Fast Crystal Scintillators for GHz Hard X-Ray Imaging

Chen Hu, Fan Yang, Liyuan Zhang, Ren-Yuan Zhu,
California Institute of Technology
Aiping Chen, Zhehui Wang,
Los Alamos National Laboratory
Lei Ying and Zongfu Yu
University of Wisconsin

September 13, 2018

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High-Energy and Ultrafast X-Ray Imaging Technologies and Applications

Organizers: Peter Denes, Sol Gruner, Michael Stevens & Zhehui (Jeff) Wang

(Location/Time: Santa Fe, NM, USA /Aug 2-3, 2016)

The goals of this workshop are to gather the leading experts in the related fields, to prioritize tasks for ultrafast hard X-ray imaging detector technology development and applications in the next 5 to 10 years, see Table 1, and to establish the foundations for near-term R&D collaborations.

Table I. High-energy photon imagers for MaRIE XFEL

<table>
<thead>
<tr>
<th>Performance</th>
<th>Type I imager</th>
<th>Type II imager</th>
</tr>
</thead>
<tbody>
<tr>
<td>X-ray energy</td>
<td>30 keV</td>
<td>42-126 keV</td>
</tr>
<tr>
<td>Frame-rate/inter-frame time</td>
<td>0.5 GHz/2 ns</td>
<td>3 GHz / 300 ps</td>
</tr>
<tr>
<td>Number of frames</td>
<td>10</td>
<td>10 - 30</td>
</tr>
<tr>
<td>X-ray detection efficiency</td>
<td>above 50%</td>
<td>above 80%</td>
</tr>
<tr>
<td>Pixel size/pitch</td>
<td>≤ 300 μm</td>
<td>&lt; 300 μm</td>
</tr>
<tr>
<td>Dynamic range</td>
<td>$10^3$ X-ray photons</td>
<td>$\geq 10^4$ X-ray photons</td>
</tr>
<tr>
<td>Pixel format</td>
<td>64 x 64 (scalable to 1 Mpix)</td>
<td>1 Mpix</td>
</tr>
</tbody>
</table>

2 ns and 300 ps inter-frame time requires very fast scintillator and sensor.
Why Crystal Scintillator?

- Detection efficiency for hard X-ray requires bulk detector.
- Scintillation light provides fast signal.
- Pixelized crystal detector is a standard for medical industry.

A detector concept:
- Pixelized fast scintillator screen;
- Pixelized fast photodetector;
- Fast electronics readout.

Challenges:
Ultra-fast crystals, photodetectors and readout.
Pixelized Crystal Detectors

Crystal panels of 300 µ pitch may be fabricated by classical mechanical processing

1 mm BGO Pixels for PET

CsI(Tl) panel of 30 x 40 X 1 cm with 0.3 mm pixels

Laser slicing and not pixelized may provide better coverage
# Candidate Scintillators for Marie

<table>
<thead>
<tr>
<th></th>
<th>LYSO (:Ce)</th>
<th>YSO:Ce</th>
<th>ZnO:Ga</th>
<th>BaF₂</th>
<th>BaF₂:Y</th>
<th>YAP:Ce</th>
<th>YAP:Yb</th>
<th>YAG:Yb</th>
<th>LuAG:Ce</th>
<th>LaBr₃ (:Ce)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density (g/cm³)</td>
<td>7.4</td>
<td>4.44</td>
<td>5.67</td>
<td>4.89</td>
<td>4.89</td>
<td>5.35</td>
<td>5.35</td>
<td>4.56</td>
<td>6.76</td>
<td>5.29</td>
</tr>
<tr>
<td>Melting points (°C)</td>
<td>2050</td>
<td>2070</td>
<td>1975</td>
<td>1280</td>
<td>1280</td>
<td>1870</td>
<td>1870</td>
<td>1940</td>
<td>2060</td>
<td>783</td>
</tr>
<tr>
<td>X₀ (cm)</td>
<td>1.14</td>
<td>3.10</td>
<td>2.51</td>
<td>2.03</td>
<td>2.03</td>
<td>2.77</td>
<td>2.77</td>
<td>3.53</td>
<td>1.45</td>
<td>1.88</td>
</tr>
<tr>
<td>Rₘ (cm)</td>
<td>2.07</td>
<td>2.93</td>
<td>2.28</td>
<td>3.1</td>
<td>3.1</td>
<td>2.4</td>
<td>2.4</td>
<td>2.76</td>
<td>2.15</td>
<td>2.85</td>
</tr>
<tr>
<td>λ₁ (cm)</td>
<td>20.9</td>
<td>27.8</td>
<td>22.2</td>
<td>30.7</td>
<td>30.7</td>
<td>22.4</td>
<td>22.4</td>
<td>25.2</td>
<td>20.6</td>
<td>30.4</td>
</tr>
<tr>
<td>Zₑff</td>
<td>64.8</td>
<td>33.3</td>
<td>27.7</td>
<td>51.6</td>
<td>51.6</td>
<td>31.9</td>
<td>31.9</td>
<td>30</td>
<td>60.3</td>
<td>45.6</td>
</tr>
<tr>
<td>dE/dX (MeV/cm)</td>
<td>9.55</td>
<td>6.57</td>
<td>8.42</td>
<td>6.52</td>
<td>6.52</td>
<td>8.05</td>
<td>8.05</td>
<td>7.01</td>
<td>9.22</td>
<td>6.90</td>
</tr>
<tr>
<td>λₚₑᵃ (nm)</td>
<td>420</td>
<td>420</td>
<td>389</td>
<td>300</td>
<td>220</td>
<td>300</td>
<td>220</td>
<td>370</td>
<td>350</td>
<td>350</td>
</tr>
<tr>
<td>Refractive Indexᵇ</td>
<td>1.82</td>
<td>1.78</td>
<td>2.1</td>
<td>1.5</td>
<td>1.5</td>
<td>1.96</td>
<td>1.96</td>
<td>1.87</td>
<td>1.84</td>
<td>1.9</td>
</tr>
<tr>
<td>Normalized Light Yieldᵃ,ᶜ</td>
<td>100</td>
<td>80</td>
<td>6.6ᵉ</td>
<td>42</td>
<td>4.8</td>
<td>1.7</td>
<td>4.8</td>
<td>9</td>
<td>32</td>
<td>0.19ᵉ</td>
</tr>
<tr>
<td>Total Light yield (ph/MeV)</td>
<td>30,000</td>
<td>24,000</td>
<td>2,000ᵉ</td>
<td>13,000</td>
<td>2,000</td>
<td>12,000</td>
<td>57ᵉ</td>
<td>110ᵉ</td>
<td>25,000ᶠ</td>
<td>46,000</td>
</tr>
<tr>
<td>Decay timeᵃ (ns)</td>
<td>40</td>
<td>75</td>
<td>&lt;1</td>
<td>600</td>
<td>0.6</td>
<td>600</td>
<td>0.6</td>
<td>191</td>
<td>25</td>
<td>1.5</td>
</tr>
<tr>
<td>Light Yield in 1ˢᵗ ns (photons/MeV)</td>
<td>740</td>
<td>318</td>
<td>610ᵉ</td>
<td>1200</td>
<td>1200</td>
<td>391</td>
<td>28ᵉ</td>
<td>24ᵉ</td>
<td>240</td>
<td>2,200</td>
</tr>
<tr>
<td>40 keV Att. Length (1/e, mm)</td>
<td>0.185</td>
<td>0.334</td>
<td>0.407</td>
<td>0.106</td>
<td>0.106</td>
<td>0.314</td>
<td>0.314</td>
<td>0.439</td>
<td>0.251</td>
<td>0.131</td>
</tr>
</tbody>
</table>


*a.* Top line: slow component, bottom line: fast component;

*b.* At the wavelength of the emission maximum;

*c.* Excited by Gamma rays;

*d.* For 0.4 at% Ca co-doping;

*e.* Excited by Alpha particles.

*f.* Ceramic with 0.3 Mg at% co-doping
LYSO and ZnO:Ga Samples

<table>
<thead>
<tr>
<th>Crystal</th>
<th>Vendor</th>
<th>ID</th>
<th>Dimension (mm³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LYSO:Ce</td>
<td>SIC</td>
<td>150210-1</td>
<td>19x19x2</td>
</tr>
<tr>
<td>YSO:Ce</td>
<td>SIC</td>
<td>51</td>
<td>25x25x5</td>
</tr>
<tr>
<td>ZnO:Ga</td>
<td>FJIRSM</td>
<td>2014-1</td>
<td>33x30x2</td>
</tr>
<tr>
<td>ZnO:Ga</td>
<td>FJIRSM</td>
<td>2014-2</td>
<td>22x22x0.3</td>
</tr>
</tbody>
</table>

Experiments

- Properties measured at room temperature: PL & Decay, Transmittance, PHS, LO & Decay kinetics
**SIC LYSO:Ce-150210-1**

- **✓ High LO, good transmittance and ER, short decay time**
- **✗ Decay time too long for X-ray frame rate of a few ns**

<table>
<thead>
<tr>
<th>ID</th>
<th>Dimension</th>
<th>EWLT (%)</th>
<th>ER (%)</th>
<th>200 ns LO (p.e./MeV)</th>
<th>Primary Decay Time (ns)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SIC LYSO-150210-1</td>
<td>19×19×2</td>
<td>80.1</td>
<td>7.1</td>
<td>4841</td>
<td>41</td>
</tr>
</tbody>
</table>

- Transmittance (%) vs Wavelength (nm)
- Number of Events vs Channel Number
- Light Output (p.e./MeV) vs Time (ns)

PMT: R1306, HV: -850, Gate = 200 ns
Grease, Tyvek wrapping

LO = 4841 p.e./MeV
ER = 7.1%

L.O = A₀ + A₁ (1 - e^(-t/τ))

A₀: 0
A₁: 4848
τ: 41 ns

September 13, 2018
Presentation by Liyuan Zhang, Caltech, in the ULTIMA 2018 Conference at ANL
SIC YSO:Ce-51 (in LANL)

- Good LO, transmittance, ER, and short decay time

- All these performance are inferior to LYSO:Ce

<table>
<thead>
<tr>
<th>ID</th>
<th>Dimension</th>
<th>EWLT (%)</th>
<th>ER (%)</th>
<th>500 ns LO (p.e./MeV)</th>
<th>Primary Decay Time (ns)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SIC YSO-51</td>
<td>25×25×5</td>
<td>72.6</td>
<td>8.1</td>
<td>3906</td>
<td>75</td>
</tr>
</tbody>
</table>
FJIRSM ZnO:Ga-2014-1

✓ Very short decay time

× Low EWLT and LO due to severe self absorption

<table>
<thead>
<tr>
<th>ID</th>
<th>Dimension</th>
<th>EWLT (%)</th>
<th>ER (%)</th>
<th>50 ns LO (p.e./MeV)</th>
<th>Primary Decay Time (ns)</th>
</tr>
</thead>
<tbody>
<tr>
<td>FJIRSM ZnO:Ga-2014-1</td>
<td>33×30×2</td>
<td>7.0</td>
<td>37.8</td>
<td>76 (α)</td>
<td>2.7</td>
</tr>
</tbody>
</table>
FJIRSM ZnO:Ga-2014-2

- Reduced self absorption due to 0.3 mm thickness
- May pursue QD, NP or thin film based solution

<table>
<thead>
<tr>
<th>ID</th>
<th>Dimension</th>
<th>EWLT (%)</th>
<th>ER (%)</th>
<th>50 ns LO (p.e./MeV)</th>
<th>Primary Decay Time (ns)</th>
</tr>
</thead>
<tbody>
<tr>
<td>FJIRSM ZnO:Ga-2014-2</td>
<td>22×22×0.3</td>
<td>10.8</td>
<td>18.2</td>
<td>296 (α)</td>
<td>3.5</td>
</tr>
</tbody>
</table>
BaF$_2$ and Other Samples

<table>
<thead>
<tr>
<th>Crystal</th>
<th>Vendor</th>
<th>ID</th>
<th>Dimension (mm$^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>BaF$_2$</td>
<td>SIC</td>
<td>1</td>
<td>50×50×5</td>
</tr>
<tr>
<td>BaF$_2$:Y</td>
<td>BGRI</td>
<td>1708</td>
<td>10×10×2</td>
</tr>
<tr>
<td>YAP:Ce</td>
<td>Dongjun</td>
<td>2102</td>
<td>Φ50×2</td>
</tr>
<tr>
<td>YAP:Yb</td>
<td>Dongjun</td>
<td>2-2</td>
<td>Φ40×2</td>
</tr>
<tr>
<td>YAG:Yb</td>
<td>Dongjun</td>
<td>4</td>
<td>10×10×5</td>
</tr>
<tr>
<td>LuAG:Ce</td>
<td>SIC</td>
<td>S2</td>
<td>25×25×0.4</td>
</tr>
</tbody>
</table>

Experiments

- Properties measured at room temperature: PL & Decay, Transmittance, PHS, LO & Decay kinetics
SIC BaF$_2$-1

✓ The highest LY in 1st ns among all non-hygroscopic scintillators

× ~600 ns slow component may be suppressed by Y doping

<table>
<thead>
<tr>
<th>ID</th>
<th>Dimension</th>
<th>EWLT (%)</th>
<th>ER (%)</th>
<th>50 ns LO (p.e./MeV)</th>
<th>Primary Decay Time (ns)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SIC BaF$_2$-1</td>
<td>50×50×5</td>
<td>85.1</td>
<td>54.9</td>
<td>209</td>
<td>0.6</td>
</tr>
</tbody>
</table>

**Figure:**
- Transmittance (%)
- Number of Events
- Light Output (p.e./MeV) vs Time (ns)

**Notes:**
- PMT: XP2554B, HV = -1800 V, Gate = 50 ns
- Na-22 source, Coincidence Trigger
- EL = 209 p.e./MeV, ER = 54.9%

Presentation by Liyuan Zhang, Caltech, in the ULITIMA 2018 Conference at ANL.
Yttrium Doping in BaF$_2$

While the fast component in BaF$_2$ keeps more or less the same, the slow component is significantly suppressed by Yttrium doping.
BGRI Y Doped/Undoped BaF$_2$

Fast/Slow ratio increased from 0.20 to 3.2
γ-ray induced damage in BaF$_2$

- BaF$_2$ crystals of 25 cm long were irradiated by Co-60 at Caltech and JPL.
- 40% fast scintillation light remains after 120 Mrad ionization dose at JPL.
Proton induced damage in BaF$_2$

- BaF$_2$ plates of 5 mm thick were irradiated by 800 MeV at LANL in 2016.
- 90% fast scintillation light remains after $10^{15}$ p/cm$^2$.

Neutron induced damage in BaF$_2$

- BaF$_2$ plates of 5 mm thick were irradiated by neutrons at LANL in 2016.
- 75% fast scintillation light remains after $3 \times 10^{15}$ n (>1 MeV)/cm$^2$.

To be published in the proceedings of CALOR2018
DJ YAP:Ce-2102

✓ Adequate LO and ER

♥ Self absorption and slow component

<table>
<thead>
<tr>
<th>ID</th>
<th>Dimension</th>
<th>EWLT (%)</th>
<th>ER (%)</th>
<th>200 ns LO (p.e./MeV)</th>
<th>Primary Decay Time (ns)</th>
</tr>
</thead>
<tbody>
<tr>
<td>DJ YAP:Ce-2102</td>
<td>Φ50×2</td>
<td>54.7</td>
<td>19.6</td>
<td>1611</td>
<td>25</td>
</tr>
</tbody>
</table>

- **Dimensions**: DJ YAP:Ce-2102  Φ 50×2 mm³
- **Light Output**: LO = A₁(1-e^{(-t/τ₁)}) + A₂(1-e^{(-t/τ₂)})
- **Channels**: LO = 1611 p.e./MeV
- **Efficiency**: E.R. = 19.6%
- **Components**: PMT:R1306, HV=-1000 V, Gate = 1000 ns
- **Source**: Na-22 Source, Coincidence Trigger
- **Net peak**: 498
- **Time**: t = τ₁ + τ₂
- **Values**: A₁ = 1341, τ₁ = 25, A₂ = 376, τ₂ = 191
DJ YAP:Yb-2-2

- **Very short decay time**
- **Low LO due to thermal quenching**

<table>
<thead>
<tr>
<th>ID</th>
<th>Dimension</th>
<th>EWLT (%)</th>
<th>ER (%)</th>
<th>50 ns LO (p.e./MeV)</th>
<th>Primary Decay Time (ns)</th>
</tr>
</thead>
<tbody>
<tr>
<td>DJ YAP:Yb-2-2</td>
<td>Φ40×2</td>
<td>77.7</td>
<td>41</td>
<td>9.1 (α)</td>
<td>1.5</td>
</tr>
</tbody>
</table>

- **Theoretical limit of transmittance:**
  - Measured transmittance:
  - EWLT=77.7%

- **Number of Events**
  - YAP:Yb 15%  DJ2-2 Φ 40×2 mm³
  - PMT:R2059, HV = -2300 V, Gate = 50 ns
  - Am 241 source, 5.03 MeV Alpha particles excited
  - Ped=82
  - Net peak=392
  - LO = 9.1 p.e./MeV
  - ER = 41%

- **Pulse Height (V)**
  - PH = A₀ + A₁e^(-t/τ)
  - A₀, A₁, τ values provided:
    - A₀ = 0
    - A₁ = 3.2
    - τ = 1.5
**DJ YAG:Yb-4**

- **Very short decay time and good transmittance**
- **Low LO due to thermal quenching**

<table>
<thead>
<tr>
<th>ID</th>
<th>Dimension</th>
<th>EWLT (%)</th>
<th>ER (%)</th>
<th>50 ns LO (p.e./MeV)</th>
<th>Primary Decay Time (ns)</th>
</tr>
</thead>
<tbody>
<tr>
<td>DJ YAG:Yb-4</td>
<td>10×10×5</td>
<td>83.1</td>
<td>22.4</td>
<td>28.4 (α)</td>
<td>3.6</td>
</tr>
</tbody>
</table>

**Graphs and Data**

- **Transmittance vs. Wavelength**
  - DJ YAG:Yb-4 10×10×5 mm³
  - EWLT=83.1%
  - Theoretical limit of transmittance

- **Number of Events vs. Channel Number**
  - YAG:Yb 13% DJ-4 10×10×5 mm³
  - PMT:R2059, HV = -2100 V, Gate = 50 ns
  - Am-241 source, 5.03 MeV Alpha particles excited
  - Ped=82
  - Net peak=583
  - LO = 28.4 p.e./MeV
  - ER = 22.4%

- **Pulse Height vs. Time**
  - YAG:Yb DJ-4 10×10×5 mm³
  - PMT:R2059, HV=-2200 V
  - Readout by DSO Agilent 9254
  - PH = A₀ + A₁e^{(-t/τ)}
  - A₀ = 0
  - A₁ = 3.8
  - τ = 3.6 ns

*September 13, 2018 Presentation by Liyuan Zhang, Caltech, in the ULITIMA 2018 Conference at ANL*
SIC LuAG:Ce-S2 Ceramics

- Good LO and ER, and short decay time
- ~ 1 μs slow component

<table>
<thead>
<tr>
<th>ID</th>
<th>Dimension</th>
<th>EWLT (%)</th>
<th>ER (%)</th>
<th>200 ns LO (p.e./MeV)</th>
<th>Primary Decay Time (ns)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SIC LuAG-S2</td>
<td>25×25×0.4</td>
<td>52.3</td>
<td>19.1</td>
<td>1531</td>
<td>51</td>
</tr>
</tbody>
</table>

September 13, 2018 Presentation by Liyuan Zhang, Caltech, in the ULTIMA 2018 Conference at ANL
A multilayer high QE photocathode coated thin fast scintillators concept was proposed for GHz hard X-ray imaging:

- Spatial resolution determined layer thickness,
- Overall efficiency defined layer number,
- Maximized conversion of scintillation photon to p.e.,
- Magnetic field extraction of p.e. and image preserving,
- Off-beam p.e. multiplication,
- On-board charge storages.

**Figure 6.** A multi-layer detector architecture for efficient and fast imaging of diffracted X rays. A guide magnetic field perpendicular to the X-ray direction guide the photoelectrons to amplification and storage. The magnetic field also preserves the image contrast due to X-ray absorption at the scintillator location.
Purcell Factor for Ag Particles

$R = \infty$ is equivalent to a infinite layer. Our simulation is greatly agree with experiments (red circles) as right figure. Agreement includes the peak value, wavelength and bandwidth.

Experimental Proposal

ZnO

Metal

SiO2

Background $\varepsilon_b \approx 2.17$ for PMMA
In recent study\cite{1}, \textit{S. Oktyabrsky, et al. report} an \textbf{ultrafast, no self-absorption, high-efficient room-temperature semiconductor scintillator} based on InAs QDs embedded in a GaAs matrix.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>BaF$_2$</th>
<th>LYSO</th>
<th>GaAs/InAs QDs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density (g/cm$^3$)</td>
<td>4.89</td>
<td>7.1</td>
<td>5.32</td>
</tr>
<tr>
<td>Radiation length, cm</td>
<td>2.03</td>
<td>1.1</td>
<td>2.3</td>
</tr>
<tr>
<td>Decay constant, ns</td>
<td>0.8 ns</td>
<td>40</td>
<td>1</td>
</tr>
<tr>
<td>Peak emission, nm</td>
<td>195; 220</td>
<td>428</td>
<td>1050</td>
</tr>
<tr>
<td>Photon Yield</td>
<td>1,400</td>
<td>34,000</td>
<td>240,000</td>
</tr>
<tr>
<td>(photons/MeV)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Time between first photons, for 1MeV</td>
<td>0.57ps</td>
<td>1.2 ps</td>
<td>2 fs</td>
</tr>
<tr>
<td>Poisson-limited energy resolution at 1MeV (keV) *</td>
<td>62</td>
<td>13</td>
<td>4.8</td>
</tr>
<tr>
<td>Radiation hardness, Gy</td>
<td>$10^4$-$10^5$</td>
<td>$10^4$-$10^5$</td>
<td>$&gt;10^4$</td>
</tr>
<tr>
<td>Coupling efficiency</td>
<td>&lt;50%</td>
<td>&lt;50%</td>
<td>~100%</td>
</tr>
</tbody>
</table>

\textit{Ref.: \cite{1} S. Oktyabrsky, et al., IEEE Trans. Nucl. Sci. 63, 656 (2016).}

\textbf{Room temperature photocurrent spectra overlapped with PL spectra of the same QD structure with reduced wetting layer placed in a p-n junction.}
It was reported that nanocrystals of cesium lead halide perovskites (CsPbX$_3$, $X = \text{Cl, Br, and I}$) shows bright emission with a tunable range by quantum size effects.

Figure 2. Colloidal perovskite CsPbX$_3$ NCs ($X = \text{Cl, Br, I}$) exhibit size- and composition-tunable bandgap energies covering the entire visible spectral region with narrow and bright emission: (a) colloidal solutions in toluene under UV lamp ($\lambda = 365$ nm); (b) representative PL spectra ($\lambda_{\text{exc}} = 400$ nm for all but 350 nm for CsPbCl$_3$ samples); (c) typical optical absorption and PL spectra; (d) time-resolved PL decays for all samples shown in (c) except CsPbCl$_3$.

Nano Lett. 2015, 15, 3692−3696. DOI: 10.1021/nl5048779
GHz hard X-ray imaging for the proposed Marie project presents an unprecedented challenge to the speed and radiation hardness of inorganic scintillators.

BaF$_2$ crystals provide sufficient fast light with sub-ns decay time and excellent radiation hardness beyond 100 Mrad and $1 \times 10^{15}$ h/cm$^2$. With its slow component effectively suppressed by yttrium doping BaF$_2$:Y promises a fast and robust front imager.

Bulk ZnO:Ga crystals suffer from serious self-absorption. Enhanced UV emission in Ag/Au ZnO core-shell nano particles hints a thin film based approach.

Our plan is to investigate along both lines: BaF$_2$:Y crystals, and ZnO QD/NP based thin film for the Marie project with a close collaboration between the NP, HEP and material science communities.

Acknowledgements: DOE Award DE-SC001192
Purcell effect for enhancing ZnO luminescence
(Theoretical framework)

Total field:

\[
\langle \bar{E}_m(r) \rangle = E^L_m(r) + \frac{\omega^2}{\varepsilon_0 c^2} \mathcal{G}(r, r_0; \omega) \cdot \mu \langle S \rangle
\]

Dipole moment: \( \langle \hat{S} \rangle = \frac{-\Omega [2\Delta - i\gamma_m]}{4\Delta^2 + 2|\Omega|^2 + \gamma_m^2} \)

Rabi frequency: \( \Omega = 2\mu \cdot E^L_m(r_0) \)

\( \Delta \) is detuning
\( \mathcal{G}(r, r_0; \omega) \) is Dyadic Green’s function

NOTE: dipole is considered as a point in theory. In experiment, ZnO is the dipole.

In numerical calculation, the dyadic Green’s function is the kernel.

\[
\gamma_m = 2\text{Im}\left[\mu \cdot E_\mu(r_0)\right] = 2\text{Im}\left[\mu \cdot \mathcal{G}(r, r_0; \omega) \cdot \mu\right]
\]

\( \gamma_m/\gamma_0 \) is Purcell enhancement factor
The Purcell factor is sensitive to the distance between ZnO and metallic nanoparticle. The bandwidth of Purcell factor is stable for a variety of distance D.
The spatial resolution Vs thickness of thin scintillator

Z. Wang et al. has proposed the following equation to determine the maximum thickness of scintillator for a scintillator camera:

\[ R_{\text{spatial}} = 2d \sin \theta_c = 2d \sin \frac{1}{n} \]

where \( R_{\text{spatial}} \) is the spatial resolution, \( d \) is the thickness of scintillator, \( \theta_c \) is the reflective index of scintillator at emission peak.

- The spatial resolution \( (R_{\text{spatial}}) \) of a scintillator camera is limited by the scintillator thickness \( (d) \); For an air-to-scintillator interface, the total internal reflection angle \( (\theta_c) \) is \( \sin(1/n) \) for a flat interface;
- \( R_{\text{spatial}} \) is limited to \( 2d \sin \theta_c \) for a thickness \( d \), assuming the light interacts with the interface only once (~ 95% of the light for incidental angles less than the Brewster’s angle of \( \text{atan}(1/n) \)). For 100-μm spatial resolution, the thickness of BaF₂ crystal with a reflective index of 1.54 @220nm cannot exceed 71μm.

The maximum thickness of thin scintillator determined by X-ray imaging spatial resolution

<table>
<thead>
<tr>
<th></th>
<th>LYSO:Ce</th>
<th>LSO:Ce, Ca</th>
<th>BaF₂</th>
<th>CsF</th>
<th>CeBr₃</th>
<th>LaBr₃:Ce</th>
<th>YAG:Yb</th>
<th>YAP:Yb</th>
<th>ZnO:Ga</th>
<th>PbI₂</th>
<th>GaAs/In</th>
<th>Plastic scintillator (BC 404)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reflective index at emission peak (n)</td>
<td>1.82</td>
<td>1.82</td>
<td>1.54</td>
<td>1.49</td>
<td>1.9</td>
<td>1.9</td>
<td>1.87</td>
<td>1.96</td>
<td>2.1</td>
<td>3.4</td>
<td>3.47</td>
<td>1.58</td>
</tr>
<tr>
<td>1/n</td>
<td>0.549</td>
<td>0.549</td>
<td>0.649</td>
<td>0.671</td>
<td>0.526</td>
<td>0.526</td>
<td>0.535</td>
<td>0.510</td>
<td>0.476</td>
<td>0.294</td>
<td>0.288</td>
<td>0.633</td>
</tr>
<tr>
<td>atan(1/n)</td>
<td>0.502</td>
<td>0.502</td>
<td>0.576</td>
<td>0.591</td>
<td>0.484</td>
<td>0.484</td>
<td>0.491</td>
<td>0.472</td>
<td>0.444</td>
<td>0.286</td>
<td>0.281</td>
<td>0.564</td>
</tr>
<tr>
<td>Maximum thickness (d) for 100um spatial resolution (mm)</td>
<td>86</td>
<td>86</td>
<td>71</td>
<td>68</td>
<td>90</td>
<td>90</td>
<td>89</td>
<td>93</td>
<td>101</td>
<td>167</td>
<td>171</td>
<td>73</td>
</tr>
</tbody>
</table>