Quest for superradiance in atomic nuclei

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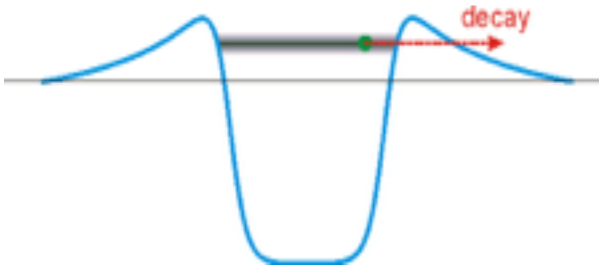
WNMP, March 2017

Quantum mechanics of decay

Why exponential decay? $\frac{dN(t)}{dt} = -\Gamma N(t) \quad N(t) = N(0)\, e^{-\Gamma t}$

Time evolution and decay in quantum mechanics

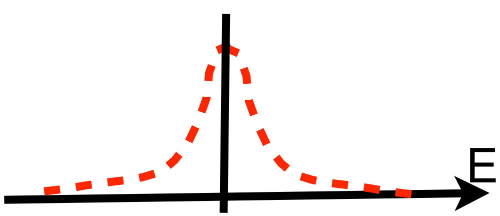


$$\psi(t) = e^{-iHt/\hbar}\psi(0)$$

Survival amplitude and probability

$$A(t) = \langle \psi(0)|e^{-iHt/\hbar}|\psi(0)\rangle \qquad P(t) = |A(t)|^2$$

Resonance wave function



$$\psi_R(t) = \exp\left[-\frac{i}{\hbar}\left(E_0 - i\frac{\Gamma}{2}\right)t\right]\psi_R(0)$$

$$|\psi_R(E)|^2 \propto \frac{\Gamma/2}{(E-E_0)^2 - \Gamma^2/4}$$

Why and when decay cannot be exponential

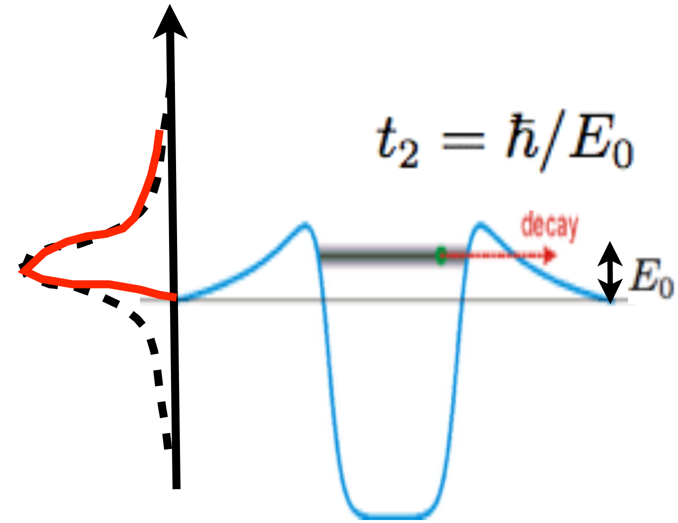
Initial state "memory" time $e^{-iHt/\hbar} \approx 1 - iHt/\hbar \ldots \quad t_1 = \hbar/(\Delta E) \quad t < t_1$

Internal motion in quasi-bound state

$$|\psi_R(E)|^2 \propto \frac{\Gamma/2}{(E-E_0)^2 - \Gamma^2/4}$$

$$t < t_2$$

$$t_2 = \hbar/E_0$$



Remote power-law $t > t_3$

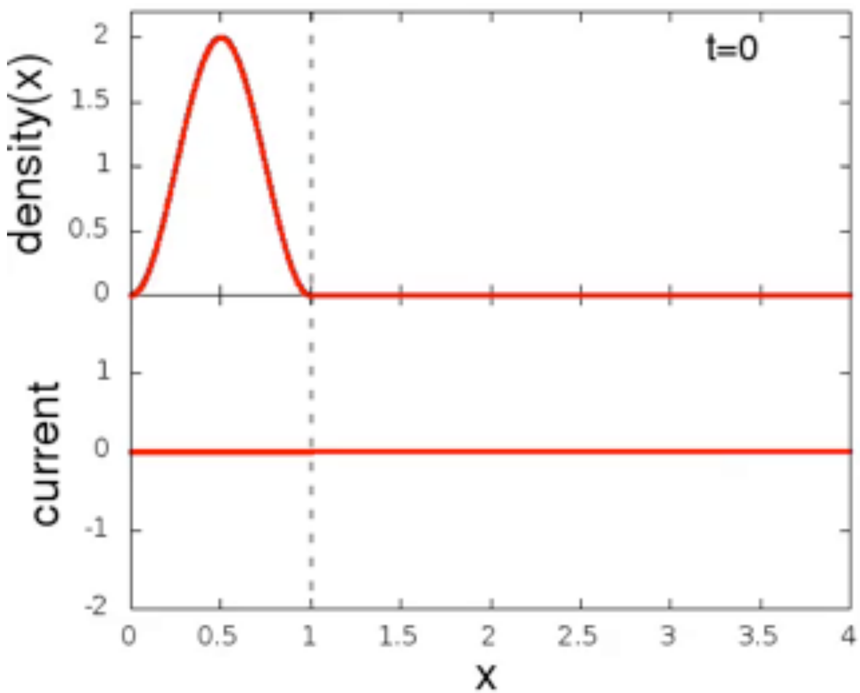
There are "free" slow-moving non-resonant particles, they escape slowly

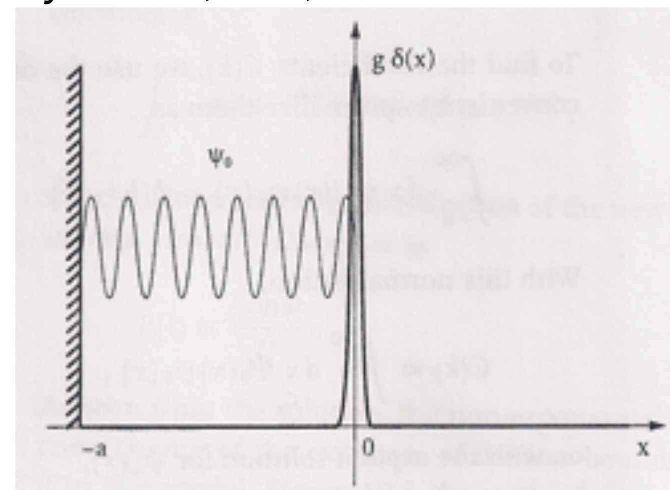
$$N(t) \propto \frac{\Delta x}{vt} = \frac{\hbar}{mv^2 t} = \frac{2\hbar}{E_0 t} \propto |\psi_N(t)|^2 \qquad \Delta x = \frac{\hbar}{mv} \qquad t_3 = \frac{\hbar}{\Gamma} \ln\left(\frac{E_0}{\Gamma}\right)$$

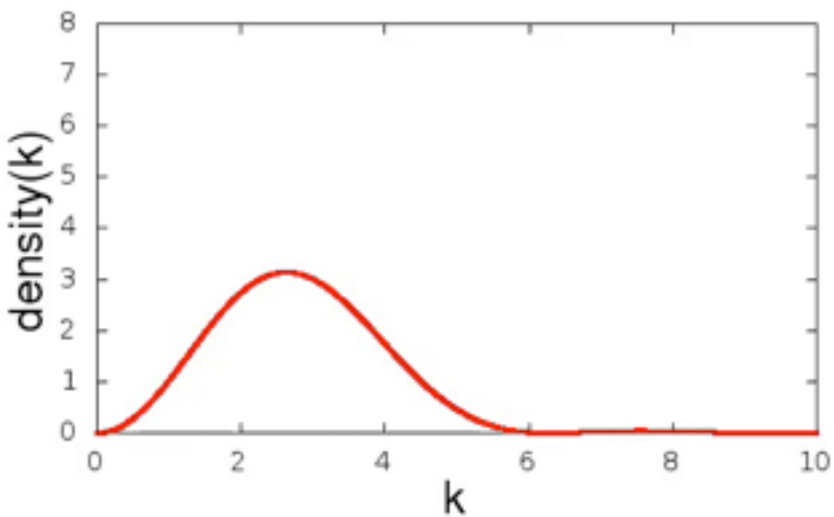
Example ^{14}C decay: E_0=0.157 MeV $t_2=10^{-21}$ s $\quad \ln\left(\frac{E_0}{\Gamma}\right)$=73

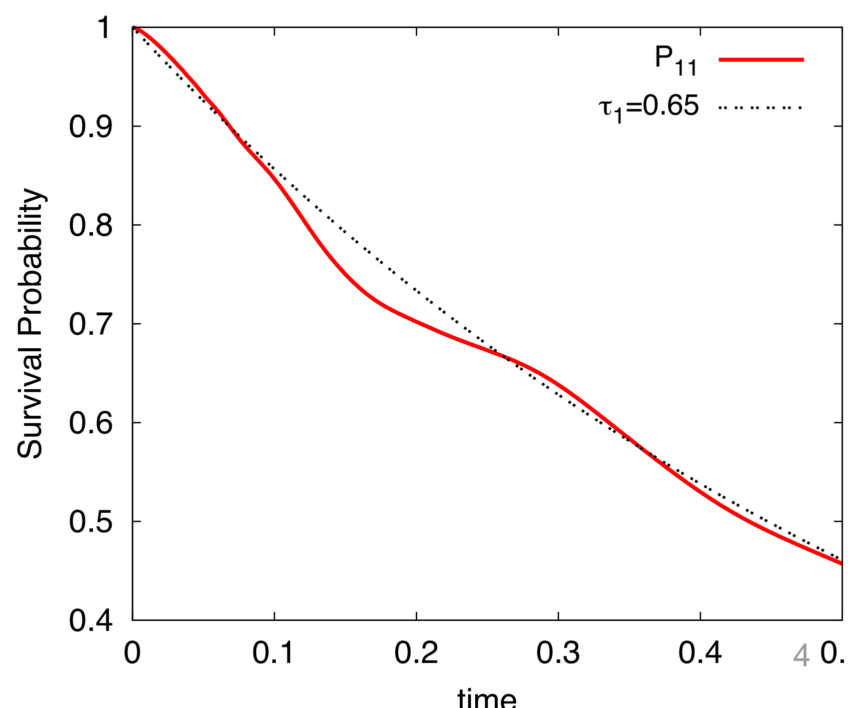
Time dependence of decay, Winter's model

Winter, Phys. Rev., ***123****,1503 1961.*

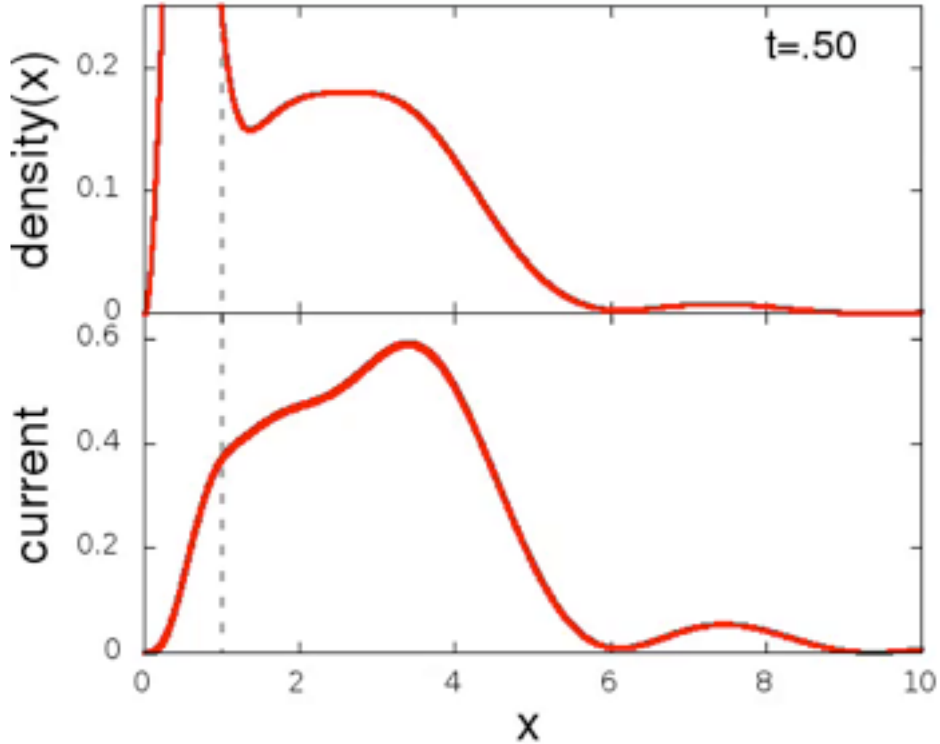


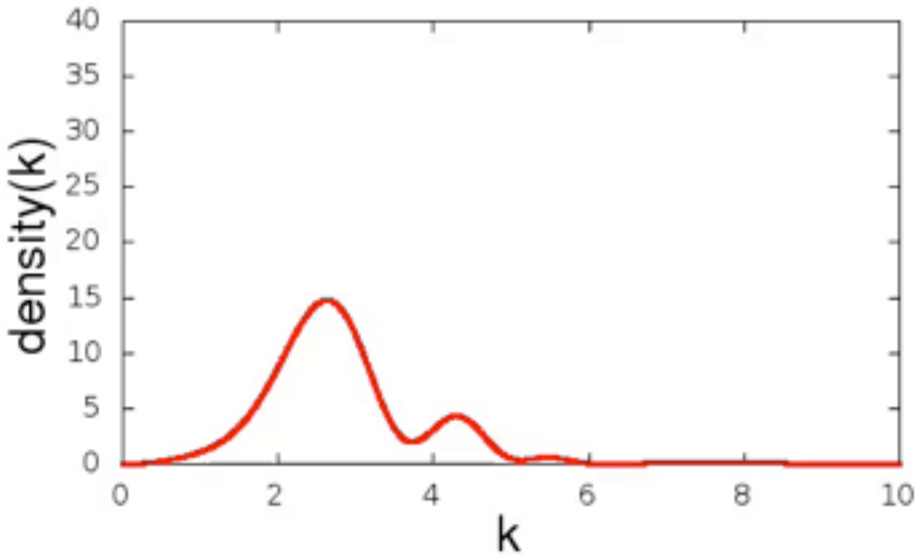


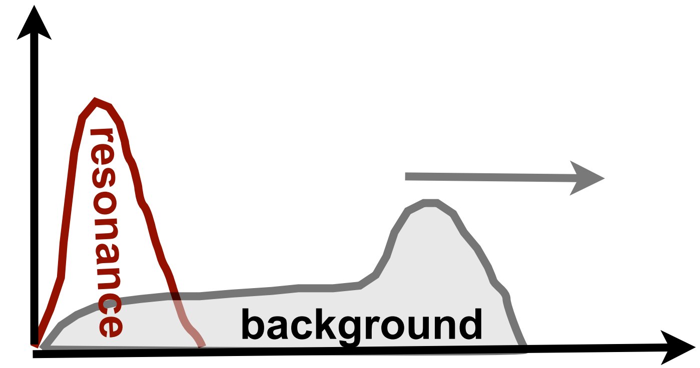


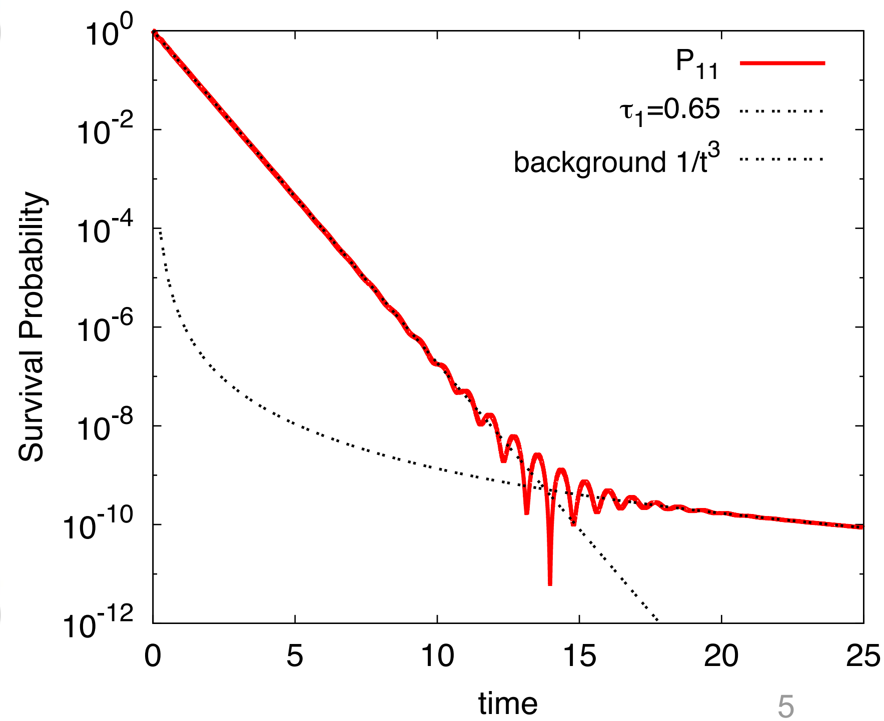


Winter's model: Dynamics at remote times

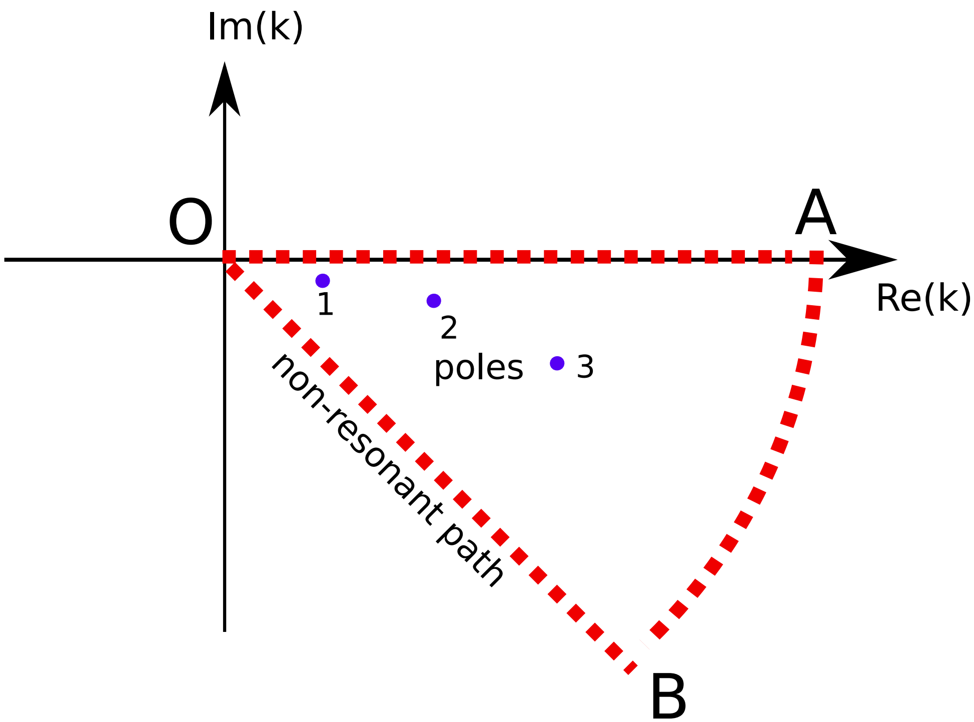




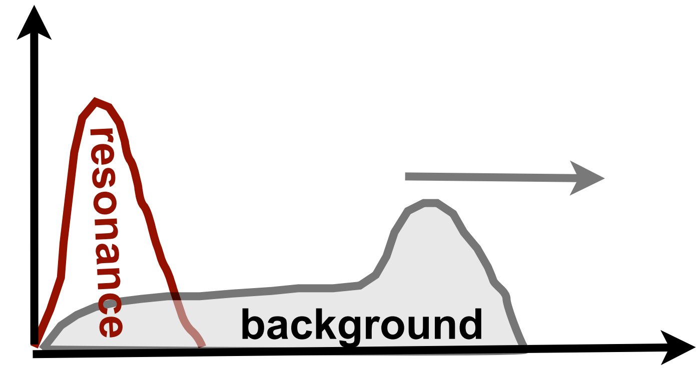


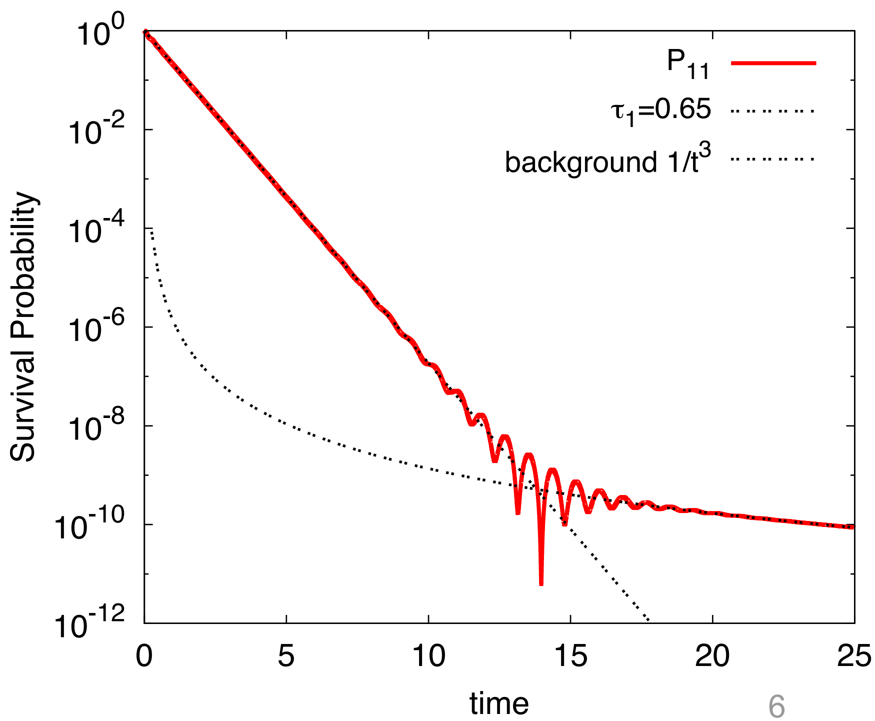


Winter's model: Dynamics at remote times



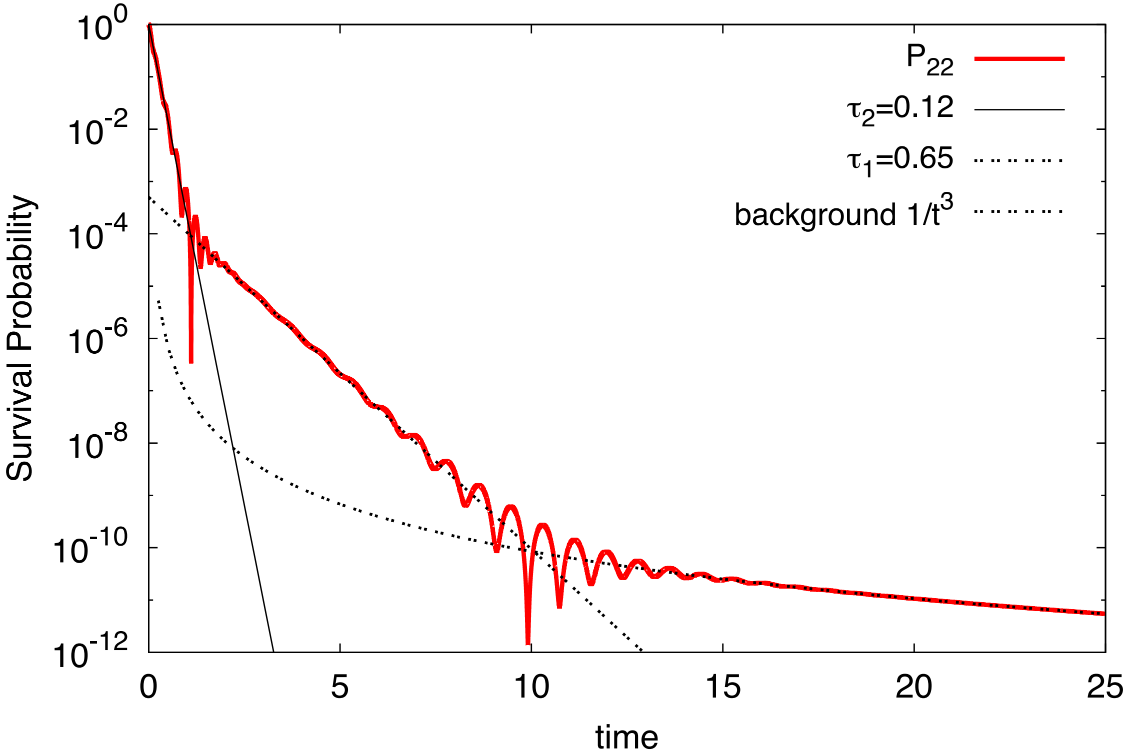
$$\langle n|e^{-iHt}|n'\rangle = \int_0^\infty e^{-ik^2 t}\langle n|k\rangle\langle k|n'\rangle \mathrm{d}k$$

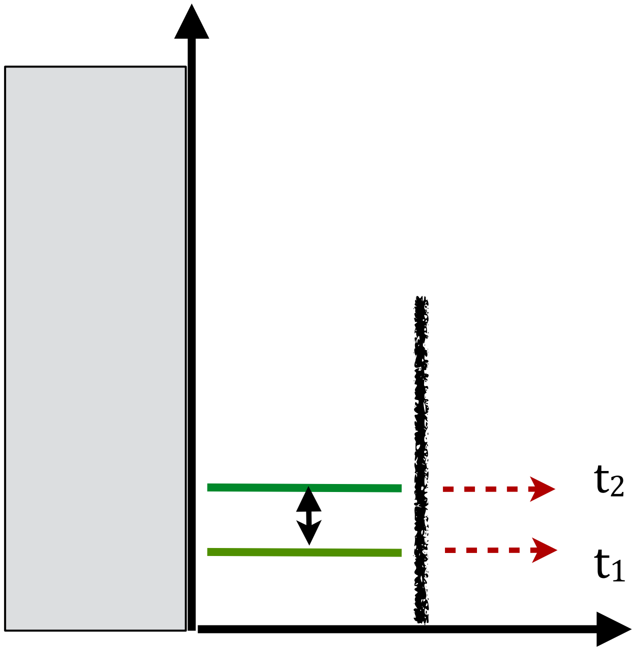




M. Peskin, AV, V. Zelevinsky, EPL, 107(4), 40001 (2014).

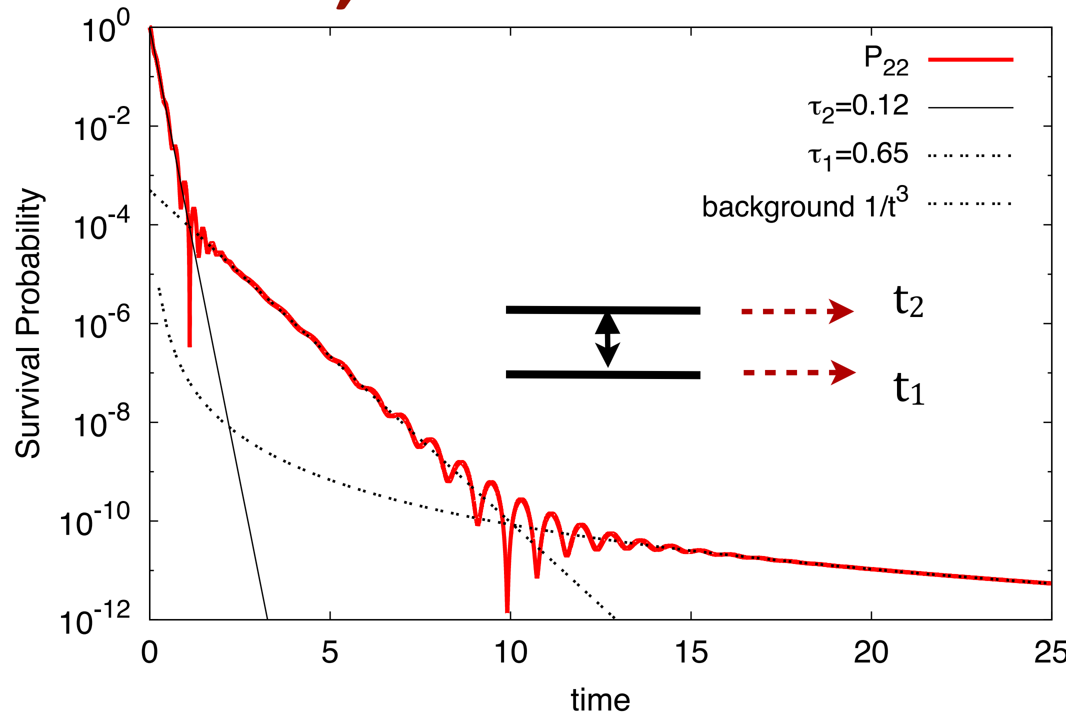
Internal dynamics in decaying system
Winter's model

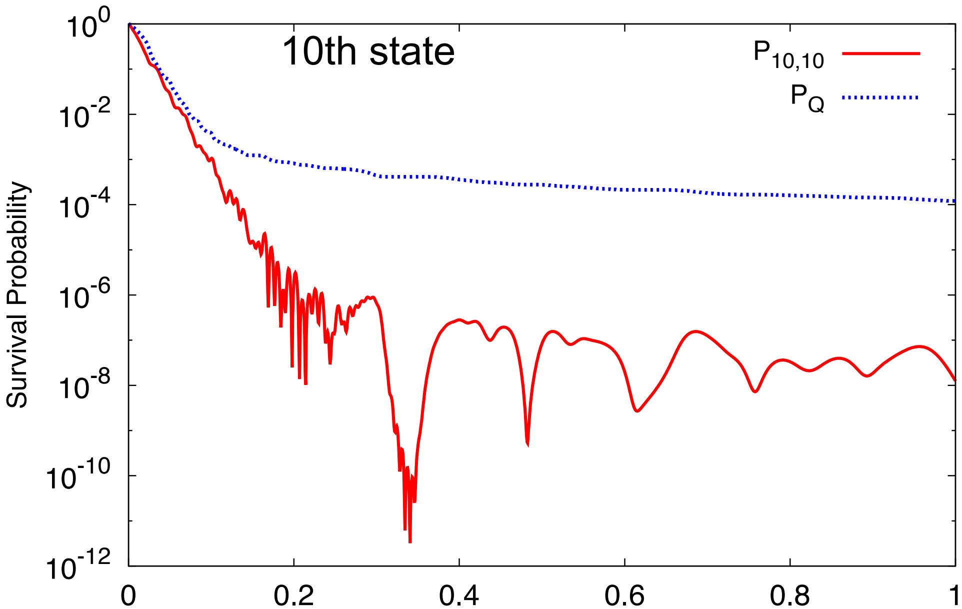


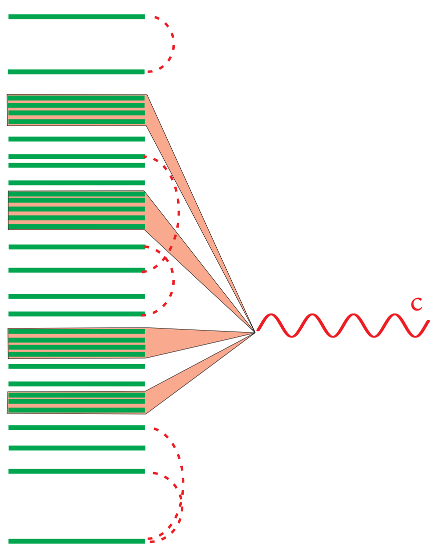


Coupling through continuum, Winter's model

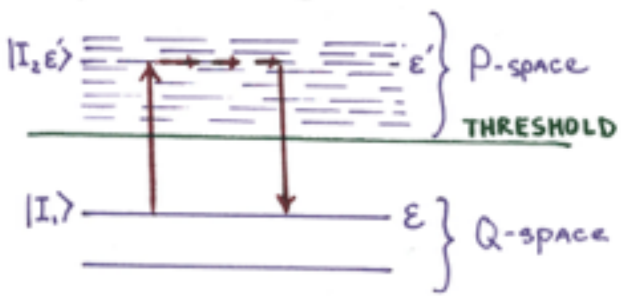
- Short time scales, internal dynamics
- Complexity of pre-exponential behavior
- Transitions driven by continuum coupling.
- Long time behavior, threshold effect
- Survival probability is to be generalized to P_Q



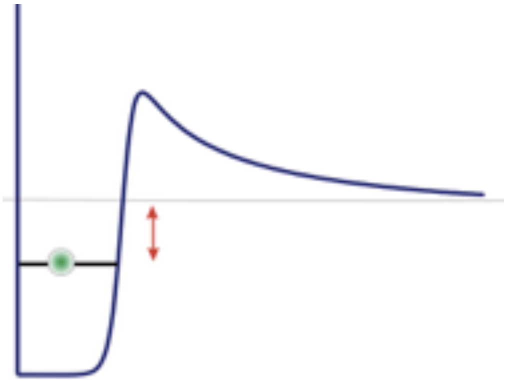




Physics of coupling to continuum

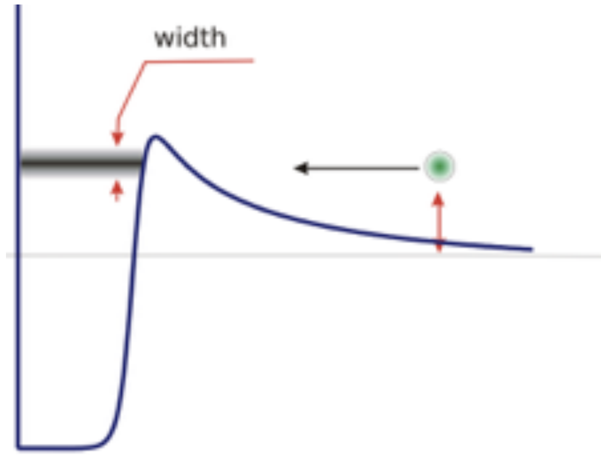


$$H'(\epsilon) = \int_0^\infty d\epsilon' \frac{|A(\epsilon')|^2}{\epsilon - \epsilon' + i0}.$$



Integration region involves no poles

$$H'(\epsilon) = \Delta(\epsilon) \qquad \Delta(\epsilon) = \fint d\epsilon' \frac{|A(\epsilon')|^2}{\epsilon - \epsilon' + i0}$$

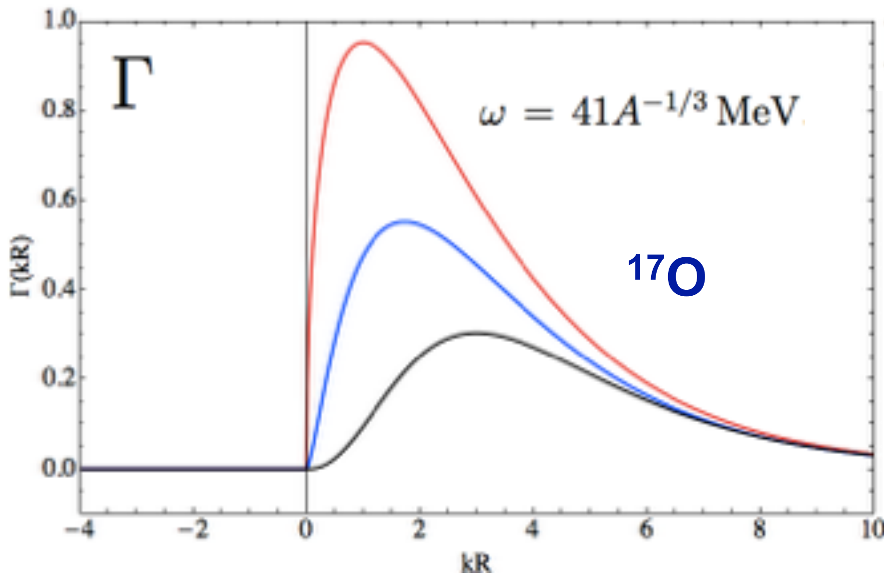


State embedded in the continuum

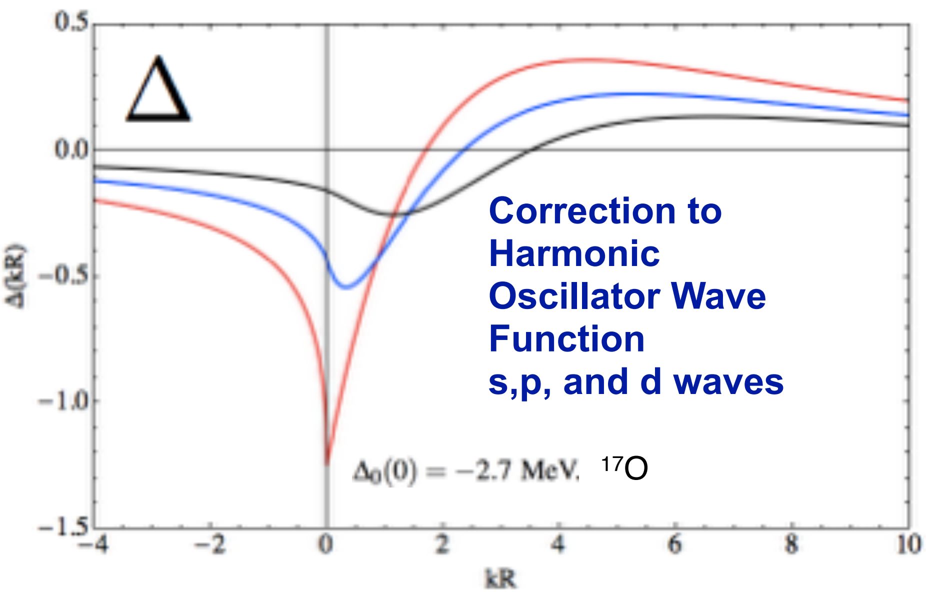
$$\frac{1}{x \pm i0} = \text{p.v.}\frac{1}{x} \mp i\pi\delta(x)$$

$$H'(\epsilon) = \Delta(\epsilon) - \frac{i}{2}\Gamma(\epsilon) \quad \Gamma(\epsilon) = 2\pi\,|A(\epsilon)|^2$$

Self energy, interaction with continuum



$\Gamma(\epsilon) \propto \epsilon^{l+1/2}$



momentum

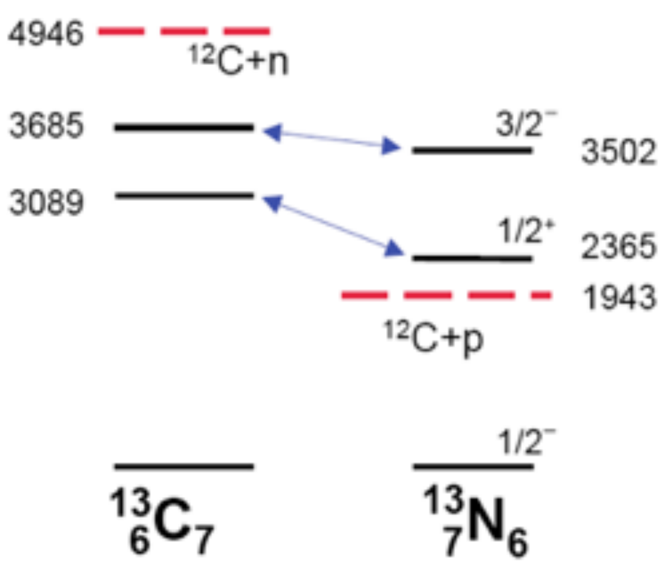
A. Volya, EPJ Web of Conf. 38, 03003 (2012).

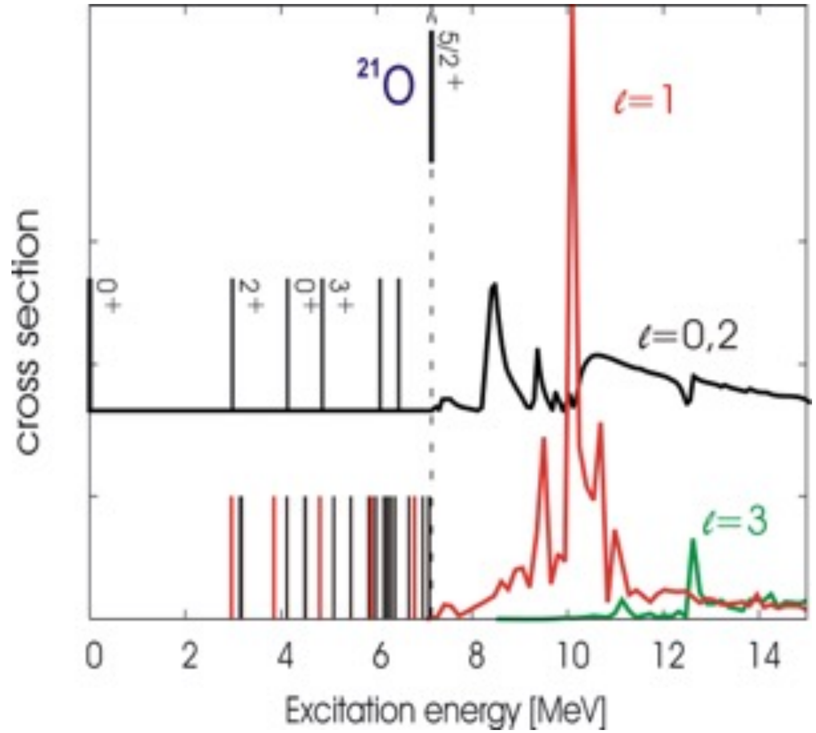
Notes:

- **Wave functions are not HO**
- **Phenomenological SM is adjusted to observation**
- **No corrections for properly solved mean field**

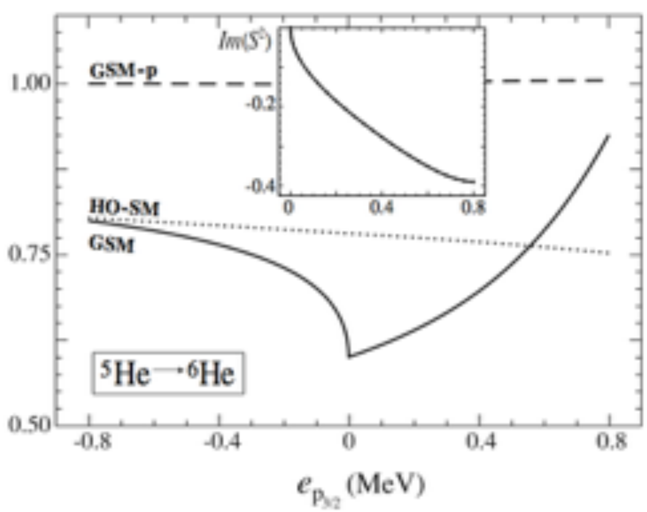
Self energy, Thomas-Ehrmann shift

Thomas-Ehrmann shift





A.V. Phys. Rev. C 79, 044308 (2009).



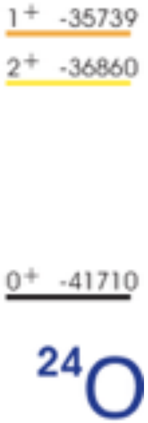
N Michel, J. Phys. G: Nucl. Part. Phys. 36 (2009) 013101

Self energy, perturbative evaluation

$$\Gamma_I = \sum_{c(open)} \Gamma_I^c, \quad \text{where} \quad \Gamma_I^c = 2\pi \left|\langle A^c(E)|I\rangle\right|^2$$

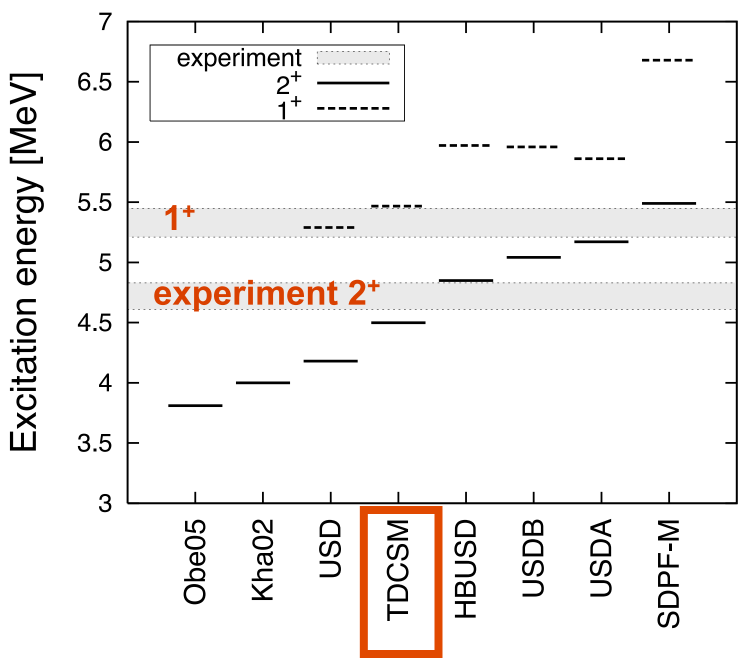
$$\Delta_I \approx \langle I|\Delta(E)|I\rangle = \sum_c \Delta^{(\ell)}(E) \left|\langle c|I\rangle\right|^2$$

	Ex (MeV) SM	Δ_I (MeV)	Ex (MeV) corrected	Ex (MeV) experiment
^{20}O				
0_1^+	0.000	−0.721	0.000	0
2_1^+	1.962	−0.789	1.894	1.674
4_1^+	3.771	−0.746	3.746	3.57
2_2^+	4.174	−1.353	3.542	4.072
0_2^+	5.038	−1.887	3.872	4.456
2_3^+	5.288	−1.616	4.394	5.234
4_2^+	5.504	−1.368	4.857	4.85
4_3^+	7.375	−1.734	6.361	5.002
2_4^+	7.970	−1.578	7.114	5.304



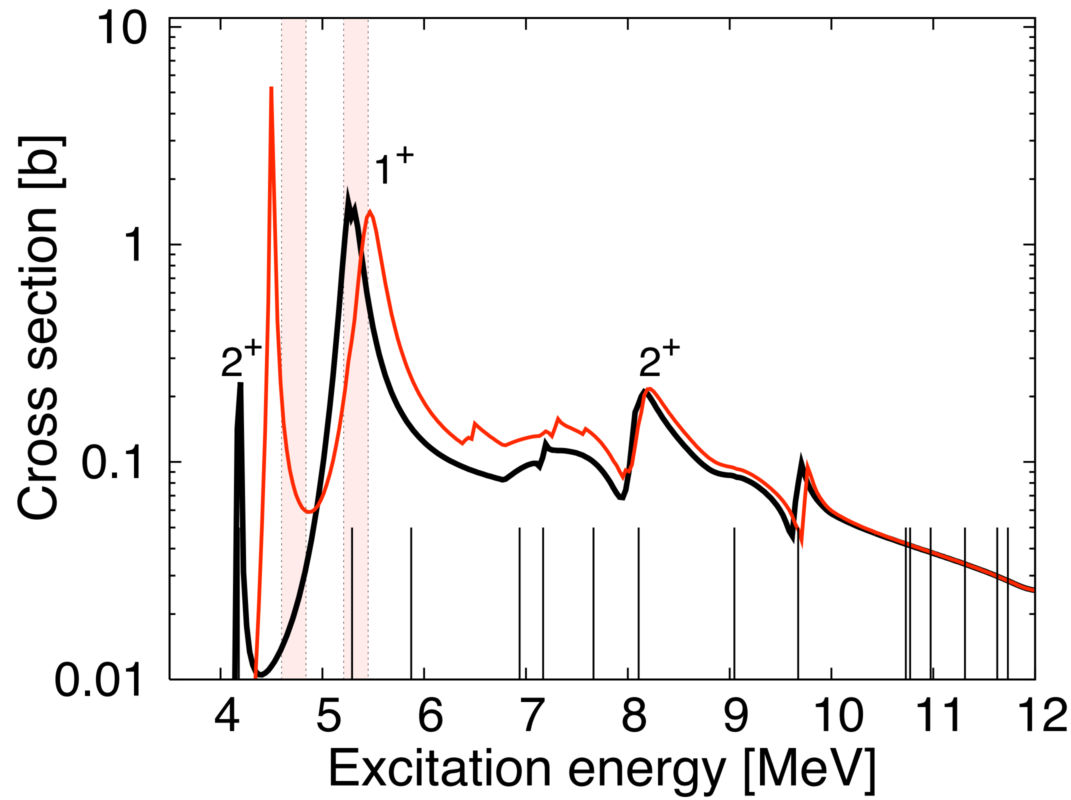
Virtual excitations into continuum

Figure: Theory predictions for states in ^{24}O



Experimental data from:
C. Hoffman, et.al. Phys. Lett. **B672**, 17 (2009)

Figure: $^{23}O(n,n)^{23}O$ Effect of self-energy term (red curve). Shaded areas show experimental values with uncertainties.



A.V. Phys. Rev. C 79, 044308 (2009).

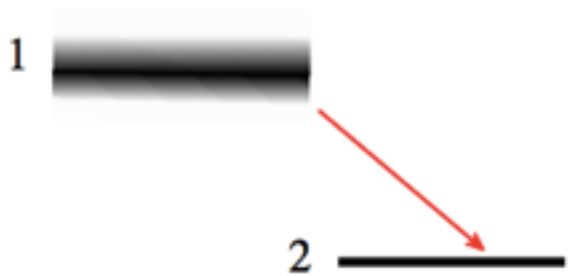
Imaginary part

Fermi Golden Rule

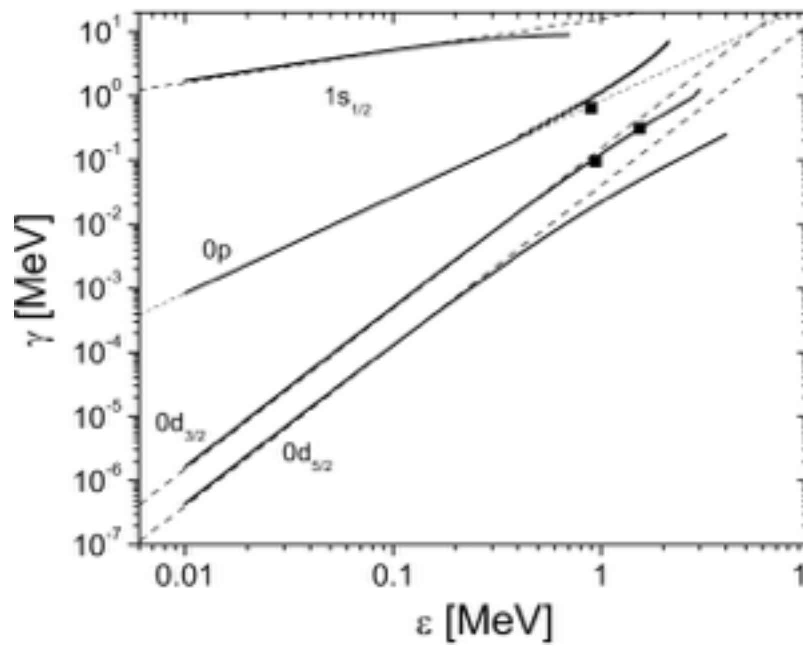
$$A_{1,2}(\epsilon) = \langle I_2, \epsilon | H_{QP_1} | I_1 \rangle$$

$$d\Gamma_{1,2}(\epsilon) = 2\pi \left| A_{1,2}(\epsilon) \right|^2 \delta(E_1 - E_2 - \epsilon) dE$$

$$\Gamma_{1,2}(\epsilon) = 2\pi \left| A_{1,2}(\epsilon) \right|^2$$



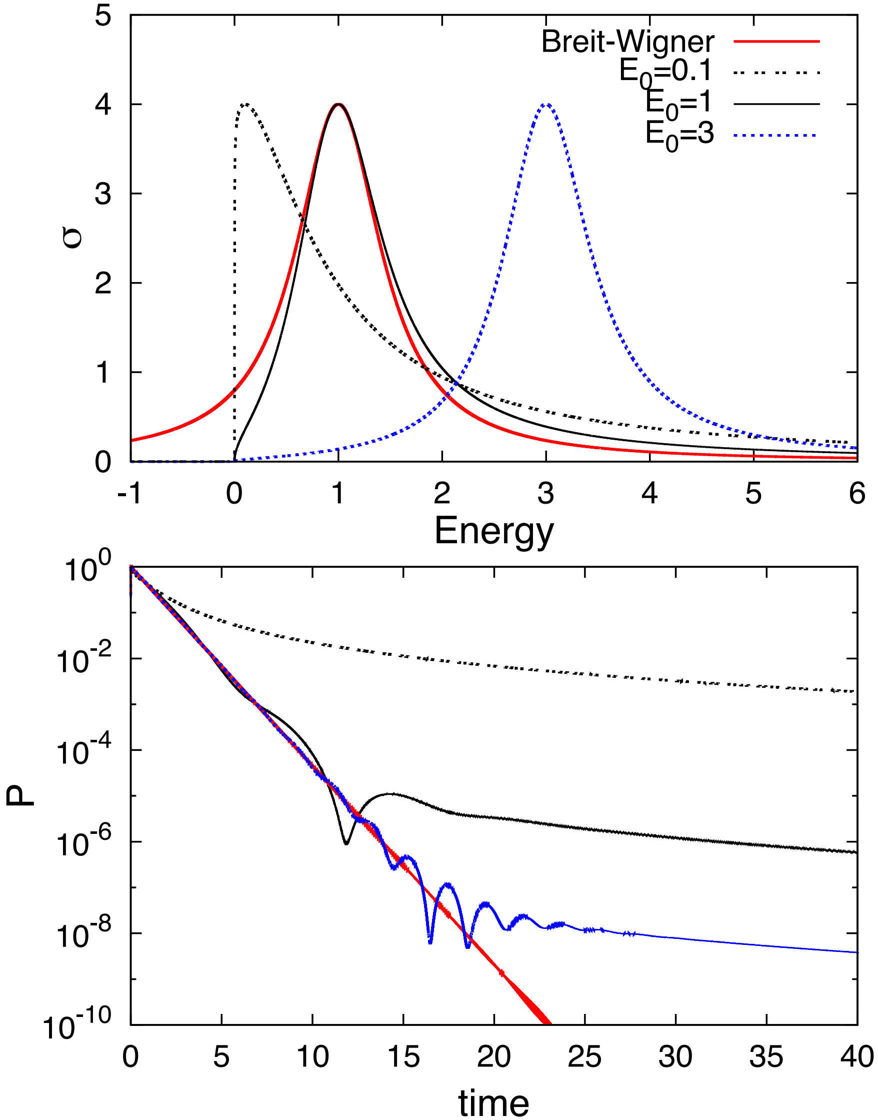
Decay and nuclear mean field



	e(MeV)	γ(keV)	r(fm)
^{5}He	0.895	648	4.5*
^{17}O	0.941	98	3.8
^{19}O	1.540	310	3.9

D. Abrahamsen, A. Volya, and I. Wiedenhoever, *Effective R-matrix parameters of the Woods-Saxon nuclear potential,* APS Volume 57, Number 16, section KA 26 (2012).

Time-dependent picture



$$\mathcal{G} = \frac{1}{E - E_o + i/2\,\Gamma(E)}$$

$$\Gamma(E) \propto \sqrt{E}$$

Power-law remote decay rate!

Physics

spotlighting exceptional research

Focus:

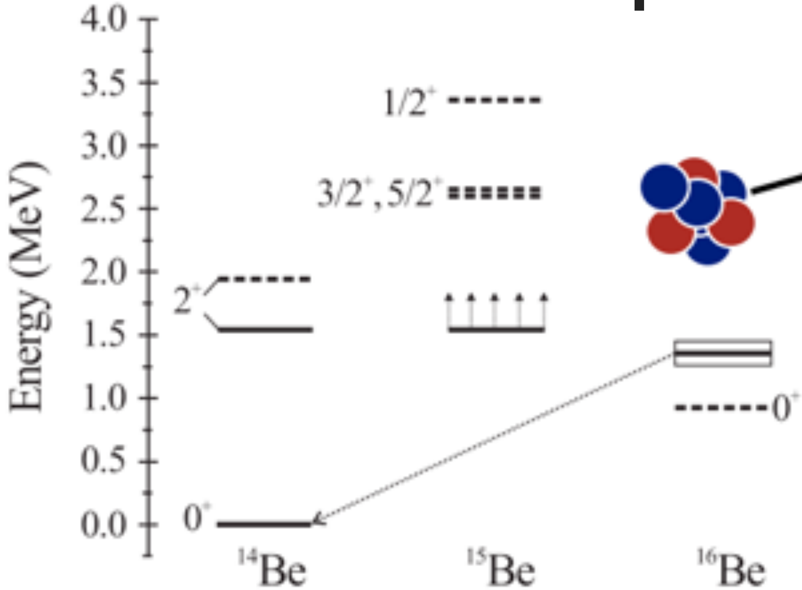
Nuclei Emit Paired-up Neutrons

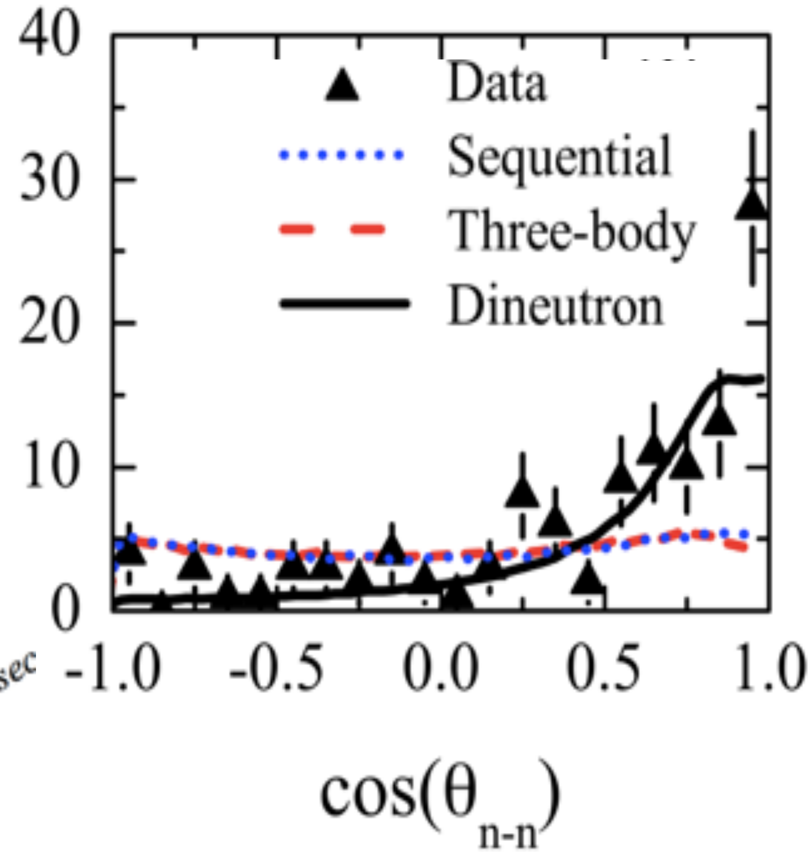
First Observation of Ground State Dineutron Decay: ^{16}Be

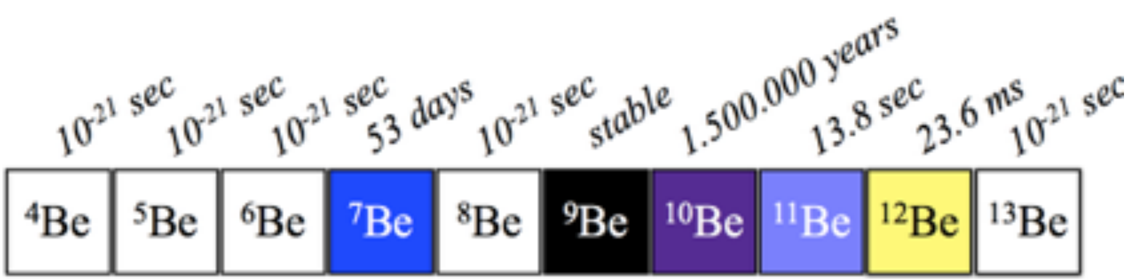
A. Spyrou, Z. Kohley, T. Baumann, D. Bazin, B. A. Brown, G. Christian, P. A. DeYoung, J. E. Finck, N. Frank, E. Lunderberg, S. Mosby, W. A. Peters, A. Schiller, J. K. Smith, J. Snyder, M. J. Strongman, M. Thoennessen, and A. Volya

Phys. Rev. Lett. **108**, 102501 (2012)

Published March 9, 2012

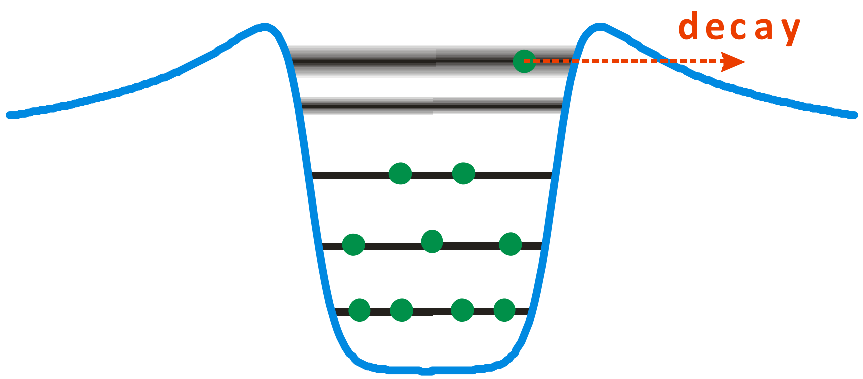






The nuclear many-body problem

The shell model or configuration interaction technique is one of the most powerful tools available to solve the quantum many-body problem.



Traditional shell-model

- Single-particles state (particle in the well)
- Many-body states (slater determinants)
- Hamiltonian and Hamiltonian matrix
- Matrix diagonalization

Continuum physics

- Effective non-hermitian energy-dependent Hamiltonian
- Bound states and resonances
- Matrix inversion at all energies (time dependent approach)

Effective Hamiltonian Formulation

The Hamiltonian in P is: $$\mathcal{H}(E) = H + \Delta(E) - \frac{i}{2}W(E)$$

Channel-vector: $$|A^c(E)\rangle = H_{QP}|c;E\rangle$$

Self-energy: $$\Delta(E) = \frac{1}{2\pi}\int dE' \sum_c \frac{|A^c(E')\rangle\langle A^c(E')|}{E-E'}$$

Irreversible decay to the excluded space: $$W(E) = \sum_{c(\mathrm{open})} |A^c(E)\rangle\langle A^c(E)|$$

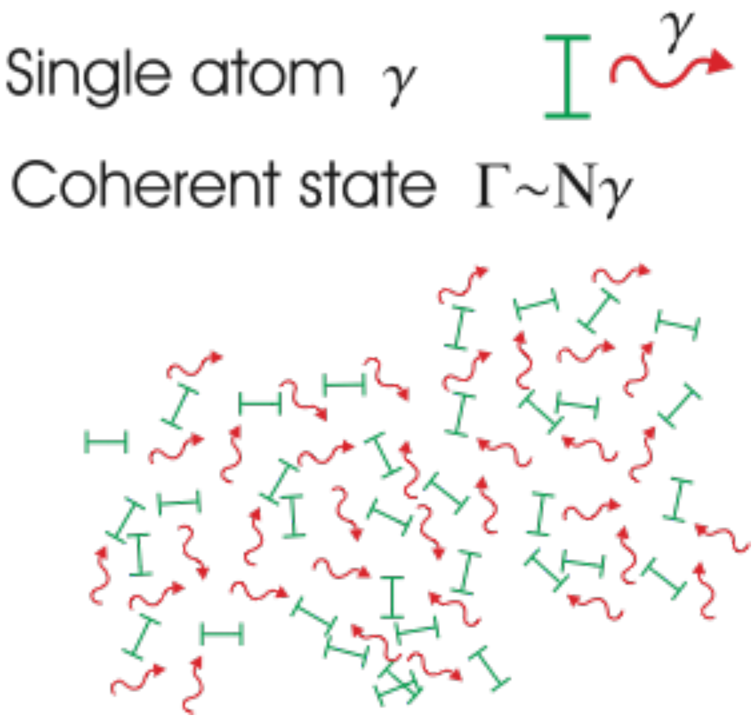
Reactions and observables

$$\mathbf{T}_{cc'}(E) = \langle A^c(E)|\left(\frac{1}{E-\mathcal{H}(E)}\right)|A^{c'}(E)\rangle$$

[1] C. Mahaux and H. Weidenmüller, *Shell-model approach to nuclear reactions*, Amsterdam 1969
[2] A. Volya and V. Zelevinsky, Phys. Rev. Lett. **94**, 052501 (2005).
[3] A. Volya, Phys. Rev. C **79**, 044308 (2009).

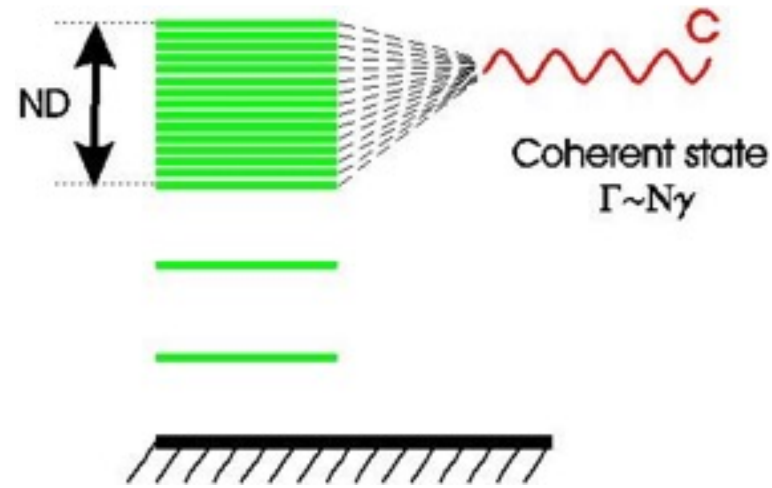
Superradiance, collectivization by decay

Dicke coherent state
N identical two-level atoms
coupled via common radiation



Volume << λ^3

Analog in nuclei
Interaction via continuum
Trapped states) self-organization



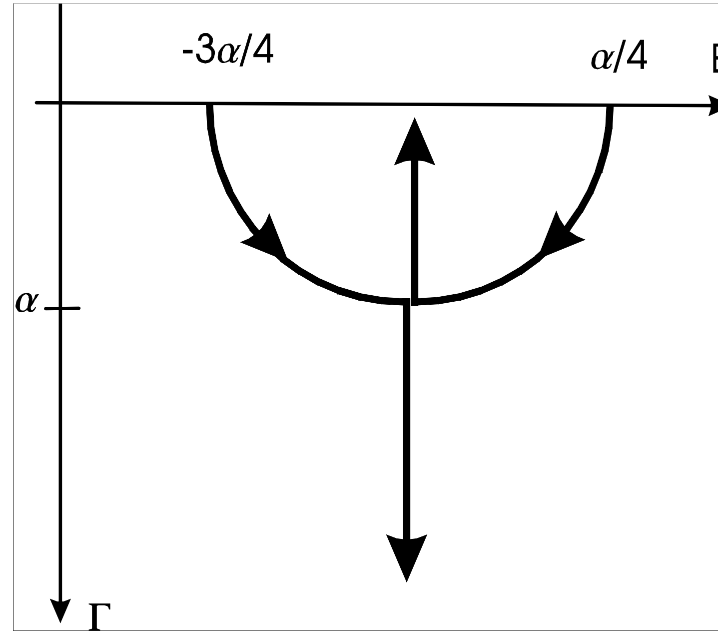
g ~ D and few channels
- Nuclei far from stability
- High level density (states of same symmetry)
- Far from thresholds

- Example Hamiltonian

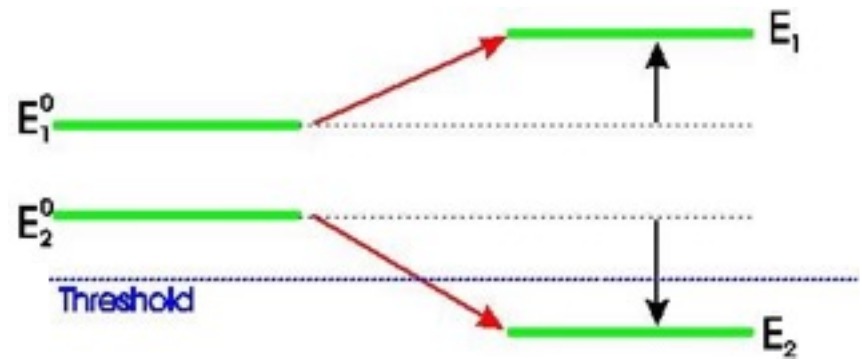
$$\mathcal{H} = -\frac{\alpha}{4} + \frac{1}{2}\begin{pmatrix} -i\gamma & \alpha \\ \alpha & 0 \end{pmatrix}$$

- Complex energies

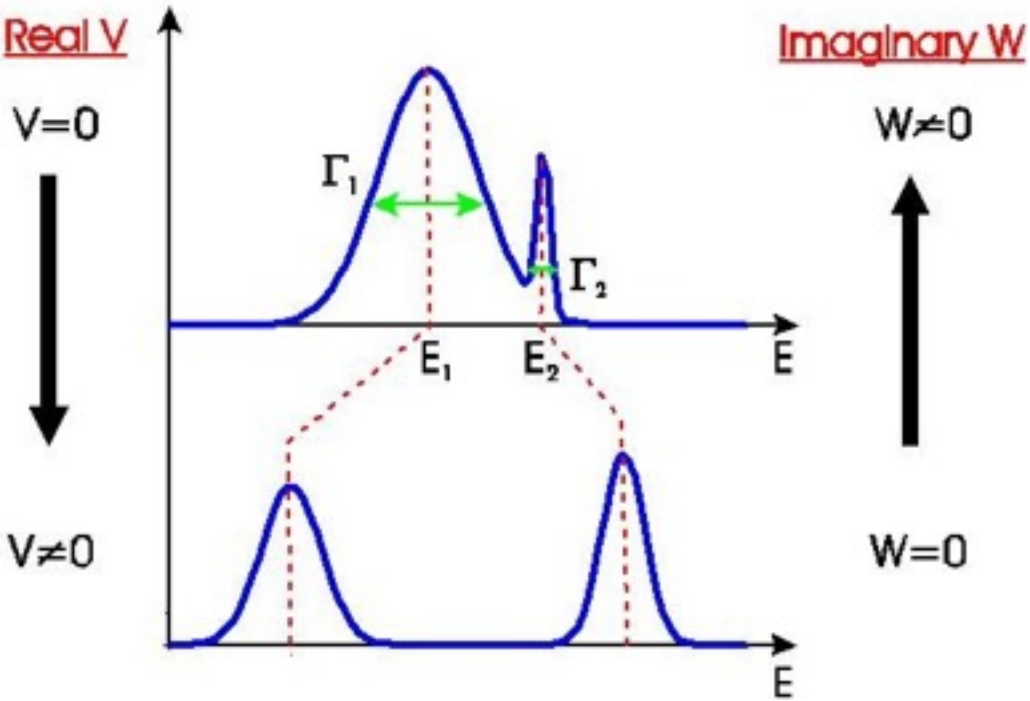
$$\mathcal{E}_{\pm} = -\frac{\alpha}{4} \pm \frac{1}{2}\sqrt{\alpha^2 - \left(\frac{\gamma}{2}\right)^2} - i\frac{\gamma}{4}$$



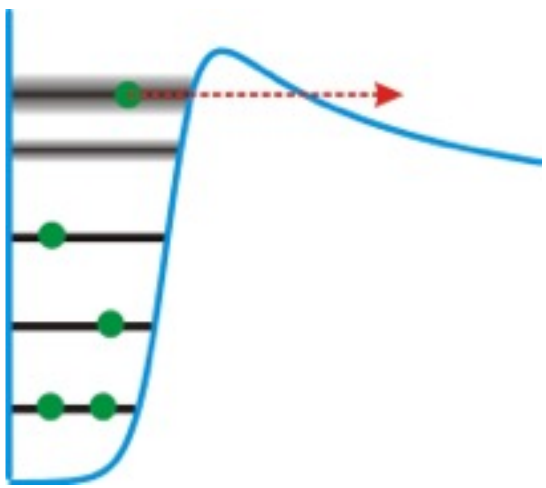
Example of interacting resonances



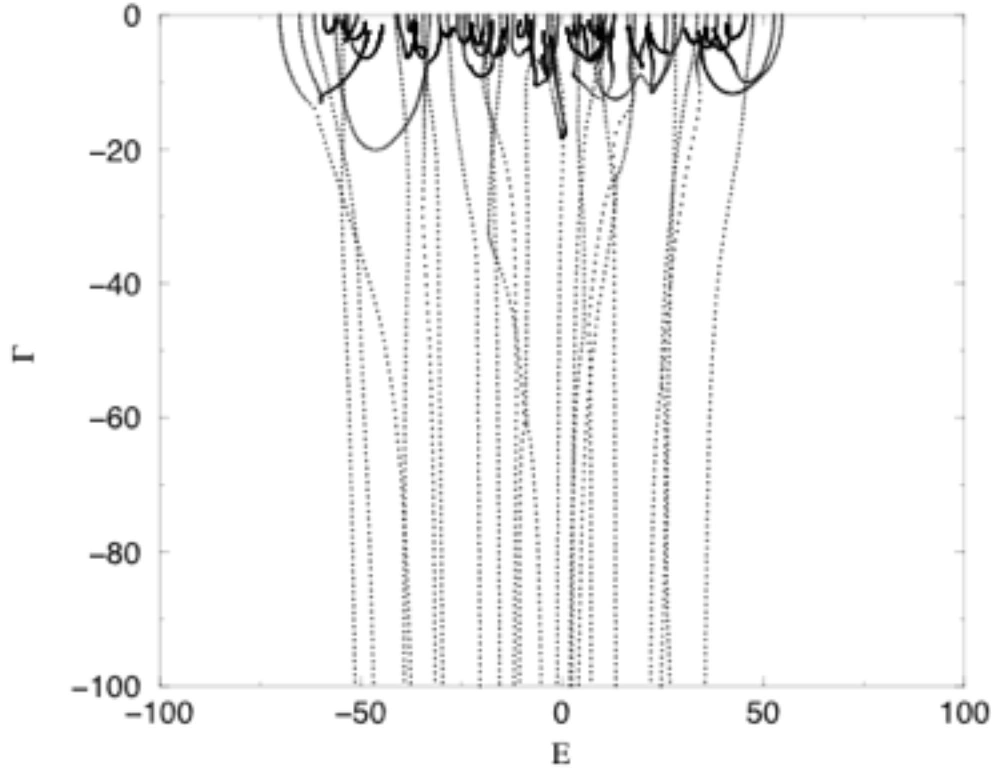
$$\mathcal{H} = H^0 + V - iW/2$$



Collectivity in decay, superradiance



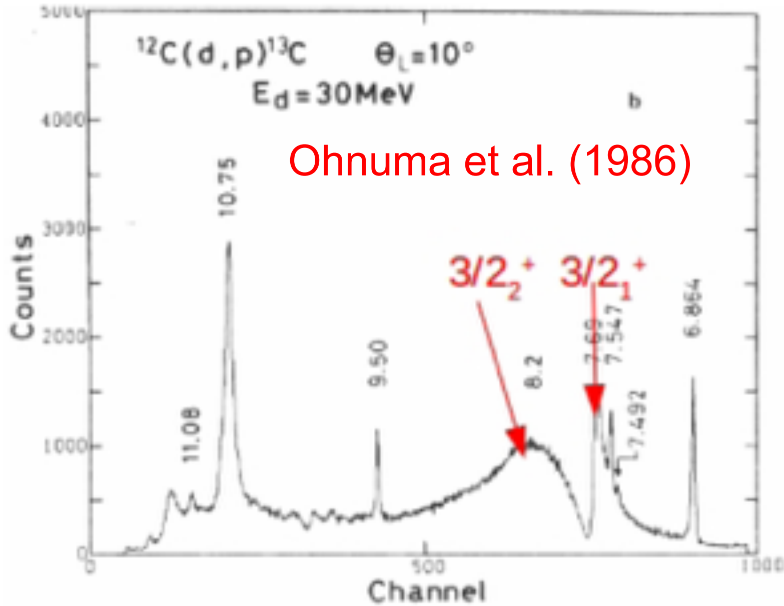
Evolution of complex energies E=E-i $\Gamma/2$ as a function of γ

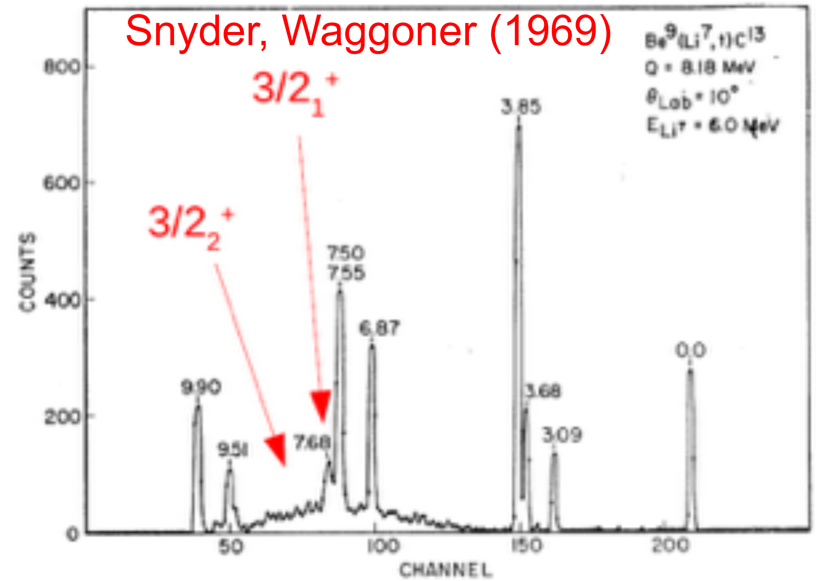


- Assume energy independent width
- Assume one channel $\gamma=A^2$
- System 8 s.p. levels, 3 particles
- One s.p. level in continuum $e=\varepsilon - i\gamma/2$

Total states 8!/(3! 5!)=56; **states that decay fast 7!/(2! 5!)=21**

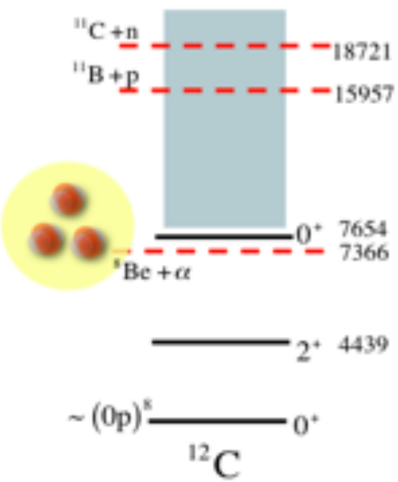
Resonance Spectroscopy: Investigation of Super-radiance

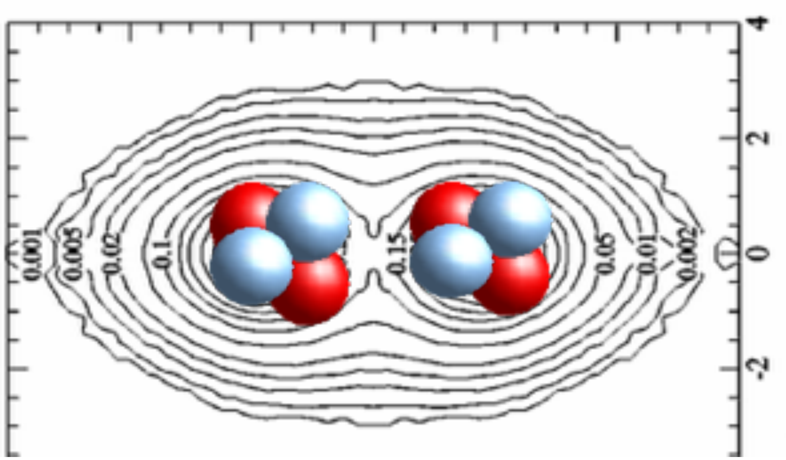




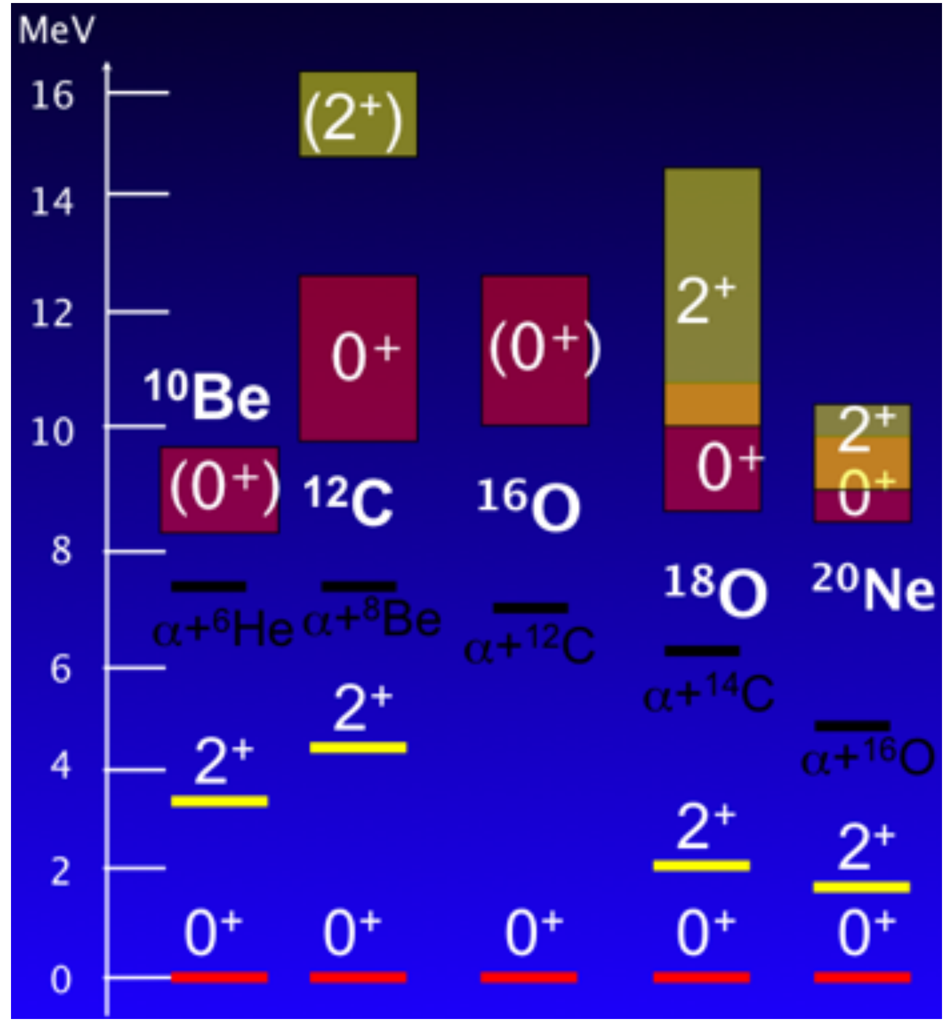
- Left: $^{12}C(d,p)^{13}C$ measure n-strength $3/2^+_1$ vs $3/2^+_2$
- Right: $^{9}Be(^{7}Li,t)^{13}C$ measure α-strength
- Repeat experiment with decay-channel coincidence (DAFNE)
- Competition between both open channels (E_x >10.7 MeV)
- Study influence of multiple open channels on bound-state wavefunctions
- Experiments: see above, also $^{12}C(^{3}He,d)^{13}N(p)$ as mirror $^{12}C(p,d)^{11}C$, $^{10}B(^{3}He,d)^{11}C$, $^{10}B(d,p)^{11}B$ as mirror

alpha clustering and superradiance



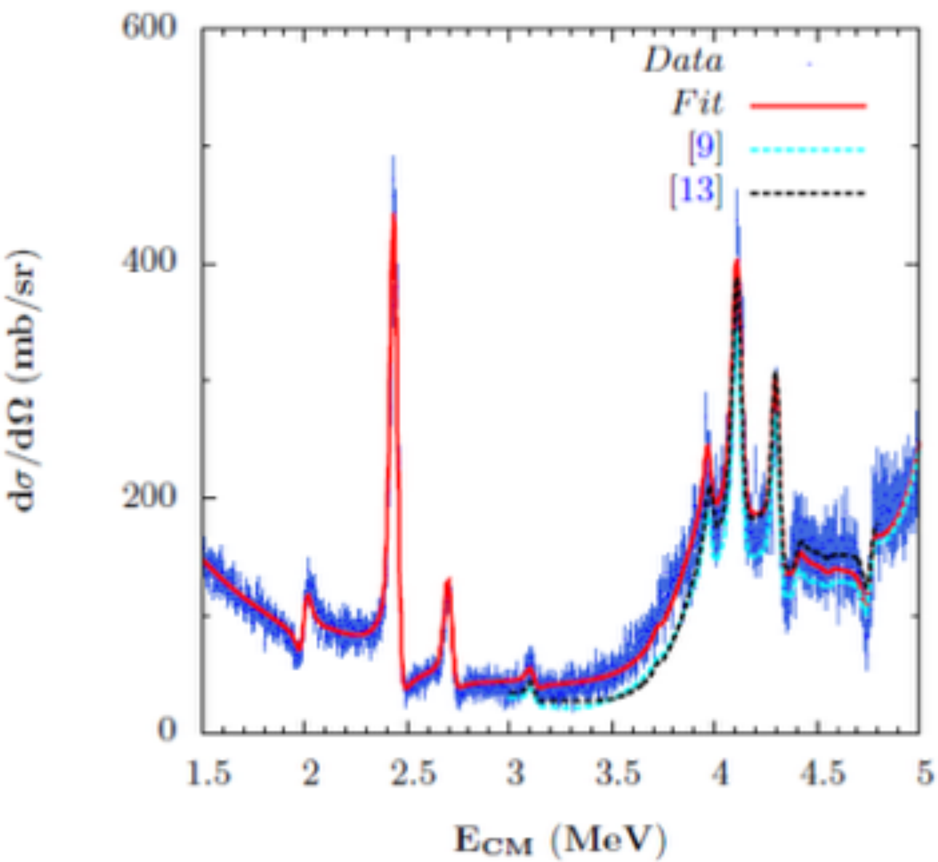


R.B. Wiringa, et al., PRC 62, 014001(2000)



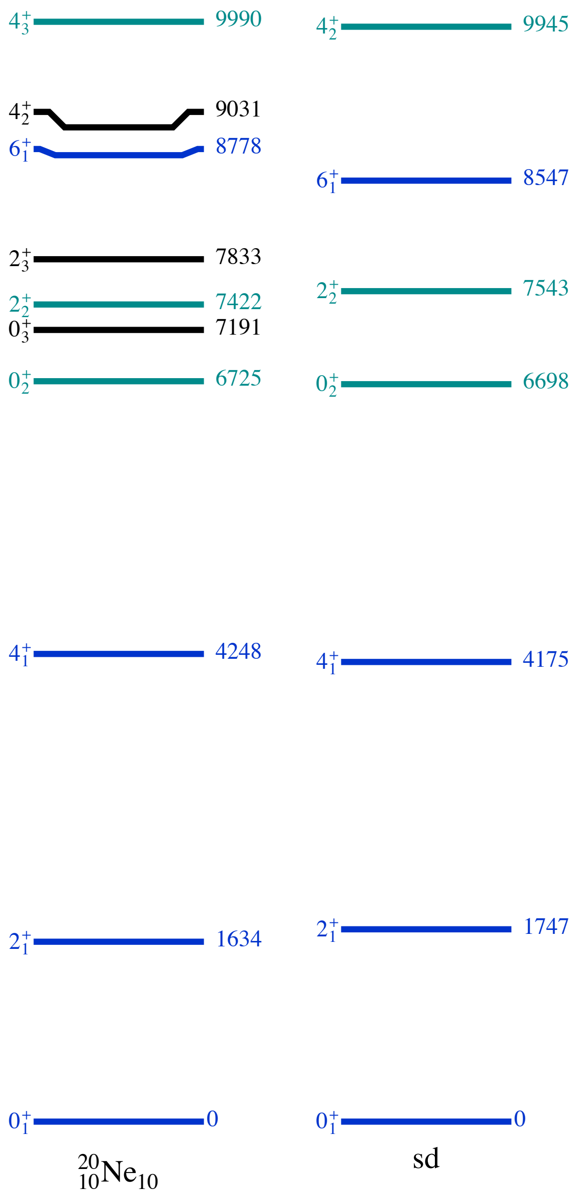
E.D. Johnson, et al., EPJA, 42 135 (2009) H. Fynbo, et al., Nature 433 (2005) 136

Clustering and superradiance

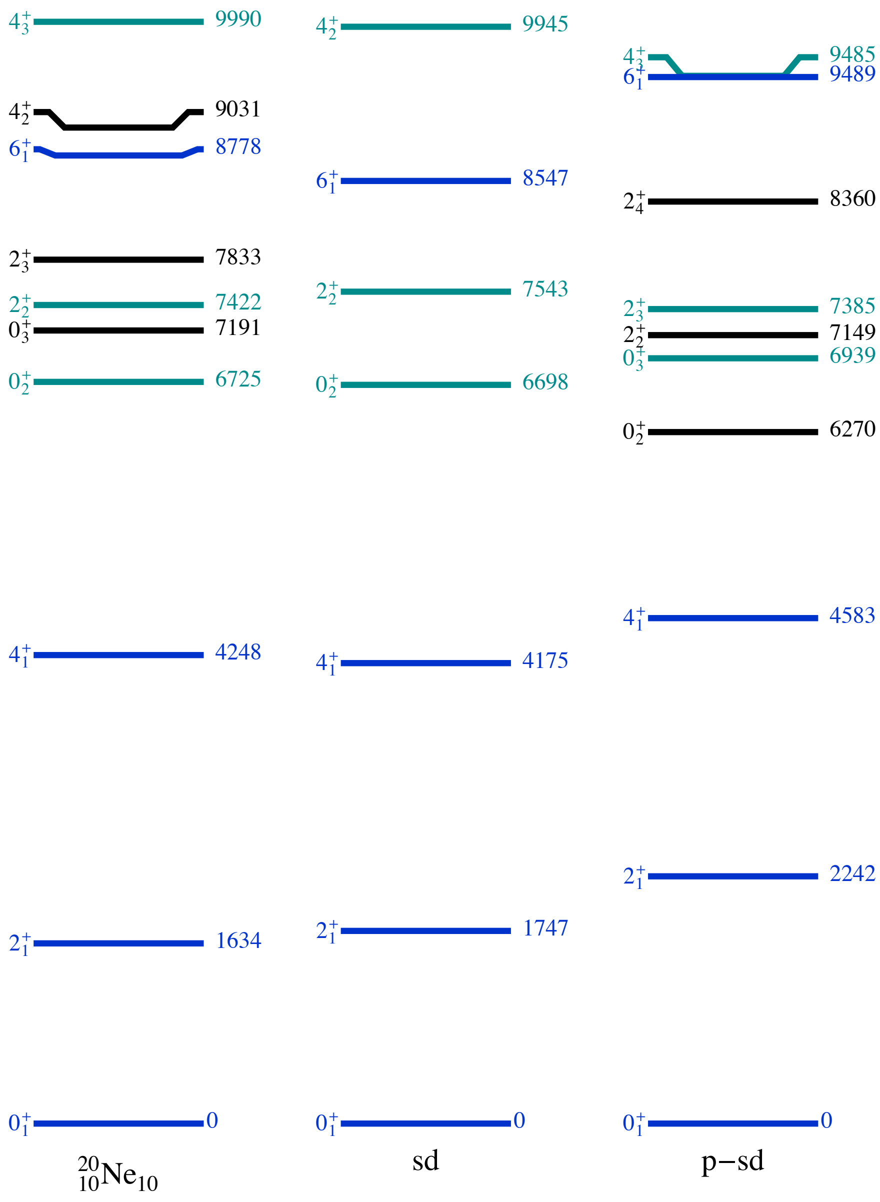


D.K. Nauruzbayev et al., (2017)

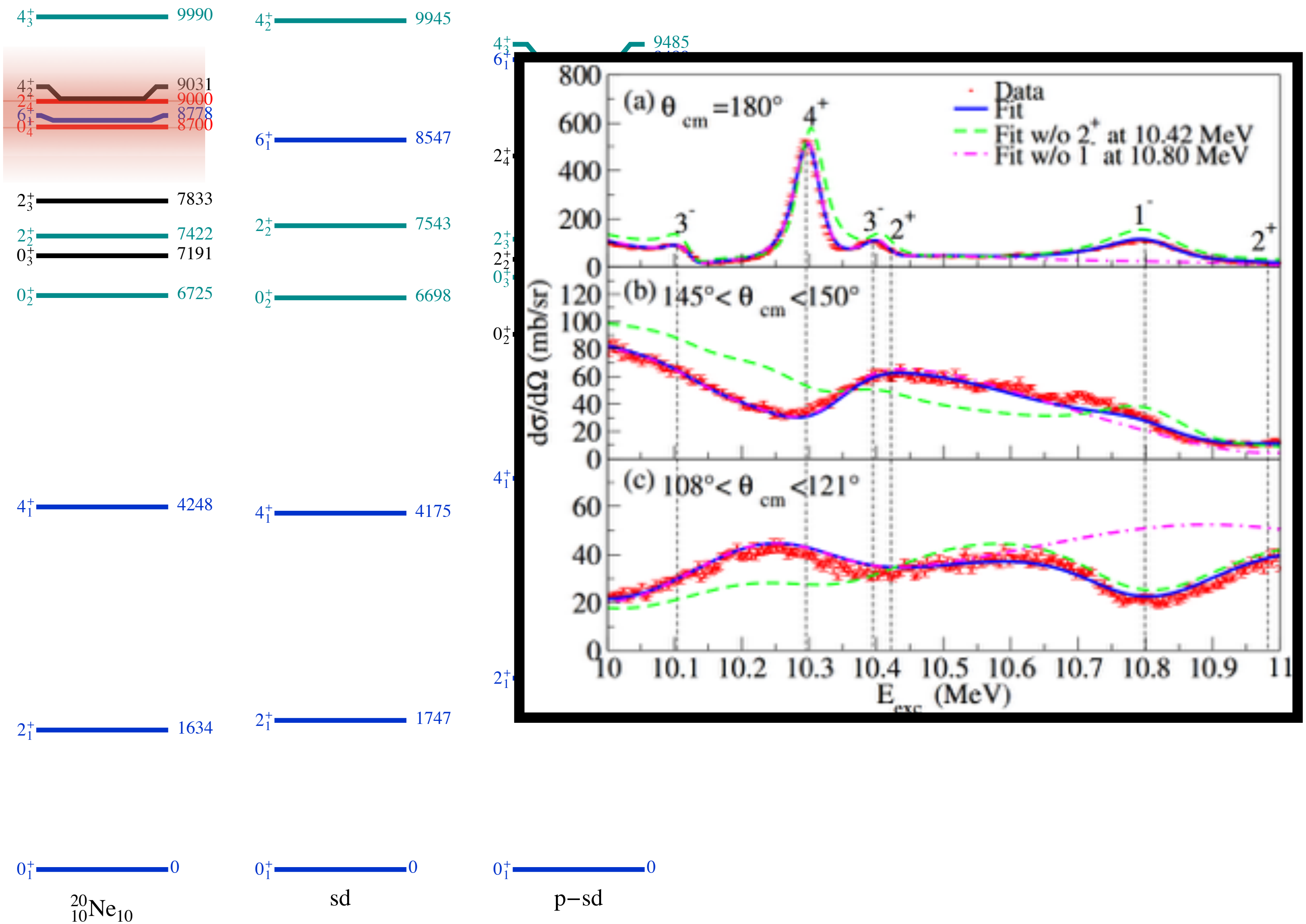
Clustering and superradiance



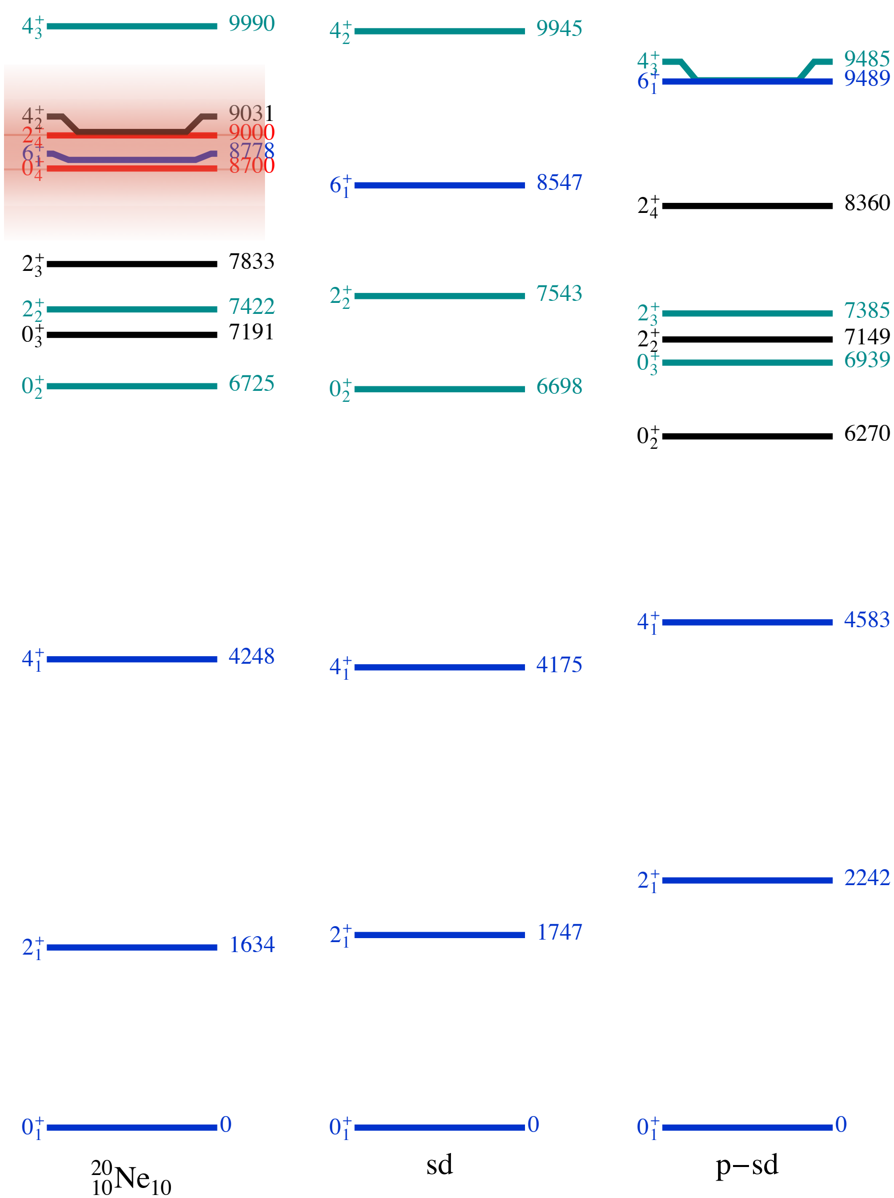
Clustering and superradiance



Clustering and superradiance

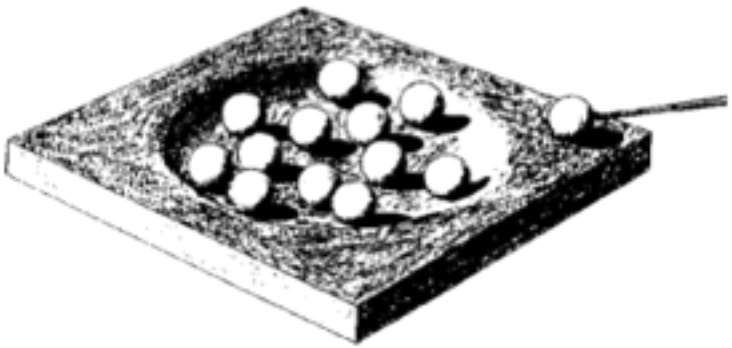


Clustering and superradiance

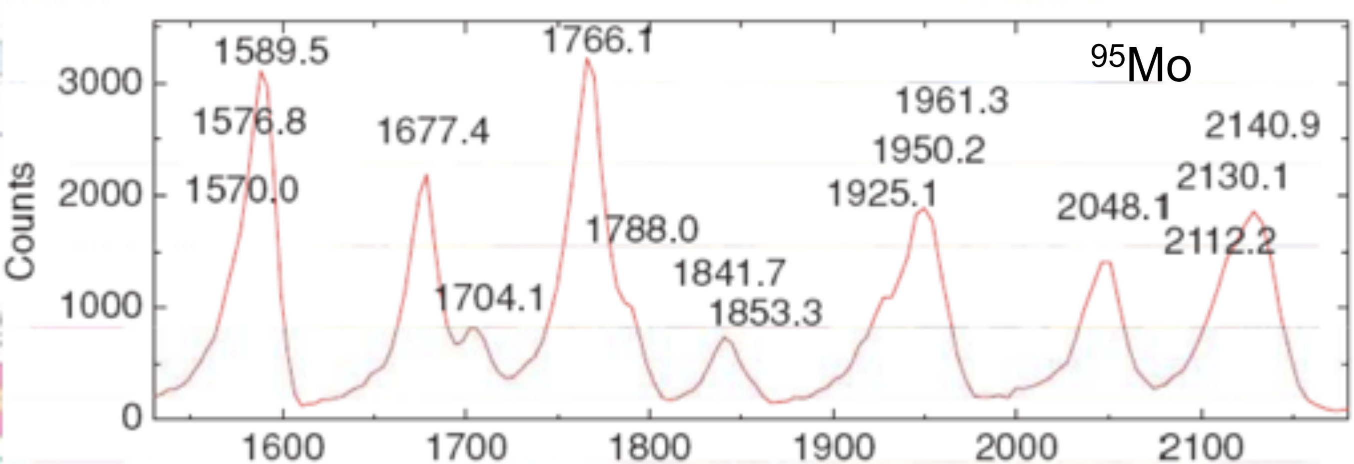


J	E MeV	Γ width	SF ex	SF th.
0+	0	0		0.73
2+	1.63	0		0.67
4+	4.25	0		0.62
0+	6.73	19	0.47	0.46
0+	7.19	3.4	0.02	0.10
2+	7.42	15	0.19	0.12
2+	7.83	2	0.01	0.09
0+	8.7	800	0.3	
6+	8.78	0.11	0.5	0.51
2+	9.00	800	0.86	

Distribution of decay widths in a chaotic system



Wooden toy model illustrating Bohr's
compound nucleus, from Nature **137**, 351 (1936)



Sheets et al., Phys. Rev. C 76, 064317

Many-body complexity and reduced widths

$|c\rangle$ Channel-vector (normalized)

$|I\rangle$ Eigenstate

Reduced width

$$\gamma_I^c = |\langle I|c\rangle|^2$$

What is the distribution of the reduced width?

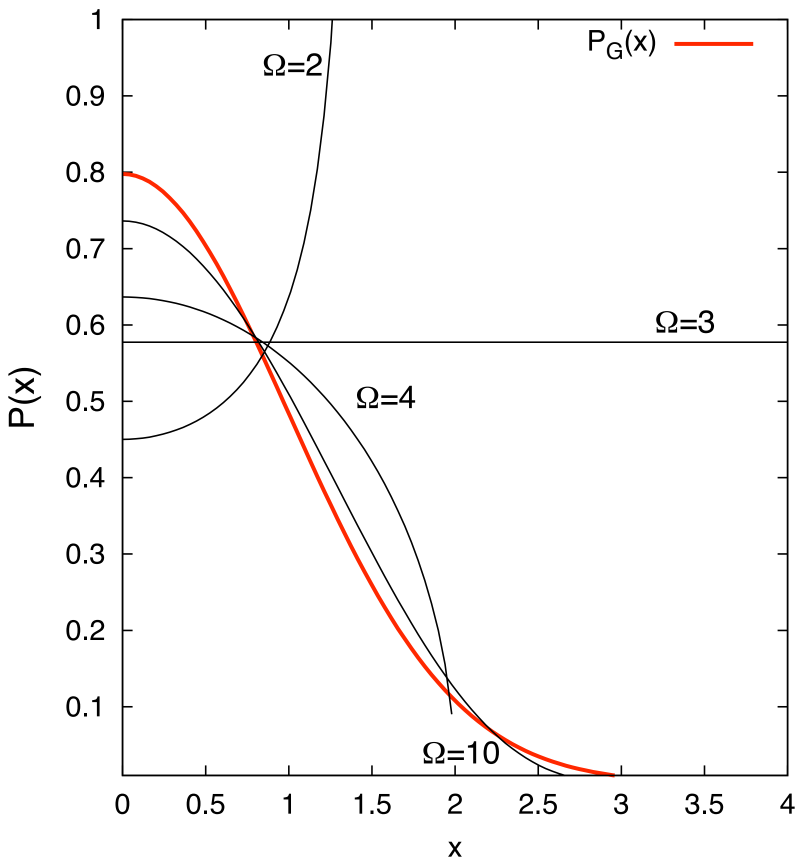
Average width $\overline{\gamma} = \frac{1}{\Omega}\sum_I \gamma_I^c = \frac{\langle c|c\rangle}{\Omega}$

Amplitude $x_I = \sqrt{\gamma_I/\overline{\gamma}}$

If any direction in the Ω-dimensional Hilbert space is equivalent

$$P(x_{I_1}, \ldots x_{I_\Omega}) \sim \delta\left(\Omega - \sum_I x_I^2\right)$$

Why Porter-Thomas Distribution?



Projection of a randomly oriented vector in Ω-dimensional space

$$P(x) = \frac{V_{\Omega-1}}{\sqrt{\Omega}V_\Omega}\left(1 - x^2/\Omega\right)^{(\Omega-3)/2}$$

$$V_\Omega = \frac{\Omega\pi^{\Omega/2}}{\Gamma(\Omega/2+1)}$$

For large Ω this leads to Gaussian

$$P_G(x) = \sqrt{\frac{2}{\pi}}\exp\left(-x^2/2\right)$$

For large ν channels

$$P_\nu(\gamma) = \frac{1}{\gamma}\left(\frac{\nu\gamma}{2\overline{\gamma}}\right)^{\nu/2}\frac{1}{\Gamma(\nu/2)}\exp\left(-\frac{\nu\gamma}{2\overline{\gamma}}\right)$$

Why PTD is so robust?

$$\gamma_I^c = |\langle I|c\rangle|^2$$

•Orthogonal invariance, in all basis $|I\rangle$ is statistically the same
→ Gaussian Orthogonal Ensemble → **PTD**

•Eigen vectors $|I\rangle$ are orthogonal and provide full coverage of the Hilbet space if $|c\rangle$s uncorrelated → **PTD**

•Central limit theorem:

$$|c\rangle = C_1^c|1\rangle + C_2^c|2\rangle + \dots \qquad |I\rangle = C_1^I|1\rangle + C_2^I|2\rangle + \dots$$

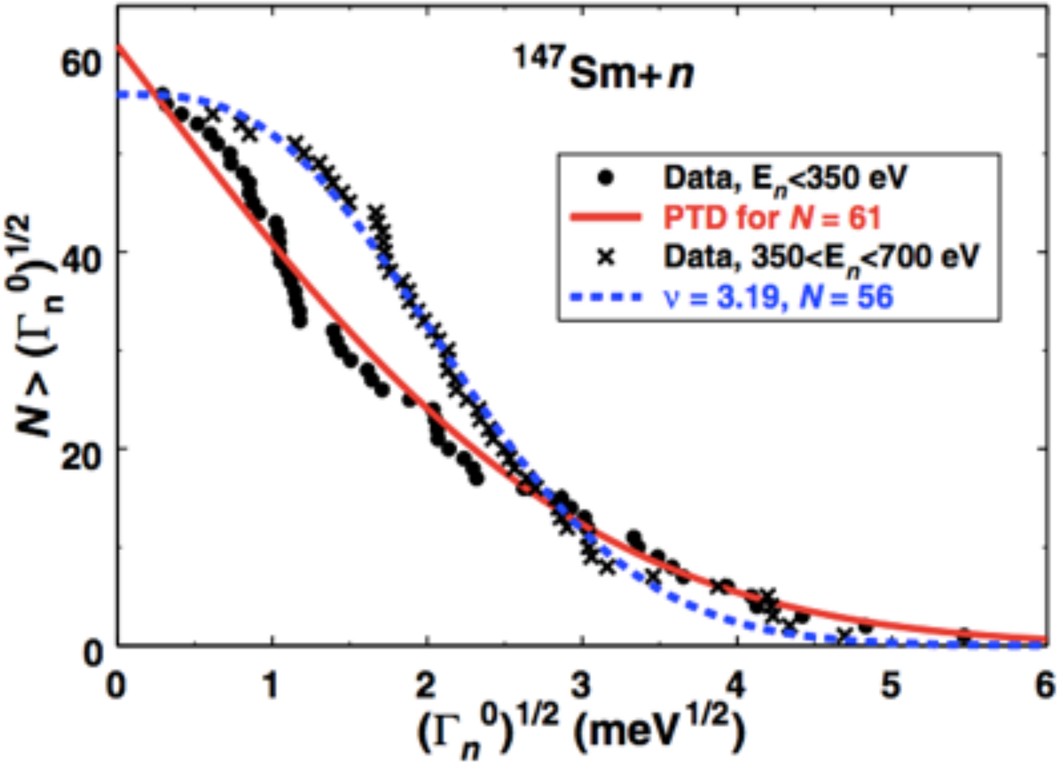
$$\langle I|c\rangle = C_1^{I*}C_1^c + C_2^{I*}C_2^c + \dots$$

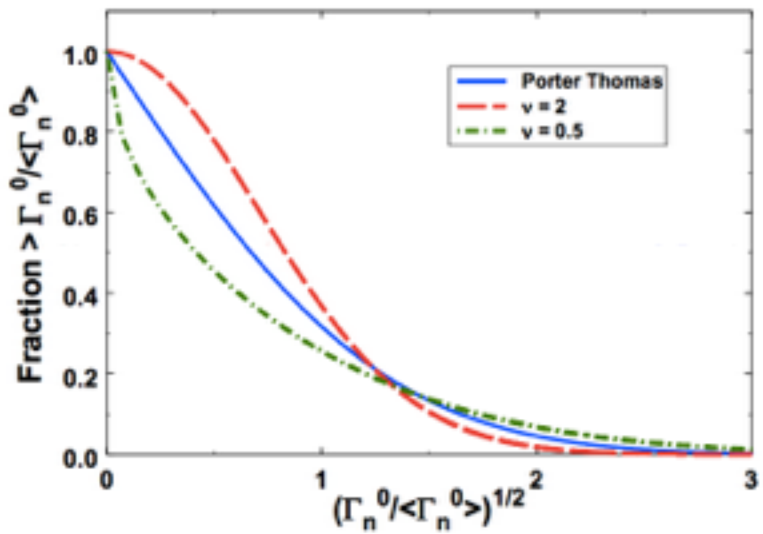
Sum of independent random variables is distributed normally → **PTD**

Nuclear theory nudged?
Violation of Porter-Thomas Distribution

Random matrix theory is rejected with 99.997% probability [Koehler, et. al. Phys. Rev. Lett. 105, 072502 (2010)] In platinum $\nu = 0.5$

Implications:
Capture rates, astrophysical reactions, nuclear reactors, critical mass, shielding...

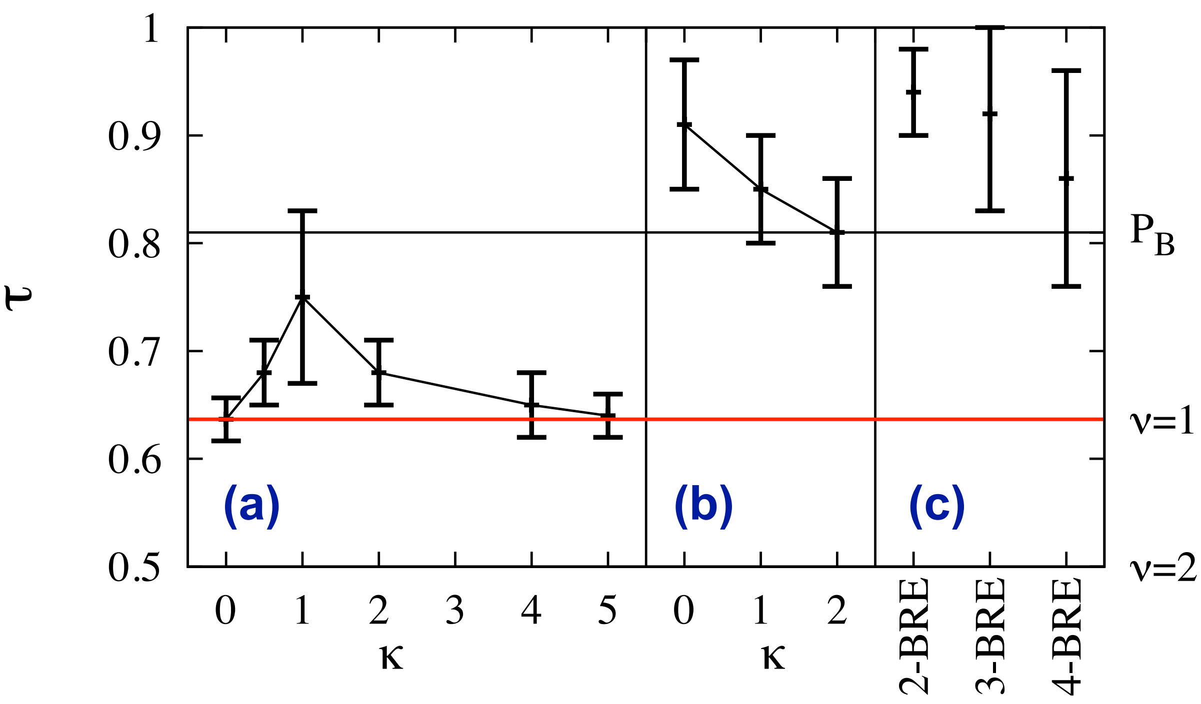


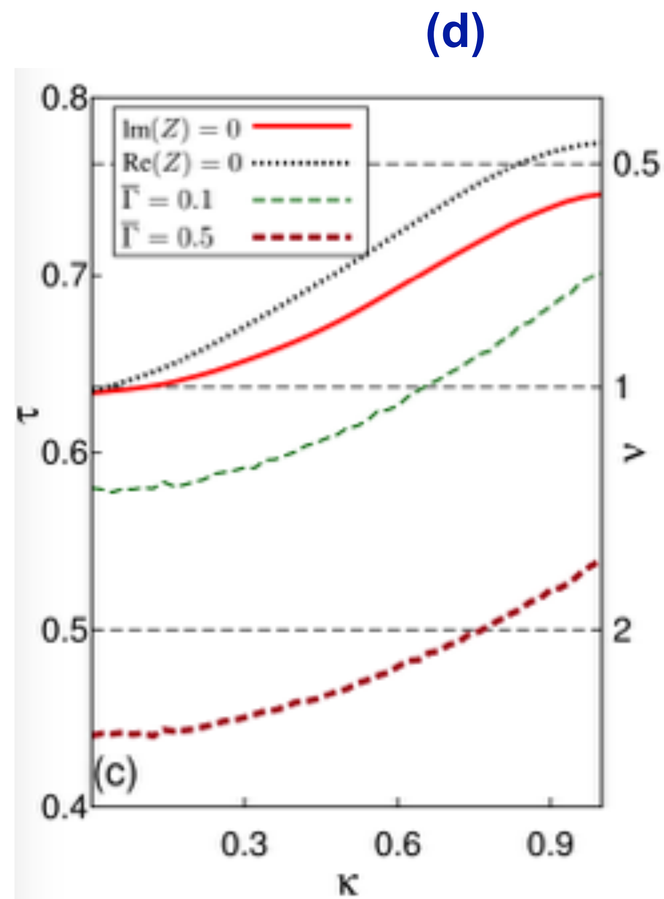


Nuclear theory nudged?
Violation of Porter-Thomas Distribution

(a) Overlapping resonances
(b) Memory effect and overlapping resonances (2-body interactions)
(c) Many-body interactions
(d) Self energy term

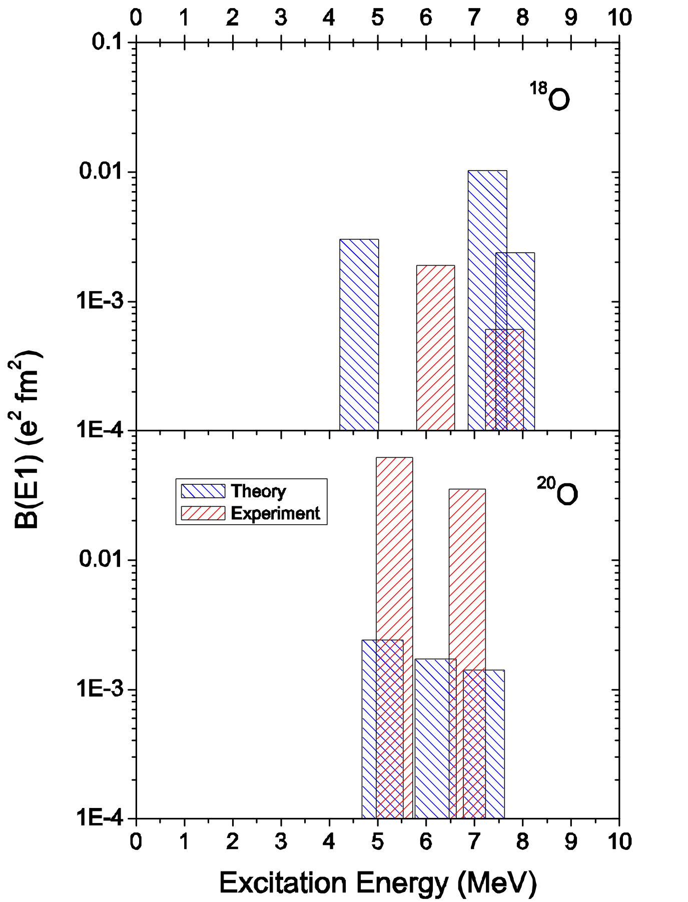
Coefficient of variation
Statistical normality test





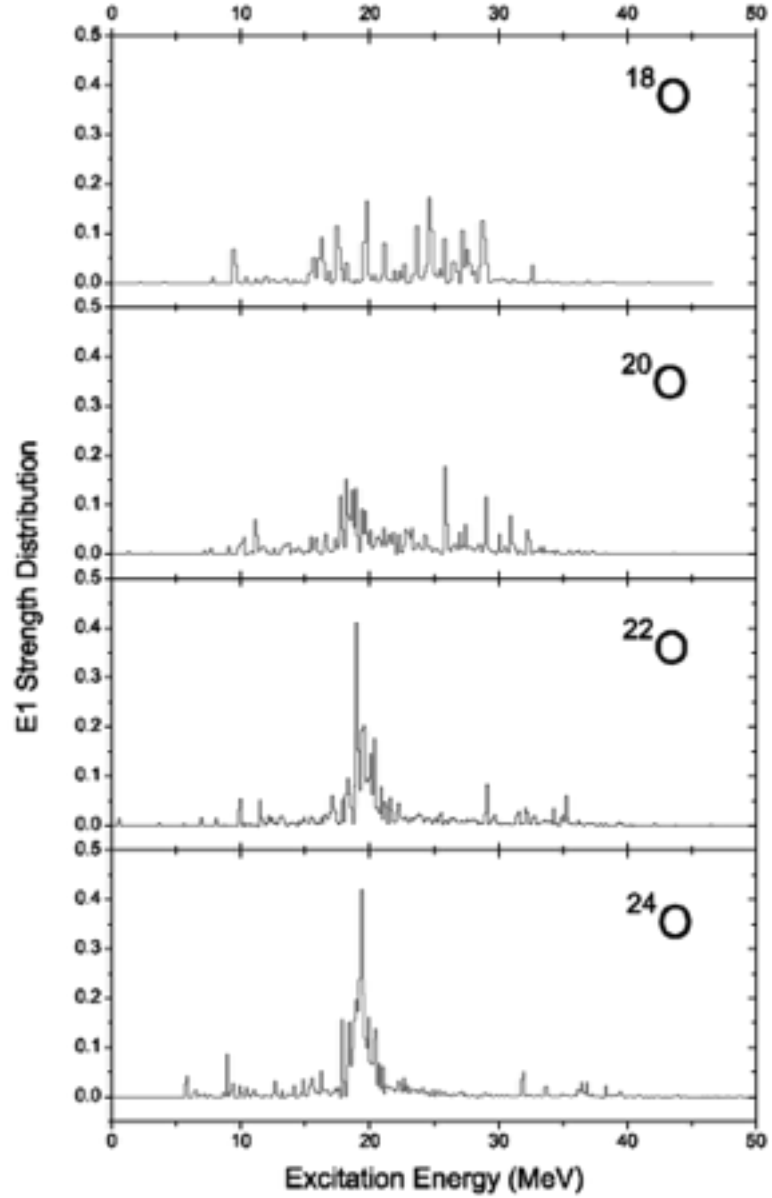
Dipole strength distribution in oxygen

Experimental evidence for pygmy strength



E. Tryggestad *et.al.* Phys. Lett. B **541,** (2002) 52
A. Leistenschneider *et.al.* Phys. Rev. Lett. 86, (2001) 5442

- Shell Model calculations
- s-p-sd-fp valence space, WBP



Interplay of collectivities

Definitions

n - labels particle-hole state

ε_n – excitation energy of state n

d_n - dipole operator

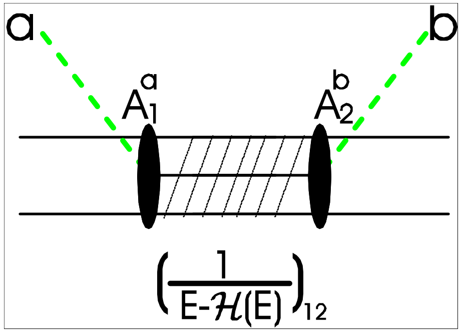
A_n – decay amplitude of n

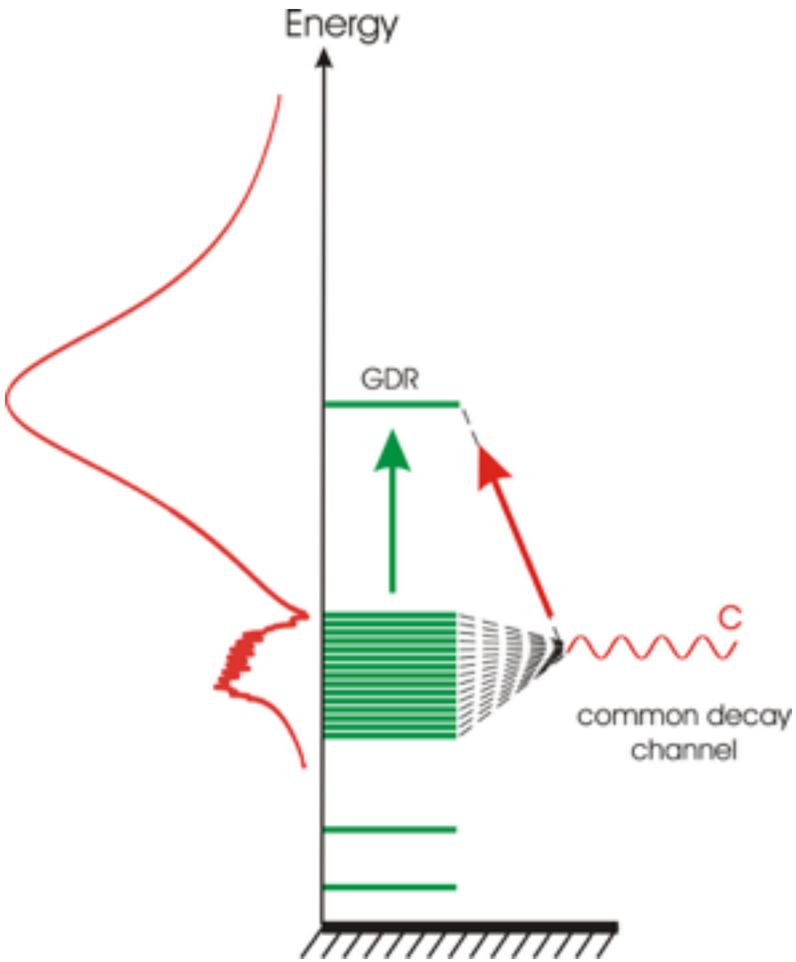
Model Hamiltonian

$$\mathcal{H}_{nn'} = \epsilon_n \delta_{nn'} + \lambda d_n d_{n'} - \frac{i}{2} A_n A_{n'},$$

Driving GDR externally

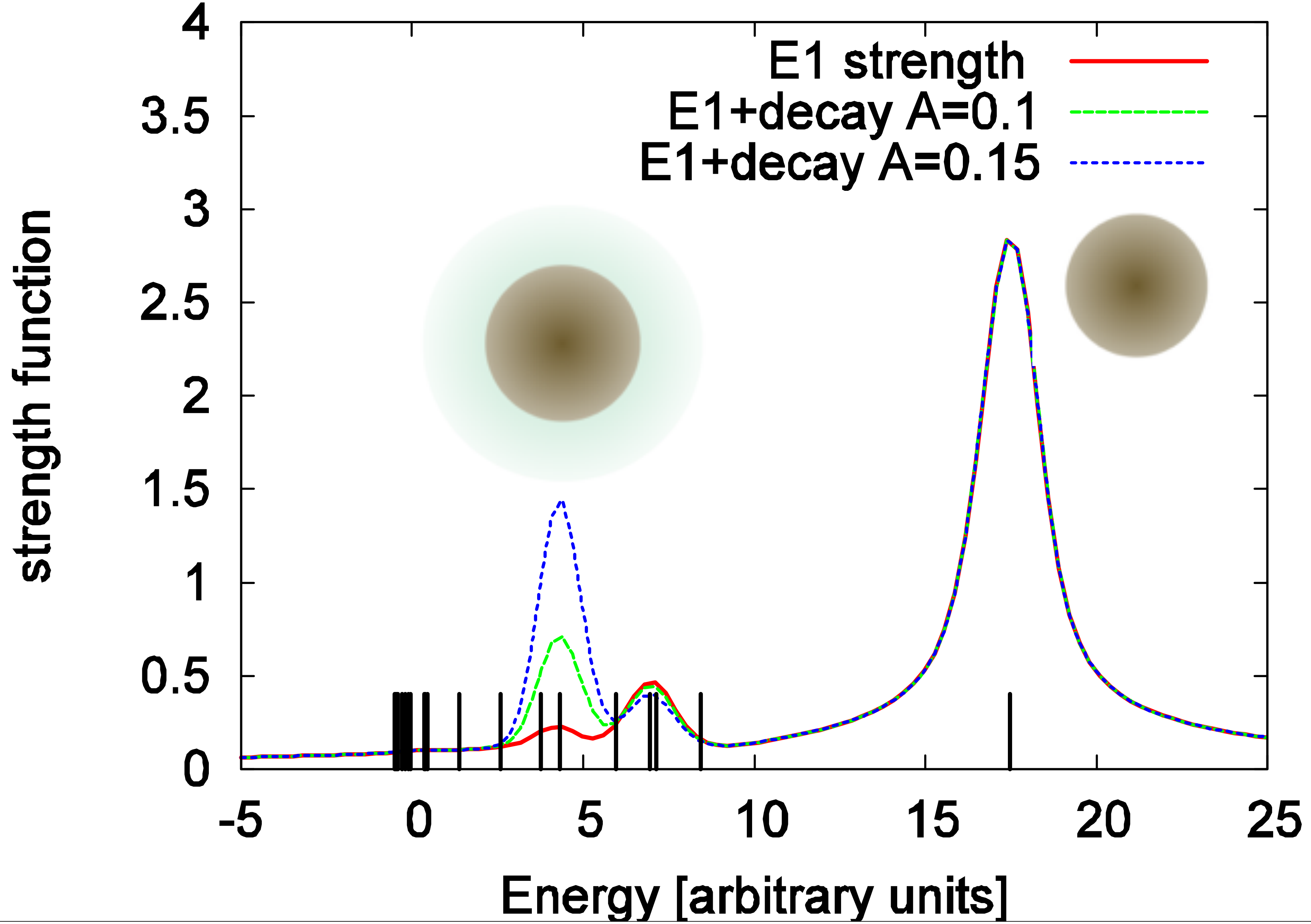
(doing scattering)





Everything depends on angle between multi dimensional vectors A and d

Model Example

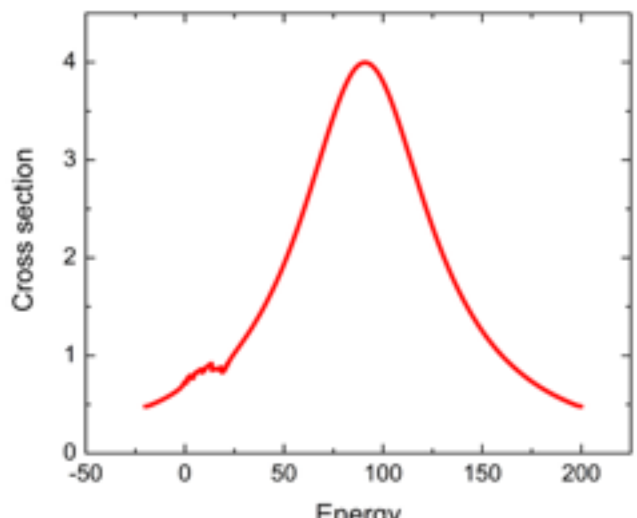


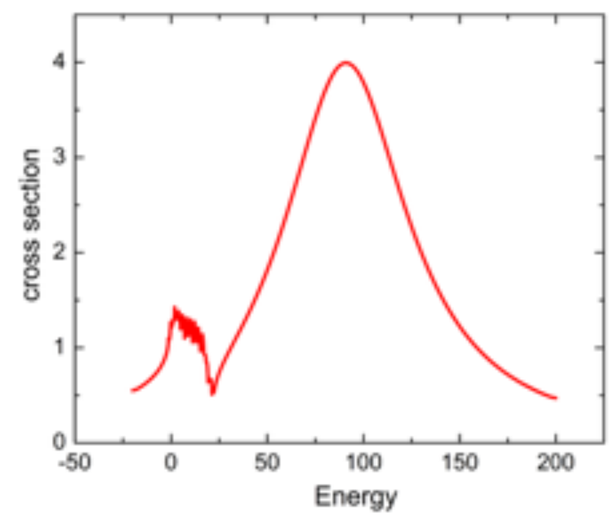
Pigmy resonance

Orthogonal:
GDR not seen

Parallel:
Most effective excitation
of GDR from continuum

At angle:
excitation of GDR
and pigmy



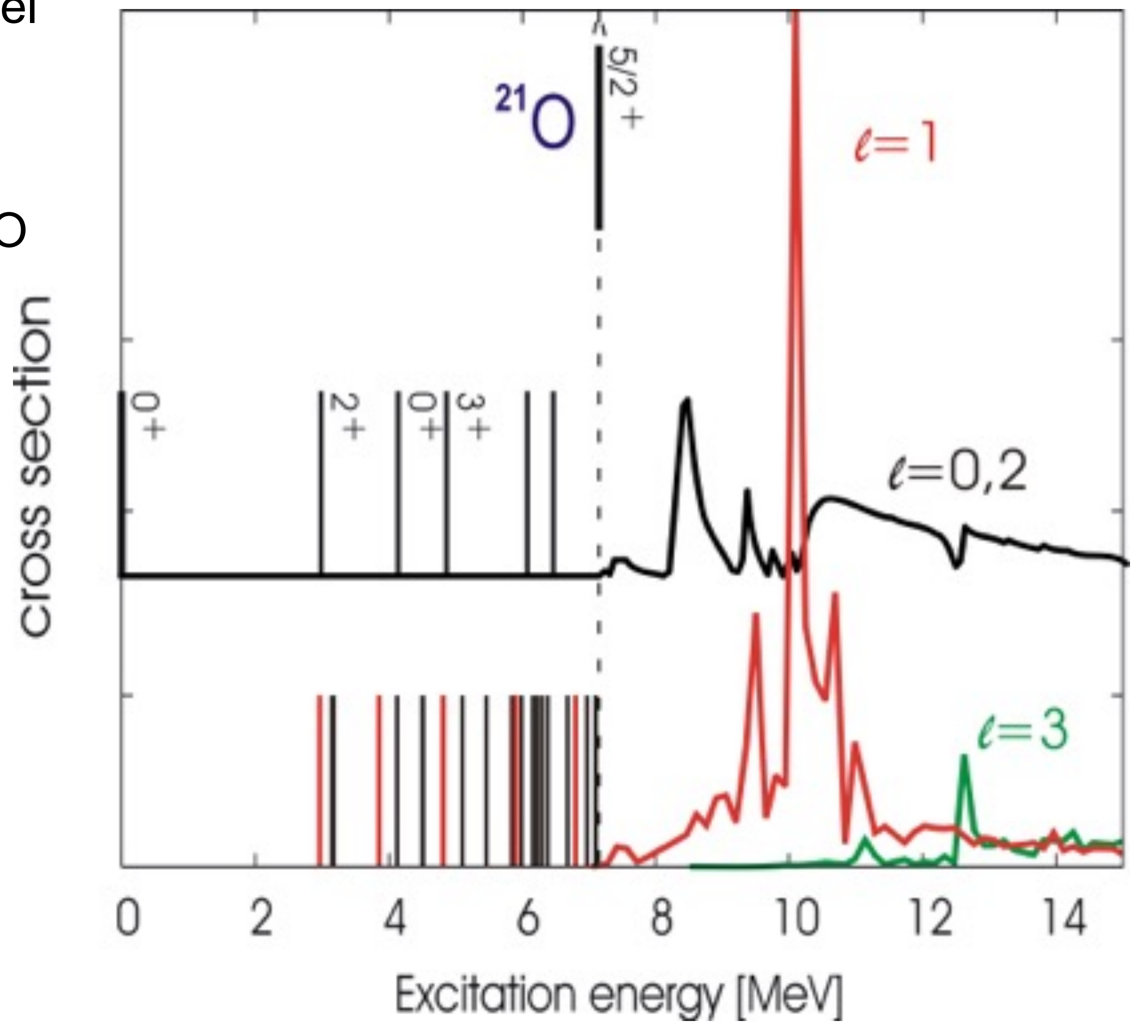


A model of 20 equally
distant levels is used

States and cross section in ^{22}O

Parameters of the model

•Internal shell s-p-sd-pf shell model with WBP interaction
•Decay channels g.s. of ^{21}O + neutron decay from fp
•EM channel: E1 strength from ^{22}O



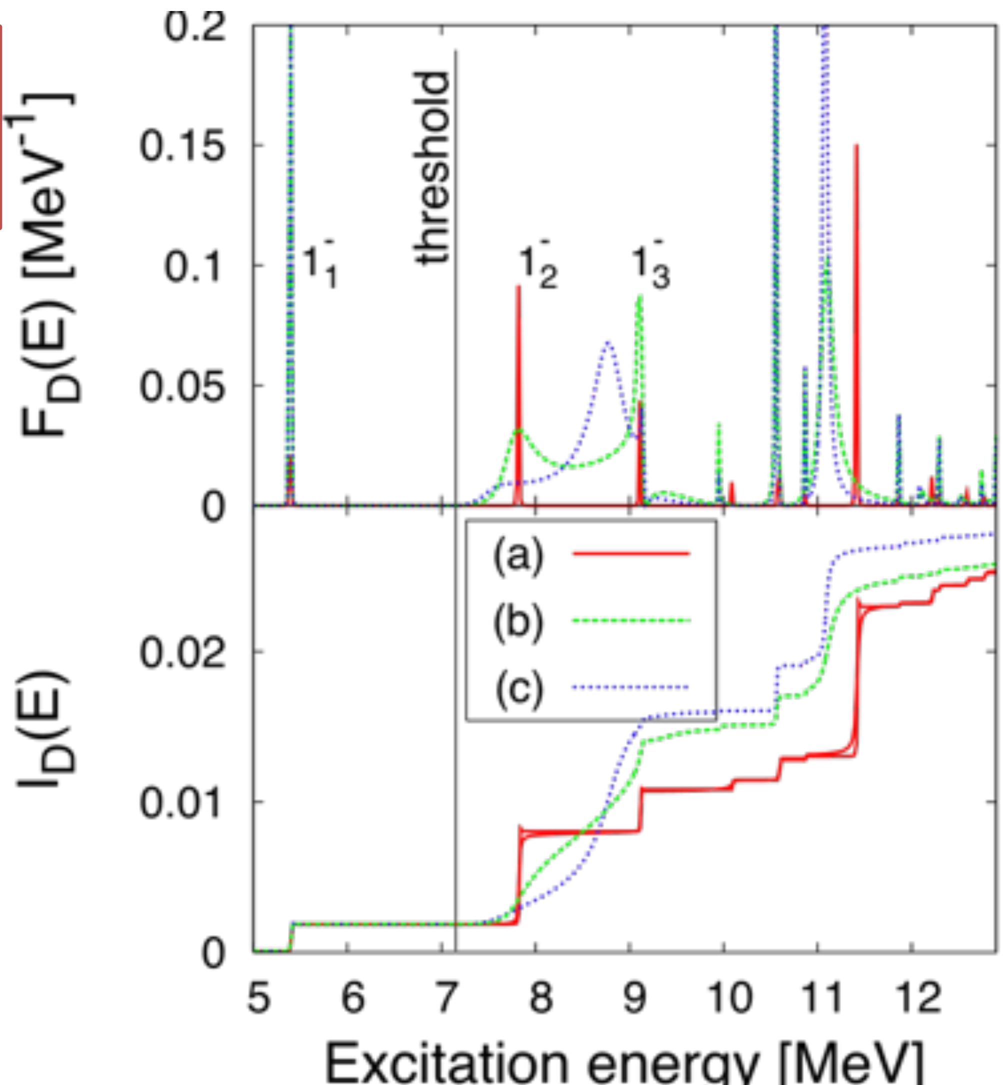
Strength function and decay in ^{22}O

Upper panel: Isovector dipole strength in ^{22}O low-energy region.
Lower panel: Integrated strength

$$I_\lambda(E) = \int_{-\infty}^{E} F_\lambda(E')dE'$$

In the limit of weak decay

$$I_D(E) = \sum_{\alpha}^{E_\alpha < E} B(\mathrm{E1}; \alpha \to 0^+_{\mathrm{g.s.}})$$



Continuum effects, summary.

- Non-exponentiallity short and long timescales
- threshold discontinuity
- Complex final states, sequential decay

- Interference of resonances
- Effects of decay on structure
- Superradiance

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Thanks to: K. Kravvaris, V. Zelevinsky, M. Peshkin, H. Weidenmüller, I. Wiedenhöver, G. Rogachev, V. Goldberg.

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