

Randomly Interacting Bosons on 2 spin levels

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NSCL March 9, 2017



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Outline Spin 3/2 talk

- 1/2 Observations
- Random H, ground state statistics, energy gaps, wave functions
- 2/2 Explorations
 - N-bosons on 2 levels, toy system
- 3/2 Explanations
 - Geometric chaoticity, group theory, random polynomials, cfp, mean fields

Real Interactions

- Real spectra, with strong features have *physical* explanations
- even-even nuclei (EE) have $J_0=0$, $J_1=2$ (mostly)
- pairing gap
- $J_1=2$ decay-strong B(E2) interactions
- Pairing, rotational/vibrational bands
- regular gamma cascades - deformed nuclei
- Quadratic yrast lines $E(J)=J(J+1)/2\mathcal{I}$
- Matrix elements from experiment/physical arguments

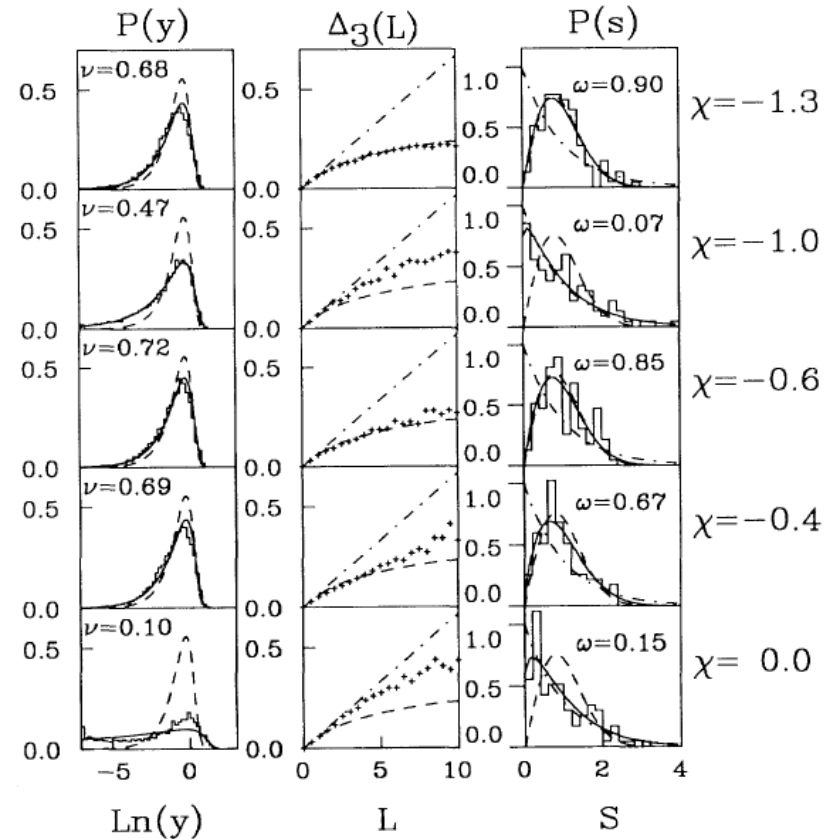
Random Hamiltonians

- RMT is working definition of Quantum Chaos
- Actually useful
- *standard conversation*
- Statistical spectroscopy $P(s)$, $\Delta_3(L)$, detect missed levels
- Quite a surprise to see physical looking regular spectra
- *New conversation* correlations between different classes
($J=3$ vs $J=4$ for example) sectors

Observations

Early signs Random IBM

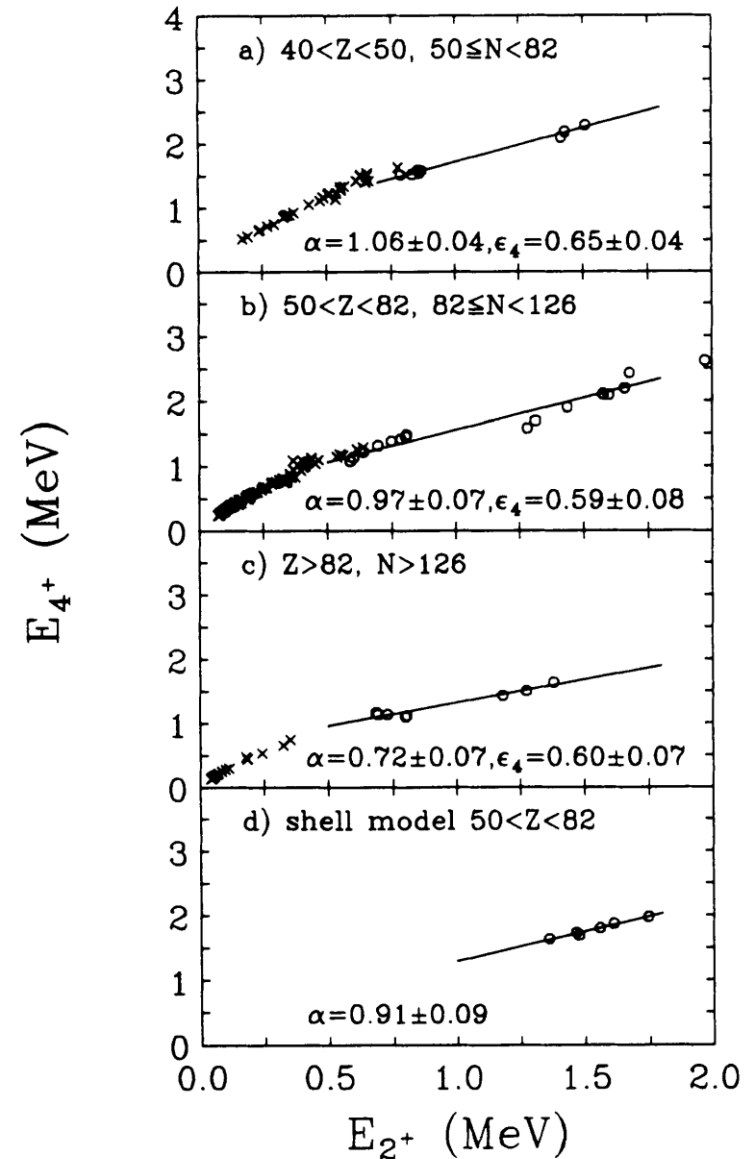
- η χ separating parts of the Hamiltonian with different symmetry
- Usual RMT analysis Regions of chaotic and non-chaotic spectra in η χ (Casten's triangle)
- Y. Alhassid and N. Whelan, Phys. Rev. Lett. 67, 816 (1991)



Observations

$$\Delta E_{4-2} \equiv E(4_1^+) - E(2_1^+) = \text{const.}$$

- Early signs Random Matrix, IBM
- J=2 states correlated with J=2 states
- $E(4^+) = \alpha E(2^+) + \epsilon$
- True for nuclei
- **reproduced in shell model for almost any interaction**
- Suggests Pair transfer collectivity
(build J+2 state by adding pair to J state)
- N. V. Zamfir, R. F. Casten, and D. S. Brenner, Phys. Rev. Lett. 72, 3480 (1994),

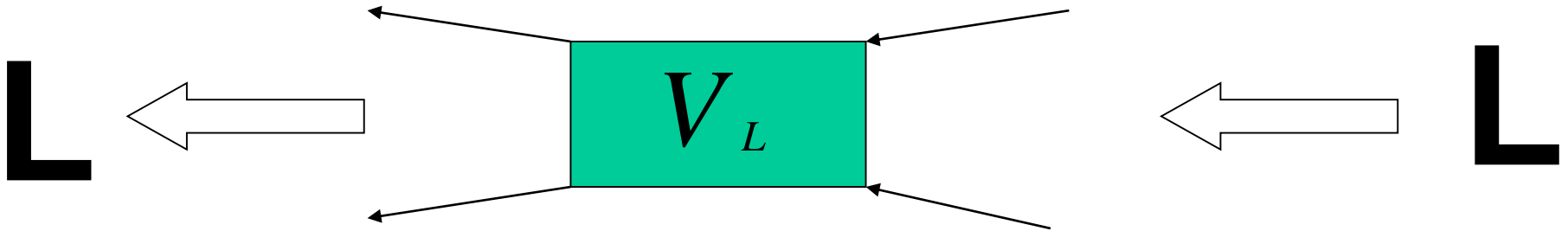


Purely Random H

- C. W. Johnson, G. F. Bertsch, and D. J. Dean, Phys. Rev. Lett. 80, 2749 (1998)
- C. W. Johnson, G. F. Bertsch, D. J. Dean, and I. Talmi, Phys. Rev. C 61, 014311
- N=6 particles, 3 or 4 levels [$j=1/2, 3/2, 5/2(7/2)$]
- $J_{gs}=0$ f_p should be small, it was comparable to 0.52 ± 0.27
- “Pairing Gap” For very collective SM states it is 0.85. S is a pair operator
- “Rotational Bands”
- “Pair Transfer Collectivity”
$$f_p = \frac{\langle A-2 | S | A \rangle^2}{\langle A | S^\dagger S | A \rangle}$$
- Quadratic yrast lines $E(J)=J(J+1)/2\mathcal{J}$

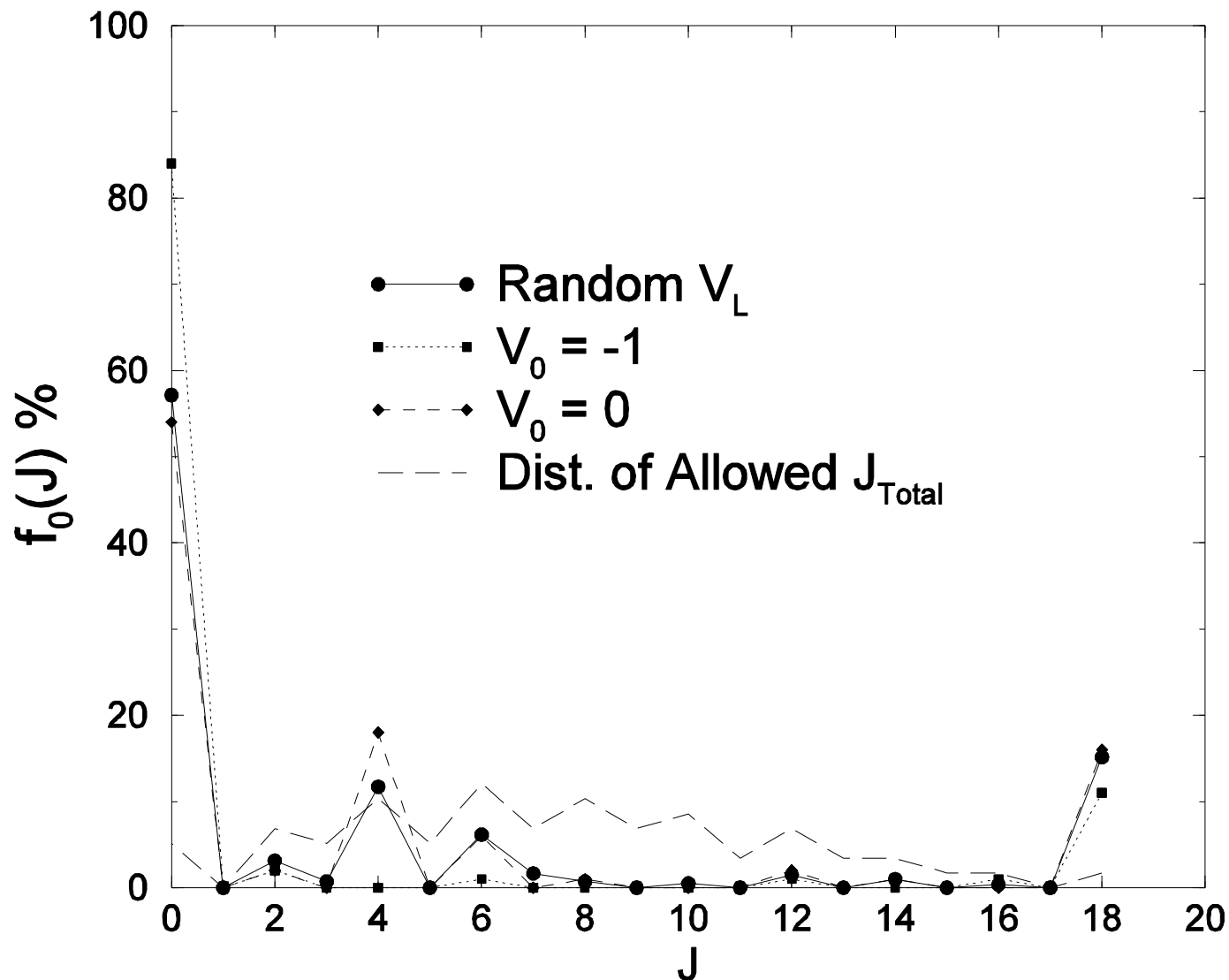
Broad range of systems show these signs
N particles, 1 level

$$H = \frac{1}{2} \sum_{L\Lambda} V_L C_{m_1 m_2}^{L\Lambda} C_{m_3 m_4}^{L\Lambda} a_2^+ a_1^+ a_3 a_4$$

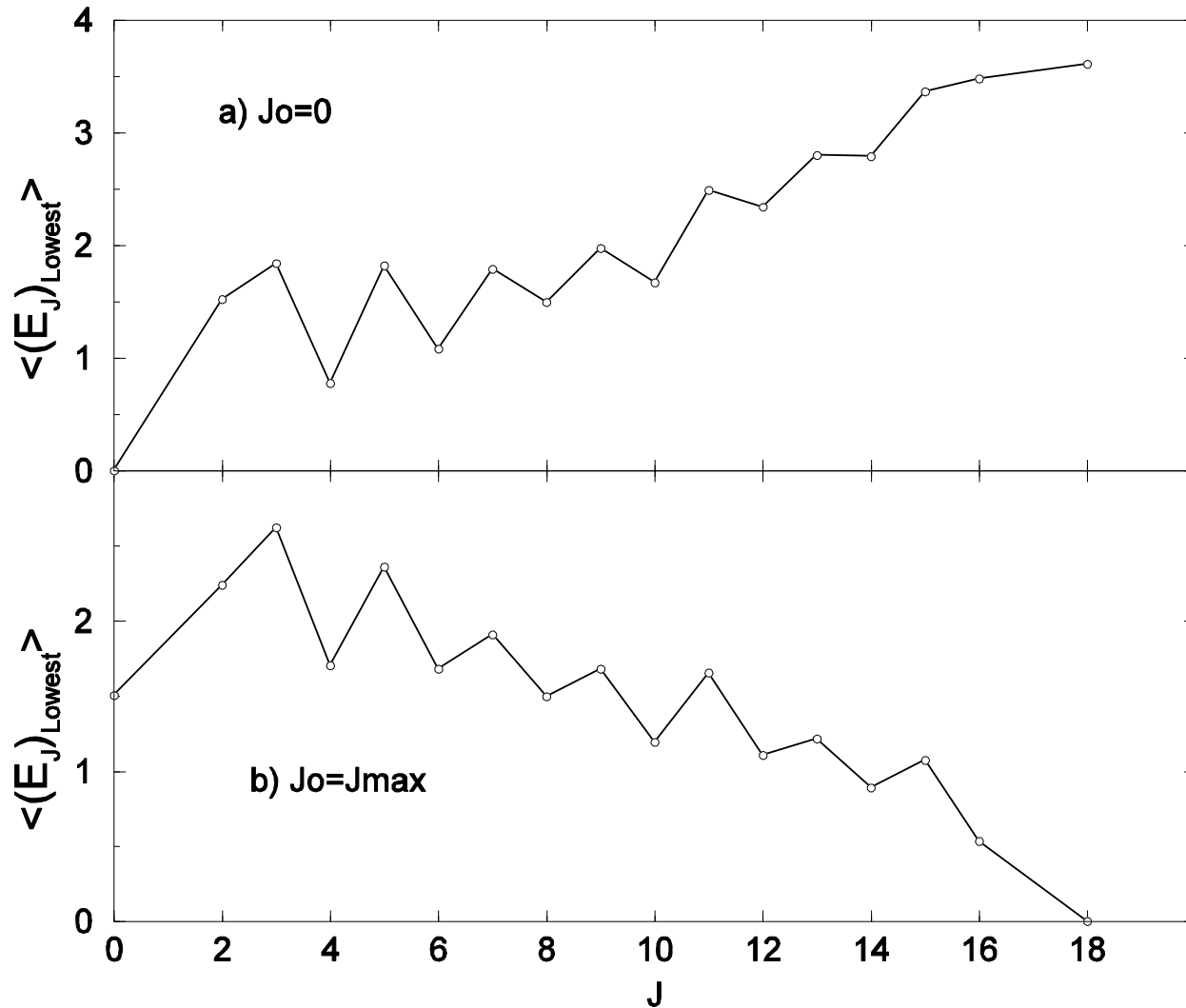


Typical system $N=6$ $j=11/2$

pairing term not crucial for $J_{gs}=0$ dominance



Yrast Line $N=6$ $j=11/2$



Purely Random H

- C. W. Johnson, G. F. Bertsch, and D. J. Dean, Phys. Rev. Lett. 80, 2749 (1998)
- C. W. Johnson, G. F. Bertsch, D. J. Dean, and I. Talmi, Phys. Rev. C 61, 014311

If $f = 1$, then the excited state is completely described as

a particle-hole excitation of the ground state. If f is very

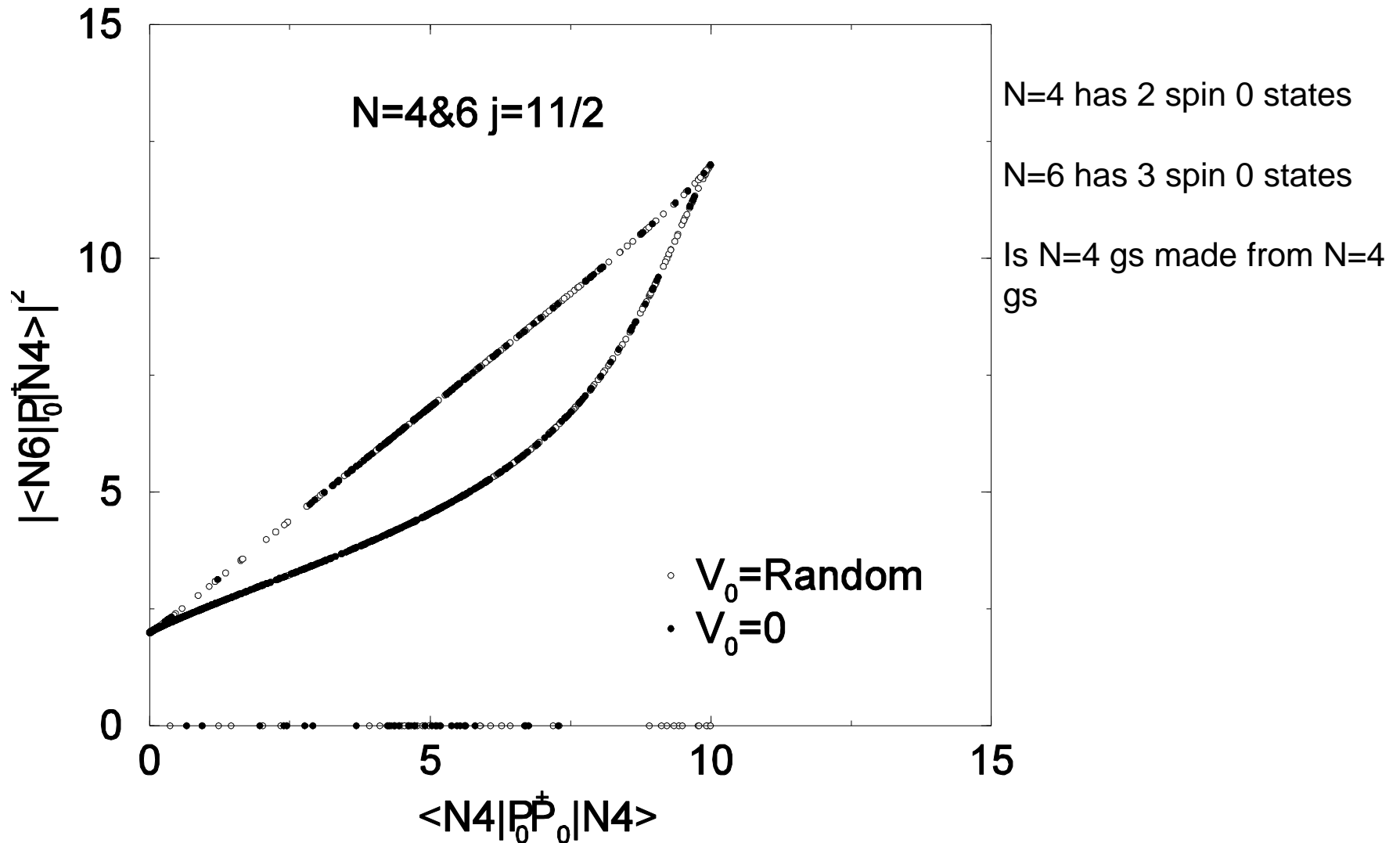
small then the two states are connected only by manybody

operators.

f_p should be small, it was
comparable to 0.52 ± 0.27
For very collective SM states
it is 0.85. S is a pair operator

$$f_p = \frac{\langle A - 2 | S | A \rangle^2}{\langle A | S^\dagger S | A \rangle}$$

Pair transfer collectivity



Rotational bands

- R. Bijker, A. Frank, and S. Pittel, Phys. Rev. C 60, 021302 (1999),
- R. Bijker and A. Frank, Phys. Rev. Lett. 84, 420 (2000)
- R. Bijker and A. Frank, Phys. Rev. C 62, 014303 (2000)
- Turn t-rev inv. off $J_{gs}=0$ increases !
- (N=16, IBM) Looked at P(R) and B(E2) ratios

TABLE I. Energies of $B(E2)$ values in the dynamical symmetry limits of the IBM [6]. In the U(5) and SO(6) limits we show the result for the leading order contribution to the rotational spectra.

	$\frac{E(4^+) - E(0^+)}{E(2^+) - E(0^+)}$	$\frac{B(E2; 4^+ \rightarrow 2^+)}{B(E2; 2^+ \rightarrow 0^+)}$
U(5)	2	$\frac{2(N-1)}{N}$
SO(6)	$\frac{5}{2}$	$\frac{10(N-1)(N+5)}{7N(N+4)}$
SU(3)	$\frac{10}{3}$	$\frac{10(N-1)(2N+5)}{7N(2N+3)}$

Rotational bands

- R. Bijker and A. Frank, Phys. Rev. Lett. 84, 420 (2000)

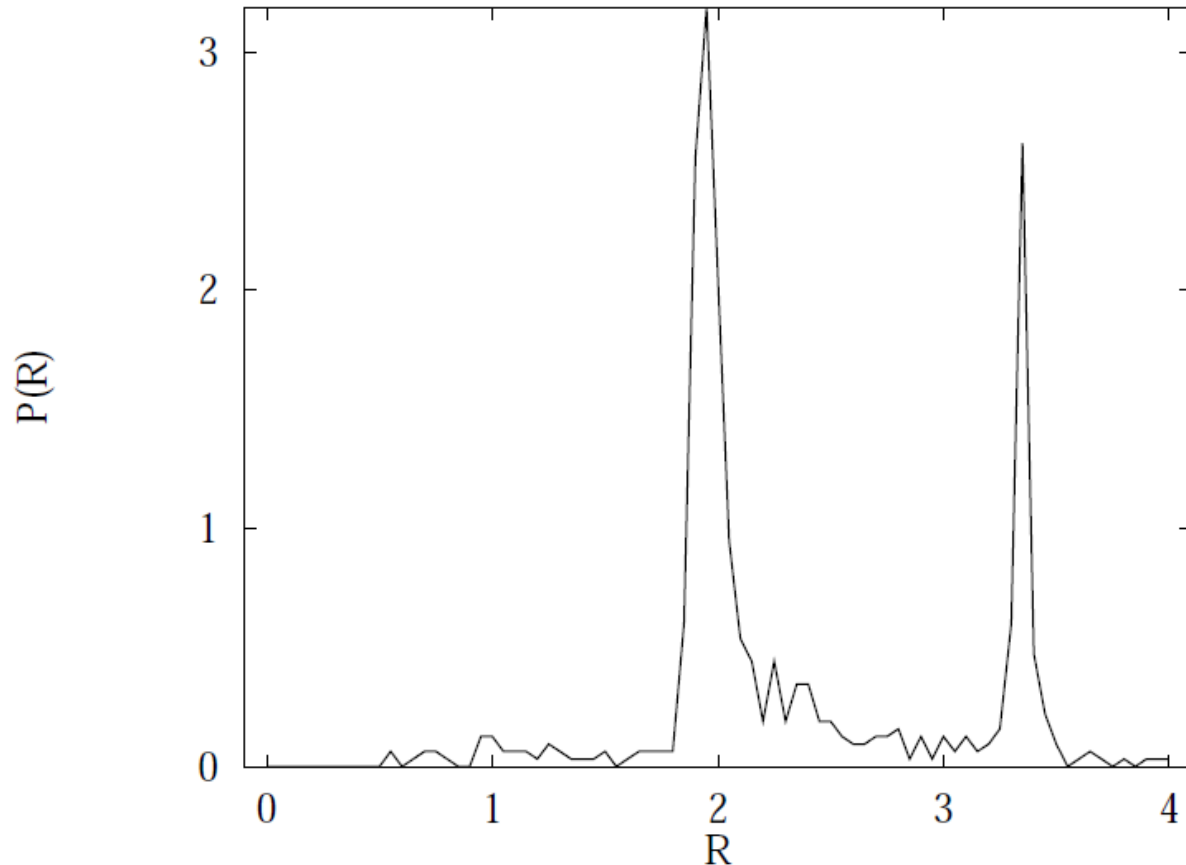


FIG. 1. Probability distribution $P(R)$ of the energy ratio $R = [E(4^+) - E(0^+)]/[E(2^+) - E(0^+)]$ with $\int P(R) dR = 1$ in the IBM with random one- and two-body interactions.

Rotational bands

– R. Bijker and A. Frank, Phys. Rev. Lett. 84, 420 (2000)

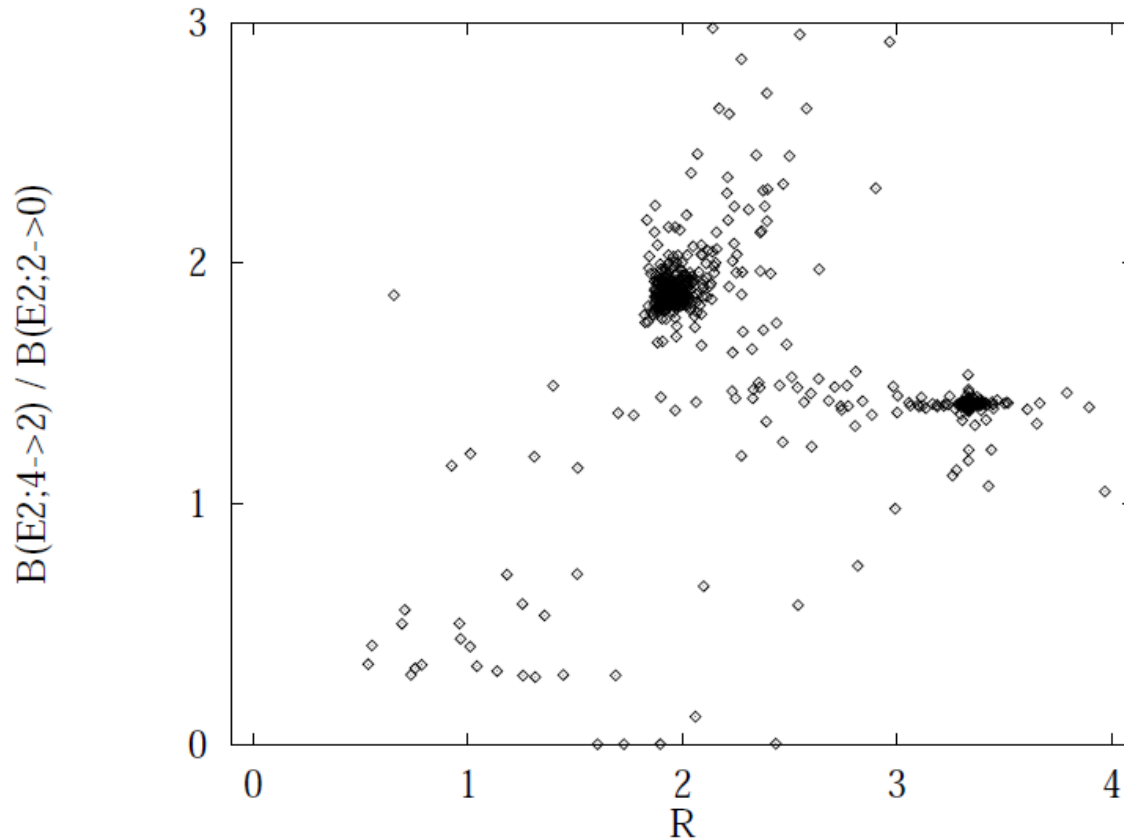


FIG. 2. Correlation between ratios of $B(E2)$ values and energies in the IBM with random one- and two-body interactions.

$J_{gs}=0$ but no pairing

- L. Kaplan, T. Papenbrock, and C. W. Johnson, Phys. Rev. C 63, 014307 (2000),

Correlation between different classes

N spin-1/2 particles on M orbitals TBRE (2 numbers C_0 and C_1)

Higher spin coupling ($C_1 > C_0$) decreases $J_{gs}=0$ (from nearly 100%)

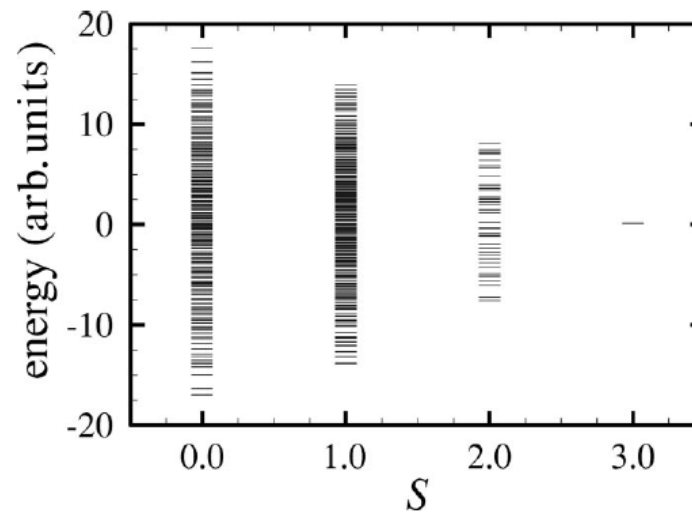


FIG. 1. Spectra for a system of six fermions on six orbitals with $S_z=0$ as a function of total spin S .

IBM vs sd Shell model

Y. M. Zhao and A. Arima, Phys. Rev. C 64, 041301 (2001)

TBRE gives vibrational but not rotational $P(E4/E2)$

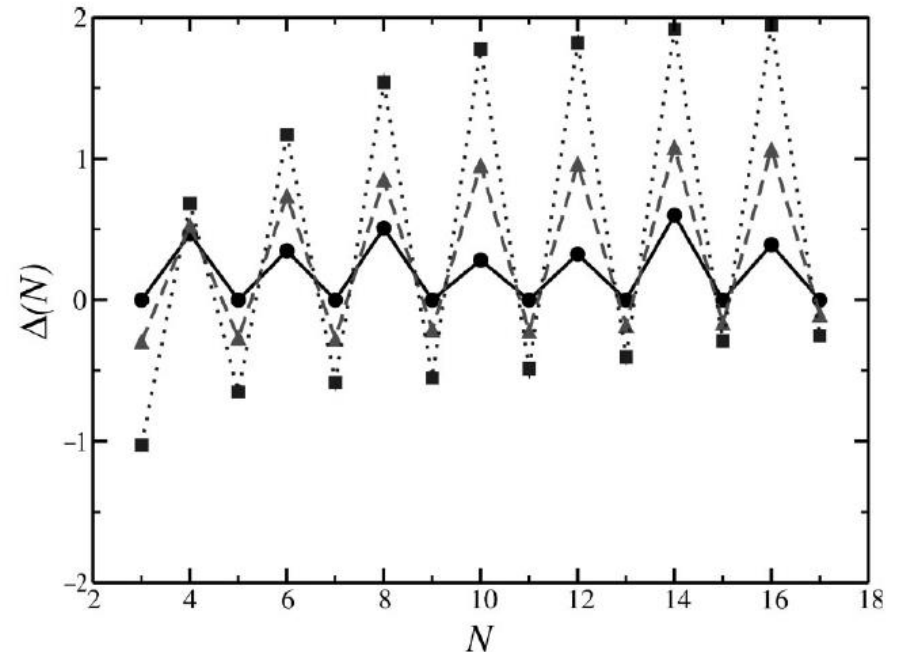
Mix realistic and TBRE

V. Velazquez, J. G. Hirsch, A. Frank, and A. P. Zuker, Phys. Rev. C 67, 034311 (2003),

- What survives when you go random?
- $H = a H_C + b H_R \quad a+b=1$
- Yrast ordering preserved
- $B(E2)$ lose quadrupole collectivity

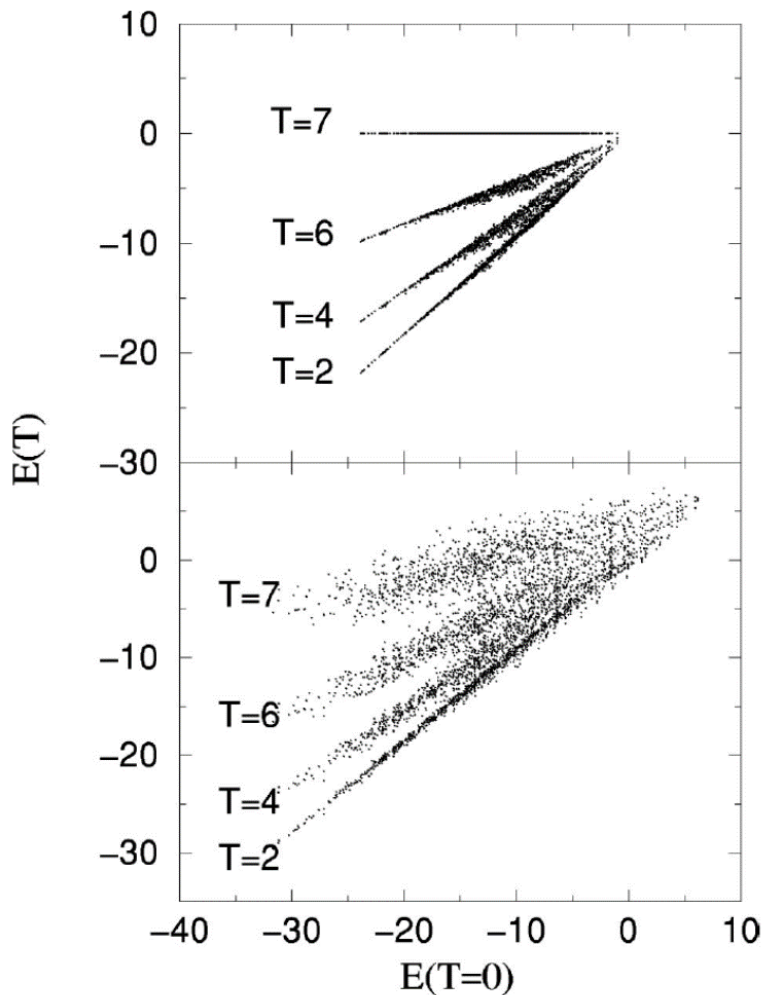
Odd even effects from TBRE

- T. Papenbrock, L. Kaplan, and G. F. Bertsch, Phys. Rev. B 65, 235120 (2002)
- N spin 1/2 particles on M levels S=0,1 pairs
- pairing gap
$$\Delta(N) \equiv \frac{1}{2} [E(N+1) - 2E(N) + E(N-1)]$$
- Turn on rTBRE slowly
- Watch $\Delta(N)$ increase



Rotational bands in Isospin space and O-E effects

M. Horoi, A. Volya, and V. Zelevinsky, Phys. Rev. C 66, 024319 (2002)

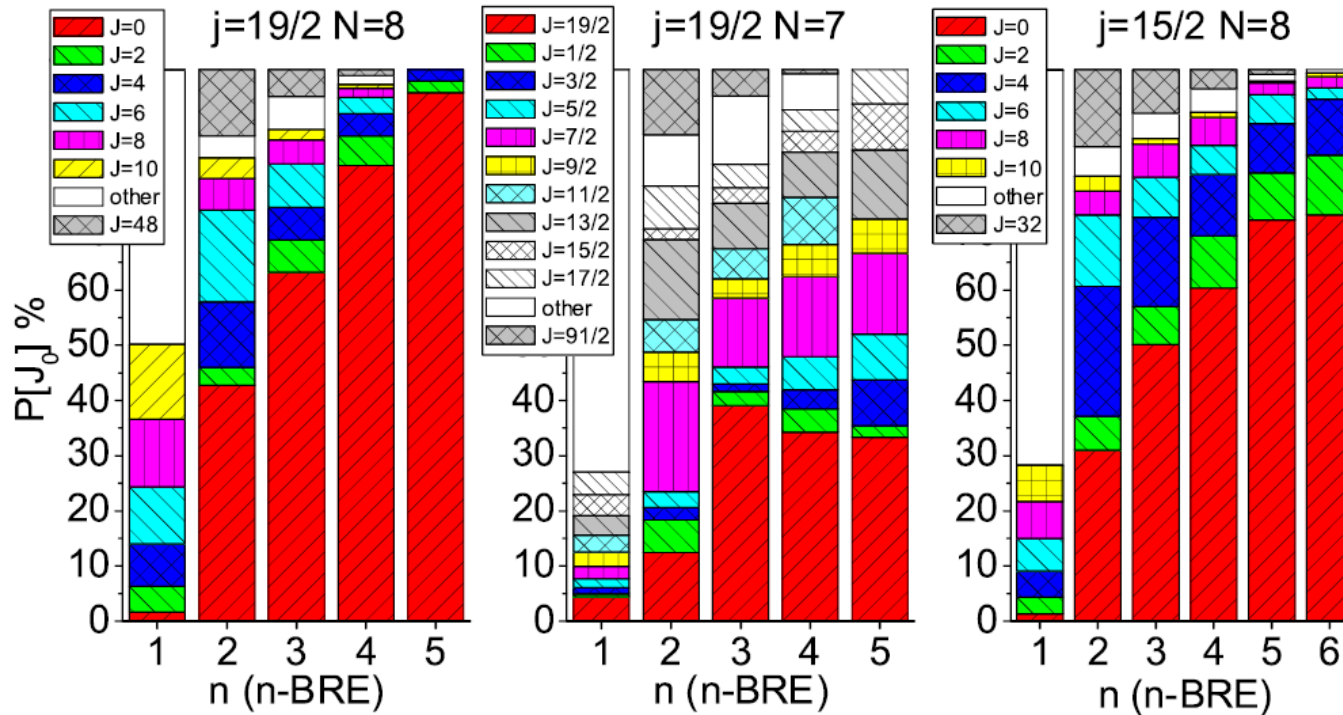


- sd shell model and single j
- fermion pairs treated like quasi bosons
- Chaotic not collective states
- rule for enhanced $J_0=0$ and $T_0=0$

$$(-1)^{J_0+T_0} = (-1)^{N/2}$$

one last thing A. Volya, Phys. Rev. Lett. 100, 162501 (2008)

- n-body interactions
- $J_{gs}=0$ stronger with increasing n
- $T_{gs}J_{gs}=00$ stronger



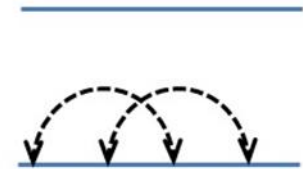
Toy model

- Work done with Dustin Frisbie NSCL
- Bosons on 2 levels
- Undergrad research
- Rich ideas, easy to see.
 - Make basis
 - Work in $M=0$
 - Get E from H , J from J^2
 - RMT analysis

Levels almost degenerate, all interactions Gaussian
RMT signatures of chaos present.



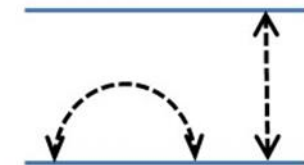
H_{2222}



H_{1111}



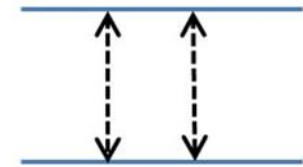
H_{1222}



H_{1112}

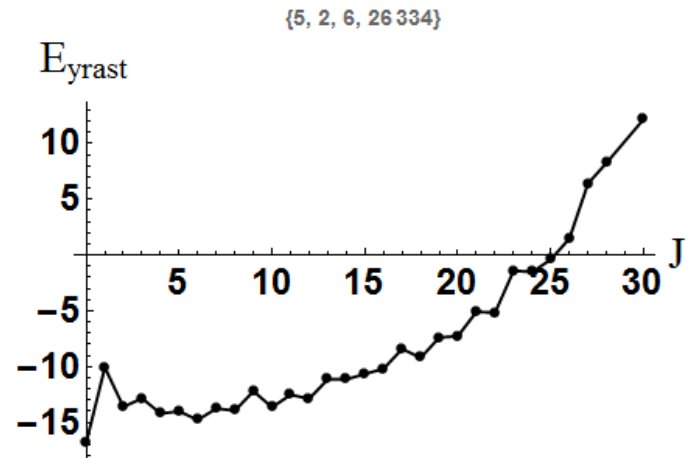
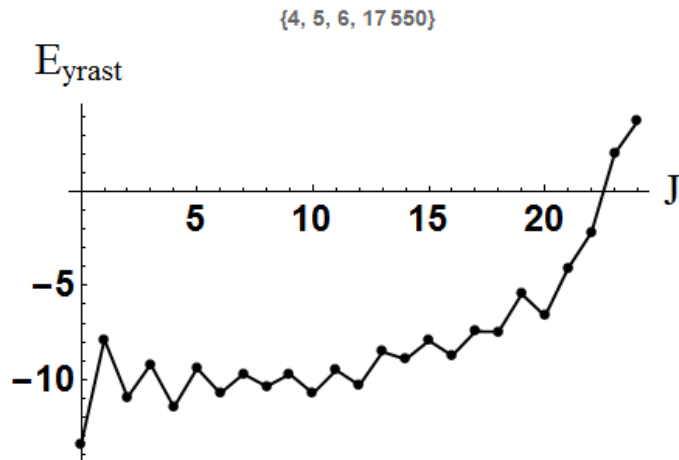
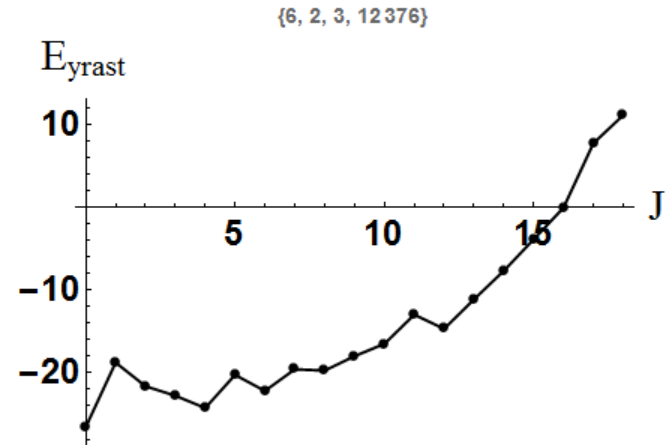
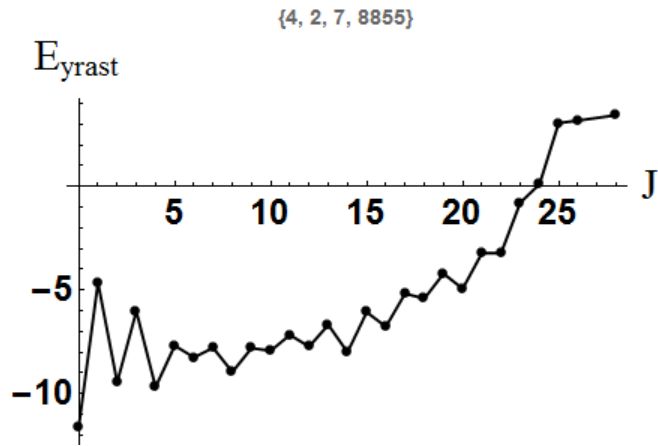


H_{1212}



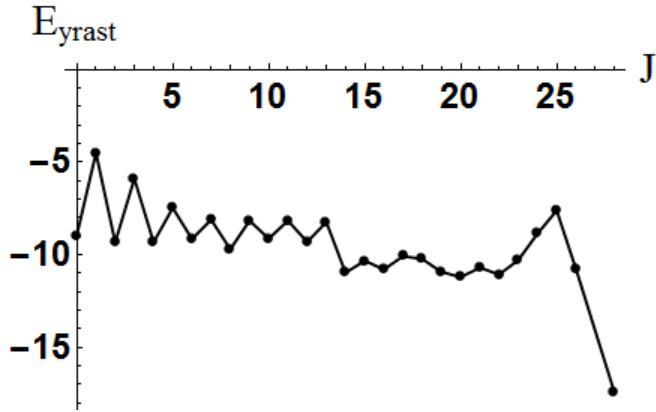
H_{1122}

$J_{gs}=0$ gives parabolic yrast lines ($j_1 > 1$)

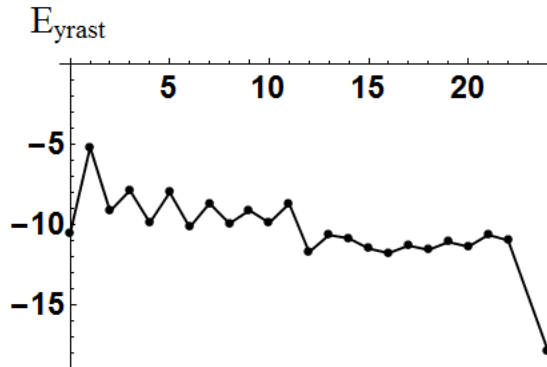


$J_{gs} = J_{max}$ gives stepped yrast lines ($j_1 > 1$)

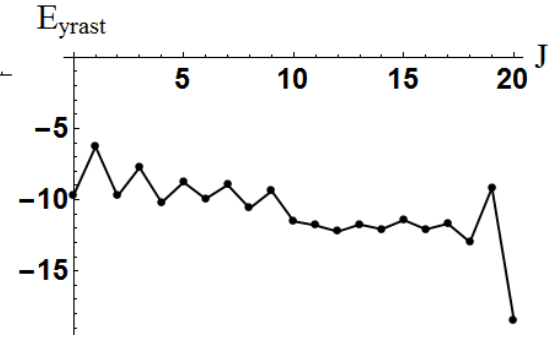
{4, 2, 7, 8855}



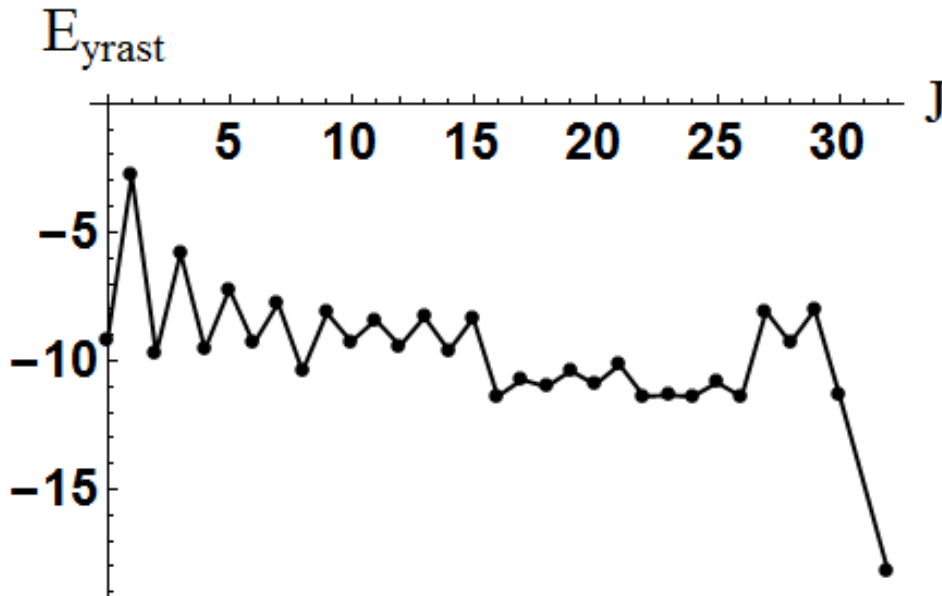
{4, 3, 6, 8855}



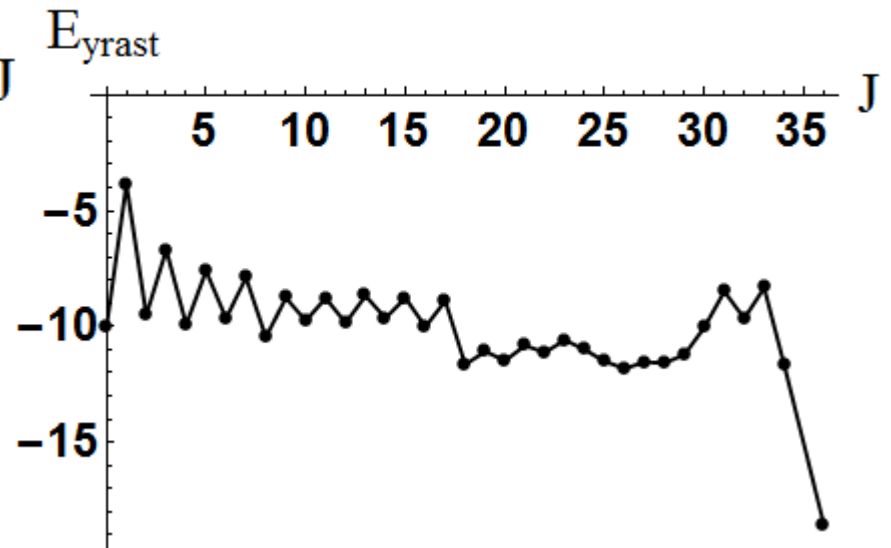
{4, 4, 5, 8855}



{4, 2, 8, 12650}

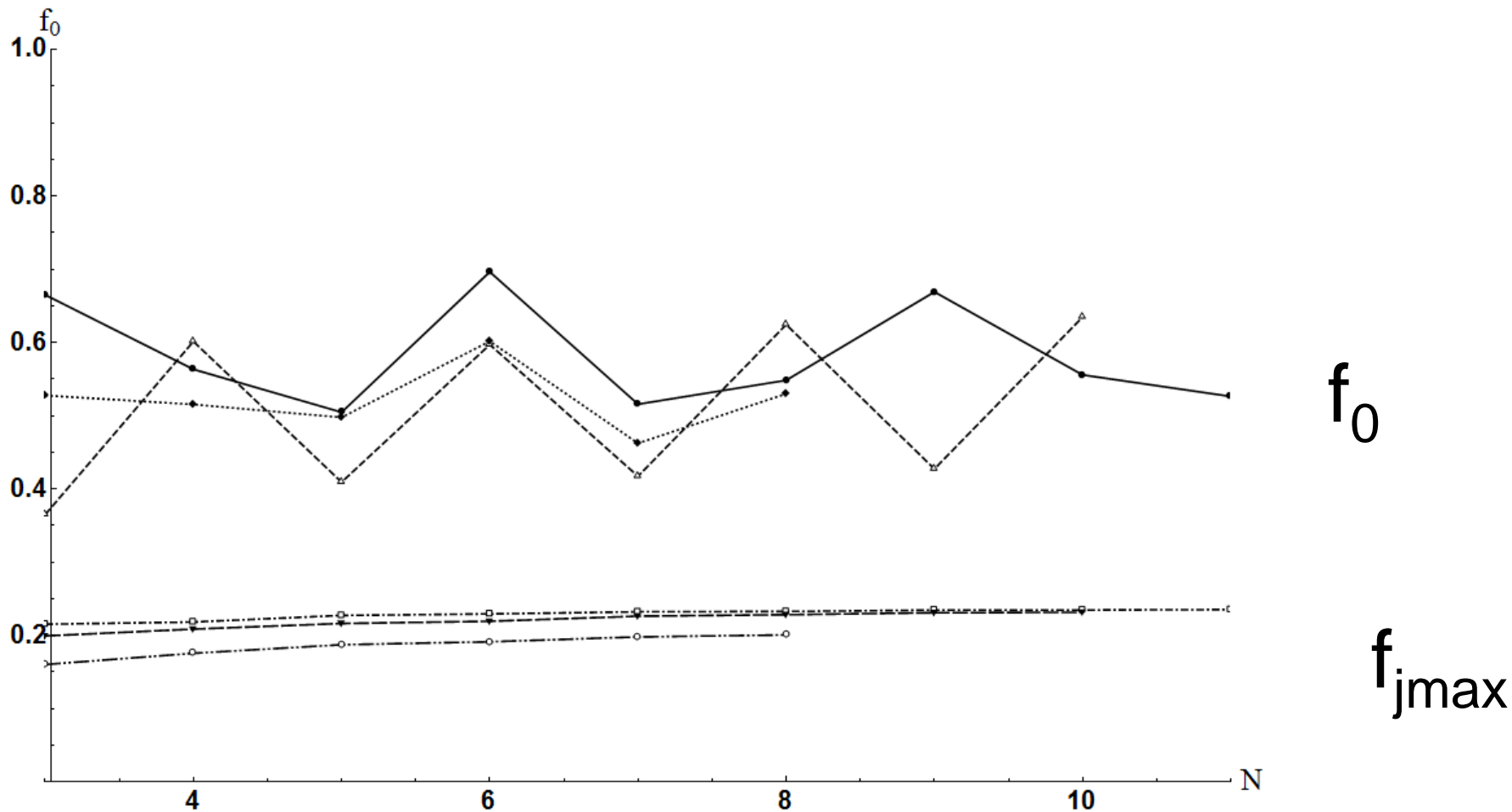


{4, 2, 9, 17550}

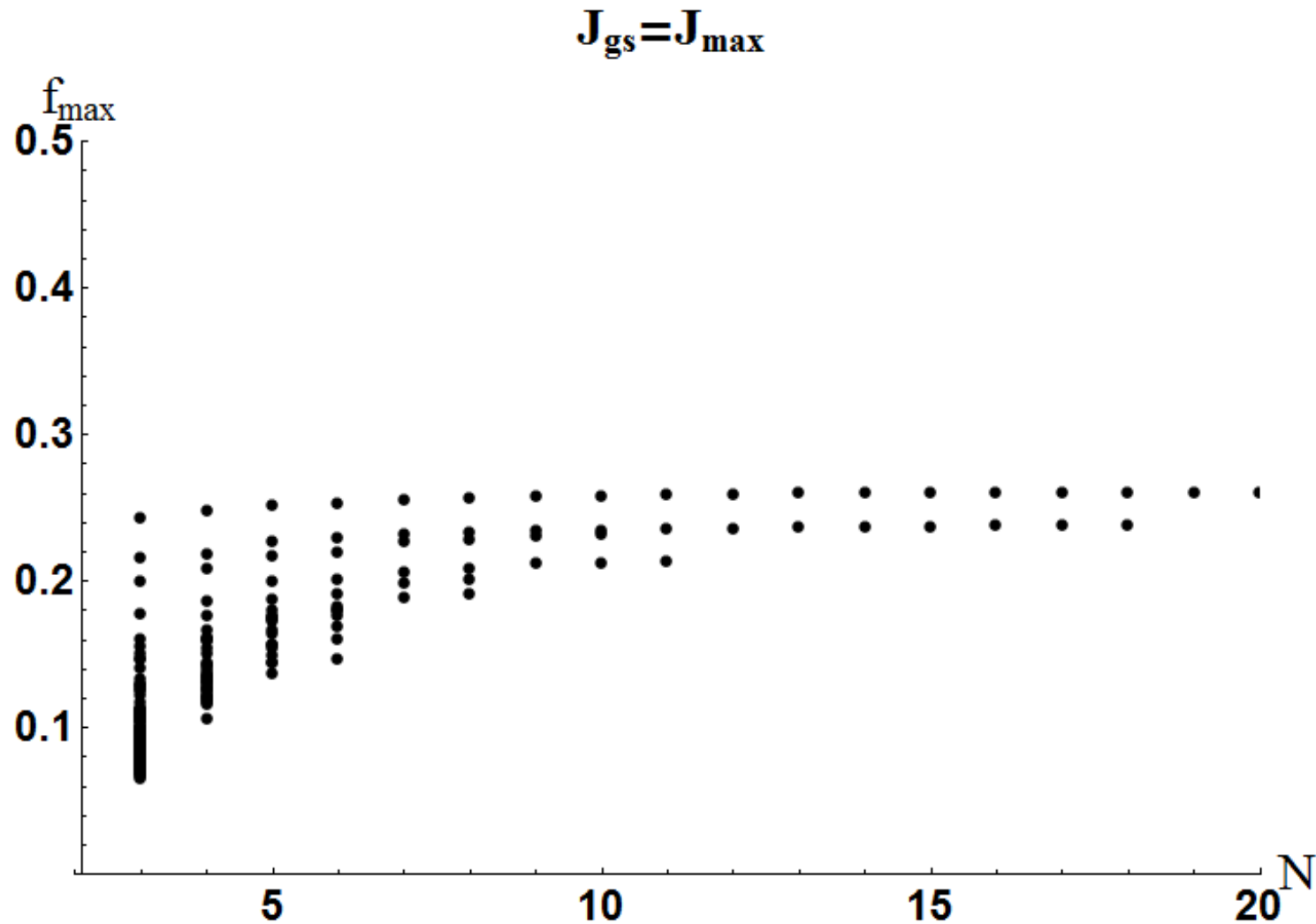


Fraction f of ground states with $J_{gs}=0$ or J_{max}
 $j_1=0, j_2 = 2,3,4$

$J_{gs}=0, J_{max}, (j_1 j_2)=(0 2) (0 3) (0 4)$

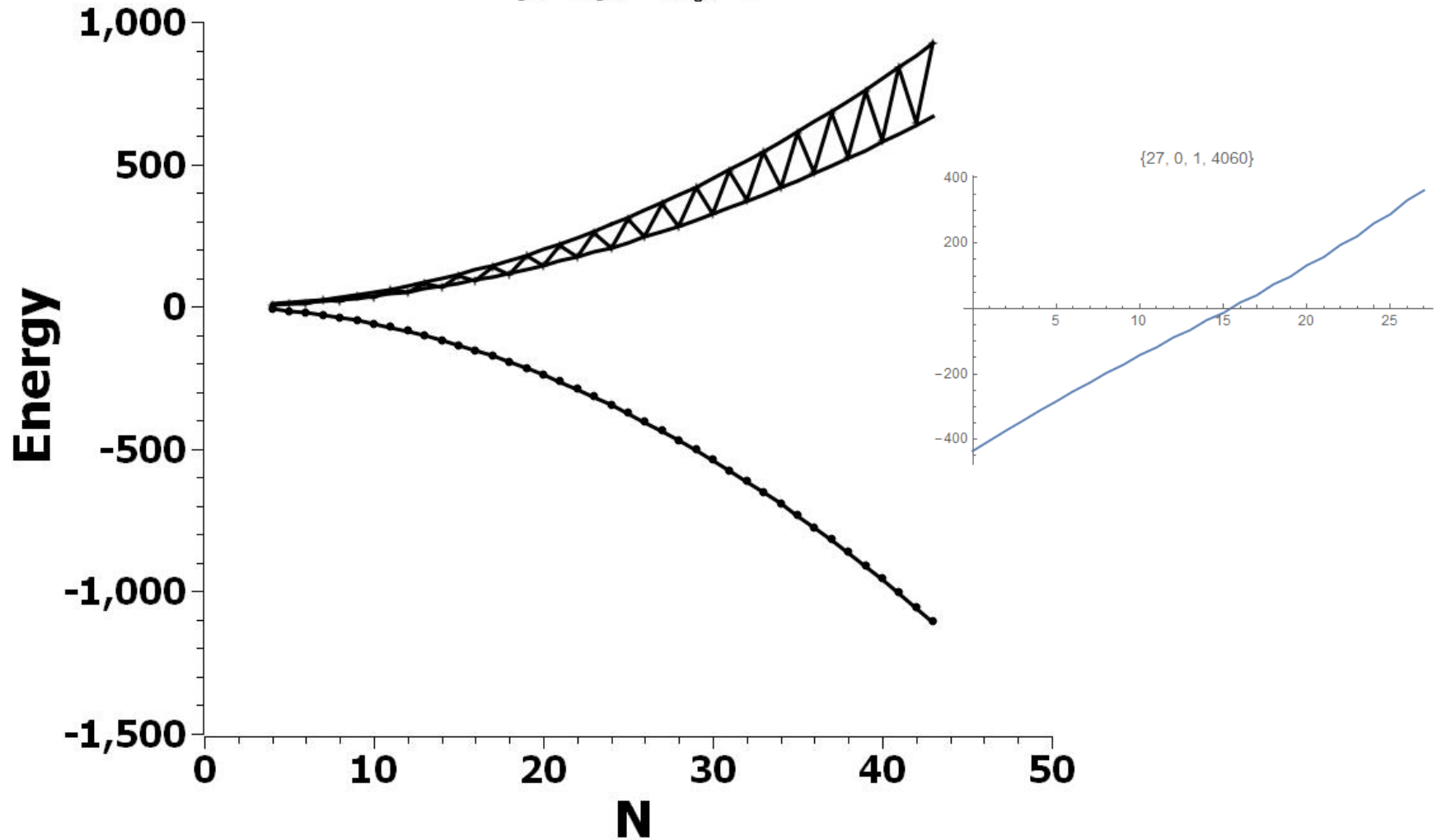


Fraction f_{\max} of ground states with $J_{\text{gs}}=J_{\text{max}}$
Around 20%, f_{\max} drops with increasing $j_1 j_2$



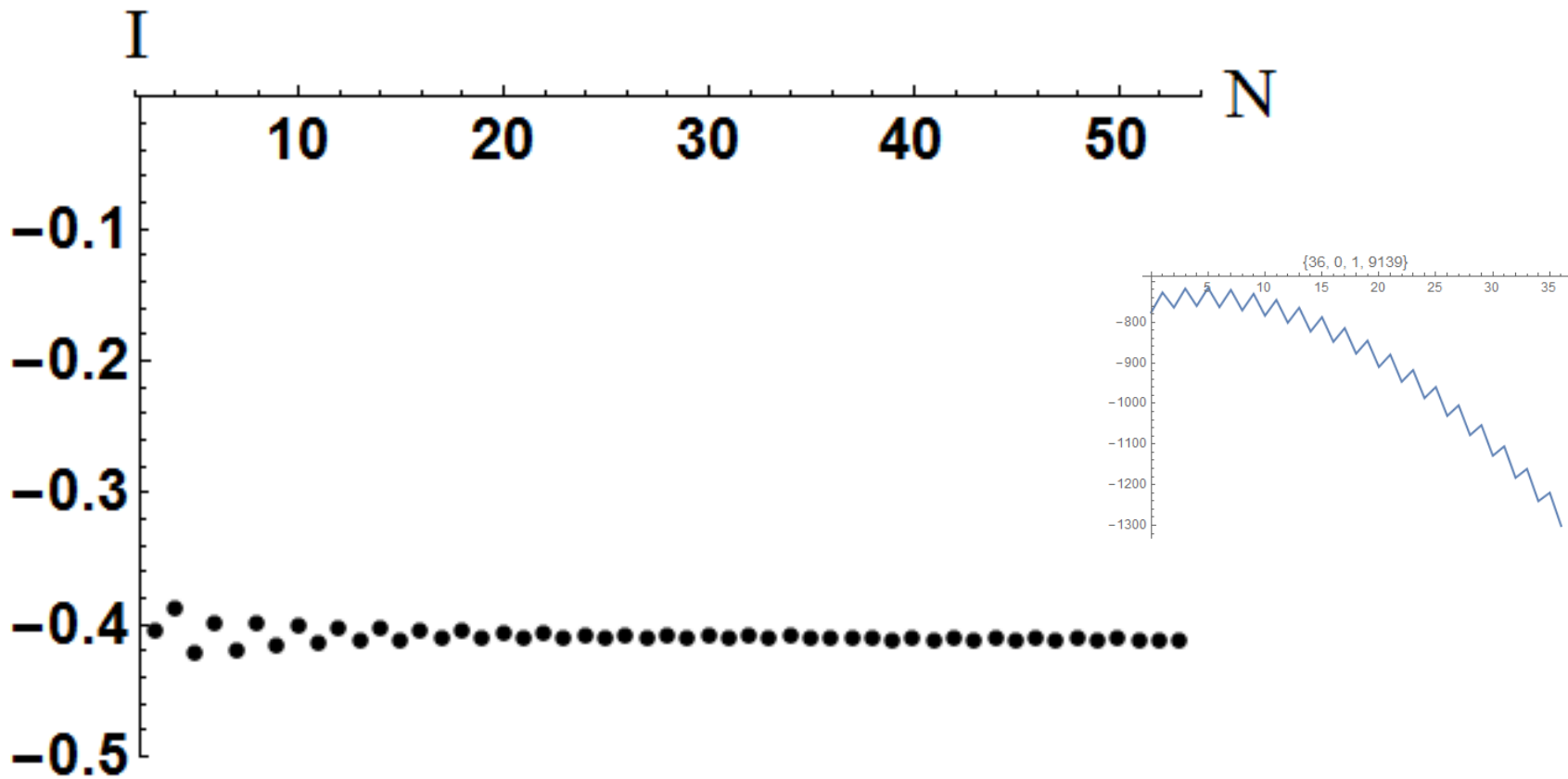
Extreme energies parabolic in N

$$j_1=0 \quad j_2=1 \quad J_{gs}=0$$



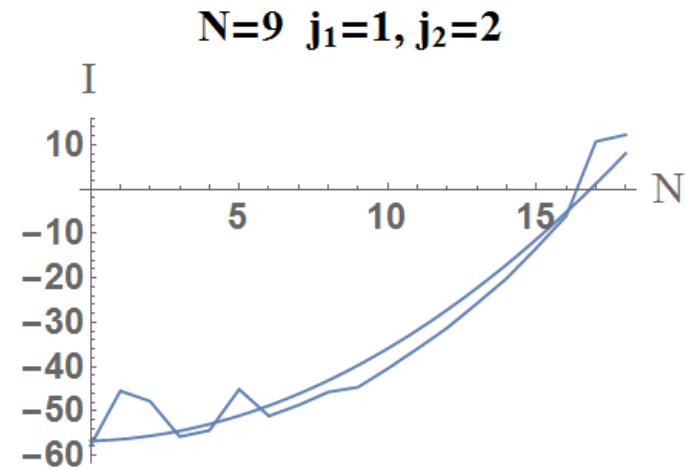
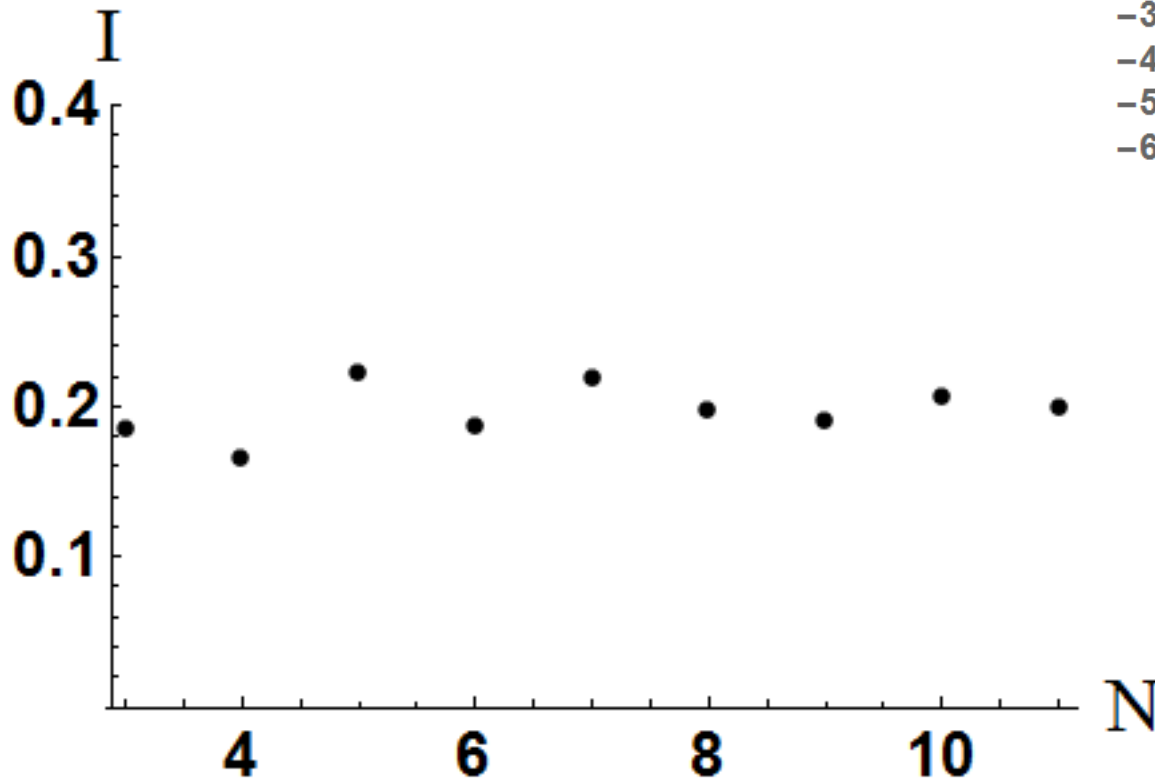
When $J_0 = 0$, the ground state energy and maximum energy for $(j_1; j_2) = (0; 1)$ are parabolic. The yrast lines were linear.

moment of inertia $J_{gs} = \max j_1 = 0, j_2 = 1$

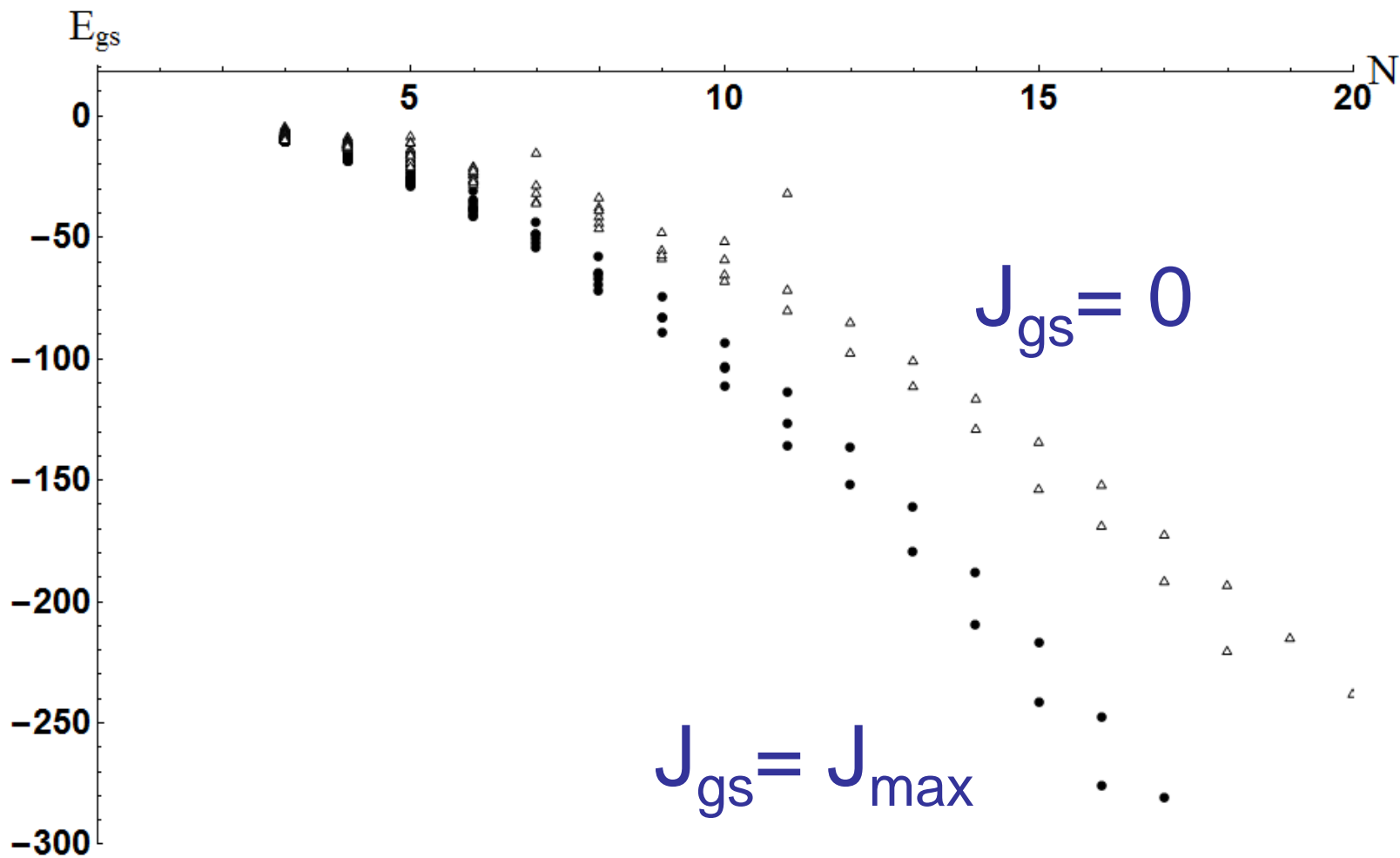


Moments of inertia are constant in N for
Many $(j_1, j_2) = (0,1), (0,2), (1,2), (1,3), (2,3)$

moment of inertia $j_1=1, j_2=2$



E_{gs} vs N for $J_{gs}=0, J_{max}$



Explanations

Q: Why *does* opium make you drowsy?

A: Because it has a *dormitive potency*!

Molière's fictional doctor who explained opium's efficacy as a soporific by attributing to it a **dormitive potency**.

Noun: dormitive virtue (plural dormitive virtues)

(idiomatic, rhetoric, logic, linguistics) A type of tautology in which an item is being explained in terms of the item itself, only put in different (usually more abstract)

https://en.wiktionary.org/wiki/dormitive_virtue#English

Kuhn's feeling is that scientific change brings about a change in the entities that are taken to be primitive and unexplained. For example, Aristotelians said that a stone fell because of its 'nature' drove it toward the center of the universe. Afterwards the normal seventeenth-century tradition of scientific practice insisted that **"the entire flux of sensory appearances, including color, taste, and even weight, was to be explained in terms of the size, shape, position, and motions the elementary corpuscles of base matter."** (p.104) The attribution of other qualities to the elementary atoms was a resort to the occult and therefore out of bounds for science. Famously, Molière ridiculed the doctor who explained opium's efficacy as a soporific by attributing to it a dormitive potency. (p.104) Kuhn sees this not as a criticism of postulating mystical entities *per se* but of postulating an entity not accepted as primitive at the time. In that vein, Kuhn remarks that **"During the last half of the seventeenth century many scientists preferred to say that the round shape of the opium particles enabled them to sooth the nerves about which they moved."**

Explanations

1 Random polynomials

D. Kusnezov, Phys. Rev. Lett. 85, 3773 (2000),

- Kusnezov looked at TBRE IBM with sp bosons (vibron model)
- Mapped problem onto random polynomials on unit circle
- $J_{gs}=0$ not from widths but **“the various interactions in H tend to put the extreme values of spin $J=0,N$ at the ends of the spectra, enhancing their chances to be the ground state”**.

2. (almost) analytical $N=4$ on $j=7/2$

Y. M. Zhao and A. Arima, Phys. Rev. C 64, 041301 (2001)

Approximation seniority (ν) is good quantum number (no mixing between different seniority states)

Analytical expression for $E_{J(\nu)}$ in terms of 2-body interaction parameters

6. Explanation: Group theory

V. K. B. Kota and K. Kar, Phys. Rev. E 65,
026130 (2002),

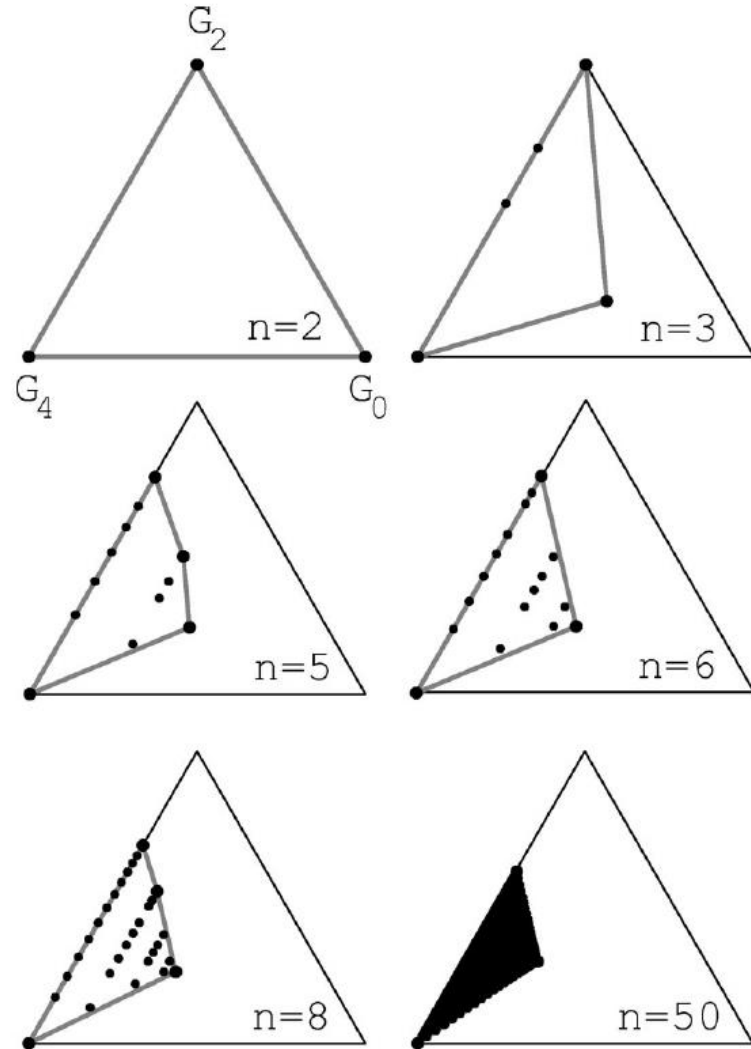
The two-body random matrix ensembles with spin possess $U(N) \supset U(N/2) \supset SU(2)$ and $U(N) \supset O(3)$ group symmetries, respectively, with N the number of single particle states.

It is shown that both these group symmetries give rise to simplicities in the ground state structures but in different ways.

8. Explanations involving a protractor

P. Chau Huu-Tai, A. Frank, N. A. Smirnova, and P. Van Isacker, Phys. Rev. C 66, 061302 (2002),

d-bosons, and $j=7/2$ fermions
“the probability for a given state to become the ground state is shown to be related to a geometric property of a polygon or polyhedron which is entirely determined by particle number, shell size, and symmetry character of the states”



9. Distribution of spectral widths

RTBE single j-shell model

derive closed expressions for the distribution of and the correlations between spectral widths of different spins.

The approximate proportionality between widths and spectral radii explains the preponderance of spin-0 ground states.

T. Papenbrock and H. A. Weidenmüller, Phys. Rev. Lett. 93, 132503 (2004)

80% Dormative potency

Explanations

3. Structure functions

N. Yoshinaga, A. Arima, and Y. M. Zhao, Journal of Physics A: Mathematical and General 35, 8575 (2002)

$$H = \sum_{J=0}^{2j-1} \sqrt{2J+1} G_J [A^{\dagger(J)} \tilde{A}^{(J)}]^{(0)}$$

$$H_{I\beta'\beta} \equiv \langle j^n I\beta' | H | j^n I\beta \rangle = \sum_J \alpha_{I\beta'\beta}^J G_J$$

$$\alpha_{I\beta'\beta}^J \equiv \langle j^n I\beta' | \sqrt{2J+1} [A^{\dagger(J)} \tilde{A}^{(J)}]^{(0)} | j^n I\beta \rangle$$

Explanations

4. Displaced Random Ensembles

V. Velazquez and A. P. Zuker, Phys. Rev. Lett. 88, 072502 (2002),

Because of the time reversal invariance of the angular momentum operator J^2 , the average energies and variances at fixed J for random two-body Hamiltonians exhibit odd-even- J staggering that may be especially strong for $J=0$. This is numerically in the yrast states.

Displaced (attractive) random ensembles lead to rotational spectra with strongly enhanced $B(E2)$ transitions for a certain class of model spaces.

Explanations

5. Mean Field IBM and vibron model

R. Bijker and A. Frank, Phys. Rev. C 64, 061303 (2001),

R. Bijker and A. Frank, Czechoslovak Journal of Physics 52, C643 (2002),

R. Bijker and A. Frank, Phys. Rev. C 65, 044316 (2002),

$\langle H \rangle$ in Coherent states $|N, \alpha\rangle = \frac{1}{\sqrt{N!}} (\cos \alpha s^\dagger + \sin \alpha d_0^\dagger)^N |0\rangle$

$$E_N(\alpha) = a_4 \sin^4 \alpha + a_3 \sin^3 \alpha \cos \alpha + a_2 \sin^2 \alpha + a_0$$

α_i written in terms of random interactions

Get an energy surface

α tells you shape and J of state

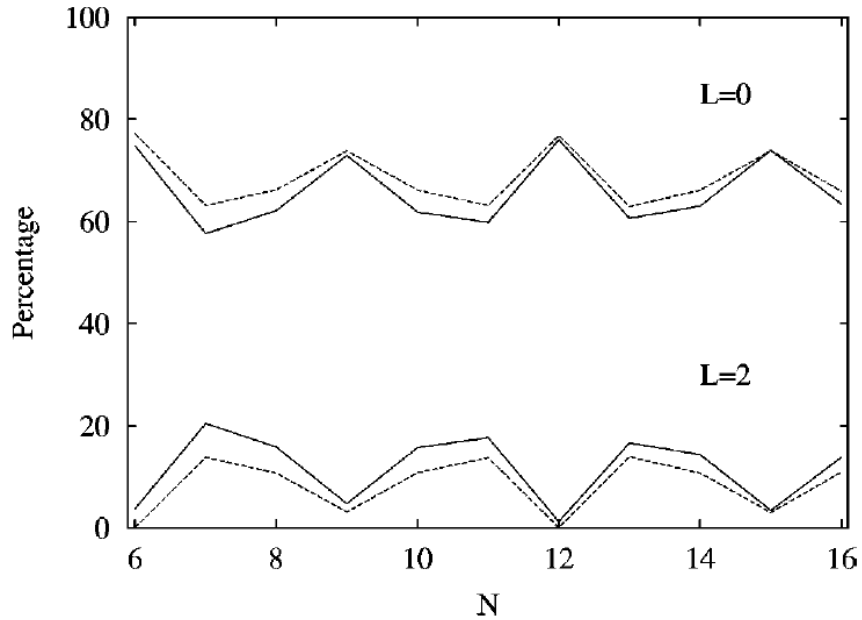


FIG. 1. Percentages of ground states with $L=0$ and $L=2$ in the IBM with random one- and two-body interactions calculated exactly for 10 000 runs (solid lines) and in mean-field approximation (dashed lines).

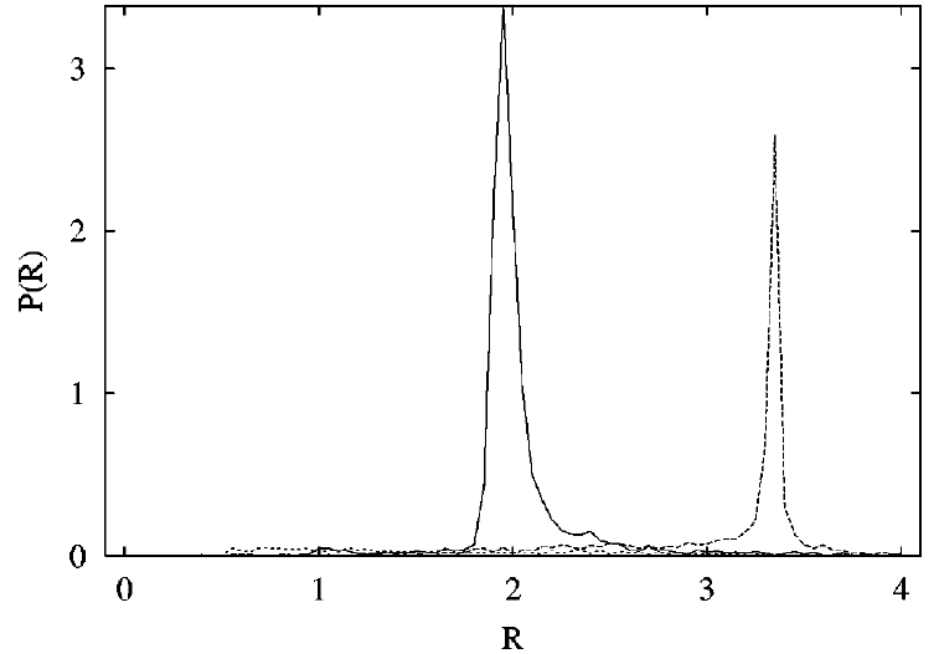


FIG. 2. Probability distribution $P(R)$ of the energy ratio R obtained for $N=16$ and 10 000 runs for the spherical (solid line), deformed (dashed line), and d -boson condensate (dotted line) equilibrium configurations, respectively.

R. Bijker and A. Frank, Phys. Rev. C 64,
061303 (2001),

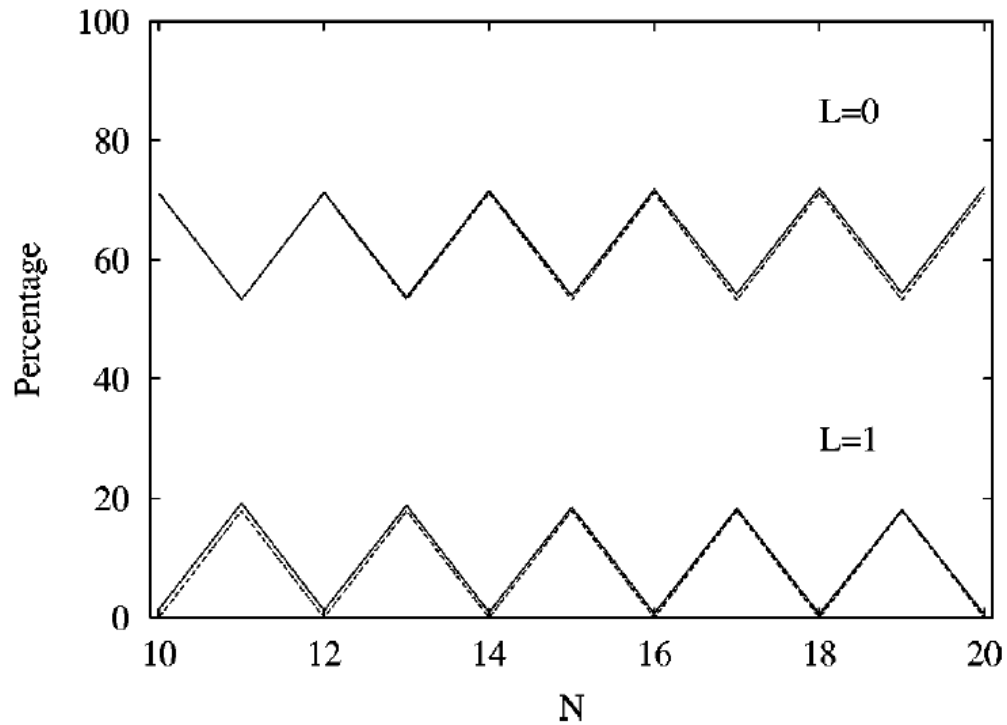
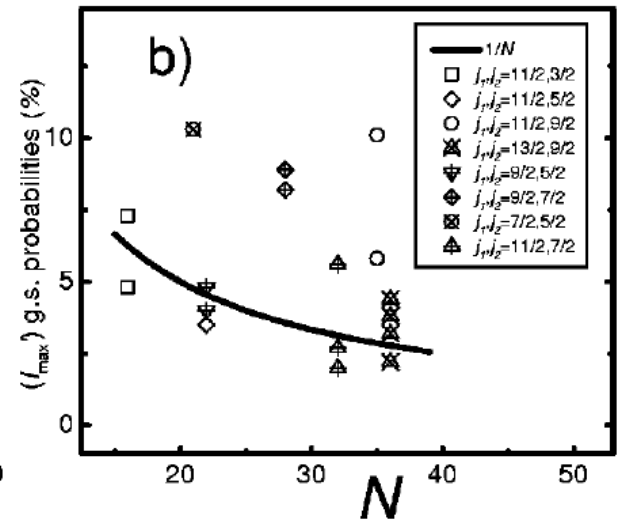
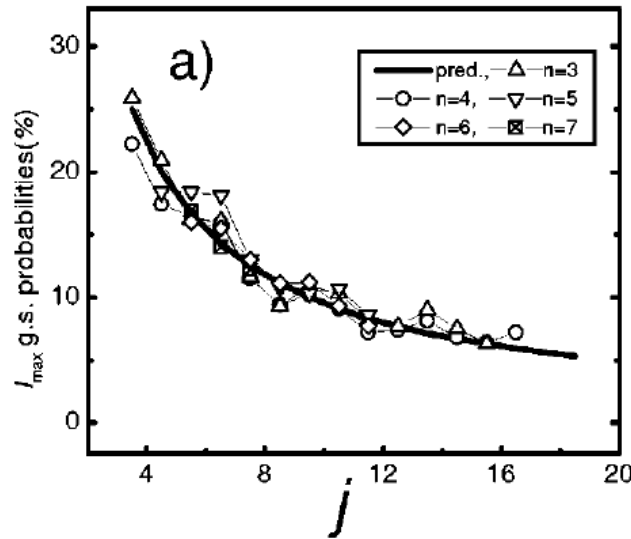
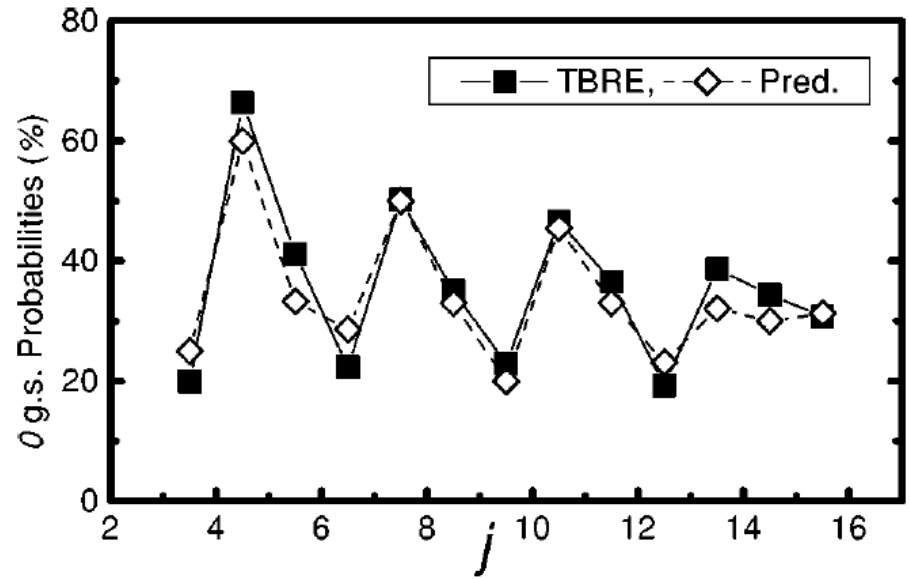


FIG. 4. Percentages of ground states with $L=0$ and $L=1$ in the vibron model with random one- and two-body interactions calculated exactly for 100 000 runs (solid lines) and in mean-field approximation (dashed lines).

R. Bijker and A. Frank, Phys. Rev. C 64,
061303 (2001),

7. Computational scheme

Turn on interactions 1 at a time: $V_L = -1$, others 0
 See which J is lowest
 Get stats for all L
 There is your distribution

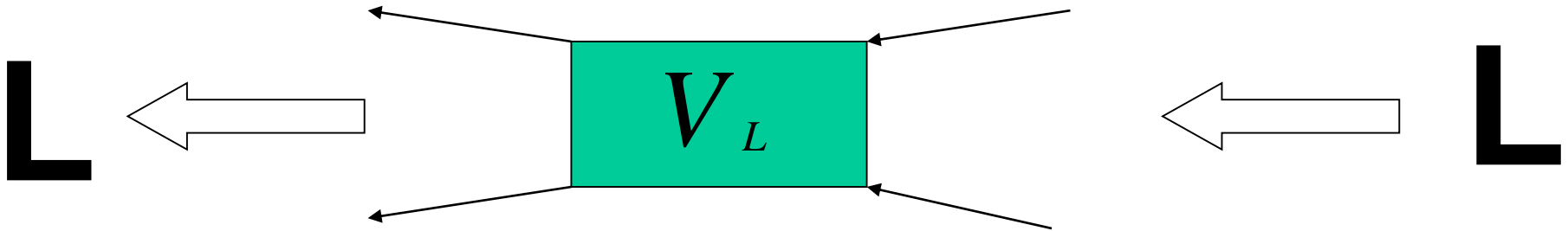


Y. M. Zhao, A. Arima, and N. Yoshinaga, Phys. Rev. C 66, 034302 (2002),
 Y. M. Zhao, A. Arima, and N. Yoshinaga, Phys. Rev. C 66, 064322 (2002)
 Y. M. Zhao, A. Arima, and N. Yoshinaga, Phys. Rev. C 66, 064323 (2002)

10. The actual answer (100% dp)

$$H = \frac{1}{2} \sum_{L\Lambda} V_L C_{m_1 m_2}^{L\Lambda} C_{m_3 m_4}^{L\Lambda} a_2^+ a_1^+ a_3 a_4$$

$$\langle H \rangle_{N,M} = \sum_{L\Lambda mm'} V_L |C_{mm'}^{L\Lambda}|^2 \langle n_m n_{m'} \rangle$$

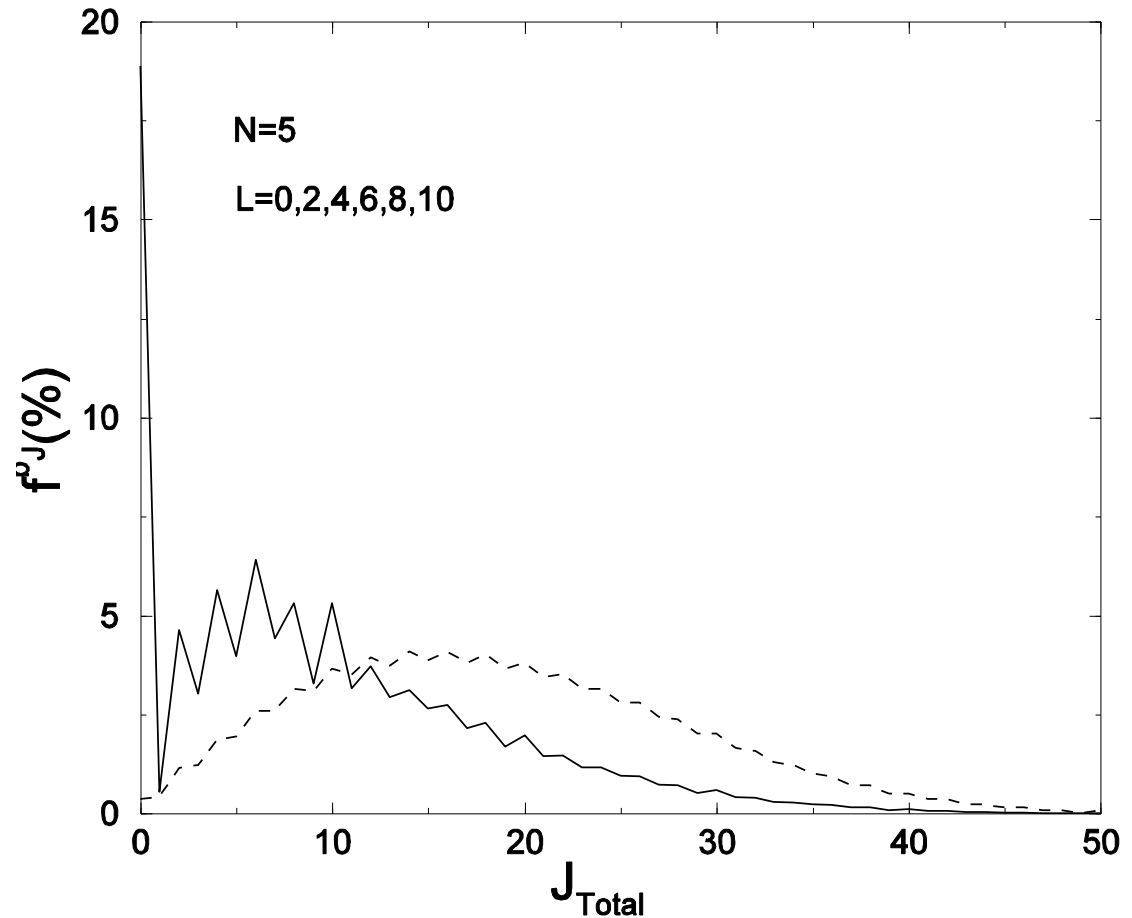


D. Mulhall, A. Volya, and V. Zelevinsky, Phys. Rev. Lett. **85**, 4016 (2000)

V. Zelevinsky and A. Volya, Physics Reports 391, 311 (2004),

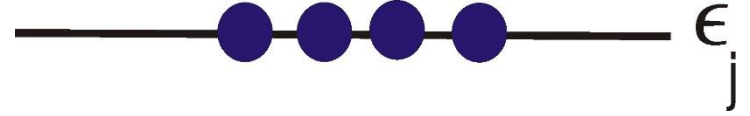
Boson Approximation $N=6$ $j=1\ 1/2$

- Pairs of fermions treated as bosons
- V_L are like single particle levels
- V_0 lowest in $j-1/2$ cases, so $J=0$
- Else $J=$ all possibilities including 0





The simple model



- Single-j level
- $\Omega=2j+1$ single-particle orbitals: $m=-j, j-1, \dots, j$
- Number of nucleons N : $0 \leq N \leq \Omega$
- Number of many-body states: $\Omega!/((N!(\Omega-N)!))$
- Many-body states classified by rotational symmetry: (J, M)

Dynamics

- Rotational invariance and two-body interactions

particle-particle pair operator $P_{LM}=(a a)_{LM}$

particle-hole pair operator $M_{K\kappa}=(a a^\dagger)_{K\kappa}$

- Hamiltonian
$$H = \sum_L V_L \sum_M P_{LM}^\dagger P_{LM}$$

- Dynamics is fully determined by $j+1/2$ parameters V_L

Statistical treatment [1,2]

Constants of motion and corresponding terms

- Particle number N * monopole (mass) term

$$\tilde{V}_0 = [\Omega(\Omega - 1)]^{-1} \sum_L (2L + 1) V_L$$

- Angular momentum J * moment of inertia

$$\tilde{V}_1 = (2j^4 \Omega^2)^{-1} \sum_L (2L + 1)(L^2 - 2j^2) V_L$$

Average energy

$$\langle H \rangle_{NJ} = \tilde{V}_0 N(N - 1) + \tilde{V}_1 J(J + 1)$$

Statistical prediction

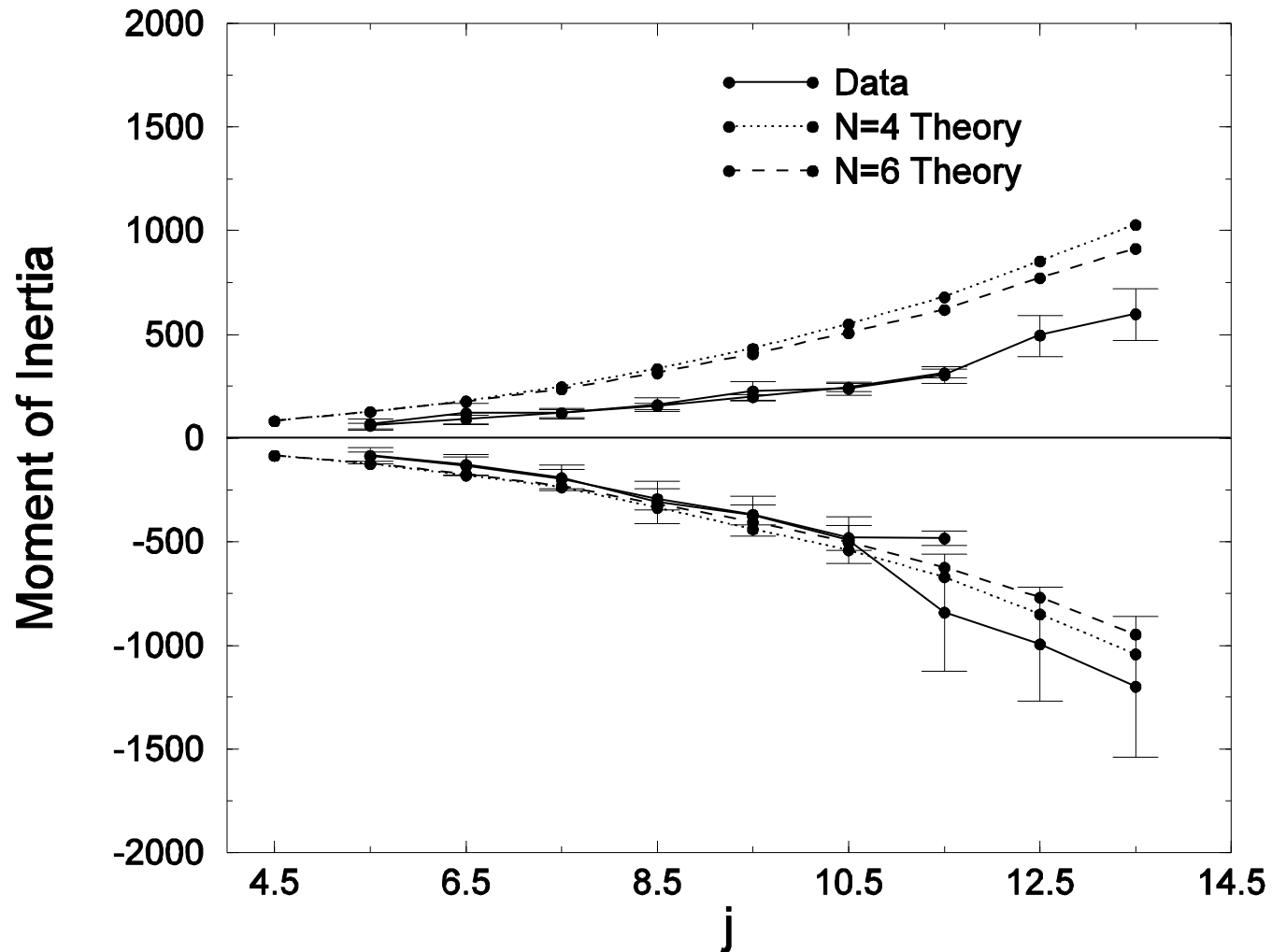
$$\tilde{V}_1 > 0 \quad \text{Ground state has } J_0=0$$

$$\tilde{V}_1 < 0 \quad \text{Ground state has } J_0=J_{\max} \text{ (maximum possible } J)$$

[1] D. Mulhall, A. Volya, and V. Zelevinsky, Phys. Rev. Lett. **85**, 4016 (2000);

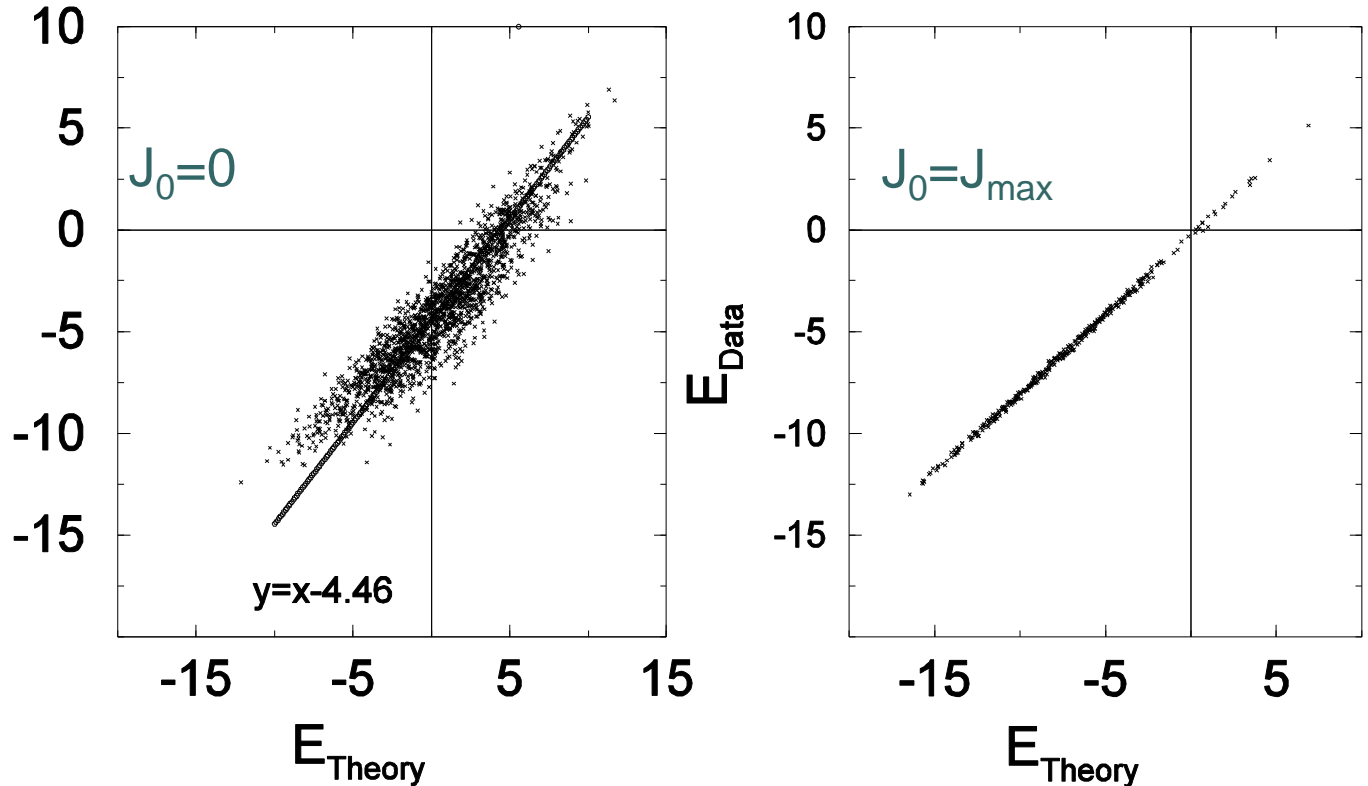
[2] V. Zelevinsky and A. Volya, Physics Reports 391, 311 (2004),

Moment of inertia

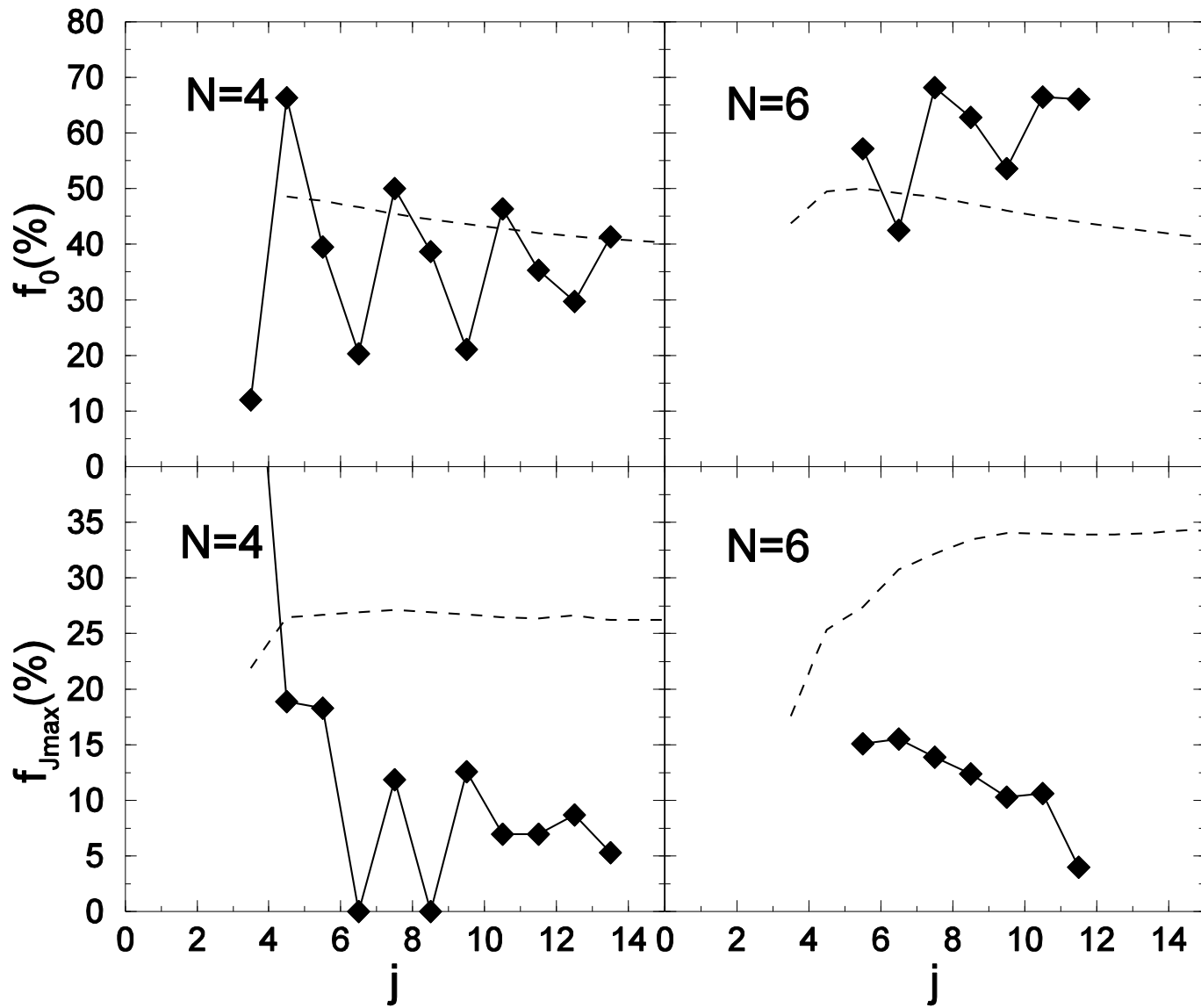


Exact energy and statistical prediction

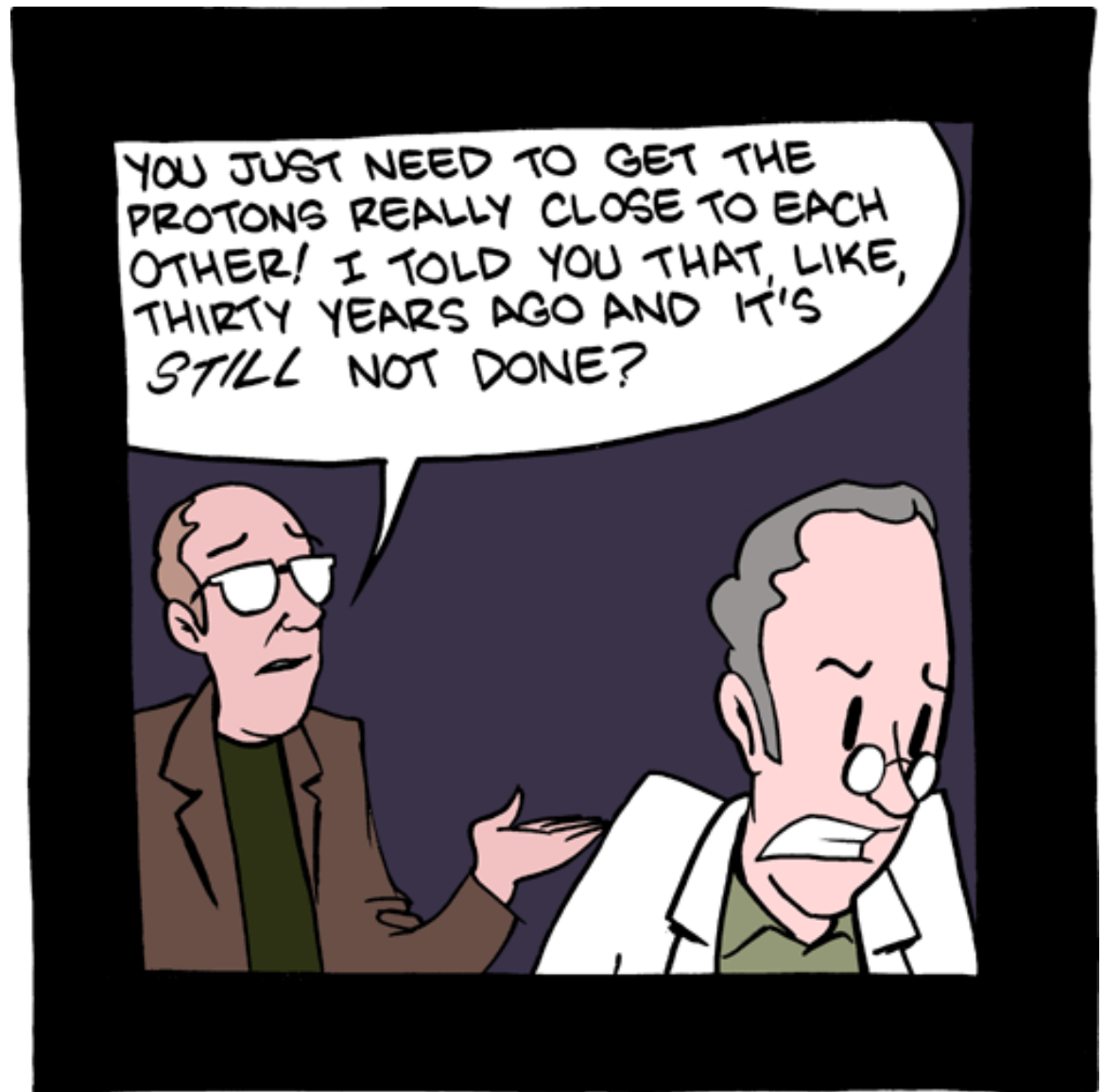
$j=17/2$
 $N=6$



To summarize, it is now clear that the round shape of the opium particles enables them to sooth the nerves about which they move.



Theorists,
don't be
like this
guy.



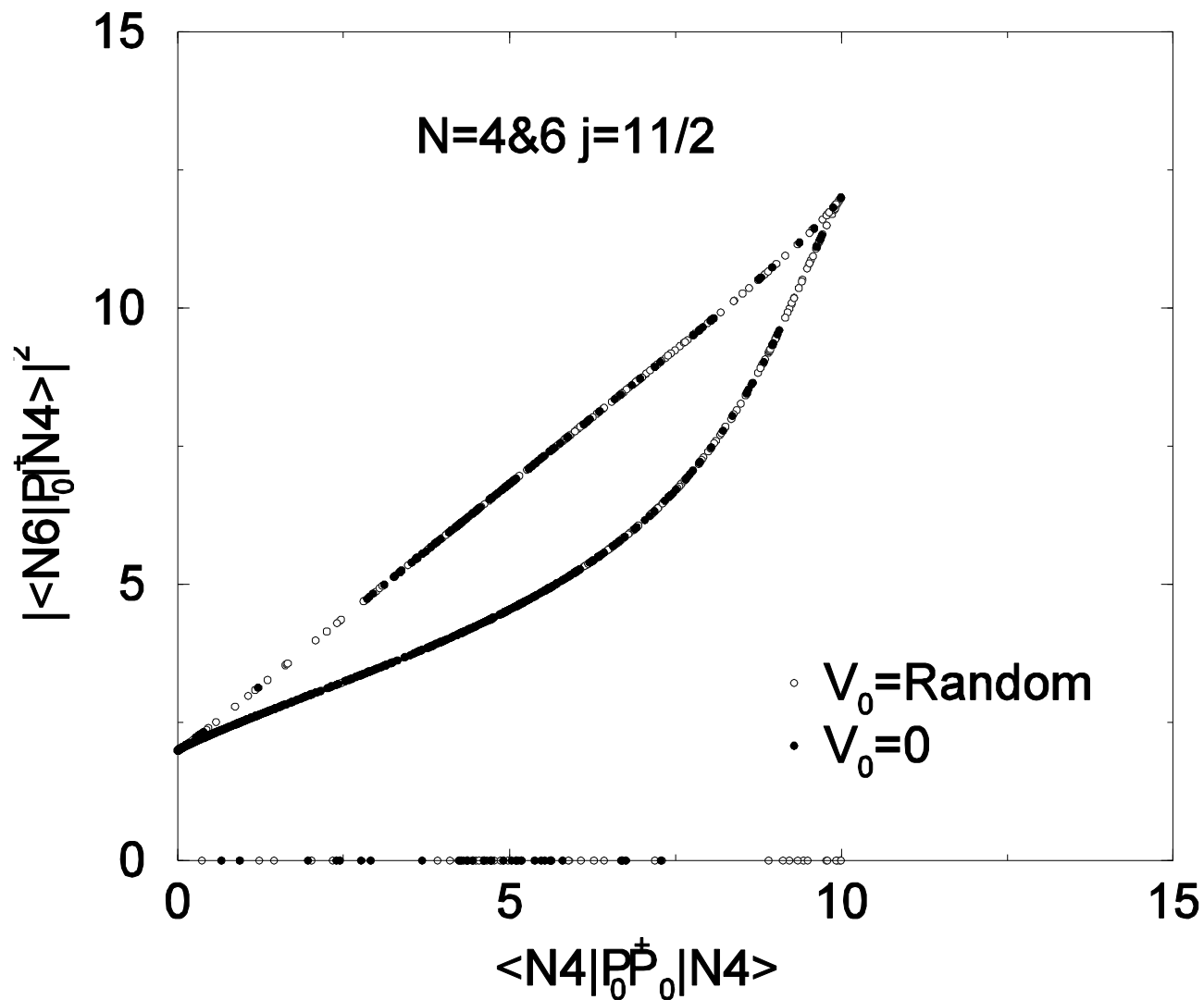
This is why experimental scientists hate theoretical scientists.

Experiments are hard

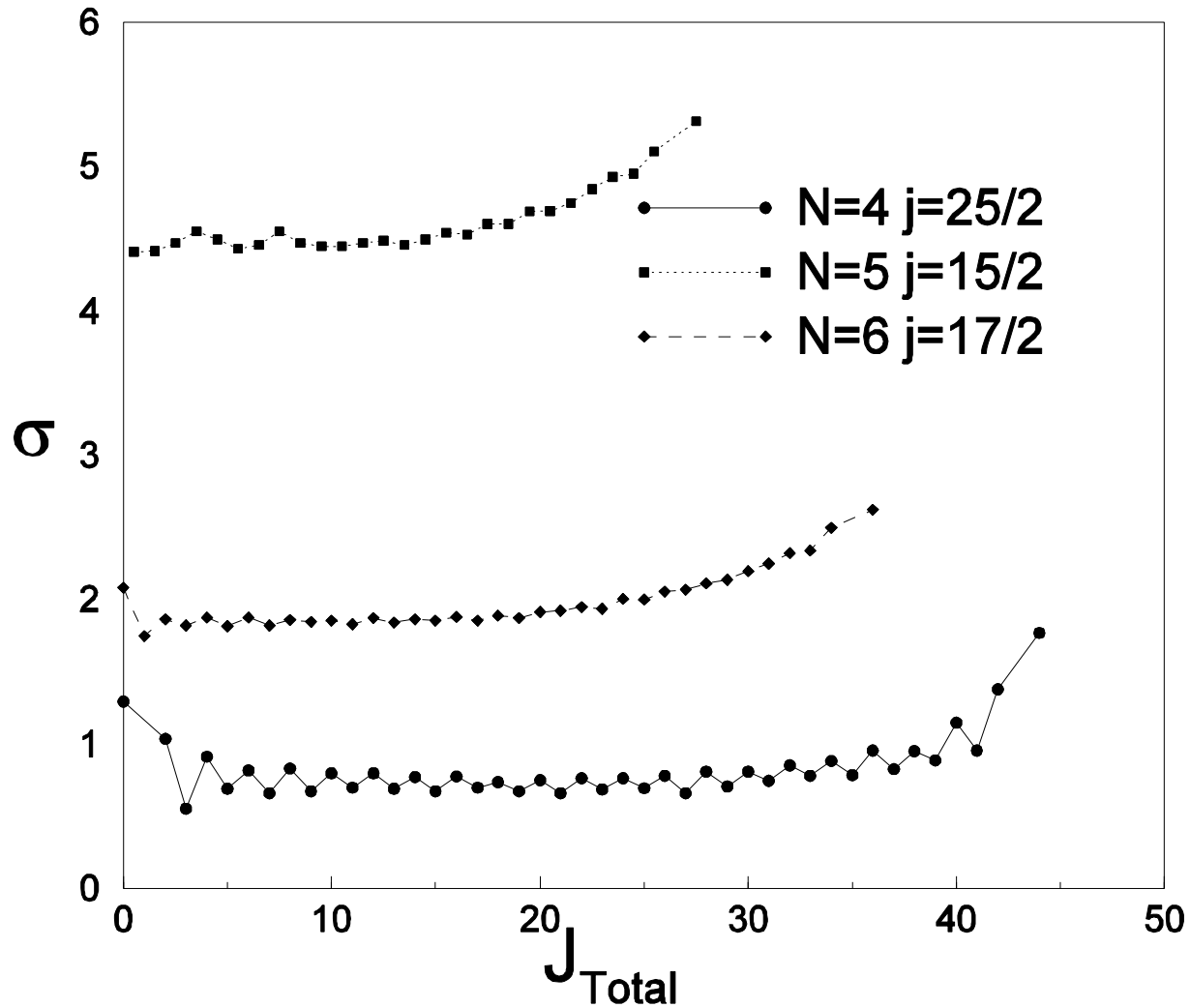


Particle physics has come
a long way since the 1700s.

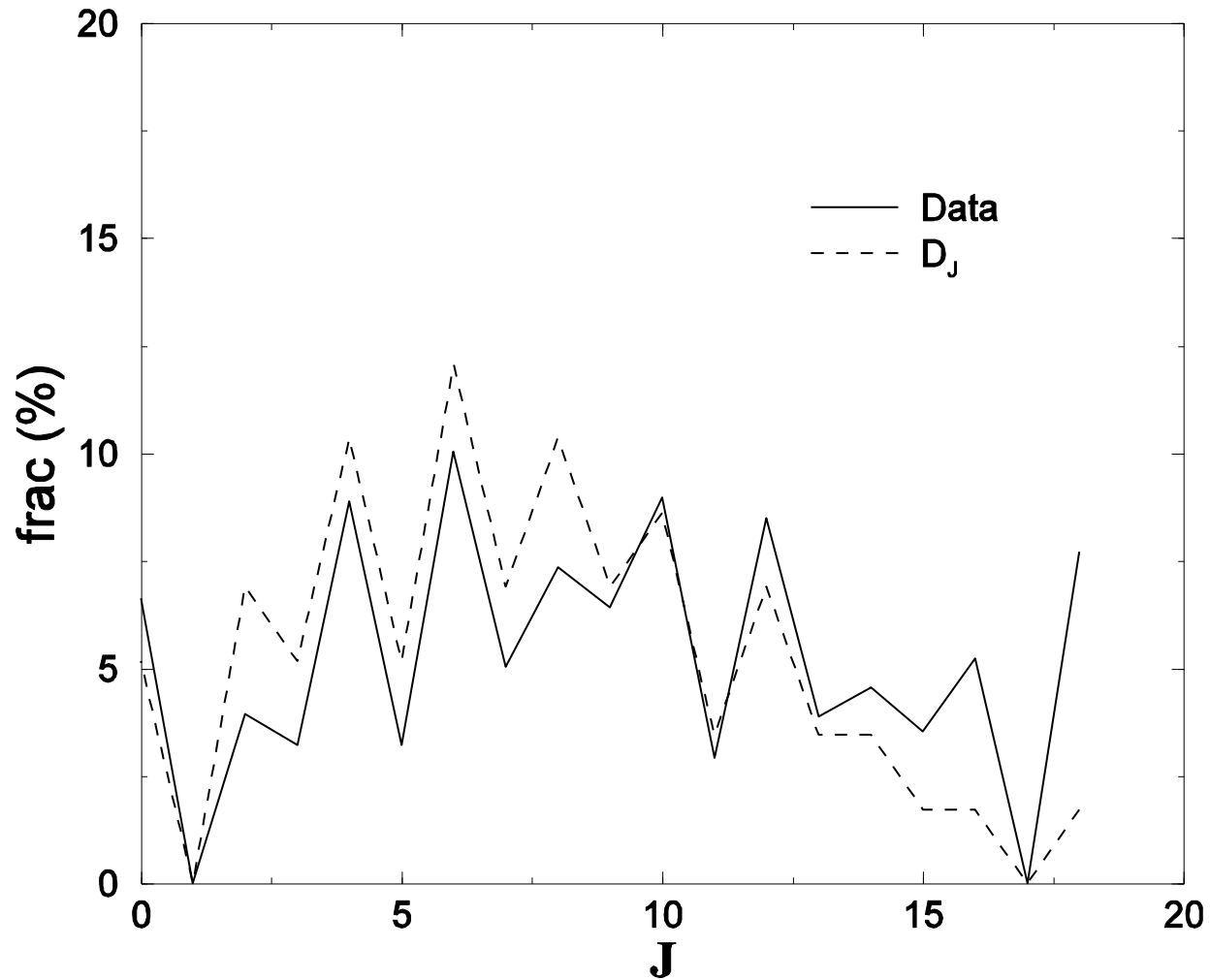
Pair transfer collectivity



Widths of Level density vs J for N=6 j=11/2 to j=23/2

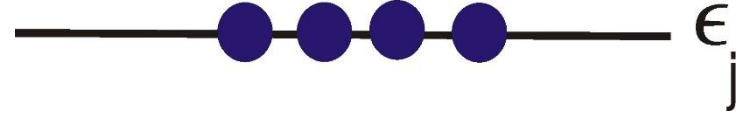


Prob $J_0=0$ fake spectra





The simple model



- Single- j level
- $\Omega=2j+1$ single-particle orbitals: $m=-j, j-1, \dots, j$
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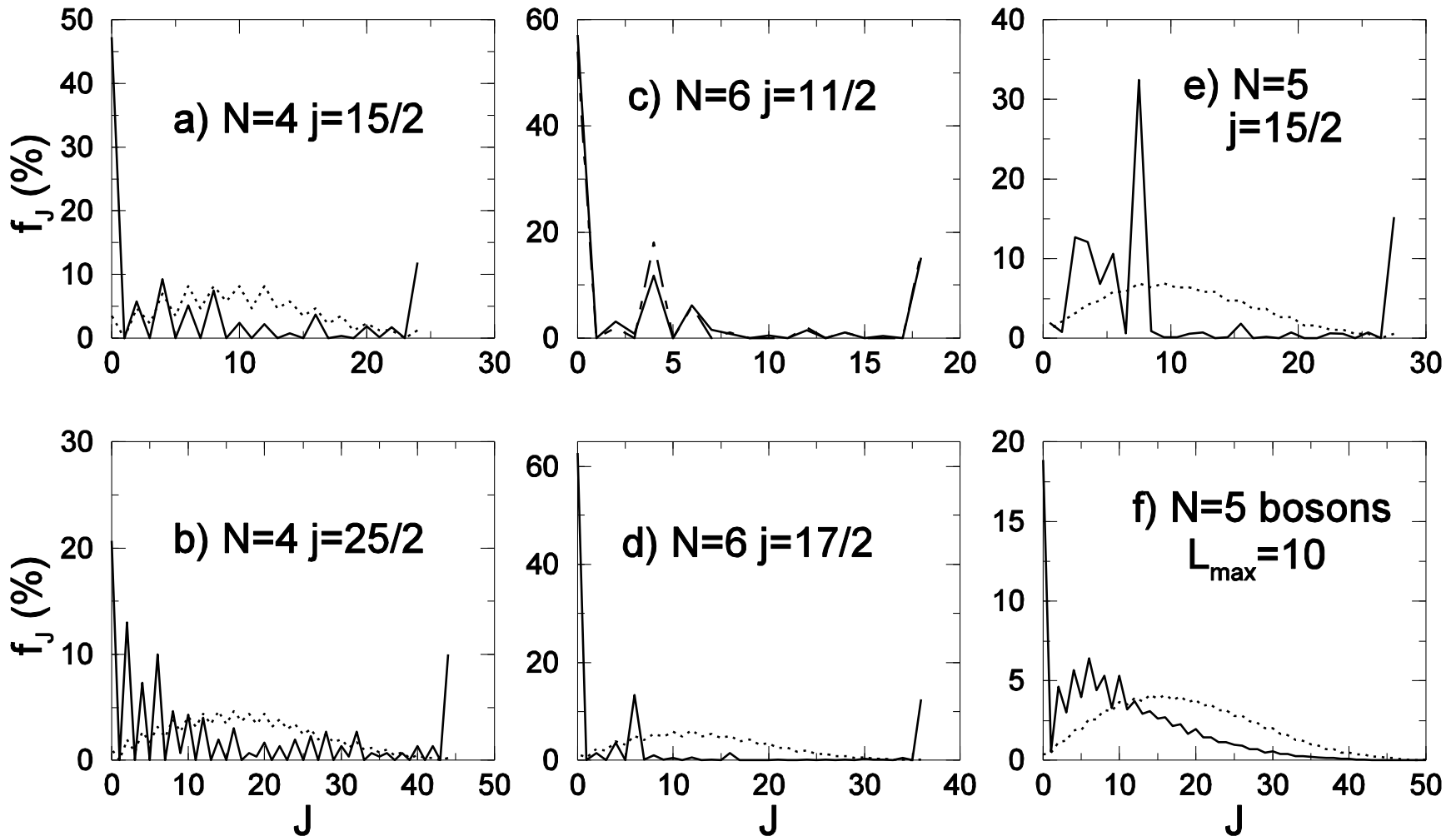
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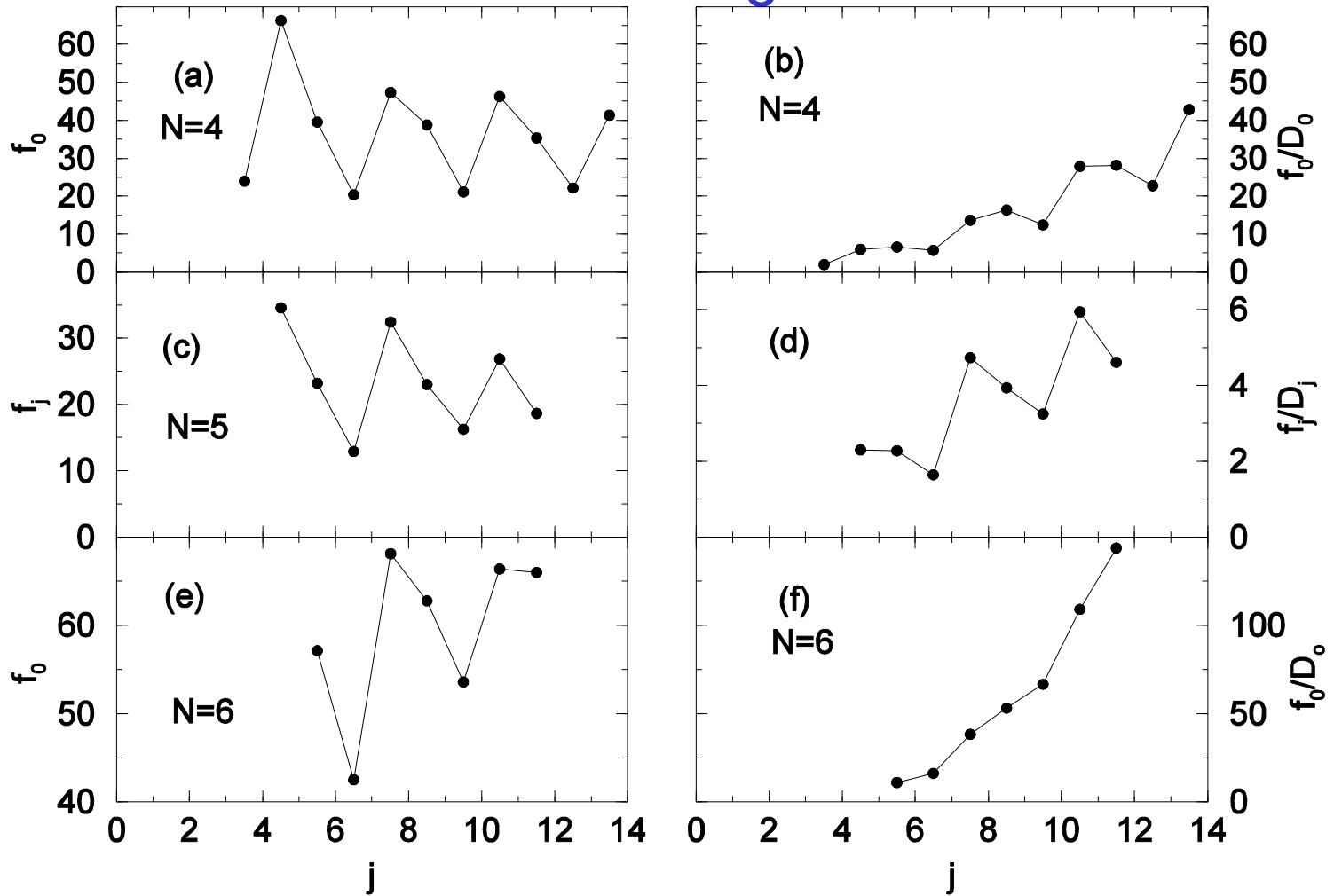
- Dynamics is fully determined by $j+1/2$ parameters V_L

Typical

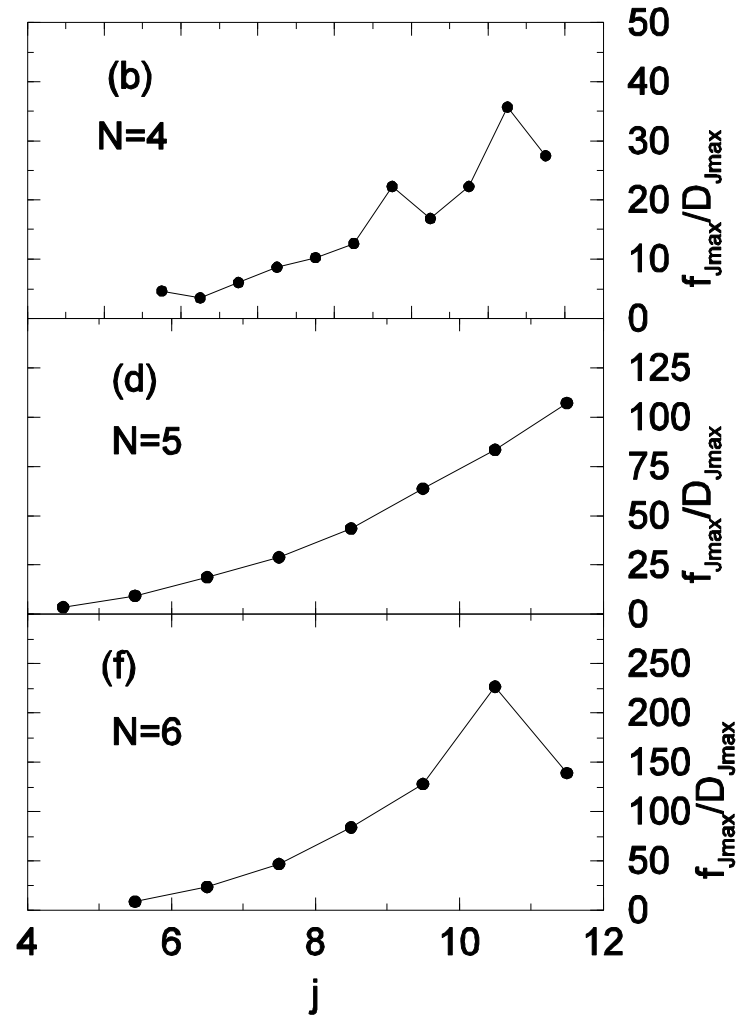
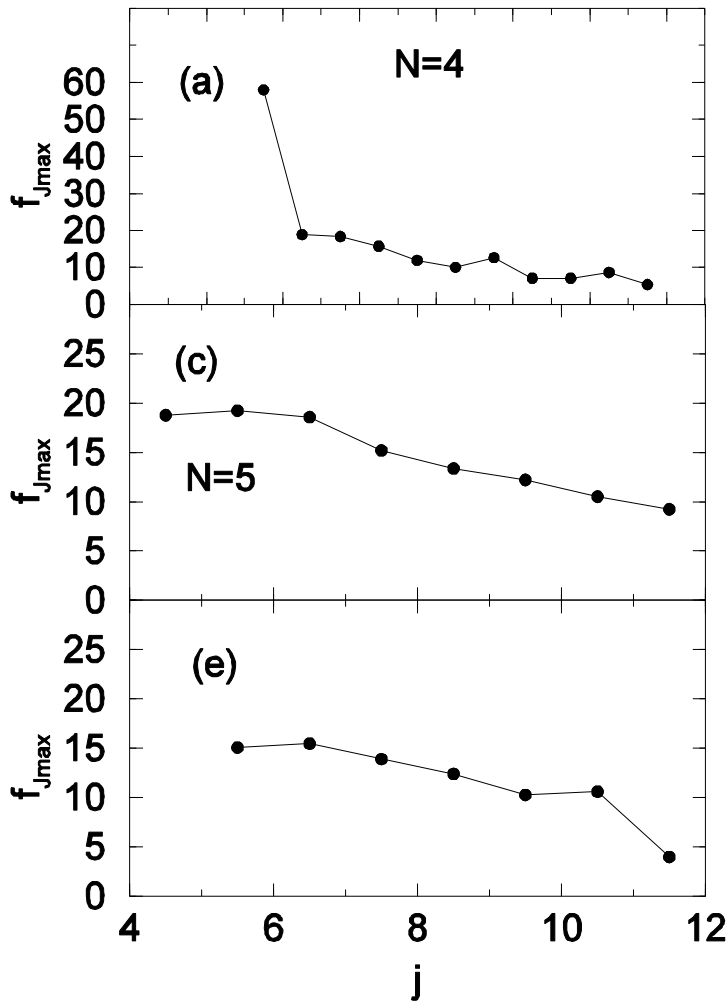
D. Mulhall, A. Volya, and V. Zelevinsky, Phys. Rev. Lett. 85 (2000)



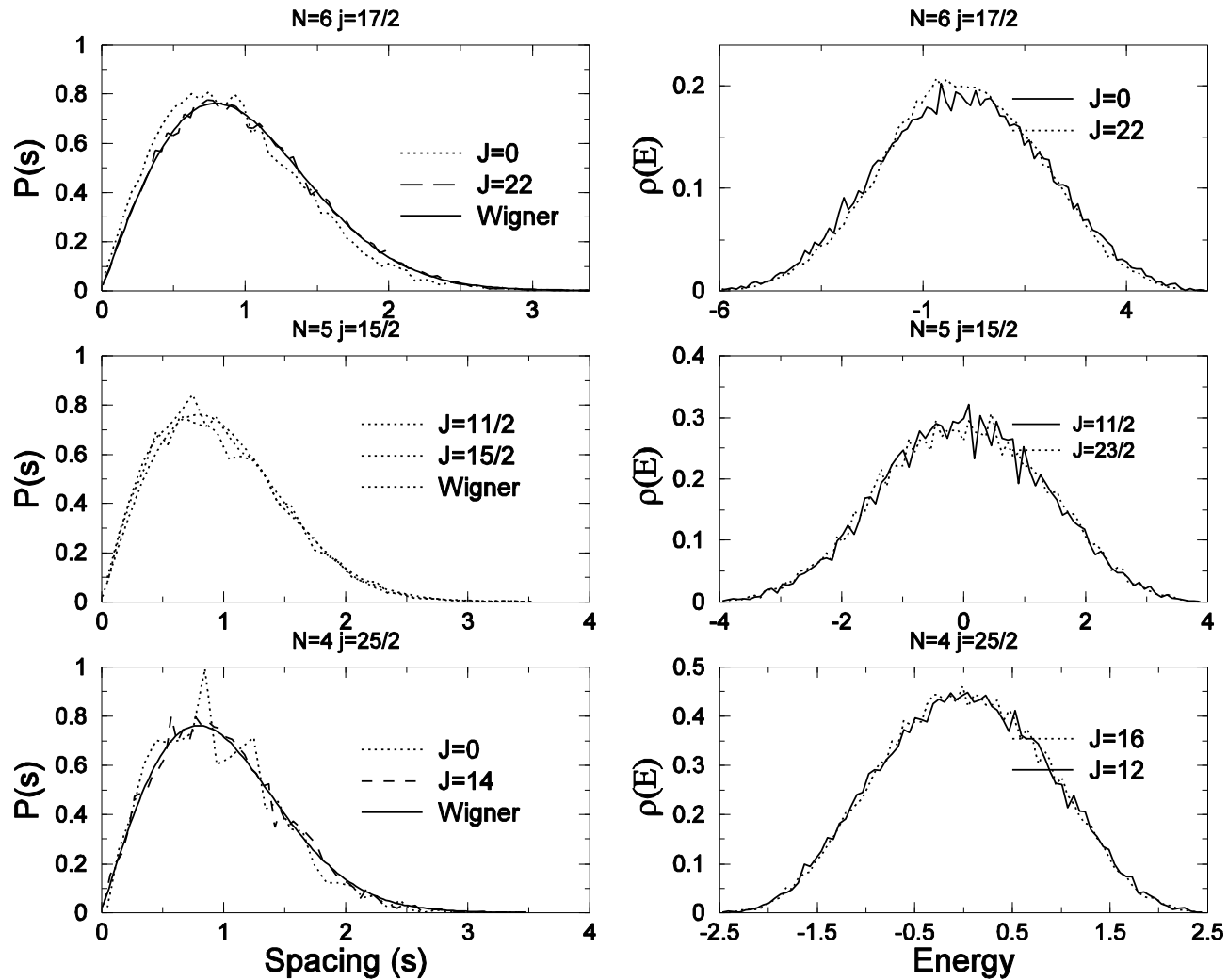
Prob $J_0 = 0$



Prob $J_0 = J_{\max}$



Level Spacing Level Density



Pairing Doesn't matter $N=6$ $j=1\ 1/2$

$$X = |\langle gs | \text{paired} \rangle|^2$$

Red is randomly oriented vector

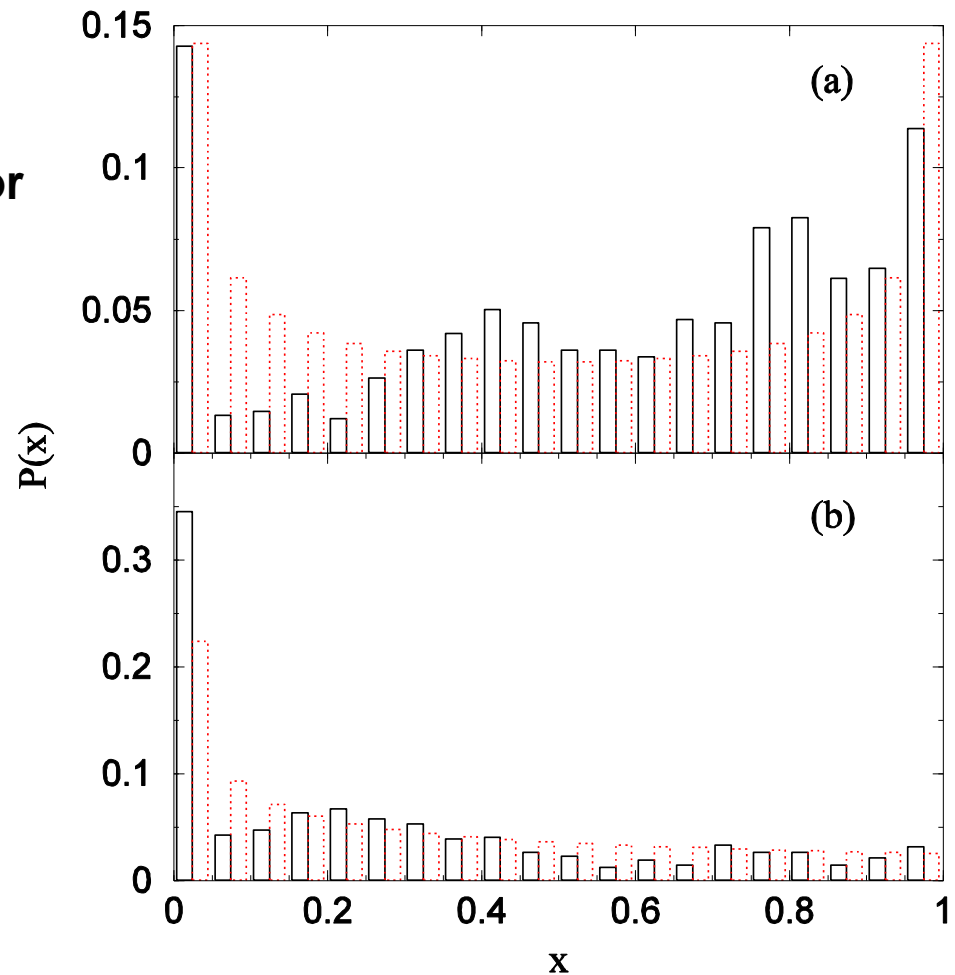
Top Panel, $V_0 = -1$

$d=2$

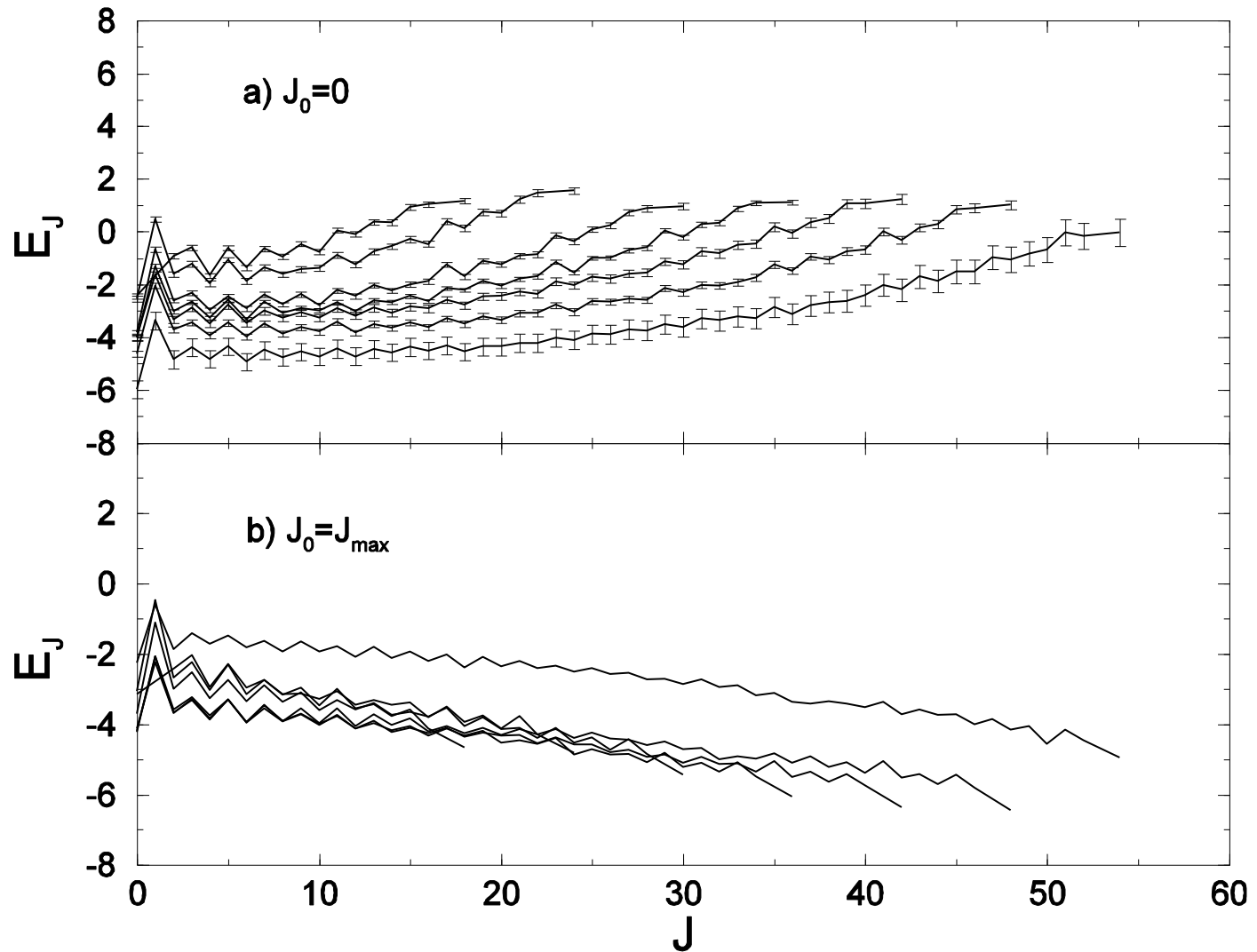
Bottom panel

$V_0 = \text{Random}$

$d=3$



Yrast levels $N=6$



Odd even effects

– T. Papenbrock, L. Kaplan, and G. F. Bertsch, Phys. Rev. B 65, 235120 (2002)

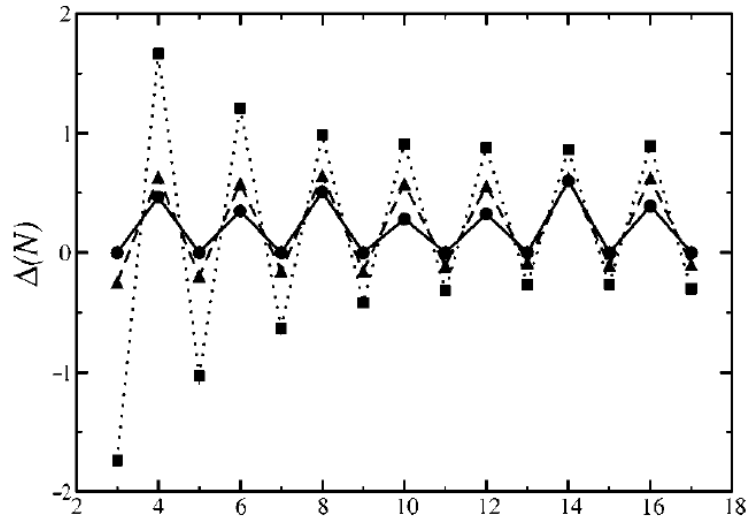


FIG. 1. Empirical pairing gap as a function of particle number for parameter $\varphi=0$ (full line), $\varphi=\pi/12$ (dashed line), and $\varphi=\pi/2$ (dotted line; graph scaled by a factor 1/2 for display purposes) shows the transition from the mean-field regime to strong interactions in the spin-0 channel. Note that the pairing gap $\Delta(N)$ is dimensionless here and in all following figures; the energy scale in our calculation is set by the overall energy scale of the Hamiltonian

S=0 channel

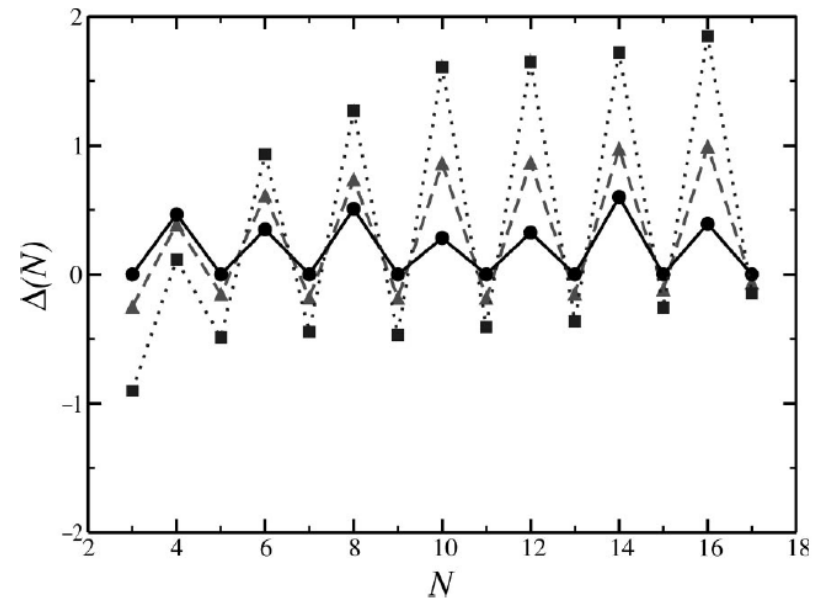


FIG. 2. Empirical pairing gap as a function of particle number for parameter $\varphi=0$ (full line), $\varphi=\pi/12$ (dashed line), and $\varphi=\pi/2$ (dotted line; graph scaled by a factor 1/2 for display purposes) shows the transition from the mean-field regime to strong interactions in the spin-1 channel.

S=1 channel

Odd even effects

- T. Papenbrock, L. Kaplan, and G. F. Bertsch, Phys. Rev. B 65, 235120 (2002)

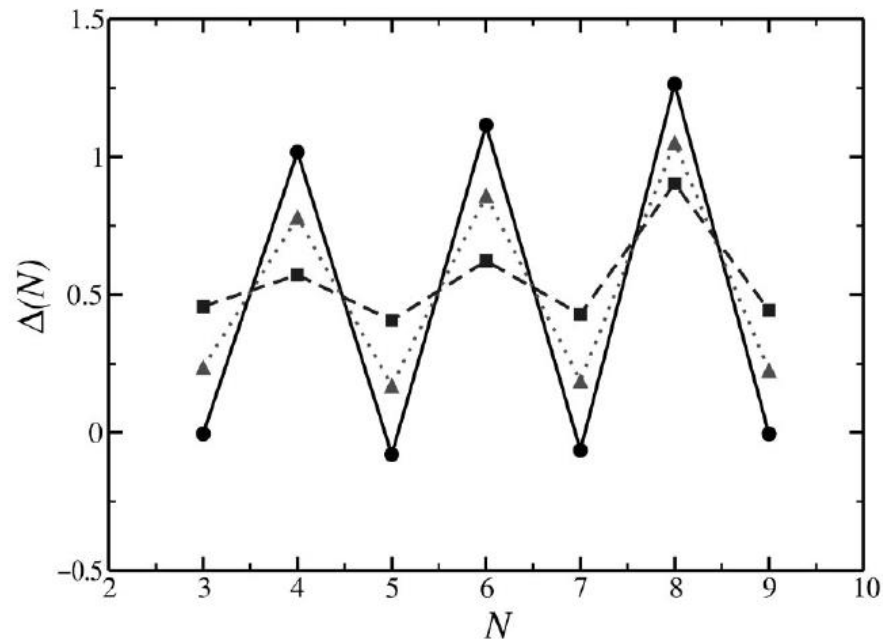


FIG. 4. Empirical pairing gap as a function of particle number for various strengths of the magnetic field: $2\mu B/\langle\varepsilon_{i+1}-\varepsilon_i\rangle = 0, 1/4, 1/2$ (full line, dotted line, dashed line).

Magnetic field breaks the effect

More O-E effects, J and Isospin

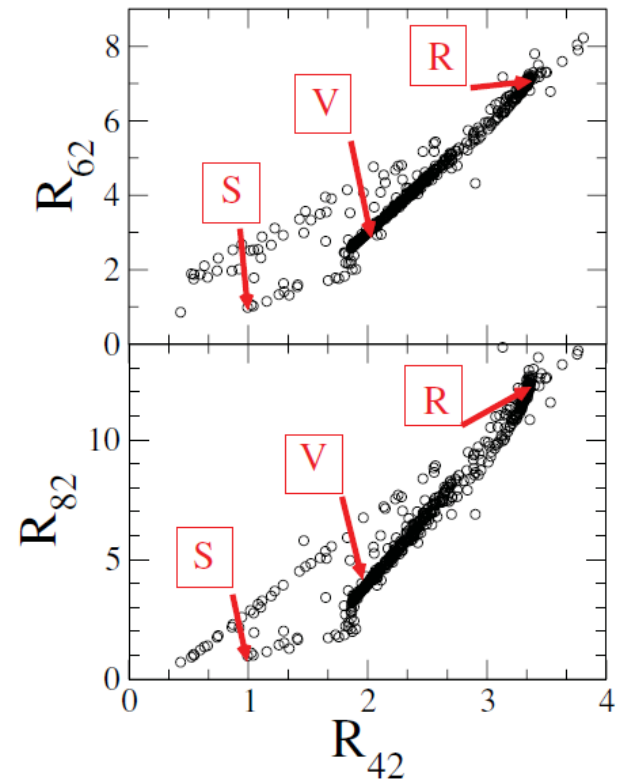
Y. M. Zhao, A. Arima, and N. Yoshinaga, Phys. Rev. C 68, 014322 (2003)

- $J_{gs}=0$ dominance not true for odd N bosons in TBRE
- Y. M. Zhao, A. Arima, N. Shimizu, K. Ogawa, N. Yoshinaga, and O. Scholten, Phys. Rev. C 70, 054322 (2004)

A few last observations

C. W. Johnson and H. A. Nam, Phys. Rev. C 75, 047305 (2007)

- $R_{42} = E_x(J=4)/E_x(J=2)$
- $R_{62} = E_x(J=6)/E_x(J=2)$
- $R_{82} = E_x(J=8)/E_x(J=2)$
- IBM, random N=16



$J_{gs}=0$ but no pairing

- L. Kaplan, T. Papenbrock, and C. W. Johnson, Phys. Rev. C 63, 014307 (2000),

N spin-1/2 particles on M orbitals TBRE (2 numbers C_0 and C_1)

Higher spin coupling ($C_1 > C_0$) decreases $J_{gs}=0$ (from nearly 100%)

Measure of collective behavior decreases with E (B is a spin flip operator, converts spin 0-pair into spin1-pair)

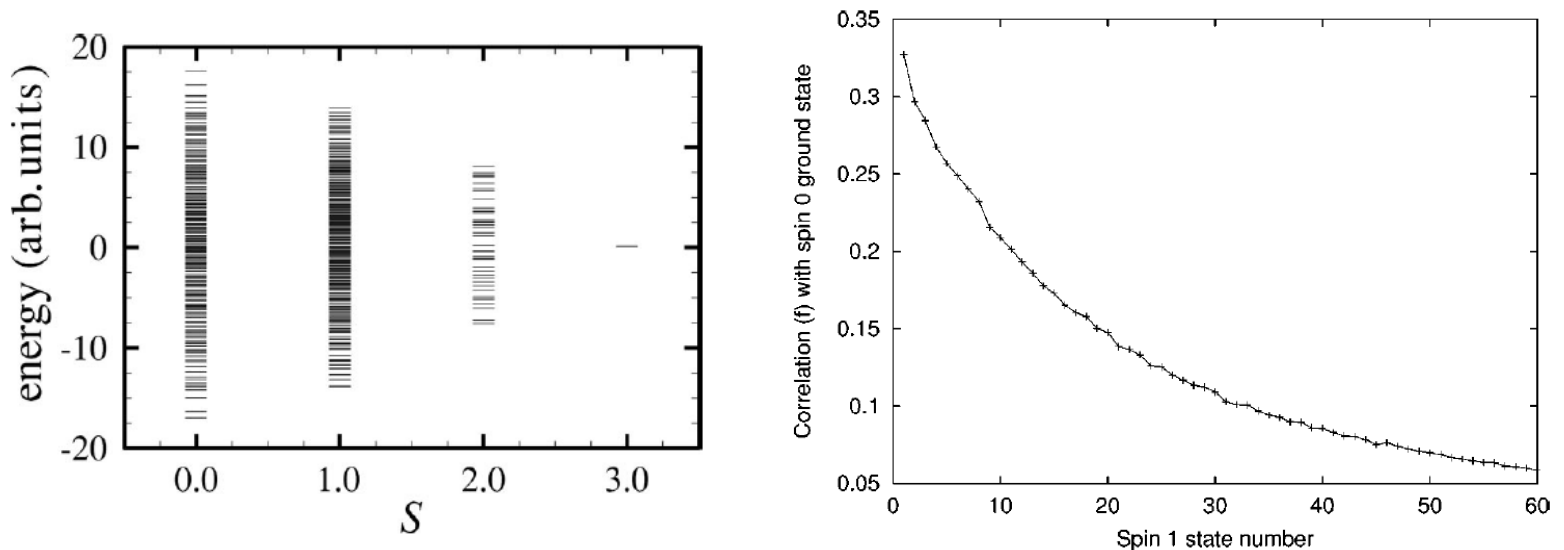


FIG. 1. Spectra for a system of six fermions on six orbitals with $S_z=0$ as a function of total spin S .

$$f \equiv \frac{|\langle S=1 | B | S=0 \rangle|^2}{\langle S=0 | B^\dagger B | S=0 \rangle}$$

Explanations-Why *does* opium make you drowsy?

Because it has a Dormitive Potency

Noun: dormitive virtue (plural dormitive virtues)

(idiomatic, rhetoric, logic, linguistics) A type of tautology in which an item is being explained in terms of the item itself, only put in different (usually more abstract)

https://en.wiktionary.org/wiki/dormitive_virtue#English

Molière ridiculed the doctor who explained opium's efficacy as a soporific by attributing to it a **dormitive potency**. (p.104)

Kuhn sees this as a criticism of postulating an entity not accepted as primitive at the time.

"During the last half of the seventeenth century many scientists preferred to say that the round shape of the opium particles enabled them to sooth the nerves about which they moved."

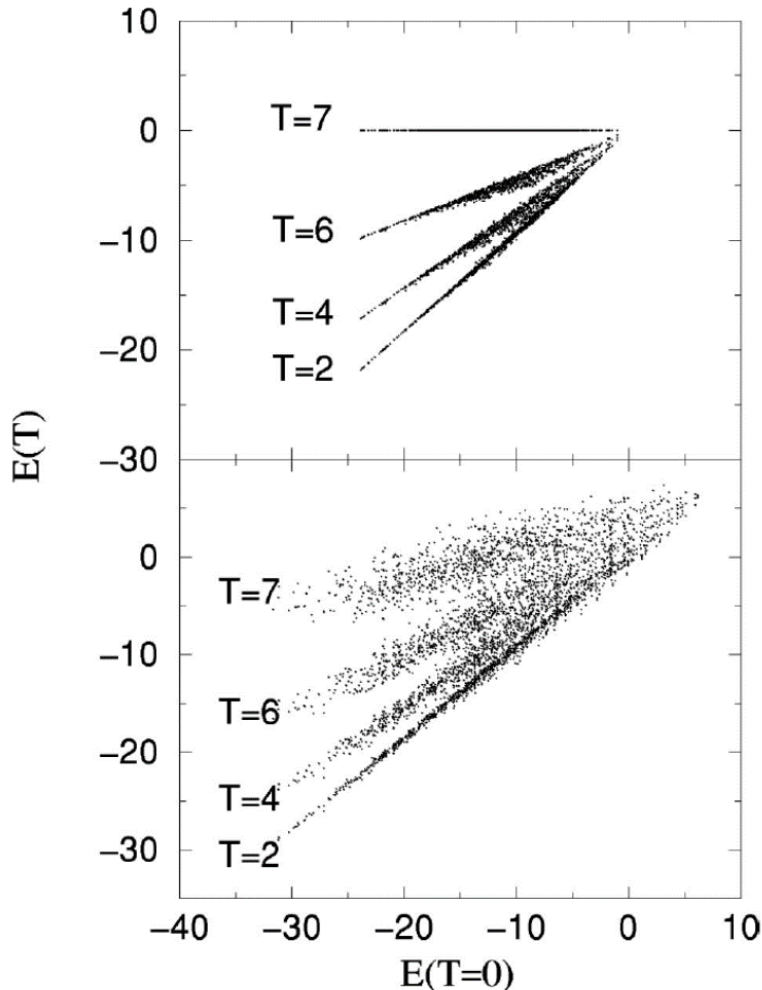
Malcolm R. Forster

<http://philosophy.wisc.edu/forster/220/kuhn.htm>

Kuhn-The Structure of Scientific Revolutions

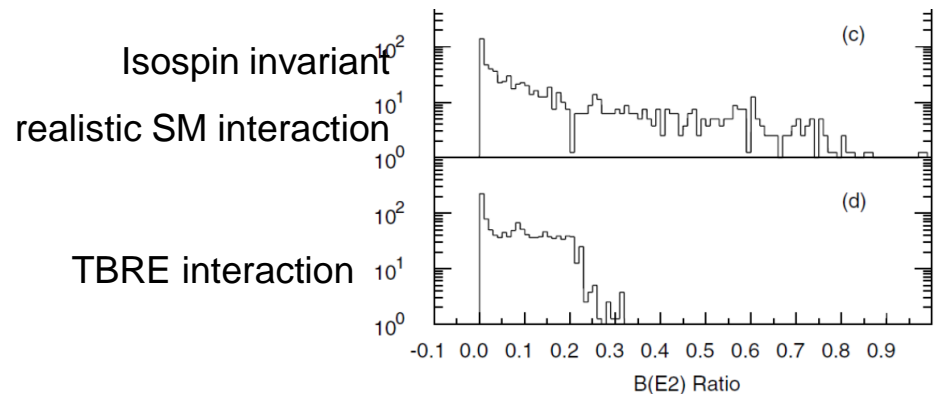
Rotational bands in Isospin space and O-E effects

M. Horoi, A. Volya, and V. Zelevinsky, Phys. Rev. C 66, 024319 (2002)



- sd shell model and single j
- fermion pairs treated like quasi bosons
- rule for enhanced $J_0=0$ and $T_0=0$

$$(-1)^{J_0+T_0} = (-1)^{N/2}$$



$$f_{B(E2)} = \frac{|\langle (2^+ 0)_1 | E2 | (0^+ 0)_{gs} \rangle|^2}{\sum_n |\langle (2^+ 0)_n | E2 | (0^+ 0)_{gs} \rangle|^2}$$

Kuhn's feeling is that scientific change brings about a change in the entities that are taken to be primitive and unexplained. For example, Aristotelians said that a stone fell because of its 'nature' drove it toward the center of the universe. Afterwards the normal seventeenth-century tradition of scientific practice insisted that **"the entire flux of sensory appearances, including color, taste, and even weight, was to be explained in terms of the size, shape, position, and motions the elementary corpuscles of base matter."** (p.104) The attribution of other qualities to the elementary atoms was a resort to the occult and therefore out of bounds for science. Famously, Molière ridiculed the doctor who explained opium's efficacy as a soporific by attributing to it a dormitive potency. (p.104) Kuhn sees this not as a criticism of postulating mystical entities *per se* but of postulating an entity not accepted as primitive at the time. In that vein, Kuhn remarks that **"During the last half of the seventeenth century many scientists preferred to say that the round shape of the opium particles enabled them to sooth the nerves about which they moved."**

Explanations

Why *does* opium make you drowsy?

Because it has a **Dormitive Potency**

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(idiomatic, rhetoric, logic, linguistics) A type of tautology in which an item is being explained in terms of the item itself, only put in different (usually more abstract)

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