Limitations of the Porter-Thomas Distribution

In collaboration with A. Volya and V. Zelevinsky
[PRL 115, 052501 (2015)]
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1. GOE Prediction

Time-reversal invariant nuclear random Hamiltonian with elements

$$H_{\mu\nu} = H_{\nu\mu} = H^*_{\mu\nu} \ , \ \mu, \nu = 1, \ldots, N \gg 1 \ .$$

Probability distribution is

$$\exp\{-\tfrac{N}{\lambda^2}\mathrm{Tr}H^2\} \prod_{\mu\leq\nu} \mathrm{d}H_{\mu\nu} \to \mathrm{d}\mathcal{O} \ \exp\{-\tfrac{N}{\lambda^2}\sum_\rho E_\rho^2\} \prod_{\mu<\nu} |E_\mu - E_\nu| \prod_\sigma \mathrm{d}E_\sigma \ .$$

Orthogonal invariance: Eigenvalues and eigenfunctions statistically uncorrelated. For $N \to \infty$, projections of eigenvectors onto fixed vector in Hilbert space have Gaussian distribution.

Partial neutron widths $\Gamma_\mu = E^{1/2}\gamma_\mu^2$ for s-wave resonances. Reduced partial width amplitudes $\gamma_\mu \propto \langle n|V|\mu\rangle$ have Gaussian distribution. Reduced partial widths γ_μ^2 have Porter-Thomas distribution (PTD)

$$\mathcal{P}(\gamma^2) = \frac{1}{\sqrt{2\pi\Gamma}} \frac{1}{2\sqrt{\gamma^2}} \exp\{-\gamma^2/(2\Gamma)\}$$

with mean value Γ. Prediction for fixed quantum numbers (spin, parity).

2. Data: Supporting Evidence and Serious Disagreement

Supporting Evidence:

(i) Analysis of (p, p' γ) reactions on nuclei with mass $A \approx 50$ yields decay amplitudes for different exit channels. Test for Gaussian distribution very well fulfilled. J. F. Shriner, Jr., G. E. Mitchell, and E. G. Bilpuch, Phys. Rev. Lett. 59, 435 (1987); J. F. Shriner, Jr., E. G. Bilpuch, and G. E. Mitchell, Z. Phys. A 332, 45 (1989).

(ii) Extensive shell-model calculations yield solid evidence for PTD in the middle of s-d shell. V. Zelevinsky et al., Phys. Rep. 276, 85 (1996).

Serious Disagreement:

(i) 158 neutron resonances in $^{192}\mathrm{Pt}$, 411 neutron resonances in $^{194}\mathrm{Pt}$.
Neutron s-wave strength function has maximum near A = 194 because 4s state of shell model here near threshold. That reduces relative contribution of p-wave resonances. Analysis: Keep only resonances with reduced widths > cutoff. Cutoff increases linearly with resonance energy. Use maximum-likelihood analysis to test for agreement with chi square distribution with nu degrees of freedom. Yields

$$\nu = 0.57 \pm 0.15 \text{ for } {}^{192}\mathrm{Pt} \ , \ \nu = 0.47 \pm 0.18 \text{ for } {}^{194}\mathrm{Pt} \ .$$

Rejects PTD with 99.997 % statistical significance.
P. E. Koehler et al., Phys. Rev. Lett. 105, 072502 (2010).

(ii) Same analysis for Nuclear Data Ensemble (1245 resonances) yields

$$\nu = 0.801 \pm 0.052 \ .$$

Rejects PTD with 98.17 % statistical significance.
P. E. Koehler, Phys. Rev. C 84, 034312 (2011).q

3. Violation of Orthogonal Invariance

Neutron scattering function at energy E above threshold (E = 0):

$$S(E) = 1 - i\pi \sum_{\mu\nu}^{N} W_\mu(E)[(E - H^{\rm eff})^{-1}]_{\mu\nu} W_\nu(E) \ ,$$

$$H^{\rm eff}_{\mu\nu} = H^{\rm GOE}_{\mu\nu} + F_{\mu\nu}(E) - i\pi W_\mu(E) W_\nu(E) - i\pi \sum_\gamma W_\mu^{(\gamma)} W_\nu^{(\gamma)} \ ,$$

$$F_{\mu\nu}(E) = P \int_0^\infty {\rm d}E' \ \frac{W_\mu(E') W_\nu(E')}{E - E'} \ .$$

All coupling terms to channels violate orthogonal invariance. But

$$W_\mu(E) \propto E^{1/4}$$

negligible near threshold. So Thomas-Ehrman shift $F_{\mu\nu}$ and coupling to gamma channels remain.

3a. Thomas-Ehrman shift

A. Volya, HAW and V. Zelevinsky, Phys. Rev. Lett. 115, 052501 (2015); E. Bogolmolny, Phys. Rev. Lett. 118, 022501 (2017).

Transformation to "superradiant state": Choose $E_0 \approx 1 \text{ MeV}$, write

$$W_\mu(E) = (E/E_0)^{1/4}\ \mathcal{W}_\mu\ ,$$

$$F_{\mu\nu}(E) - i\pi W_\mu(E) W_\nu(E) \approx \mathcal{W}_\mu \mathcal{W}_\nu\ (P \int \mathrm{d}E'\ \frac{(E/E_0)^{1/2}}{E - E_0} - i\pi \sqrt{\frac{E}{E_0}})\ .$$

Matrix $\mathcal{W}_\mu \mathcal{W}_\nu$ has single nonzero eigenvalue $\sum_\mu \mathcal{W}_\mu^2$. Transform to that basis. Transformation leaves $H^{\rm GOE}$ unchanged. So

$$H^{\rm eff}_{\mu\nu} = H^{\rm GOE}_{\mu\nu} + \delta_{\mu 1}\delta_{\nu 1} \sum_\rho \mathcal{W}_\rho^2\ (P \int \mathrm{d}E'\ \frac{(E/E_0)^{1/2}}{E - E_0} - i\pi \sqrt{\frac{E}{E_0}})\ .$$

Far above neutron threshold we have

$$\sum_\rho \mathcal{W}_\rho^2 = \frac{N dx}{\pi^2} \approx \frac{\lambda}{\pi}\ .$$

That is very much larger than $\langle (H_{11}^{\rm GOE})^2 \rangle^{1/2} = \lambda \sqrt{2/N}$. Thomas-Ehrman shift is big at neutron threshold, especially for Pt isotopes! A doorway state!

(i) Effect on average level density: Doorway state has spreading width

$$\Gamma^{\downarrow} = \frac{2\pi}{d} \langle (H^{\rm GOE}_{1\mu})^2 \rangle = 2\lambda \ .$$

The doorway state is not observable in the spectrum. Completely smeared out.

(ii) Effect on distribution of partial neutron widths: Effective Hamiltonian is

$$H^{\rm eff}_{\mu\nu} = H^{\rm GOE}_{\mu\nu} + \delta_{\mu 1} \delta_{\nu 1} Z$$

Perturbation has rank one. Calculate distribution of reduced widths $\gamma^2_\mu \propto \langle 1 | \Phi_\mu \rangle^2$ exactly (Bogomolny) or by numerical diagonalization (VWZ). Strength parameter is $\kappa = Z / \lambda$. Compare with PTD. Variable is $x = \gamma^2 / \Gamma$.

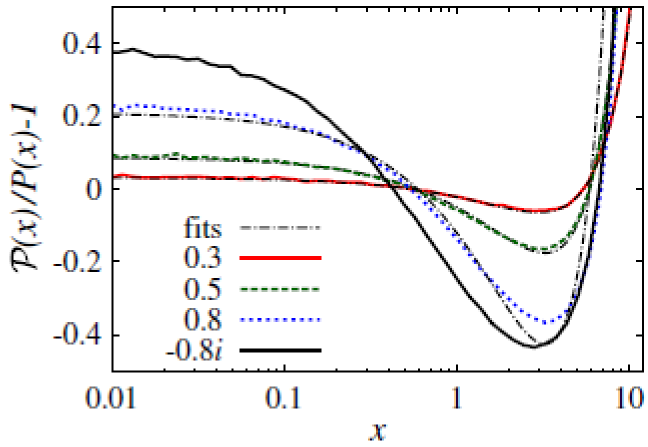

FIG. 1 (color online). Relative difference $[\mathcal{P}(x)/P(x)] - 1$ as a function of x for several values of κ as indicated in the figure and for $N = 1000$. The dashed-dotted lines are fits using expression (8).

$$\mathcal{P}(x) = [1 + A(1 - x) + B(x^2 - 6x + 3)]P(x) \qquad (8)$$

That same dependence on x is obtained analytically
(E. Bogomolny, Phys. Rev. Lett. 118, 022501 (2017)).

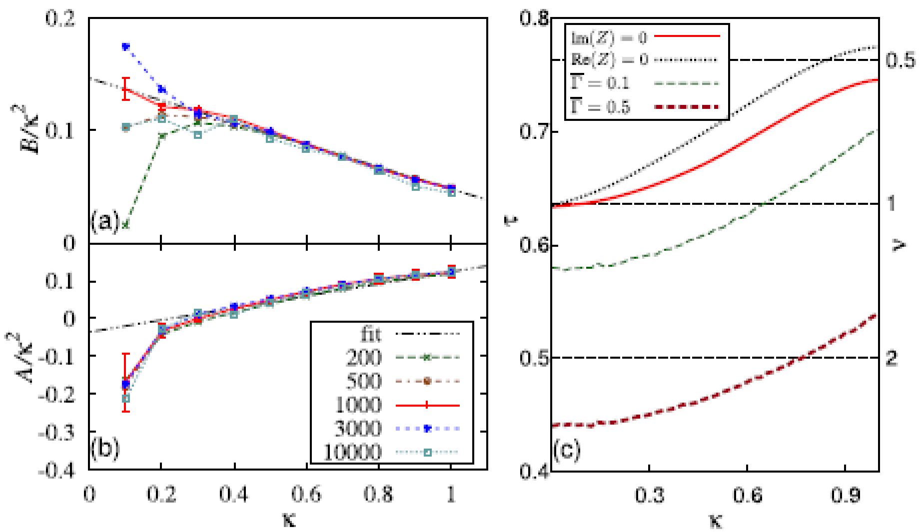

FIG. 2 (color online). Scaled fit coefficients: (a) B/κ^2 and (b) A/κ^2 with A and B defined in Eq. (8) are plotted as functions of κ for GOE ensembles of different dimensions N as indicated in the figure. For $N = 1000$ the error in the fit is shown by error bars. The errors for the other curves are similar. The double-dot-dashed black line shows the linear fits [Eq. (9)] of A/κ^2 and B/κ^2 as functions of κ. Right: (c) The coefficient τ defined in Eq. (10) is plotted versus κ for different widths of multiple random γ channels as explained in the text. The scale on the right shows the corresponding ν values.

$$\tau = \frac{1}{2\bar{x}} \int dx \int dx' |x - x'| \mathcal{P}(x) \mathcal{P}(x'). \qquad (10)$$

Gini coefficient ranges from zero to unity and decreases with increasing nu.

$$\nu = 1 \ : \ \tau = 2/\pi \ ; \ \nu = 2 \ : \ \tau = 1/2$$

3b. Gamma Channels

Koehler reports deviations of distribution of total gamma widths of neutron resonances from GOE prediction. Is Thomas-Ehrman shift responsible? Or similar effect due to gamma channels themselves? Orthogonality of gamma channels $\sum_\mu W_\mu^\gamma W_\mu^{\gamma'} = Z_\gamma \; \delta_{\gamma\gamma'}$ with $\gamma = 2, 3, \ldots, \Lambda$, $\Lambda \gg 1$ can be used to write

$$H_{\mu\nu}^{\rm eff} = H_{\mu\nu}^{\rm GOE} + \delta_{\mu 1}\delta_{\nu 1} Z - i\pi\delta_{\mu\gamma}\delta_{\nu\gamma} Z_\gamma \ .$$

Individual coupling strengths obey

$$Z_\gamma / \lambda = x_\gamma \ll 1$$

Investigate influence of $Z/\lambda = x_n$ and of x_γ on distribution of reduced gamma widths for $N \to \infty$ and for $\Lambda \gg 1$.

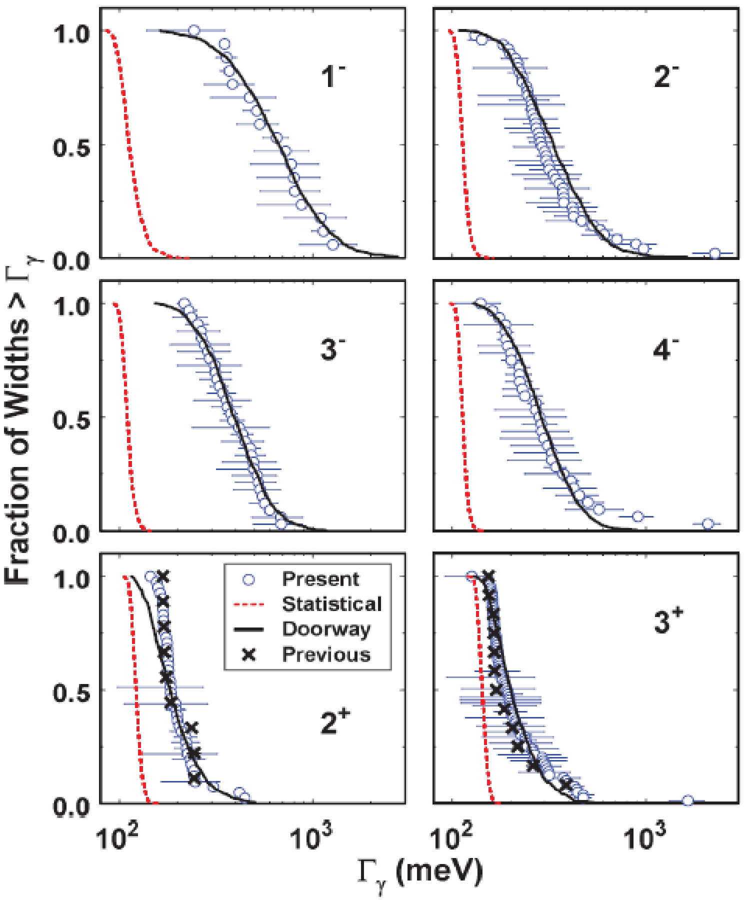

FIG. 6. (Color online) Cumulative Γ_γ distributions for ^{95}Mo + n resonances. Each panel shows the fraction of resonances having Γ_γ larger than a certain size vs the size. Data from the present work and those based on previous firm J^π values are shown as open blue circles and black x's, respectively. Error bars depict one-standard-deviation uncertainties as reported by SAMMY. Statistical-model simulations with and without an added doorway are shown as solid black and dashed red curves, respectively. Simulated Γ_γ values have been normalized by factors of 0.75, 0.9, 0.85, 1.1, 1.0, and 1.2 for 1^-, 2^-, 3^-, 4^-, 2^+, and 3^+ resonances, respectively. Not shown is the smallest 2^- Γ_γ (39.7 ± 2.9 meV).

P. E. Koehler, Phys. Rev. C 88, 041305 (2013).

Y. Alhassid and P. Fanto, private communication

Typically choose $N = 1000, \Lambda = 100, x_\gamma = 0.01$ all equal. Numerical matrix diagonalization. Plot logarithm of PTD with width normalized to unity. Fig. 1 shows that distribution of reduced gamma widths differs from PTD. Fig. 2 shows cross-channel effect on reduced neutron width. No cross-channel effect on gamma channels from neutron channel. But modified distribution of reduced gamma widths does not alter noticeably cumulative fraction of total gamma decay widths.

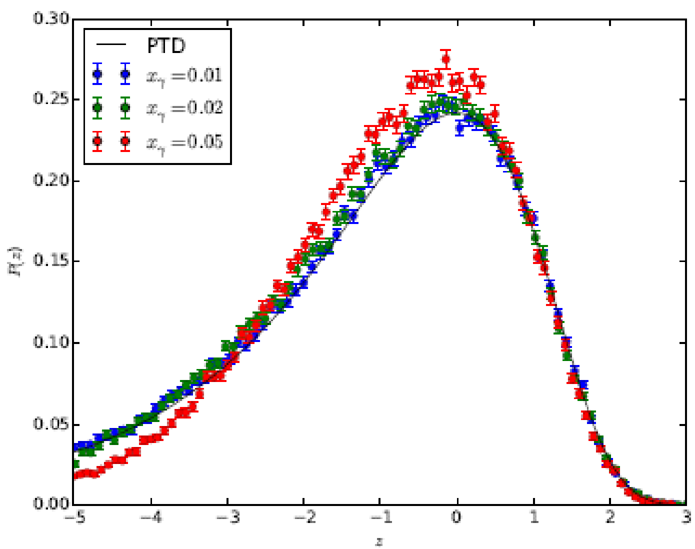

FIG. 1: The distributions of $\ln y = \ln\left[C\,|V_{2\mu}|^2\right]$ in the gamma channel for various shifts x_γ in $\Lambda = 100$ gamma channels. The solid black line is the PTD.

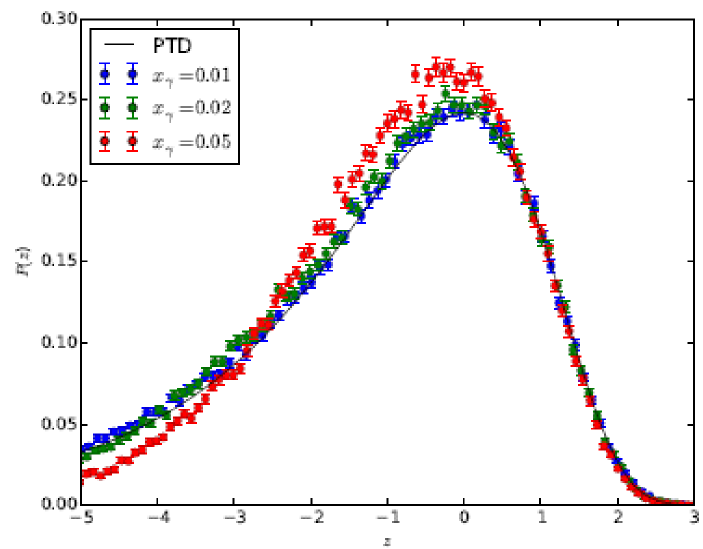

FIG. 2: The distributions of $\ln y = \ln\left[C\,|V_{2\mu}|^2\right]$ in the neutron channel for various shifts x_γ in $\Lambda = 100$ gamma channels. The solid black line is the PTD.

4. Summary

PTD is based upon orthogonal invariance of effective Hamiltonian. We have identified two possible sources for violation of that invariance: Thomas-Ehrman shift and large number of gamma channels. Thomas-Ehrman shift: Reasonable estimates show big effect on distribution of neutron widths. Ranges of nu values found in our analysis overlap with nu values found for Pt isotopes and for NDE. But only detailed analysis in each nucleus using all available experimental information can lead to definitive conclusions.

Many weakly coupled gamma channels do affect PTD distribution in every channel (including neutron channel). But effect is too weak to account for observed deviations of distribution of total gamma decay widths in Mo from GOE prediction. Remains open question.

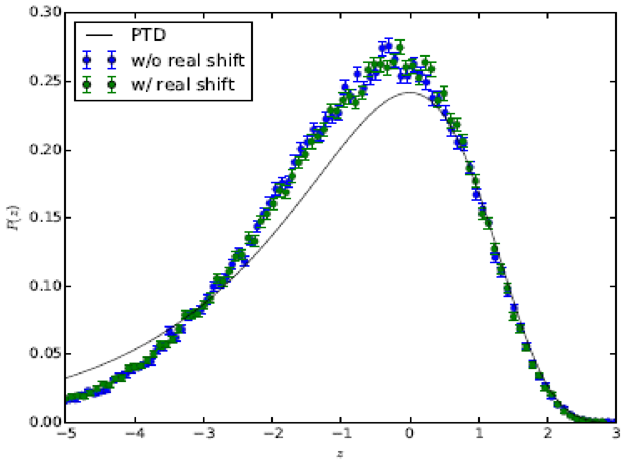

FIG. 3: The distributions of $\ln y = \ln\left[C\,|V_{2\mu}|^2\right]$ in the gamma channel for shifts $x_\gamma = 0.05$ in $\Lambda = 100$ gamma channels and $x_n = 1.0$ in one neutron channel. The solid black line is the PTD.

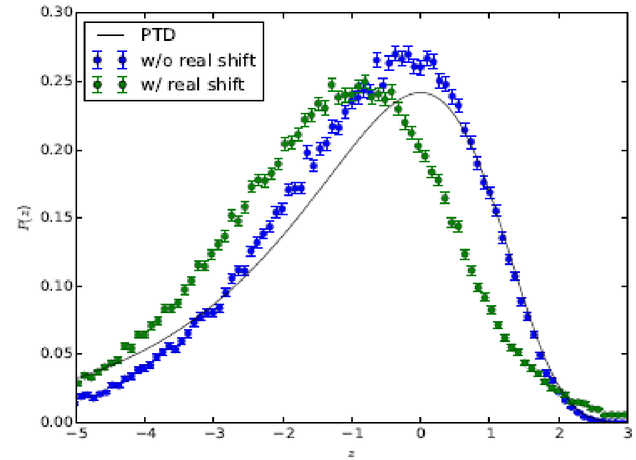

FIG. 4: The distributions of $\ln y = \ln\left[C\,|V_{2\mu}|^2\right]$ in the neutron channel for shifts $x_\gamma = 0.05$ in $\Lambda = 100$ gamma channels and $x_n = 1.0$ in one neutron channel. The solid black line is the PTD.

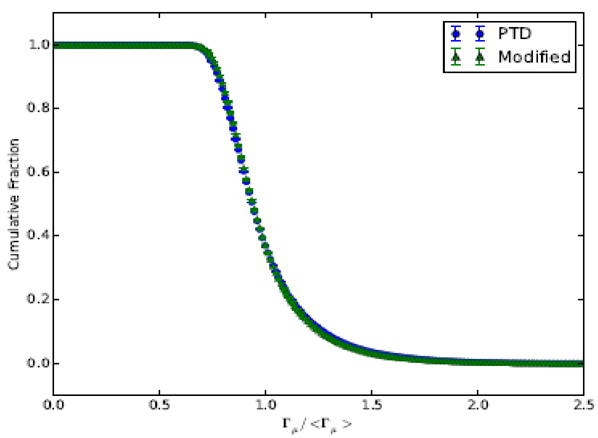

FIG. 11: The average cumulative fraction of the total gamma widths. The result obtained by using the PTD as the partial width distribution is compared with the result obtained by using the modified partial width distribution.