Measurement of anisotropic radial flow in relativistic heavy ion collisions

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1. Motivation

**Elliptic flow:**

**Coordinate-Space**

**Momentum-Space**

Azimuthal multiplicity distribution:

\[
\frac{dN}{d\phi} \propto 1 + \sum_{n=1}^{\infty} 2v_n(N) \cos(n(\phi - \psi_r)) \\
\phi = \tan^{-1}\left(\frac{p_y}{p_x}\right)
\]

Define elliptic flow:

\[
v_2(N) = \langle \cos(2(\phi - \psi_r)) \rangle
\]

➢ It measures the anisotropy of azimuthal multiplicity distribution!
Radial expansion

3 velocities:
- Average radial expansion (isotropic) velocity ($v_r$)
- Anisotropic velocity ($v_a$)
- Thermal velocity

Radial expansion + Elliptic flow

Particle mass splitting of differential elliptic flow at low transverse momentum region.


- Radial velocities are input parameters of hydrodynamic calculations!
Generalized Blast-wave parameterization

**Cooper-Frye formula:**

\[
E \frac{d^3N}{d^3p_t} \propto \frac{1}{(2\pi)^3} \int p^\mu d\sigma_u(x) f(x, p)
\]

**Radial flow:**

\[
\rho = \tilde{r}(\rho_0 + \rho_2 \cos(\phi_m))
\]

\[\rho_0: \text{the isotropic radial flow rapidity; } \rho_2: \text{the anisotropic radial flow rapidity}\]

\(\rho_0 \text{ and } \rho_2 \text{ are determined by fitting:}\)

1. Transverse momentum spectrum
2. Elliptic flow

Such extracted parameters are model dependent!
2. Measure of radial expansion

◆ Total transverse momentum in a given azimuthal angle bin:

\[
\langle P_t(\phi_m) \rangle = \frac{1}{N_{\text{event}}} \sum_{j=1}^{N_{\text{event}}} \left( \sum_{i=1}^{N_m} (p_{t,i}^j(\phi_m)) \right) \frac{dP_t}{d\phi}
\]

\(p_{t,i}^j\): transverse momentum of the \(i\)th particle in the \(m\)th angular bin.

\(N_m\): total number of particles in the \(m\)th angular bin.

➢ It contains the information from both kinetic expansion and multiplicity distribution!

◆ Mean transverse momentum in a given azimuthal angle bin:

\[
\langle p_t(\phi_m) \rangle = \frac{1}{N_{\text{event}}} \sum_{j=1}^{N_{\text{event}}} \left( \frac{1}{N_m} \sum_{i=1}^{N_m} (p_{t,i}^j(\phi_m)) \right) \frac{dp_t}{d\phi}
\]

➢ It measures the radial expansion only!
Definitions of various flows:

**Azimuthal multiplicity distribution:**

\[ \frac{dN}{d\phi} \propto 1 + \sum_{n=1}^{\infty} 2v_n(N) \cos(n(\phi - \psi_r)) \]

**Azimuthal total transverse momentum distribution:**

\[ \frac{dP_t}{d\phi} \propto 1 + \sum_{n=1}^{\infty} 2v_n(P_t) \cos(n(\phi - \psi_r)) \]

**Azimuthal mean transverse momentum distribution:**

\[ \frac{dp_t}{d\phi} \propto 1 + \sum_{n=1}^{\infty} 2v_n(p_t) \cos(n(\phi - \psi_r)) \]
Centrality dependence of various anisotropic flows

AMPT with string melting for Au+Au coll. at 200GeV

- They show similar centrality dependence.
- $v_2(\langle \langle p_t \rangle \rangle)$ is the smallest, $v_2(N)$ is in the middle, and $v_2(P_t)$ is largest, as it counts the anisotropy from both multiplicity and transverse momentum distributions.

Azimuthal distribution of mean transverse momentum can measure the radial expansion.
Suggested measurement: azimuthal dis. of mean transverse rapidity

Similarly, we can define the mean transverse rapidity:

\[
\langle y_t(\phi_m) \rangle = \frac{1}{N_{\text{event}}} \sum_{j=1}^{N_{\text{event}}} \left( \frac{1}{N_m} \sum_{i=1}^{N_m} (y^j_{t,i}(\phi_m)) \right)
\]

It relates to the total radial flow rapidity.

AMPT with string melting for Au+Au coll. at 200GeV

\[
y^j_{t,i} = \ln \left( \frac{m^j_{t,i} + p^j_{t,i}}{m_0} \right)
\]

It is well fitted by:

\[
\langle y_t(\phi) \rangle = y_{t0} + y_{t2} \cos(2\phi)
\]

with

\[
y_{t0} = 1.3371 \pm 0.0001
\]

Isotropic mean transverse rapidity: isotropic expansion + thermal motion

\[
y_{t2} = 0.0334 \pm 0.0002
\]

Anisotropic mean transverse rapidity: anisotropic rapidity.
3. The physics of measured $dy_t/d\phi$

**Particle mass dependence of suggested distribution**

AMPT with string melting for Au+Au coll. at 200GeV

Thermal motion: **temperature**

- particle mass

At fixed $T$: **lighter particle,**

- **larger thermal velocity**

<table>
<thead>
<tr>
<th>Mass</th>
<th>Particles</th>
<th>$Y_{t0}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Pions</td>
<td></td>
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<tr>
<td></td>
<td>Kaons</td>
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<tr>
<td></td>
<td>Protons</td>
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</tbody>
</table>

- Their isotropic parts are ordered as expected random thermal motion!
Centrality dependence of suggested distribution:

The distributions is almost azimuthal angle independent in central collisions, but dependent in non-central coll. 

Large anisotropy in mid-central collisions, and small anisotropy in peripheral collisions.

Consistent with the fact that anisotropic expansion appears in non-central collisions, and is the largest in mid-central collisions!
4. Measured $y_{t2}$ and extracted $\rho_2$

- **Extracted $\rho_2$ by blast-wave parameterization:**

  Fitting $p_t$ spectrum of 6 particles

  ![Graphs showing $\pi^+$, $\pi^-$, $K^+$, $K^-$, $p$, and $\bar{p}$ distributions with fits]

  Fitting differential elliptic flow

  ![Graph showing $V_2$ vs $p_T$ for $\pi^+\pi^-$, $K^+K^-$, and $p\bar{p}$ with fits]

  Extracted parameters:

  \[ T = 96.1 \pm 1.0 \]

  \[ \rho_0 = 0.73 \pm 0.01 \]

  \[ \rho_2 = 0.035 \pm 0.003 \]

  Measured: \[ y_{t2} = 0.0334 \pm 0.0002 \]

- Extracted anisotropic rapidity parameter is consistent with measured anisotropic part of mean transverse rapidity!
Centrality dependence of extracted $\rho_2$ and measured $y_{t2}$

AMPT with string melting for Au+Au coll. at 200GeV

- At each of centrality, the extracted anisotropic radial flow parameter is close to that from measured anisotropic part of mean transverse rapidity.
- They show consistent centrality dependence.

Provides a model independent way to get the anisotropic rapidity!
5. Summary

- We suggest the measurements for the azimuthal distribution of mean transverse rapidity.
- It consists of two parts: isotropic, and anisotropic mean transverse rapidity.
  - Isotropic part: isotropic radial expansion + thermal motion
    Consistent with the mass ordering
  - Anisotropic part: anisotropic radial expansion
    Centrality dependence is consistent with extracted anisotropic radial rapidity
- It provides a model independent way to get anisotropic rapidity. It is helpful for hydrodynamic calculations, and a model independent determination of shear viscosity.