The Left Hand of the Electron in Superfluid ^{3}He

J. A. Sauls

Northwestern University

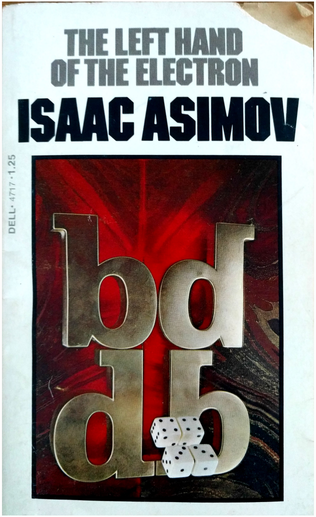
- Oleksii Shevtsov

- Parity violation
- Superfluid ^{3}He
- Edge States & Currents
- Electron Bubbles in ^{3}He
- Anomalous Hall Effect
- Electron Transport in ^{3}He

▶ NSF Grant DMR-1508730

The Left Hand of the Electron, Issac Asimov, circa 1971

- An Essay on the Discovery of Parity Violation by the Weak Interaction



Experimental Test of Parity Conservation in Beta Decay*

C. S. Wu, *Columbia University, New York, New York*

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E. Ambler, R. W. Hayward, D. D. Hoppes, and R. P. Hudson,
National Bureau of Standards, Washington, D. C.

(Received January 15, 1957)

Parity Violation in Beta Decay of ^{60}Co - Physical Review 105, 1413 (1957)

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$$^{60}Co \rightarrow \ ^{60}Ni + e^{-} + \bar{\nu}$$

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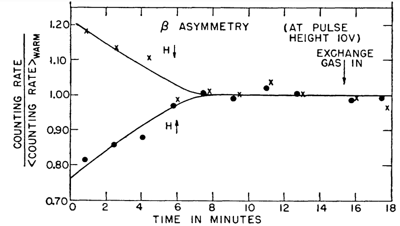
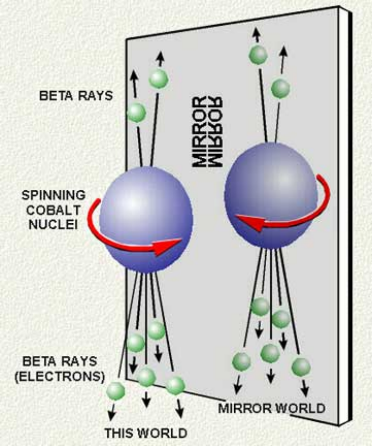


Fig. 2. Gamma anisotropy and beta asymmetry for polarizing field pointing up and pointing down.



- Current of Beta electrons is (anti) correlated with the Spin of the ^{60}Co nucleus. $\langle \vec{S} \cdot \vec{p} \rangle \neq 0 \rightsquigarrow$ Parity violation

Chiral P-wave BCS Condensate

$$|\,\Phi_N\,\rangle = \left[\iint d\mathbf{r}_1 d\mathbf{r}_2\; \varphi_{s_1 s_2}(\mathbf{r}_1 - \mathbf{r}_2)\; \psi^\dagger_{s_1}(\mathbf{r}_1)\psi^\dagger_{s_2}(\mathbf{r}_2)\right]^{N/2} |\,\text{vac}\,\rangle$$

$$\varphi_{s_1 s_2}(\mathbf{r}) = f(|\mathbf{r}|/\xi)\;(x + iy)\;\chi_{s_1 s_2}$$

- P.W. Anderson & P. Morel, Phys. Rev. 123, 1911 (1961)

Chiral P-wave BCS Condensate

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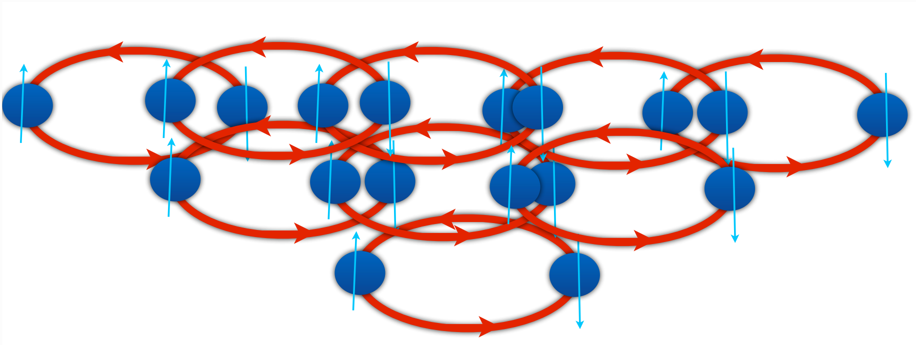

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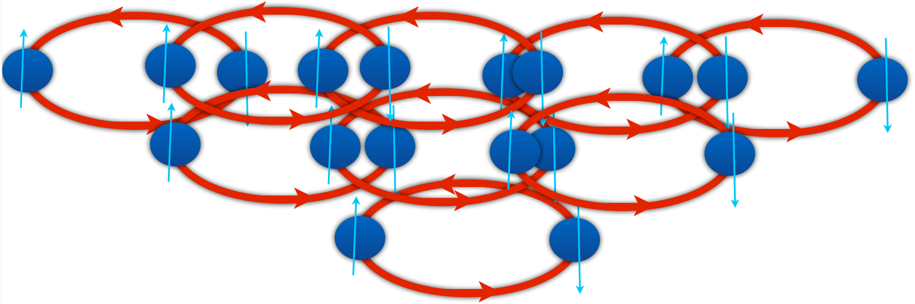
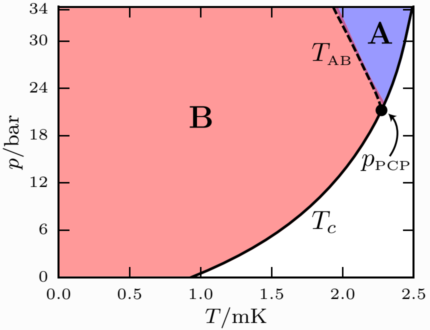


$$\text{SO}(3)_\text{S} \times \text{SO}(3)_\text{L} \times \text{U}(1)_\text{N} \times \text{T} \times \text{P} \longrightarrow \text{SO}(2)_\text{S} \times \text{U}(1)_{\text{N-L}_z} \times \text{Z}_2$$

Realized in the Superfluid Ground State of Liquid ^{3}He

BCS Condensate Amplitude: $\Psi_{\alpha\beta}(p) = \langle \psi_\alpha(p)\psi_\beta(-p) \rangle$

J. Wiman & J. A. Sauls, PRB 92, 144515 (2015)



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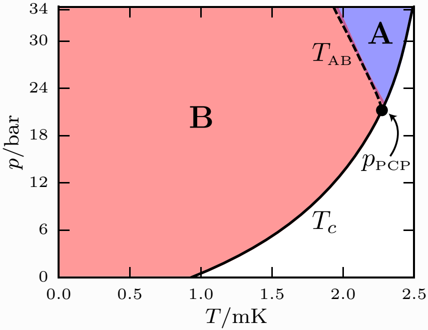
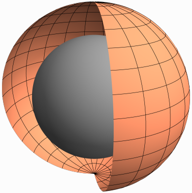


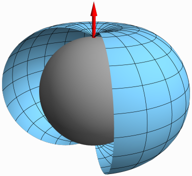
"Isotropic" BW State



$J = 0,\ J_z = 0$

$H = \mathtt{SO(3)_J} \times \mathtt{T}$

Chiral AM State $\vec{l} = \hat{\mathbf{z}}$



$L_z = 1,\ S_z = 0$

$H = \mathtt{U(1)_S} \times \mathtt{U(1)_{L_z\text{-}N}} \times \mathtt{Z_2}$

$$\begin{pmatrix} \Psi_{\uparrow\uparrow} & \Psi_{\uparrow\downarrow} \\ \Psi_{\uparrow\downarrow} & \Psi_{\downarrow\downarrow} \end{pmatrix}_{BW} = \begin{pmatrix} p_x - ip_y \sim e^{-i\phi} & p_z \\ p_z & p_x + ip_y \sim e^{+i\phi} \end{pmatrix}$$

$$\begin{pmatrix} \Psi_{\uparrow\uparrow} & \Psi_{\uparrow\downarrow} \\ \Psi_{\uparrow\downarrow} & \Psi_{\downarrow\downarrow} \end{pmatrix}_{AM} = \begin{pmatrix} p_x + ip_y \sim e^{+i\phi} & 0 \\ 0 & p_x + ip_y \sim e^{+i\phi} \end{pmatrix}$$

What is the Signature & Evidence for Chirality of Superfluid ^{3}He-A?

Signatures of Broken T and P Symmetry in ^{3}He-A

What is the Signature & Evidence for Chirality of Superfluid ^{3}He-A?

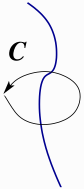
Spontaneous Symmetry Breaking $\rightsquigarrow$ Emergent Topology of ^{3}He-A

Chirality + Topology $\rightsquigarrow$ Edge States & Chiral Edge Currents

Broken T and P $\rightsquigarrow$ Anomalous Hall Effect for electrons in ^{3}He-A

Topology in Real Space

$$\Psi(\mathbf{r}) = |\Psi(r)|\, e^{i\vartheta(\mathbf{r})}$$



Phase Winding

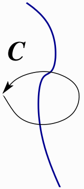
$$N_C = \frac{1}{2\pi} \oint_C d\vec{l} \cdot \frac{1}{|\Psi|} \text{Im}[\boldsymbol{\nabla}\Psi] \in \{0, \pm1, \pm2, \ldots$$

- Massless Fermions confined in the Vortex Core

Real-Space vs. Momentum-Space Topology

Topology in Real Space

$\Psi(\mathbf{r}) = |\Psi(r)|\, e^{i\vartheta(\mathbf{r})}$



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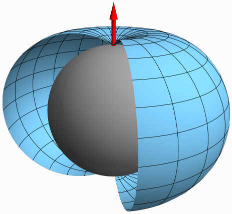
Chiral Symmetry $\rightsquigarrow$

Topology in Momentum Space

$\Psi(\mathbf{p}) = \Delta(p_x \pm i p_y) \sim e^{\pm i\varphi_{\mathbf{p}}}$

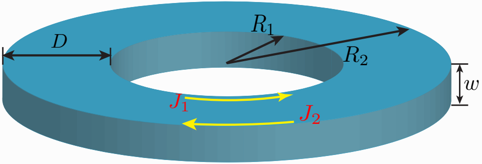


Topological Quantum Number: $L_z = \pm 1$

$$N_{2D} = \frac{1}{2\pi}\oint d\mathbf{p}\cdot \frac{1}{|\Psi(\mathbf{p})|}\mathrm{Im}[\boldsymbol{\nabla}_{\mathbf{p}}\Psi(\mathbf{p})] = L_z$$

- Massless Chiral Fermions
  - Nodal Fermions in 3D
  - Edge Fermions in 2D

^{3}He-A confined in a toroidal cavity



- $R_1, R_2, R_1 - R_2 \gg \xi_0$

- Sheet Current: $J = \frac{1}{4}\, n\, \hbar$ ($n = N/V =$ ^{3}He density)
- Counter-propagating Edge Currents: $J_1 = -J_2 = \frac{1}{4}\, n\, \hbar$
- Angular Momentum:

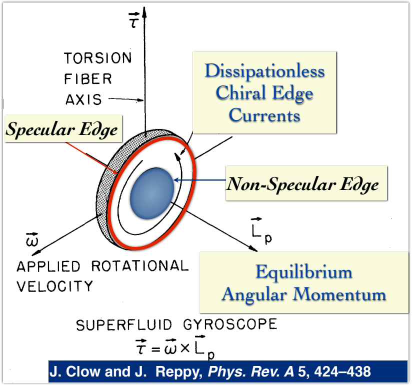
$$L_z = 2\pi\, h\, (R_1^2 - R_2^2) \times \frac{1}{4}\, n\, \hbar = (N/2)\, \hbar$$

McClure-Takagi Result

▶ J. A. Sauls, Phys. Rev. B 84, 214509 (2011)

Possible Gyroscopic Experiment to Measure of $L_z(T)$

- Hyoungsoon Choi (KAIST) [sub-micron mechanical gyroscope @ 200 μK]



Thermal Signature of Chiral Edge States

- Power Law for $T \lesssim 0.5 T_c$

$$L_z = (N/2)\hbar \, (1 - c\,(T/\Delta)^2)$$

Toroidal Geometry with Engineered Surfaces

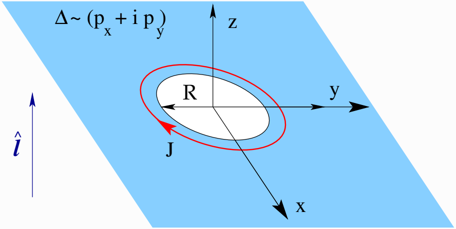
- Incomplete Screening

$$L_z > (N/2)\hbar$$

Direct Signature of Edge Currents

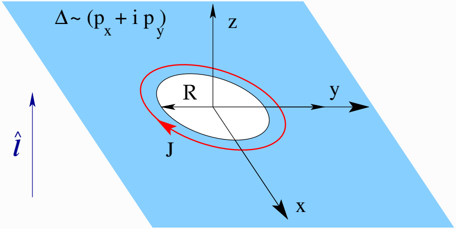
- J. A. Sauls, Phys. Rev. B 84, 214509 (2011)
- Y. Tsutsumi, K. Machida, JPSJ 81, 074607 (2012)

Unbounded Film of ^{3}He-A perforated by a Hole



- $R \gg \xi_0 \approx 100\,\mathrm{nm}$

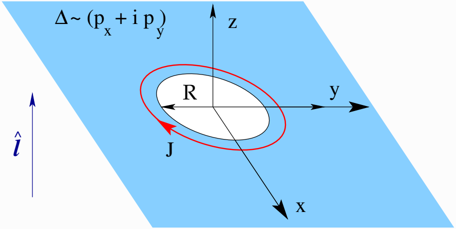
Unbounded Film of ^{3}He-A perforated by a Hole



- $R \gg \xi_0 \approx 100\,\text{nm}$

- Magnitude of the Sheet Current: $\frac{1}{4}\, n\, \hbar$ ($n = N/V =$ ^{3}He density)
- Edge Current *Counter*-Circulates: $J = -\frac{1}{4}\, n\, \hbar$ w.r.t. Chirality: $\hat{\mathbf{l}} = +\mathbf{z}$

Unbounded Film of ^{3}He-A perforated by a Hole



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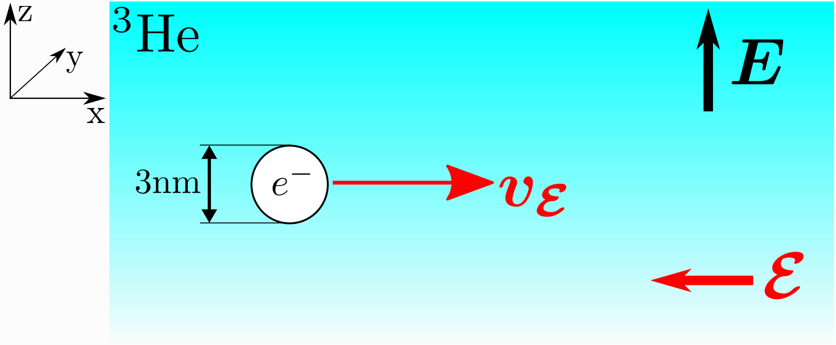
- Edge Current *Counter*-Circulates: $J = -\frac{1}{4}\,n\,\hbar$ w.r.t. Chirality: $\hat{\mathbf{l}} = +\mathbf{z}$
- Angular Momentum: $L_z = 2\pi\,h\,R^2 \times (-\frac{1}{4}\,n\,\hbar) = -(N_{\text{hole}}/2)\,\hbar$

N_{hole} = Number of ^{3}He atoms excluded from the Hole

$\therefore$ An object in ^{3}He-A *inherits* angular momentum from the Condensate of Chiral Pairs!

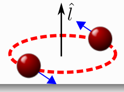
Electron bubbles in the Normal Fermi liquid phase of ^{3}He



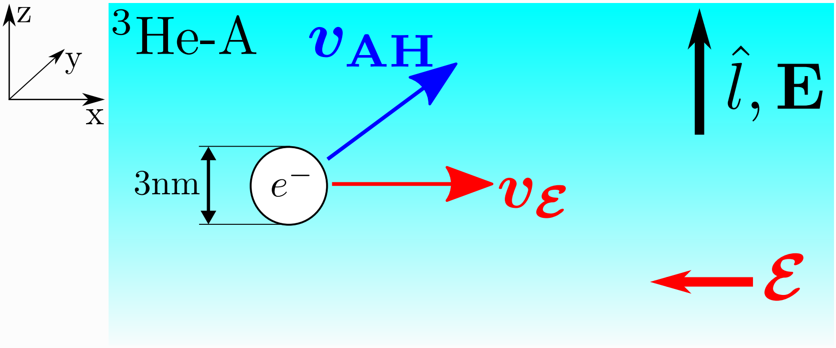
- Bubble with $R \simeq 1.5$ nm, $0.1\,\text{nm} \simeq \lambda_f \ll R \ll \xi_0 \simeq 80\,\text{nm}$
- Effective mass $M \simeq 100 m_3$ (m_3 – atomic mass of ^{3}He)
- QPs mean free path $l \gg R$
- Mobility of ^{3}He is *independent of* T for $T_c < T < 50$ mK

  B. Josephson and J. Leckner, PRL 23, 111 (1969)

Electron bubbles in chiral superfluid ^{3}He-A

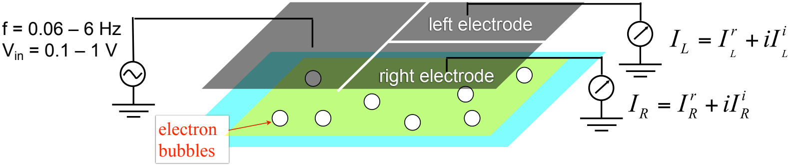


$$\Delta_{\text{A}}(\hat{\mathbf{k}}) = \Delta \frac{k_x + i k_y}{k_f} = \Delta\, e^{i\phi_{\mathbf{k}}}$$



- Current: $\mathbf{v} = \overbrace{\mu_\perp \boldsymbol{\mathcal{E}}}^{\mathbf{v}_{\mathcal{E}}} + \overbrace{\mu_{\text{AH}} \boldsymbol{\mathcal{E}} \times \hat{\mathbf{l}}}^{\mathbf{v}_{\text{AH}}}$ R. Salmelin, M. Salomaa & V. Mineev, PRL **63**, 868 (1989)
- Hall ratio: $\tan\alpha = v_{\text{AH}}/v_{\mathcal{E}} = |\mu_{\text{AH}}/\mu_\perp|$

Measurement of the Transverse e⁻ mobility in Superfluid ^{3}He Films



H. Ikegami, Y. Tsutsumi, K. Kono, Science 341, 59-62 (2013)

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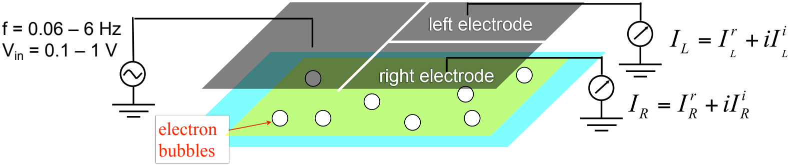
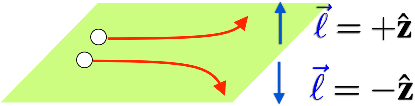


Transverse Force from ***Skew* Scattering**

$$\rightsquigarrow \quad \Delta I = I_R - I_L \neq 0$$

$$\vec{v} = \left[\mu_\perp \vec{E} + \mu_{xy}\,\hat{\ell} \times \vec{E}\right]$$



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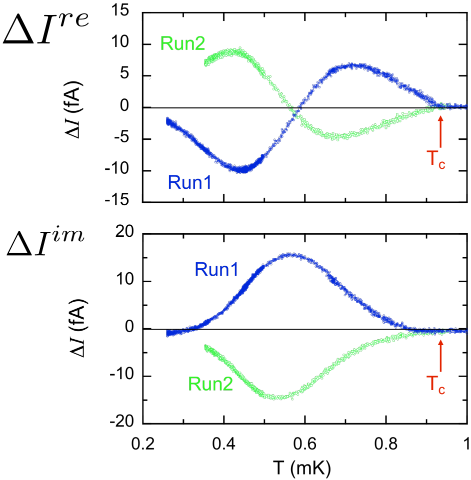
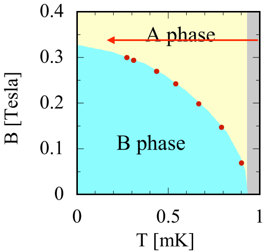
Transverse e⁻ bubble current in ^{3}He-A $\Delta I = I_R - I_L$





Single Domains:

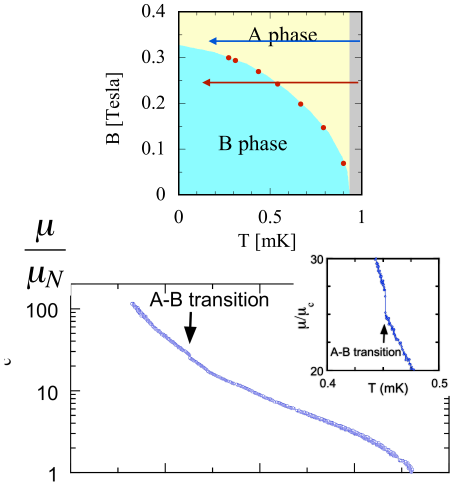
Run 1 $\vec{\ell} = +\hat{\mathbf{z}}$

Run 2 $\vec{\ell} = -\hat{\mathbf{z}}$

$$\frac{|I_R - I_L|}{I_R + I_L} \approx 6\,\%$$

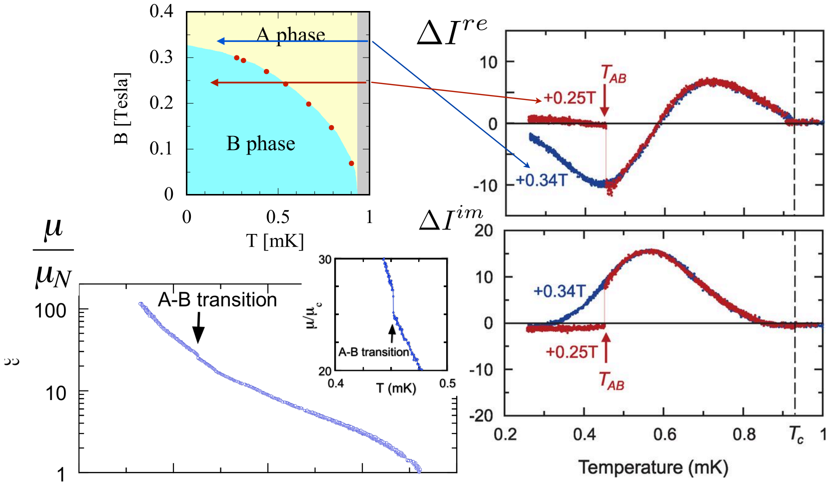
H. Ikegami, Y. Tsutsumi, K. Kono, Science 341, 59-62 (2013)

Zero Transverse e⁻ current in ^{3}He-B (*T - symmetric phase*)



H. Ikegami, Y. Tsutsumi, K. Kono, Science 341, 59-62 (2013)

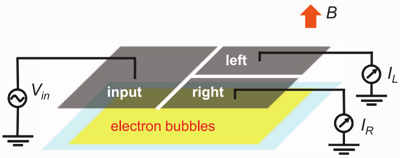
Zero Transverse e^- current in ^{3}He-B (*T - symmetric phase*)

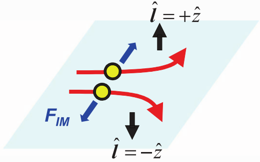


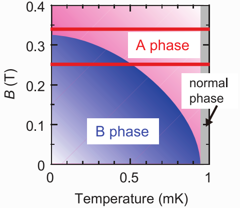
H. Ikegami, Y. Tsutsumi, K. Kono, **Science 341, 59-62 (2013)**

Mobility of Electron Bubbles in ^{3}He-A

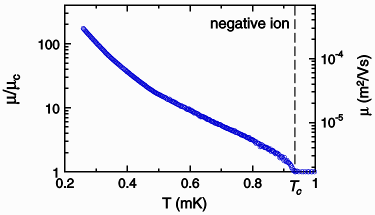
- H. Ikegami et al., Science **341**, 59 (2013); JPSJ **82**, 124607 (2013); JPSJ **84**, 044602 (2015)

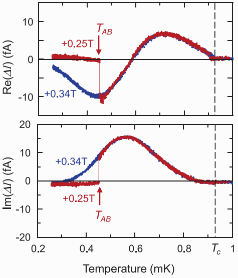


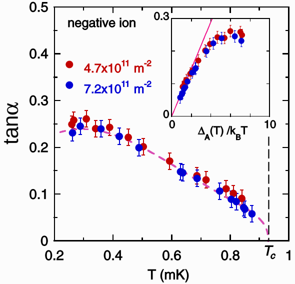




Electric current: $\mathbf{v} = \overbrace{\mu_{\perp}\boldsymbol{\mathcal{E}}}^{\mathbf{v}_{\mathcal{E}}} + \overbrace{\mu_{\text{AH}}\boldsymbol{\mathcal{E}} \times \hat{\mathbf{l}}}^{\mathbf{v}_{\text{AH}}}$ Hall ratio: $\tan\alpha = v_{\text{AH}}/v_{\mathcal{E}} = |\mu_{\text{AH}}/\mu_{\perp}|$







Forces on the Electron bubble in ^{3}He-A:

- $M\dfrac{d\mathbf{v}}{dt} = e\boldsymbol{\mathcal{E}} + \mathbf{F}_{\rm QP}$, $\mathbf{F}_{QP}$ – force from quasiparticle collisions
- $\mathbf{F}_{QP} = -\overleftrightarrow{\eta}\cdot\mathbf{v}$, $\overleftrightarrow{\eta}$ – generalized Stokes tensor

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- $\mathbf{F}_{QP} = -\overleftrightarrow{\eta}\cdot\mathbf{v}$, $\overleftrightarrow{\eta}$ – generalized Stokes tensor
- $\overleftrightarrow{\eta} = \begin{pmatrix} \eta_{\perp} & \eta_{\text{AH}} & 0 \\ -\eta_{\text{AH}} & \eta_{\perp} & 0 \\ 0 & 0 & \eta_{\parallel} \end{pmatrix}$ for chiral symmetry with $\hat{\mathbf{l}} \parallel \mathbf{e}_z$

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- $M\dfrac{d\mathbf{v}}{dt} = e\boldsymbol{\mathcal{E}} - \eta_\perp\mathbf{v} + \dfrac{e}{c}\mathbf{v}\times\mathbf{B}_{\text{eff}}$, for $\boldsymbol{\mathcal{E}} \perp \hat{\mathbf{l}}$

Forces on the Electron bubble in ^{3}He-A:

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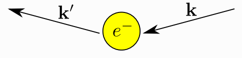
- $M\dfrac{d\mathbf{v}}{dt} = e\boldsymbol{\mathcal{E}} - \eta_\perp\mathbf{v} + \dfrac{e}{c}\mathbf{v}\times\mathbf{B}_{\text{eff}}$, for $\boldsymbol{\mathcal{E}} \perp \hat{\mathbf{l}}$
- $\mathbf{B}_{\text{eff}} = -\dfrac{c}{e}\eta_{\text{AH}}\hat{\mathbf{l}}$ $\quad B_{\text{eff}} \simeq 10^3 - 10^4\ \text{T}$!!!

Forces on the Electron bubble in ^{3}He-A:

- $M\frac{d\mathbf{v}}{dt} = e\boldsymbol{\mathcal{E}} + \mathbf{F}_{\text{QP}}$, $\mathbf{F}_{QP}$ – force from quasiparticle collisions
- $\mathbf{F}_{QP} = -\overleftrightarrow{\eta}\cdot\mathbf{v}$, $\overleftrightarrow{\eta}$ – generalized Stokes tensor
- $\overleftrightarrow{\eta} = \begin{pmatrix} \eta_\perp & \eta_{\text{AH}} & 0 \\ -\eta_{\text{AH}} & \eta_\perp & 0 \\ 0 & 0 & \eta_\parallel \end{pmatrix}$ for chiral symmetry with $\hat{\mathbf{l}} \parallel \mathbf{e}_z$
- $M\frac{d\mathbf{v}}{dt} = e\boldsymbol{\mathcal{E}} - \eta_\perp\mathbf{v} + \frac{e}{c}\mathbf{v}\times\mathbf{B}_{\text{eff}}$, for $\boldsymbol{\mathcal{E}} \perp \hat{\mathbf{l}}$
- $\mathbf{B}_{\text{eff}} = -\frac{c}{e}\eta_{\text{AH}}\hat{\mathbf{l}}$ $\quad B_{\text{eff}} \simeq 10^3 - 10^4\ \text{T}$!!!
- $\frac{d\mathbf{v}}{dt} = 0 \rightsquigarrow \mathbf{v} = \overleftrightarrow{\mu}\boldsymbol{\mathcal{E}}$, where $\overleftrightarrow{\mu} = e\,\overleftrightarrow{\eta}^{-1}$

▶ O. Shevtsov and JAS, Phys. Rev. B 96, 064511 (2016)



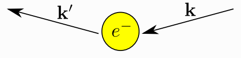
- Lippmann-Schwinger equation for the T-matrix ($\varepsilon = E + i\eta\,;\ \eta \to 0^+$):

$$\hat{T}_S^R(\mathbf{k}', \mathbf{k}, E) = \hat{T}_N^R(\mathbf{k}', \mathbf{k}) + \int \frac{d^3k''}{(2\pi)^3} \hat{T}_N^R(\mathbf{k}', \mathbf{k}'') \Big[\hat{G}_S^R(\mathbf{k}'', E) - \hat{G}_N^R(\mathbf{k}'', E) \Big] \hat{T}_S^R(\mathbf{k}'', \mathbf{k}, E)$$

$$\hat{G}_S^R(\mathbf{k}, E) = \frac{1}{\varepsilon^2 - E_{\mathbf{k}}^2} \begin{pmatrix} \varepsilon + \xi_k & -\Delta(\hat{\mathbf{k}}) \\ -\Delta^\dagger(\hat{\mathbf{k}}) & \varepsilon - \xi_k \end{pmatrix}, \quad E_{\mathbf{k}} = \sqrt{\xi_k^2 + |\Delta(\hat{\mathbf{k}})|^2}, \ \ \xi_k = \frac{\hbar^2 k^2}{2m^*} - \mu$$

- Normal-state T-matrix:

$$\hat{T}_N^R(\hat{\mathbf{k}}', \hat{\mathbf{k}}) = \begin{pmatrix} t_N^R(\hat{\mathbf{k}}', \hat{\mathbf{k}}) & 0 \\ 0 & -[t_N^R(-\hat{\mathbf{k}}', -\hat{\mathbf{k}})]^\dagger \end{pmatrix} \quad \text{in p-h (Nambu) space}$$



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$$t_N^R(\hat{\mathbf{k}}',\hat{\mathbf{k}}) = -\frac{1}{\pi N_f} \sum_{l=0}^{\infty} (2l+1) e^{i\delta_l} \sin\delta_l P_l(\hat{\mathbf{k}}' \cdot \hat{\mathbf{k}}), \quad P_l(x)$$ – Legendre function

- Hard-sphere potential $\rightsquigarrow \tan\delta_l = j_l(k_f R)/n_l(k_f R)$ – spherical Bessel functions

  - $k_f R$ – determined by the Normal-State Mobility

$$\hat{\mathcal{G}}_S^R(\mathbf{r}',\mathbf{r},E) = \int \frac{d^3k}{(2\pi)^3} \int \frac{d^3k'}{(2\pi)^3} e^{i\mathbf{k}'\mathbf{r}'} e^{-i\mathbf{k}\mathbf{r}} \hat{\mathcal{G}}_S^R(\mathbf{k}',\mathbf{k},E)$$

$$\hat{\mathcal{G}}_S^R(\mathbf{k}',\mathbf{k},E) = (2\pi)^3 \hat{G}_S^R(\mathbf{k},E)\delta_{\mathbf{k}',\mathbf{k}} + \hat{G}_S^R(\mathbf{k}',E)\hat{T}_S(\mathbf{k}',\mathbf{k},E)\hat{G}_S^R(\mathbf{k},E)$$

$$\hat{G}_S^R(\mathbf{k},E) = \frac{1}{\varepsilon^2 - E_{\mathbf{k}}^2} \begin{pmatrix} \varepsilon + \xi_k & -\Delta(\hat{\mathbf{k}}) \\ -\Delta^\dagger(\hat{\mathbf{k}}) & \varepsilon - \xi_k \end{pmatrix}, \quad \varepsilon = E + i\eta, \quad \eta \to 0^+$$

$$N(\mathbf{r},E) = -\frac{1}{2\pi} \mathrm{Im}\left\{ \mathrm{Tr}\left[\hat{\mathcal{G}}_S^R(\mathbf{r},\mathbf{r},E) \right] \right\}$$

$$\mathbf{j}(\mathbf{r}) = \frac{\hbar}{4mi} k_B T \sum_{n=-\infty}^{\infty} \lim_{\mathbf{r}\to\mathbf{r}'} \mathrm{Tr}\left[(\boldsymbol{\nabla}_{\mathbf{r}'} - \boldsymbol{\nabla}_{\mathbf{r}}) \hat{\mathcal{G}}^M(\mathbf{r}',\mathbf{r},\epsilon_n) \right]$$

$$\hat{\mathcal{G}}_S^R(\mathbf{r}',\mathbf{r},E) = \hat{\mathcal{G}}_S^M(\mathbf{r}',\mathbf{r},\epsilon_n)\Big|_{i\epsilon_n \to \varepsilon}, \text{ for } n \geq 0$$

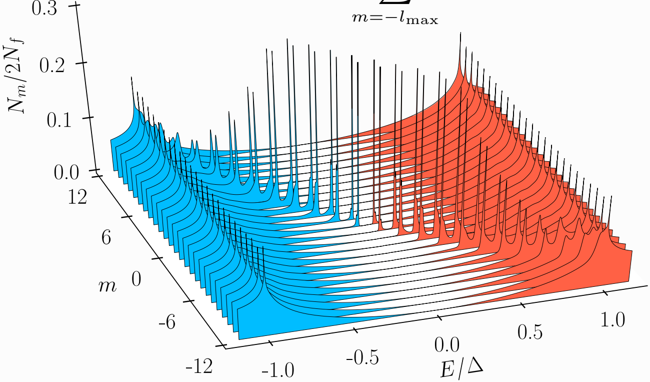
$$\hat{\mathcal{G}}_S^M(\mathbf{k},\mathbf{k}',-\epsilon_n) = \left[\hat{\mathcal{G}}_S^M(\mathbf{k}',\mathbf{k},\epsilon_n) \right]^\dagger$$

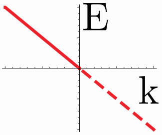
Weyl Fermion Spectrum bound to the Electron Bubble

$$\mu_{\text{N}} = \frac{e}{n_3 p_f \sigma_{\text{N}}^{\text{tr}}} \quad \Leftarrow \quad \mu_{\text{N}}^{\text{exp}} = 1.7 \times 10^{-6} \, \frac{m^2}{V\, s}$$

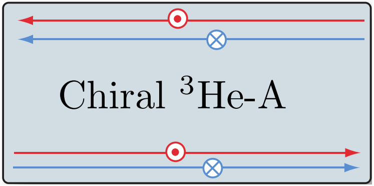
$$\tan\delta_l = j_l(k_f R)/n_l(k_f R) \quad \Rightarrow \quad \sigma_{\text{N}}^{\text{tr}} = \frac{4\pi}{k_f^2}\sum_{l=0}^{\infty}(l+1)\sin^2(\delta_{l+1}-\delta_l) \quad \rightsquigarrow \quad k_f R = 11.17$$

$$N(\mathbf{r}, E) = \sum_{m=-l_{\text{max}}}^{l_{\text{max}}} N_m(\mathbf{r}, E), \quad l_{\text{max}} \simeq k_f R$$

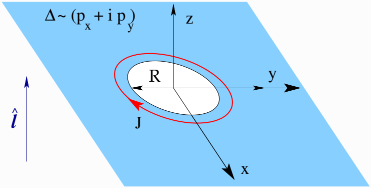


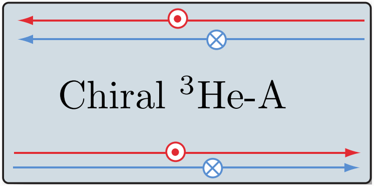


Current density bound to an electron bubble ($k_f R = 11.17$)

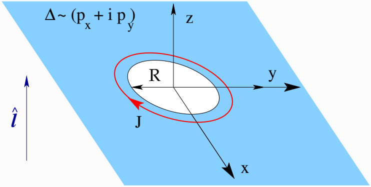


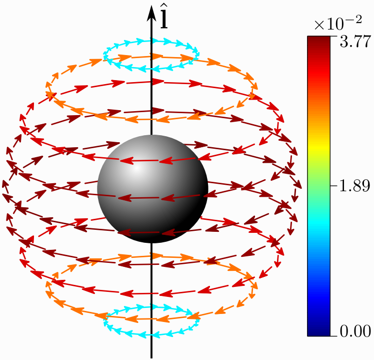
$\Longrightarrow$

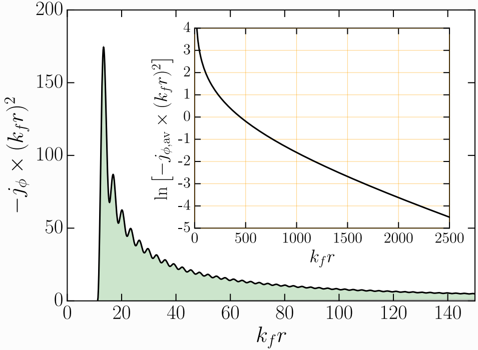




$\Longrightarrow$







$$\mathbf{j}(\mathbf{r})/v_f N_f k_B T_c = j_\phi(\mathbf{r})\hat{\mathbf{e}}_\phi \quad \Longrightarrow \quad \mathbf{L}(T \to 0) \approx -\hbar N_{\text{bubble}}\hat{\mathbf{l}}/2$$

Determination of the Stokes Tensor from the QP-Ion T-matrix

(i) Fermi's golden rule and the QP scattering rate:

$$\Gamma(\mathbf{k}',\mathbf{k}) = \frac{2\pi}{\hbar} W(\hat{\mathbf{k}}',\hat{\mathbf{k}})\delta(E_{\mathbf{k}'} - E_{\mathbf{k}}),$$

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(ii) Drag force from QP-ion collisions (linear in $\mathbf{v}$): ▶ Baym et al. PRL **22**, 20 (1969)

$$\mathbf{F}_{\mathsf{QP}} = -\sum_{\mathbf{k},\mathbf{k}'} \hbar(\mathbf{k}'-\mathbf{k})\left[\hbar\mathbf{k}'\mathbf{v}f_{\mathbf{k}}\left(-\frac{\partial f_{\mathbf{k}'}}{\partial E}\right) - \hbar\mathbf{k}\mathbf{v}(1-f_{\mathbf{k}'})\left(-\frac{\partial f_{\mathbf{k}}}{\partial E}\right)\right]\Gamma(\mathbf{k}',\mathbf{k})$$

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(iii) Microscopic reversibility condition: $W(\hat{\mathbf{k}}',\hat{\mathbf{k}}:+\mathbf{l}) = W(\hat{\mathbf{k}},\hat{\mathbf{k}}':-\mathbf{l})$

Broken T and mirror symmetries in ^{3}He-A $\Rightarrow$ fixed $\hat{\mathbf{l}}$ $\rightsquigarrow$ $W(\hat{\mathbf{k}}',\hat{\mathbf{k}}) \neq W(\hat{\mathbf{k}},\hat{\mathbf{k}}')$

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(iv) Generalized Stokes tensor:

$$\mathbf{F}_{\text{QP}} = -\overset{\leftrightarrow}{\eta}\cdot\mathbf{v}$$

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(iv) Generalized Stokes tensor:

$$\mathbf{F}_{\text{QP}} = -\overleftrightarrow{\eta}\cdot\mathbf{v} \;\;\rightsquigarrow\;\; \eta_{ij} = n_3 p_f \int_0^\infty dE\left(-2\frac{\partial f}{\partial E}\right)\sigma_{ij}(E),\quad \overleftrightarrow{\eta} = \begin{pmatrix} \eta_\perp & \eta_{\text{AH}} & 0 \\ -\eta_{\text{AH}} & \eta_\perp & 0 \\ 0 & 0 & \eta_\parallel \end{pmatrix}$$

$n_3 = \dfrac{k_f^3}{3\pi^2}$ – ^{3}He particle density, $\sigma_{ij}(E)$ – transport scattering cross section,

$f(E) = [\exp(E/k_B T) + 1]^{-1}$ – Fermi Distribution

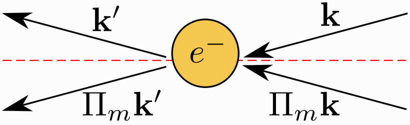
Mirror-symmetric scattering $\Rightarrow$ longitudinal drag force

$$\mathbf{F}_{\text{QP}} = -\overleftrightarrow{\eta}\cdot\mathbf{v}, \qquad \eta_{ij} = n_3 p_f \int_0^\infty dE \left(-2\frac{\partial f}{\partial E}\right)\sigma_{ij}(E)$$

Subdivide by mirror symmetry:

$$W(\hat{\mathbf{k}}',\hat{\mathbf{k}}) = W^{(+)}(\hat{\mathbf{k}}',\hat{\mathbf{k}}) + W^{(-)}(\hat{\mathbf{k}}',\hat{\mathbf{k}}),$$

$$\sigma_{ij}(E) = \sigma_{ij}^{(+)}(E) + \sigma_{ij}^{(-)}(E),$$



$$\sigma_{ij}^{(+)}(E) = \frac{3}{4}\int_{E\ge|\Delta(\hat{\mathbf{k}}')|} d\Omega_{\mathbf{k}'}\int_{E\ge|\Delta(\hat{\mathbf{k}})|}\frac{d\Omega_{\mathbf{k}}}{4\pi}\left[(\hat{\mathbf{k}}_i' - \hat{\mathbf{k}}_i)(\hat{\mathbf{k}}_j' - \hat{\mathbf{k}}_j)\right]\frac{d\sigma^{(+)}}{d\Omega_{\mathbf{k}'}}(\hat{\mathbf{k}}',\hat{\mathbf{k}};E)$$

Mirror-symmetric cross section: $W^{(+)}(\hat{\mathbf{k}}',\hat{\mathbf{k}}) = [W(\hat{\mathbf{k}}',\hat{\mathbf{k}}) + W(\hat{\mathbf{k}},\hat{\mathbf{k}}')]/2$

$$\frac{d\sigma^{(+)}}{d\Omega_{\mathbf{k}'}}(\hat{\mathbf{k}}',\hat{\mathbf{k}};E) = \left(\frac{m^*}{2\pi\hbar^2}\right)^2 \frac{E}{\sqrt{E^2 - |\Delta(\hat{\mathbf{k}}')|^2}} W^{(+)}(\hat{\mathbf{k}}',\hat{\mathbf{k}}) \frac{E}{\sqrt{E^2 - |\Delta(\hat{\mathbf{k}})|^2}}$$

$\rightsquigarrow$ Stokes Drag $\eta_{xx}^{(+)} = \eta_{yy}^{(+)} \equiv \eta_\perp,\ \eta_{zz}^{(+)} \equiv \eta_\parallel$, No transverse force $\left[\eta_{ij}^{(+)}\right]_{i\neq j} = 0$

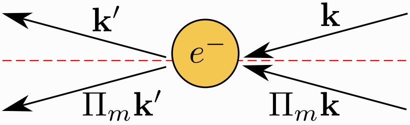
Mirror-antisymmetric scattering ⇒ transverse force

$$\mathbf{F}_{\text{QP}} = -\overleftrightarrow{\eta} \cdot \mathbf{v}, \qquad \eta_{ij} = n_3 p_f \int_0^\infty dE \left(-2\frac{\partial f}{\partial E}\right) \sigma_{ij}(E)$$

Subdivide by mirror symmetry:

$$W(\hat{\mathbf{k}}', \hat{\mathbf{k}}) = W^{(+)}(\hat{\mathbf{k}}', \hat{\mathbf{k}}) + W^{(-)}(\hat{\mathbf{k}}', \hat{\mathbf{k}}),$$

$$\sigma_{ij}(E) = \sigma_{ij}^{(+)}(E) + \sigma_{ij}^{(-)}(E),$$



$$\sigma_{ij}^{(-)}(E) = \frac{3}{4}\int_{E\geq|\Delta(\hat{\mathbf{k}}')|} d\Omega_{\mathbf{k}'} \int_{E\geq|\Delta(\hat{\mathbf{k}})|} \frac{d\Omega_{\mathbf{k}}}{4\pi} \left[\epsilon_{ijk}(\hat{\mathbf{k}}' \times \hat{\mathbf{k}})_k\right] \frac{d\sigma^{(-)}}{d\Omega_{\mathbf{k}'}}(\hat{\mathbf{k}}', \hat{\mathbf{k}}; E)\left[f(E) - \frac{1}{2}\right]$$

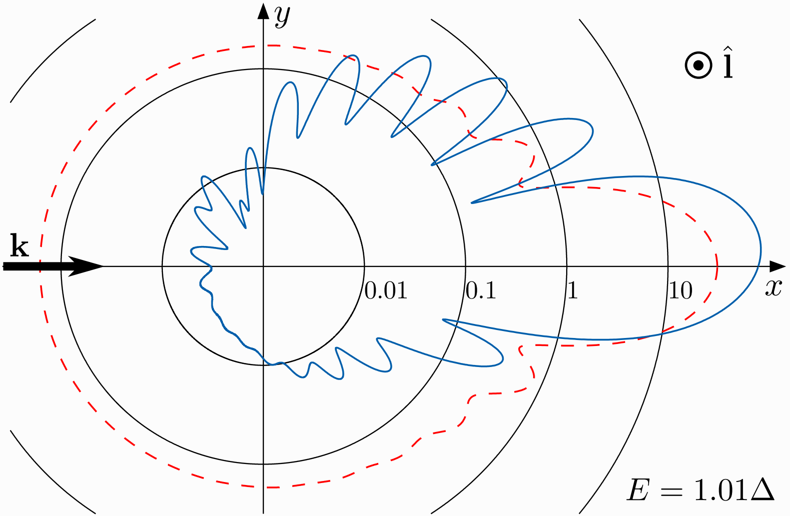
Mirror-antisymmetric cross section: $W^{(-)}(\hat{\mathbf{k}}', \hat{\mathbf{k}}) = [W(\hat{\mathbf{k}}', \hat{\mathbf{k}}) - W(\hat{\mathbf{k}}, \hat{\mathbf{k}}')]/2$

$$\frac{d\sigma^{(-)}}{d\Omega_{\mathbf{k}'}}(\hat{\mathbf{k}}', \hat{\mathbf{k}}; E) = \left(\frac{m^*}{2\pi\hbar^2}\right)^2 \frac{E}{\sqrt{E^2 - |\Delta(\hat{\mathbf{k}}')|^2}} W^{(-)}(\hat{\mathbf{k}}', \hat{\mathbf{k}}) \frac{E}{\sqrt{E^2 - |\Delta(\hat{\mathbf{k}})|^2}}$$

Transverse force $\eta_{xy}^{(-)} = -\eta_{yx}^{(-)} \equiv \eta_{\text{AH}}$ ⇒ anomalous Hall effect

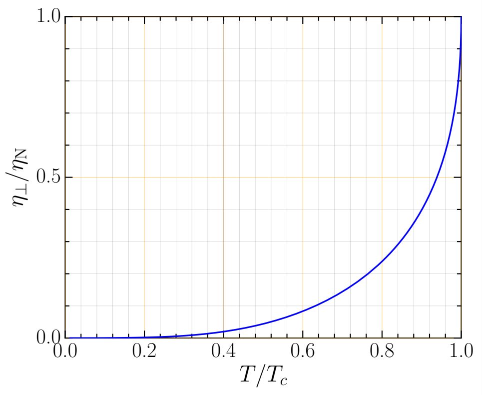
▶ O. Shevtsov and JAS, Phys. Rev. B 96, 064511 (2016)

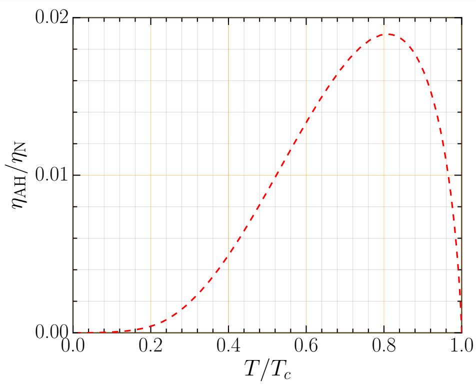
Differential cross section for Bogoliubov QP-Ion Scattering $k_f R = 11.17$



- O. Shevtsov and JAS, Phys. Rev. B 96, 064511 (2016)

Theoretical Results for the Drag and Transverse Forces

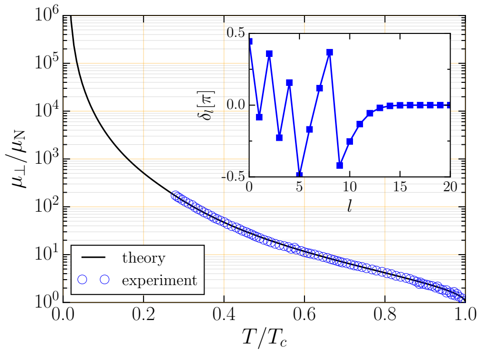


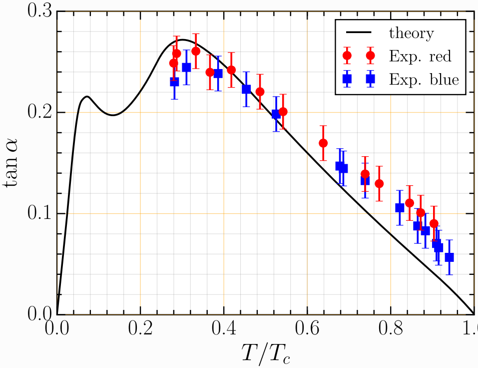


- $\Delta p_x \approx p_f \qquad \sigma_{xx}^{\text{tr}} \approx \sigma_{\text{N}}^{\text{tr}} \approx \pi R^2$
- $F_x \approx n\, v_x\, \Delta p_x\, \sigma_{xx}^{\text{tr}}$
  $\approx n\, v_x\, p_f\, \sigma_{\text{N}}^{\text{tr}}$
- $\Delta p_y \approx \hbar/R\, \sigma_{xy}^{\text{tr}} \approx (\Delta(T)/k_{\text{B}}T_c)^2 \sigma_{\text{N}}^{\text{tr}}$
- $F_y \approx n\, v_x\, \Delta p_y\, \sigma_{xy}^{\text{tr}}$
  $\approx n\, v_x\, (\hbar/R)\, \sigma_{\text{N}}^{\text{tr}} (\Delta(T)/k_{\text{B}}T_c)^2$

$$|F_y/F_x| \approx \frac{\hbar}{p_f R}\, (\Delta(T)/k_B T_c)^2 \qquad k_f R = 11.17$$

▶ O. Shevtsov and JAS, Phys. Rev. B 96, 064511 (2016)



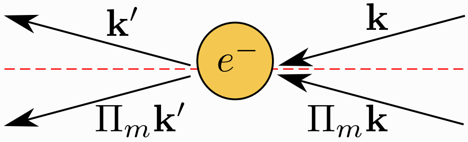


- $\mu_\perp = e\,\dfrac{\eta_\perp}{\eta_\perp^2 + \eta_{AH}^2}$
- $\mu_{AH} = -e\,\dfrac{\eta_{AH}}{\eta_\perp^2 + \eta_{AH}^2}$
- $\tan\alpha = \left|\dfrac{\mu_{AH}}{\mu_\perp}\right| = \dfrac{\eta_{AH}}{\eta_\perp}$
- Hard-Sphere Model: $k_f R = 11.17$

▶ O. Shevtsov and JAS, Phys. Rev. B 96, 064511 (2016)

Summary

- Electrons in ^{3}He-A are "dressed" by a spectrum of Weyl Fermions
- Electrons in ^{3}He-A are "Left handed" in a Right-handed Chiral Vacuum
  $\rightsquigarrow L_z \approx -(N_{bubble}/2)\hbar \approx -100\,\hbar$
- Experiment: RIKEN mobility experiments $\rightsquigarrow$ Observation an AHE in ^{3}He-A
- Scattering of Bogoliubov QPs by the dressed Ion
  $\rightsquigarrow$ Drag Force $(-\eta_{\perp}\mathbf{v})$ and Transverse Force $(\frac{e}{c}\mathbf{v}\times\mathbf{B}_{eff})$ on the Ion
- *Anomalous Hall Field:* $\mathbf{B}_{\text{eff}} \approx \frac{\Phi_0}{3\pi^2}\,k_f^2\,(k_f R)^2\left(\frac{\eta_{\text{AH}}}{\eta_{\text{N}}}\right)\mathbf{l} \simeq 10^3 - 10^4\,\text{T}\,\mathbf{l}$
- Mechanism: Skew/Andreev Scattering of Bogoliubov QPs by the dressed Ion
- Origin: Broken Mirror & Time-Reversal Symmetry $\rightsquigarrow W(\mathbf{k},\mathbf{k}') \neq W(\mathbf{k}',\mathbf{k})$
- Theory: $\rightsquigarrow$ Quantitative account of RIKEN mobility experiments
- Ongoing: New directions for Transport in ^{3}He-A & Chiral Superconductors



(1) Broken TRS: $\mathrm{T}\hat{\mathbf{l}} = -\hat{\mathbf{l}}$

(2) Broken mirror symmetry: $\Pi_m \hat{\mathbf{l}} = -\hat{\mathbf{l}}$

(3) Chiral symmetry: $\mathrm{C} = \mathrm{T} \times \Pi_m$

(4) Microscopic reversibility for chiral superfluids: $W(\hat{\mathbf{k}}', \hat{\mathbf{k}}; \hat{\mathbf{l}}) = W(\hat{\mathbf{k}}, \hat{\mathbf{k}}'; -\hat{\mathbf{l}})$

(5) $\therefore$ For BTRS: the chiral axis $\hat{\mathbf{l}}$ is fixed $\rightsquigarrow W(\hat{\mathbf{k}}', \hat{\mathbf{k}}) \neq W(\hat{\mathbf{k}}, \hat{\mathbf{k}}')$

Symmetry or Normal Liquid ^{3}*He*: $\mathtt{G} = \mathtt{SO(3)_S} \times \mathtt{SO(2)_L} \times \mathtt{U(1)_N} \times \mathtt{P} \times \mathtt{T}$

- Length Scale for Strong Confinement:
  $\xi_0 = \hbar v_f / 2\pi k_B T_c \approx 20 - 80\,\text{nm}$

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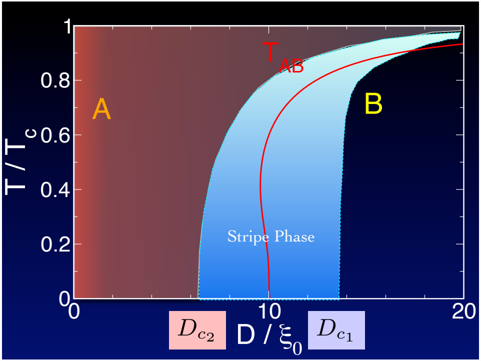
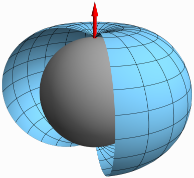
  A. Vorontsov & JAS, PRL, 2007

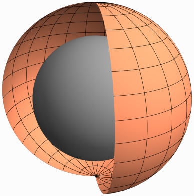


Chiral AM State $\vec{l} = \hat{\mathbf{z}}$



$L_z = 1,\ S_z = 0$

"Isotropic" BW State

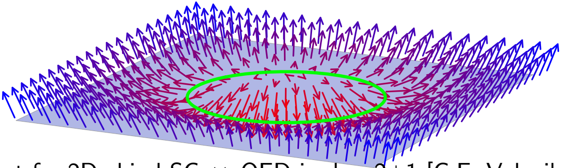


$J = 0,\ J_z = 0$

Hamiltonian for quasi-2D Chiral Superconductor (Sr_2RuO_4 & ^{3}He-A Film):

$$\widehat{H} = \begin{pmatrix} (|\mathbf{p}|^2/2m^* - \mu) & c(p_x + ip_y) \\ c(p_x - ip_y) & -(|\mathbf{p}|^2/2m^* - \mu) \end{pmatrix} = \vec{\mathbf{m}}(\mathbf{p}) \cdot \widehat{\vec{\tau}}$$

$\vec{\mathbf{m}} = (\, cp_x \,,\, \mp cp_y \,, \xi(\mathbf{p}))$ with $|\vec{\mathbf{m}}(\mathbf{p})|^2 = \left(|\mathbf{p}|^2/2m - \mu\right)^2 + c^2|\mathbf{p}|^2 > 0 \;, \mu \neq 0$



Topological Invariant for 2D chiral SC ↔ QED in d = 2+1 [G.E. Volovik, JETP 1988]:

$$N_{2D} = \pi \int \frac{d^2p}{(2\pi)^2}\, \hat{\mathbf{m}}(\mathbf{p}) \cdot \left(\frac{\partial \hat{\mathbf{m}}}{\partial p_x} \times \frac{\partial \hat{\mathbf{m}}}{\partial p_y} \right) = \begin{cases} \pm 1\,; & \mu > 0 \text{ and } \Delta \neq 0 \\ 0\,; & \mu < 0 \text{ or } \Delta = 0 \end{cases}$$

"Vacuum" $(\Delta = 0)$ with $N_{2D} = 0$ | ^{3}He-A $(\Delta \neq 0)$ with $N_{2D} = 1$

Zero Energy Fermions ↑ Confined on the Edge

Determination of the Electron Bubble Radius

(i) Energy required to create a bubble:

$$E(R,P) = E_0(U_0,R) + 4\pi R^2\gamma + \frac{4\pi}{3}R^3 P, \quad P \text{ – pressure}$$

(ii) For $U_0 \to \infty$:

$E_0 = -U_0 + \pi^2\hbar^2/2m_e R^2$ – ground state energy

(iii) Surface Energy: hydrostatic surface tension $\rightsquigarrow \gamma = 0.15\,\mathrm{erg/cm^2}$

(iv) Minimizing E w.r.t. $R \rightsquigarrow P = \pi\hbar^2/4m_e R^5 - 2\gamma/R$

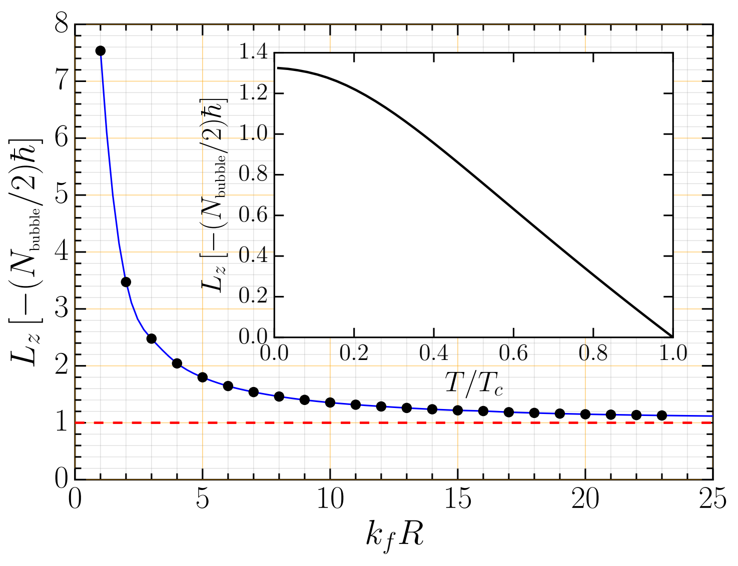
(v) For zero pressure, $P = 0$:

$$R = \left(\frac{\pi\hbar^2}{8m_e\gamma}\right)^{1/4} \approx 2.38\,\mathrm{nm} \quad \rightsquigarrow \quad k_f R = 18.67$$

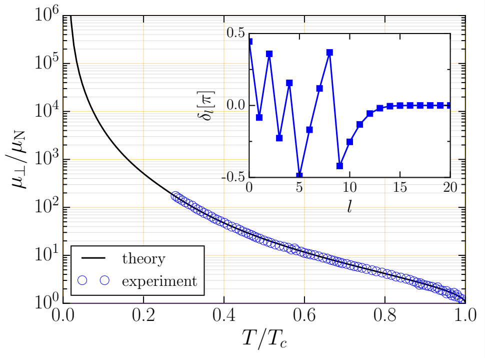
Transport $\rightsquigarrow k_f R = 11.17$

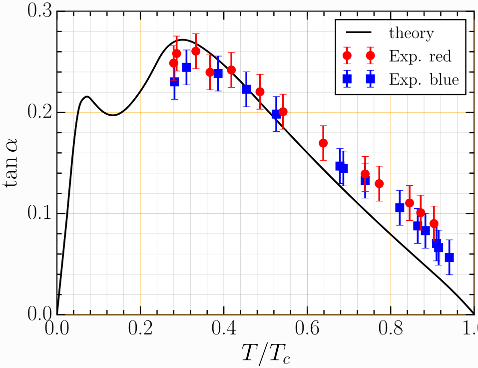
- A. Ahonen et al., J. Low Temp. Phys., 30(1):205228, 1978

$\mathbf{L}(T \to 0) \approx -\hbar N_{\text{bubble}} \hat{\mathbf{l}}/2\,; \qquad N_{\text{bubble}} = n_3 \frac{4\pi}{3} R^3 \approx 200$ ^{3}He atoms



Comparison between Theory and Experiment for the Drag and Transverse Forces





- $\mu_\perp = e\,\dfrac{\eta_\perp}{\eta_\perp^2 + \eta_{\mathsf{AH}}^2}$
- $\mu_{\mathsf{AH}} = -e\,\dfrac{\eta_{\mathsf{AH}}}{\eta_\perp^2 + \eta_{\mathsf{AH}}^2}$
- $\tan\alpha = \left|\dfrac{\mu_{\mathsf{AH}}}{\mu_\perp}\right| = \dfrac{\eta_{\mathsf{AH}}}{\eta_\perp}$
- Hard-Sphere Model: $k_f R = 11.17$

▶ O. Shevtsov and JAS, Phys. Rev. B 96, 064511 (2016)

Theoretical Models for the QP-ion potential

- $U(r) = \begin{cases} U_0, & r < R, \\ -U_1, & R < r < R', \\ 0, & r > R'. \end{cases}$

- $\rightsquigarrow$ Hard-Sphere Potential: $U_1 = 0$, $R' = R$, $U_0 \to \infty$

- $U(x) = U_0\,[1 - \tanh[(x-b)/c]], \quad x = k_f r$

- $U(x) = U_0/\cosh^2[\alpha x^n], \quad x = k_f r$ (Pöschl-Teller-like potential)

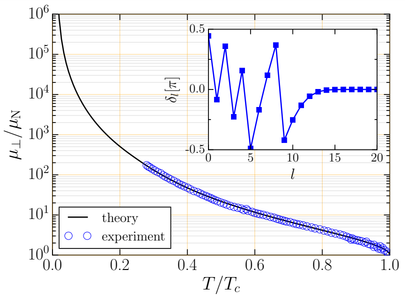
- Random phase shifts: $\{\delta_l | \, l = 1 \dots l_{\text{max}}\}$ are generated with δ_0 is an adjustable parameter

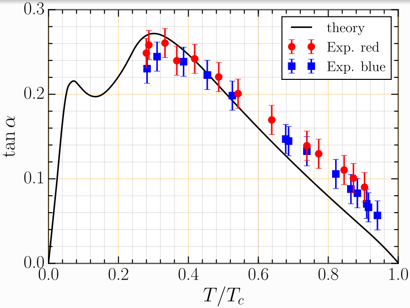
- Parameters for all models are chosen to fit the experimental value of the normal-state mobility, $\mu_N^{\text{exp}} = 1.7 \times 10^{-6}\, m^2/V \cdot s$

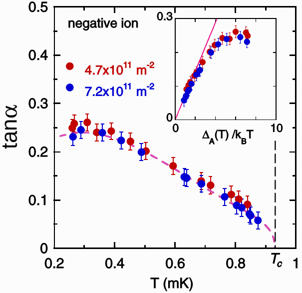
Theoretical Models for the QP-ion potential

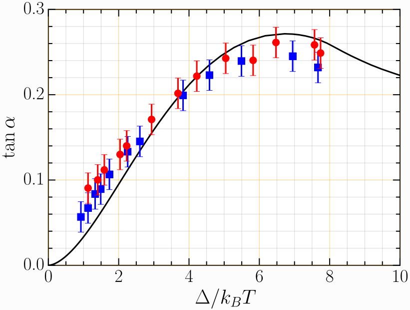
Label	Potential	Parameters
Model A	hard sphere	$k_f R = 11.17$
Model B	repulsive core & attractive well	$U_0 = 100E_f, U_1 = 10E_f, k_f R' = 11, R/R' = 0.36$
Model C	random phase shifts model 1	$l_{\max} = 11$
Model D	random phase shifts model 2	$l_{\max} = 11$
Model E	Pöschl-Teller-like	$U_0 = 1.01E_f, k_f R = 22.15, \alpha = 3 \times 10^{-5}, n = 4$
Model F	Pöschl-Teller-like	$U_0 = 2E_f, k_f R = 19.28, \alpha = 6 \times 10^{-5}, n = 4$
Model G	hyperbolic tangent	$U_0 = 1.01E_f, k_f R = 14.93, b = 12.47, c = 0.246$
Model H	hyperbolic tangent	$U_0 = 2E_f, k_f R = 14.18, b = 11.92, c = 0.226$
Model I	soft sphere 1	$U_0 = 1.01E_f, k_f R = 12.48$
Model J	soft sphere 2	$U_0 = 2E_f, k_f R = 11.95$

Hard-sphere model with $k_f R = 11.17$ (Model A)









Comparison with Experiment for Models for the QP-ion potential

Label	Potential	Parameters
Model A	hard sphere	$k_f R = 11.17$
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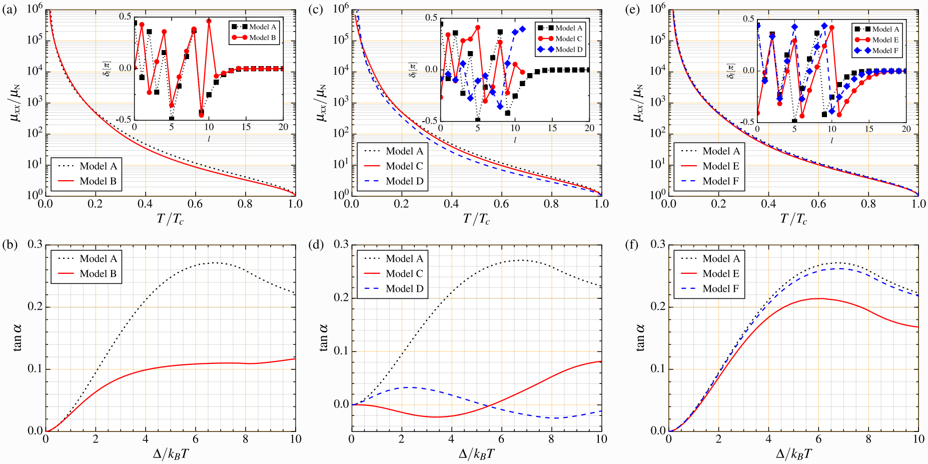

Mobility of an electron bubble in the Normal Fermi Liquid

(i) $t^R_{\text{N}}(\hat{\mathbf{k}}', \hat{\mathbf{k}}; E) = \sum_{l=0}^{\infty} (2l+1) t^R_l(E) P_l(\hat{\mathbf{k}}' \cdot \hat{\mathbf{k}})$

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(iv) $\sigma^{\mathsf{tr}}_{\mathsf{N}} = \int \frac{d\Omega_{\mathbf{k}'}}{4\pi} (1 - \hat{\mathbf{k}} \cdot \hat{\mathbf{k}}') \frac{d\sigma}{d\Omega_{\mathbf{k}'}} = \frac{4\pi}{k_f^2} \sum_{l=0}^{\infty} (l+1) \sin^2(\delta_{l+1} - \delta_l)$

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(v) $\mu_{\mathsf{N}} = \frac{e}{n_3 p_f \sigma^{\mathsf{tr}}_{\mathsf{N}}}, \quad p_f = \hbar k_f, \quad n_3 = \frac{k_f^3}{3\pi^2}$

$$\hat{\mathcal{G}}^R_S(\mathbf{r}',\mathbf{r},E) = \int \frac{d^3k}{(2\pi)^3}\int \frac{d^3k'}{(2\pi)^3} e^{i\mathbf{k}'\mathbf{r}'} e^{-i\mathbf{k}\mathbf{r}} \hat{\mathcal{G}}^R_S(\mathbf{k}',\mathbf{k},E)$$

$$\hat{\mathcal{G}}^R_S(\mathbf{k}',\mathbf{k},E) = (2\pi)^3 \hat{G}^R_S(\mathbf{k},E)\delta_{\mathbf{k}',\mathbf{k}} + \hat{G}^R_S(\mathbf{k}',E)\hat{T}_S(\mathbf{k}',\mathbf{k},E)\hat{G}^R_S(\mathbf{k},E)$$

$$\hat{G}^R_S(\mathbf{k},E) = \frac{1}{\varepsilon^2 - E^2_{\mathbf{k}}}\begin{pmatrix} \varepsilon + \xi_k & -\Delta(\hat{\mathbf{k}}) \\ -\Delta^\dagger(\hat{\mathbf{k}}) & \varepsilon - \xi_k \end{pmatrix}, \quad \varepsilon = E + i\eta, \quad \eta \to 0^+$$

$$N(\mathbf{r},E) = -\frac{1}{2\pi}\mathrm{Im}\left\{\mathrm{Tr}\left[\hat{\mathcal{G}}^R_S(\mathbf{r},\mathbf{r},E)\right]\right\}$$

$$\mathbf{j}(\mathbf{r}) = \frac{\hbar}{4mi}k_B T \sum_{n=-\infty}^{\infty} \lim_{\mathbf{r}\to\mathbf{r}'} \mathrm{Tr}\left[(\boldsymbol{\nabla}_{\mathbf{r}'} - \boldsymbol{\nabla}_{\mathbf{r}})\hat{\mathcal{G}}^M(\mathbf{r}',\mathbf{r},\epsilon_n)\right]$$

$$\hat{\mathcal{G}}^R_S(\mathbf{r}',\mathbf{r},E) = \hat{\mathcal{G}}^M_S(\mathbf{r}',\mathbf{r},\epsilon_n)\Big|_{i\epsilon_n\to\varepsilon}, \text{ for } n \geq 0$$

$$\hat{\mathcal{G}}^M_S(\mathbf{k},\mathbf{k}',-\epsilon_n) = \left[\hat{\mathcal{G}}^M_S(\mathbf{k}',\mathbf{k},\epsilon_n)\right]^\dagger$$

Temperature scaling of the Stokes tensor components

- For $1 - \frac{T}{T_c} \to 0^+$:

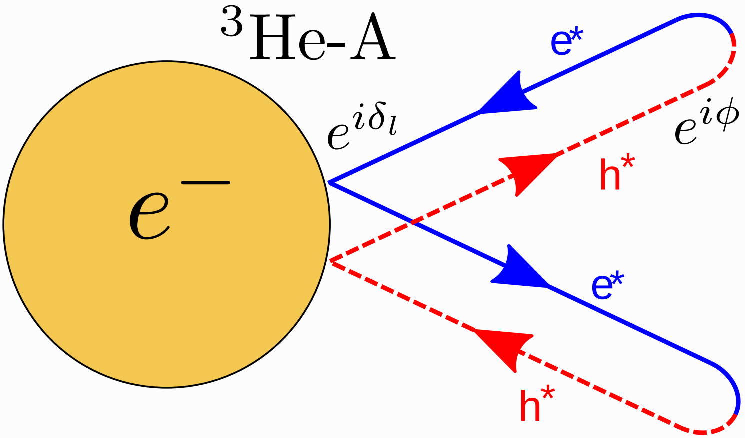
$$\frac{\eta_\perp}{\eta_\mathsf{N}} - 1 \propto -\Delta(T) \propto \sqrt{1 - \frac{T}{T_c}}$$

$$\frac{\eta_\mathsf{AH}}{\eta_\mathsf{N}} \propto \Delta^2(T) \propto 1 - \frac{T}{T_c}$$

- For $\frac{T}{T_c} \to 0^+$:

$$\frac{\eta_\perp}{\eta_\mathsf{N}} \propto \left(\frac{T}{T_c}\right)^2$$

$$\frac{\eta_\mathsf{AH}}{\eta_\mathsf{N}} \propto \left(\frac{T}{T_c}\right)^3$$



$$\Delta(\hat{\mathbf{k}}) = \Delta \sin\theta e^{i\phi}$$

Local Density of States: $N(\mathbf{p}, x; \varepsilon) = -\dfrac{1}{\pi}\,\mathrm{Im}\,\mathfrak{g}^{\mathsf{R}}(\mathbf{p}, x; \varepsilon)$

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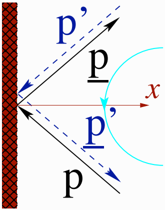

Pair Time-Reversed Trajectories

Spectral Current Density:

$$\vec{J}(\mathbf{p}, x; \varepsilon) = 2N_f\, \mathbf{v}(\mathbf{p}) \left[N(\mathbf{p}, x; \varepsilon) - N(\mathbf{p}', x; \varepsilon)\right]$$

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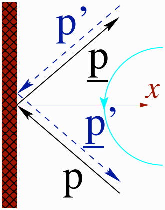

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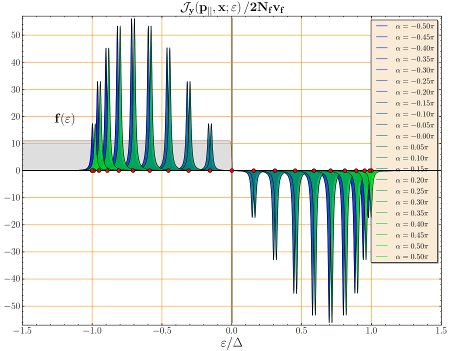
Bound-State Edge Current at $x = 0$



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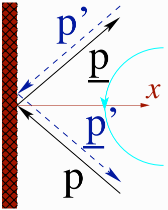

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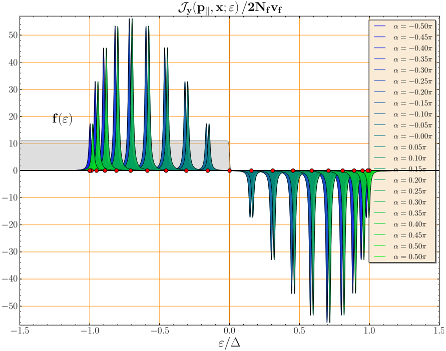
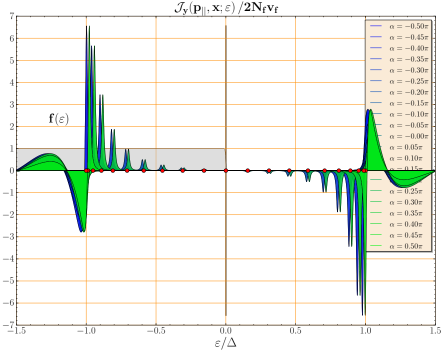
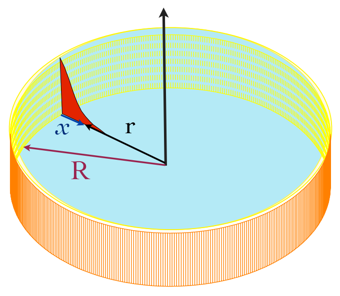


Continuum Edge Current at $x = 10\xi_0$



Ground-State Current Density: $\vec{J}(x) = \int_{-p_f}^{+p_f} \frac{dp_{||}}{p_f} \int_{-\infty}^{0} \vec{J}(\mathbf{p}, x; \varepsilon)$



Bound-State Contribution ($R \gg \xi_\Delta$):

$$\begin{aligned} J_\varphi(\mathbf{p}, x; \varepsilon) &= 2N_f\, v_f\, \Delta\, |p_x|\, p_\varphi\, e^{-x/\xi_\Delta} \\ &\times \left[\delta(\varepsilon - \varepsilon_{\mathsf{bs}}(\mathbf{p}_{||})) - \delta(\varepsilon - \varepsilon_{\mathsf{bs}}({\mathbf{p}'}_{||}))\right] \end{aligned}$$

Bound-State Edge Current: $\int_0^\infty dx J_\varphi(x) = \frac{1}{2}\, n\, \hbar$

Mass Current: $v_f \to p_f \rightsquigarrow \vec{J} \to \vec{g}$

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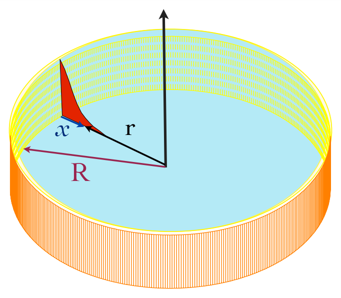

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- $L_z^{\text{bs}} = \int_V d^2r\; [r\, g_\varphi(\mathbf{r})]$ $= N\,\hbar$ $\times 2$ Too Large vs. MT

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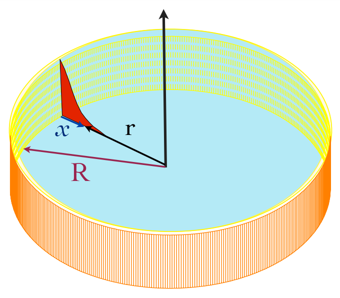

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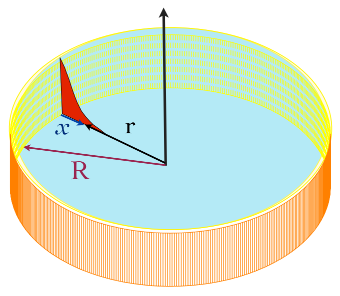
Bound-State Edge Current: $\int_0^\infty dx J_\varphi(x) = \frac{1}{2}\, n\, \hbar$

Mass Current: $v_f \rightarrow p_f \rightsquigarrow \vec{J} \rightarrow \vec{g}$

- $L_z^{\text{bs}} = \int_V d^2r\, [r\, g_\varphi(\mathbf{r})] = N\,\hbar$ ×2 Too Large vs. MT
- Continuum ($\varepsilon < -\Delta$): $J_\varphi^{\text{C}} = 2N_f\, v_f\, |p_x| \left(\frac{\Delta^2\, p_\varphi^2}{\varepsilon^2 - \varepsilon_{\text{bs}}^2(\mathbf{p}_{||})}\right) \sin\left(2\sqrt{\varepsilon^2 - \Delta^2}\, x/v_x\right)$

Ground-State Current Density: $\vec{J}(x) = \int_{-p_f}^{+p_f} \frac{dp_{||}}{p_f} \int_{-\infty}^{0} \vec{J}(\mathbf{p}, x; \varepsilon)$



Bound-State Contribution ($R \gg \xi_\Delta$):

$$\begin{aligned} J_\varphi(\mathbf{p}, x; \varepsilon) &= 2N_f\, v_f\, \Delta\, |p_x|\, p_\varphi\, e^{-x/\xi_\Delta} \\ &\times \left[\delta(\varepsilon - \varepsilon_{\text{bs}}(\mathbf{p}_{||})) - \delta(\varepsilon - \varepsilon_{\text{bs}}(\mathbf{p}'_{||}))\right] \end{aligned}$$

Bound-State Edge Current: $\int_0^\infty dx J_\varphi(x) = \frac{1}{2}\, n\, \hbar$

Mass Current: $v_f \to p_f \rightsquigarrow \vec{J} \to \vec{g}$

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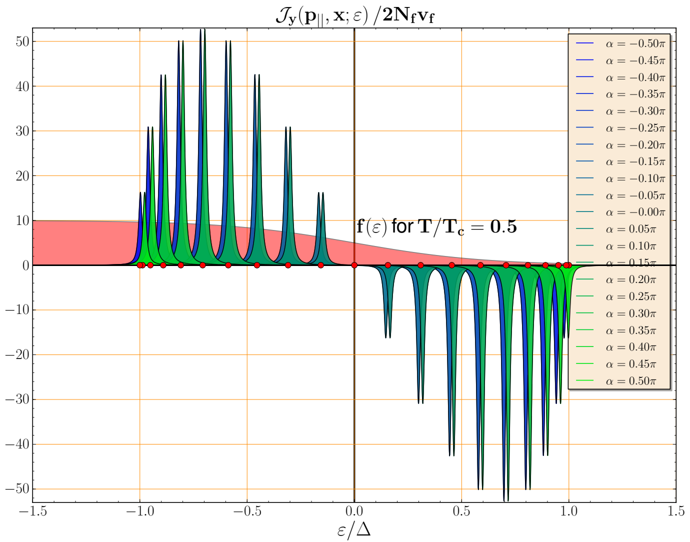
- $L_z^{\text{C}} = \int_V d^2r\, \left[r\, g_\varphi^{\text{C}}(\mathbf{r})\right] = -\frac{1}{2} N\, \hbar \rightsquigarrow L_z^{\text{total}} = (N/2)\hbar$ - MT Result Recovered!

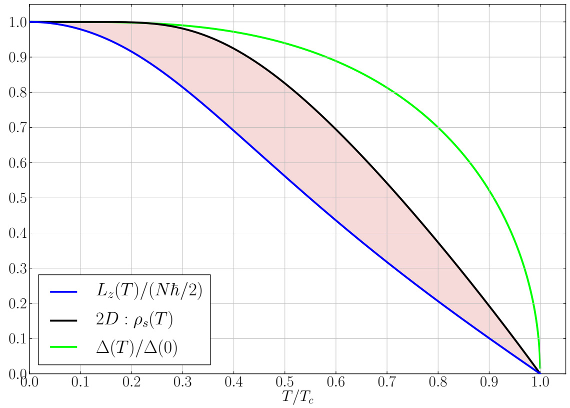
Thermally Excited Edge Fermions Carry the Opposite Current



$$J = \frac{1}{4}\, n\, \hbar \times \mathcal{Y}_{\text{edge}}(T) \qquad \mathcal{Y}_{\text{edge}}(T) \approx 1 - c\,(T/\Delta)^2\,, \quad T \ll \Delta$$

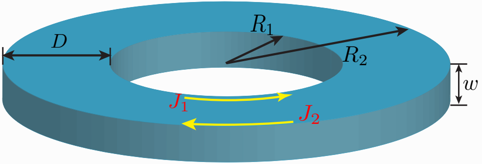
- Thermal Signature of the Chiral Edge States

$$\rho_s(T)/\rho = \mathcal{Y}_{\text{bulk}}(T) - 1 \propto -\, e^{-\Delta/T}\,, \quad T \ll \Delta$$



- JAS, Phys. Rev. B 84, 214509 (2011)
- Y. Tsutsumi et al., PRB 85, 100506(R) (2012)

^{3}He-A confined in a toroidal cavity



- $R_1, R_2, R_1 - R_2 \gg \xi_0$

- Sheet Current: $J = \dfrac{1}{4}\, n\, \hbar$ ($n = N/V =$ ^{3}He density)
- Counter-propagating Edge Currents: $J_1 = -J_2 = \dfrac{1}{4}\, n\, \hbar$
- Angular Momentum:

$$L_z = 2\pi\, h\, (R_1^2 - R_2^2) \times \frac{1}{4}\, n\, \hbar = (N/2)\, \hbar$$

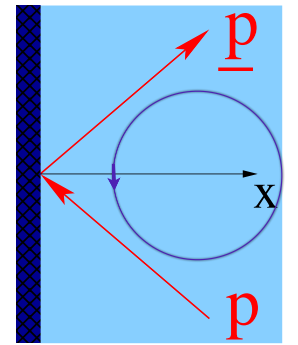
McClure-Takagi Result

▶ J. A. Sauls, Phys. Rev. B 84, 214509 (2011)

Magnitude of Edge Currents are Protected by Symmetry

Magnitude of Edge Currents are Protected by Symmetry

Specular Reflection



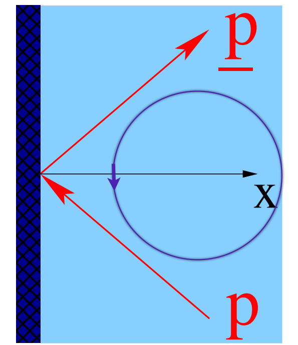
Propagating Chiral Fermions:

$$\mathfrak{g}^{\mathsf{R}}(\mathbf{p},\varepsilon;x) = \frac{\pi\Delta|\mathbf{p}_x|}{\varepsilon + i\gamma - \varepsilon_{\mathsf{bs}}(\mathbf{p}_{||})}\, e^{-x/\xi_\Delta}$$

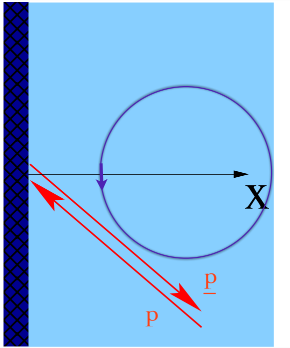
Edge Current: $J = \frac{1}{4}\, n\, \hbar$

Magnitude of Edge Currents are Protected by Symmetry

Specular Reflection



Retro Reflection



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Magnitude of Edge Currents are Protected by Symmetry

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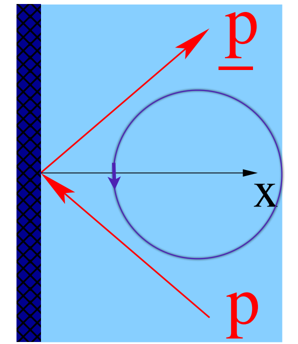
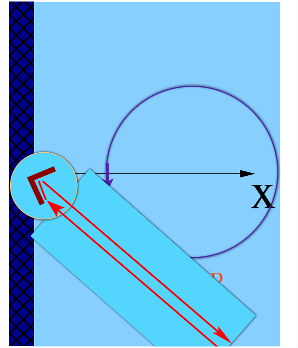


Retro Reflection



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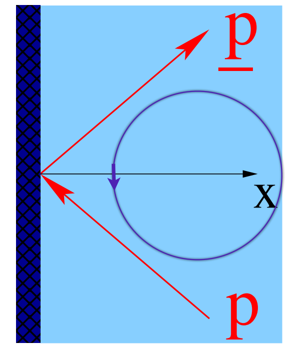

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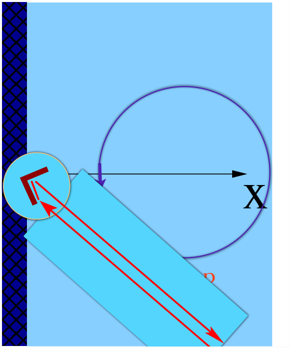
Retro Reflection

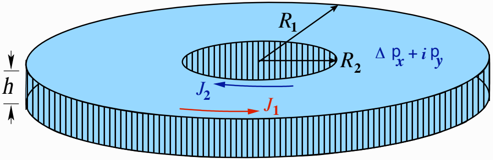


Zero-Energy Fermions for all $\mathbf{p}$:

$$\mathfrak{g}^{\mathsf{R}}(\mathbf{p},\varepsilon;x) = \frac{\pi\Delta}{\varepsilon + i\gamma}\, e^{-2\Delta x/v_x}$$

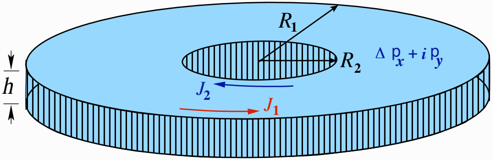
$\rightsquigarrow$ Edge Current: $J = 0$

Magnitude of Edge Currents are Protected by Symmetry



- Sheet Current: $J = f \times \frac{1}{4} n \hbar$
- Non-Specular Surfaces $0 \leq f \leq 1$

Magnitude of Edge Currents are Protected by Symmetry



- Sheet Current: $J = f \times \frac{1}{4} n\, \hbar$
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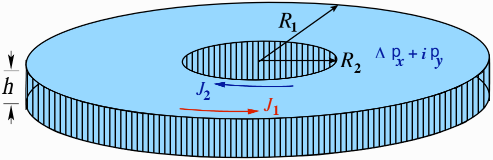
Incomplete Screening of Counter-Propagating Currents

Scaling of L_z with $r = (R_2/R_1)^2 \qquad 0 < r < 1$

Magnitude of Edge Currents are Protected by Symmetry



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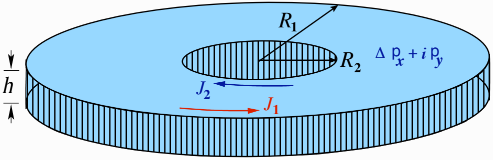
- $f_1 = 1,\; f_2 = 0$

$$L_z = (N/2)\,\hbar \times \left(\frac{1}{1-r}\right) \gg (N/2)\,\hbar$$

Magnitude of Edge Currents are Protected by Symmetry



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Incomplete Screening of Counter-Propagating Currents

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- $f_1 = 1,\ f_2 = 0$

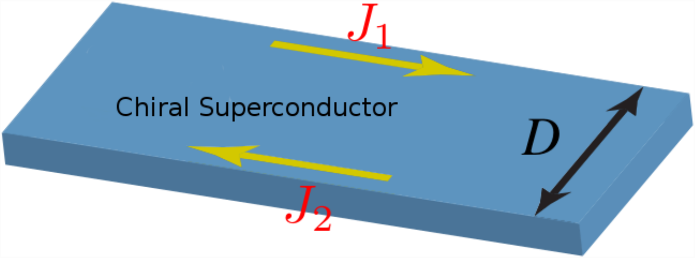
$$L_z = (N/2)\,\hbar \times \left(\frac{1}{1-r}\right) \gg (N/2)\,\hbar$$

- $f_1 = 0,\ f_2 = 1$

$$L_z = (N/2)\,\hbar \times \left(\frac{-r}{1-r}\right) \ll -(N/2)\,\hbar$$

- Strong violations of the McClure-Takagi Result

- Mesoscopic geometries: Edge states are important for transport



- Surface states, edge currents, and the angular momentum of chiral p-wave superfluids and superconductors,
  JAS, Phys. Rev. B 84, 214509 (2011) [arXiv:1209.5501]

- Symmetry Protected Topological Superfluids and Superconductors — From the Basics to ^{3}He,
  T. Mizushima, Y. Tsutsumi, T. Kawakami, M. Sato, M. Ichioka, K. Machida
  [arXiv:1508.00787]