Kaon Freeze-out Dynamics
in $\sqrt{s_{NN}}=200$ GeV Au+Au Collisions at RHIC*

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Correlation function of two identical bosons/fermions at small momentum difference $q$ shows effect of quantum statistics.

Height/depth of the B-E/F-D bump $\lambda$ is related to the fraction $(\lambda^{1/2})$ of particles participating in the enhancement.

Its width scales with the emission radius as $R^{-1}$. 
Correlation femtoscopy in a nutshell (2/3)

The correlation is determined by the size of region from which particles with roughly the same velocity are emitted.

⇒ Femtoscopy measures size, shape, and orientation of homogeneity regions.
Kernel $K(q,r)$ is independent of freeze-out conditions
$S(r)$ is often assumed to be Gaussian $\Rightarrow$ HBT radii
Other option: Inversion of linear integral equation to obtain source function
$\Rightarrow$ Model-independent analysis of emission shape (goes beyond Gaussian shape assumption)
Source Imaging

Geometric information from imaging.

$$R(q) = \int K(q,r) S(r) r^2 \, dr$$

**General task:**
From data w/errors, $R(q)$, determine the source $S(r)$.
Requires inversion of the kernel $K$.

**Optical recognition:** $K$ - blurring function, max entropy method

Any determination of source characteristics from data, unaided by reaction theory, is an imaging.
\[ R(q) \equiv C(q) - 1 = 4\pi \int drr^2 K(q,r)S(r) \]

\[ K(q,r) = \frac{1}{2} \int d\cos\theta_{\vec{q},\vec{r}} \left[ \left| \phi(\vec{q},\vec{r}) \right|^2 - 1 \right] \]

Freeze-out occurs after the last scattering. \( \Rightarrow \) Only Coulomb & quantum statistics effects included in the kernel.

**Expand into B-spline basis**

\[ S(r) = \sum S_j \cdot B_j(r) \]

\[ C^{Th}(q_i) = \sum_j K_{ij} \cdot S_j \]

\[ K_{ij} = \int dr \cdot K(q_i,r)B_j(r) \]

**Vary \( S_j \) to minimize \( \chi^2 \)**

\[ \chi^2 = \frac{\left( C^{Expt}(q_i) - \sum_j K_{ij} \cdot S_j \right)^2}{\left( \Delta C^{Expt}(q_i) \right)^2} \]

D. A. Brown, P. Danielewicz: UCRL-MA-147919
Why Kaons?

- Pion source shows a heavy, non-Gaussian tail
- Interpretation is problematic
  Tail attributed to decays of long-lived resonances, non-zero emission duration etc.
- Kaons: cleaner probe
  less contribution from resonances
- PHENIX 1D kaon result shows also a long non-Gaussian tail
The STAR Experiment

- **Time Projection Chamber**
  - ID via energy loss \((dE/dx)\)
  - Momentum \((p)\)

- Full azimuth coverage

- Uniform acceptance
  for different energies and particles

The Solenoidal Tracker At RHIC
Kaon femtoscopy analyses

Au+Au @ $\sqrt{s_{NN}}$=200 GeV
Mid-rapidity $|y|<0.5$

1. Source shape: 20% most central
   Run 4: 4.6 Mevts, Run 7: 16 Mevts

2. $m_T$-dependence: 30% most central
   Run 4: 6.6 Mevts

$0.2<k_T<0.36$ GeV/c

$0.36<k_T<0.48$ GeV/c
1. Source shape analysis
   - $dE/dx$: $n\sigma(Kaon) < 2.0$ and $n\sigma(Pion) > 3.0$ and $n\sigma(electron) > 2.0$
     
     $n\sigma(X)$: deviation of the candidate $dE/dx$ from the normalized distribution of particle type $X$ at a given momentum
   - $0.2 < p_T < 0.4$ GeV/c

2. $m_T$-dependent analysis
   - $-1.5 < n\sigma(Kaon) < 2.0$
     
     $0.2 < k_T < 0.36$ GeV/c
   - $-0.5 < n\sigma(Kaon) < 2.0$
     
     $0.36 < k_T < 0.48$ GeV/c
34M+83M=117M (K⁺K⁺ & K⁻K⁻) pairs

STAR data well described by a single Gaussian. Contrary to PHENIX no non-gaussian tails observed. May be due to a different k_T-range: STAR bin is 4x narrower.
Expansion of $R(q)$ and $S(r)$ in Cartesian Harmonic basis


$$R(q) = \sum_l \sum_{\alpha_1 \ldots \alpha_l} R^l_{\alpha_1 \ldots \alpha_l}(q) A^l_{\alpha_1 \ldots \alpha_l}(\Omega_q) \quad (1)$$

$$S(r) = \sum_l \sum_{\alpha_1 \ldots \alpha_l} S^l_{\alpha_1 \ldots \alpha_l}(r) A^l_{\alpha_1 \ldots \alpha_l}(\Omega_q) \quad (2)$$

$\alpha_i = x, y$ or $z$

$x = \text{out-direction}$

$y = \text{side-direction}$

$z = \text{long-direction}$

3D Koonin-Pratt:

Plug (1) and (2) into (3) \(\Rightarrow\)

$$R(q) = C(q) - 1 = 4\pi \int d^3K(q, r)S(r) \quad (3)$$

$$R^l_{\alpha_1 \ldots \alpha_l}(q) = 4\pi \int d^3K_l(q, r)S^l_{\alpha_1 \ldots \alpha_l}(r) \quad (4)$$

Invert (1) \(\Rightarrow\)

$$R^l_{\alpha_1 \ldots \alpha_l}(q) = \frac{(2l + 1)!!}{l!} \int \frac{d\Omega_q}{4\pi} A^l_{\alpha_1 \ldots \alpha_l}(\Omega_q) R(q)$$

Invert (2) \(\Rightarrow\)

$$S^l_{\alpha_1 \ldots \alpha_l} = \frac{(2l + 1)!!}{l!} \int \frac{d\Omega_q}{4\pi} A^l_{\alpha_1 \ldots \alpha_l}(\Omega_q) S(q)$$
**Shape analysis**

- \( \ell=0 \) moment agrees 1D \( C(q) \)
  - Higher moments relatively small
- Trial function form for \( S(r) \):
  - 4-parameter ellipsoid (3D Gauss)
  
\[
S^G(x, y, z) = \frac{\lambda}{(2\sqrt{\pi})^3 r_x r_y r_z} \exp \left[ -\left( \frac{x^2}{4r_x^2} + \frac{y^2}{4r_y^2} + \frac{z^2}{4r_z^2} \right) \right]
\]
- Fit to \( C(q) \): technically a simultaneous fit on 6 independent moments
  
\[
R_\alpha^\ell, \ 0 \leq \ell \leq 4
\]
- Result: statistically good fit

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**Run4+Run7**

200 GeV Au+Au

Centrality < 20%

0.2 < \( k_T \) < 0.36 GeV/c

\[
\begin{align*}
\lambda &= 0.48 \pm 0.01 \\
r_x &= (4.8 \pm 0.1) \text{ fm} \\
r_y &= (4.3 \pm 0.1) \text{ fm} \\
r_z &= (4.7 \pm 0.1) \text{ fm}
\end{align*}
\]
Correlation profiles and source

\[ C(q_x) = C(q_x,0,0) \]
\[ C(q_y) = C(0,q_y,0) \]
\[ C(q_z) = C(0,0,q_z) \]

\[ S(r_x) = S_{0}A_{0} + S_{2}A_{2}^{2} + S_{4}A_{4}^{4} \]
\[ S(r_y) = S_{0}A_{0} + S_{2}A_{2}^{2} + S_{4}A_{4}^{4} \]
\[ S(r_z) = S_{0}A_{0} + S_{2}A_{2}^{2} + S_{4}A_{4}^{4} \]

Gaussian source fit with error band

*N.B.:* Low statistics shows up as systematic uncertainty on shape assumption
Source: Data comparison

kaon vs. pion: different shape

- Long pion tail caused by resonances and/or emission duration?
- Sign of different freeze-out dynamics?
Source: Model comparison

Therminator

- Blast-wave model (STAR tune):
  - Expansion: \( v_t(\rho) = (\rho/\rho_{\text{max}})/(\rho/\rho_{\text{max}} + v_t) \)
  - Freeze-out occurs at \( \tau = \tau_0 + \alpha \rho \).
  - Finite emission duration \( \Delta \tau \)
- Kaons: Instant freeze-out
  (\( \Delta \tau = 0 \), compare to \( \Delta \tau \sim 2 \text{ fm}/c \) of pions) at \( \tau_0 = 0.8 \text{ fm}/c \)
- Resonances are needed for proper description

Hydrokinetic model

- Hybrid model
  - Glauber initial+Hydro+UrQMD
- Consistent in “side”
- Slightly more tail (\( r > 15 \text{ fm} \)) in “out” and “long”

\[ (a) S^0 A^0 + S^2_{xx} A^2_{xx} + S^4_{xxxx} A^4_{xxxx} \]
\[ (b) S^0 A^0 + S^2_{yy} A^2_{yy} + S^4_{yyyy} A^4_{yyyy} \]
\[ (c) S^0 A^0 + S^2_{zz} A^2_{zz} + S^4_{zzzz} A^4_{zzzz} \]

\( \text{Au+Au } \sqrt{s} = 200 \text{ AGeV} \)
\( 0 < \text{centrality} < 20 \% \)

PHENIX pions
\( S(r_x) \text{ (fm}^{-3}\) \)
\( S(r_y) \text{ (fm}^{-3}\) \)
\( S(r_z) \text{ (fm}^{-3}\) \)

\( \text{STAR kaons } \)
3D Gaussian source fit
\( 0.20 < k_T < 0.36 \text{ GeV}/c \)
\( -0.5 < y < 0.5 \)

\( a = 0, \rho_{\text{max}} = 9.0 \text{ fm} \)
\( \tau_0 = 8.0 \text{ fm}/c, \Delta \tau = 0 \)


HKM: PRC81, 054903 (2010)
data from Shapoval, Sinyukov, private communication
Excellent description of PHENIX pion data (PRL 93:152302, 2004) using exact solutions of perfect fluid hydrodynamics (Buda-Lund). Ideal hydro has inherent $m_T$-scaling $\Rightarrow$ predicts kaon radii $m_T$-dependence.
SPS results on pions and kaons

- “The kaon radii are fully consistent with pions and the hydrodynamic expansion model.”

- “Pions and kaons seem to decouple simultaneously.”

Radii: rising trend at low $m_T$
- Strongest in “long”

Buda-Lund model
- Perfect hydrodynamics, inherent $m_T$-scaling
- Works perfectly for pions
- Deviates from kaons in the “long” direction in the lowest $m_T$ bin

HKM (Hydro-kinetic model)
- Describes all trends
- Some deviation in the “out” direction
- Note the different centrality definition

HKM: PRC81, 054903 (2010)
Summary

- First model-independent extraction of kaon 3D source shape presented
- No significant non-Gaussian tail is observed in RHIC $\sqrt{s_{NN}}=200$ GeV central Au+Au data
- Model comparison indicates that kaons and pions may be subject to different dynamics
- The $m_T$-dependence of the Gaussian radii indicates that $m_T$-scaling is broken in the “long” direction
Thank You!

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STAR Collaboration
3D pions, PHENIX and STAR

Elongated source in “out” direction
Therminator Blast Wave model suggests non-zero emission duration

Very good agreement of PHENIX and STAR 3D pion source images
Fit to correlation moments

Dataset #2
Run4 Cent<30%

0.2<kT<0.36 GeV/c

(a) $R^0_{Au+Au}$

\[ \chi^2_{ndf} = 316/283 = 1.1 \]

(b) $R^2_{x2}$

(c) $R^2_{y2}$

(d) $R^4_{x2y}$

(e) $R^4_{x4}$

(f) $R^4_{y4}$

0.36<kT<0.48 GeV/c

(a) $R^0_{Au+Au}$

\[ \chi^2_{ndf} = 363/283 = 1.3 \]

(b) $R^2_{x2}$

(c) $R^2_{y2}$

(d) $R^4_{x2y}$

(e) $R^4_{x4}$

(f) $R^4_{y4}$
Source parameters

<table>
<thead>
<tr>
<th>Year</th>
<th>2004+2007</th>
<th>2004</th>
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<tbody>
<tr>
<td>Centrality</td>
<td>0–20%</td>
<td>0–30%</td>
</tr>
<tr>
<td>$k_T$ [GeV/c]</td>
<td>0.2–0.36</td>
<td>0.2–0.36</td>
</tr>
<tr>
<td>$R_x$ [fm]</td>
<td>4.8±0.1±0.2</td>
<td>4.3±0.1±0.4</td>
</tr>
<tr>
<td>$R_y$ [fm]</td>
<td>4.3±0.1±0.1</td>
<td>4.0±0.1±0.3</td>
</tr>
<tr>
<td>$R_z$ [fm]</td>
<td>4.7±0.1±0.2</td>
<td>4.3±0.2±0.4</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.49±0.02±0.05</td>
<td>0.39±0.01±0.09</td>
</tr>
<tr>
<td>$\chi^2/ndf$</td>
<td>497/289</td>
<td>316/283</td>
</tr>
</tbody>
</table>

TABLE I. Parameters obtained from the 3-D Gaussian source function fits for the different datasets. The first errors are statistical, the second errors are systematic.
Cartesian harmonics basis

- Based on the products of unit vector components, $n_{\alpha 1} n_{\alpha 2}$, ..., $n_{\alpha \ell}$. Unlike the spherical harmonics they are real.
- Due to the normalization identity $n^2_x + n^2_y + n^2_z = 1$, at a given $\ell \geq 2$, the different component products are not linearly independent as functions of spherical angle.
- At a given $\ell$, the products are spanned by spherical harmonics of rank $\ell' \leq \ell$, with $\ell'$ of the same evenness as $\ell$.

| $A^{(1)}_x$ | $A^{(3)}_{xyz}$ |
| $A^{(2)}_{xx}$ | $A^{(4)}_{xxxx}$ |
| $A^{(2)}_{xy}$ | $A^{(4)}_{xxyy}$ |
| $A^{(3)}_{xxx}$ | $A^{(4)}_{xxyz}$ |
| $A^{(3)}_{xyy}$ |

$A^{(1)}_x = n_x$

$A^{(2)}_{xx} = n_x^2 - 1/3$

$A^{(2)}_{xy} = n_x n_y$

$A^{(3)}_{xxx} = n_x^3 - (3/5)n_x$

$A^{(3)}_{xyy} = n_x^2 n_y - (1/5)n_y$

$A^{(3)}_{xyz} = n_x n_y n_z$

$A^{(4)}_{xxxx} = n_x^4 - (6/7)n_x^2 + 3/35$

$A^{(4)}_{xxyy} = n_x^3 n_y - (3/7)n_x n_y$

$A^{(4)}_{xxyz} = n_x^2 n_y n_z - (1/7)n_y n_z$
Spherical Harmonics basis

\[ R_{\ell m}(q) = (4\pi)^{-1/2} \int d\Omega_q Y^*_{\ell m}(\Omega_q) R(q), \]
\[ S_{\ell m}(r) = (4\pi)^{-1/2} \int d\Omega_r Y^*_{\ell m}(\Omega_r) S(r). \]

- Disadvantage: connection between the geometric features of the real source function \( S(r) \) and the complex valued projections \( S_{\ell m}(r) \) is not transparent.

- \( Y_{\ell m} \) harmonics are convenient for analyzing quantum angular momentum, but are clumsy for expressing anisotropies of real-valued functions.