>$[0.0002, 2.9] \times 10^{-9}$</td>
</tr>
<tr>
<td>$B \to K^*e^+\mu^+$</td>
<td>$5.1 \times 10^{-7}$</td>
<td>$[0.001, 7.2] \times 10^{-9}$</td>
</tr>
<tr>
<td>$B_s \to e^+\mu^+$</td>
<td>$1.1 \times 10^{-8}$</td>
<td>$[0.001, 8.6] \times 10^{-12}$</td>
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<tr>
<td>$B \to \pi e^+\mu^+$</td>
<td>$9.2 \times 10^{-8}$</td>
<td>$[0.0001, 1.2] \times 10^{-10}$</td>
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<tr>
<td>$B \to \rho e^+\mu^+$</td>
<td>$3.2 \times 10^{-6}$</td>
<td>$[0.0003, 3.1] \times 10^{-10}$</td>
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<tr>
<td>$B^0 \to e^+\mu^+$</td>
<td>$2.8 \times 10^{-9}$</td>
<td>$[0.0002, 3.0] \times 10^{-13}$</td>
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<tr>
<td>$B^+ \to K^+ e^+\tau^+$</td>
<td>$3.0 \times 10^{-5}$</td>
<td>$[0.02, 8.1] \times 10^{-9}$</td>
</tr>
<tr>
<td>$B^+ \to K^*+ e^+\tau^+$</td>
<td>$- $</td>
<td>$[0.003, 1.4] \times 10^{-8}$</td>
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<tr>
<td>$B_s \to e^+\tau^+$</td>
<td>$- $</td>
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<td>$B^+ \to \pi^+ e^-\tau^+$</td>
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<td>$[0.004, 1.8] \times 10^{-10}$</td>
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<td>$[0.01, 6.5] \times 10^{-10}$</td>
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<td>$B^0 \to e^-\tau^+$</td>
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<td>$[0.005, 2.2] \times 10^{-10}$</td>
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<tr>
<td>$B^+ \to K^+ \mu^+\tau^+$</td>
<td>$4.8 \times 10^{-5}$</td>
<td>$[1, 3] \times 10^{-9}$</td>
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<td>$4.8 \times 10^{-5}$</td>
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<tr>
<td>$B^0 \to \mu^+\tau^+$</td>
<td>$2.2 \times 10^{-5}$</td>
<td>$[3, 9] \times 10^{-11}$</td>
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<tr>
<td>$K_L \to e^+\mu^+$</td>
<td>$4.7 \times 10^{-12}$</td>
<td>$[-0, 1.4] \times 10^{-12}$</td>
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<td></td>
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<td>IO</td>
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<tr>
<td>$B \to Ke^+\mu^+$</td>
<td>$3.8 \times 10^{-8}$</td>
<td>$[-0, 3.2] \times 10^{-9}$</td>
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<tr>
<td>$B \to K^*e^+\mu^+$</td>
<td>$5.1 \times 10^{-7}$</td>
<td>$[-0, 8.2] \times 10^{-9}$</td>
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<td>$B_s \to e^+\mu^+$</td>
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<td>$[-0, 2.9] \times 10^{-13}$</td>
</tr>
</tbody>
</table>
In the Z’ MFV framework, not possible to solve $R_{K(*)}$ and $R_{D(*)}$ simultaneously!
Bring $R(D(\ast))$ and $b \rightarrow s \mu \mu$ anomalies together

A role of a leptoquark scalar


$$L_{int} = \frac{1}{2m_\phi^2} \left[ x_{ij} x_{i'j'} \bar{\nu}_{L}^j \gamma^\mu \nu_{L}^i \bar{d}_{L}^{j'} \gamma_\mu d_{L}^i + z_{ij} z_{i'j'} \bar{e}_{L}^{j'} \gamma^\mu e_{L}^i \bar{\mu}_{L}^{j'} \gamma_\mu \mu_{L}^i \right]$$

$K \rightarrow \pi \nu \nu, B \rightarrow K(\ast) \nu \nu$  \hspace{1cm}  $D \rightarrow \mu \mu, \pi \mu \mu$

$$- \frac{1}{2m_\phi^2} \left[ x_{ij} z_{i'j'} \bar{\nu}_{L}^j \gamma^\mu e_{L}^i \bar{d}_{L}^{j'} \gamma_\mu u_{L}^i - x_{ij} z_{i'j'} \bar{e}_{L}^{j'} \gamma^\mu e_{L}^i \bar{\mu}_{L}^{j'} \gamma_\mu u_{L}^i \right]$$

$R(D(\ast))$, $B \rightarrow D(\ast) (\rho, \pi) \nu \nu, B_c \rightarrow \tau \nu$

$$+ \frac{1}{2m_\phi^2} \left[ y_{ij} y_{i'j'} \bar{e}_{R}^{j'} \gamma^\mu e_{R}^i \bar{\nu}_{R}^{j'} \gamma_\mu \nu_{R}^i \left( \bar{d}_{R}^{j'} \gamma^\mu \nu_{R}^i \bar{\nu}_{R}^{j'} \gamma_\mu d_{L}^i - \frac{1}{2} \right) \right]$$

$D \rightarrow \mu \mu, \pi \mu \mu$  \hspace{1cm}  $R(D(\ast))$, $B \rightarrow D(\ast) (\rho, \pi) \nu \nu, B_c \rightarrow \tau \nu$

One loop contributions for $b \rightarrow s \mu \mu$

Significant contribution needed from $\bar{e}_{R} Y U_{R}^{c} \phi$ coupling! Can also solve $(g-2)_\mu$ anomaly!
The protophobic fifth force by a 17 MeV X boson

$^8Be^* \rightarrow ^8Bee^+e^-$ is larger than expected $e^+e^-$ opening angle at 140°. The anomaly is at 6.9σ level. (Rev. Lett.116, 042501 (2016))

A. J. Krasznahorkay et al. found that the observed excess in shape and size are fit by a new boson with mass $m_X = 16.7 \pm 0.35 (\text{stat}) \pm 0.5 (\text{sys}) \text{ MeV}$.

J. Feng et al., PRL117, 071803; PRD95, 035017

The X boson is likely a vector boson of mass 16.7 MeV,

$$L = -\frac{1}{4}X_{\mu\nu}X^{\mu\nu} + \frac{1}{2}m_X^2X_{\mu}X^{\mu} - X_{\mu}J_{\mu}^{X}, \quad J_{\mu} = \sum_{f=u,d,e,\nu,e,\nu,e,\ldots} e\bar{e}_{f}\gamma_{\mu}f.$$

In general there are also axial current coupling: $\sum_{f=u,d,e,\nu,e,\ldots} e\bar{e}_{f}\gamma_{5}\gamma_{\mu}f$.

Assuming $^8Be^* \rightarrow ^8B eX$ followed by $X \rightarrow e^+e^-$ saturating $X$ decay

Using the fact that the interaction matrix element of $X$ with $^8Be$ and $^8Be^*$ is iso-scalar and proportional to $\epsilon_{u}^v + \epsilon_{d}^v$: $\epsilon_{u}^v = 2\epsilon_{u}^2 + \epsilon_{d}^2$ and $\epsilon_{d}^v = \epsilon_{u}^2 + 2\epsilon_{d}^2$.

Data can be explained with: $\epsilon_{u}^v + \epsilon_{d}^v \approx 0.011$.

Constraint: $B(\pi^0 \rightarrow X\gamma) < 8 \times 10^{-4}$ leads to: $-0.067 < \epsilon_{p}^v/\epsilon_{n}^v < 0.078$.

A protophobic interaction: $\epsilon_{p}^v = 0$, is consistent!

Satisfying beam damp and $(g-2)_\mu$ and also $e-\nu$ scattering data,

$$\epsilon_{u}^v = \pm 3.7 \times 10^{-3}, \quad \epsilon_{d}^v = \mp 7.4 \times 10^{-3}$$
$$2 \times 10^{-4} < |\epsilon_{u}^v| < 1.3 \times 10^{-3}, \quad |\epsilon_{e}\epsilon_{\nu_e}|^{1/2} < 7 \times 10^{-5}.$$  

For $B(X \rightarrow e^+e^-)$ not equal to 1, the couplings are scaled by $1/\sqrt{B(X \rightarrow e^+e^-)}$. 

[Graph and equation images]
A realistic renormalizable model


A protophobic gauge model $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{Y'}$

$Q^1_L : (3, 2, 1/6)(-1)$, \quad $u^1_R : (3, 1, 2/3)(5)$, \quad $d^1_R : (3, 1, -1/3)(-7)$,

$L^1_L : (1, 2, -1/2)(\beta)$, \quad $e^1_R : (1, 1, -1)(\beta)$,

$Q^2_L : (3, 2, 1/6)(1)$, \quad $u^2_R : (3, 1, 2/3)(-5)$, \quad $d^2_R : (3, 1, -1/3)(7)$,

$L^2_L : (1, 2, -1/2)(-\beta)$, \quad $e^2_R : (1, 1, -1)(-\beta)$,

and the third generation does not have any $U(1)_{Y'}$ charges.

Can change the 2nd generation to 3rd generation.

$Y'$ boson as the desired $X$ boson yet!

The Higgs scalars have non-trivial $U(1)_Y$ and $U(1)_{Y'}$ numbers.

The VEV should be much smaller than electroweak scale.

The couplings of neutrinos to $Y'$ are too large to satisfy the constraints.
Introduce an additional $U(1)_X$ which do not couple to SM fermions. But has a kinetic mixing of $U(1)_Y$, and $U(1)_X$ gauge fields, $-(\epsilon/2)Y'_{\mu\nu}X^{\mu\nu}$, The $X$ boson to the $J_{Y'}^\mu$, current as, $\epsilon X_\mu J_{Y'}^\mu$.

$$\epsilon J_{Y'}^\mu = \epsilon g_{Y'} \left[ \bar{u}\gamma^\mu (4 + 6\gamma_5)u - \bar{d}\gamma^\mu (8 + 6\gamma_5)d + \beta \bar{e}\gamma^\mu e + \frac{\beta}{2} \bar{\nu}_e \gamma^\mu (1 - \gamma_5)\nu_e \right],$$

$$- \epsilon g_{Y'} \left[ \bar{c}\gamma^\mu (4 + 6\gamma_5)c - \bar{s}\gamma^\mu (8 + 6\gamma_5)s + \beta \bar{\mu}\gamma^\mu \mu + \frac{\beta}{2} \bar{\nu}_\mu \gamma^\mu (1 - \gamma_5)\nu_\mu \right],$$

$g_{Y'}$ being the $U(1)_Y$ gauge coupling.

The full gauge group is: $SM \times U(1)_Y \times U(1)_X$. 
Neutrino couplings still too large.

Make $\nu_e$ mixes with a vector-like fermion $S = S_L + S_R$ with $U(1)_X$ charge $x_S$.

With three iso-doublet Higgs scalars $\phi_e(0, 0)$, $\phi_{\nu_e}(\beta, 0)$, and $\eta(\beta, -x_S)$,

$$L = -y_e \bar{L}^1_L \phi_e e_R - y_N \bar{L}^1_L \phi_{\nu_e} N_R - \frac{1}{2} M_N \bar{\tilde{N}}^c_R N_R - f_S \bar{L}^1_L \eta S_R - m_S \bar{S}_L S_R$$

![Diagram]

FIG. 1: The effects of the $\nu_L - S$ mixing, generation of the $U_s$ matrix (left) and modification of the interaction of the $X$ boson with the left-handed neutrinos (right).

One finds the effective coupling $\nu_e$ to the $X$ boson should be

$$\frac{\epsilon g_{\gamma' \beta}}{2} \bar{\nu}_e \gamma_\mu (1 - \gamma_5) \nu_e X^\mu \rightarrow \frac{\epsilon g_{\gamma' \beta}}{2} \frac{1 - \frac{g_X x_S}{\epsilon g_{\gamma' \beta}} U_s}{1 + U_s} \bar{\nu}_e \gamma_\mu (1 - \gamma_5) \nu_e X^\mu.$$
The parameters $\varepsilon^v_f$ in the effective theory are given by

\[
\varepsilon^v_u = -\frac{4\epsilon g_{Y'}}{e}, \quad \varepsilon^v_d = \frac{8\epsilon g_{Y'}}{e}, \quad \varepsilon^v_e = -\frac{\epsilon \beta g_{Y'}}{e}, \quad \varepsilon^v_{\nu_e} = \frac{\epsilon g_{Y'} \beta}{2e} \frac{1 - g_X U_S/(\epsilon g_{Y'} \beta)}{1 + U_S}.
\]

and also non-zero $\varepsilon^a_f$

\[
\varepsilon^a_u = -\frac{6\epsilon g_{Y'}}{e}, \quad \varepsilon^a_d = \frac{6\epsilon g_{Y'}}{e}, \quad \varepsilon^a_e = 0, \quad \varepsilon^a_{\nu_e} = -\frac{\epsilon g_{Y'} \beta}{2e} \frac{1 - g_X U_S/(\epsilon g_{Y'} \beta)}{1 + U_S}.
\]

Enough freedom to fit requird $\varepsilon^v$. Vector like interaction is protophobic! However, the model above also contains axial-vector current interactions Needs to check if OK!
$^8\text{Be}^* \rightarrow ^8\text{Be}X$ is induced by iso-scalar, in our model proportional to $[(\epsilon_u^a + \epsilon_d^a)/2](\bar{u}\gamma^\mu\gamma_5 u + \bar{d}\gamma^\mu\gamma_5 d)$. It is identically equal to Zero! Do not affect previous discussions!

No $\pi^0 \rightarrow e^+e^-$ decay, because $X$ coupling to $\bar{e}\gamma_\mu e$.

Parity violating-quark scattering. $L_{PV} = (G_F/\sqrt{2})C_{2f}\bar{e}\gamma^\mu e\bar{f}\gamma_\mu\gamma_5 f$. 

Data: $2C_{2u} - C_{2d} = -0.145 \pm 0.086, 2C_{2u} - C_{2d} = -0.095\left(1 - 1.7 \frac{\epsilon_e^V}{10^{-3}} \frac{\epsilon_n^V}{10^{-2}}\right)$

To produce the experimental central value for $2C_{2u} - C_{2d}$, $\epsilon_e^V\epsilon_n^V$ needs to be $-0.3 \times 10^{-5}$.

If one keeps as large a $\epsilon_n^V$ close to its upper bound, $\epsilon_e^V$ needs to be $0.21 \times 10^{-3}$.

But can be as large as $1.4 \times 10^{-3}$, contributes $\Delta a_\mu = 152 \times 10^{-11}$.

**A viable model for $^8\text{Be}^* \rightarrow ^8\text{Be} e^+e^-$ anomaly!**
2. Flavor Physics with Higgs

In the SM, there is just one Higgs doublet $H : (1, 2)(-12) \rightarrow (0, (v + h)/\sqrt{v})^T$

$$L_4 = -\bar{f}_L Y_f^{SM} H f_R + H.C. \rightarrow -\bar{f}(M_f + Y_f^{SM} \frac{h}{\sqrt{2}}) f,$$

$$M_f = \text{diag}(m_f^1, m_f^2, m_f^3), \quad Y_f^{SM} = \frac{M_f}{v/\sqrt{2}}.$$

Going beyond SM, Yukawa couplings will in general be modified.

Example

$$L_6 = -\frac{1}{\Lambda^2}(H^\dagger H)\bar{f}_L g_f H f_R, \quad \rightarrow \quad L_6 = -\frac{v^2}{2\sqrt{2}\Lambda^2} \bar{f}_L g_f v(1 + 3\frac{h}{v}) f_R, \quad Y_l = \begin{pmatrix} Y_{ee} & Y_{e\mu} & Y_{e\tau} \\ Y_{\mu e} & Y_{\mu\mu} & Y_{\mu\tau} \\ Y_{\tau e} & Y_{\tau\mu} & Y_{\tau\tau} \end{pmatrix}$$

$$L = -\bar{f}(M_f + (Y_f + i\gamma_5 \bar{Y}_f) \frac{h}{\sqrt{2}}) f, \quad M_f = S_f^\dagger \frac{v}{\sqrt{2}} (Y_f^{SM} + \frac{v^2}{2\Lambda^2} g_f) T_f, \quad \bar{Y}_l = \begin{pmatrix} \bar{Y}_{ee} & \bar{Y}_{e\mu} & \bar{Y}_{e\tau} \\ \bar{Y}_{\mu e} & \bar{Y}_{\mu\mu} & \bar{Y}_{\mu\tau} \\ \bar{Y}_{\tau e} & \bar{Y}_{\tau\mu} & \bar{Y}_{\tau\tau} \end{pmatrix}$$

$$Y_f = \sqrt{2} \frac{M_f}{v} + (\delta Y_f + \delta Y_f^\dagger), \quad \bar{Y}_f = -i(\delta Y_f - \delta Y_f^\dagger), \quad \delta Y = \frac{v^2}{2\Lambda^2} S_f^i g_f T_f$$

Flavor Physics with Higgs is largely related to the Yukawa couplings.
Higgs to $\mu \tau$ anomaly is going away!

2016 CMS result: $\text{Br}(h \rightarrow \mu \tau \mu) = (-0.76 \pm 0.81)\%$ in May

<table>
<thead>
<tr>
<th>Channel</th>
<th>Coupling</th>
<th>Pre-LHC</th>
<th>CMS</th>
<th>ATLAS</th>
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</thead>
<tbody>
<tr>
<td>$H \rightarrow \mu e$</td>
<td>$\sqrt{</td>
<td>Y_{\mu e}</td>
<td>^2 +</td>
<td>Y_{e\mu}</td>
</tr>
<tr>
<td>$H \rightarrow \mu \tau$</td>
<td>$\sqrt{</td>
<td>Y_{\mu \tau}</td>
<td>^2 +</td>
<td>Y_{\tau \mu}</td>
</tr>
<tr>
<td>$H \rightarrow e \tau$</td>
<td>$\sqrt{</td>
<td>Y_{e \tau}</td>
<td>^2 +</td>
<td>Y_{\tau e}</td>
</tr>
</tbody>
</table>
Going back to the normal

H→mutau 2.6 sigma is ruled out by the 35.9 fb^{-1} @CMSexperiment #LHCP2017

Results of H→μτ and H→eτ searches

- No excess of data
  - Best fit branching fraction: 0.00 ± 0.12%
  - B(H→μτ) < 0.25% at 95% CL

- Slight excess of data (1.6 σ)
  - Best-fit branching fraction: 0.30 ± 0.18%
  - B(H→eτ) < 0.61% at 95% CL

\[ BR(H → eτ) < 0.61\% (0.37\% expected) \]
\[ BR(H → μτ) < 0.25\% (0.25\% expected) \]
\[ \sqrt{|Y_{μτ}|^2 + |Y_{τμ}|^2} < 1.43 \times 10^{-3} \]
\[ \sqrt{|Y_{eτ}|^2 + |Y_{τe}|^2} < 2.26 \times 10^{-3} \]
$\bar{B}(B_s \to \mu^+\mu^-)_{\text{LHCb}} = (3.0 \pm 0.6^{+0.3}_{-0.2}) \times 10^{-9}$,

$\bar{B}(B_s \to \mu^+\mu^-)_{\text{CMS}} = (3.0^{+1.0}_{-0.9}) \times 10^{-9}$,

$\bar{B}(B_s \to \mu^+\mu^-)_{\text{average}} = (3.0 \pm 0.5) \times 10^{-9}$,

$\bar{B}(B_s \to \mu^+\mu^-)_{\text{SM}} = (3.44 \pm 0.19) \times 10^{-9}$.

$\bar{B}(B_s \to \mu^+\mu^-)_{\text{avg}}/\bar{B}(B_s \to \mu^+\mu^-)_{\text{SM}} \approx 0.87$.

Constraints and Implications on Higgs FCNC Couplings from Precision Measurement of $B_s \to \mu^+\mu^-$ Decay

$\mathcal{L}_{h\bar{f}f} \equiv -\frac{1}{\sqrt{2}} \bar{f} (Y_f + i \gamma_5 \tilde{Y}_f) f h$

$Y_f = \sqrt{2} \hat{M}_f / v + (\delta Y_f + \delta Y_f^\dagger)$ and $\tilde{Y}_f = -i(\delta Y_f - \delta Y_f^\dagger)$

Chiang, He, Ye and Yuan, arXiv: 1703.06289

See Xingbo Yuan’s talk Friday

LHCb arXiv:1705.03274
CP violation in $h \rightarrow \tau\tau$

Models beyond SM usually generate correction to $h \rightarrow \tau\tau$ coupling. If the corrections is CP violating, effects can show up in $h \rightarrow \tau\tau$ decay.


$$L_Y = -\bar{L}_L[y \frac{v}{\sqrt{2}} + (y + \delta y) \frac{h}{\sqrt{2}}]E_R$$

Diagonalizing the mass term, $S^\dagger_e y T_e(v/\sqrt{2}) = \tilde{M}$,

the $h$ interaction becomes $L_h = -\bar{l}_i(\frac{\tilde{M}}{v} + \frac{1}{\sqrt{2}}S^\dagger_e \delta y T_e)l_j h$

If there is CP violation, the Higgs $h$ coupling to tauon becomes

$$L_{h\tau\tau} = -\frac{h}{v} m_\tau \bar{\tau}(r_\tau + i\tilde{r}_\tau \gamma_5)\tau, \quad r_\tau = 1 + \epsilon_\tau$$

For $\tau \rightarrow \pi^- \nu_\tau$, $\bar{\tau} \rightarrow \pi^+ \bar{\nu}_\tau$,

one can construct T odd operator $O_\pi = \vec{p}_\tau \cdot (\vec{p}_\pi^+ \times \vec{p}_\pi^-)$, $\text{Br}(h \rightarrow \tau\tau) \sim 5 \times 10^{-2}$, $\text{Br}(\tau \rightarrow \pi \nu) \sim 0.1$

$10^6$ Higgs bosons, sensitivity to $A_\pi$ can be 10% at CEPC.

(He, Ma, McKellar, Mod. Phys Lett. A9, 205(1994); Berge, Bereuther, Kirchner, PRD92,096012(2015))
3. Flavor Physics with Neutrino Mixing

Model for $\delta_{CP} = -\pi/2$ and $\theta_{23} = \pi/4$? Grand Unification prediction?

T2K Experiment

- $v_e$ appearance results
  - 2 degenerate best fit points:
    - NH, $\delta_{CP} = 1.48\pi$, $\sin^2\theta_{23} = 0.404$
    - NH, $\delta_{CP} = 0.74\pi$, $\sin^2\theta_{23} = 0.623$
  - Inverted hierarchy slightly disfavored - $\Delta\chi^2 = 0.47$
  - Lower octant in the IH is disfavored at 93% CL
  - arXiv:1703.03328
<table>
<thead>
<tr>
<th>$\delta_{\text{CP}}$</th>
<th>$\delta_{\text{CP}} = -\pi/2$</th>
<th>$\delta_{\text{CP}} = 0$</th>
<th>$\delta_{\text{CP}} = +\pi/2$</th>
<th>$\delta_{\text{CP}} = \pi$</th>
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</thead>
<tbody>
<tr>
<td>$\nu_e$</td>
<td>28.7</td>
<td>24.2</td>
<td>19.6</td>
<td>24.2</td>
</tr>
<tr>
<td>$\bar{\nu}_e$</td>
<td>6.0</td>
<td>6.9</td>
<td>7.7</td>
<td>6.8</td>
</tr>
</tbody>
</table>

Global 6-parameter fit [http://www.nu-fit.org](http://www.nu-fit.org)
Esteban, Maltoni, Martinez-Soler, Schwetz, MCG-G ArXiv:1611.01514
Neutrino mixing ata summary

\[ \Delta m^2 = m_3^2 - (m_2^2 + m_1^2)/2. \] Thus, \( \Delta m^2 = \Delta m_{31}^2 - \Delta m_{21}^2/2 > 0 \), if \( m_1 < m_2 < m_3 \) and \( \Delta m^2 = \Delta m_{32}^2 + \Delta m_{21}^2/2 < 0 \) for \( m_3 < m_1 < m_2 \).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>best-fit</th>
<th>3σ</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta m_{21}^2 ) [10^{-5} \text{ eV}^2]</td>
<td>7.37</td>
<td>6.93 – 7.97</td>
</tr>
<tr>
<td>(</td>
<td>\Delta m^2</td>
<td>) [10^{-3} \text{ eV}^2]</td>
</tr>
<tr>
<td>( \sin^2 \theta_{12} )</td>
<td>0.297</td>
<td>0.250 – 0.354</td>
</tr>
<tr>
<td>( \sin^2 \theta_{23} ), ( \Delta m^2 &gt; 0 )</td>
<td>0.437</td>
<td>0.379 – 0.616</td>
</tr>
<tr>
<td>( \sin^2 \theta_{23} ), ( \Delta m^2 &lt; 0 )</td>
<td>0.569</td>
<td>0.383 – 0.637</td>
</tr>
<tr>
<td>( \sin^2 \theta_{13} ), ( \Delta m^2 &gt; 0 )</td>
<td>0.0214</td>
<td>0.0185 – 0.0246</td>
</tr>
<tr>
<td>( \sin^2 \theta_{13} ), ( \Delta m^2 &lt; 0 )</td>
<td>0.0218</td>
<td>0.0186 – 0.0248</td>
</tr>
<tr>
<td>( \delta/\pi )</td>
<td>1.35 (1.32)</td>
<td>(0.92 – 1.99)</td>
</tr>
</tbody>
</table>

Leading approximation: \( \delta_{CP} = -\pi/2 \) and \( \theta_{23} = \pi/4 \). Models???
SO(10) Predictions

$$16_F(Y_{10}10_H + Y_{126}126_H + Y_{120}120_H)16_F$$

Minimal SO(10) Model without 120

$$\mathcal{L}_{\text{Yukawa}} = Y_{10} 16 16 10_H + Y_{126} 16 16 126_H$$

Two Yukawa matrices determine all fermion masses and mixings, including the neutrinos

$$M_u = \kappa_u Y_{10} + \kappa'_u Y_{126} \quad M_{\nu R} = \langle \Delta_R \rangle Y_{126}$$

$$M_d = \kappa_d Y_{10} + \kappa'_d Y_{126} \quad M_{\nu L} = \langle \Delta_L \rangle Y_{126}$$

$$M_{\nu}^D = \kappa_u Y_{10} - 3\kappa'_u Y_{126}$$

$$M_l = \kappa_d Y_{10} - 3\kappa'_d Y_{126}$$

Model has only 11 real parameters plus 7 phases

Babu, Mohapatra (1993)  
Fukuyama, Okada (2002)  
Babu, Macesanu (2005)  
Bertolini, Malinsky, Schwetz (2006)  
Dutta, Mimura, Mohapatra (2007)  
Bajc, Dorsner, Nemevsek (2009)  
Jushipura, Patel (2011)

Good prediction for $\theta_{13}$  
$\delta$ Away from $-\pi/2$!! Tobe tested!!
Neutrino models for $\delta_{\text{CP}}=-\pi/2$ and $\theta_{23}=\pi/4$

Assuming that the charged lepton mass matrix $M_l$ is diagonalized from left by $U_l$,

$$M_l = U_l \tilde{m}_l U_r, \quad U_l = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix},$$

where $\omega = \exp(i2\pi/3)$ and $\omega^2 = \exp(i4\pi/3)$.

A$_4$ models usually have the above characteristic $U_l$.

$U_r$ is a unitary matrix, but does not play a role in determining $V_{\text{PMNS}}$.

If neutrinos are Majorana particles, the most general mass matrix is

$$M_\nu = \begin{pmatrix} w_1 & x & y \\ x & w_2 & z \\ y & z & w_3 \end{pmatrix},$$

In the basis where charged lepton is diagonalized,

$$m_\nu = U_l^\dagger M_\nu U_l,$$

If all $w_i$, $x$, $y$ and $z$ are all real

$$A = \frac{1}{3}(w_1 + w_2 + w_3 + 2(x + y + z)),$$

$$B = \frac{1}{3}(w_1 + w_2 + w_3 - x - y - z),$$

$$C = \frac{1}{3}(w_1 + \omega^2 w_2 + \omega w_3 - \omega x - \omega^2 y - z),$$

$$D = \frac{1}{3}(w_1 + \omega^2 w_2 + \omega w_3 + 2(\omega x + \omega^2 y + z)).$$

A$_4$ models

(X-G He, Chin. J. Phys 53, 100101(2015);
X-G He and G-N Li, Phys. Lett. B750, 620(2015);
E Ma, Phys. Rev. D92, 051301(2015))

Naturally give $\delta = \pm \pi/2$ and $\theta_{23} = \pi/4$. 

How to obtain such an mixing pattern?

$\mu$–$\tau$ conjugate symmetry

(Grimus, Lavoura, Phys. Lett. B579, 113(2004))

$$m_\nu = \begin{pmatrix} A & C & C^* \\ C & D^* & B \\ C^* & B & D \end{pmatrix}. $$

In the basis where charged lepton is diagonalized, $m_\nu = U_l^\dagger M_\nu U_l$, If all $w_i$, $x$, $y$ and $z$ are all real

$A = \frac{1}{3}(w_1 + w_2 + w_3 + 2(x + y + z)),$

$B = \frac{1}{3}(w_1 + w_2 + w_3 - x - y - z),$ 

$C = \frac{1}{3}(w_1 + \omega^2 w_2 + \omega w_3 - \omega x - \omega^2 y - z),$ 

$D = \frac{1}{3}(w_1 + \omega^2 w_2 + \omega w_3 + 2(\omega x + \omega^2 y + z)).$
Three Types of Discrete Groups

Discrete (flavor) symmetry $G$

Type II: one can impose a physical CP transformation

Type I groups $G_I$:
- generic settings based on $G_I$ do not allow for a physical CP transformation

Type II A groups $G_{II A}$:
- there is a CP basis in which all CG's are real

Type II B groups $G_{II B}$:
- there is no basis in which all CG's are real

Bickerstaff-Damhus Automorphism

non-BDA, class-inverting automorphism

no class-inverting involutory automorphism
Examples

M.-C.C., M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner (2014)

- **Type I**: all odd order non-Abelian groups

<table>
<thead>
<tr>
<th>group</th>
<th>$\mathbb{Z}_5 \times \mathbb{Z}_4$</th>
<th>$T_7$</th>
<th>$\Delta(27)$</th>
<th>$\mathbb{Z}_9 \times \mathbb{Z}_3$</th>
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<tbody>
<tr>
<td>SG</td>
<td>(20,3)</td>
<td>(21,1)</td>
<td>(27,3)</td>
<td>(27,4)</td>
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</table>

- **Type IIA**: dihedral and all Abelian groups

<table>
<thead>
<tr>
<th>group</th>
<th>$S_3$</th>
<th>$Q_8$</th>
<th>$A_4$</th>
<th>$\mathbb{Z}_3 \times \mathbb{Z}_8$</th>
<th>$T'$</th>
<th>$S_4$</th>
<th>$A_5$</th>
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<tr>
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<td>(6,1)</td>
<td>(8,4)</td>
<td>(12,3)</td>
<td>(24,1)</td>
<td>(24,3)</td>
<td>(24,12)</td>
<td>(60,5)</td>
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- **Type IIB**

<table>
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<th>group</th>
<th>$\Sigma(72)$</th>
<th>$((\mathbb{Z}_3 \times \mathbb{Z}_3) \times \mathbb{Z}_4) \times \mathbb{Z}_4$</th>
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</thead>
<tbody>
<tr>
<td>SG</td>
<td>(72,41)</td>
<td>(144,120)</td>
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</table>
A model with Type II seesaw

Particle contents and their transformation properties under standard $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge and $A_4$ family symmetry properties

\[
l_L : (1, 2, -1)(3) \, , \quad l_R : (1, 1, -2)(1 + 1'' + 1') \, ,
\]
\[
\phi : (1, 2, -1)(1) \, , \quad \Phi : (1, 2, -1)(3) \, ,
\]
\[
\Delta^{0,''} : (1, 3, -2)(1 + 1' + 1'') \, , \quad \chi : (1, 3, -2)(3) .
\]

The Lagrangian responsible for the lepton mass matrix is

\[
L = y_e \bar{l}_L \tilde{\Phi} l_R^1 + y_\mu \bar{l}_L \tilde{\Phi} l_R^2 + y_\tau \bar{l}_L \tilde{\Phi} l_R^3
\]
\[
+ Y^0_\nu \bar{l}_L \Delta^0 l^c_L + Y'_\nu \bar{l}_L \Delta' l^c_L + Y''_\nu \bar{l}_L \Delta'' l^c_L + y_\nu \bar{l}_L \chi l^c_L + H.C.
\]
If the structure of the vacuum expectation value (vev) is of the form
\[ < \Phi_{1,2,3} > = v_{1,2,3}^{\Phi}, < \chi_i > = v_i^\chi, < \phi > = v_\phi, \text{ and } < \Delta_{0,1}^{0,1} > = v_{\Delta}^{0,1}, \]

\[
M_l = U_l \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_{\mu} & 0 \\ 0 & 0 & m_\tau \end{pmatrix}, \quad M_\nu = \begin{pmatrix} w_1 & x & y \\ x & w_2 & z \\ y & z & w_3 \end{pmatrix},
\]

In general \( w_i, x, y, z \) are complex!

where \( m_{e,\mu,\tau} = \sqrt{3} y_{e,\mu,\tau} v^{\Phi} \) and

\[
\begin{align*}
w_1 &= Y_\nu^0 v_\Delta^0 + Y_\nu^0 v_\Delta^0 + Y_\nu^0 v_\Delta^0, \\
w_2 &= Y_\nu^0 v_\Delta^0 + \omega^2 Y_\nu^0 v_\Delta^0 + \omega Y_\nu^0 v_\Delta^0, \\
w_3 &= Y_\nu^0 v_\Delta^0 + \omega Y_\nu^0 v_\Delta^0 + \omega^2 Y_\nu^0 v_\Delta^0, \\
x &= y_\nu v_3^\chi, \quad y = y_\nu v_2^\chi, \quad z = y_\nu v_1^\chi.
\end{align*}
\]

In general \( w_i, x, y, z \) are complex!

If \( w_i, x, y \) and \( z \) are real \( \delta_{\text{CP}} = -\pi/2 \) and \( \theta_{23} = \pi/4 \),

Complex ones, a natural way to be away from the real limit value.
Diagonalizing the mass matrices, we have

$$V_{PMNS} = \frac{1}{\sqrt{3}} \begin{pmatrix} c + se^{i\rho} & 1 - ce^{i\rho} & s \\ c + \omega se^{i\rho} & \omega^2 - \omega ce^{i\rho} & s \\ c + \omega^2 se^{i\rho} & \omega & \omega^2 ce^{i\rho} - s \\ \end{pmatrix}$$

$$\tan \rho = \text{Im}(yw_1^* + y^*w_3)/\text{Re}(yw_1^* + y^*w_3),$$

$$s = \sin \theta \text{ and } c = \cos \theta,$$

$$\tan 2\theta = \frac{2|yw_1^* + w_3y^*|}{|w_1|^2 - |w_3|^2}.$$

Majorana phases $\alpha_i$ of $m_i$

$$\alpha_{1,3} = \text{Arg}(w_i(1 \pm \cos 2\theta) + w_2e^{-i2\rho}(1 \mp \cos 2\theta) \pm 2 \sin 2\theta ye^{-i\rho}, \quad \alpha_2 = \text{Arg}(w_2)$$

Translate into standard parameterization

$$s_{12} = \frac{1}{\sqrt{2(1 + cs \cos \rho)^{1/2}}}, \quad s_{23} = \frac{(1 + cs \cos \rho + \sqrt{3}cs \sin \rho)^{1/2}}{\sqrt{2(1 + cs \cos \rho)^{3/2}}},$$

$$s_{13} = \frac{(1 - 2cs \cos \rho)^{1/2}}{\sqrt{3}}.$$

and

$$\sin \delta = (1 + \frac{4c^2s^2 \sin^2 \rho}{(c^2 - s^2)^2})^{-1/2}(1 - \frac{3c^2s^2 \sin^2 \rho}{(1 + cs \cos \rho)^2})^{-1/2} \times \left\{ \begin{array}{ll} -1, & \text{if } c^2 > s^2, \\ +1, & \text{if } s^2 > c^2. \end{array} \right.$$
4. Conclusions

There are a few anomalies in B and lepton flavor physics. Models can be constructed to explain the anomalies. Too many models on the market, MFV provides a good framework for model independent analysis for flavor physics.

Protophobic model explaining anomaly in $^{8}\text{Be}^\ast$ to $^{8}\text{Be}$ $e\,e$ can be realized.

Flavor physics with Higgs is now becoming a precision test for SM. Consistent with SM Higgs sector. But there are rooms for new physics

It is important to get experimental data confirmed. LHCb and BELLE II can provide data to further confirm anomalies in B decays. Experimental measurement of muon g-2, $\mu \rightarrow e\,\gamma$, $\mu$-e conversion, edm ... can provide much needed information about flavor physics in lepton sector.

Most of data can be accommodated by SM. For sure there are physics beyond minimal SM from neutrino mixing data.

Mixing pattern is emerging with $\delta_{\text{CP}} = -\pi/2$ and $\theta_{23}=\pi/4$. Theoretical models have been constructed to realize such mixing pattern.

A lot can be lernt from flavor physics with leptons and Higgs.