Universality of Emergent States in Diverse Physical Systems

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Textbook states and real states

Physics textbooks are dominated by simple, weakly-interacting microscopic states. Actual physical states are often *emergent.*

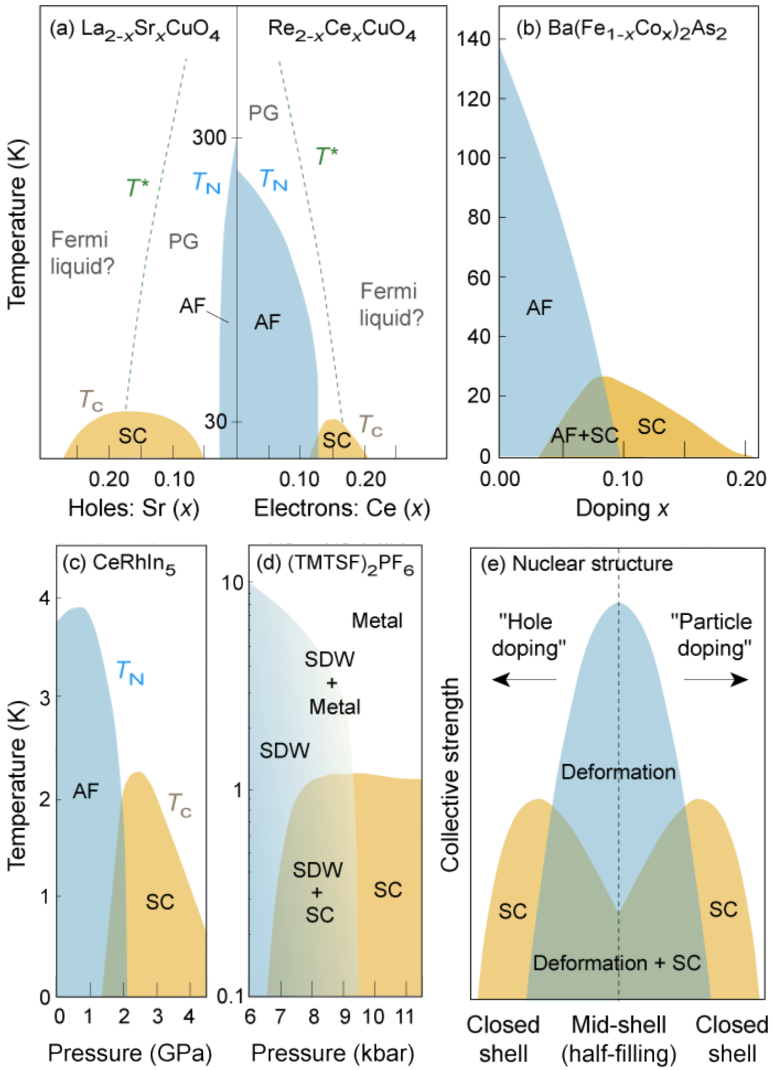
- Emergent states are strongly-correlated and dominated by properties that *emerge* as a consequence of interactions.

- Emergent states are *not part of the description of the corresponding weakly-interacting system.*

- Such states often have a phenomenological description but *no clear microscopic connection to the simple states described in our textbooks.*

This talk proposes a connection of weakly-interacting states and emergent states through dynamical symmetries implying unique truncations of the full Hilbert space to an emergent subspace.

Example: How are superconductivity and superfluidity related across fields?



Unifying Principle:

Dynamical symmetry at the emergent collective level that *depends only parametrically on underlying microscopic physics* of the normal state.

Gravity: a property of spacetime and not of objects in that spacetime

In 1907 Albert Einstein was sitting in the Swiss Patent Office in Bern when the following thought came to him:

> *If I were to fall freely in a gravitational field, I would be unable to feel my own weight.*

Einstein said that this was the "happiest thought of my life". Why? Because he saw in it how to formulate a covariant theory of gravity.

Weak Equivalence Principle: *gravitational mass = inertial mass* (all objects feel the same gravitational acceleration)

Strong Equivalence Principle: In a local, freely-falling frame, *gravity can be transformed away.*

Gravity: a property of spacetime and not of objects in that spacetime

It took Einstein 8 more years to work out the new ideas and the new math necessary to formulate the general theory of relativity.

- The essential new idea was the insight that

> *Because of the universality of gravity implied by the equivalence principle, gravity must be a property of spacetime and not of objects in that spacetime.*

- This led Einstein to the realization that *gravity is spacetime curvature*.

- New math was required: tensor calculus on Riemannian manifolds.

Our Goal: generalize the idea of *universality as a property of a space*, rather than of objects in that space, to manybody systems.

Emergence: a property of a space and not of objects in that space

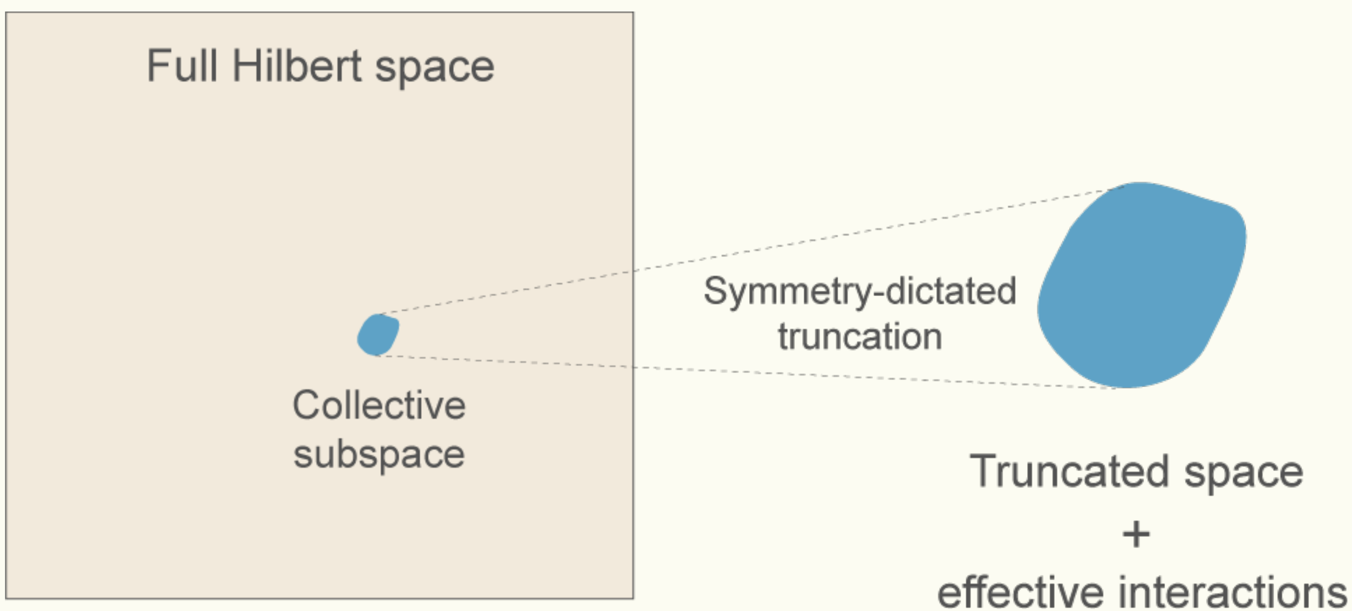
Gravity lives on a 4D curved Riemannian manifold with indefinite metric. *Natural mathematical expression:* the *tensor calculus of Riemannian manifolds and metric connections on that manifold.*

Emergence lives in a truncated linear vector space (subspace of Hilbert space) that is in some sense also “curved”. *Natural mathematical expression: Lie algebras on the vector subspace.*

Emergence becomes universal to the degree that it is a property of the space in which it lives and not of the microphysics for specific systems in that space.

Fermion dynamical symmetry methods

Dynamical symmetries truncate the Hilbert space to a subspace of specific structure (given by the symmetry).



Fermion dynamical symmetry methods

- Find subgroup chains ending with symmetries reflecting the conservation laws of the system. Each defines a *dynamical symmetry*.

$$\begin{tikzcd}
 & & SO(4) \times U(1) \arrow[dr] & \\
SO(8) \arrow[r] & SU(4) \arrow[ur] \arrow[r] \arrow[dr] & SO(5) \arrow[r] & SU(2)_S \times U(1) \\
 & & SU(2)_P \times SU(2)_S \arrow[ur] &
\end{tikzcd}$$

Each dynamical symmetry specifies an *exact manybody solution*

- The most general Hamiltonian for the subspace is a *sum of Casimir invariants C_g for all groups* in all subgroup chains.

$$H = a_1 C_{g1} + a_2 C_{g2} + a_3 C_{g3} + \ldots + a_n C_{gn}$$

- Coefficients a_i are functions of the *effective interaction.*

Solutions not in dynamical symmetry limits: *Generalized coherent states*

The inspiration: Glauber coherent states for the photon field

Generalize to define coherent states for *fermion fields* specified by a Lie Algebra

Lie Algebra

Identify minimal complete set of microscopic, physical operators

Generalized Hartree-Fock-Bogoliubov quasiparticle theory, subject to symmetry constraints

or

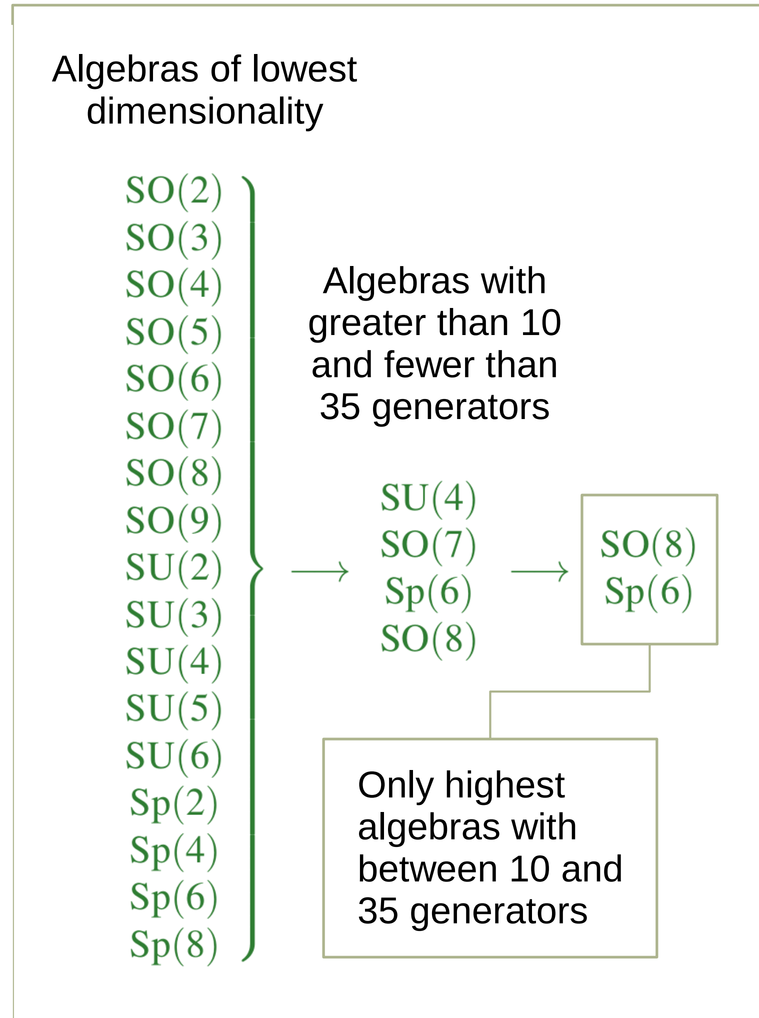
Microscopically-derived Ginzburg-Landau energy surfaces for symmetry-constrained solutions

“Quantization” of compact Lie algebras

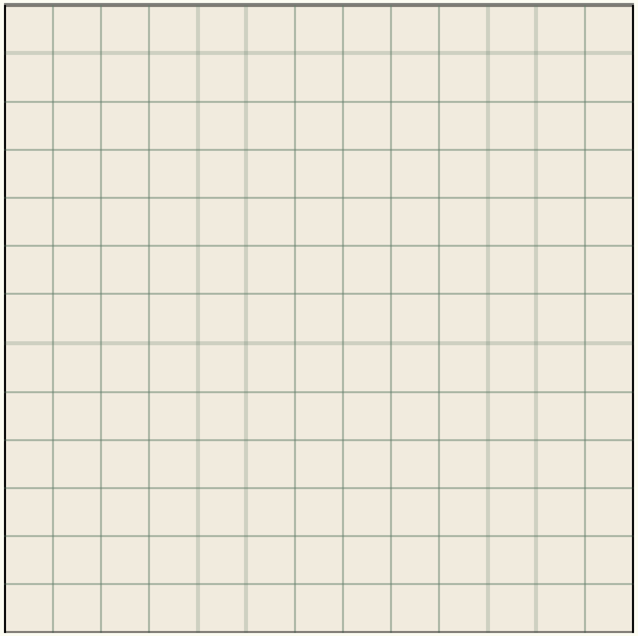
The Compact Lie Algebras

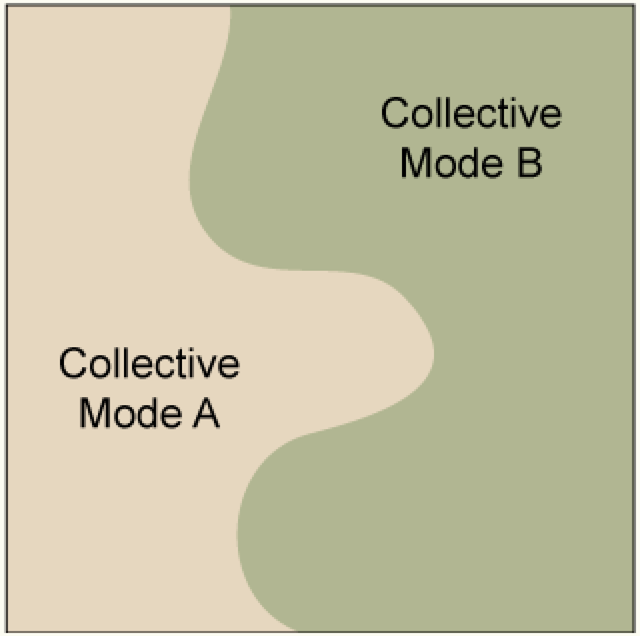
Group	Instances	Generators	Dimensionality*
$SO(2n+1)$	$SO(3)$	3	$\mathrm{Dim} = 2n^2 + n$
	$SO(5)$	10	
	$SO(7)$	21	
	$SO(9)$	36	
$SO(2n)$	$SO(2)$	1	$\mathrm{Dim} = 2n^2 - n$
	$SO(4)$	6	
	$SO(6)$	15	
	$SO(8)$	28	
	$SO(10)$	45	
$SU(n)$	$SU(2)$	3	$\mathrm{Dim} = n^2 - 1$
	$SU(3)$	8	
	$SU(4)$	15	
	$SU(5)$	25	
	$SU(6)$	35	
	$SU(7)$	48	
$Sp(2n)$	$Sp(2)$	3	$\mathrm{Dim} = 2n^2 + n$
	$Sp(4)$	10	
	$Sp(6)$	21	
	$Sp(8)$	36	

*General expression for dimensionality of the generator space.

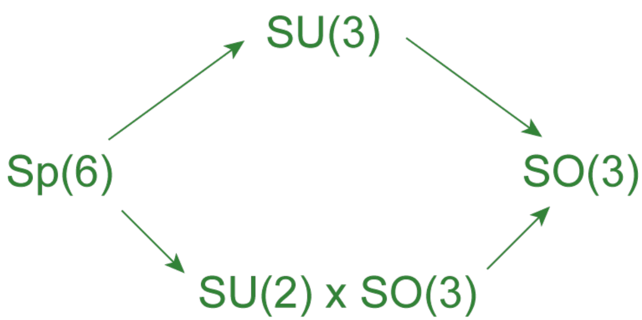


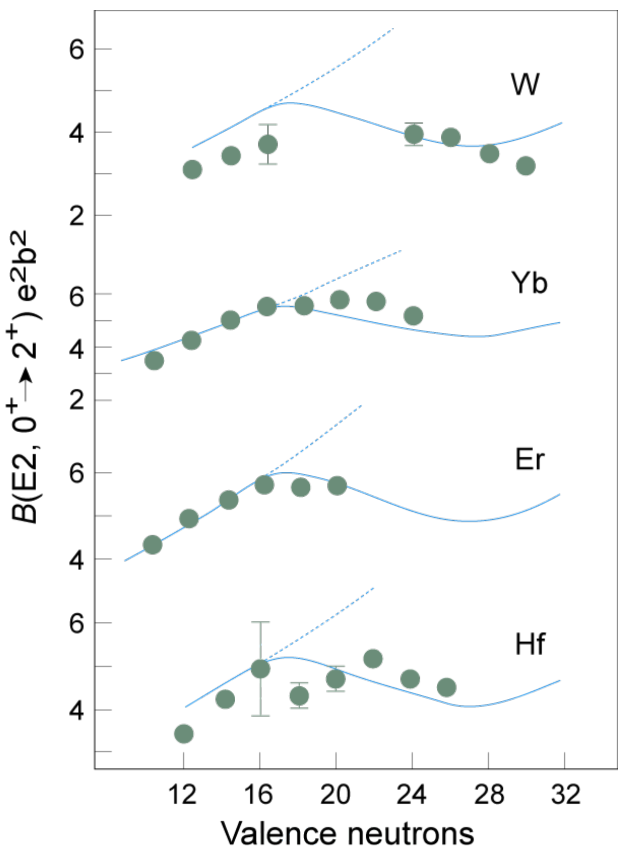
Analogy: Constraints required by closure of compact Lie algebras



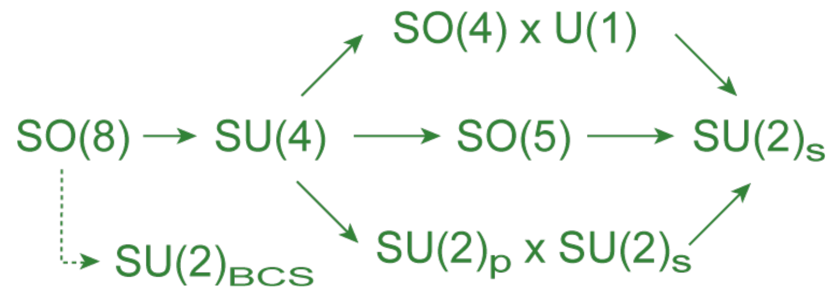


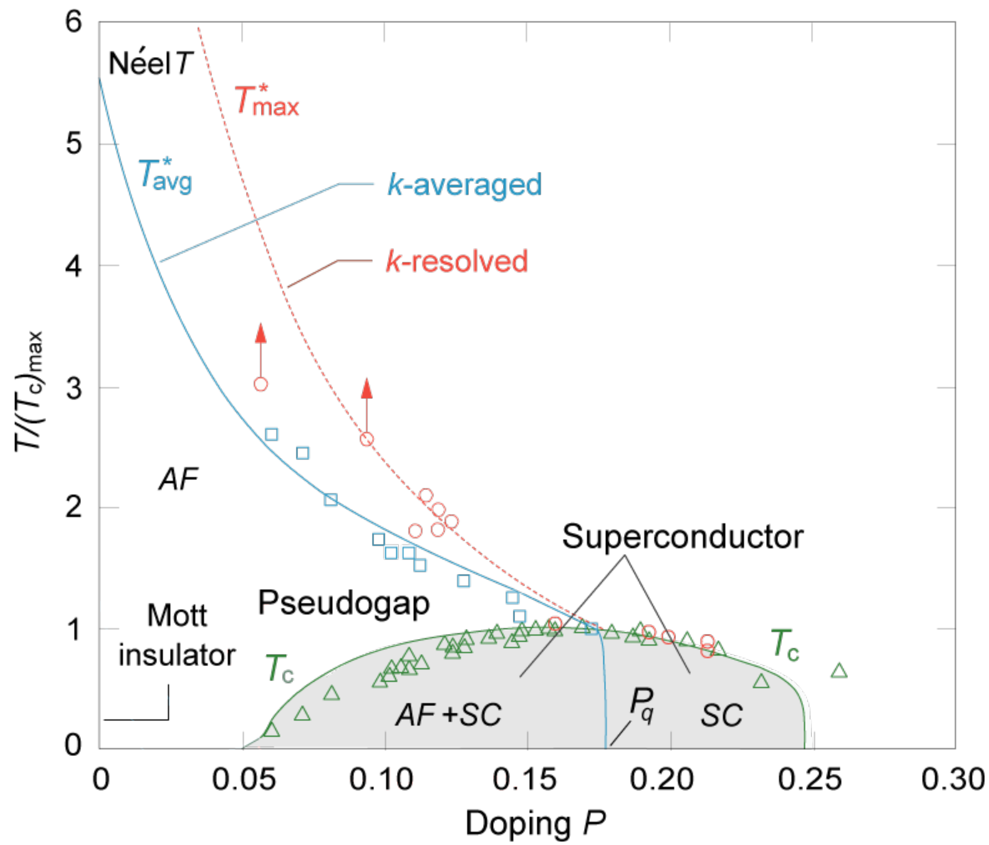
Electromagnetic transition rates in rare earth nuclei





Temperature-doping phase diagram for cuprate superconductors



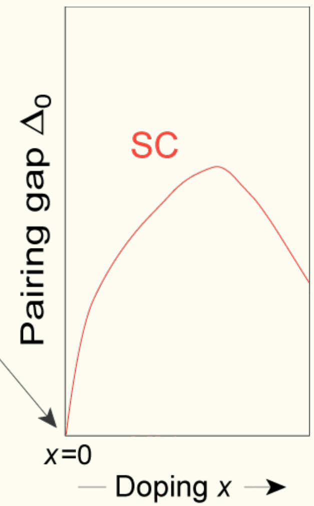


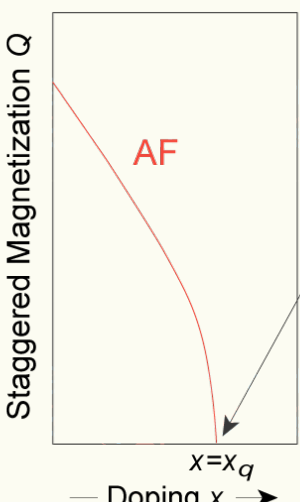
New Insights: Quantum phase transitions in doped Mott insulators

Superconducting instability

$$\left.\frac{\partial\Delta}{\partial x}\right|_{x=0} = \frac{1}{4}\left.\frac{x_q^{-1}-2x}{(x(x_q^{-1}-x))^{1/2}}\right|_{x=0} = \infty$$

Generalization of the Cooper instability to doped Mott insulators.





Antiferromagnetic instability

$$\left.\frac{\partial Q}{\partial x}\right|_{x=x_q} = -\frac{1}{4}\left.\frac{x_q+x_q^{-1}-2x}{[(x_q-x)(x_q^{-1}-x)]^{1/2}}\right|_{x=x_q} = -\infty$$

Quantum phase transition between ground states having finite AF correlation and those having vanishing AF correlations.

Dynamical symmetries imply *quantum phase transitions.*

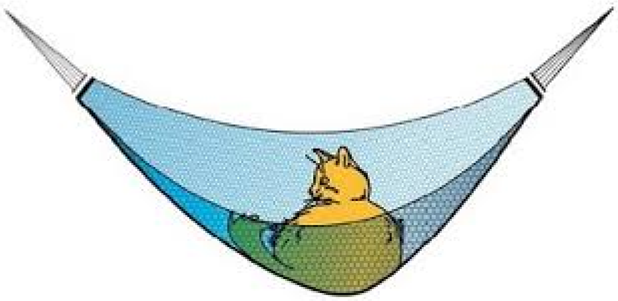
In some cases these lead to *quantum critical points* and to *quantum critical phases.*

Example: A great deal of cuprate behavior is explained by the two SU(4) instabilities illustrated adjacent left.

Quantum critical behavior does not “cause” SC, but it is a consequence of the emergent dynamical symmetry that does.

Quantum Hall states of graphene in a strong magnetic field

A sheet of graphene the thickness of ordinary wrapping film could support an *elephant balanced on a pencil.*

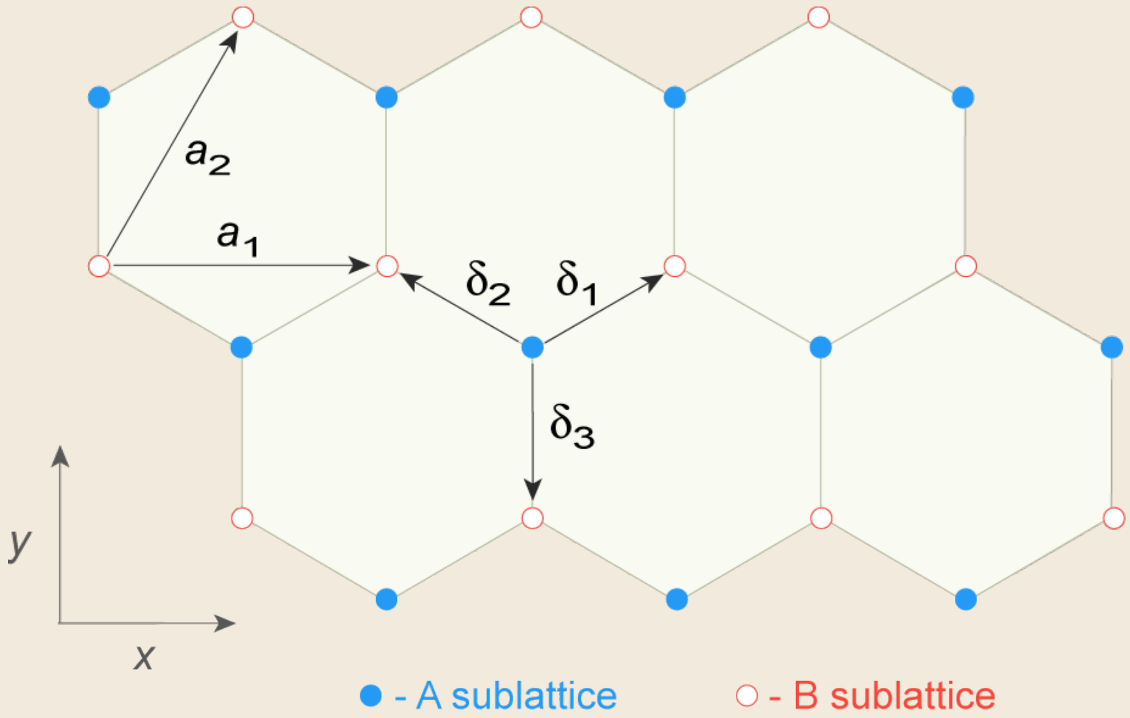


A hammock made from a 1 m^2 single sheet of graphene could *support a 4 kg cat.* The hammock would be

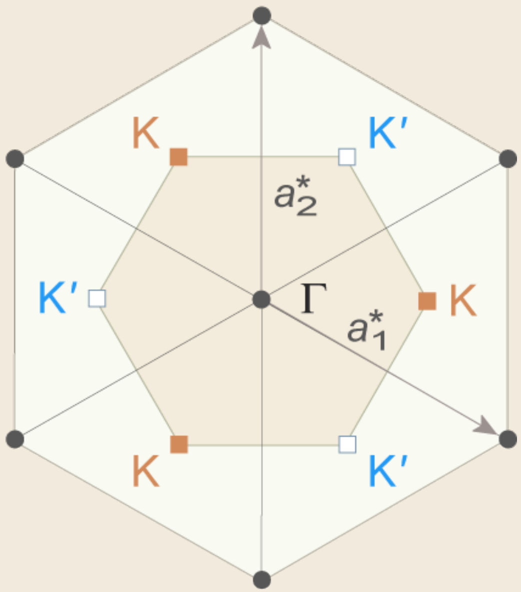
- One atom thick,
- almost invisible,
- weigh less than a single cat whisker (< 1 mg)

Real space and reciprocal lattices

(a) Direct lattice



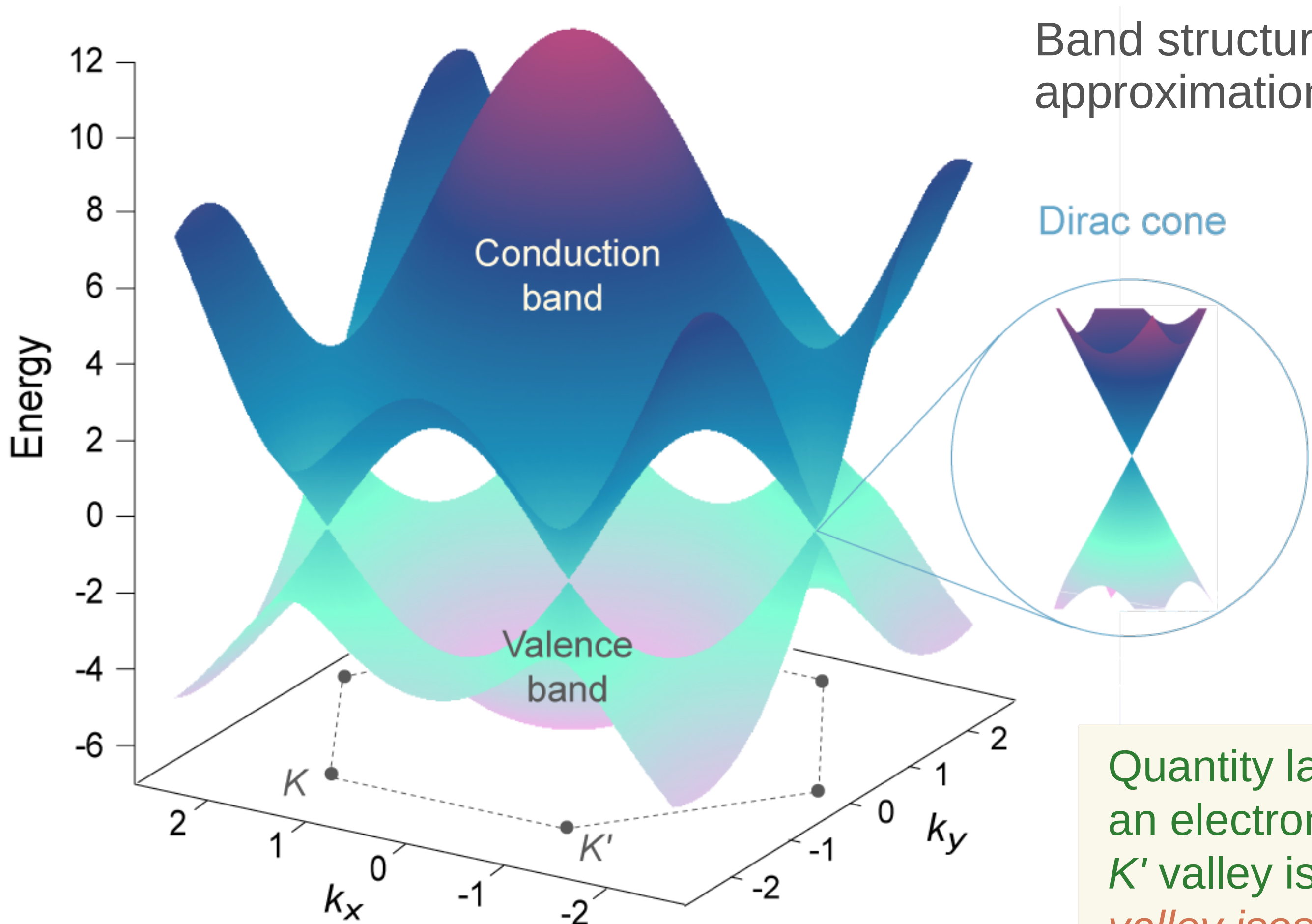
(a) Reciprocal lattice



Sublattice pseudospin: whether on A or B sublattice

Valley isospin: whether on K or K' site

Dirac cones, massless electrons, and valley isospin



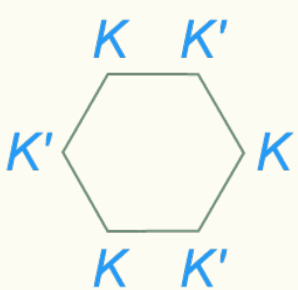
Quantity labeling whether an electron is in the *K* or *K′* valley is called the *valley isospin.*

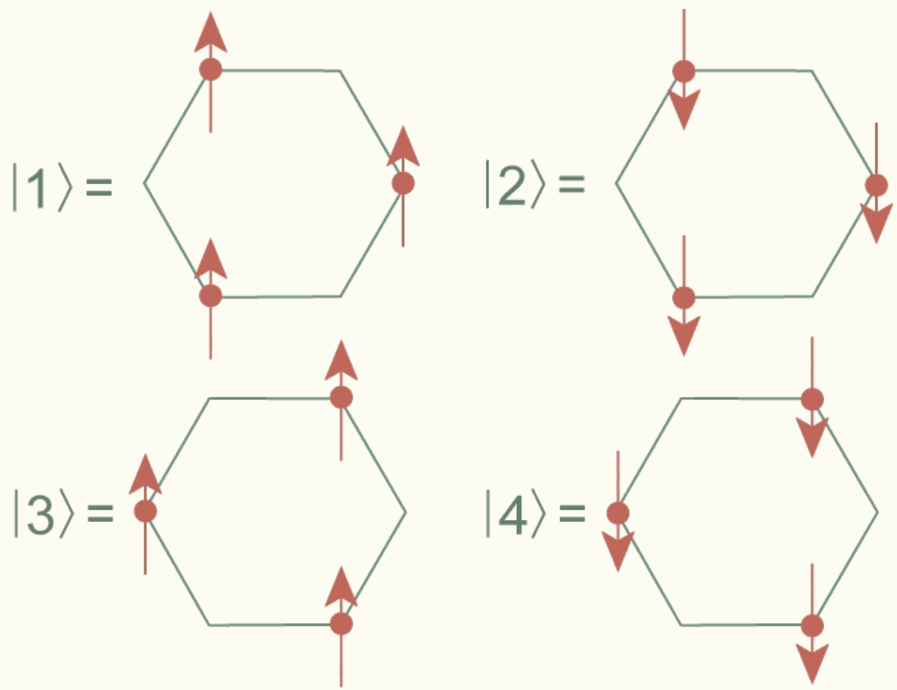
The 4-fold spin and isospin basis

For low-energy excitations, valley isospin τ and sublattice pseudospin are equivalent labels. Using *isospin projection τ and spin projection σ:*

Valley	τ	σ	a
K	+	$\uparrow$	1
K	+	$\downarrow$	2
K'	–	$\uparrow$	3
K'	–	$\downarrow$	4

Valley isospin labels

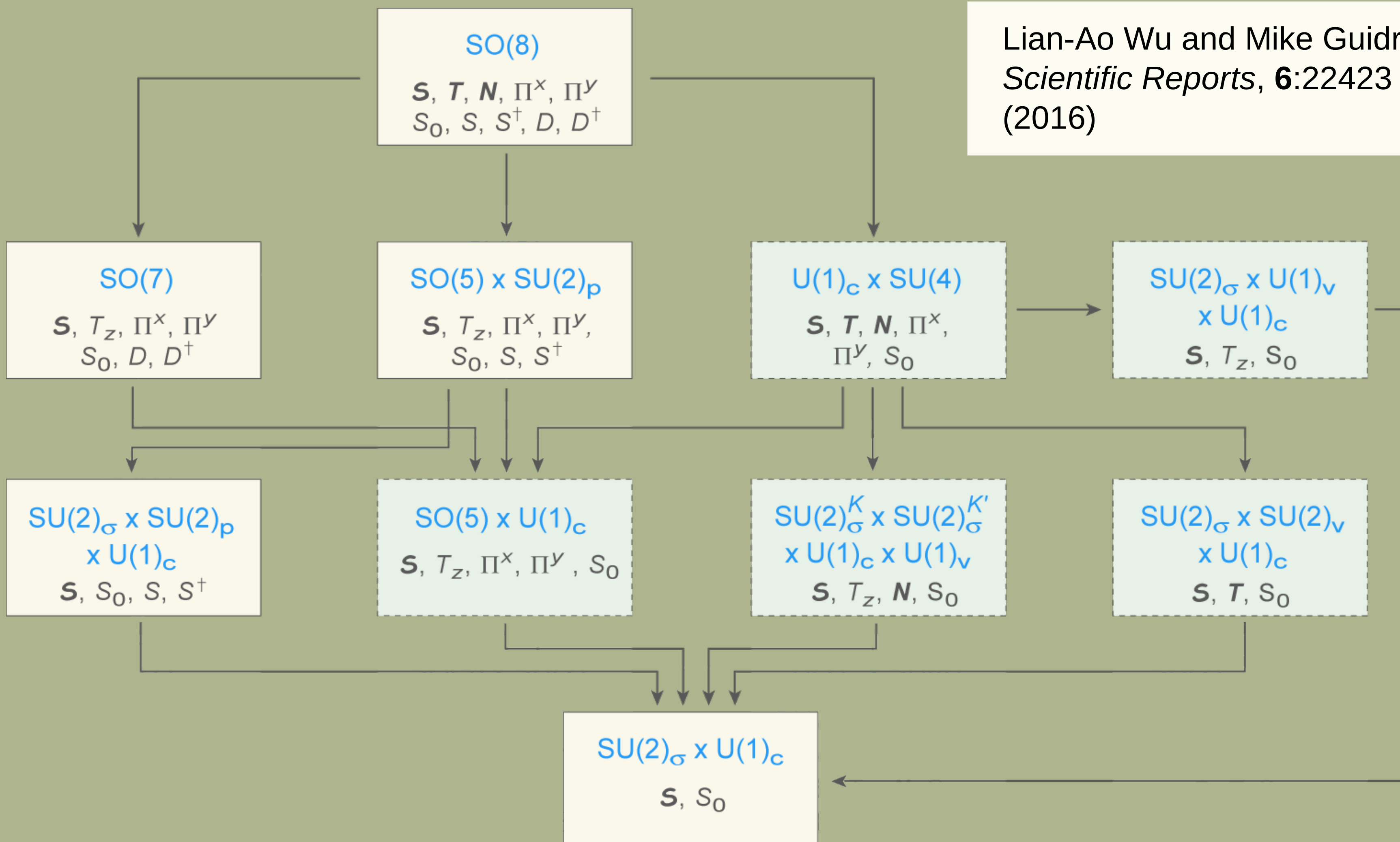




Basis $|a\rangle$

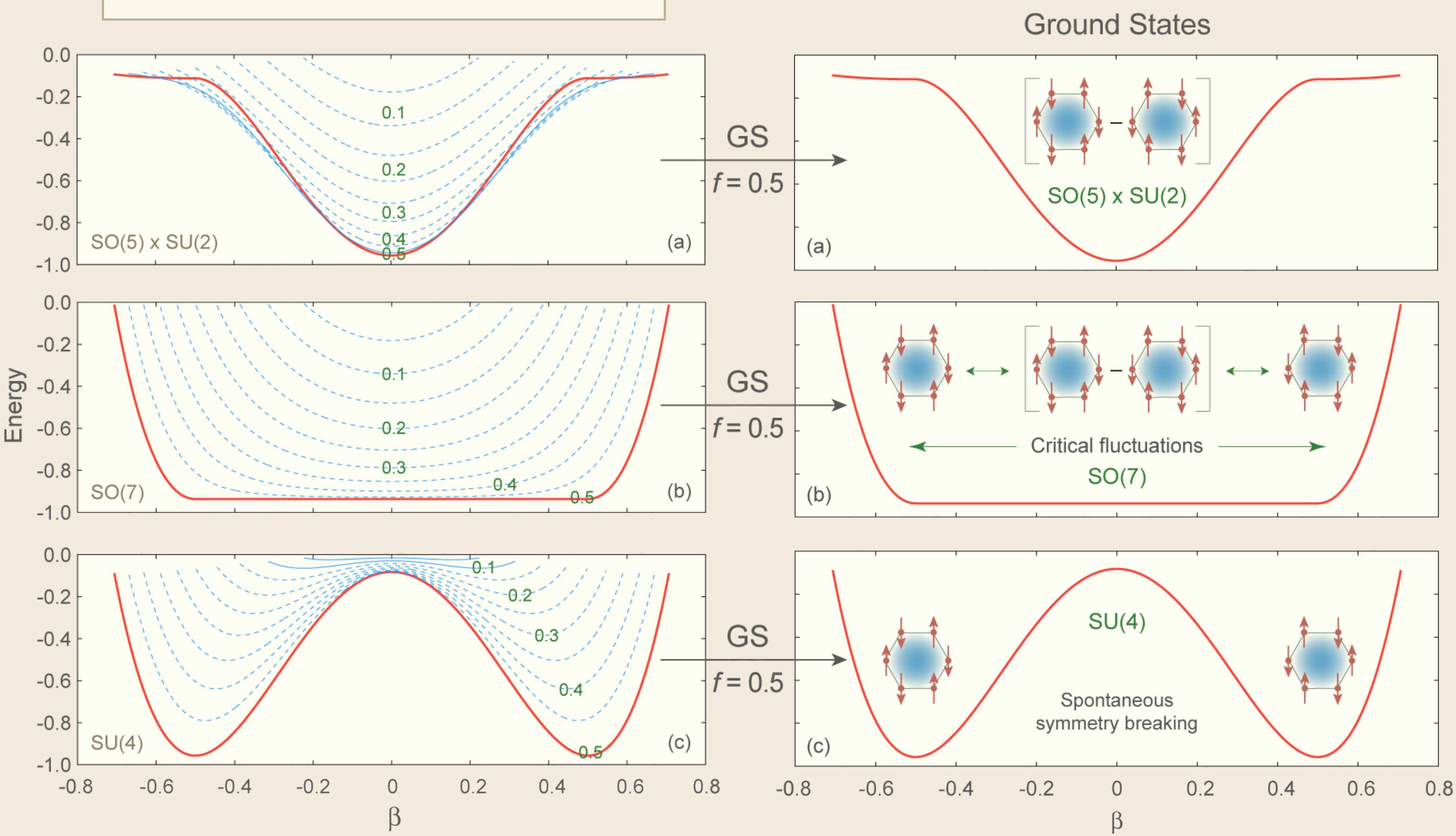
The SO(8) dynamical symmetry group chains in graphene

Lian-Ao Wu and Mike Guidry, *Scientific Reports*, **6**:22423 (2016)



Graphene dynamical symmetry limits

$$\begin{tikzcd}
& SO(5) \times SU(2) \arrow[dr] & \\
SO(8) \arrow[ur] \arrow[r] \arrow[dr] & SU(4) \arrow[r] & SO(5) \\
& SO(7) \arrow[ur] &
\end{tikzcd}$$

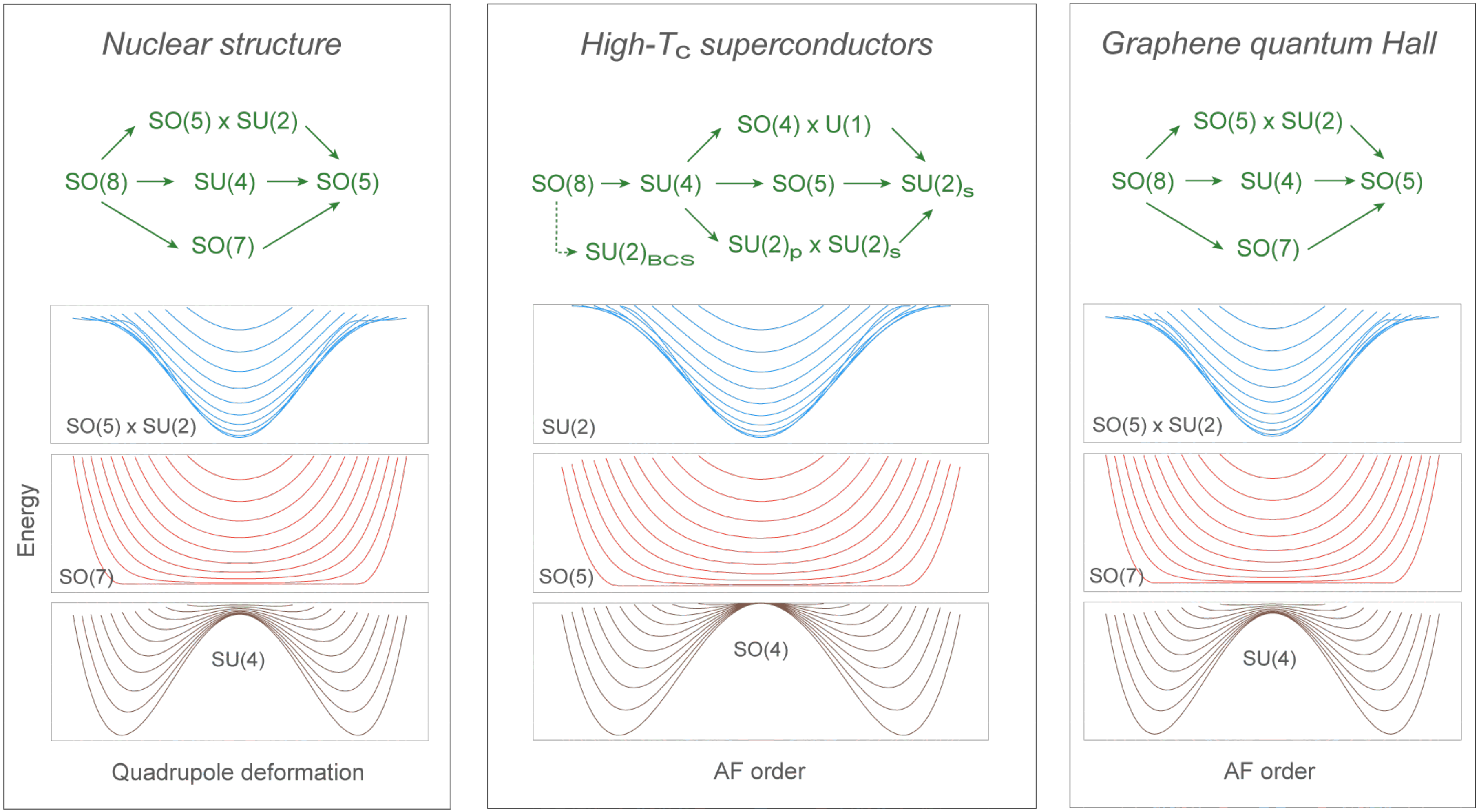


SO(8) quantum Hall states in graphene

"The Ground State of Monolayer Graphene in a Strong Magnetic Field", Lian-Ao Wu and Mike Guidry, *Scientific Reports,* **6**:22423 (2016)

"SO(8) fermion dynamical symmetry and quantum hall states for graphene in a strong magnetic field", Mike Guidry, *Fortschr. Phys.,* 1–8 (2016) / DOI 10.1002/prop.201600057 Published online 28 November 2016 (page numbers pending)

"SO(8) Fermion Dynamical Symmetry and Strongly-Correlated Quantum Hall States in Monolayer Graphene", Lian-Ao Wu, Matthew Murphy, and Mike Guidry, accepted for publication, *Phys. Rev. B* (arXiv:1607.08857)



Particles: *neutrons & protons*
Force: *screened strong*
Interaction strength: *MeV*
Distance scale: *10^{-13} cm*
Single particle: *nuclear shell*

Particles: *nonrelativistic electrons*
Force: *screened Coulomb*
Interaction strength: *eV*
Distance scale: *10^{-8} cm*
Single particle: *electronic band*

Particles: *relativistic electrons*
Force: *screened Coulomb*
Interaction strength: *eV*
Distance scale: *10^{-8} cm*
Single particle: *Landau levels*

Summary

(1) Basic properties of emergent states are governed by properties of a severely-truncated Hilbert space, just as gravity is a property of spacetime and not of specific objects in that spacetime.

(2) The natural expression of this idea is compact Lie algebras that transcend specific microscopic structure.

(3) Symmetry truncates the Hilbert space to a tiny emergent subspace with properties governed by its Lie algebras.

(4) Universality arises because only a few Lie algebras can satisfy the highly-restrictive conditions imposed by generic emergent states on the subspace.

(5) Emergent states depend only weakly on the microphysics, which affects parameter values but not generic behavior. In the language of object oriented programming: Symmetry defines the class; microphysics gives variety to specific instances of the class.

(6) Collective nuclear states, states of high-*T* superconductors, and graphene quantum Hall states are essentially the *same states* when viewed in this way. This is remarkable!