Multiferroic skyrmions

Maxim Mostovoy

University of Groningen
Zernike Institute
for Advanced Materials

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Outline

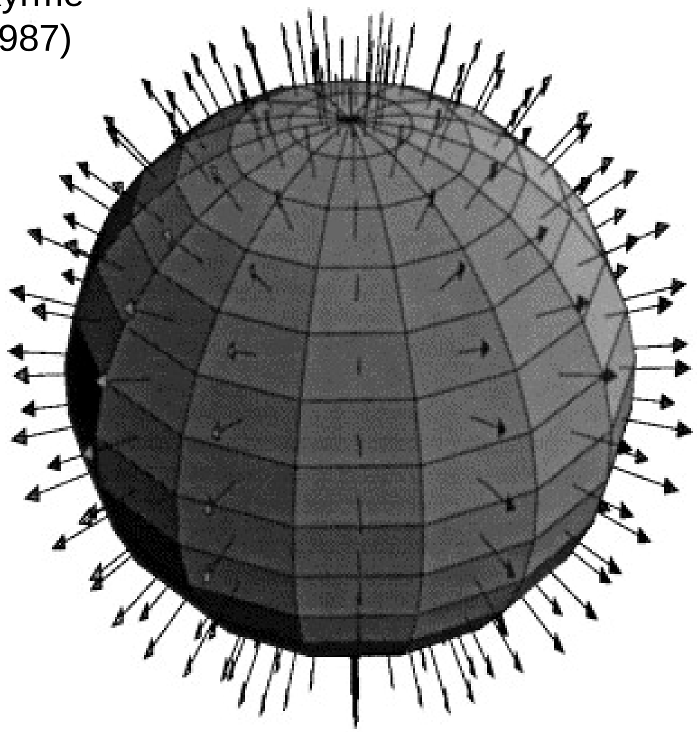
- **Skyrmions in nuclear physics**
- **Skyrmions in chiral and frustrated magnets**
- **Topology and magnetoelectricity**
- **Topological edge states and skyrmion exchange in nanostructures**

A. Leonov & MM, Nature Commun. ***6****, 8275 (2015)*
A. Leonov & MM, Nature Commun. ***8****, 14394 (2017)*
Y. Kharkov, O.P. Sushkov & MM (unpublished)

Tony Skyrme
(1922-1987)

Skyrmion

T. H. R. Skyrme, A Nonlinear field theory,
*Proc. Roy. Soc. London A **260**, 127 (1961)*



$$\pi_1^2 + \pi_2^2 + \pi_3^2 + \sigma^2 = 1$$

$$S_3 \to S_3$$

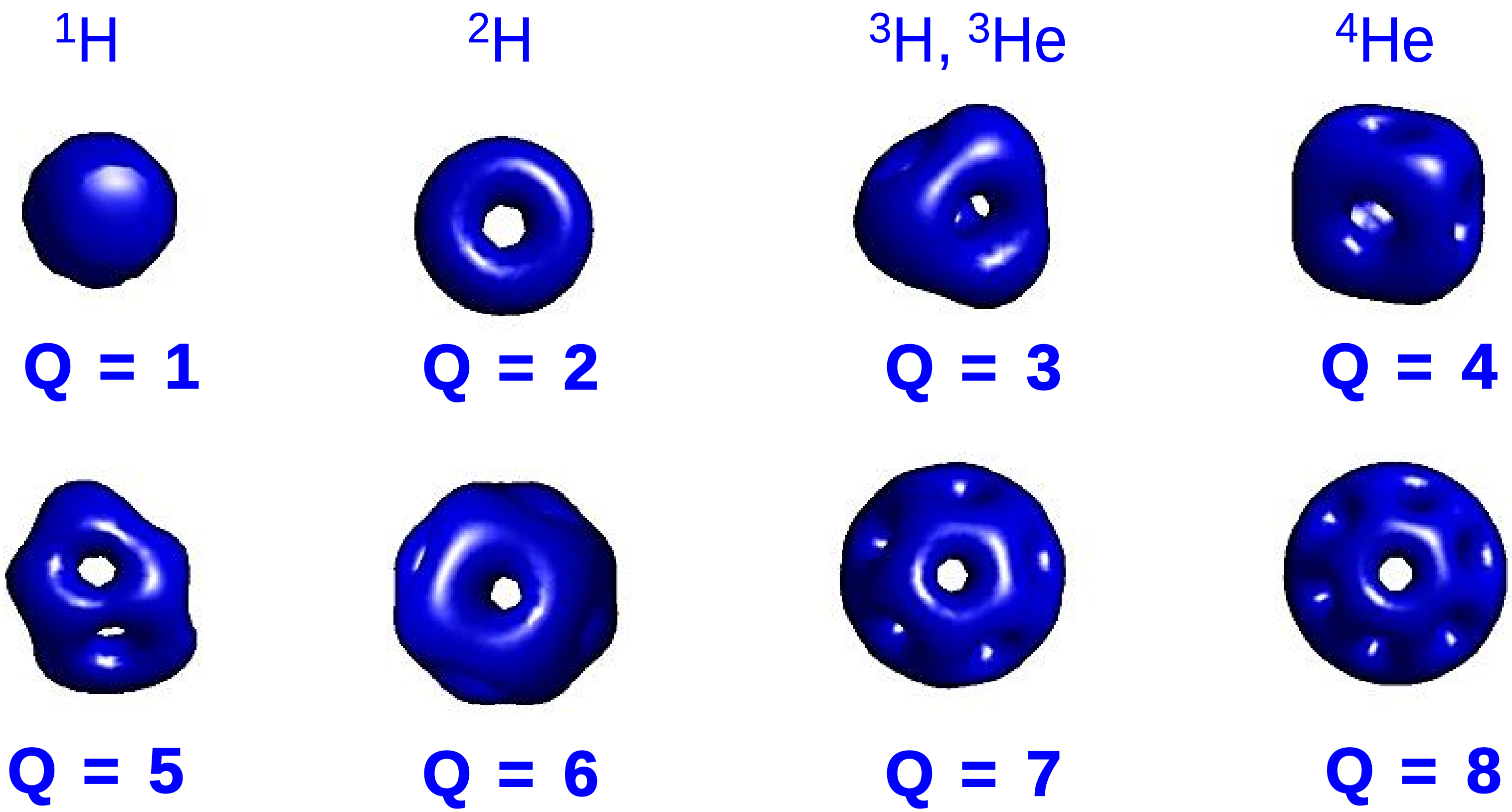
Topological charge = Baryon number

Skyrme model

$$E = \int d^3x \left[\frac{F_\pi^2}{8} \sum_{\alpha,i} (\partial_i n^\alpha)^2 + \frac{1}{4e^2} \sum_{\alpha,\beta,i,j} (\partial_i n^\alpha \partial_j n^\beta - \partial_i n^\beta \partial_j n^\alpha)^2 \right]$$

$$n = (\pi_1, \pi_2, \pi_3, \sigma) \qquad n^\alpha n_\alpha = 1$$

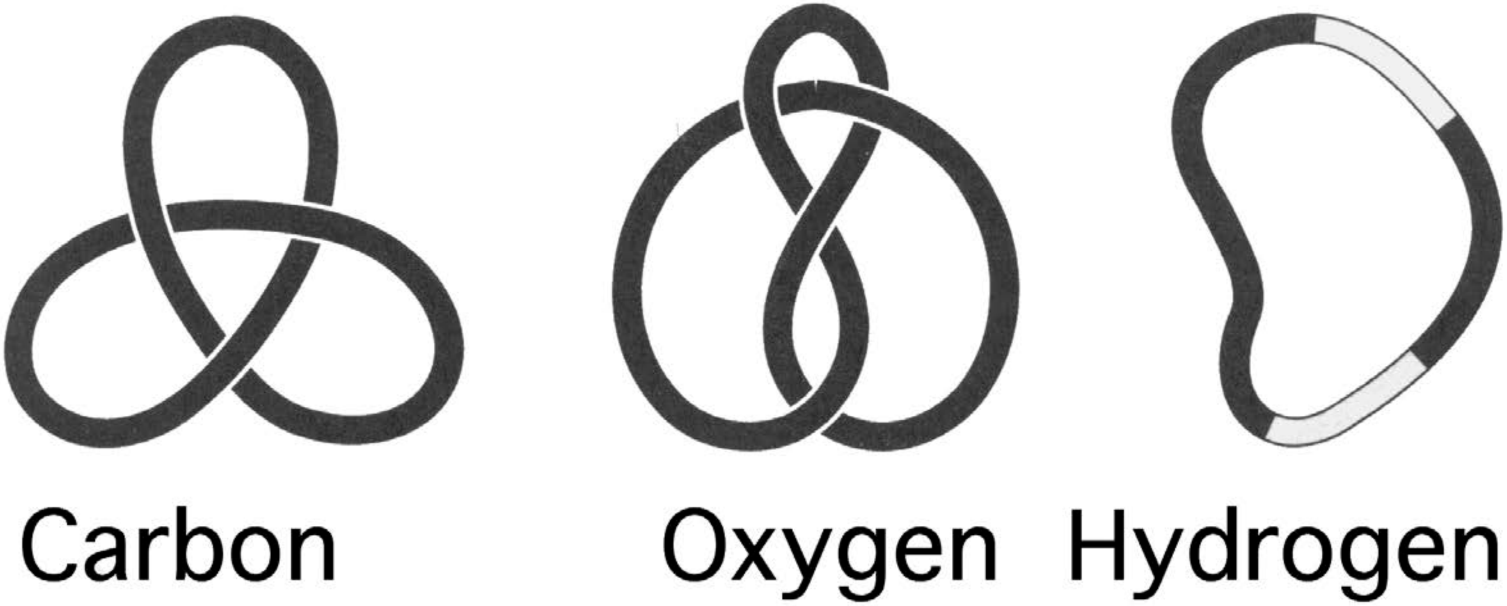
Nuclei as multi-Q skyrmions



*R.A. Battye & P.M. Sutcliffe, PRL **79** 363 (1997)*

Atoms as vortices

Lord Kelvin
(1824-1907)



Nuclear matter as a skyrmion crystal

Simple cubic crystal of skyrmions

I. Klebanov, Nucl. Phys. ***B262****, 133 (1985)*

Crystal of half-skyrmions has a lower energy

E. Wüst, G.E. Brown & A.D. Jackson, Nucl. Phys. ***A468****, 450 (1985)*

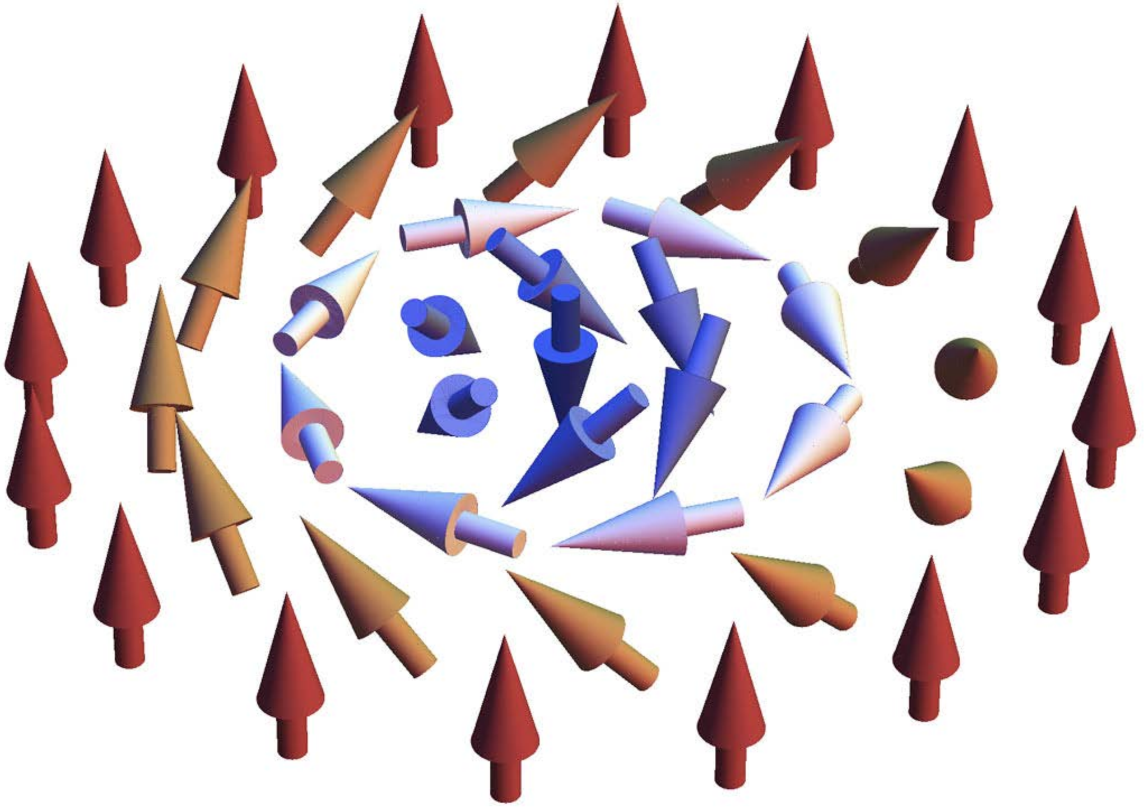
A.S. Goldhaber & N. Manton, Phys. Lett. B ***198****, 231 (1987)*

M. Kugler & S. Shtrikman, Phys. Rev. D ***40****, 3421 (1989)*

Skyrmions in chiral magnets

Magnetic skyrmion

$$n_x^2 + n_y^2 + n_z^2 = 1$$

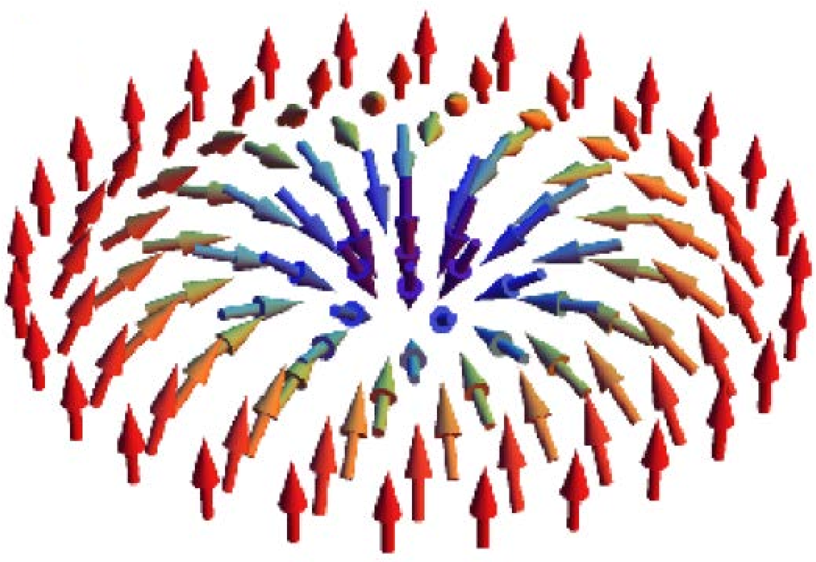


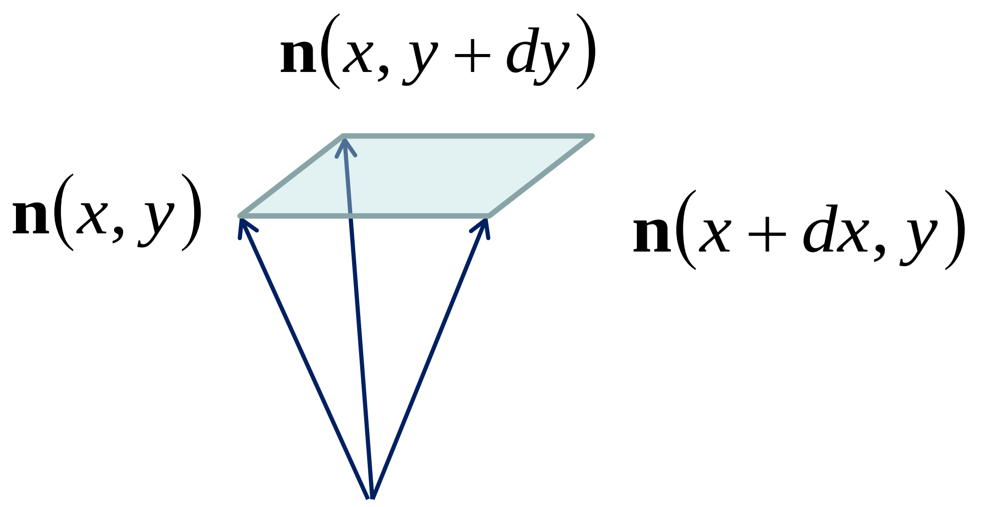
$$S_2 \to S_2$$

Topological charge

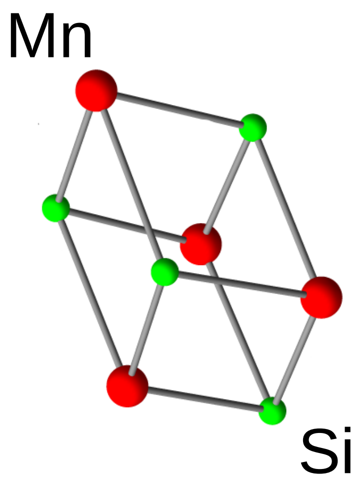
Pontryagin number:

$$Q = \frac{1}{4\pi}\int d^2x\left(\mathbf{n}\cdot\left[\partial_x\mathbf{n}\times\partial_y\mathbf{n}\right]\right) = \frac{1}{4\pi}\int d\Omega = \text{integer }\#$$

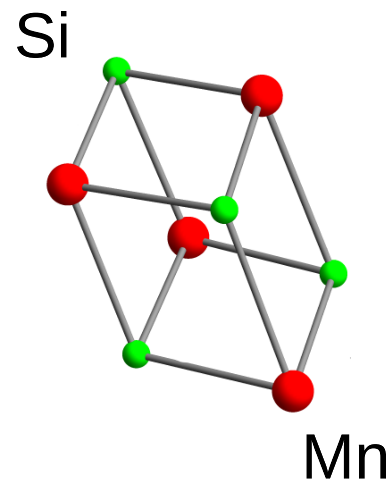




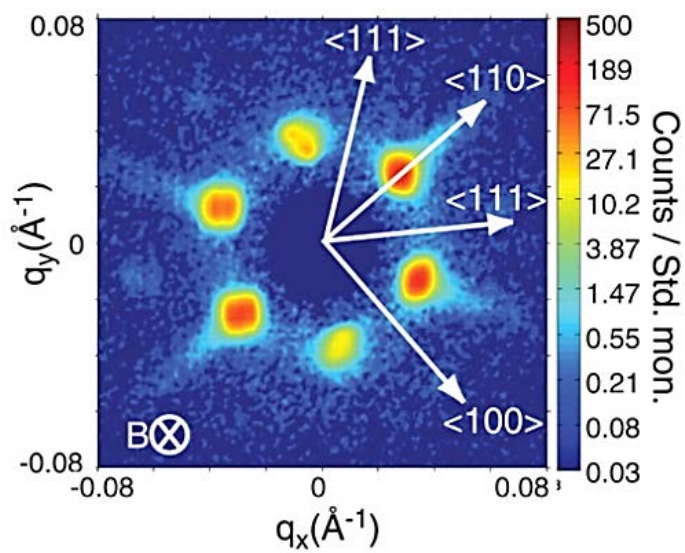
Skyrmions in chiral magnets



$$\mathbf{x} \to -\mathbf{x}$$

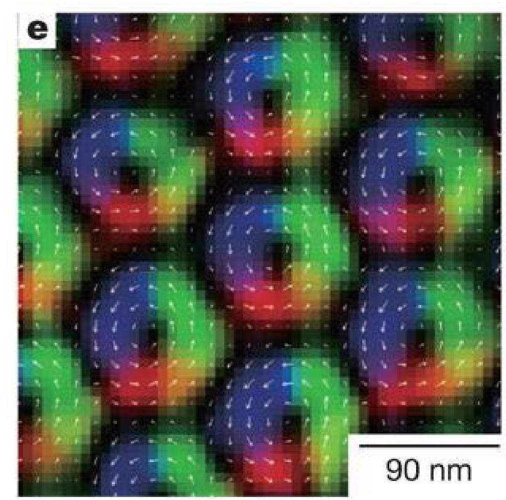


Neutron scattering



Mühlbauer et al, Science (2009)

Lorentz microscopy



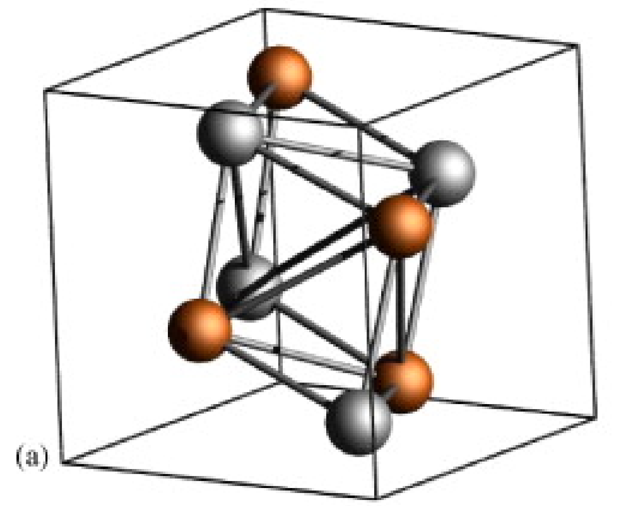
X. Yu et al. Nature (2010)

Non-centrosymmetric magnets

MnSi, $Fe_{1-x}Co_xSi$, FeGe
$CuOSeO_3$

Cubic B20 structure

Class 32



$$f = \frac{J}{2}(\nabla \mathbf{n})^2 + D\,\mathbf{n}\cdot[\nabla \times \mathbf{n}]$$

Exchange energy

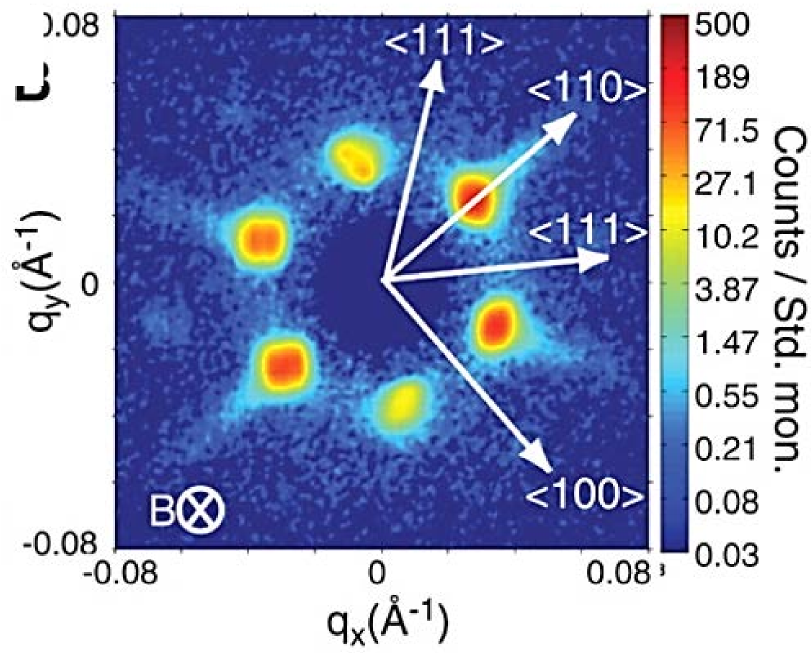
DM energy

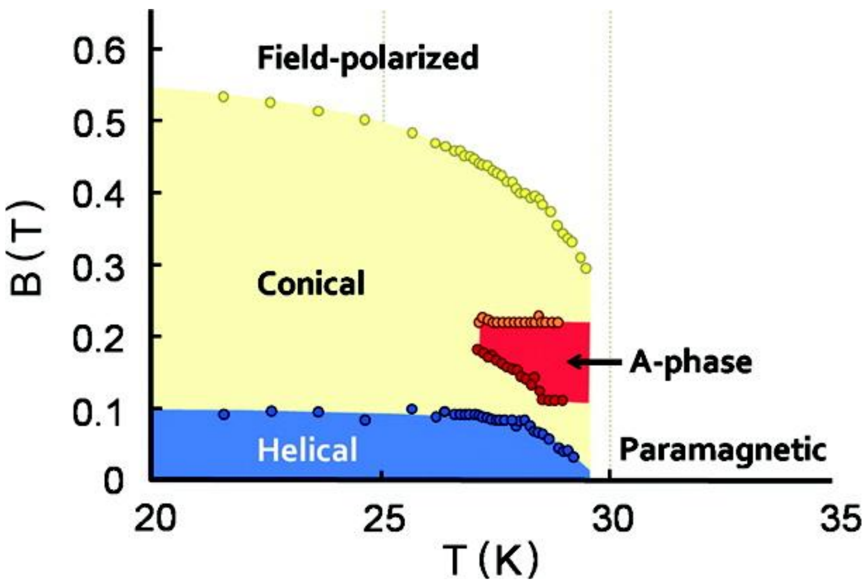
helical spiral wave vector

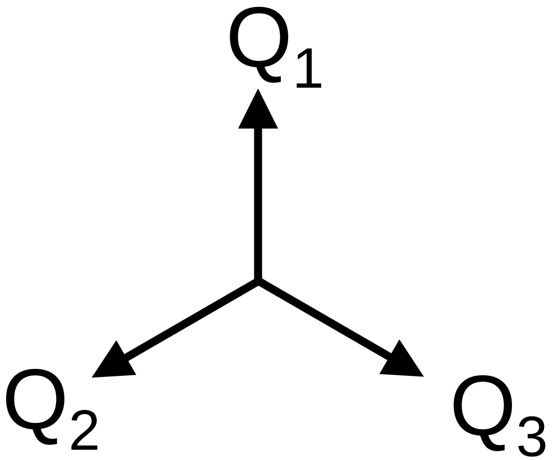
$$q = D / J$$

3Q-state

SANS on MnSi







Mühlbauer et al, Science (2009)

Physical meaning of topological charge

Adiabatic motion of spin-polarized electrons

$$i\hbar\frac{\partial\psi}{\partial t} = \left(-\frac{\hbar^2}{2m}\triangle - J\boldsymbol{\sigma}\cdot\boldsymbol{n}\right)\psi \qquad \boldsymbol{n} = \boldsymbol{n}(\boldsymbol{x}, t)$$

Adiabatic projection

$$|\psi\rangle = \chi|\boldsymbol{n}\rangle \qquad \boldsymbol{\sigma}\cdot\boldsymbol{n}|\boldsymbol{n}\rangle = |\boldsymbol{n}\rangle$$

$$i\hbar\frac{\partial\chi}{\partial t} = \left(-ea_0 + \frac{1}{2m}\left(\frac{\hbar}{i}\boldsymbol{\nabla} + \frac{e}{c}\boldsymbol{a}\right)^2 - J + \frac{\hbar^2}{8m}\partial_i\boldsymbol{n}\cdot\partial_i\boldsymbol{n}\right)\chi$$

Effective gauge potentials

$$a_0 = \frac{i\hbar}{e}\langle\boldsymbol{n}|\frac{\partial}{\partial t}|\boldsymbol{n}\rangle \qquad \boldsymbol{a} = -\frac{i\hbar c}{e}\langle\boldsymbol{n}|\boldsymbol{\nabla}|\boldsymbol{n}\rangle$$

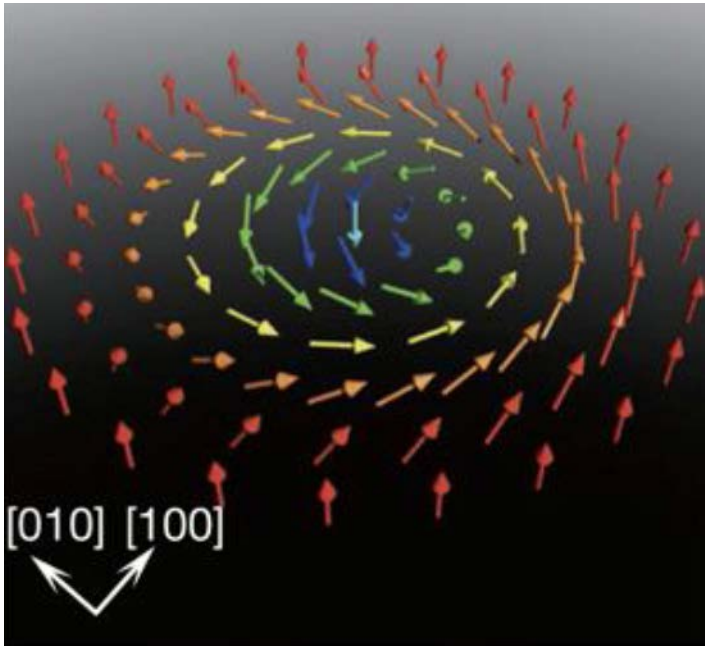
Effective gauge fields

$$\begin{cases} e_i = -\partial_i a_0 - \frac{1}{c}\partial_t a_i = \frac{\hbar}{2e}\left(\mathbf{n}\cdot\left[\partial_i \mathbf{n}\times\partial_t \mathbf{n}\right]\right) \\ h_i = \left[\nabla\times\mathbf{a}\right]_i = \frac{\hbar c}{4e}\varepsilon_{ijk}\left(\mathbf{n}\cdot\left[\partial_j \mathbf{n}\times\partial_k \mathbf{n}\right]\right) \end{cases}$$

Lorentz force acting on spin-polarized electron

$$\Delta\mathbf{F} = -e\,\mathbf{e} - \frac{e}{c}[\mathbf{v}\times\mathbf{h}]$$

Physical meaning of Skyrmion charge



Topological charge:

$$Q = \frac{1}{4\pi} \int d^2x \left(\mathbf{n} \cdot \left[\partial_x \mathbf{n} \times \partial_y \mathbf{n} \right] \right)$$

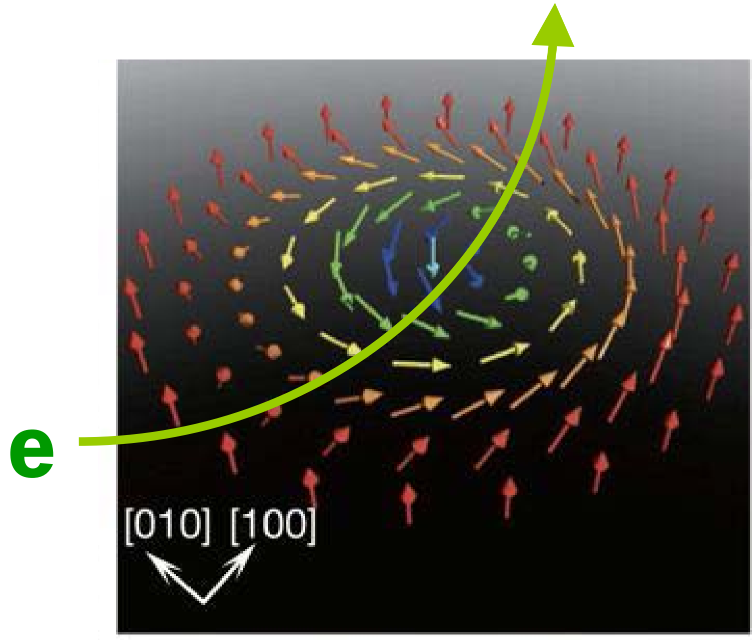
$$Q = \frac{e}{hc} \int d^2x h_z$$

Flux of internal magnetic field:

$$\Phi = \int d^2x h_z = Q\Phi_0 \qquad \Phi_0 = \frac{hc}{e}$$

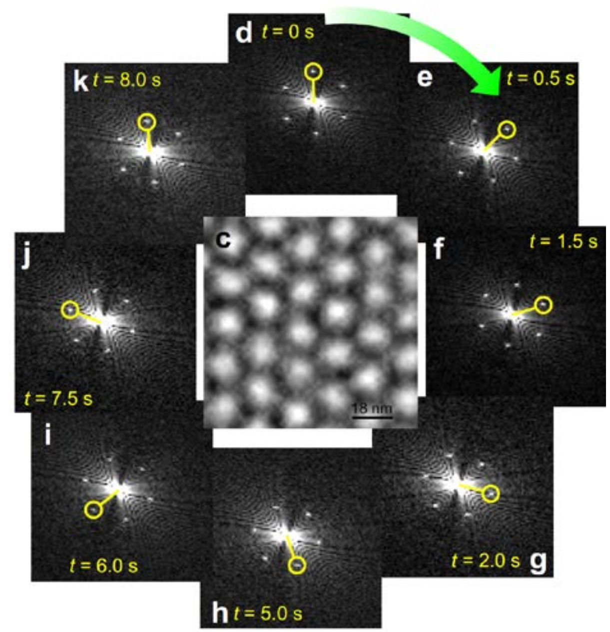
Emergent electromagnetism

Topological Hall effect



A. Neubauer et al PRL (2009)

Heat-induced rotation of skyrmion crystal



M. Mochizuki, MM et al Nature Materials (2014)

Electric field of moving skyrmion

$$\mathbf{E} = -\frac{1}{c}[\mathbf{V} \times \mathbf{H}]$$

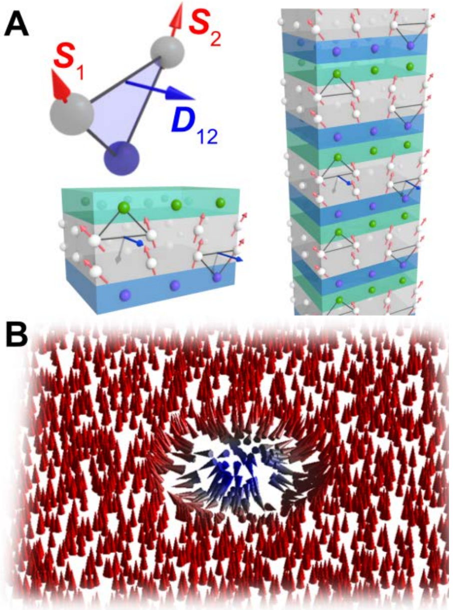
J. Zang, MM et al PRL (2011)

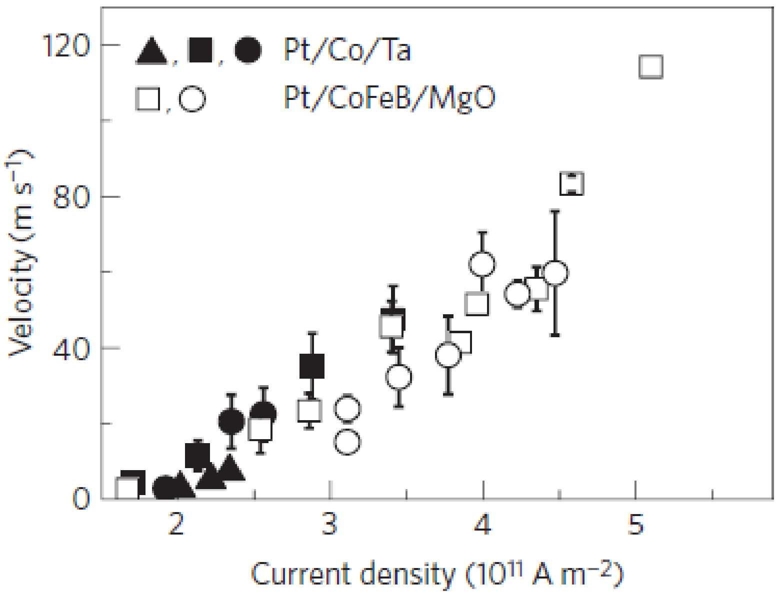
T. Schultz et al Nature Physics (2012)

Skyrmionics

- **Nanometer size**
- **Topological protection**
- **Low critical currents**

Albert Fert et al Nature Nano (2013)

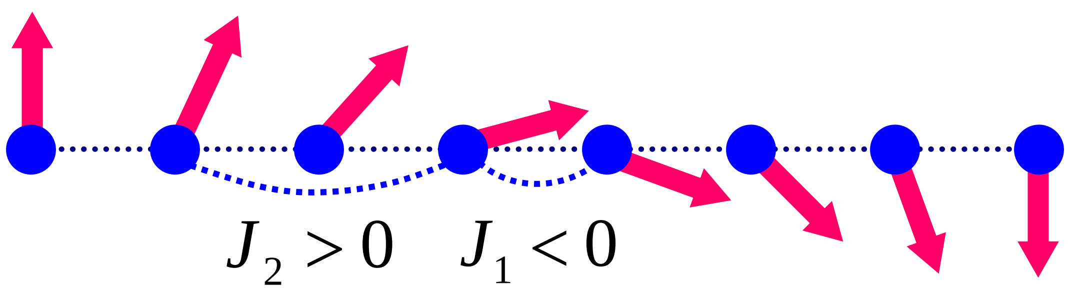




C. Moreau-Luchaire et al Nature Nano 11, 444 (2016)

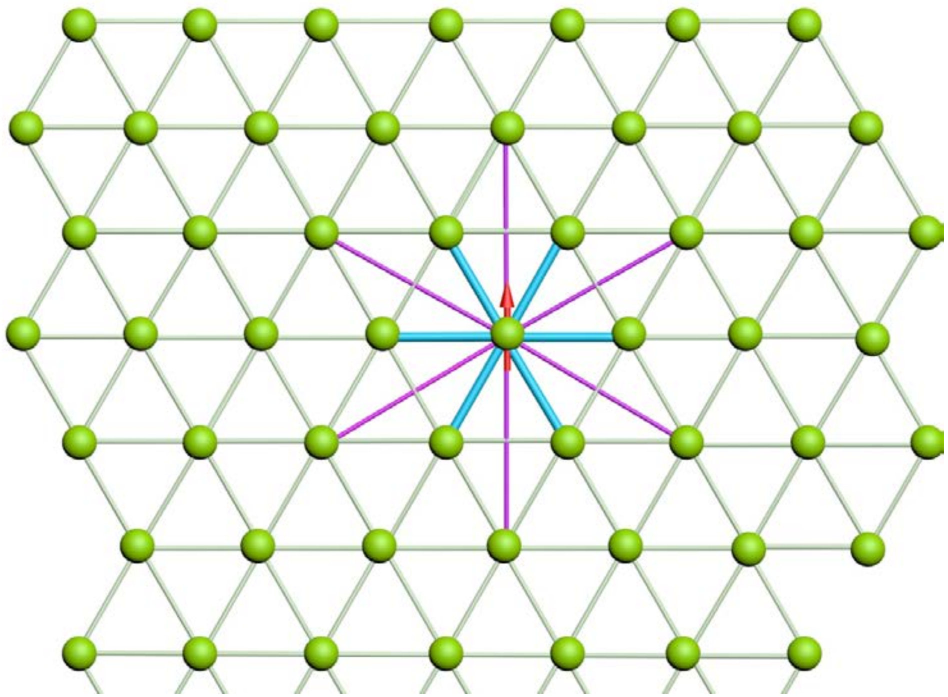
S. Woo et al, Nature Materials (2016)

Skyrmions in frustrated magnets



Frustrated triangular magnet with anisotropy

$$E = -J_1 \sum_{<i,j>} \boldsymbol{S}_i \boldsymbol{S}_j + J_2 \sum_{\ll i,j \gg} \boldsymbol{S}_i \boldsymbol{S}_j - h \sum_i S_{iz} - \frac{K}{2} \sum_i (S_{iz})^2$$



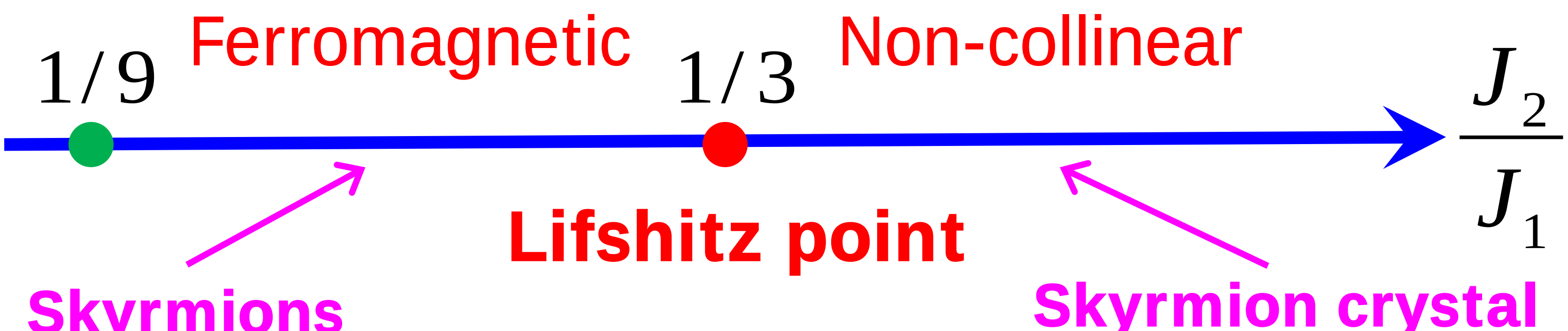
Continuum model

$$E = \frac{1}{2}\int d^2x\left(\rho\partial_i\mathbf{n}\cdot\partial_i\mathbf{n} + \mu\Delta\mathbf{n}\cdot\Delta\mathbf{n} - 2hn_z - Kn_z^2\right)$$

Spin stiffness

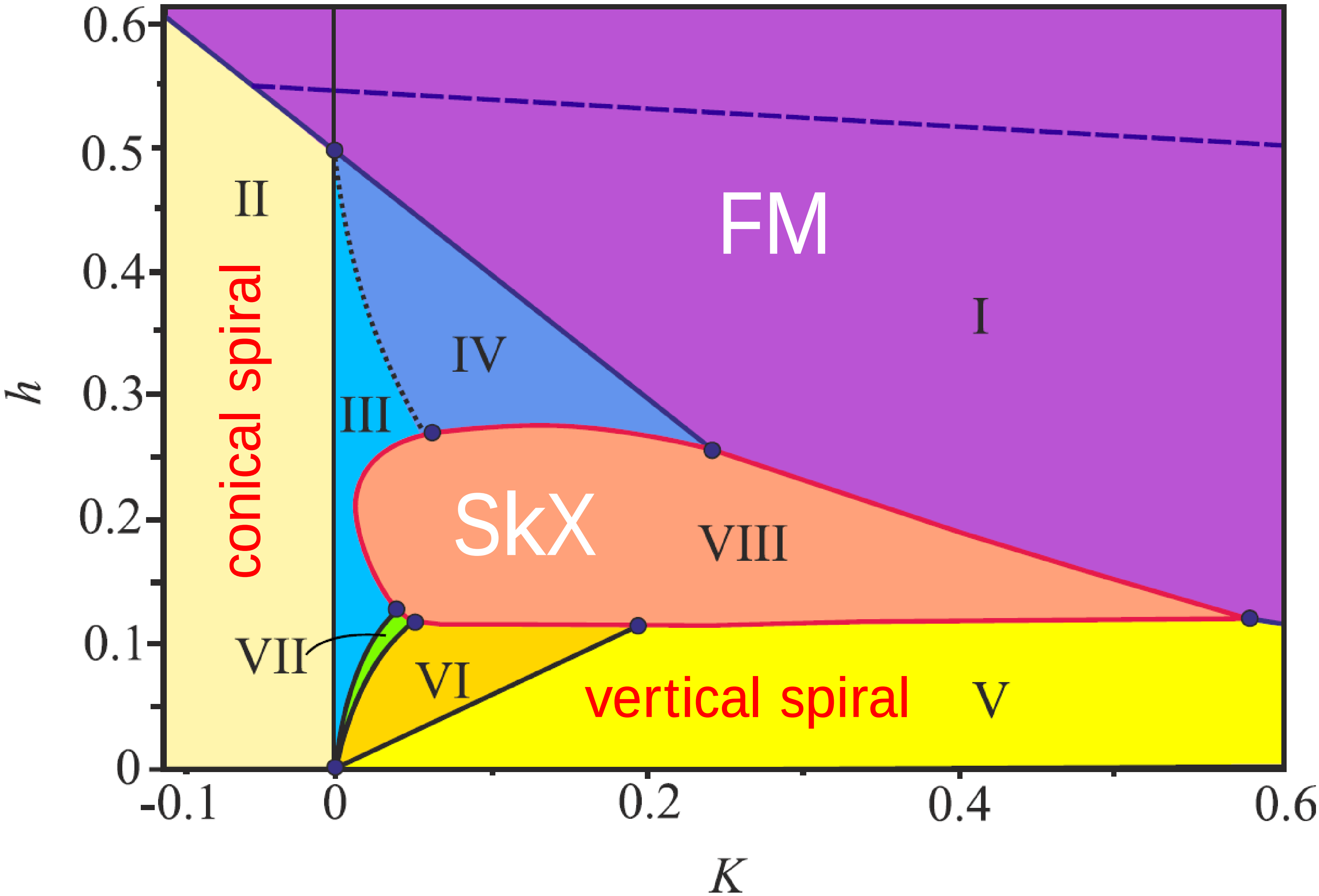
$$\rho = \frac{3}{2}\left(J_1 - 3J_2\right) \qquad \mu = \frac{3}{32}\left(9J_2 - J_1\right)$$

$$\rho > 0 \qquad \rho < 0$$

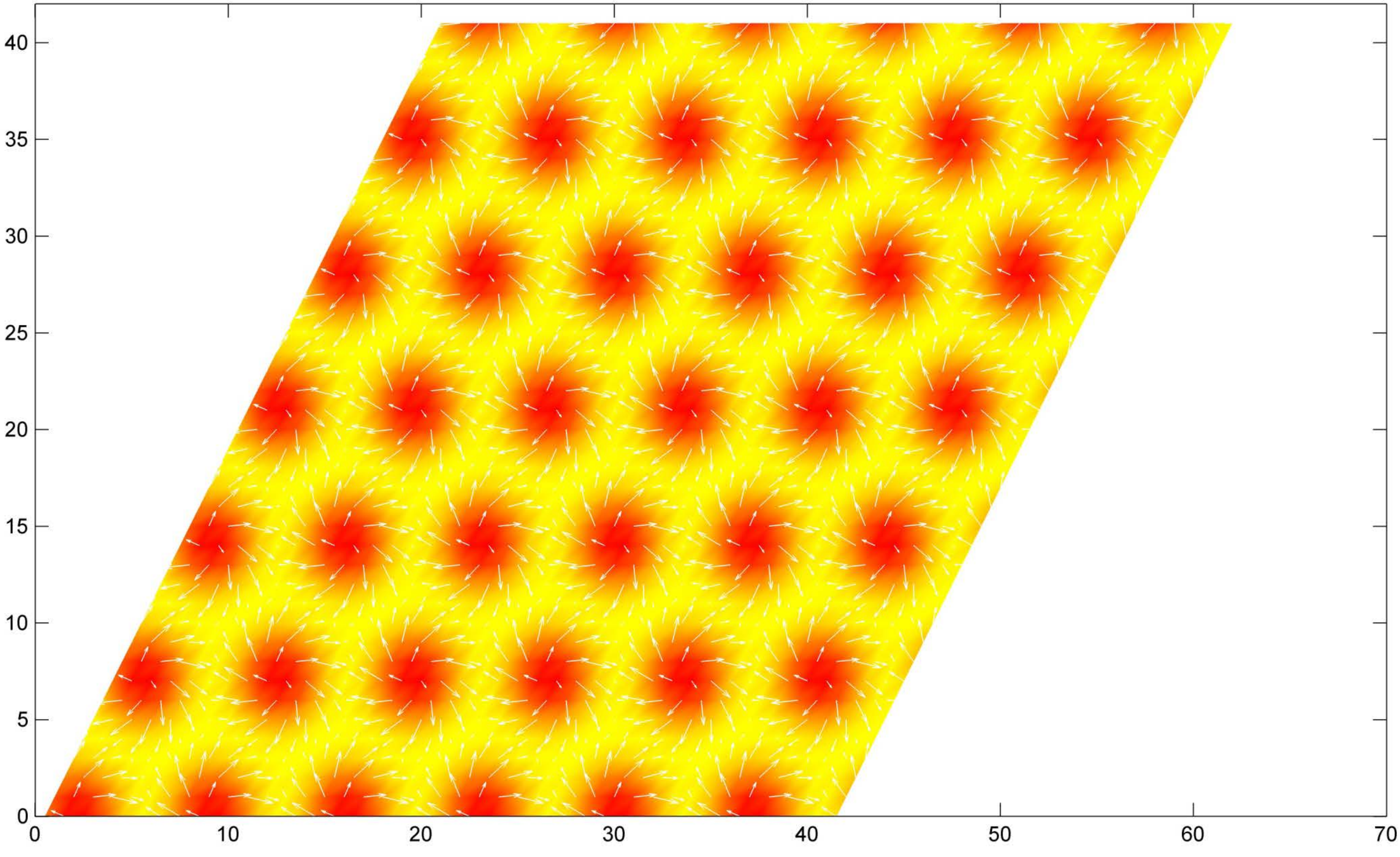


Phase Diagram ($\rho < 0$)

A. Leonov and MM, Nature Commun. 6, 8275 (2015)



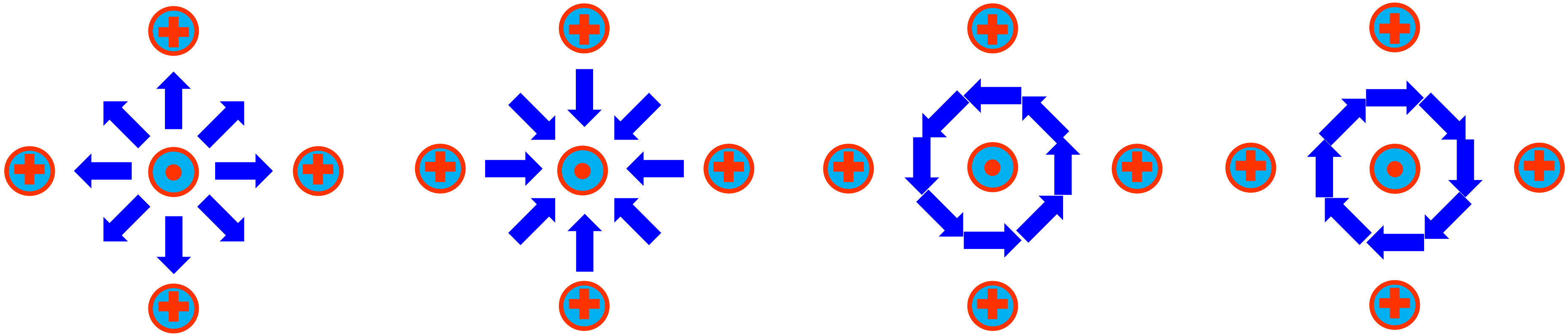
Skyrmion crystal



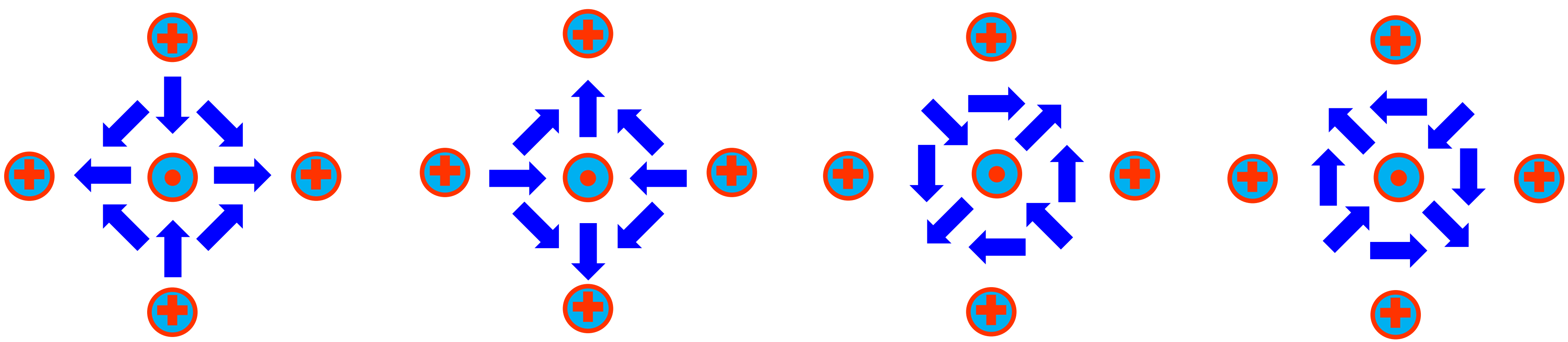
Skyrmion charge, vorticity & helicity

Q = +1

$\chi = 0$ $\chi = \pi$ $\chi = +\pi/2$ $\chi = -\pi/2$



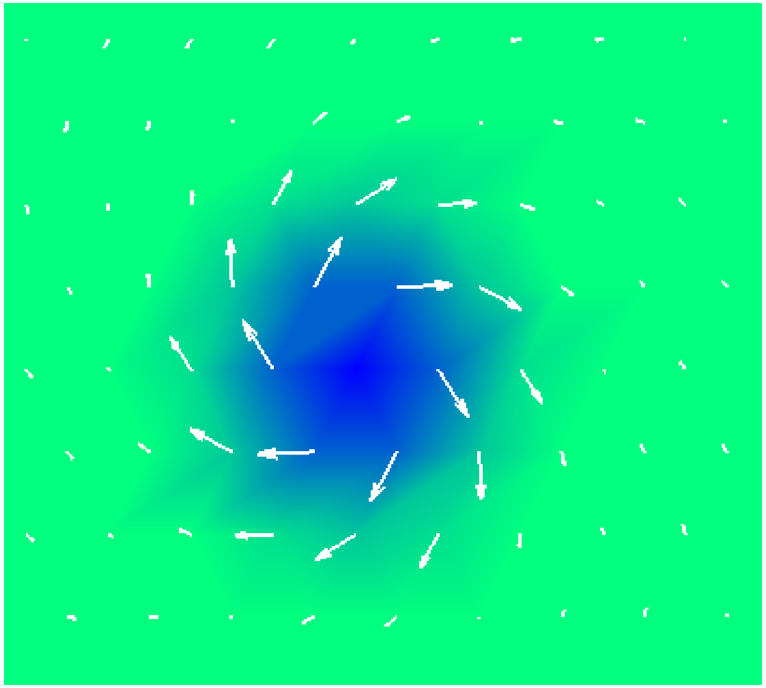
Q = -1

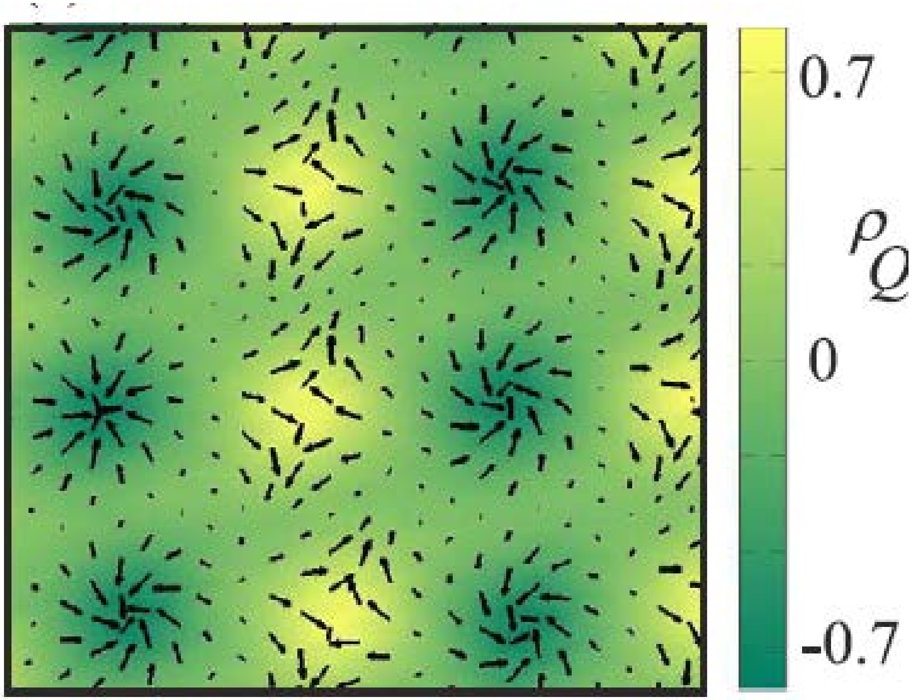


Arbitrary helicity and vorticity

$$\Theta = \Theta(\rho) \qquad \Phi = \mathrm{v}\,\varphi + \chi$$

$$Q = \mathrm{v}\,\frac{\left(n_z(0) - n_z(\infty)\right)}{2}$$

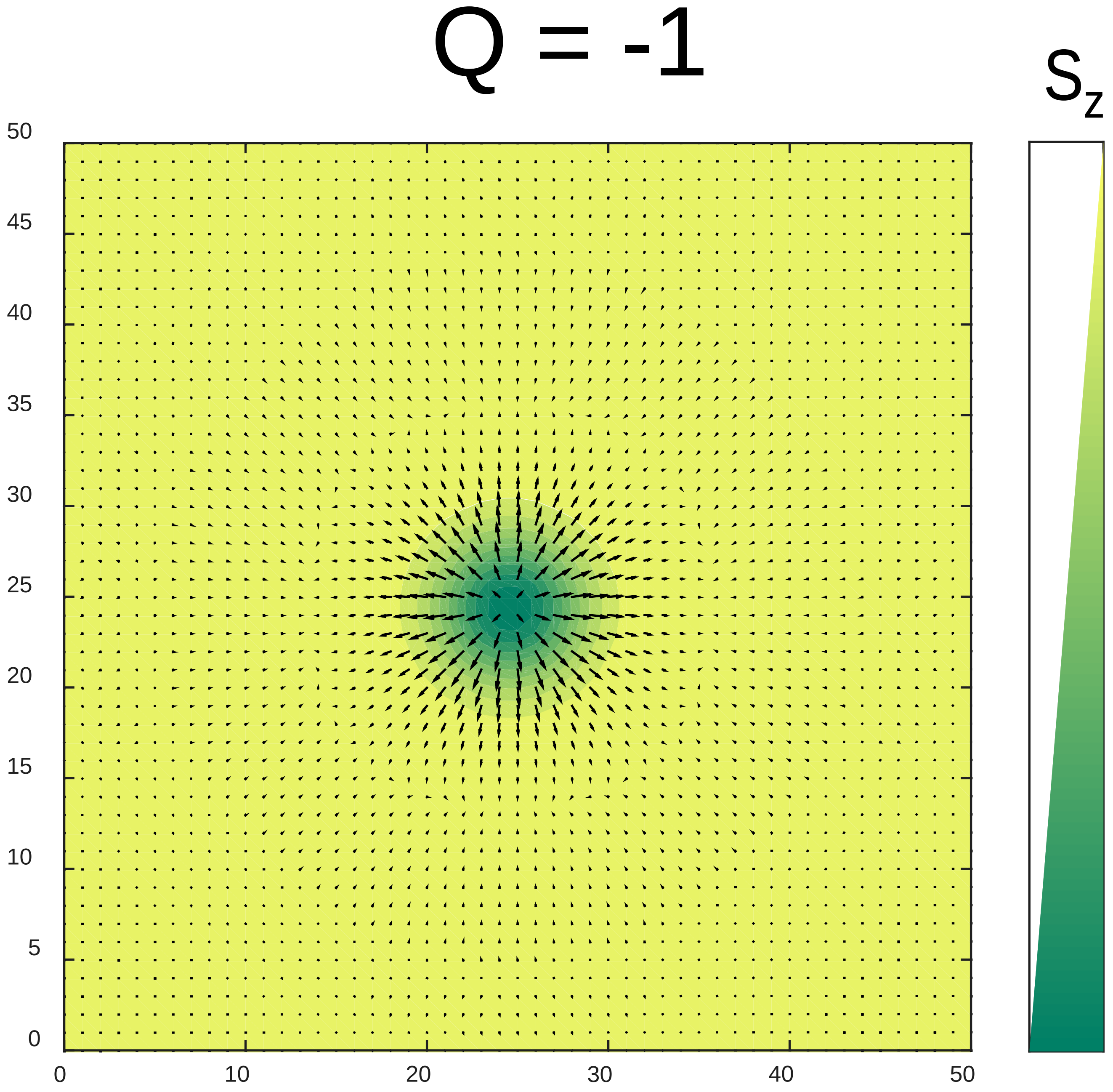


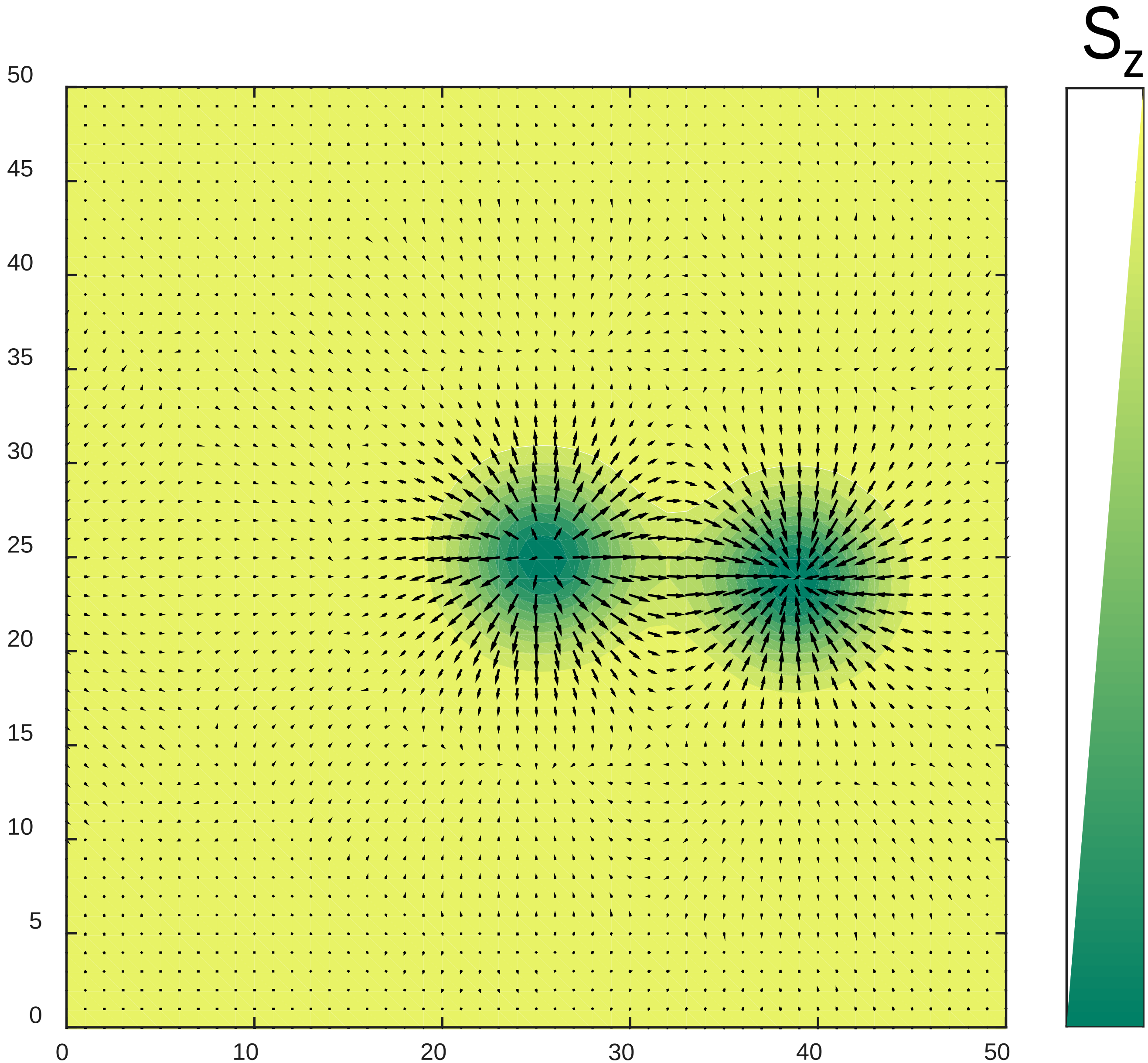


Multi-Q skyrmion states

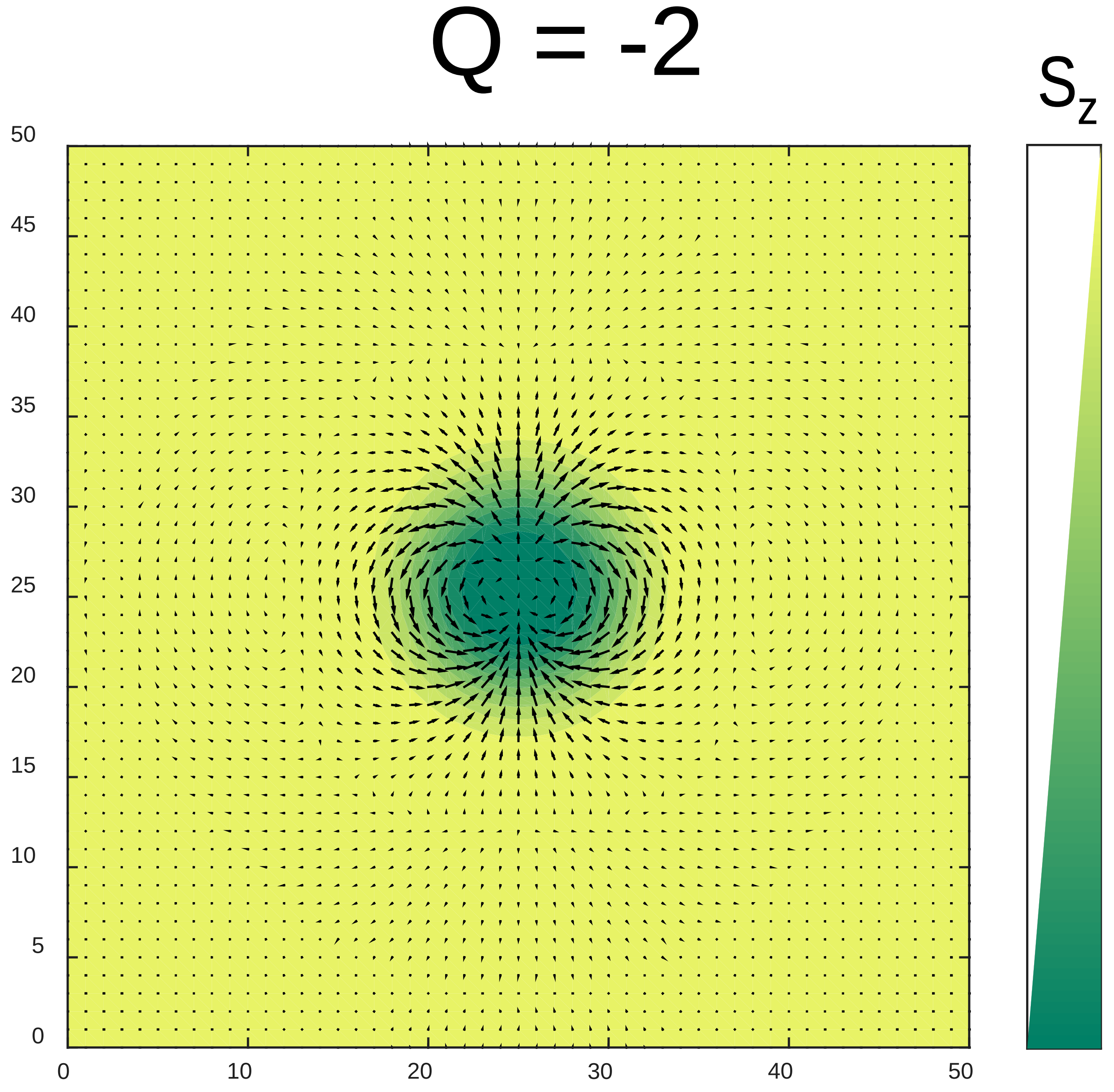
$$\rho > 0, K > 0$$

Y. Kharkov, O.P. Sushkov & MM (unpublished)

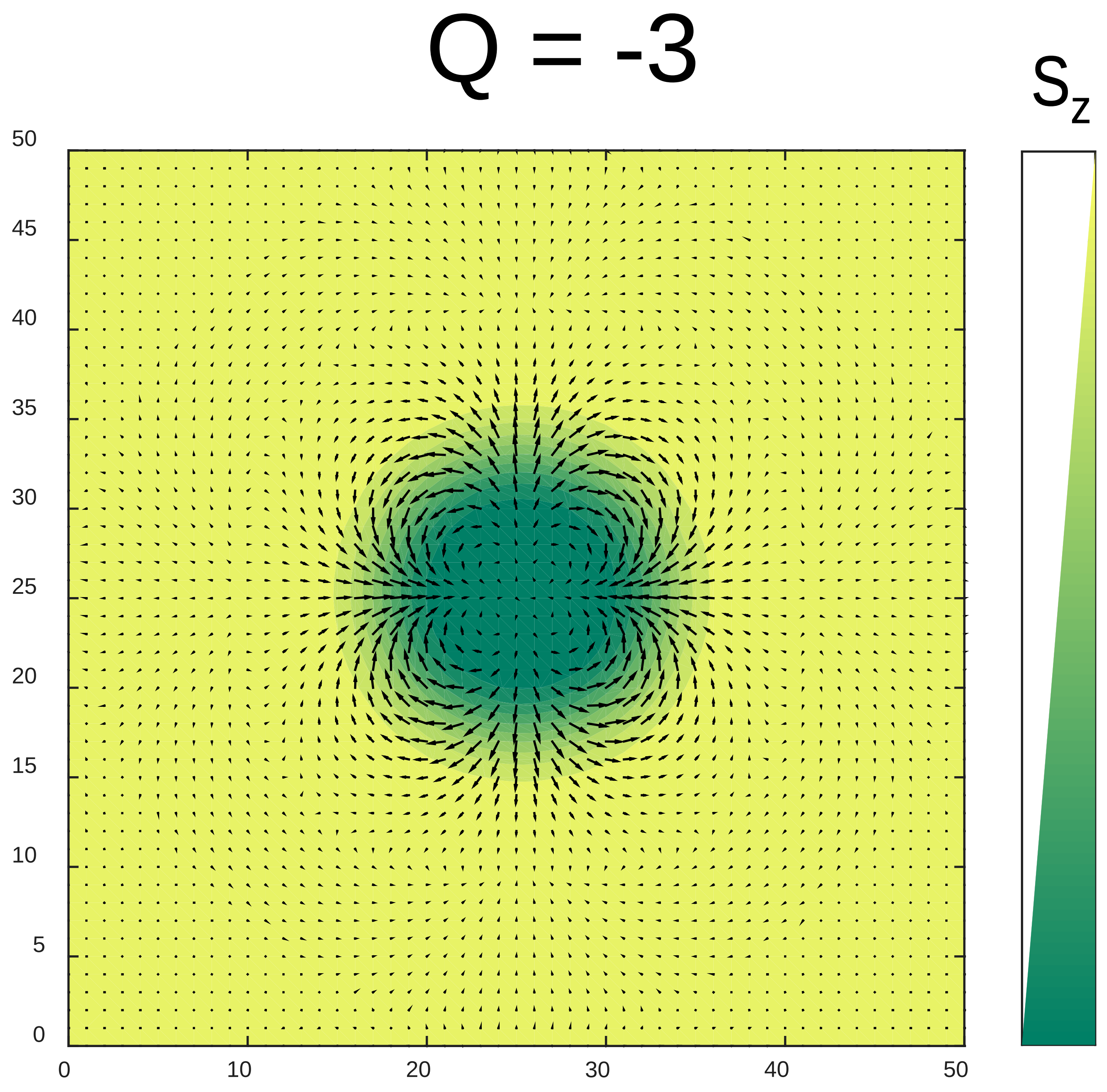

Q = -1
S_z
50
45
40
35
30
25
20
15
10
5
0
0
10
20
30
40
50

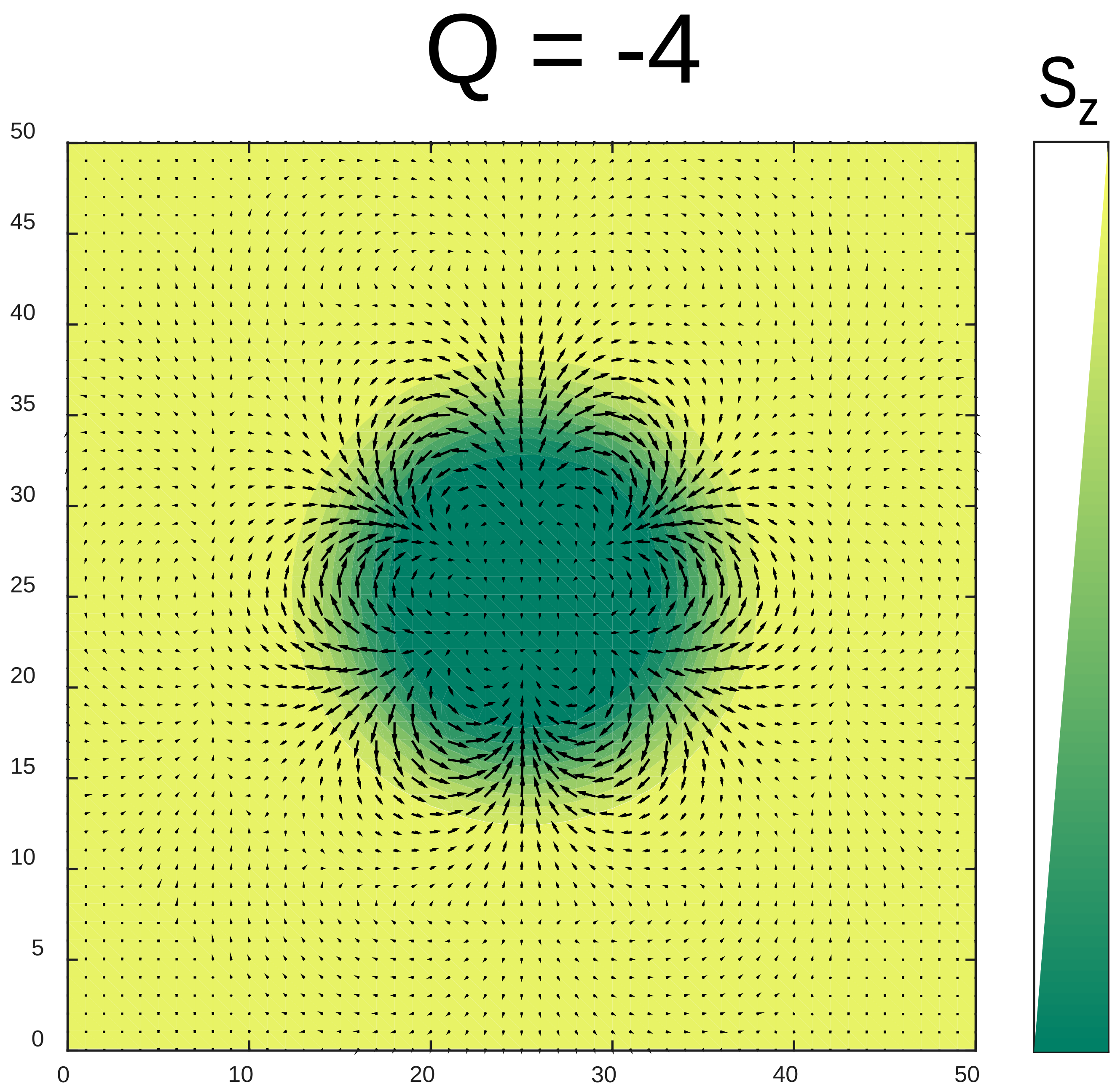


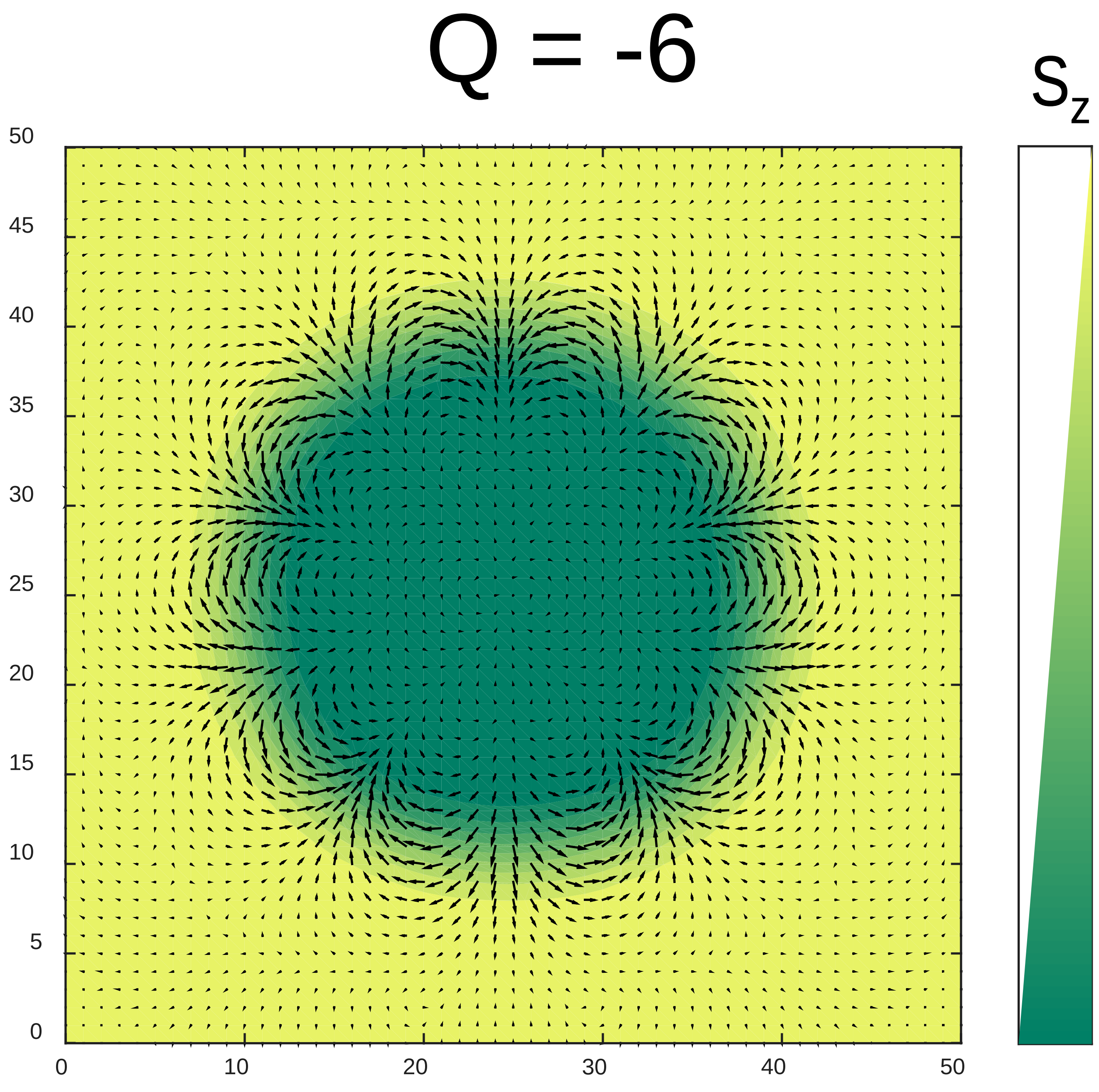
S_z
0
5
10
15
20
25
30
35
40
45
50
0
10
20
30
40
50



Q = -2
S_z
0
5
10
15
20
25
30
35
40
45
50
0
10
20
30
40
50

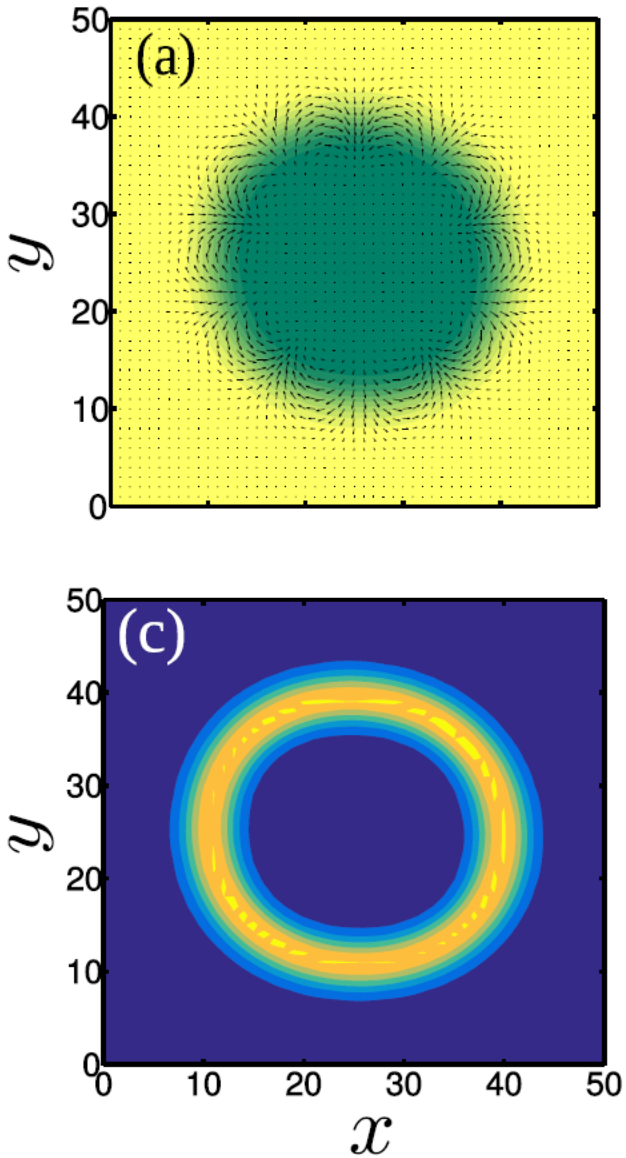

Q = -3
S_z
50
45
40
35
30
25
20
15
10
5
0
0
10
20
30
40
50

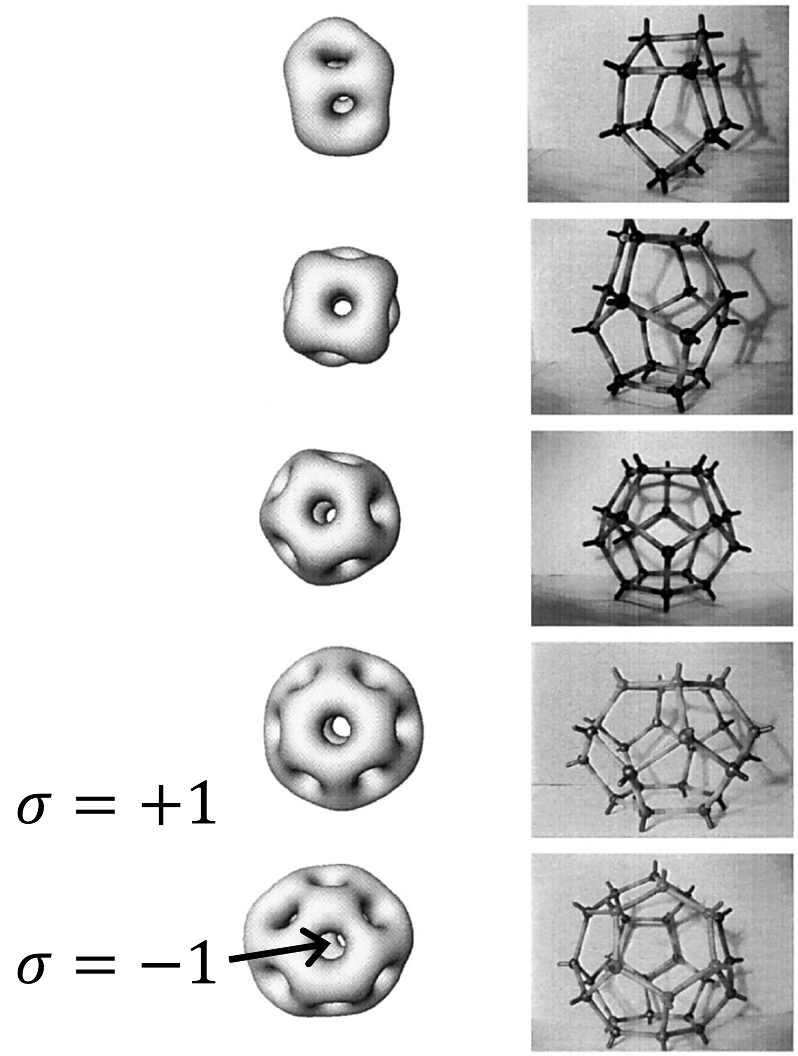

Q = -4
S_z
50
45
40
35
30
25
20
15
10
5
0
0
10
20
30
40
50



Q = -6
S_z
50
45
40
35
30
25
20
15
10
5
0
0
10
20
30
40
50

Holographic multi-skyrmions

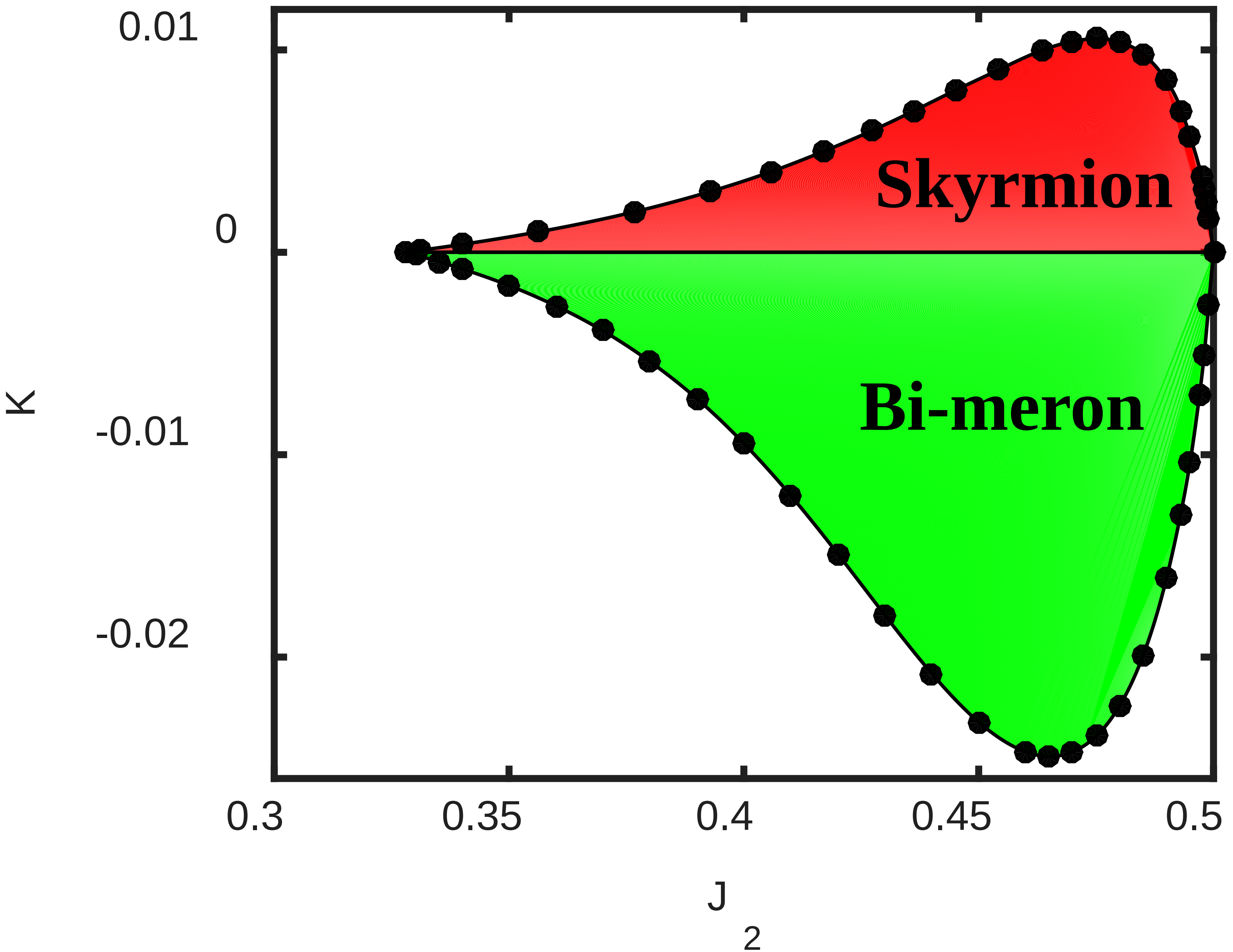




R. A. Battye & P.M. Sutcliffe, PRL **79**, 363 (1997)

Half-skyrmions

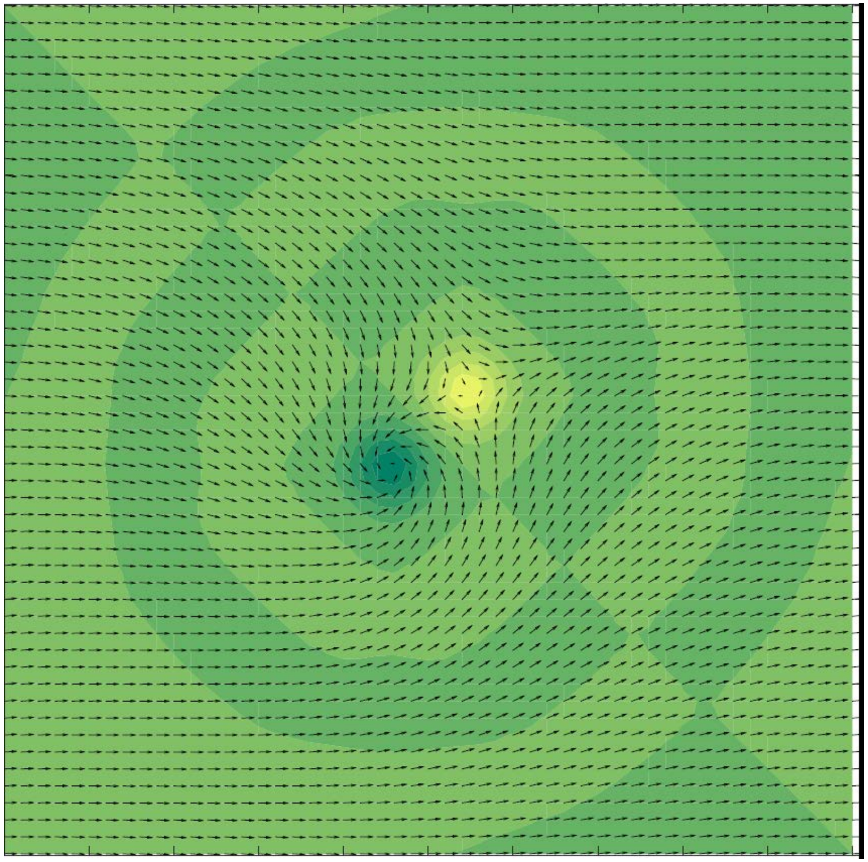
$$\rho > 0, K < 0$$



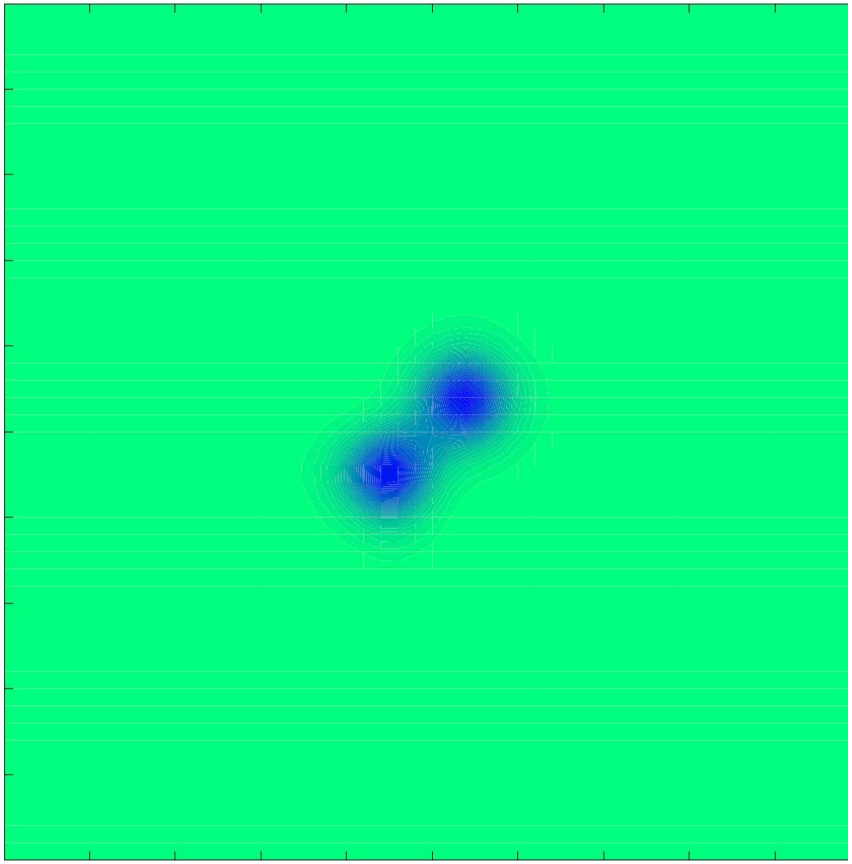
0.01
0
-0.01
-0.02
K
0.3
0.35
0.4
0.45
0.5
J
2
Skyrmion
Bi-meron

Bi-merons

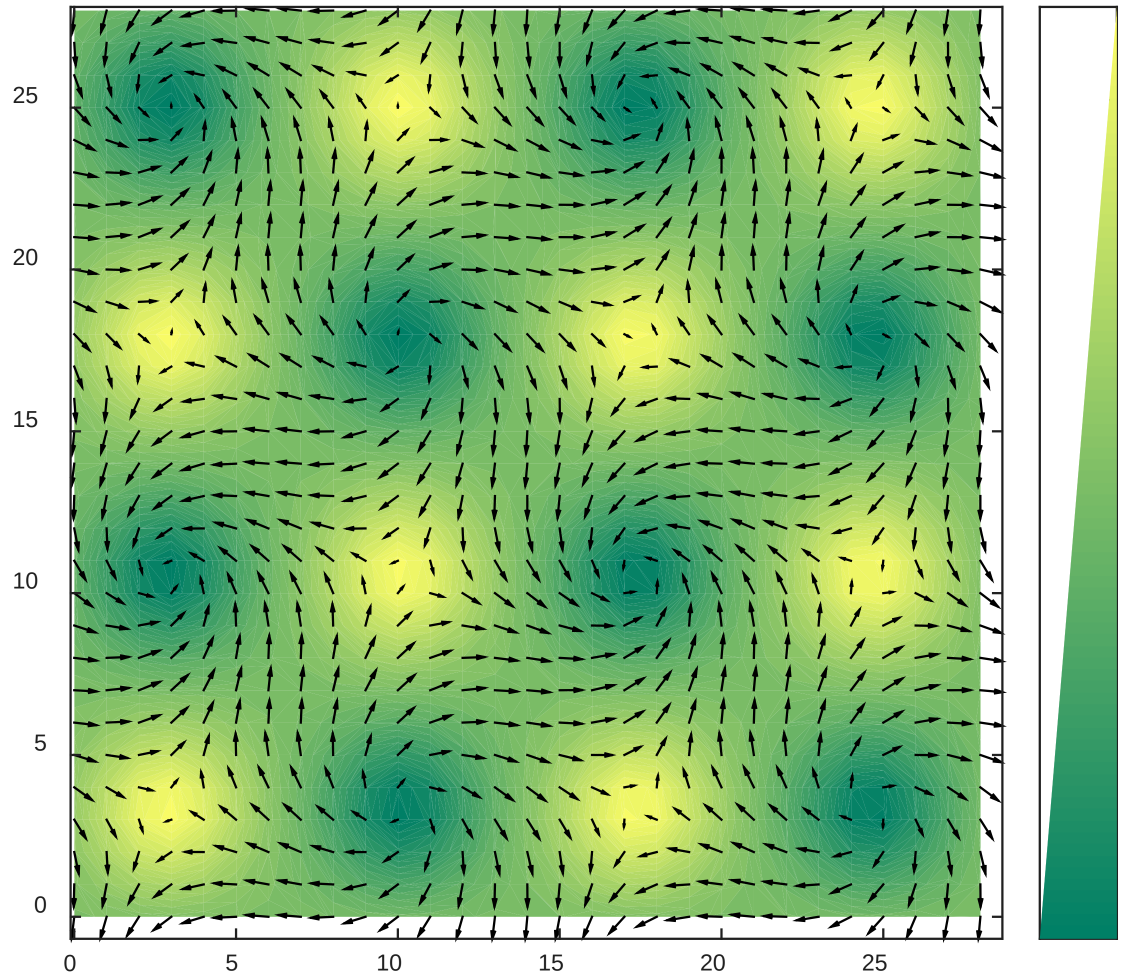
Spin configuration



Topological density

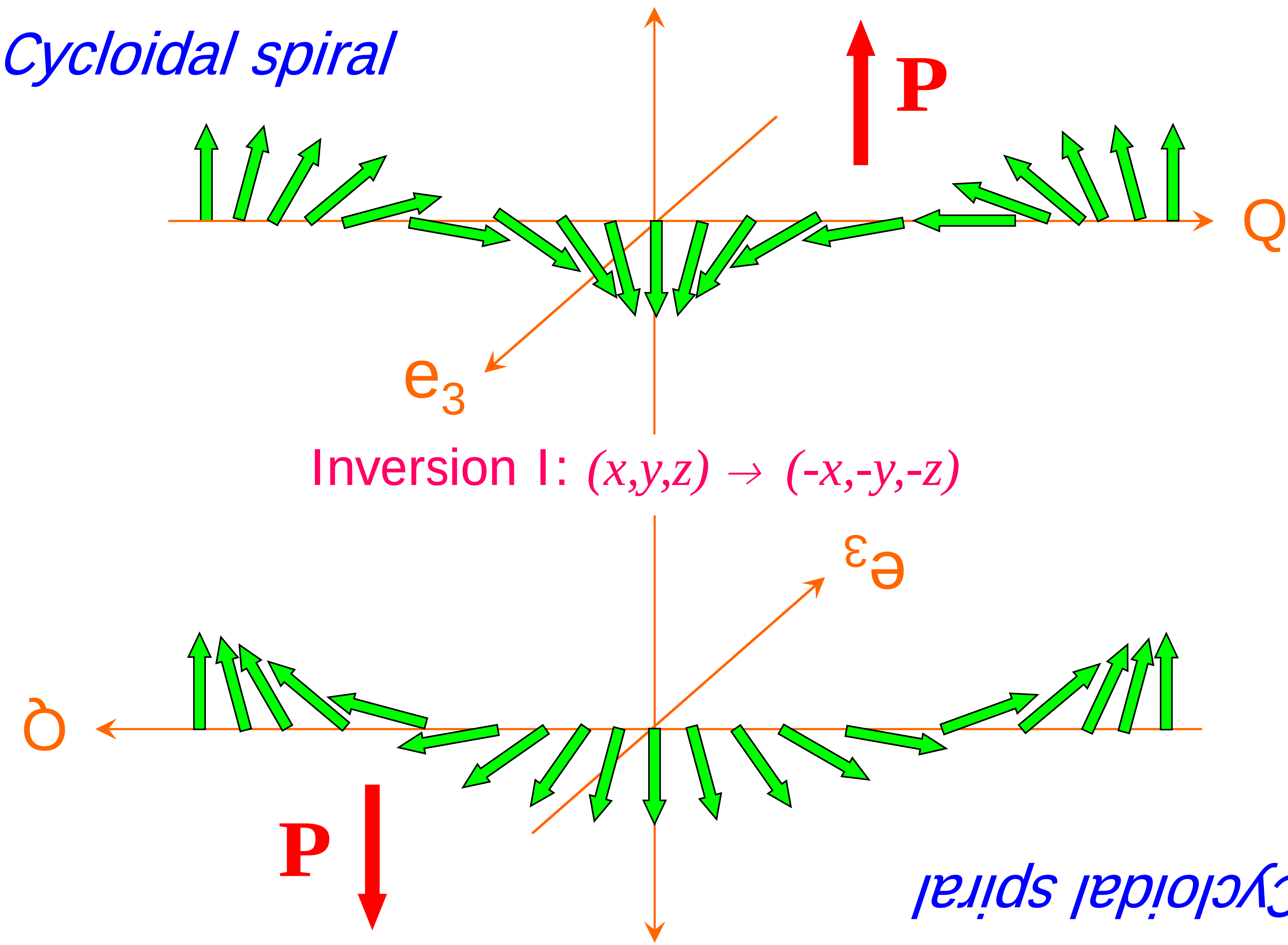


Meron-antimeron crystal



Multiferroic properties

Breaking of inversion symmetry by spin ordering



Skyrmion electric dipole moment

Magneto-electric coupling *MM PRL **96**, 067601 (2006)*

$$f_{me} = -g\,\mathbf{E}\cdot\left[\mathbf{n}(\nabla\cdot\mathbf{n}) - (\mathbf{n}\cdot\nabla)\mathbf{n}\right]$$

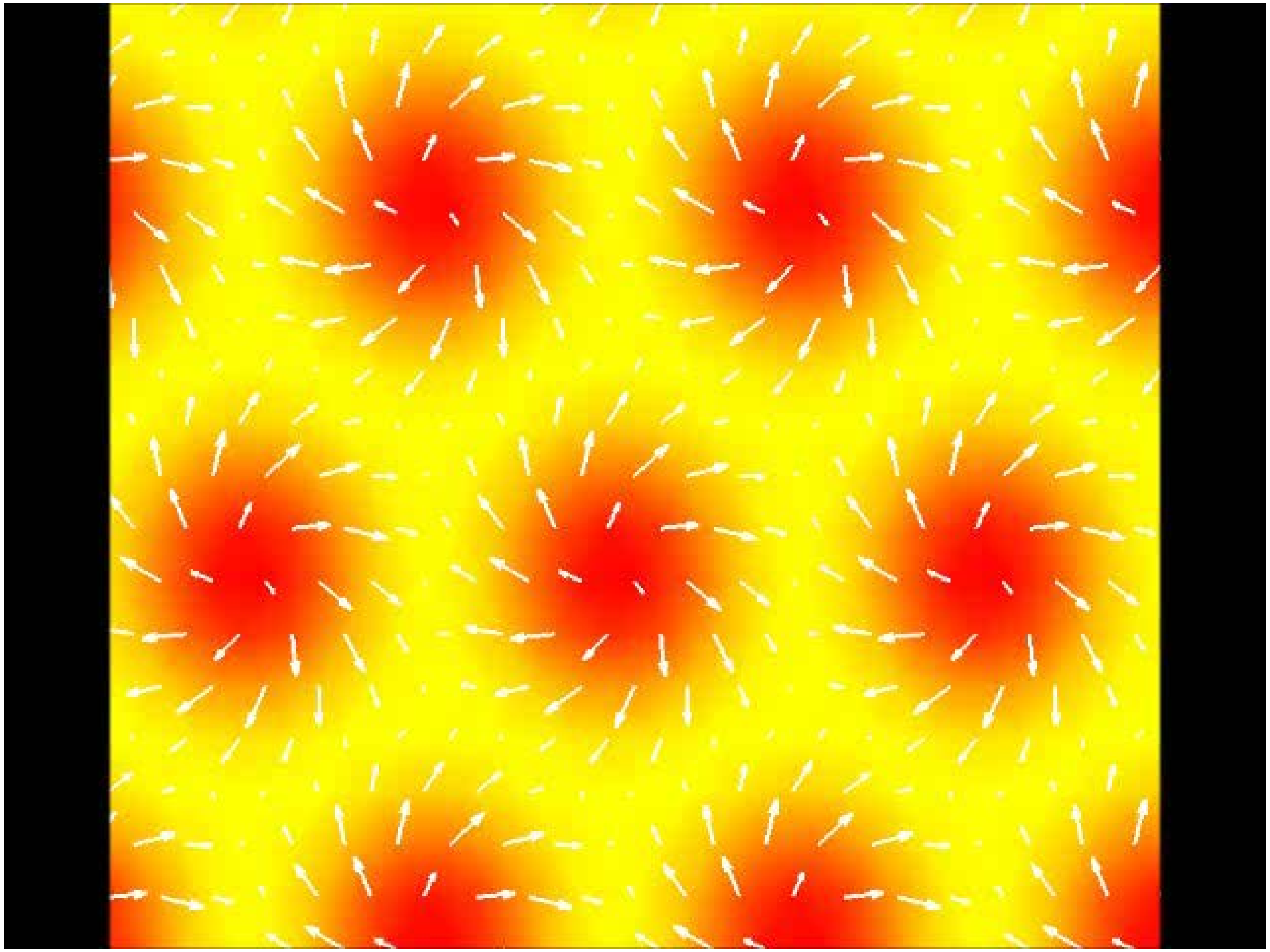
Polarization induced by skyrmion

$$P_z = -\frac{\partial f}{\partial E_z} = g\left[\frac{d\Theta}{dr} + \mathrm{v}\frac{\sin 2\Theta}{2r}\right]\cos\left((\mathrm{v}-1)\varphi + \chi\right)$$

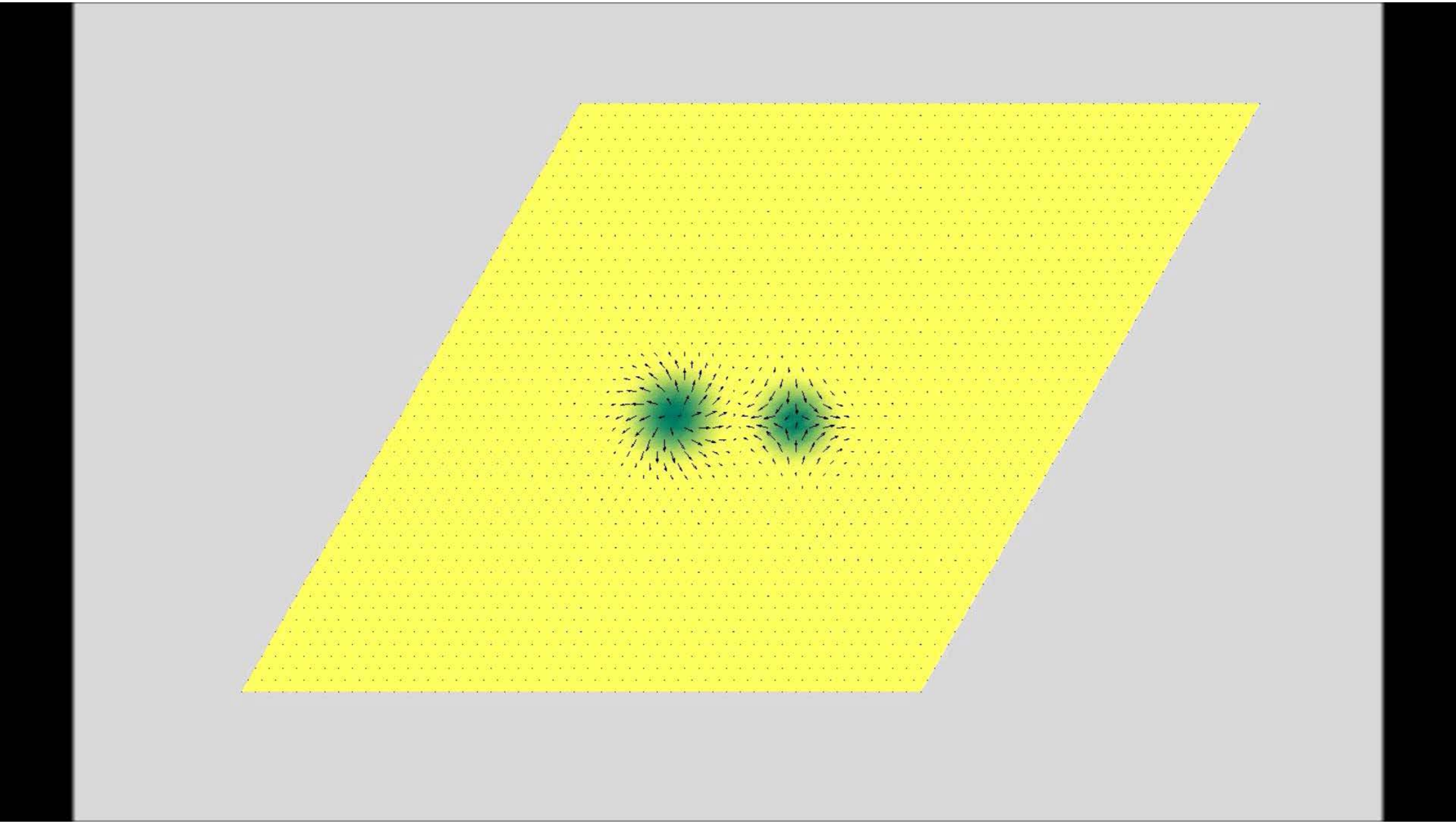
Dipole moment for vorticity v = +1

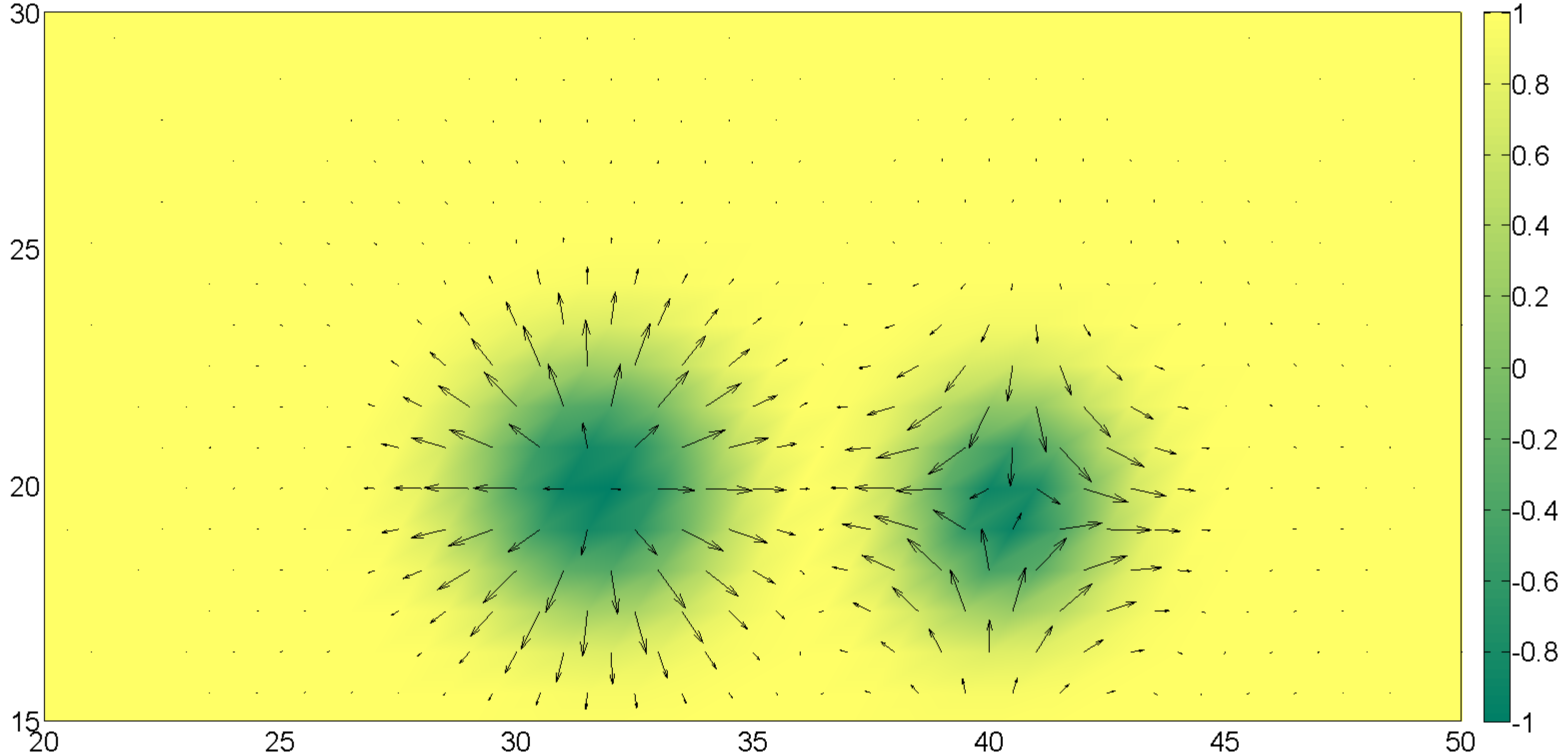
$$D_z(\chi) = D(0)\cos\chi$$

Magnetically-induced electric oscillations



E_z-induced motion of s-a pair




30
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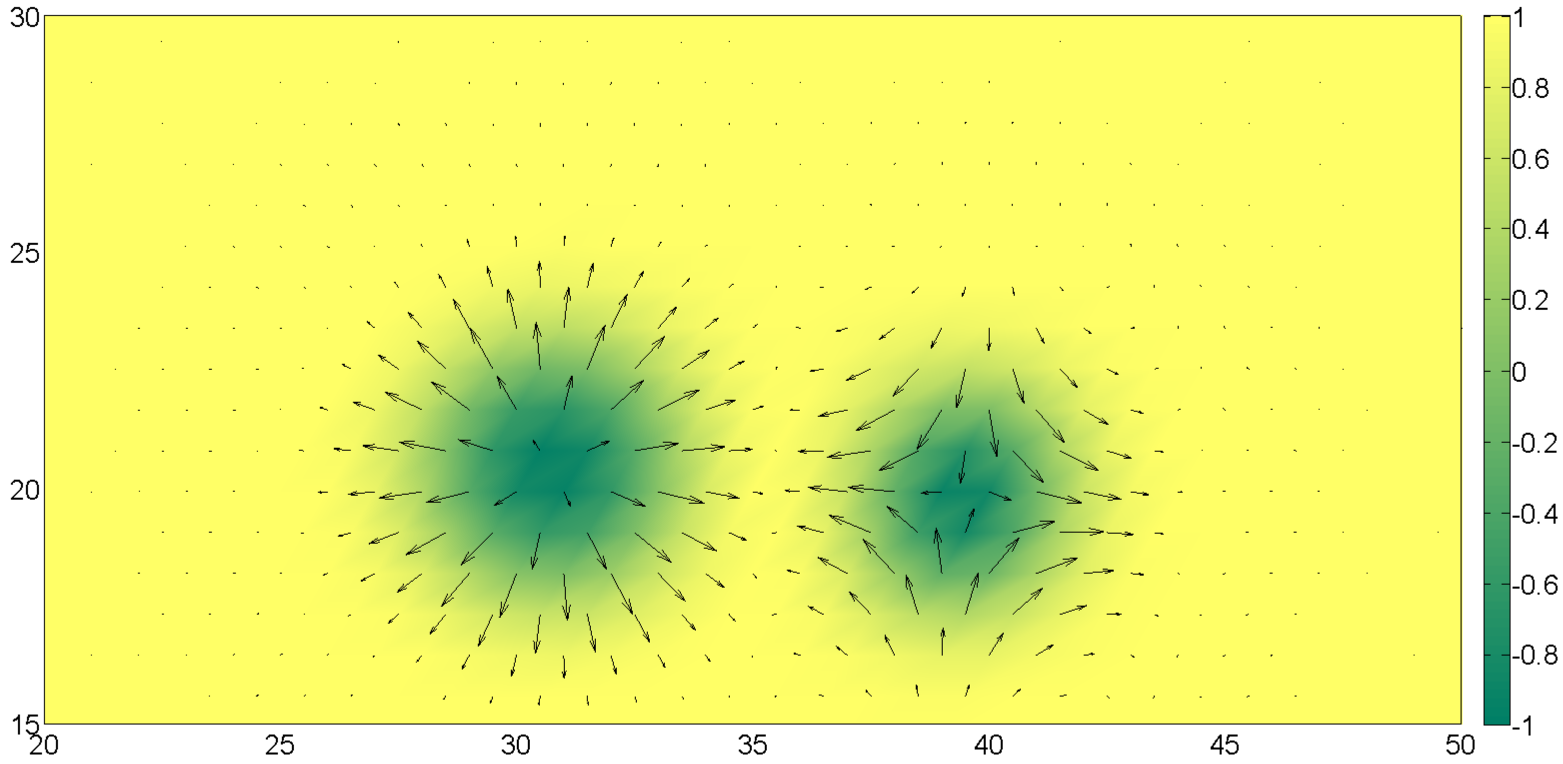
25
30
35
40
45
50
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1



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45
50
1
0.8
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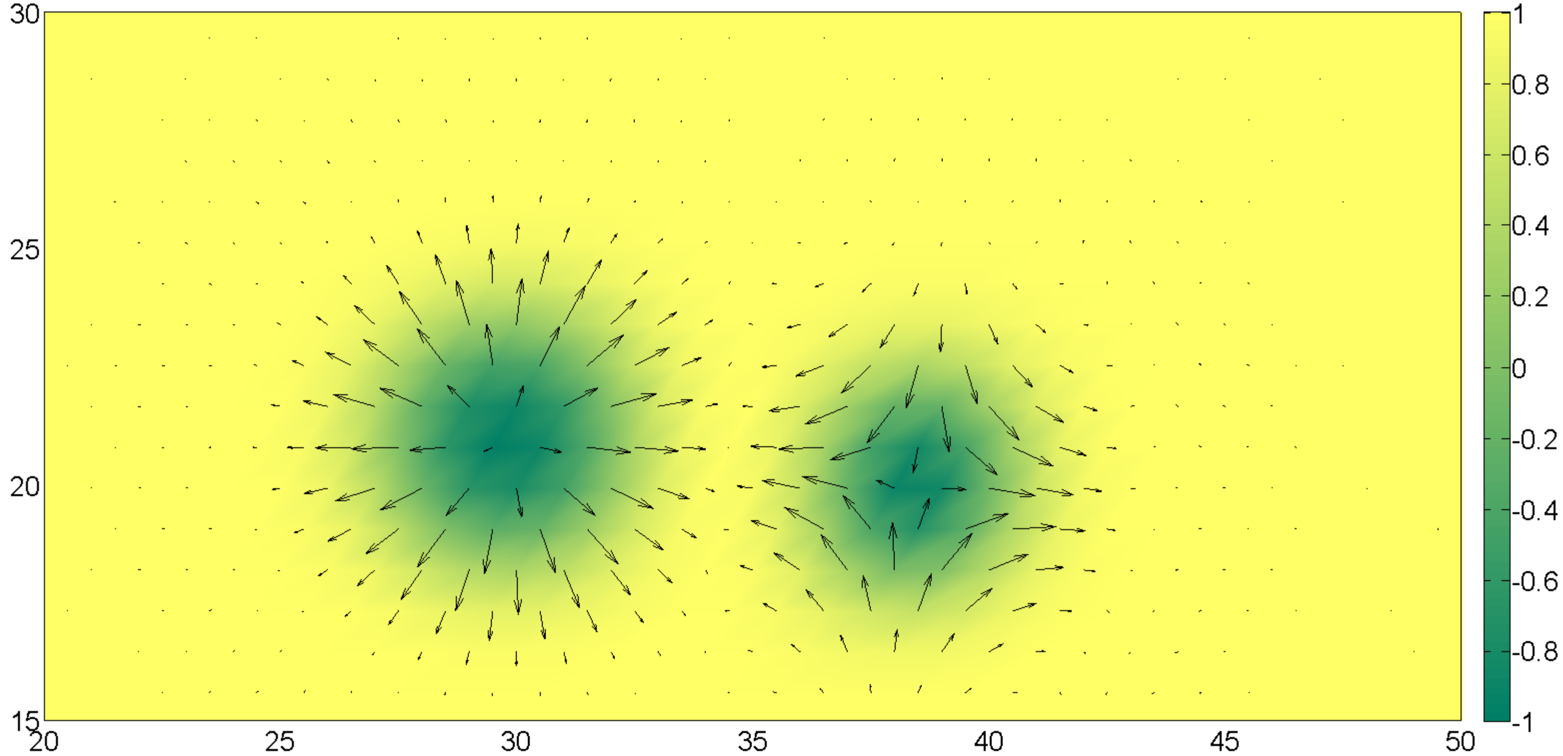
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1



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0.8
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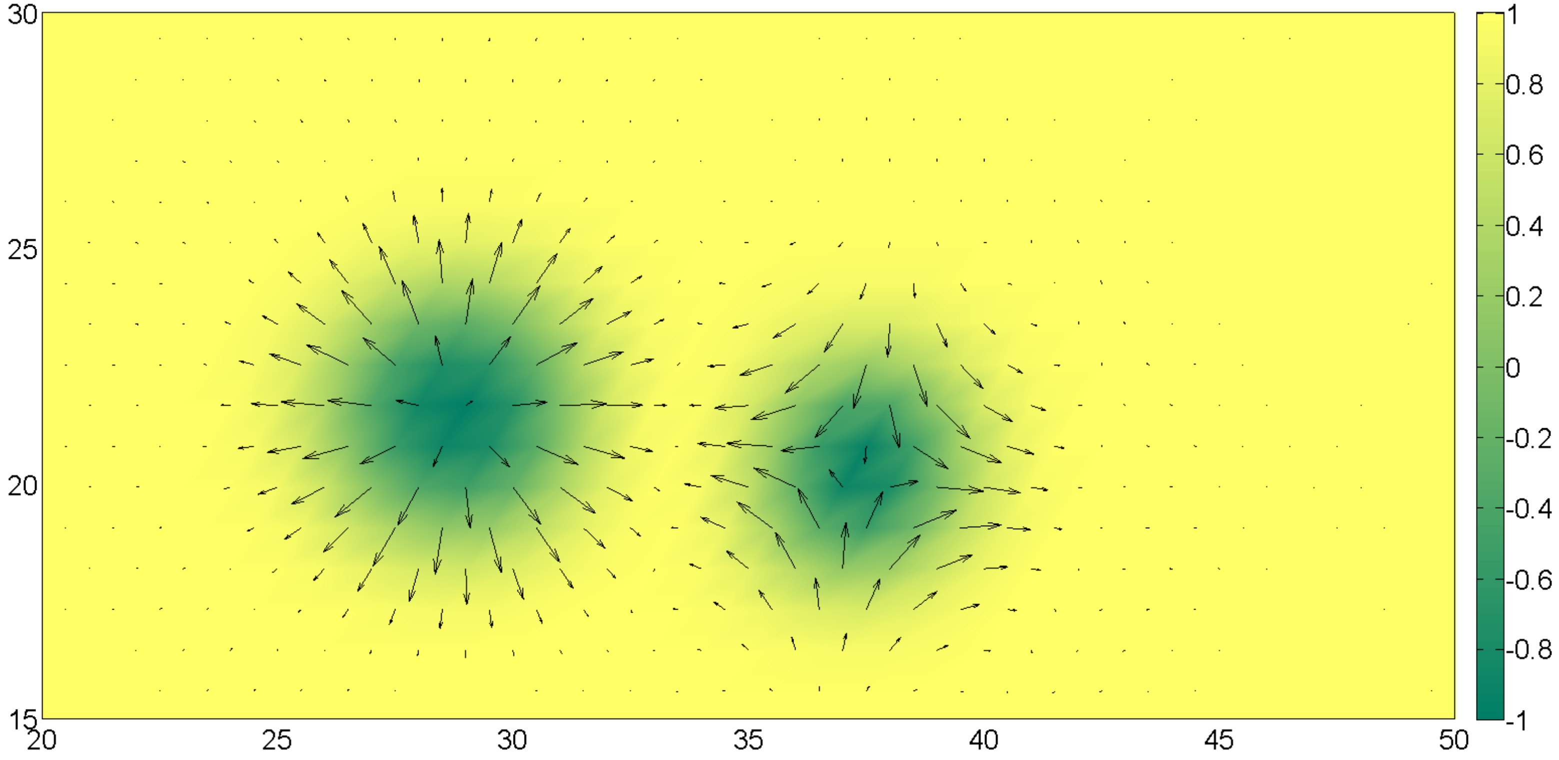
0
-0.2
-0.4
-0.6
-0.8
-1



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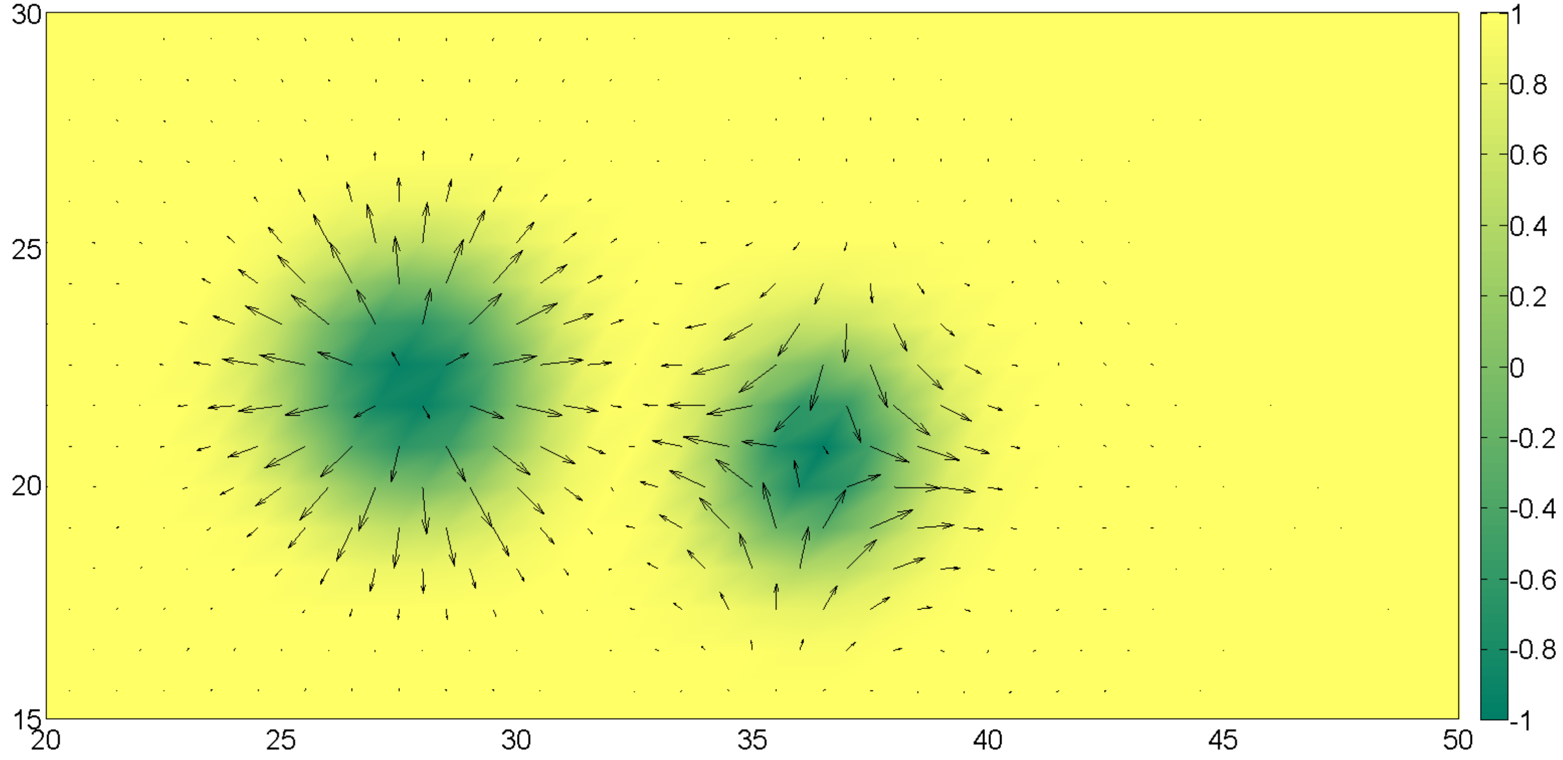
25
30
35
40
45
50
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1


30
25
20
15
20
25
30
35
40
45
50
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1

Topological and electric charge

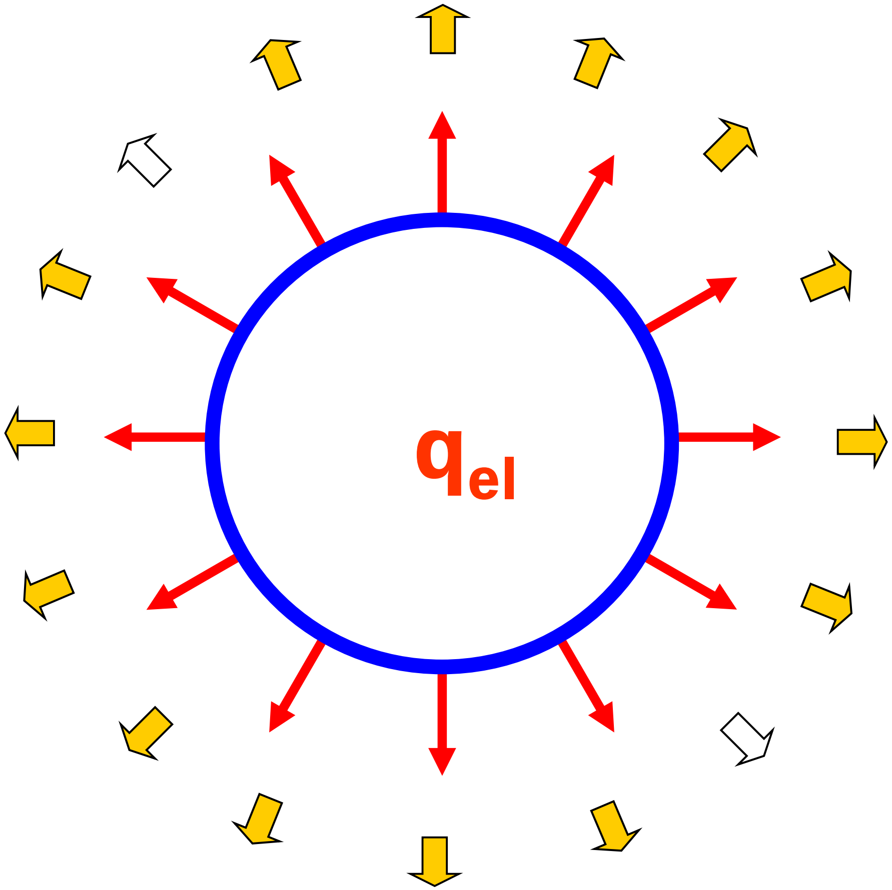
Magnetically-induced electric polarization & charge density

$$P_x = g n_x \overleftrightarrow{\partial}_y n_y \qquad P_y = g n_y \overleftrightarrow{\partial}_x n_x$$

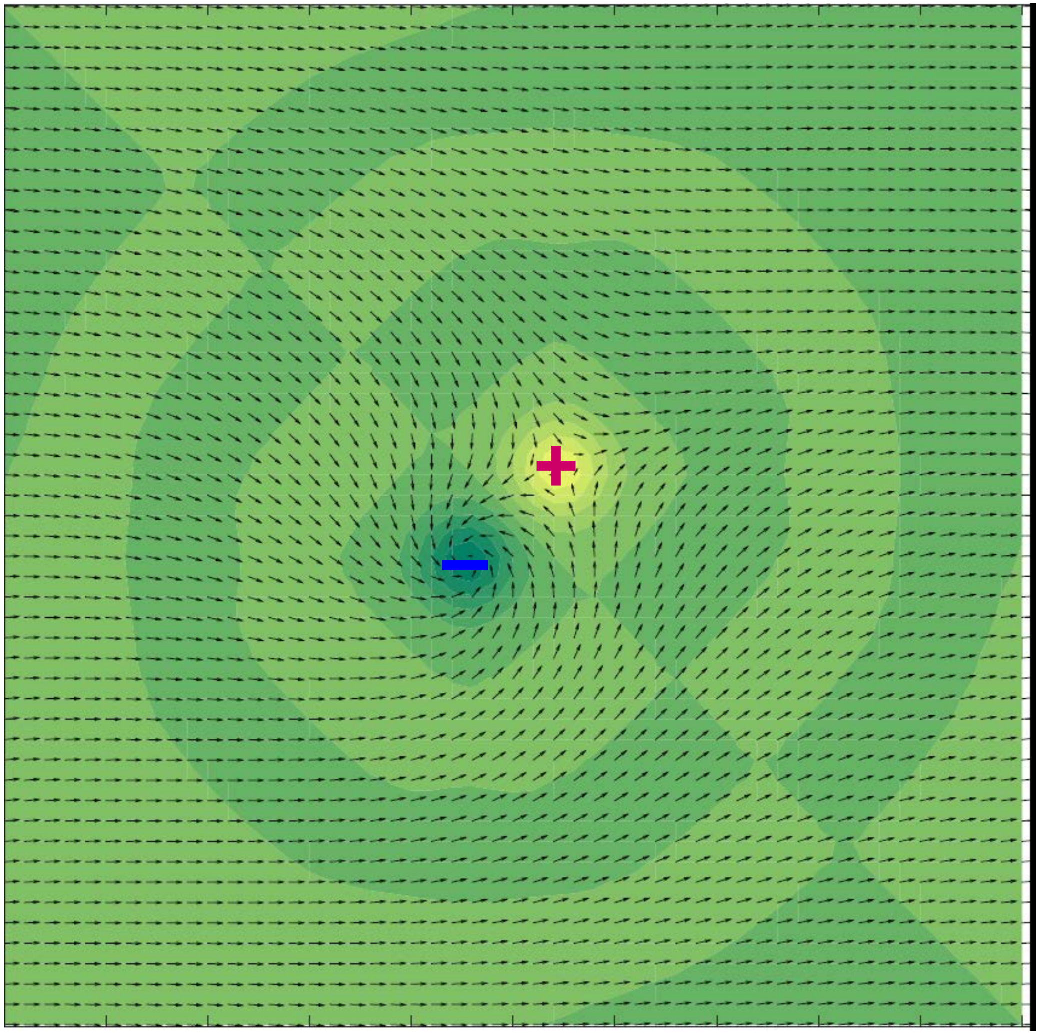
$$\rho = -\nabla \cdot \mathbf{P} = -\left(\partial_x P_x + \partial_y P_y \right)$$

$$Q = -\frac{3}{8\pi g} \int d^2 x \rho n_z$$

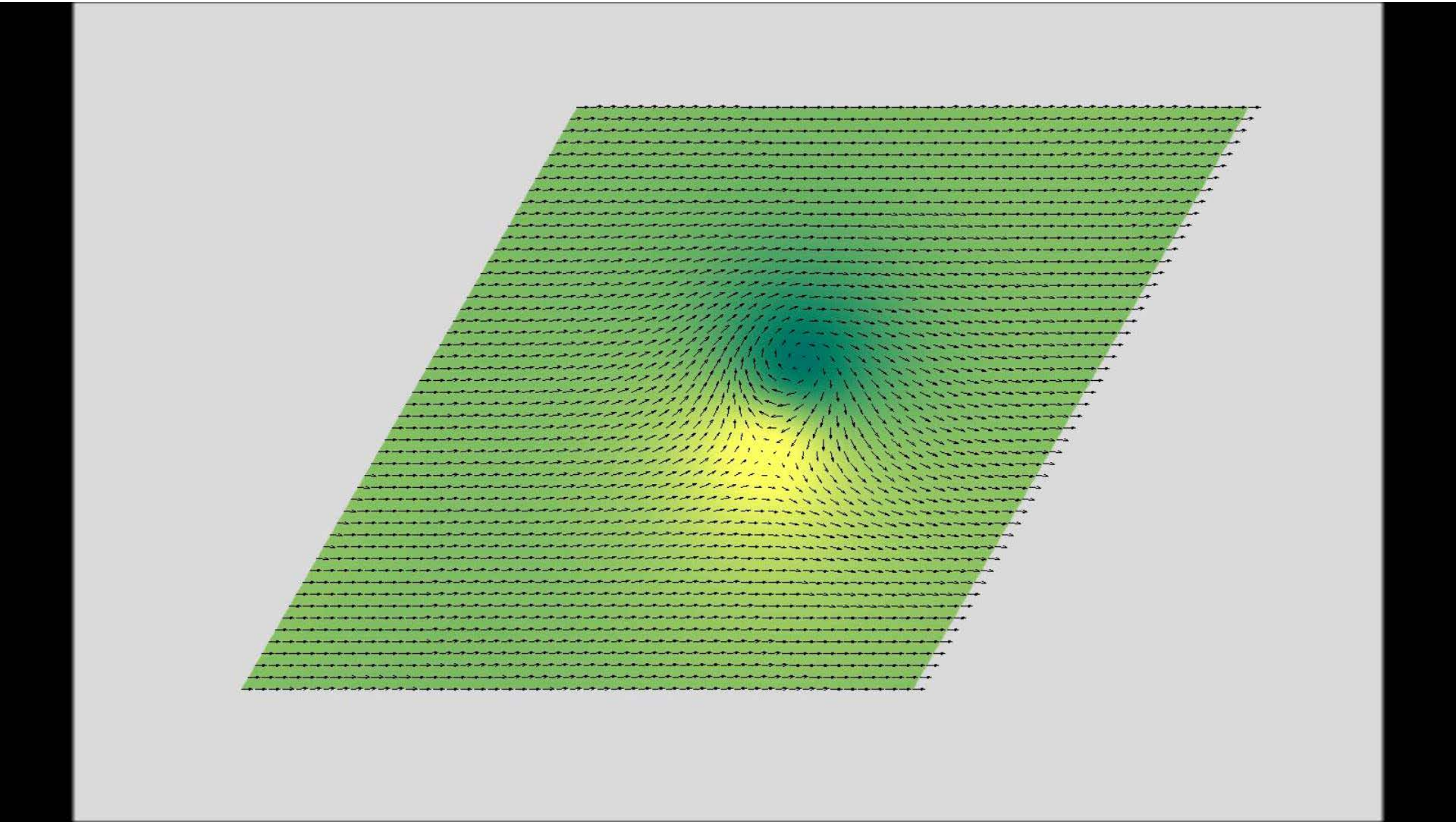
Electric charge of magnetic vortex



Electrically-charged merons



E_x-induced rotation of merons



Conclusions

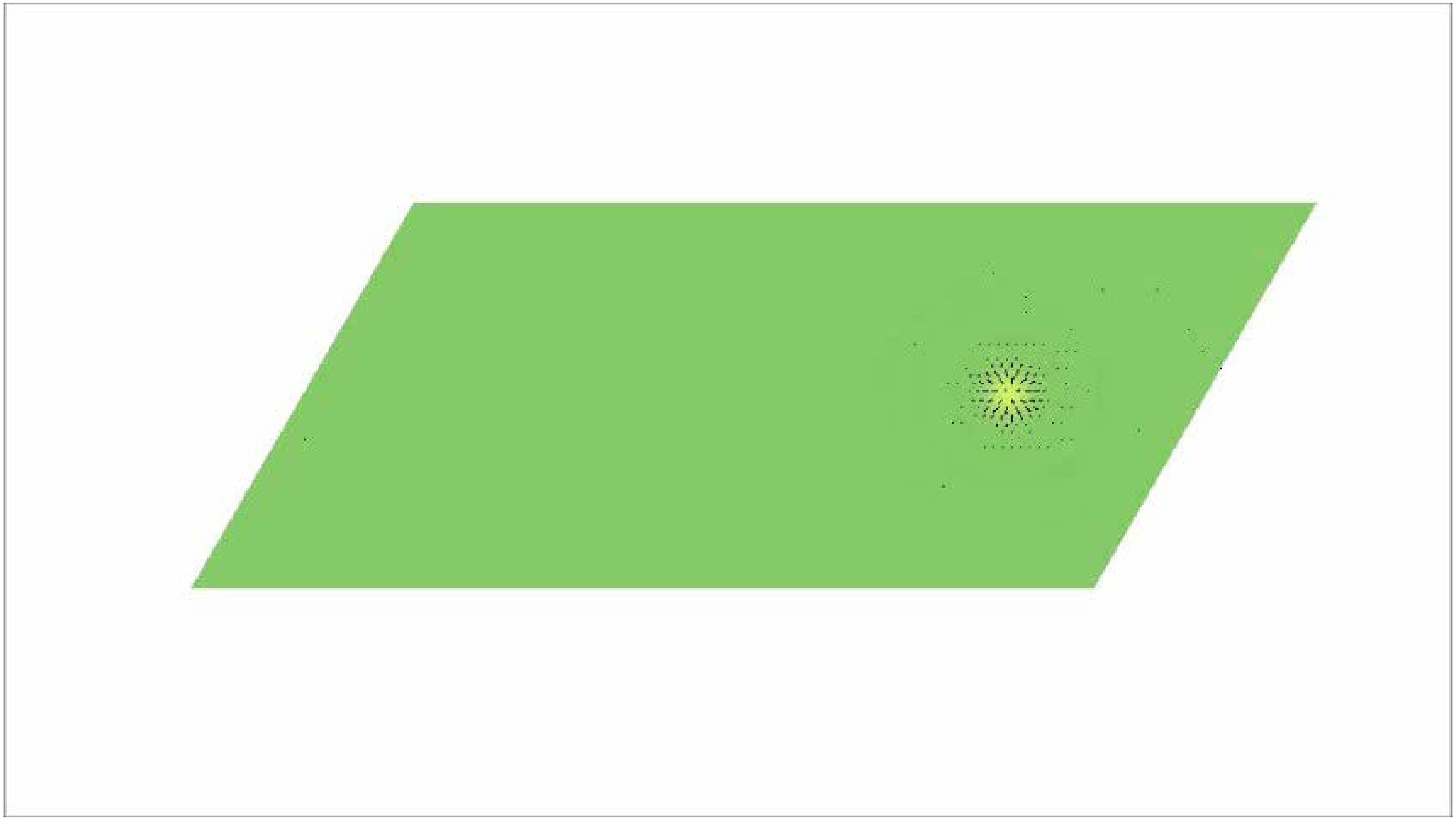
- Unusual multi-Q skyrmions
- Electric field control of skyrmions in frustrated magnetic insulators

A. Leonov & MM, Nature Commun. ***6****, 8275 (2015)*
Y. Kharkov, O. Sushkov & MM (unpublished)

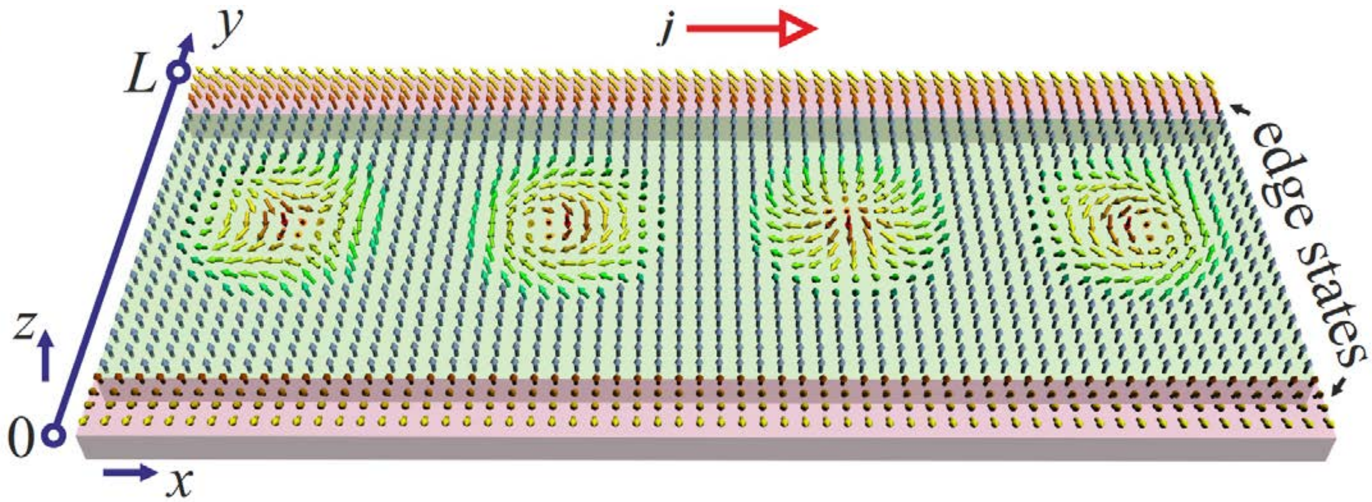
Topological edge states

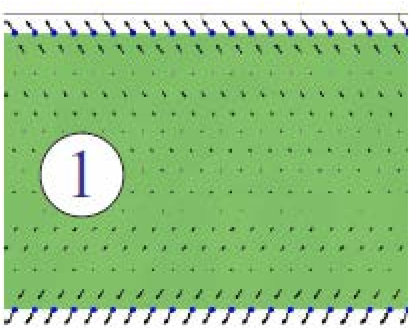
A. Leonov & MM Nature Commun. ***8****, 14394 (2017)*

Skyrmion Hall Effect

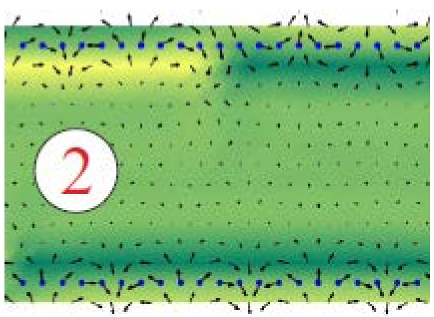


Edge States

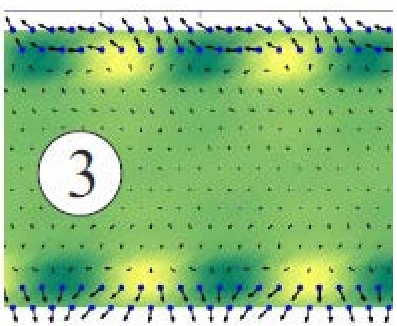




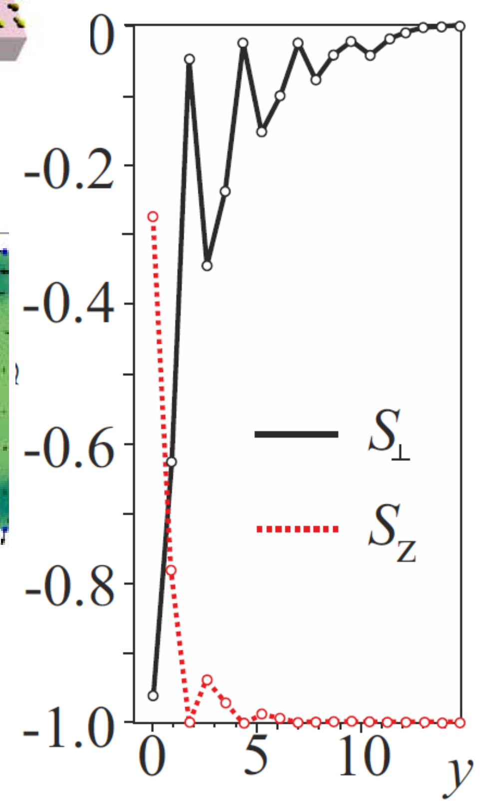
Parallel in-plane spins

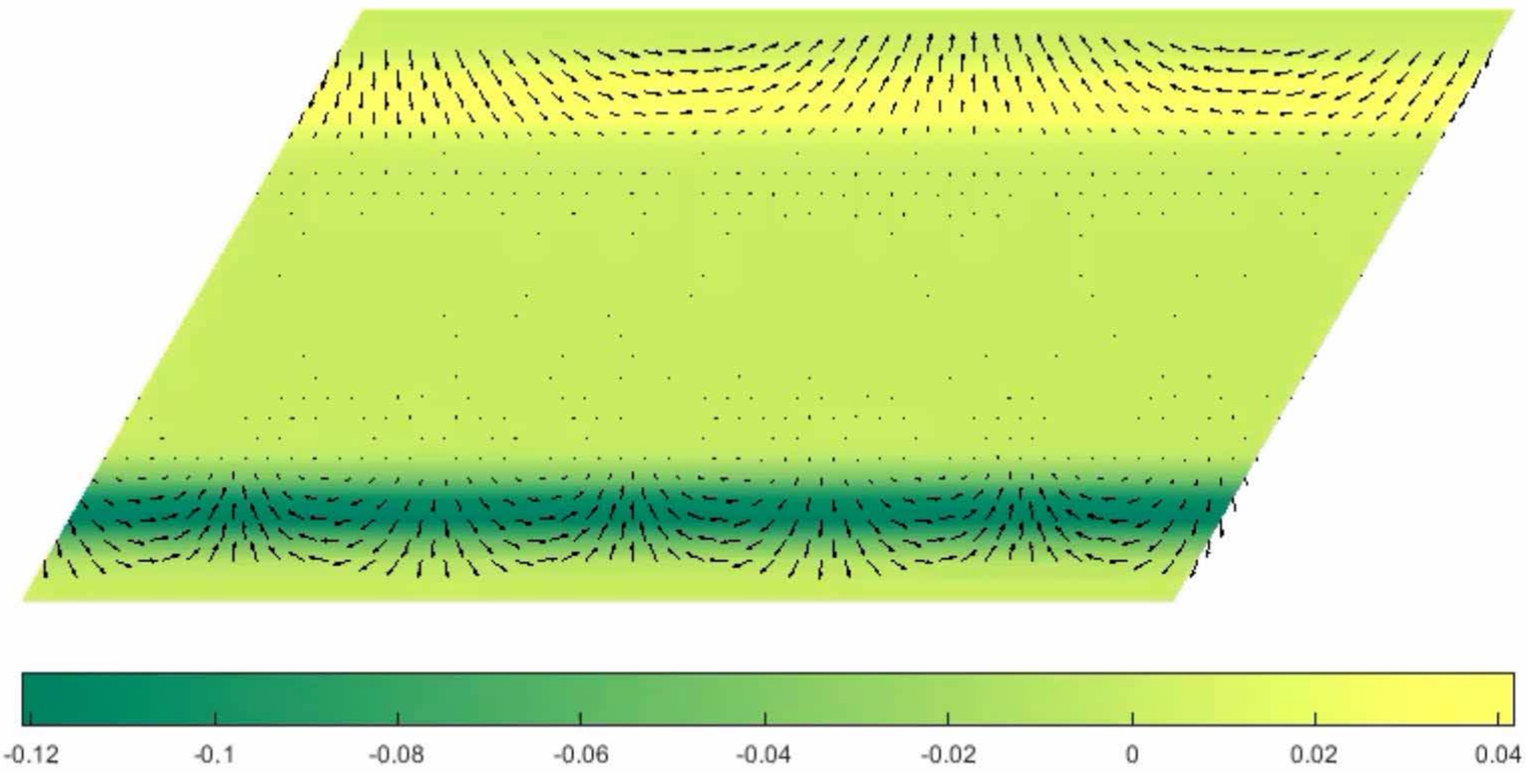


Conical spiral



Fan state





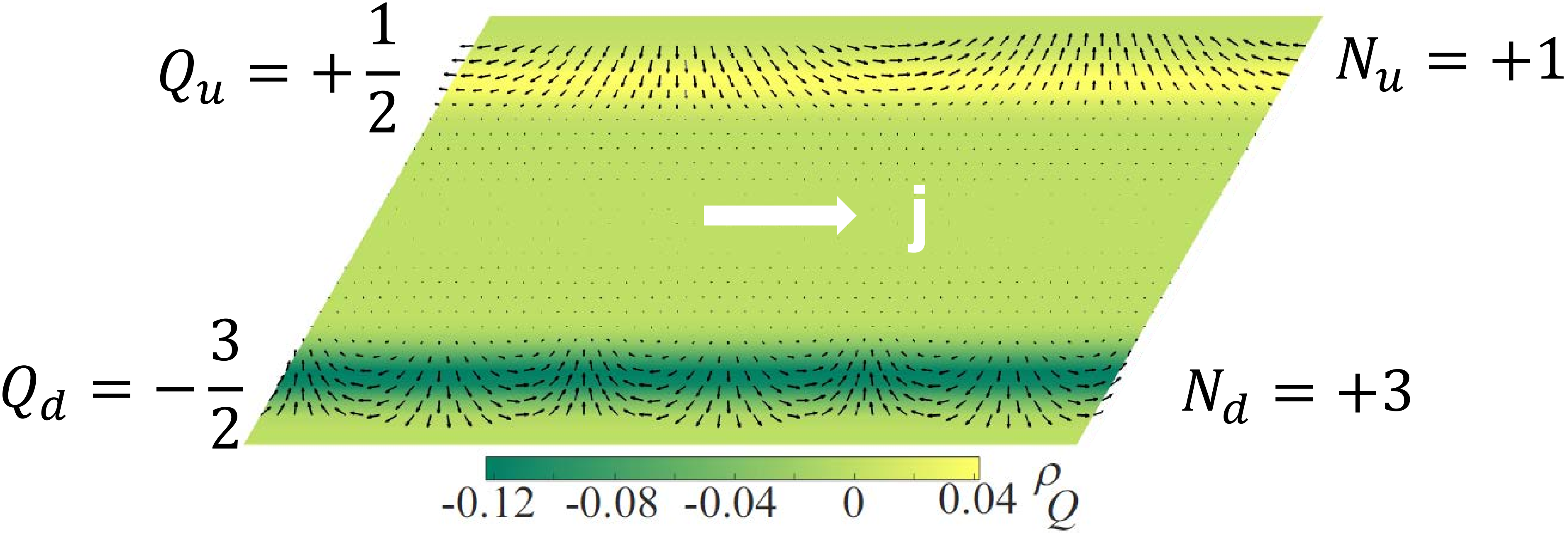
-0.12
-0.1
-0.08
-0.06
-0.04
-0.02
0
0.02
0.04

Edge state instability

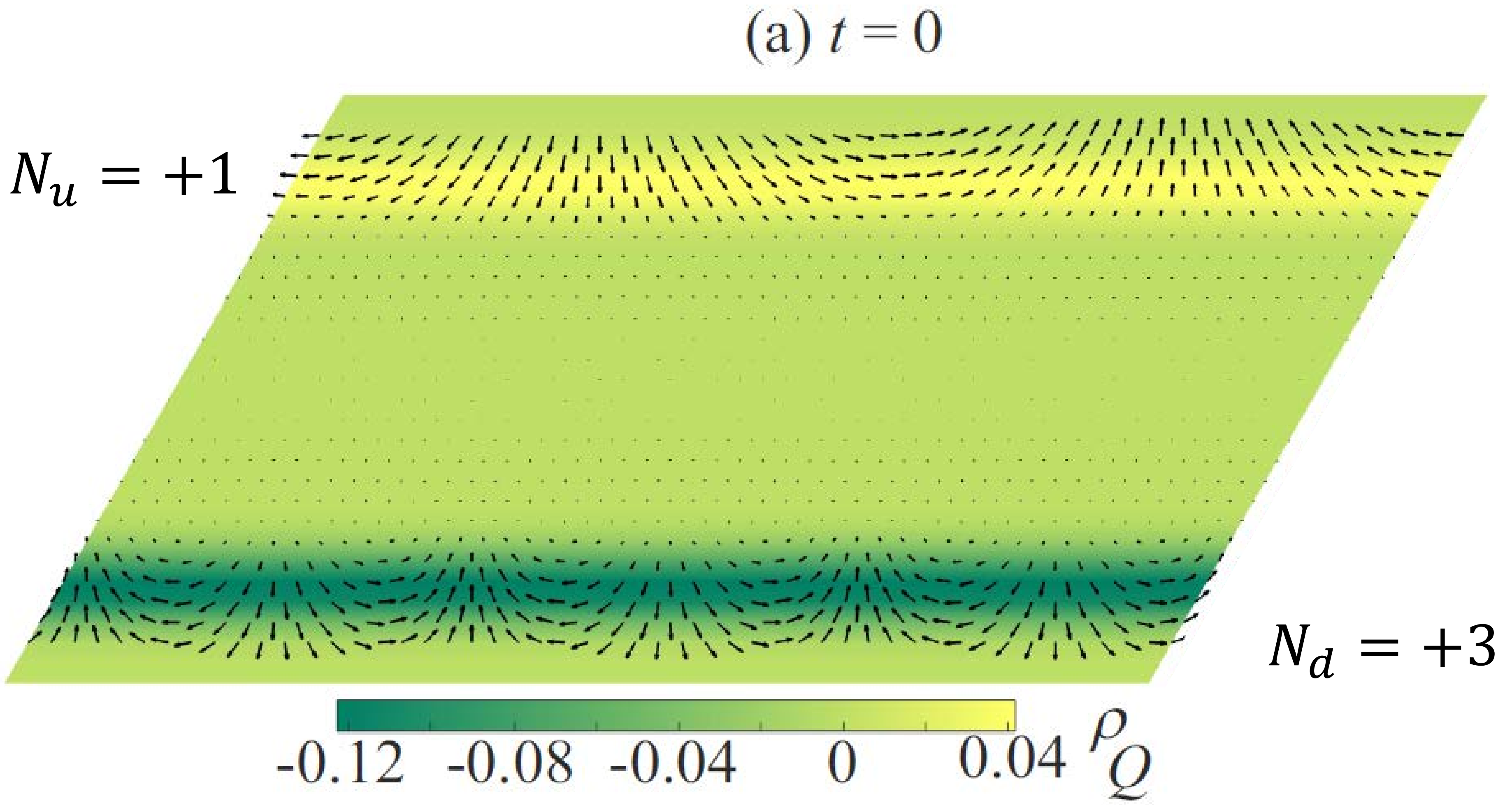
Skyrmion charge of the conical spiral edge state

Skyrmion charge $$Q_{edge} = \pm\frac{1}{2}N_{edge}$$ **Winding number**

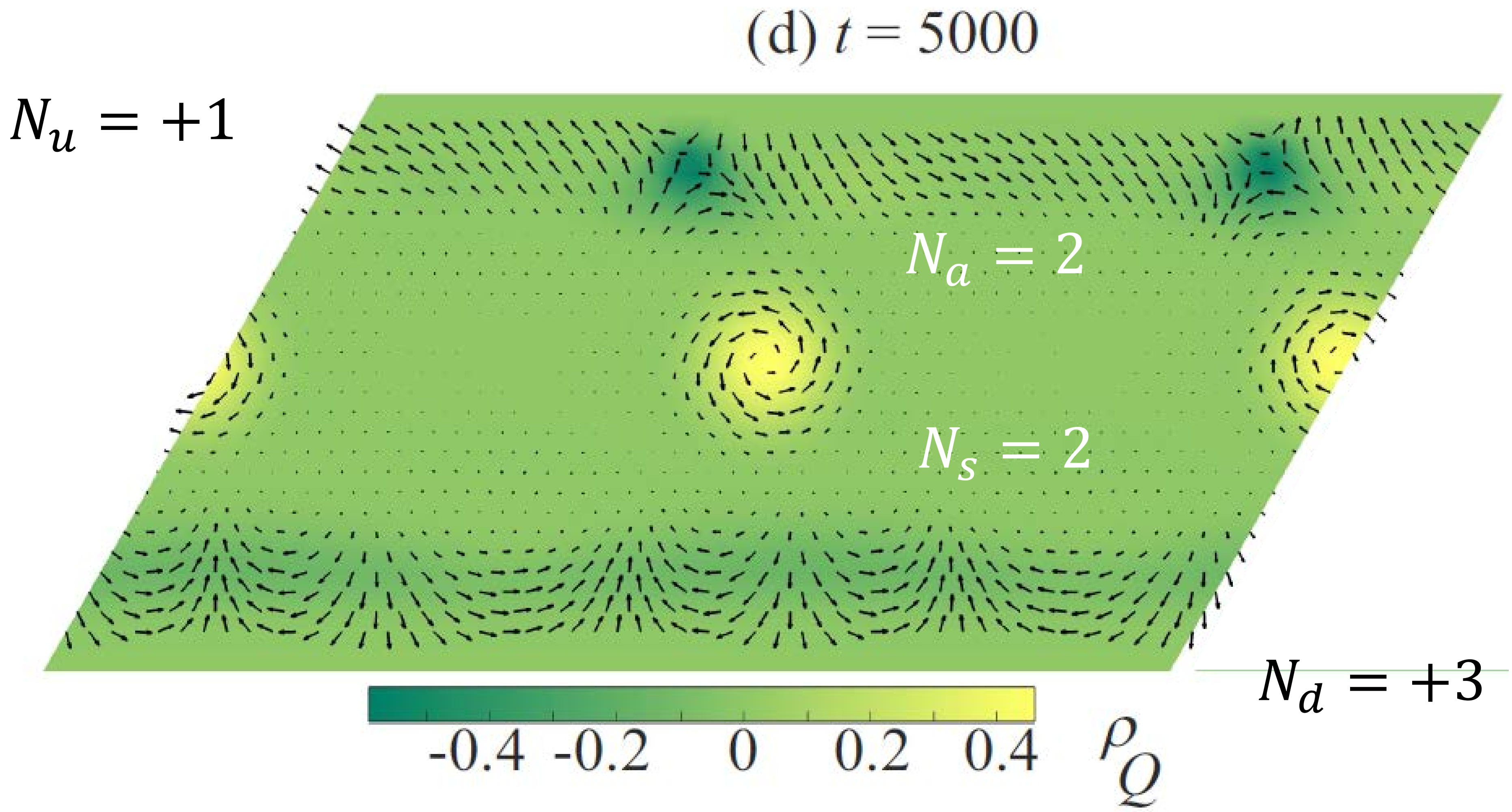
(a) $t = 0$



Initial state



Creation of s-a pairs at the upper edge



Vorticity conservation

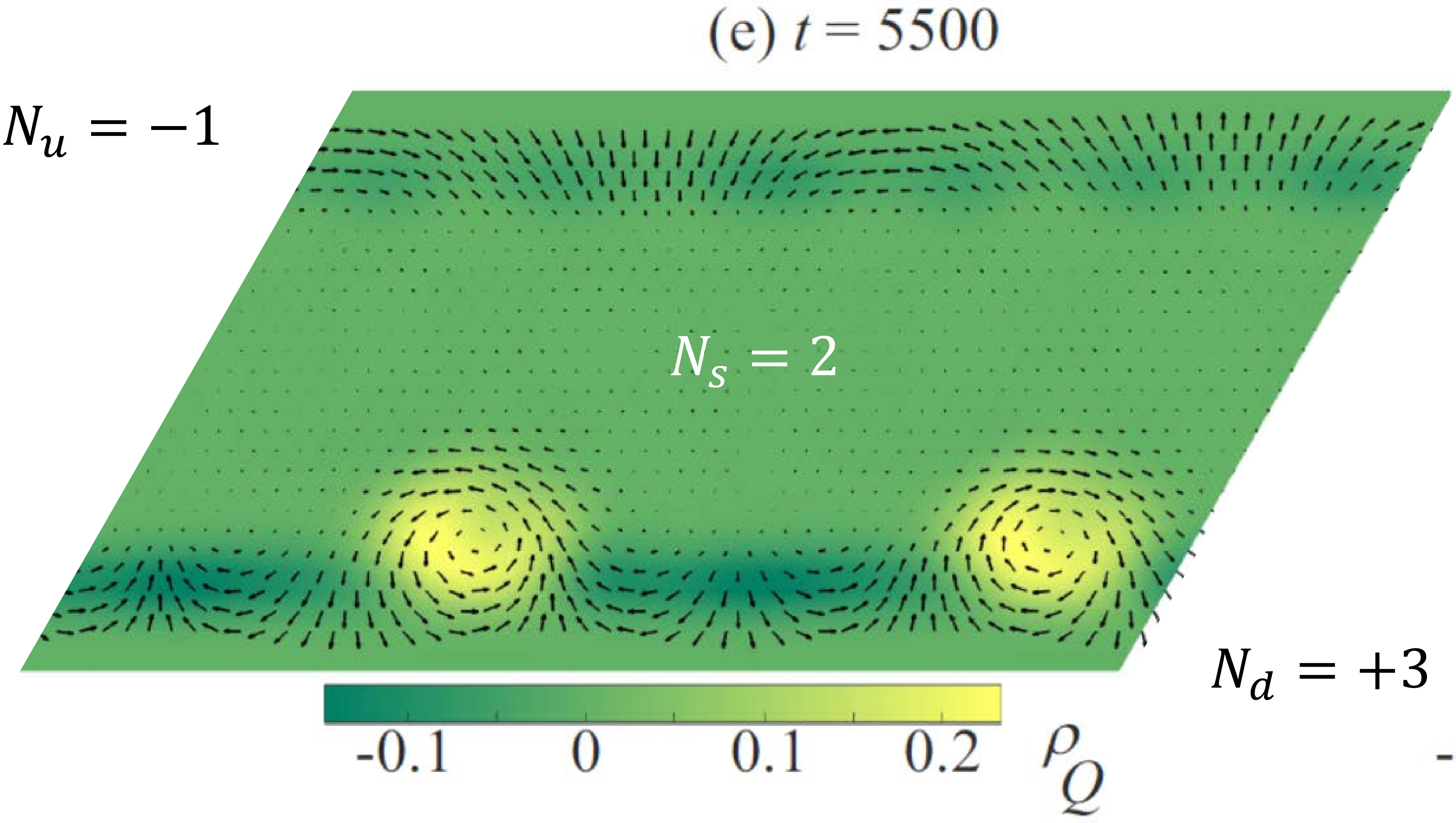
$$V = N_s - N_a + N_u - N_d$$

Number of skyrmions (N_s)

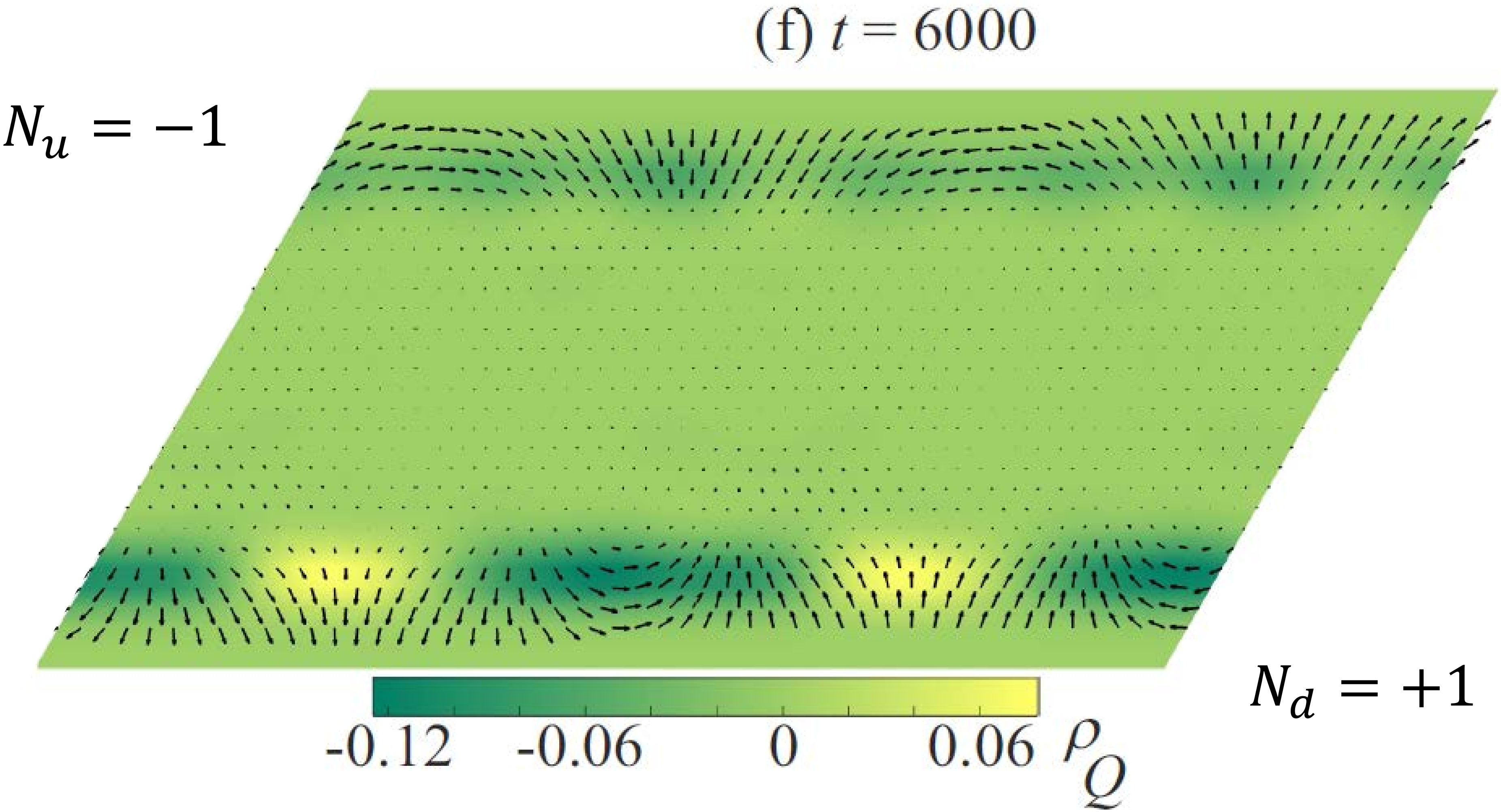
Number of antiskyrmions (N_a)

Winding numbers of edge states (N_u, N_d)

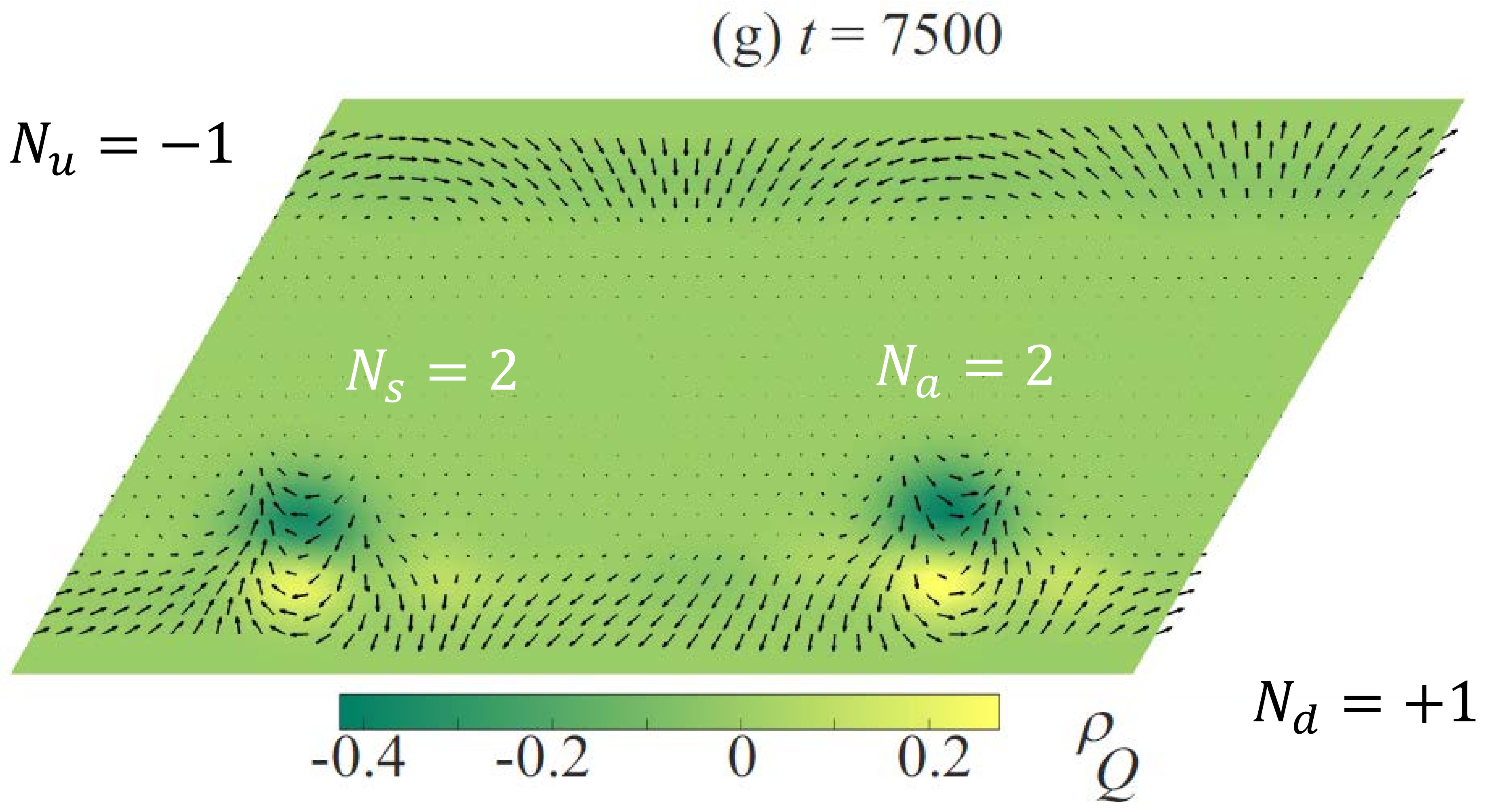
2 antiskyrmions disappeared at the upper edge



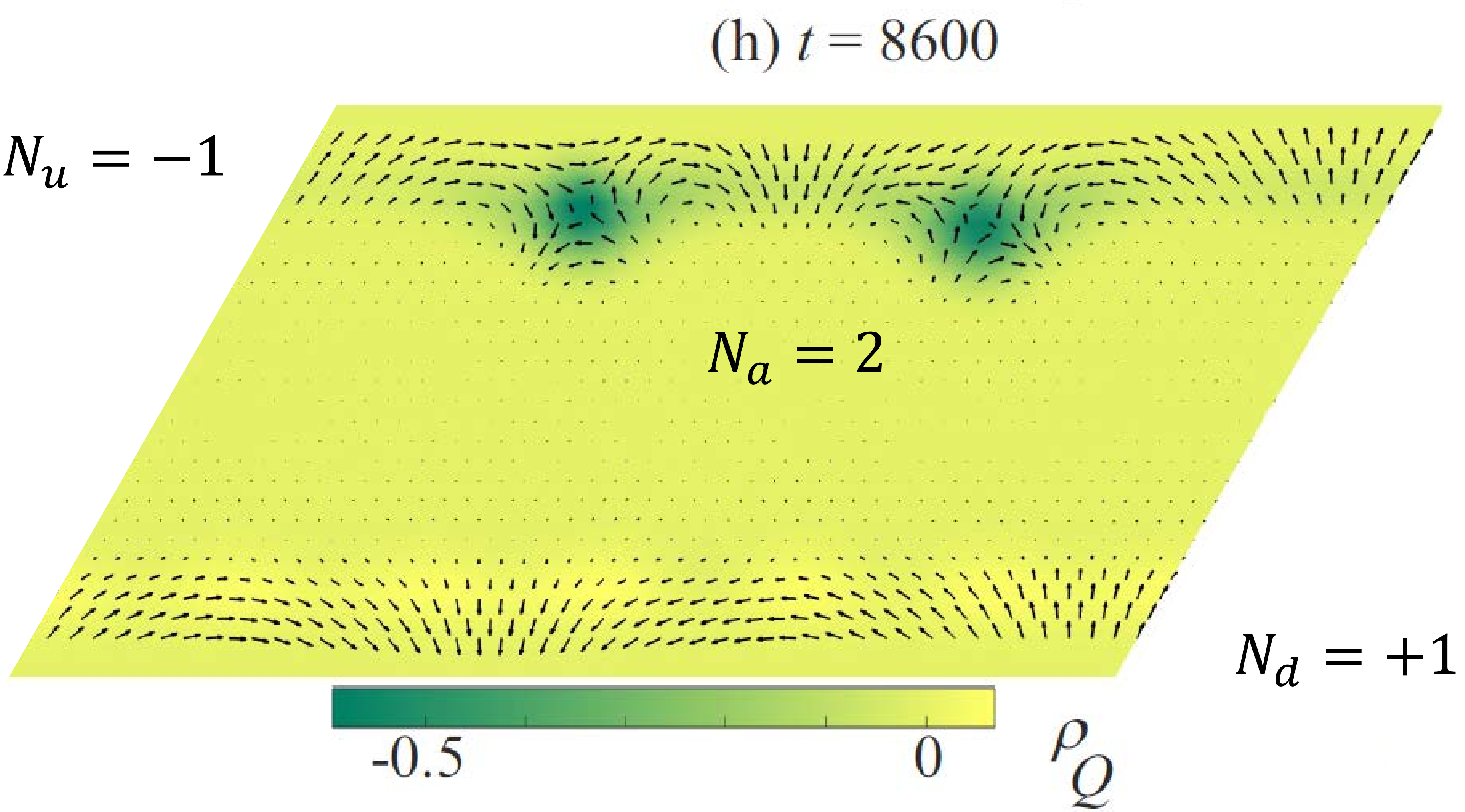
2 skyrmions disappeared at the lower edge



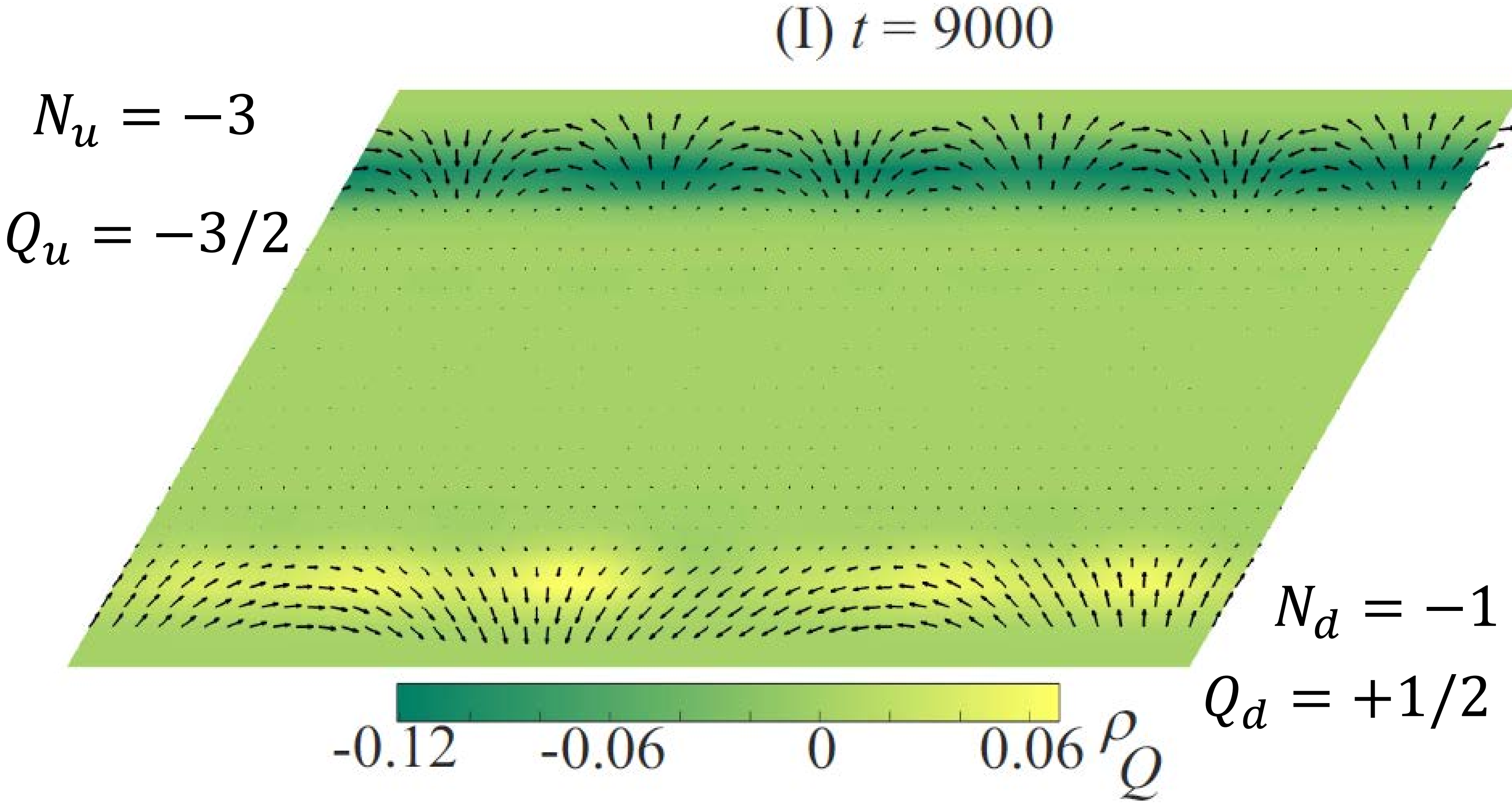
Creation of 2 s-a pairs at the lower edge



2 skyrmions disappeared at the lower edge



2 antiskyrmions disappeared at the upper edge



Conclusions

- Unusual multi-Q skyrmions
- Electric field control of skyrmions in frustrated magnetic insulators
- Skyrmion exchange between topological edge states

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