Introduction to the SM (2)

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Yesterday...

- Yesterday: QFT and model building
- Today
  - QED
  - QCD
  - The gauge sector of SM

The SM

Input: Symmetries and fields

- Symmetry: 4d Poincare and

$$SU(3)_C \times SU(2)_L \times U(1)_Y$$

- Fields:
  - 3 copies of QUDLE fermions and one scalar

$$Q_L(3,2)_{1/6} \quad U_R(3,1)_{2/3} \quad D_R(3,1)_{-1/3}$$
$$L_L(1,2)_{-1/2} \quad E_R(1,1)_{-1} \quad \phi(1,2)_{+1/2}$$

- Then Nature is described by the most general $\mathcal{L}$ up to dim 4

$$\mathcal{L} = \mathcal{L}_{kin} + \mathcal{L}_\phi + \mathcal{L}_\psi + \mathcal{L}_{Yukawa}$$

The gauge interactions

The gauge part

$$SU(3)_C \times SU(2)_L \times U(1)_Y \ \to \ SU(3)_C \times U(1)_{EM}$$

Three parts, each look so different...

- QED - photon interaction: Perturbation theory
- QCD - gluon interaction: Confinement and asymptotic freedom
- Electro-weak: SSB and massive gauge bosons

QED

Lets “built” a simple QED model based on our rules

- Gauge group: $U(1)$
- Fields: E_L and E_R with charges -1 and $+1$
- No scalars and no SSB

The most general renormalizable Lagrangian

$$
\begin{aligned}
\mathcal{L} &= \overline{E_L} i \not{D} E_L + \overline{E_R} i \not{D} E_R - m \overline{E_L} E_R - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} \\
&= \overline{E_L}(i\not{\partial} - q\not{A}) E_L + \overline{E_R}(i\not{\partial} - q\not{A}) E_R - m \overline{E_L} E_R - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} \\
&= \overline{E}(i\not{\partial} - q\not{A} - m) E - \frac{1}{4} F^{\mu\nu} F_{\mu\nu}
\end{aligned}
$$

Remarks

$$\mathcal{L} = \overline{E}(i\not{D} - m)E - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} \qquad D_\mu = \partial_\mu + iqA_\mu$$

- The interaction term is part of the kinetic term. Universality!
- In QED we can work with 4-components fields
- The electron has a mass
- We call such theory “vector”. This is in contrast to a “chiral” theory

An aside: small electron mass

In QED the electron mass is a free parameter. So we measure it. What do we expect?

- It is a free parameter. We do not expect anything
- Well, we know there is a "UV cutoff" where new theory come in (BTW, what is this new theory?)
- The electron mass is "technically natural." If it were zero we will have an enhanced symmetry
- The enhance symmetry is "chiral symmetry." E_L and E_R rotate differently

QCD

Lets “built” a simple QCD model based on our rules

- Gauge group: $SU(3)$
- Fields: q_L and q_R. Both are triplets of $SU(3)$
- No scalars and no SSB

The most general renormalizable Lagrangian

$$\mathcal{L} = \overline{q}(i\not{D} - m_q)q - \frac{1}{4}G^{\mu\nu}G_{\mu\nu}$$

QCD: remarks

$$\mathcal{L} = \overline{q}(i\not{D} - m_q)q - \frac{1}{4}G^{\mu\nu}G_{\mu\nu}$$

- It looks just like QED. And yes, it is very much the same
- There are 8 gluons DOFs. Can we tell them apart?
- There are gluon self interactions. Very important
- Running is important. Asymptotic freedom and confinement
- Dynamical generated scale, $\Lambda_{QCD} \sim \text{few} \times 10^2~\text{MeV}$

SSB

Breaking a symmetry

SSB

- By choosing a ground state we break the symmetry
- We choose to expand around a point that does not respect the symmetry
- PT only works when we expand around a minimum

What is the different between a broken symmetry and no symmetry?

SSB implies relations between parameters

SSB

Symmetry is $x \to -x$ and we keep up to x^4

$$f(x) = a^2x^4 - 2b^2x^2 \qquad x_{\rm min} = \pm b/a$$

We choose to expand around $+b/a$ and use $u \to x - b/a$

$$f(x) = 4b^2u^2 + 4bau^3 + a^2u^4$$

- No $u \to -u$ symmetry
- The $x \to -x$ symmetry is hidden
- A general function has 3 parameters $c_2u^2 + c_3u^3 + c_4u^4$
- SSB implies a relation between them

$$c_3^2 = 4c_2c_4$$

SSB in QFT

- When we expand the field around a minimum that is not invariant under a symmetry

$$\phi \to v + h$$

- It breaks the symmetries that ϕ is not a singlet under
- Masses to other fields via Yukawa interactions

$$\phi X^2 \to (v+h)X^2 = vX^2 + ...$$

- Gauge fields of the broken symmetries also get mass

$$|D_\mu \phi|^2 = |\partial_\mu \phi + iqA_\mu \phi|^2 \ni A^2\phi^2 \to v^2 A^2$$

$SU(2) \times U(1)$ and leptons

Electroweak theory

Lets "built" a simple EW model for leptons

- Gauge group: $SU(2) \times U(1)$
- Fields:

$$L_L(2)_{-1/2} \qquad E_R(1)_{-1}$$

- One scalar $\phi(2)_{1/2}$, with negative $\mu^2\phi^2$ term

The most general renormalizable Lagrangian

$$\mathcal{L} = \mathcal{L}_{\rm kin} + \mathcal{L}_{\rm Yuk} + \mathcal{L}_\phi + \mathcal{L}_\psi$$

$\mathcal{L}_{kin}$ and $SU(2) \times U(1)$

- Four gauge bosons DOFs

$$W_a^\mu(1,3)_0 \qquad B^\mu(1,1)_0$$

- The covariant derivative is

$$D^\mu = \partial^\mu + igW_a^\mu T_a + ig'YB^\mu$$

  - Two parameters g and g'
  - Y is the $U(1)$ charge of the field D_μ work on
  - T_a is the $SU(2)$ representation
  - $T_a = 0$ for singlets. $T_a = \sigma_a/2$ for doublets
- Write D_μ for $L(1,2)_{-1/2}$ and $E(1,1)_{-1}$

Explicit examples

$$D^\mu = \partial^\mu + igW_a^\mu T_a + ig'YB^\mu$$

- Write D_μ for $L(1,2)_{-1/2}$ and $E(1,1)_{-1}$

$$D^\mu L = \left(\partial^\mu + \frac{i}{2}gW_a^\mu \sigma_a - \frac{i}{2}g'B^\mu\right) L$$
$$D^\mu E = \left(\partial^\mu - ig'B^\mu\right) E$$

- HW: Using $\phi(1,2)_{1/2}$ write $D^\mu \phi$

QED

- Where is QED in all of this?

$$Q = T_3 + Y$$

- We can write explicitly for $L(1,2)_{-1/2}$ and $\phi(1,2)_{1/2}$

$$L_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \qquad \phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$

- This is arbitrary. It becomes usefull once we have SSB

SSB in the SM

$$-\mathcal{L}_{Higgs} = \lambda\phi^4 - \mu^2\phi^2 = \lambda(\phi^2 - v^2)^2$$

- We measure the fact that $\mu^2 > 0$ by having SSB
- The minimum is at $|\phi| = v$
- ϕ has 4 DOFs. We can choose

$$\langle\phi_1\rangle = \langle\phi_2\rangle = \langle\phi_4\rangle = 0 \qquad \langle\phi_3\rangle = v$$

- It leads to: $SU(2)_L \times U(1)_Y \ \rightarrow \ U(1)_{EM}$
- We call the remaining symmetry EM
- Could we "choose" the vev in the neutral direction?
- We left with one real scalal field: the Higgs boson

Spectrum

Gauge boson masses

From the kinetic term of the Higgs we get mass for the gauge bosons

$$|D^\mu\phi|^2 \sim \frac{1}{8}\left|\begin{pmatrix} gW_3 + g'B & g(W_1 - iW_2) \\ g(W_1 + iW_2) & -gW_3 + g'B \end{pmatrix}\begin{pmatrix} 0 \\ v \end{pmatrix}\right|^2$$

which gives for mass terms

$$\frac{1}{4}g^2v^2W^+W^- + \frac{1}{8}v^2(gW_3 - g'B)^2$$

Masses

Define the mass eigenstates

$$W^\pm = \frac{1}{\sqrt{2}}(W_1 \mp iW_2)$$

$$Z = \cos\theta_W W_3 - \sin\theta_W B$$

$$A = \sin\theta_W W_3 + \cos\theta_W B$$

$$\tan\theta_W \equiv \frac{g'}{g}$$

The masses are

$$M_W^2 = \frac{1}{4}g^2v^2 \qquad M_Z^2 = \frac{1}{4}(g^2 + g'^2)v^2 \qquad M_A^2 = 0$$

We have a rotation from W_3, B to the mass basis Z, A

Remarks

- $W^{\pm}$ are charged under EM. A and Z are not
- We have a mechanism for $SU(2)_L \times U(1)_Y \to U(1)_{EM}$
- $M_A^2 = 0$ is not a prediction, it is a consistency check on our calculation
- Note that we get the following testable relation:

$$\rho \equiv \frac{M_W^2}{M_Z^2 \cos^2 \theta_W} = 1$$

- Out of the four scalar degrees of freedom, three are the would-be Goldstone bosons eaten by the $W_{\pm}$ and Z, and one is the physical Higgs boson with $m_H^2 = 2\lambda v^2$

$\rho = 1$

Very non-trivial prediction:

$$\frac{M_W^2}{M_Z^2} = \frac{g^2}{g^2 + g'^2}$$

- Tested experimentally
- $\rho = 1$ is a prediction of the SM with a Higss doublet
- Quantum corrections
- Related to a symmetry: Custodial symmetry

$\mathcal{L}_{\text{Yuk}}$ and fermion masses

- There is no way to write a mass term, that is $\mathcal{L}_\psi = 0$
- The Yukawa part of the leptons

$$\mathcal{L}_{Yuk} = y_{ij}\overline{L_{Li}}E_{Rj}\,\phi \Rightarrow m_{ij}\overline{L_{Li}}E_{Rj} \qquad m_{ij} = v y_{ij}$$

- $i, j = 1, 2, 3$ are flavor indices
- y is a general complex 3×3 matrix and we can choose a basis where m is diagonal and real

$$m_{ij} = y\,v = \text{diag}(m_e, m_\mu, m_\tau)$$

- Neurinos are massless

Interactions

Interactions

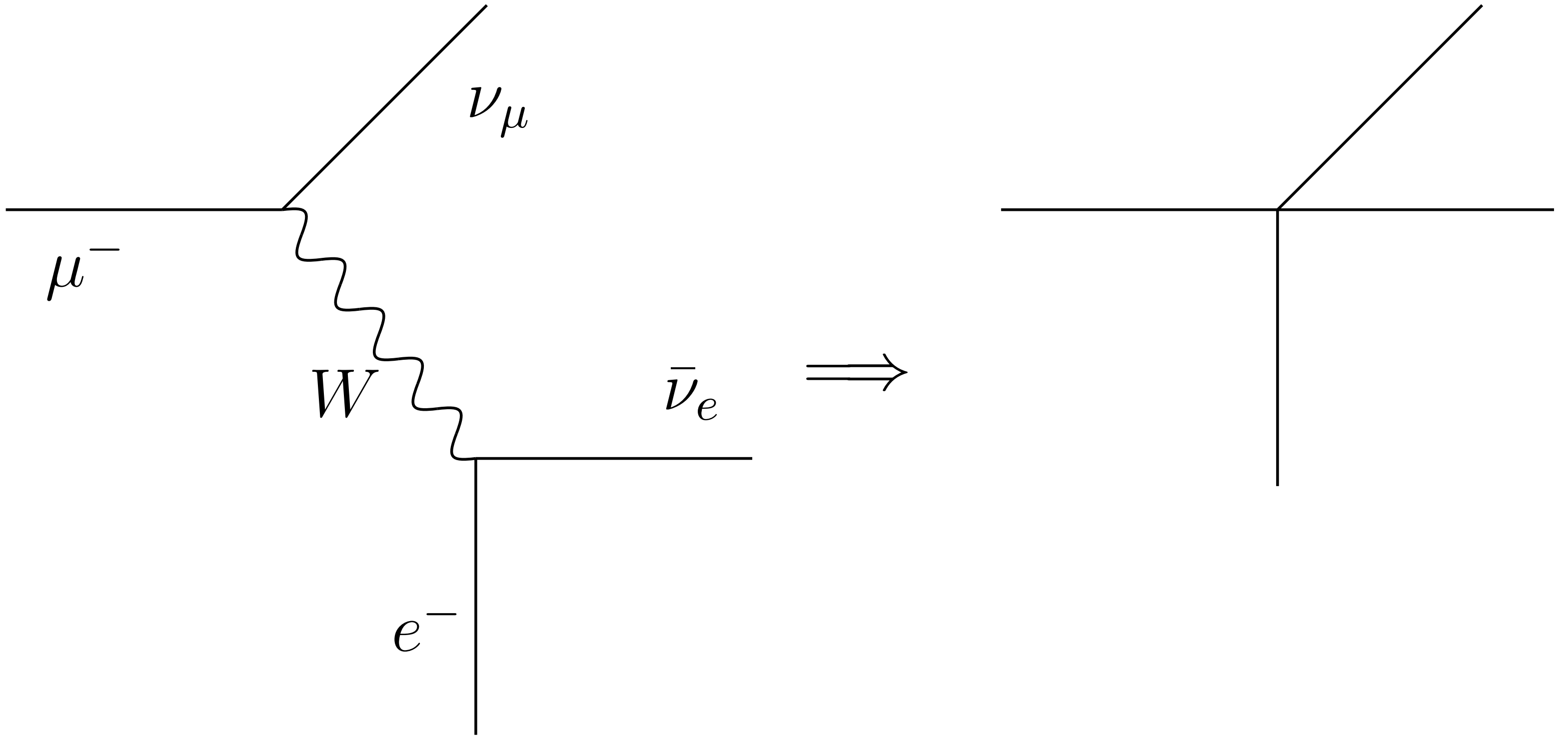
$$-\frac{g}{\sqrt{2}}\,\overline{\nu_{eL}}\,W^{\mu}\gamma_{\mu}e_L^{-} + h.c.$$

- Only left-handed particles take part in charged-current interactions. Therefore the W interaction violate parity
- Universality: the couplings of the W to $\tau\bar{\nu}_\tau$, to $\mu\bar{\nu}_\mu$ and to $e\bar{\nu}_e$ are equal
- At low energy we can "integrate out" the W

$$G_F = \frac{\sqrt{2}g^2}{8M_W^2} = \frac{1}{\sqrt{2}v^2}$$

- Almost direct measurement of the vev, $v = 246\ GeV$
- Instead of g, g', v we can use $G_F, m_Z, \sin^2\theta_W$

Muon decay



$$\mathcal{A} \sim \frac{g^2}{p^2 - m_W^2} \sim \frac{g^2}{m_W^2} \sim G_F$$

Neutral currents

$$\mathcal{L}_{\rm int} = \frac{e}{\sin\theta\cos\theta}(T_3 - \sin^2\theta_W Q)\,\bar\psi \not{Z} \psi\,,$$

- Photon and Z. The Z is the extra stuff
- Both LH and RH coupling. Still Z is parity violating
- Diagonal couplings. No flavor violation at tree level
- Processes involving the Z can be used to measure $\sin^2\theta_W$
- Together with m_W and G_F we can get the two parameters of the model, g and g'

Experimental tests

Of course, the model was built from experimental data...

- High energy: Open your pdg and check W and Z decays to leptons. What do you expect to see?
- Z decays to lepton gives $\sin^2\theta_W \approx 0.23$
- Based on universality, what do we expect for $Z \to \mu\mu$ vs $Z \to \tau\tau$ decays?
- More low energy data:
  - pion decay: proof of spin one nature of the weak interaction
  - neutrino scattering: proof of the left-handedness of it

Neutrino scattering

$$\sigma(\nu e^- \to \nu e^-) = \frac{G_F^2 s}{\pi} \qquad \sigma(\bar{\nu} e^- \to \bar{\nu} e^-) = \frac{G_F^2 s}{3\pi}$$

- Note the factor of 3
- Think about backward scattering:
  - νe: Both LH and thus, $J_Z = 0$ before and after. Can go
  - $\bar{\nu} e$: One LH and one RH: $J_Z = +1$ before and $J_Z = -1$ after. Cannot go.

Some summary

- The SM gauge sector has three parts:
  - QED: perturbation theory
  - QCD: Confinement and asymptotic freedom
  - Electroweak: SSB, masses and parity violation

Gauge interactions are universal!