Particle-phonon coupling effects within the self-consistent theory of finite Fermi systems

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The report is based on these articles:

1. N. V. Gnezdilov, I. N. Borzov, E. E. Saperstein, and S. V. Tolokonnikov, Phys. Rev. C 89, 034304 (2014).
Self-consistent description of single-particle levels of magic nuclei.

2. E.E. Saperstein, M. Baldo, S.S. Pankratov, S.V.Tolokonnikov. JETP Lett., 104, 609 (2016).
Phonon-particle coupling effects in single-particle energies of semi-magic nuclei.

3. E.E. Saperstein, S. Kamerdzhiev, S. Krewald, J. Speth, and S.V. Tolokonnikov, EPL, 103, 42001 (2013)

A model for phonon coupling contributions to electromagnetic moments of odd spherical nuclei.

4. E.E. Saperstein, S. Kamerdzhiev, D.S. Krepish, S.V. Tolokonnikov, and D. Voitenkov, arXiv: 1608.07962v2 [nucl-th]

Self-consistent account for phonon induced corrections to quadrupole moments of odd semi-magic nuclei.

5. E. E. Saperstein, I. N. Borzov, and S. V. Tolokonnikov, JETP Lett., 104, 218 (2016).

On the Anomalous *A* Dependence of the Charge Radii of Heavy Calcium Isotopes.

Plan:

1. Fayans energy density functional (EDF) vs Skyrme EDFs.
2. The double counting problem for particle-phonon coupling (PC) effects.
3. PC effects in single-particle energies of magic (semi-magic) nuclei.
4. PC effects in charge radii.
5. PC effects in electromagnetic moments of odd magic (semi-magic) nuclei.

Fayans EDF vs Skyrme EDFs

We use the Fayans EDF:

the main in-volume term is

$$\mathcal{E}(\rho) = \frac{a\rho^2}{2}\frac{1+\alpha\rho^\sigma}{1+\gamma\rho}. \qquad m^* = m$$

For Skyrme EDFs: $\gamma = 0, m^* \neq m$

Both peculiarities of the Fayans EDF ***(*** the Fayans denominator and bare mass) are related to the self-consistent theory of finite Fermi systems (sc TFFS)
V. A. Khodel and E. E. Saperstein, Phys. Rep. **92**, 183 (1982).

In hidden form, they describe the energy dependence effects of the sc TFFS.

S-c TFFS *[V. A. Khodel, E. E. Saperstein, Phys. Rep. 92, 183 (1982)] starts from the quasiparticle mass operator:*

$$G_q = (\varepsilon - \varepsilon_k - \Sigma_q)^{-1}. \quad \varepsilon_k = k^2/2m$$

$$\Sigma_q(\mathrm{r}, k^2; \varepsilon) = \Sigma_0(\mathrm{r}) + \frac{1}{2m\varepsilon_{\mathrm{F}}^0}\,\mathrm{k}\,\Sigma_1(\mathrm{r})\mathrm{k} + \Sigma_2(\mathrm{r})\frac{\varepsilon}{\varepsilon_{\mathrm{F}}^0},$$

$$\varepsilon_{\mathrm{F}}^0 = (k_{\mathrm{F}}^0)^2/2m$$

k-dependence and energy dependence on equal footing

By definition,

$$\Sigma_1(\mathbf{r}) = \varepsilon_{\mathrm{F}}^0 \left.\frac{\partial\Sigma(\mathbf{r})}{\partial\varepsilon_k}\right|_0,$$

$$\Sigma_2(\mathbf{r}) = \varepsilon_{\mathrm{F}}^0 \left.\frac{\partial\Sigma(\mathbf{r})}{\partial\varepsilon}\right|_0,$$

$$Z(\mathbf{r}) = \left(1 - \Sigma_2(\mathbf{r})/\varepsilon_{\mathrm{F}}^0\right)^{-1},$$

It determines the in-volume term of spectroscopic factors

$$\frac{m}{m^*(\mathbf{r})} = \frac{\left(1 + \Sigma_1(\mathbf{r})/\varepsilon_{\mathrm{F}}^0\right)}{\left(1 - \Sigma_2(\mathbf{r})/\varepsilon_{\mathrm{F}}^0\right)}.$$

In s-c TFFS, m*(r) includes 'k-mass' and 'E-mass', and the two effects strongly cancel each other, the effective mass being close to the bare one.

In the sc TFFS the EDF contains

$$Z^2(\rho), Z^3(\rho)$$

$$Z(\mathbf{r}) = \frac{2}{1 + \sqrt{1 - 4C_0\lambda_{02}\rho(\mathbf{r})/\varepsilon_F^0}},$$

Rather comlpicate density dependence of the EDF. Fayans with coauthors found that it can be approximated with the above fractional function. In the sc TFFS, the effective mass is close to the bare one. In the Fayans EDF the bare mass is used. Thus, the Fayans EDF could be interpreted as an approximate version of the sc TFFS

Reasons why $m^* \approx m$

In the s-c TFFS, for nuclear matter,

$$Z_0 = 0.8,\ m^* = 0.95m$$

In any non-relativistic many-body theory of nuclear matter

$$\Sigma(\varepsilon, \varepsilon_k) = \Sigma[\hat{V}, G_0]$$

Where V is the free NN-potential and $G_0 = (\varepsilon - \varepsilon_k)^{-1}$.

V is short-range: $r_{eff} << 1/k_F$

Put it to be zero-range: $V(r_1 - r_2) = V_0 \delta(r_1 - r_2)$

All the momentum integrals diverge: cut-off at $k_{cut} >> k_F$

With accuracy $(k_F / k_{cut})^2$ ***k- and E-dep. due to*** G_0

$$G_0 = (\varepsilon - \varepsilon_k)^{-1}.$$

$$\frac{\partial \Sigma}{\partial \varepsilon_k} = \frac{\delta \Sigma}{\delta G_0} \frac{\partial G_0}{\partial \varepsilon_k},$$

$$\frac{\partial \Sigma}{\partial \varepsilon} = \frac{\delta \Sigma}{\delta G_0} \frac{\partial G_0}{\partial \varepsilon}.$$

$$\frac{\partial \Sigma}{\partial \varepsilon_k} = -\frac{\partial \Sigma}{\partial \varepsilon}.$$

$m^* = m$, ***with accuracy of*** $(r_{eff} k_F)^2$

The double counting problem

Such a problem exists for the study of PC corrections in any theory starting from the EDF with phenomenological parameters.

Indeed, in the EDF approach,

the PC contributions are taken into account on average.

If we include all the PC contributions, we must readjust these parameters anew.

Our idea is to separate the fluctuating parts of the PC corrections that behave in a non-regular way, depending significantly on the nucleus under consideration and the single-particle state of the odd nucleon. Their contributions to observables are, as a rule, rather small on average but are often important to reproduce the specific experimental value in a nucleus under consideration.

Single-particle energies of seven magic nuclei: 40,48 Ca, 56,78 Ni, 100,132 Sn, 208 Pb

N.V. Gnezdilov, I. N. Borzov, E. E. Saperstein, and S. V. Tolokonnikov, Phys. Rev. 89, 034304 (2014).

In addition to the old DF3 and DF3-a EDFs, a new one, DF3-b, is found for better description of 35 experimental spin-orbit differences!

The data from

H. Grawe, K. Langanke, and G. Mart´ınez-Pinedo, Rep. Prog.Phys. **70**, 1525 (2007) [105 levels]

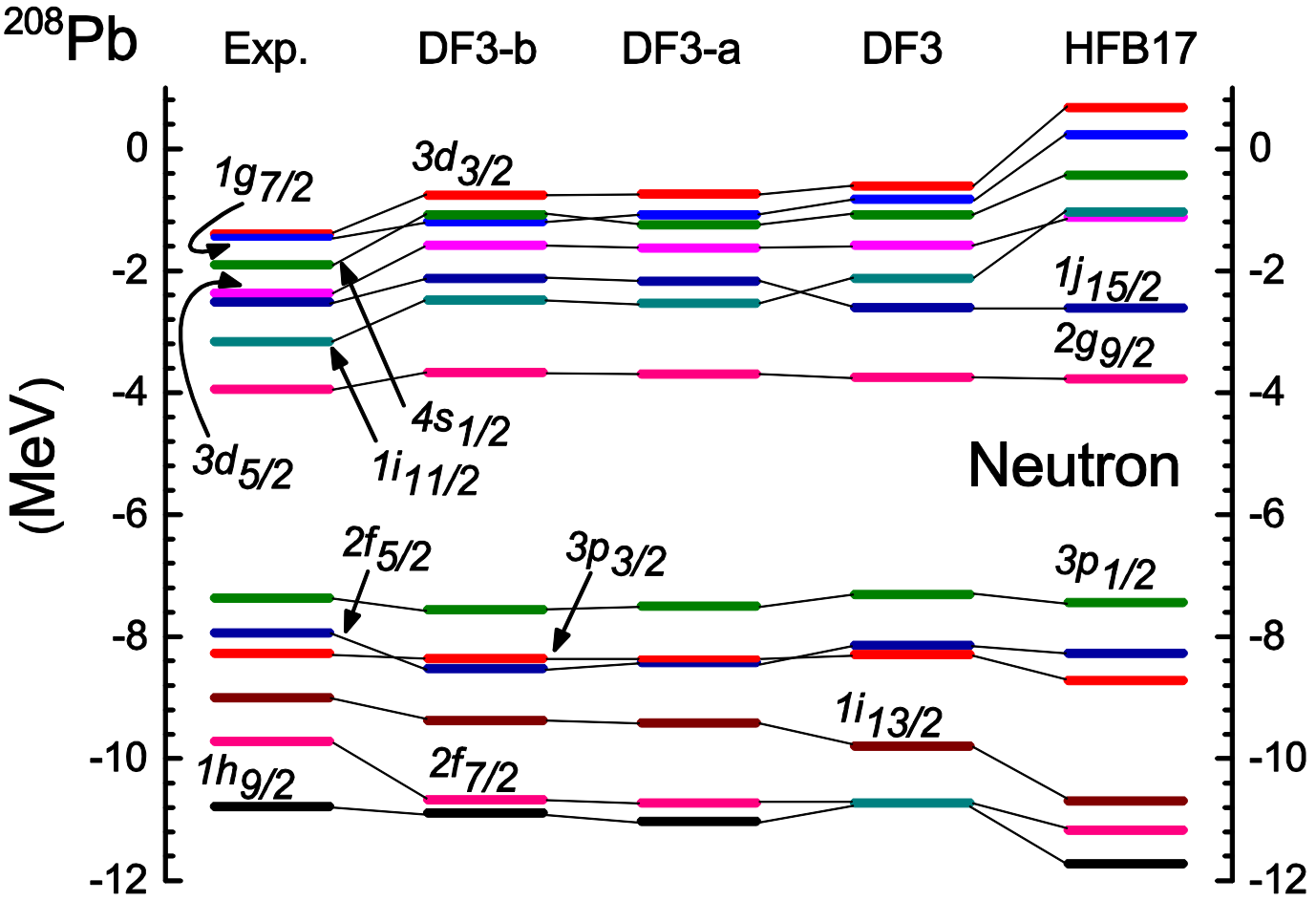
In magic nuclei, the perturbation theory (PT) in $\delta\Sigma^{PC}$

is valid. Another situation there is in semi-magic nuclei, where the Dyson equation with $\Sigma(\varepsilon) = \Sigma_0 + \delta\Sigma^{PC}(\varepsilon)$

Sould be solved directly, without PT

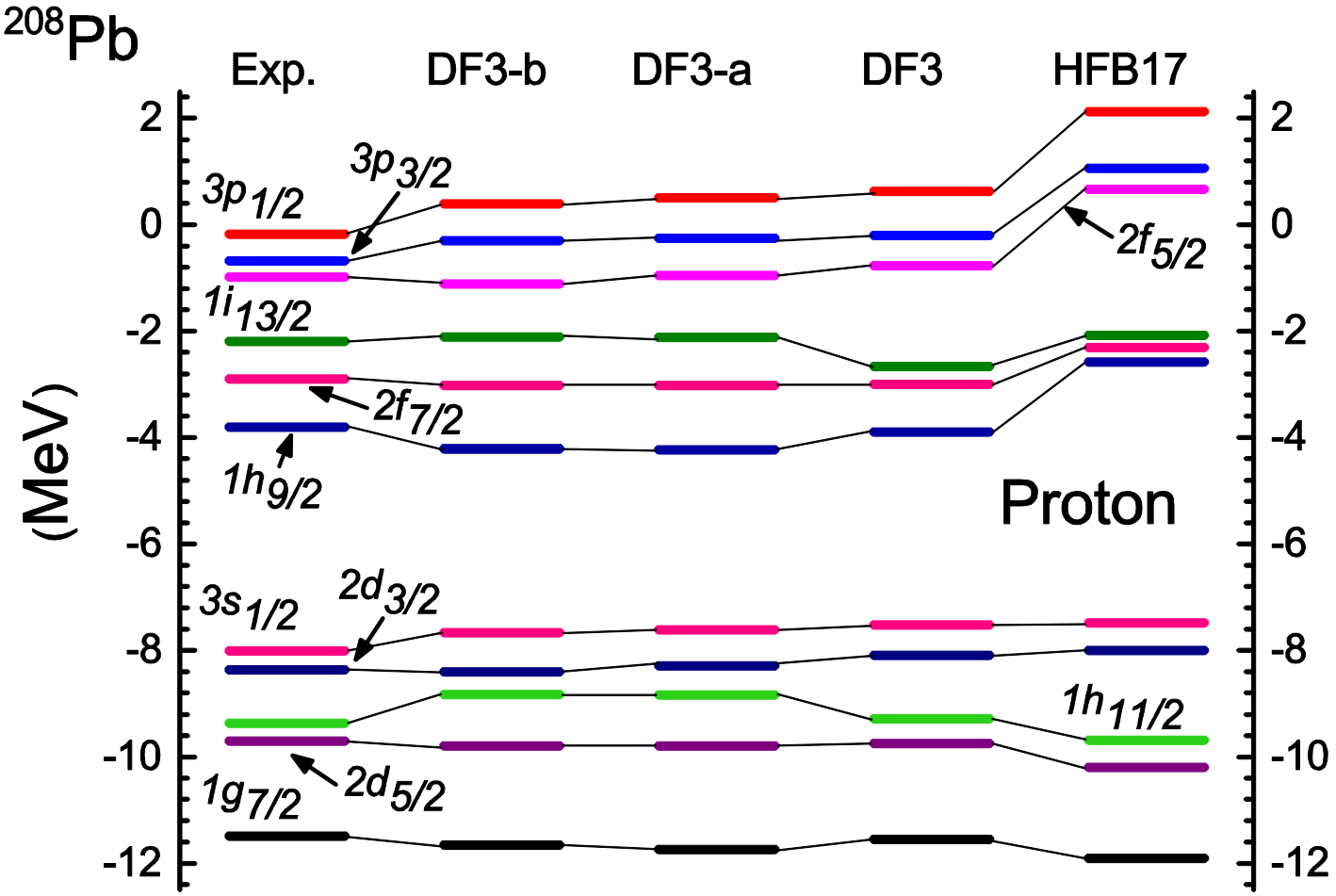
208 Pb, neutron levels

N.V. Gnezdilov, I. N. Borzov, E. E. Saperstein, and S. V. Tolokonnikov, Phys. Rev. 89, 034304 (2014).



208 Pb, proton levels

N.V. Gnezdilov, I. N. Borzov, E. E. Saperstein, and S. V. Tolokonnikov, Phys. Rev. 89, 034304 (2014).

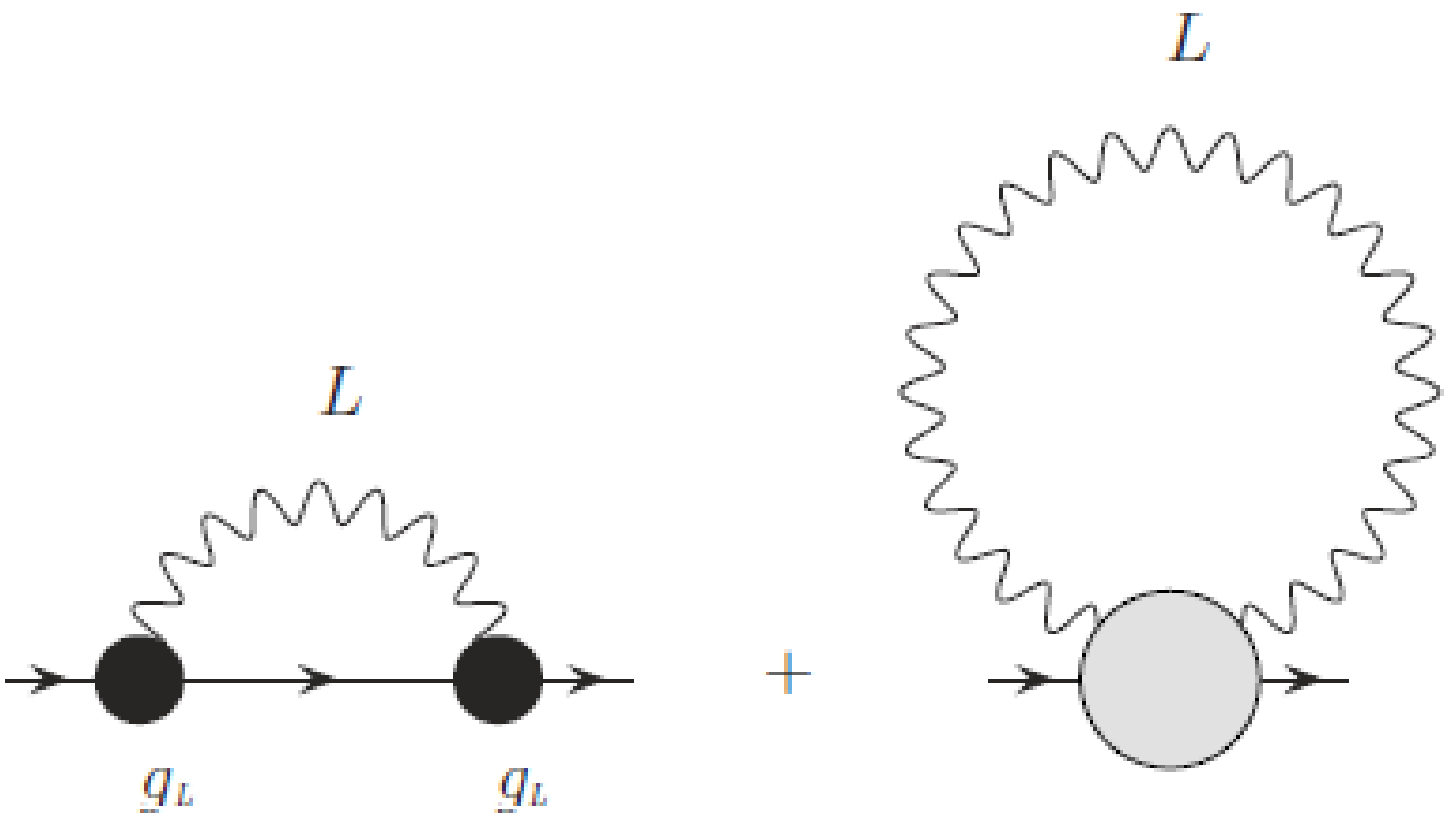


Nucleus	N	DF3-b	DF3-a	DF3	HFB17
^{40}Ca	14	1.08	1.25	1.35	1.64
^{48}Ca	12	0.89	1.00	1.01	1.70
^{56}Ni	14	1.00	0.97	0.85	1.40
^{78}Ni	11	1.24	1.41	1.09	1.32
^{100}Sn	13	1.09	1.17	1.01	1.56
^{132}Sn	17	0.58	0.66	0.55	1.15
^{208}Pb	24	0.44	0.51	0.43	1.15
Total	105	0.89	0.98	0.89	1.40

Average deviations $\langle \delta\varepsilon_\lambda \rangle_{\rm rms}$ (MeV)

of predictions for SPEs without the PC corrections from the data (N is the total number of neutron and proton states). The effective mass effect !

PC corrections to SPEs



pole ***tadpole***

PC corrections to the mass operator

Low-laying collective states are mainly surface

$$g_L(\omega) = \mathcal{F}A(\omega)g_L(\omega),$$

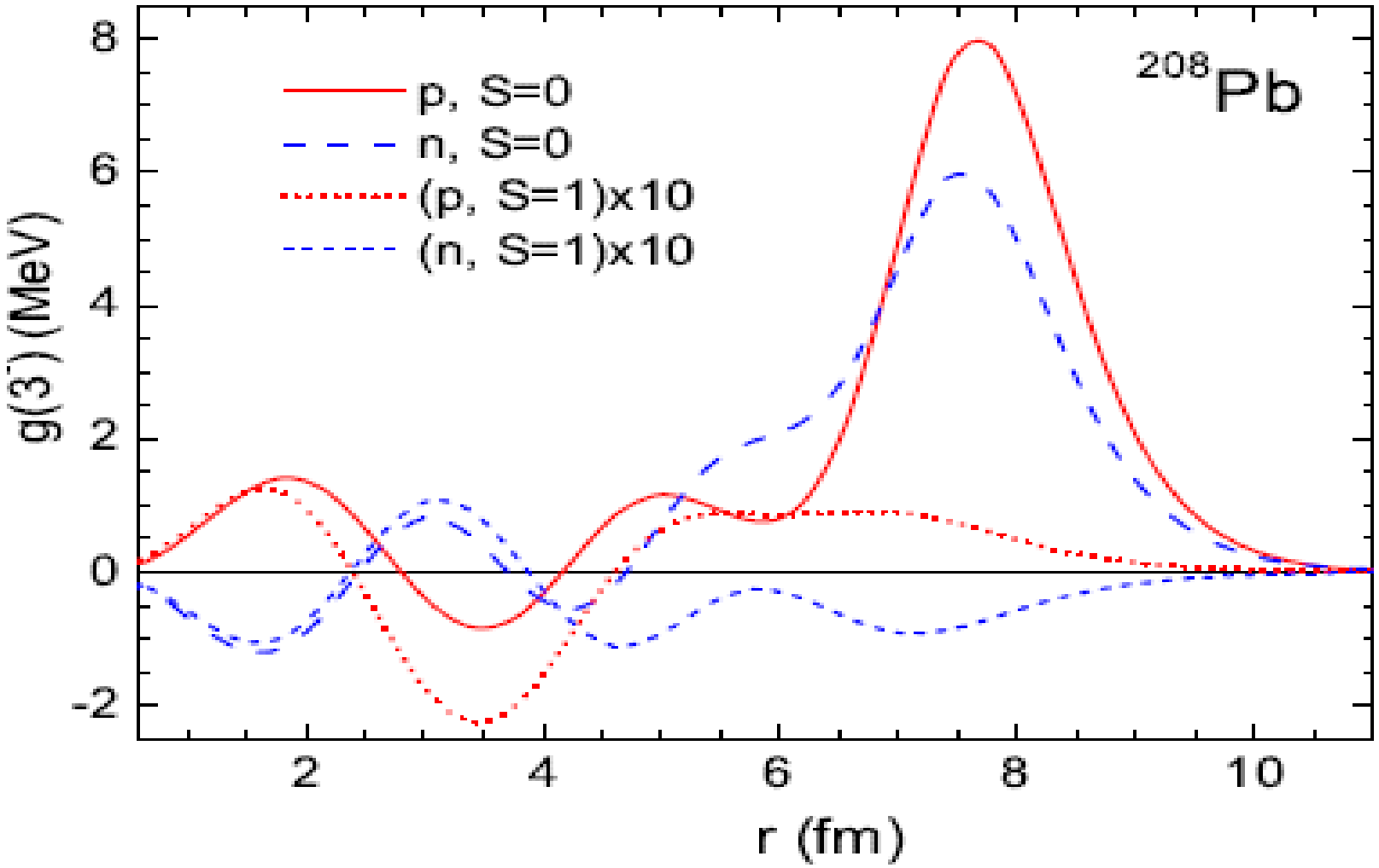
$$\delta\Sigma^{\text{tad}} = \int \frac{d\omega}{2\pi i}\delta_L g_L D_L(\omega),$$

$$\delta_L g_L = \delta_L \mathcal{F}A(\omega_L)g_L + \mathcal{F}\delta_L A(\omega_L)g_L + \mathcal{F}A(\omega_L)\delta_L g_L.$$

$$g_L(r) = \alpha_L \frac{dU}{dr} + \chi_L(r), \qquad \chi_L(r) \quad \textbf{\textit{is small}}$$

$$\delta\Sigma_L^{\text{tad}} = \frac{\alpha_L^2}{2}\frac{2L+1}{3}\Delta U(r).$$

3- in 208 Pb



Pole and tadpole contributions to proton SPEs in 208 Pb

λ	$\delta\varepsilon_\lambda^{\text{pole}}$	$\delta\varepsilon_\lambda^{\text{tad}}$	$\delta\varepsilon_\lambda$
$3p_{1/2}$	-0.375	0.153	-0.222
$3p_{3/2}$	-0.371	0.152	-0.219
$2f_{5/2}$	-0.278	0.168	-0.110
$1i_{13/2}$	-0.534	0.266	-0.268
$2f_{7/2}$	-0.409	0.168	-0.240
$1h_{9/2}$	-0.054	0.222	0.168
$3s_{1/2}$	-0.310	0.143	-0.167
$2d_{3/2}$	-0.241	0.146	-0.095
$1h_{11/2}$	-0.017	0.246	0.229
$2d_{5/2}$	0.435	0.147	0.582
$1g_{7/2}$	-0.271	0.197	-0.074

The tadpole term is always positive,

The pole one, as a rule, negative.

Account for the pole term alone overestimates the PC correction to SPEs significantly.

For DF3-a EDF, in 208 Pb,

$\langle \delta\bar{\varepsilon}_\lambda \rangle_{\rm rms}$ = 0.51 MeV without PC,

= 0.38 MeV with PC

[RMF+PC = 0.85 MeV]

E. Litvinova and P. Ring, Phys. Rev. C 73, 044328 (2006).

Recent calculations of PC corrections to SPEs of magic nuclei with Skyrme EDFs

1. L.-G. Cao, G. Coló, H. Sagawa, and P. F. Bortignon,

Phys. Rev. C **89**, 044314 (2014).

2. D. Tarpanov, J. Dobaczewski, J. Toivanen, and B. G. Carlsson, Phys. Rev. Lett. **113**, 252501 (2014).

In the second one, the SPEs were examined with a family of Skyrme EDFs with various m^* values, firstly, without PC and then + PC

Conclusions: without PC, the EDFs with $m^* \approx m$

are preferable, in accordance with our results, BUT

inclusion of PC corrections spoils the agreement.

I believe, it happens because of omitting the tadpole term.

PC corrections to charge radii

Recently, in

R. F. Garcia Ruiz, et al. (Collab.), Nature Phys. **12**, 594 (2016),

a puzzle was announced of ‘anomanolously big’ charge radii of heavy Ca isotopes. We explain it with the PC effect.

In [Nucl. Phys. A **676**, 49 (2000)] , Fayans et al. explained rather fancy (crown-like) behavior of R_{ch} in the Ca (40 – 48) chain

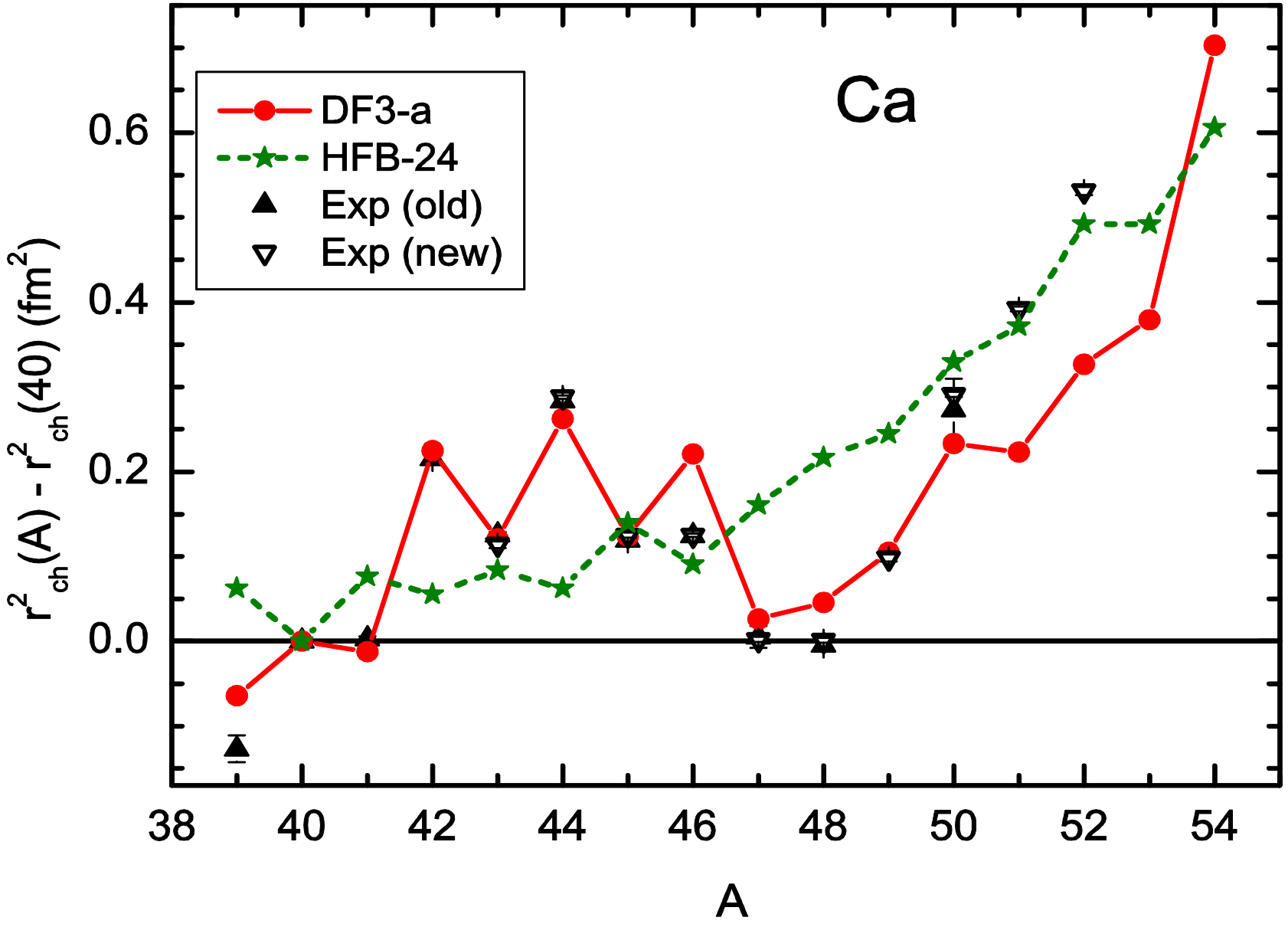
without PC, Fayans tried to find the best EDF without phonons.

EDF DF3: $R_{ch}(40Ca) \approx R_{ch}(48Ca)$

For a strong odd-even effect, the gradient term in paring force was added

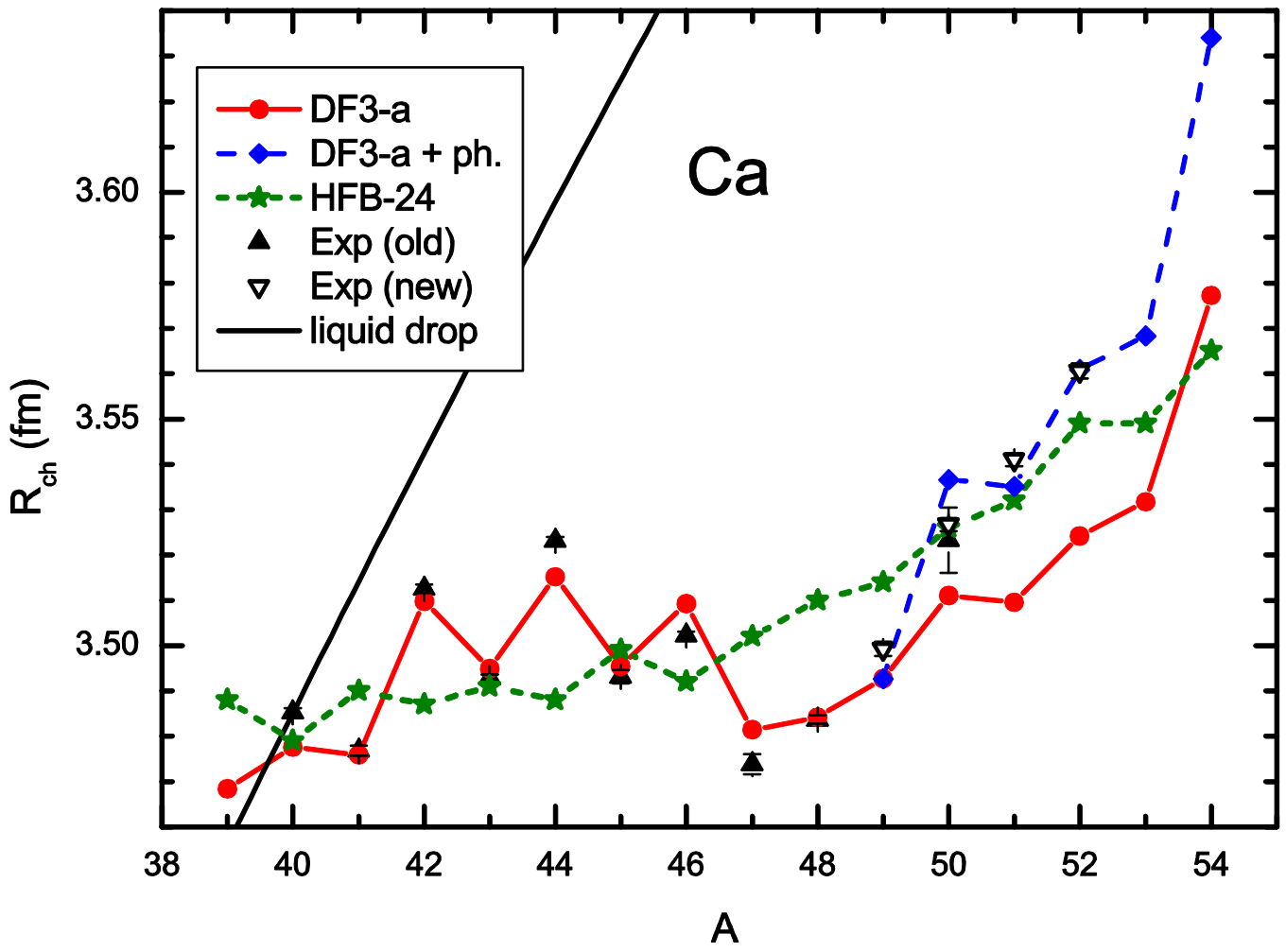
$$\mathcal{F}^{\xi} = C_0 f^{\xi} = C_0 \left(f_{ex}^{\xi} + h^{\xi} x^{2/3} + f_{\nabla}^{\xi} r_0^2 (\nabla x)^2 \right).$$

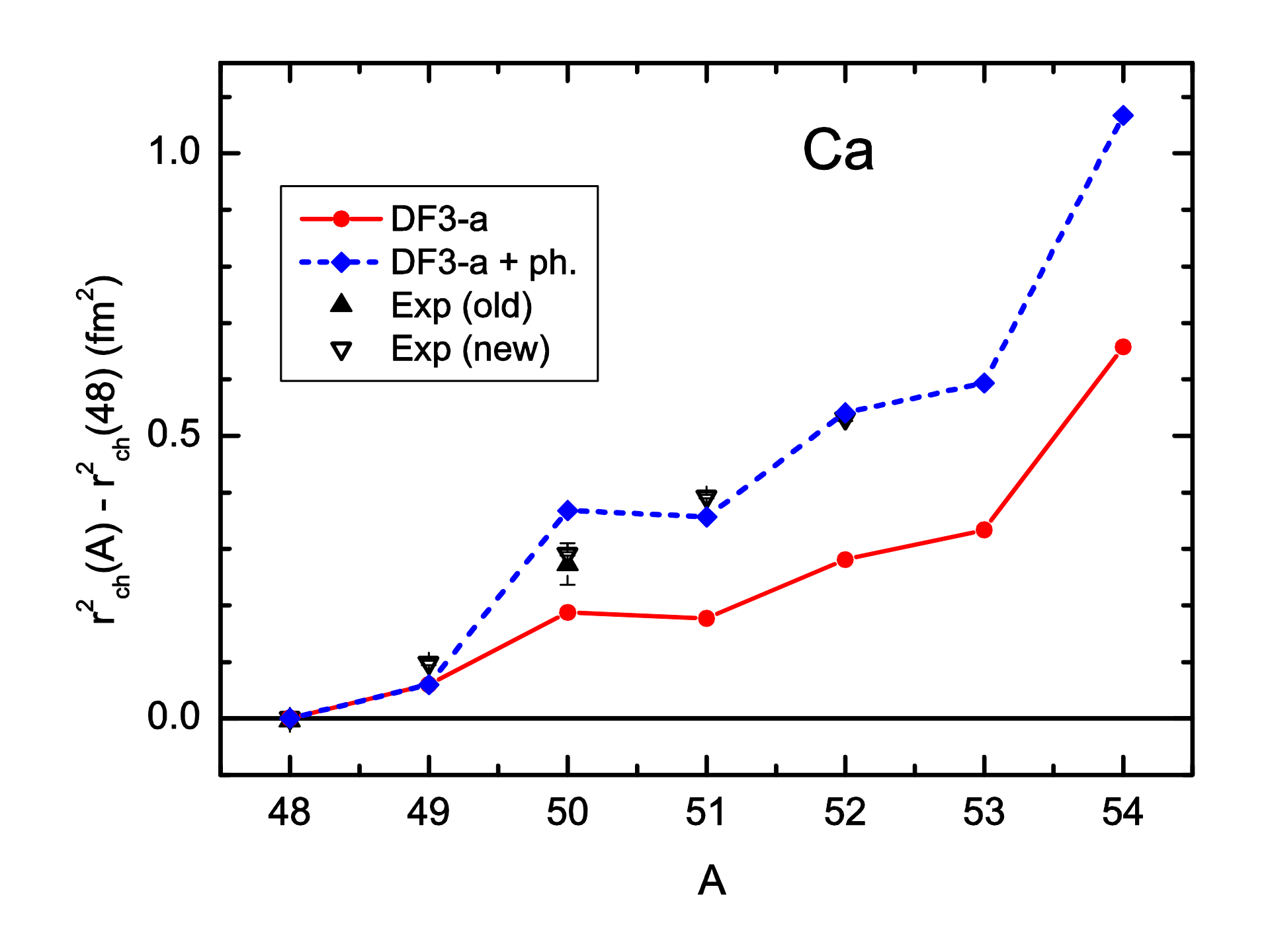
:



JETP Lett., 104, 218 (2016).

$$\delta\langle r_{\rm ch}^2\rangle(48,A) \equiv \langle r_{\rm ch}^2\rangle(^{A}{\rm Ca}) - \langle r_{\rm ch}^2\rangle(^{48}{\rm Ca}),$$




Ca
DF3-a
DF3-a + ph.
Exp (old)
Exp (new)
$r^2_{ch}(A) - r^2_{ch}(48)$ (fm^2)
1.0
0.5
0.0
48
49
50
51
52
53
54
A

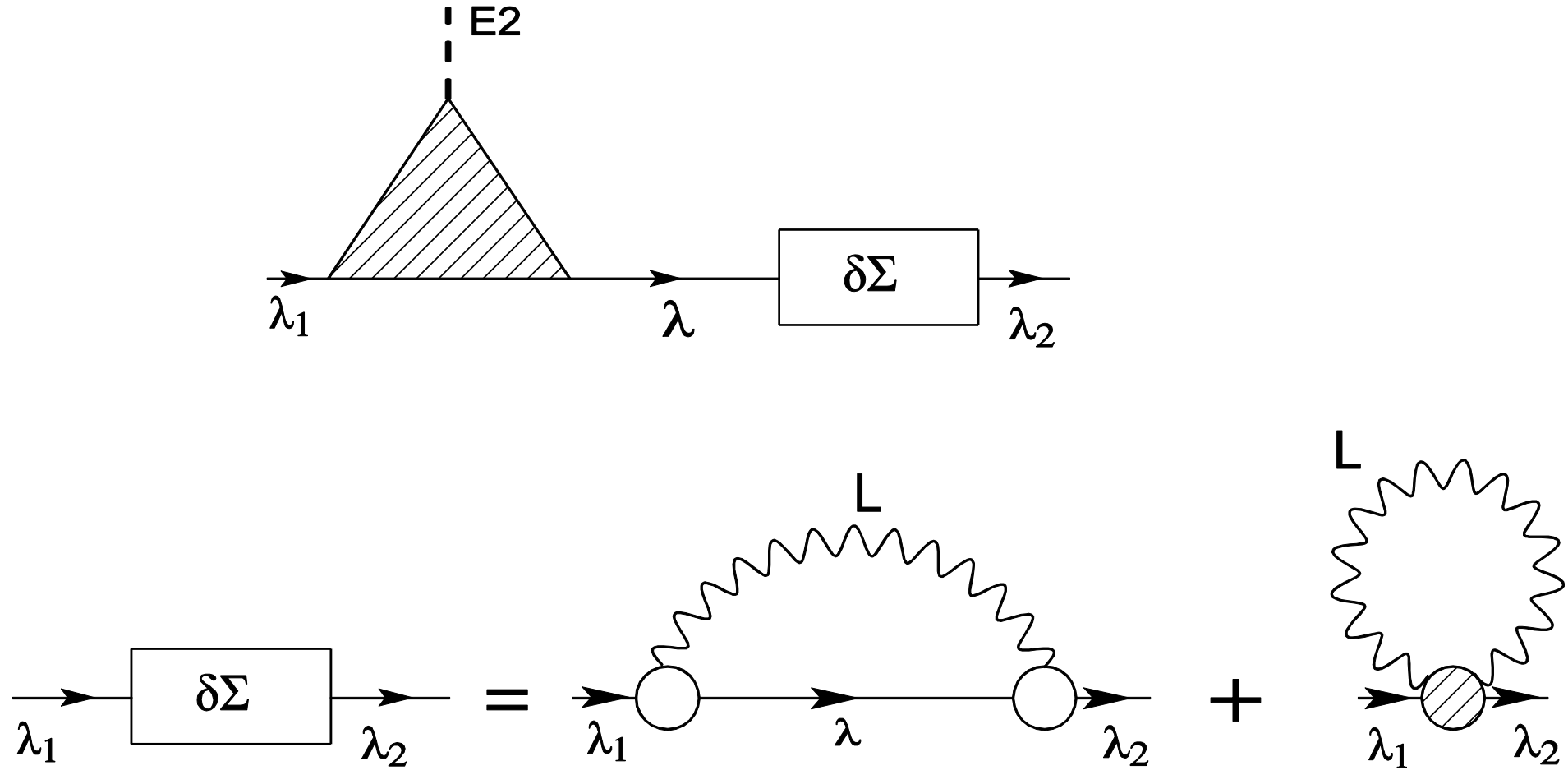
PC corrections to electromagnetic moments of odd semi-magic nuclei

Quadrupole moments of proton-odd neighbors of even Sn isotopes

$$Q_\lambda = (\lambda|V|\lambda)_{m=j},$$

$$V = e_q V_0 + \mathcal{F} A(\omega = 0) V,$$

PC corrections to different (not all) elements of these formulas which behave in a non-regular way. Two main terms (due to the Z-factors and the induced interaction) are of opposite signs and cancel each other significantly leaving some room for ‘small’ corrections.



End corrections

The main ‘end correction’ is due to the diagonal term with $\lambda = \lambda_2$

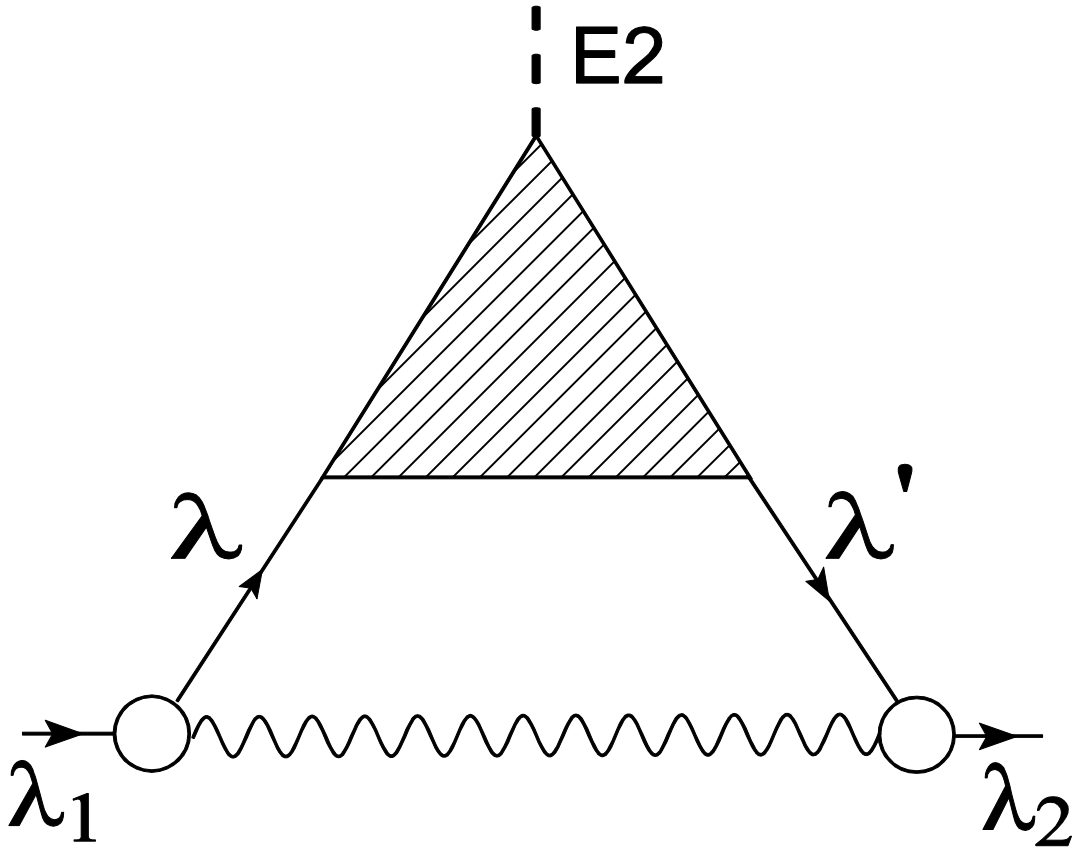
It results in a ‘renormalization’ of the ends: $|\lambda\rangle \to \sqrt{Z_\lambda}|\lambda\rangle$, where

$$Z_\lambda = \left(1 - \left.\frac{\partial \delta\Sigma_{\lambda\lambda}(\varepsilon)}{\partial\varepsilon}\right|_{\varepsilon=\varepsilon_\lambda}\right)^{-1}$$

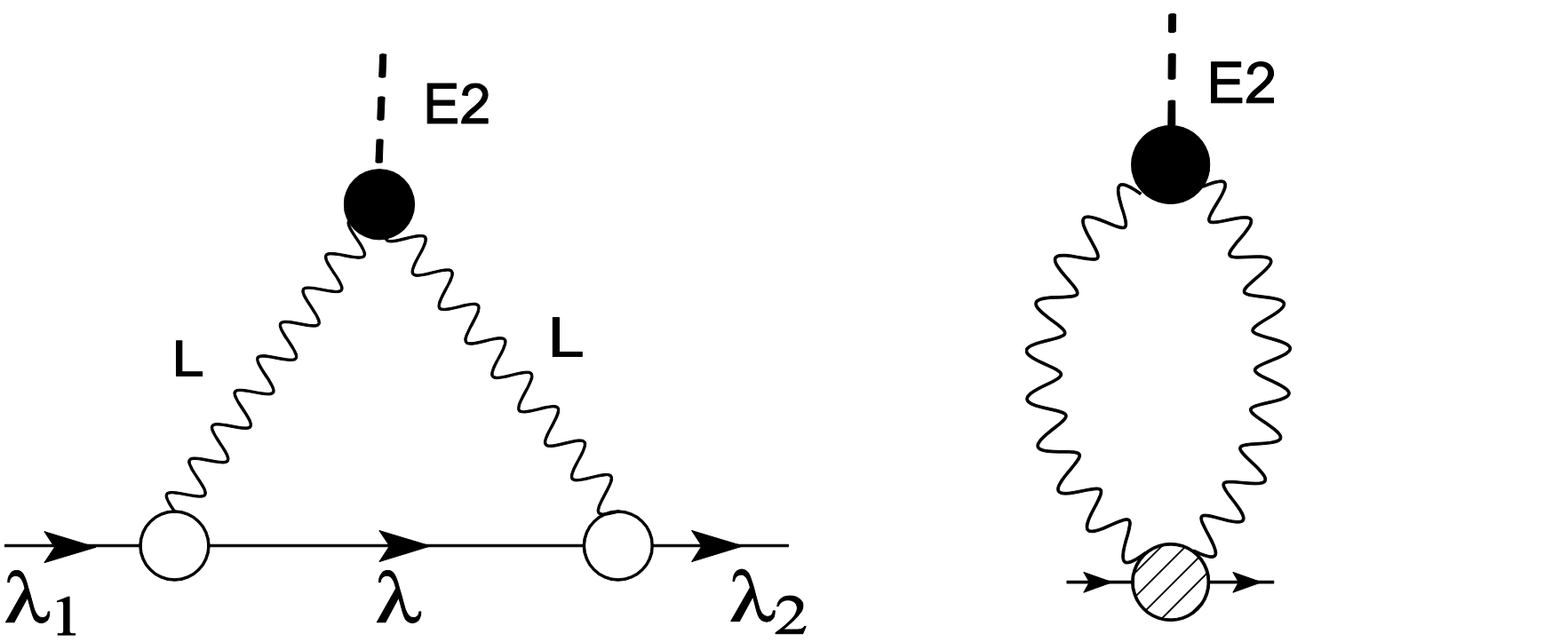
The non-diagonal term, the sum over $\lambda \neq \lambda_2$

denoted as $\delta V'_{\mathrm{end}}$

is rather small but sometimes is also important.



Phonon-induced interaction (GGD triangle)



Diagrams of the phonon quadrupole moment, pole (GDD-triangle) and tadpole

Final formula for the PC corrected matrix element:

$$\tilde{V}_{\lambda\lambda}=Z_\lambda \left(V+\delta V_{GGD}+\delta V^{(1)}_{GDD}+\delta V'_{\text{end}}\right)_{\lambda\lambda}.$$

$$\delta V_{GDD}=\delta V^{(1)}_{GDD}+\delta V^{(2)}_{GDD},$$

two terms with different behavior at $\omega_L \to 0.$

regular and singular: $1/\omega_L$

The tadpole term possesses the same singularity (of opposite sign). If the tadpole 'blob' is a constant, [the model of EPL, 103, 42001 (2013)] these two singular terms cancel each other exactly.

p-odd neighbors of even Sn isotopes (In and Sb), Q in b

nucl.	λ	Q	Z	$\delta Q^{Z}_{\rm ptb}$	δQ_{GGD}	δQ_{GDD}	$\delta Q'_{\rm end}$	$\delta Q_{\rm ptb}$	$\delta Q_{\rm ph}$
^{105}In	$1g_{9/2}$	+0.833	0.675	-0.400	0.231	0.055	0.014	-0.100	-0.067
^{107}In	$1g_{9/2}$	+0.976	0.584	-0.692	0.404	0.094	0.021	-0.172	-0.100
^{109}In	$1g_{9/2}$	+1.113	0.573	-0.826	0.487	0.128	0.023	-0.188	-0.108
^{111}In	$1g_{9/2}$	+1.165	0.488	-1.220	0.722	0.163	0.034	-0.301	-0.147
^{113}In	$1g_{9/2}$	+1.117	0.576	-0.820	0.484	0.071	0.025	-0.240	-0.138

113 In : phonon corrections

$$\delta Q(Z) = -0.840b,\ \delta Q(ind) = 0.484b,\ \delta Q(tot) = -0.138b$$

In the final table,

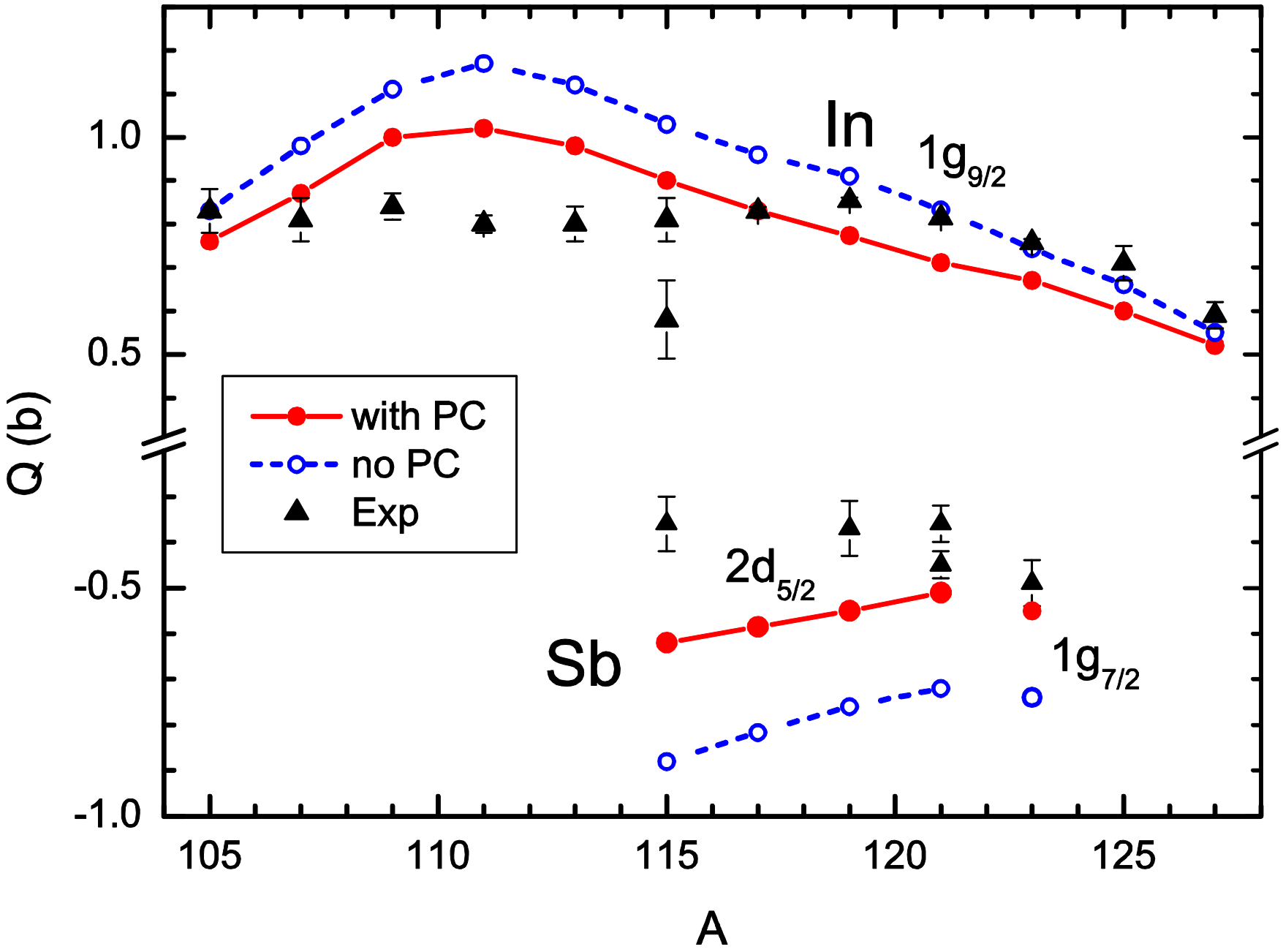
$$\delta Q = Q(no\ ph) - Q(\exp),$$

$$\delta \tilde{Q} = Q(with\ ph) - Q(\exp)$$

$$< \delta Q >_{\rm rms} = 0.27\ {\rm b} \qquad < \delta \tilde{Q} >_{\rm rms} = 0.15\ {\rm b}$$

Results:

nucl.	λ	Q_{exp}	Q_0	Q_{th}	$\tilde{Q}_{\mathrm{th}}$	δQ	$\delta\tilde{Q}$
^{105}In	$1g_{9/2}$	+0.83(5)	0.18	+0.83	0.76	0.00	-0.07
^{107}In	$1g_{9/2}$	+0.81(5)	0.18	+0.98	0.87	0.17	0.06
^{109}In	$1g_{9/2}$	+0.84(3)	0.18	+1.11	1.00	0.27	0.16
^{111}In	$1g_{9/2}$	+0.80(2)	0.19	+1.17	1.02	0.37	0.22
^{113}In	$1g_{9/2}$	+0.80(4)	0.19	+1.12	0.98	0.32	0.16
^{115}In	$1g_{9/2}$	+0.81(5)	0.19	+1.03	0.90	0.22	0.09
		0.58(9)				0.45	0.32
^{117}In	$1g_{9/2}$	+0.829(10)	0.19	+0.96	0.83	0.131	0.001
^{119}In	$1g_{9/2}$	+0.854(7)	0.19	+0.91	0.773	0.056	-0.081
^{121}In	$1g_{9/2}$	+0.814(11)	0.19	+0.833	0.711	0.019	-0.103
^{123}In	$1g_{9/2}$	+0.757(9)	0.19	+0.743	0.670	-0.014	-0.087
^{125}In	$1g_{9/2}$	+0.71(4)	0.19	+0.66	0.60	-0.05	-0.11
^{127}In	$1g_{9/2}$	+0.59(3)	0.19	+0.55	0.52	-0.04	-0.07
^{115}Sb	$2d_{5/2}$	-0.36(6)	-0.14	-0.88	-0.62	-0.52	-0.26
^{117}Sb	$2d_{5/2}$	-	-0.14	-0.817	-0.585	-	-
^{119}Sb	$2d_{5/2}$	-0.37(6)	-0.14	-0.76	-0.55	-0.39	-0.18
^{121}Sb	$2d_{5/2}$	-0.36(4)	-0.14	-0.72	-0.51	-0.36	-0.15
		-0.45(3)				-0.27	-0.06
^{123}Sb	$1g_{7/2}$	-0.49(5)	-0.17	-0.74	-0.55	-0.25	-0.06


1.0
0.5
-0.5
-1.0
Q (b)
105
110
115
120
125
A
In
1g9/2
Sb
2d5/2
1g7/2
with PC
no PC
Exp

Conclusions

For the Fayans EDF DF3-a (m*=m, the ‘Fayans denominator’), account for the PC corrections makes agreement with experiment better (in fact, with a record accuracy) for

1. SPEs of magic nuclei,
2. Quadrupole moments of odd semi-magic nuclei.
3. Charge radii of Ca isotopes.

In the first two cases,

The tadpole term plays a crucial role.

Thank you