Universal quasi-steady states in periodically driven many-body systems

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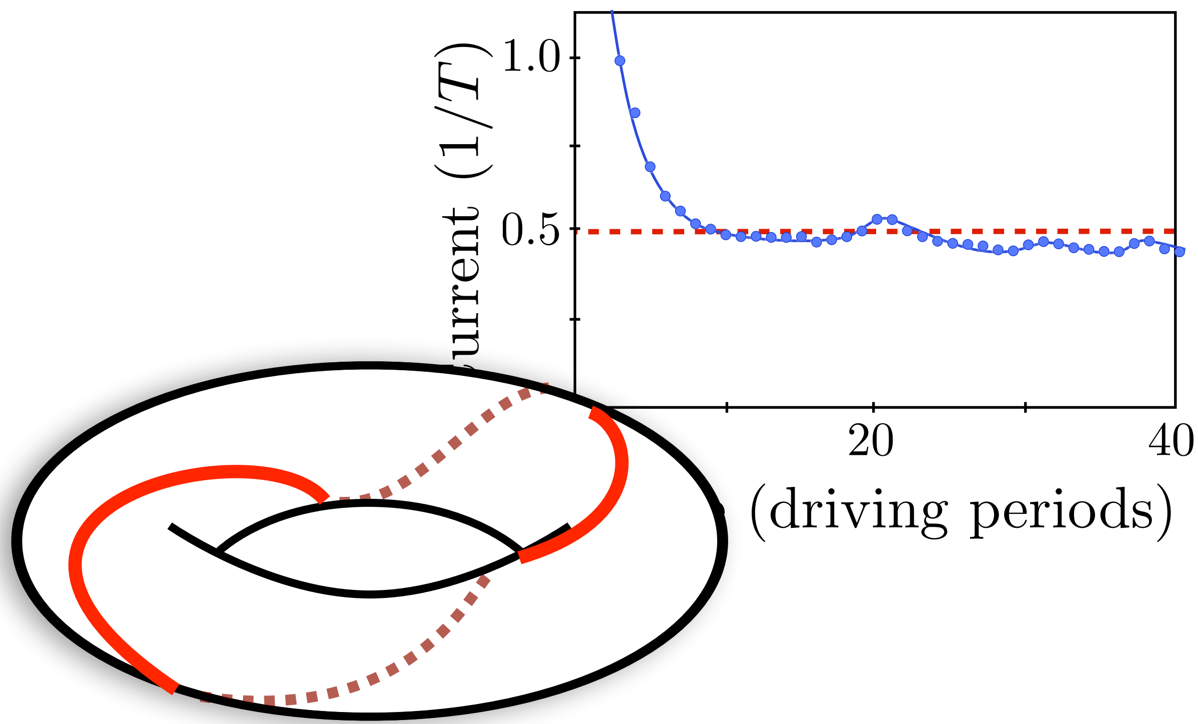
NMP17

8 March 2017



In collaboration with Erez Berg and Netanel Lindner

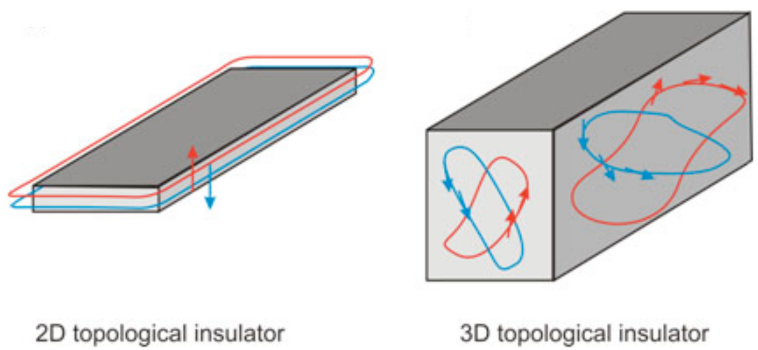
*For details see: N. Lindner, E. Berg, and MR, PRX **7**, 011018 (2017)

VILLUM FONDEN

Advances of the past decade bring new challenges, new tools

Theory

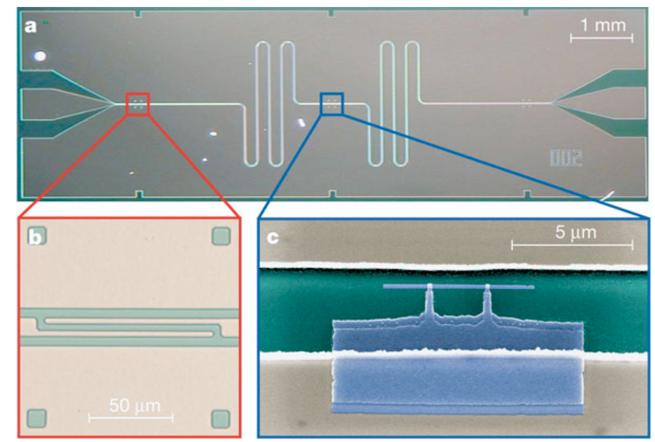
New phases, topological phenomena



M. Z. Hasan, SSRL Science Highlight, March 2009

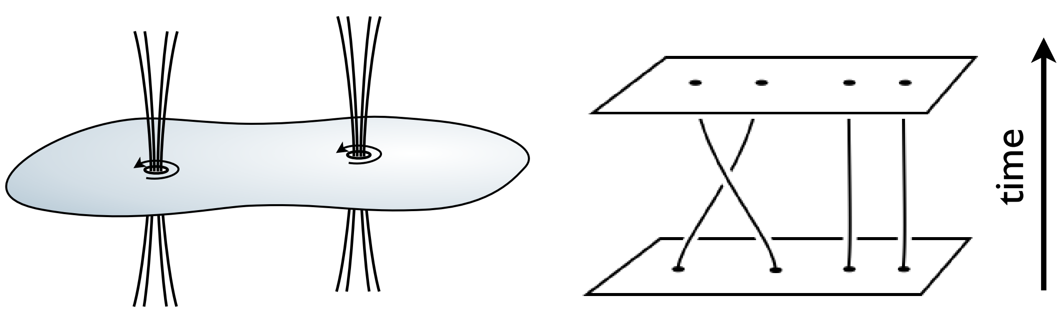
Experiment

Quantum control: MWs, lasers



A. Wallraff. *et al.*, Nature **431**, 162 (2004)

Fault-tolerant quantum computation



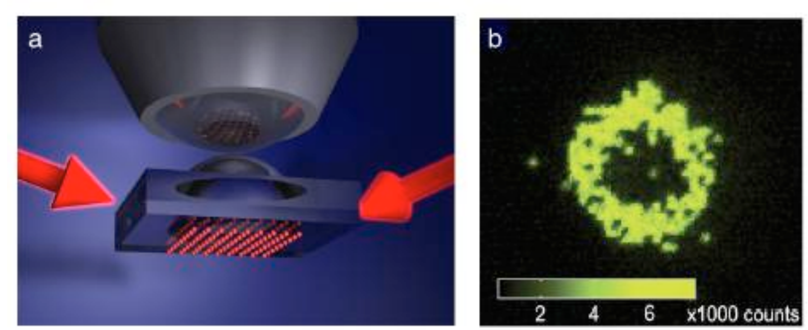


Image from `http://greiner.physics.harvard.edu`

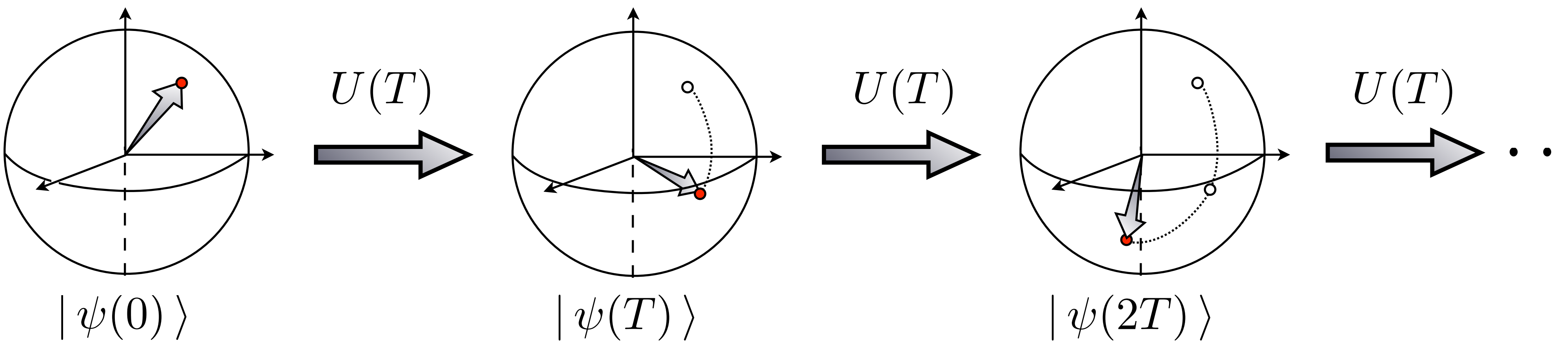
No ground state, energy conservation for driven system

$$i\frac{d}{dt}|\,\psi\,\rangle = H(t)|\,\psi\,\rangle; \quad H(t+T) = H(t)$$

periodic driving

Quasi-energy is conserved for system with discrete time translation symmetry

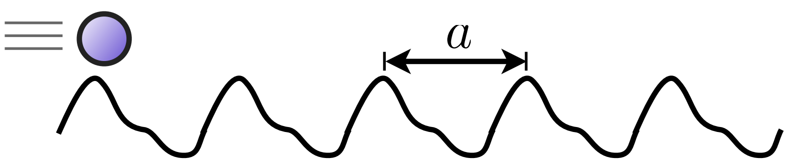
$$\boxed{U(T) = \mathcal{T} e^{-i\int_0^T H(t)dt}} \qquad H(t+T) = H(t)$$



$$\boxed{U(T)|\,\psi_n\,\rangle = e^{-i\varepsilon_n T}|\,\psi_n\,\rangle}$$

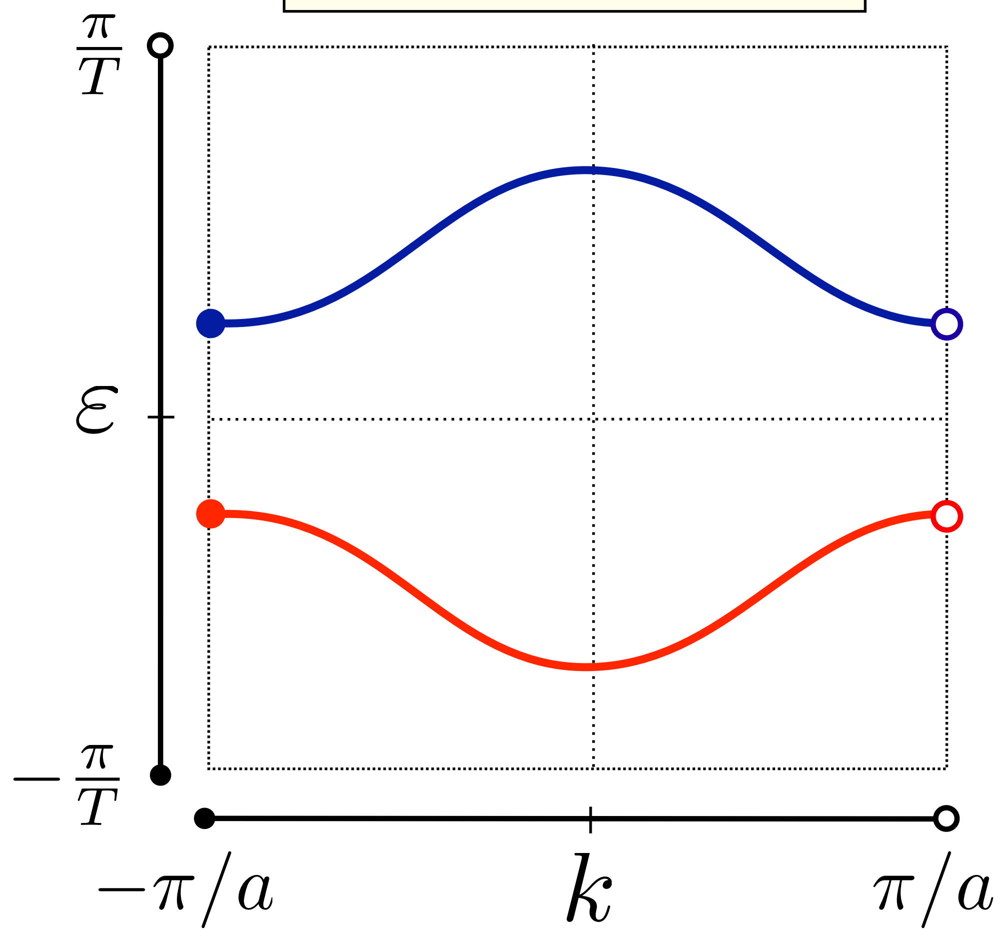
Eigenvalue invariant under $\varepsilon_n \to \varepsilon_n + 2\pi N/T$: quasi-energy lives on a circle

On a lattice find Floquet bands, similar to static system



$$V(x+a) = V(x)$$

$$\boxed{U(T) = e^{-iH_{\text{eff}}T}}$$

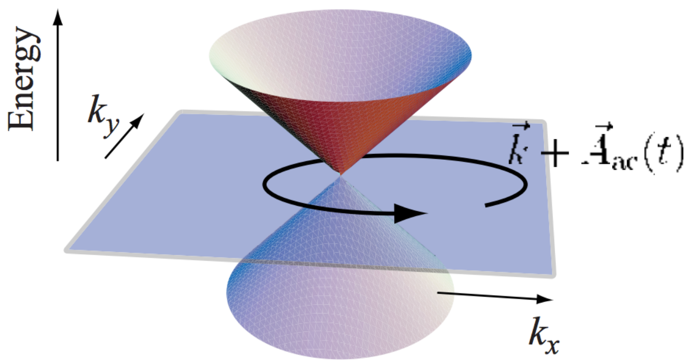


Suggests analogues of topological phenomena from static systems in driven systems

T. Kitagawa, E. Berg, MR, and E. A. Demler, Phys. Rev. B 82, 235114 (2010).

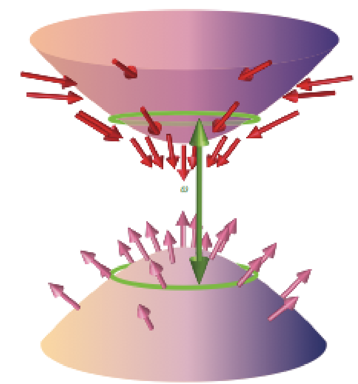
Optical control of band topology proposed for many setups

Circularly-polarized light to generate gap



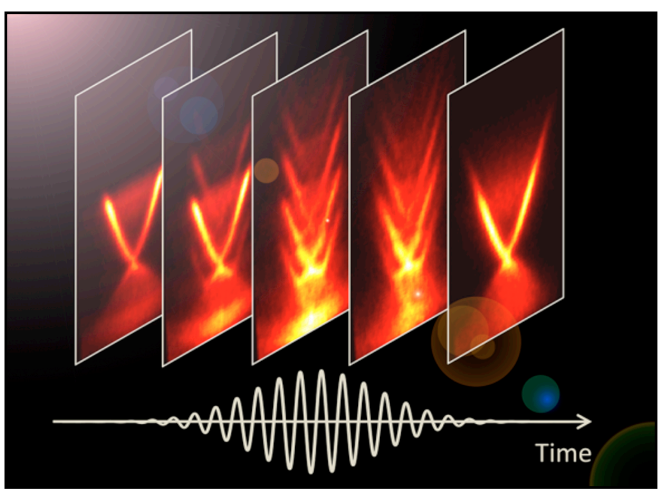
T. Oka and H. Aoki, Phys. Rev. B **79**, 081406 (2009).

Resonant driving to create band inversion



N. Lindner, G. Refael, and V. Galitski, Nature Physics **7**, 490 (2011).

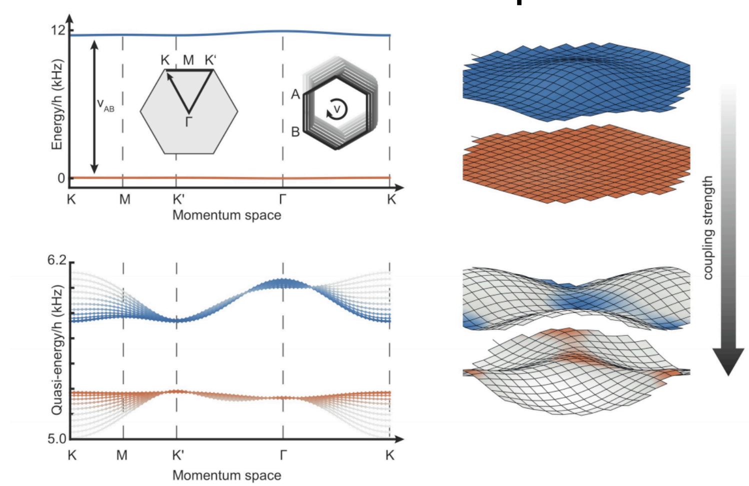
Gapped states on TI surface



Gedik group, MIT

Cold atoms: G. Jotzu, et al., Nature **515**, 237 (2014).

Tune Chern numbers in optical lattices



N. Fläschner, et al., arXiv:1509.05763.

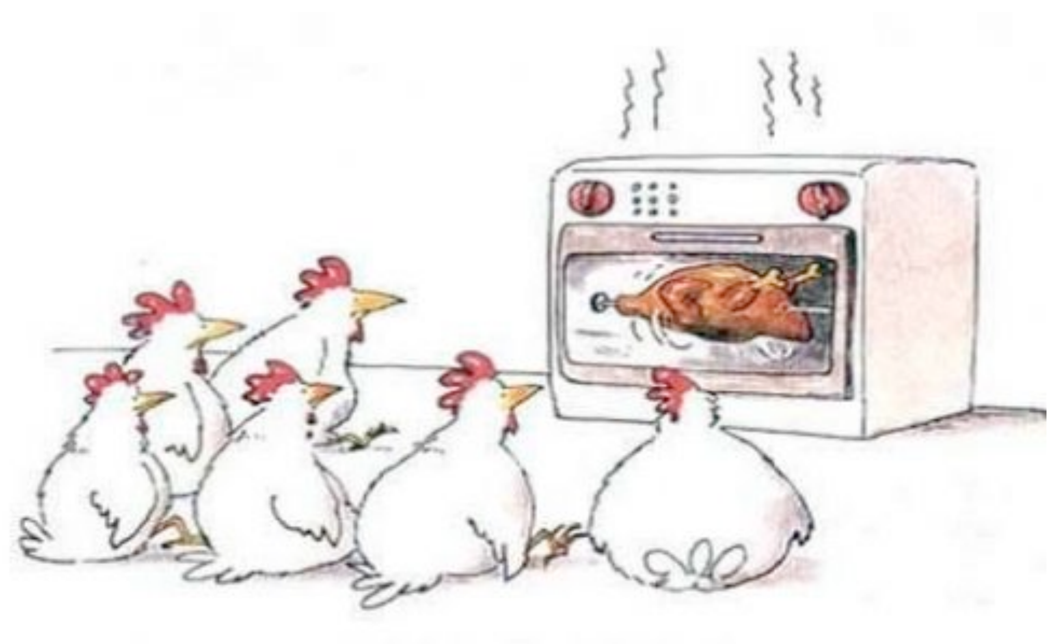
Chiral/topological transport of light

M. C. Rechtsman et al., Nature 496, 196 (2013).

T. Kitagawa, M. Broome, A. Fedrizzi, MR, et al., Nature Comm. 3, 882 (2012).

W. Hu et al., Phys. Rev. X 5, 011012 (2015).

A <u>closed</u>, interacting, periodically driven many-body system generically heats to infinite temperature



From images.google.com

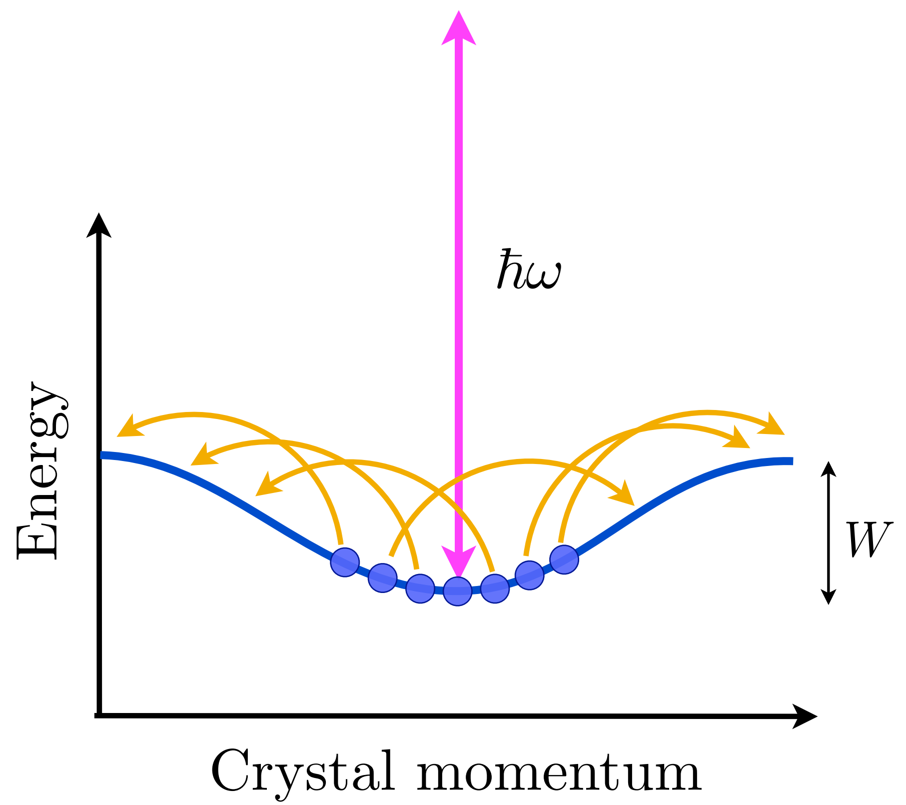
See for example:

L. D'Alessio, M. Rigol, Phys. Rev. X **4**, 041048 (2014).

A. Lazarides, A. Das, and R. Moessner, Phys. Rev. Lett. **112**, 150401 (2014).

P. Ponte, A. Chandran, Z. Papic, D. A. Abanin, Annals of Physics **353**, 196 (2015).

Energy absorption exponentially suppressed at high frequency



Single photon absorption requires high-order ($\hbar\omega/W$) rearrangement

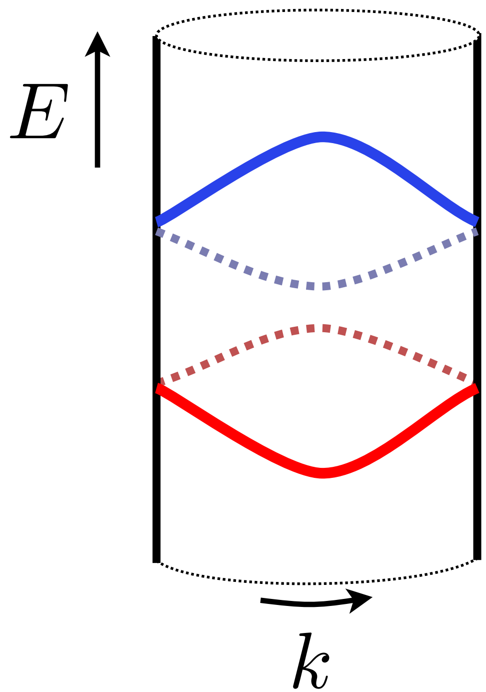
D. A. Abanin, W. De Roeck, and F. Huveneers, Phys. Rev. Lett. **115**, 256803 (2015).

M. Bukov, L. D'Alessio, A. Polkovnikov, Adv. in Phys. **64**, 139 (2015)

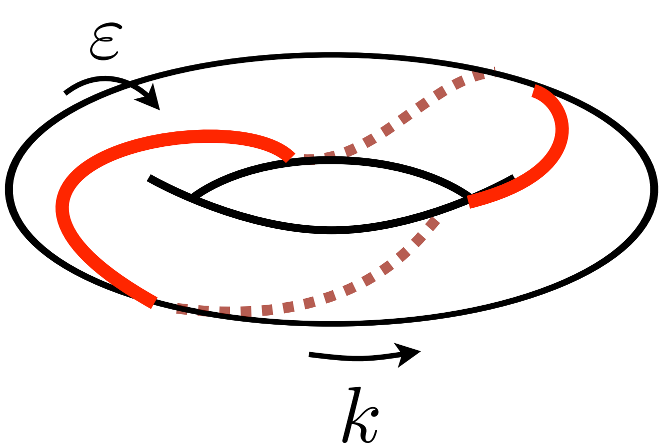
Universal chiral quasi-steady states emerge from interactions and unique topology of Floquet bands

New topological configurations possible in driven systems

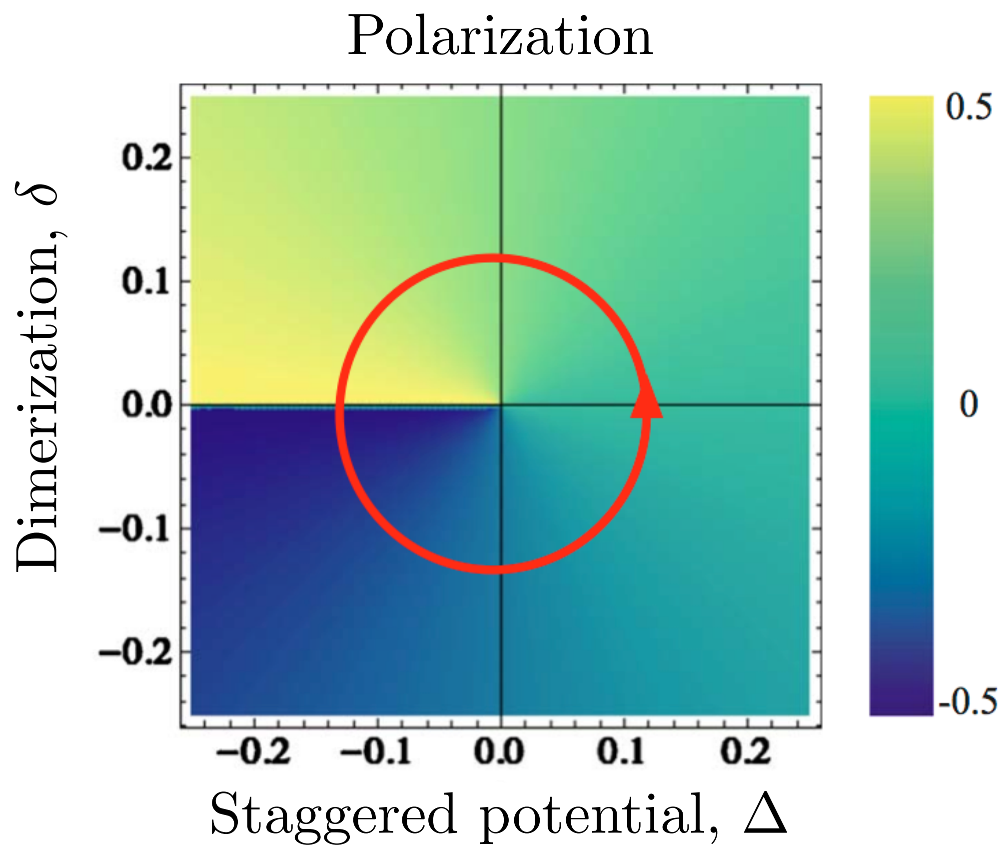
Normal band structure: <u>cylinder</u>



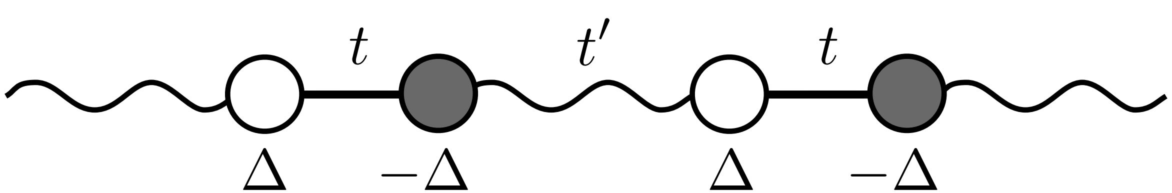
Quasi-band structure: <u>torus</u>



Gapped system: charge pumped via adiabatic cycle is quantized



Rev. Mod. Phys. **82**, 1959 (2010)



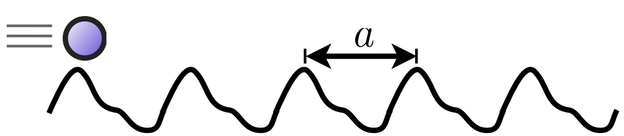
$$t = t_0 + \delta$$
$$t' = t_0 - \delta$$

D. J. Thouless, Phys. Rev. B **27**, 6083 (1983).

Quasi-energy winding related to quantized adiabatic transport

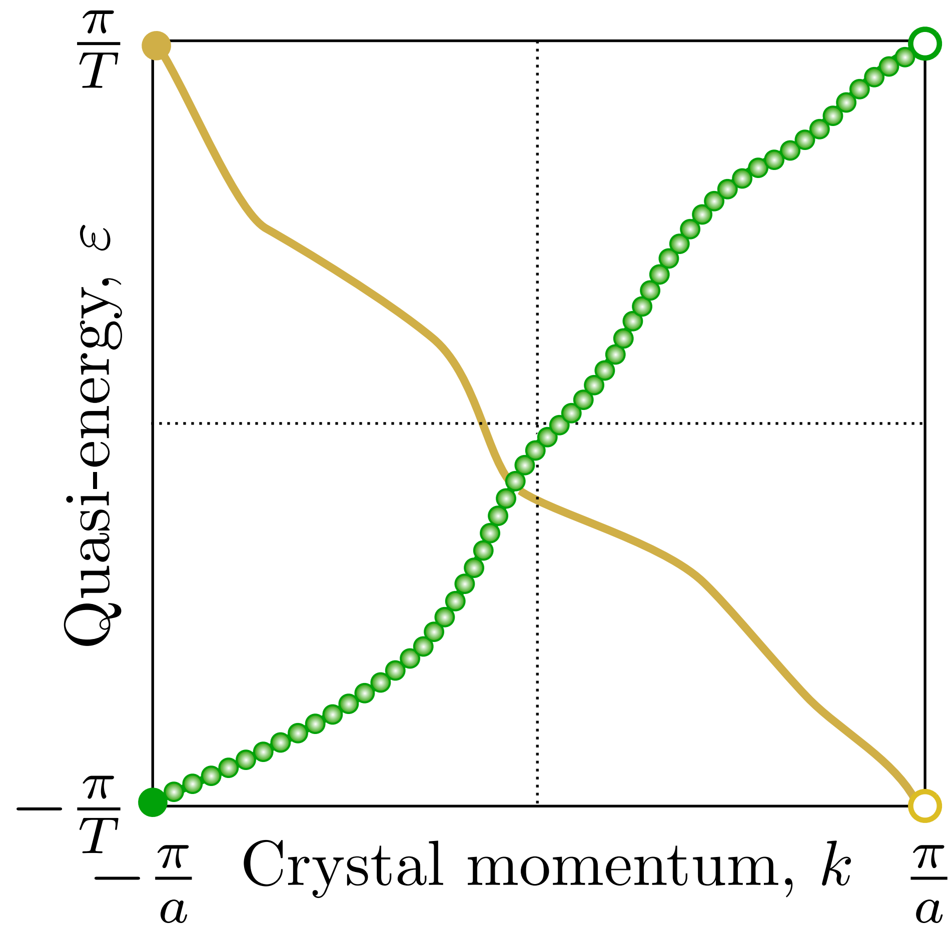
Average group velocity <u>quantized</u>

$$\bar{v}_g = \overline{\frac{d\varepsilon_k}{dk}} = a/T$$



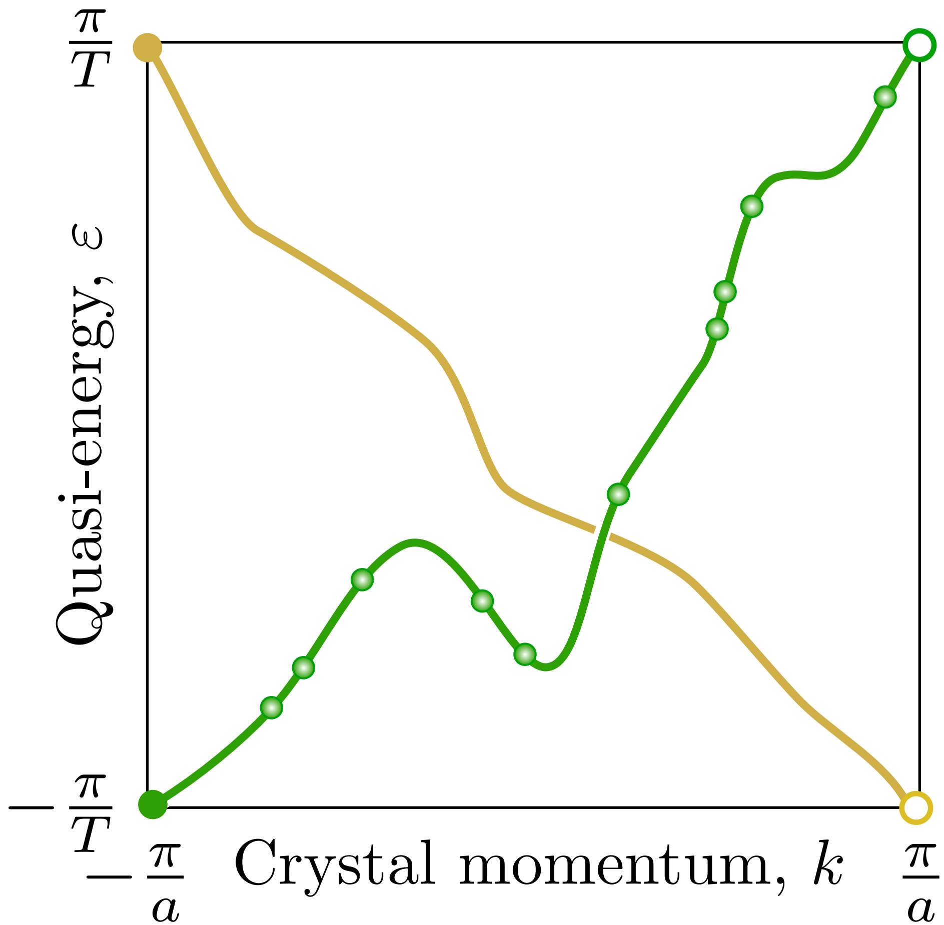
$$V(x+a) = V(x)$$

$$U(T)|\,\psi(k)\,\rangle = e^{-i\varepsilon_k T}|\,\psi(k)\,\rangle$$

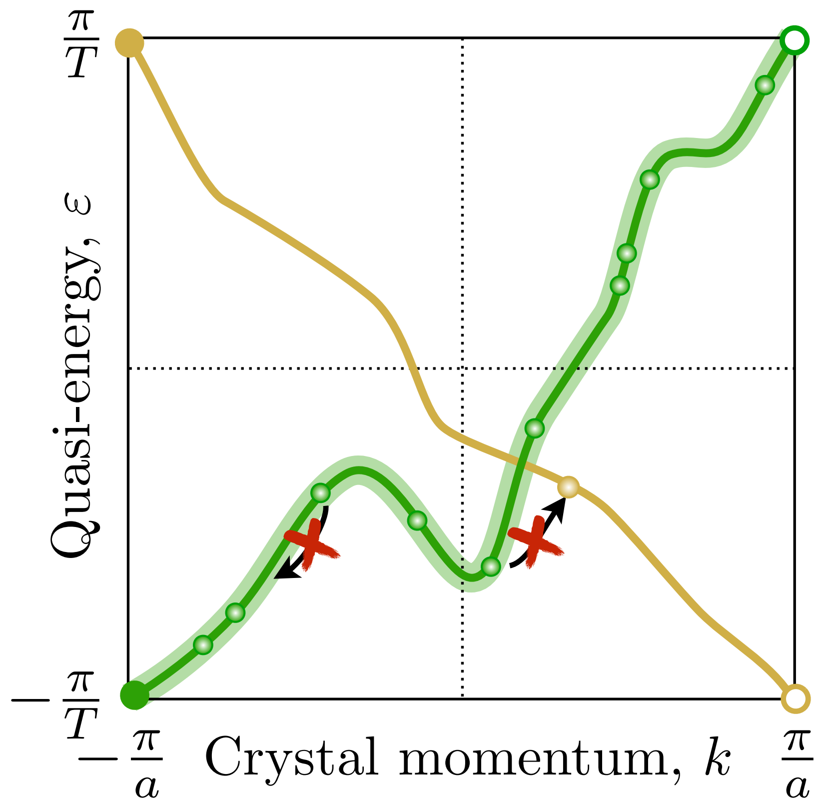


T. Kitagawa, E. Berg, MR, and E. A. Demler, Phys. Rev. B 82, 235114 (2010).

Current carried by partially-filled band can be anything



Restricted infinite-temperature-like state within a band yields uniform averages, restores universality

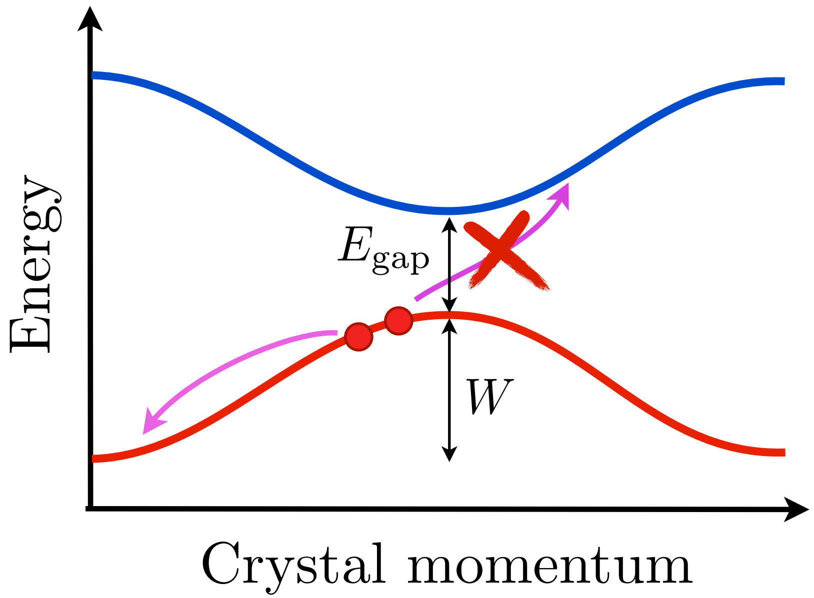


* Universal quantized pumping coefficient!

$$\boxed{(\text{current}) = (1/T) \times (\text{density})}$$

Must suppress both <u>direct</u> and <u>photon-assisted</u> processes

Direct scattering

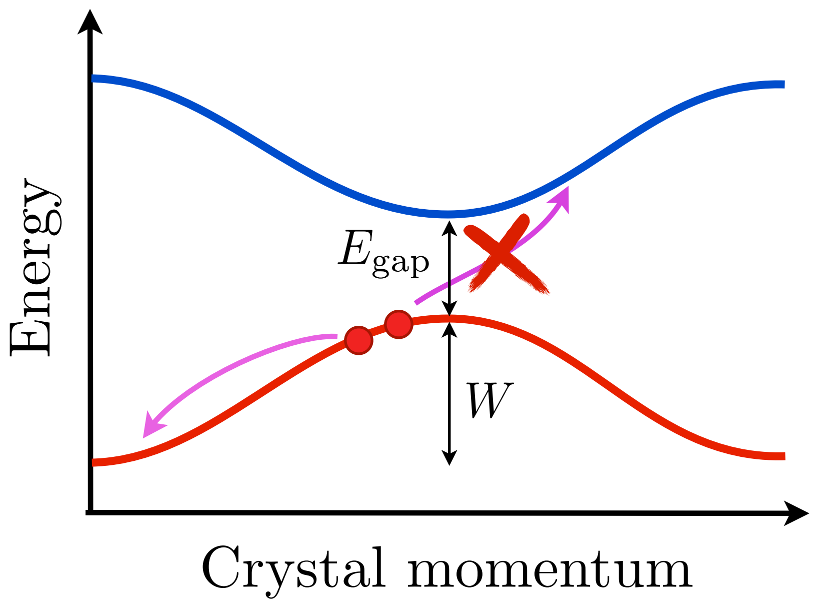


Small bandwidth

$$W/E_{\text{gap}} \ll 1$$

Must suppress both direct and photon-assisted processes

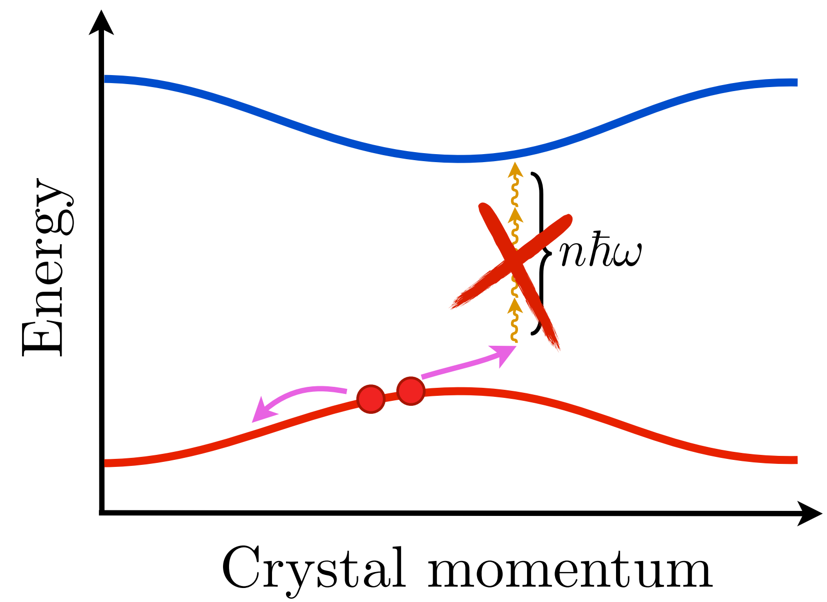
Direct scattering



Small bandwidth

$$W/E_{\mathrm{gap}} \ll 1$$

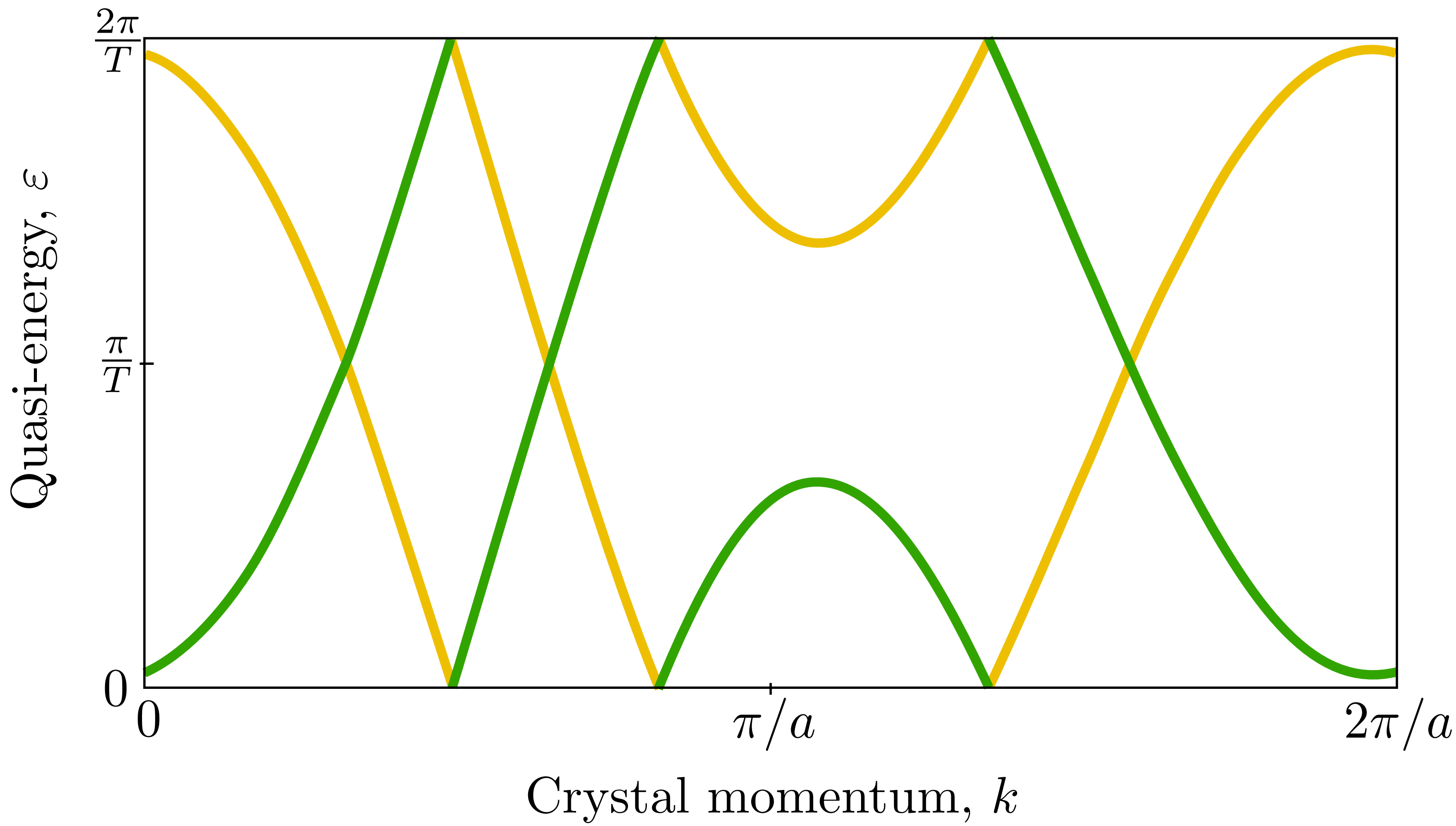
Photon-assisted scattering



Single-particle adiabaticity

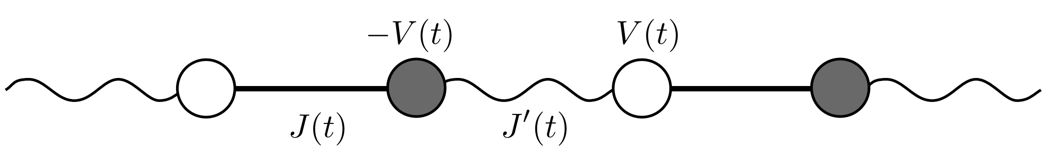
$$\hbar\omega/E_{\mathrm{gap}} \ll 1$$

Floquet bands exhibit nontrivial winding



$$\frac{V_1}{\hbar\omega} = 8.2$$

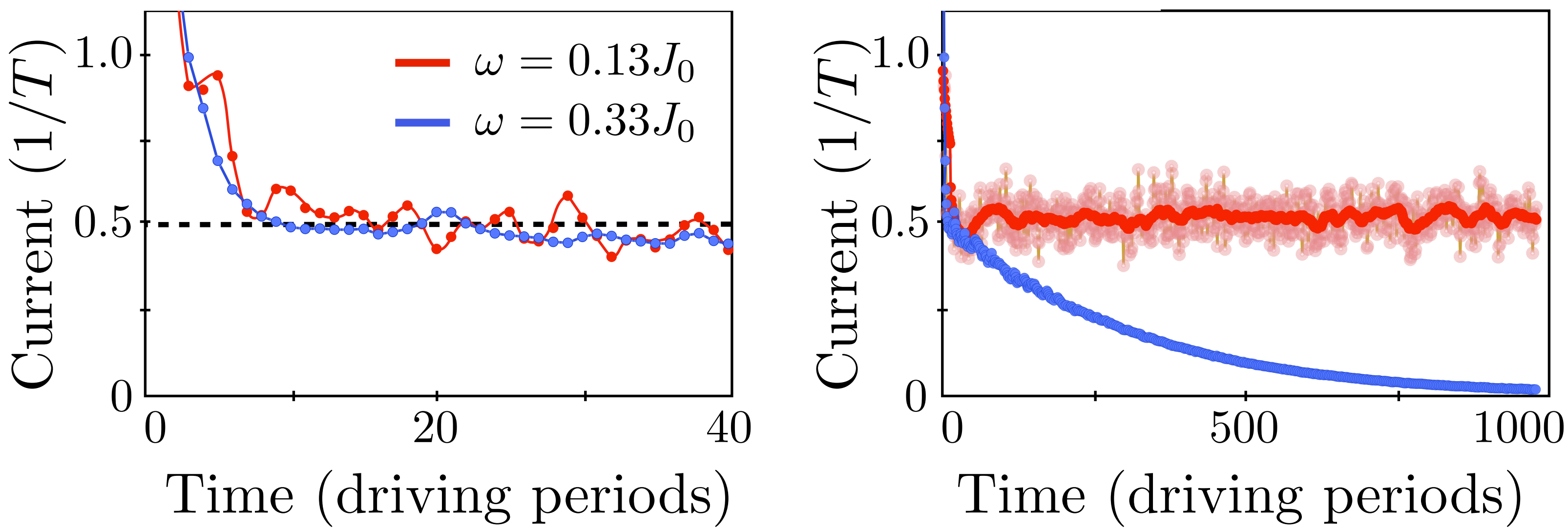
$$\frac{J_0}{\hbar\omega} = 4.1$$

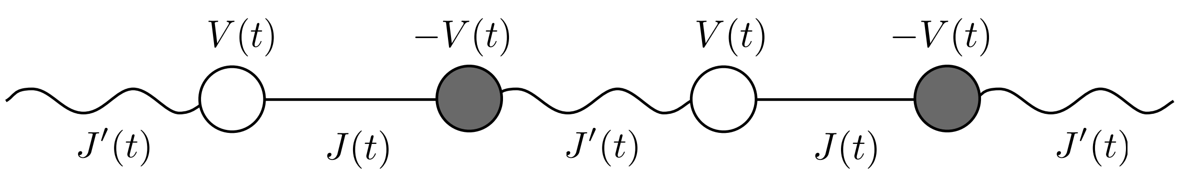


$V(t) = V_1 \sin \omega t$ $\quad J(t) = J_0(1 + \cos \omega t)$ $\quad J'(t) = J_0(1 - \cos \omega t)$

Two timescales emerge for intraband equilibration and interband scattering (decay of current)

Numerics: 8 fermionic particles, 32 sites (16 unit cells)

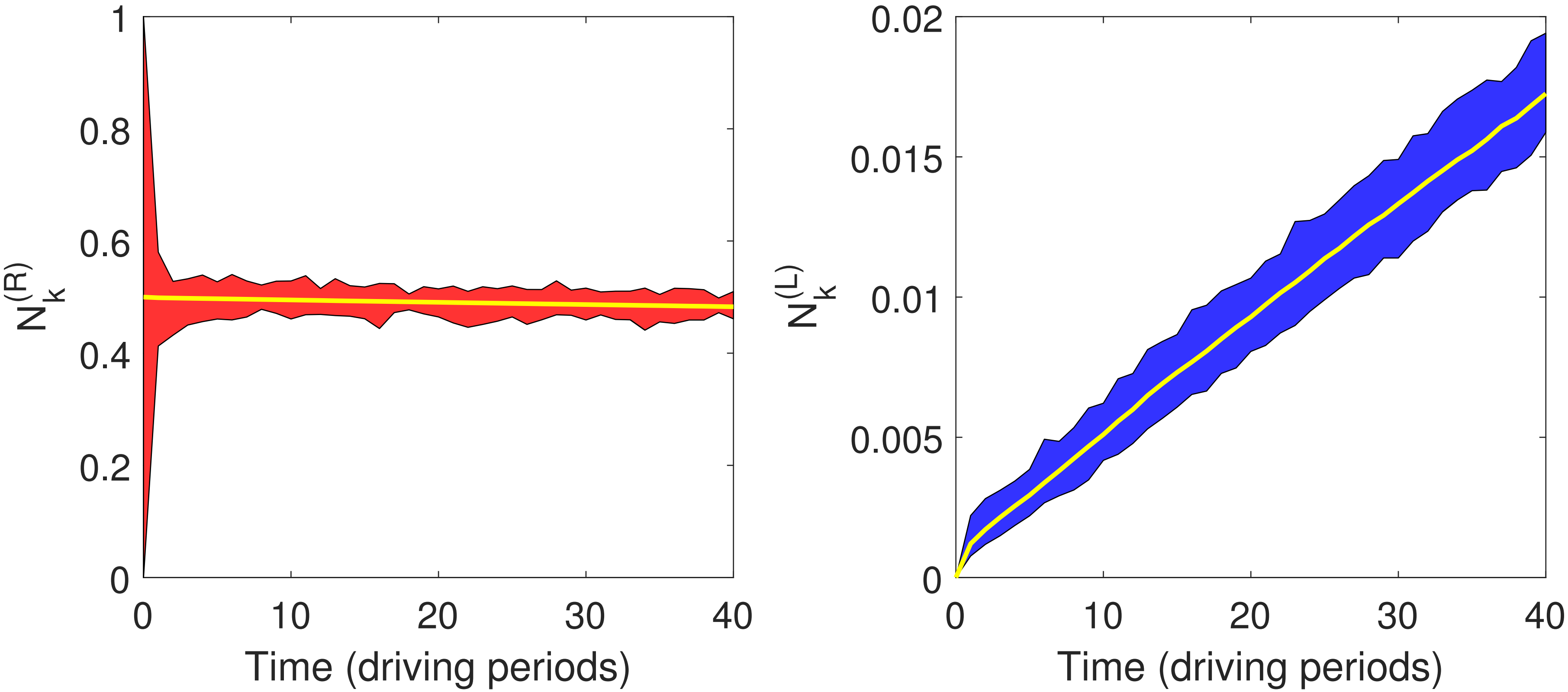




$$J_1 = \frac{2}{3} J_0$$
$$V_1 = 2 J_0$$
$$\omega = \frac{1}{3} J_0$$

$J(t) = J_0 + J_1 \cos \omega t \quad J'(t) = J_0 - J_1 \cos \omega t \quad V(t) = V_1 \sin \omega t$

Populations rapidly converge to quasi-steady distribution



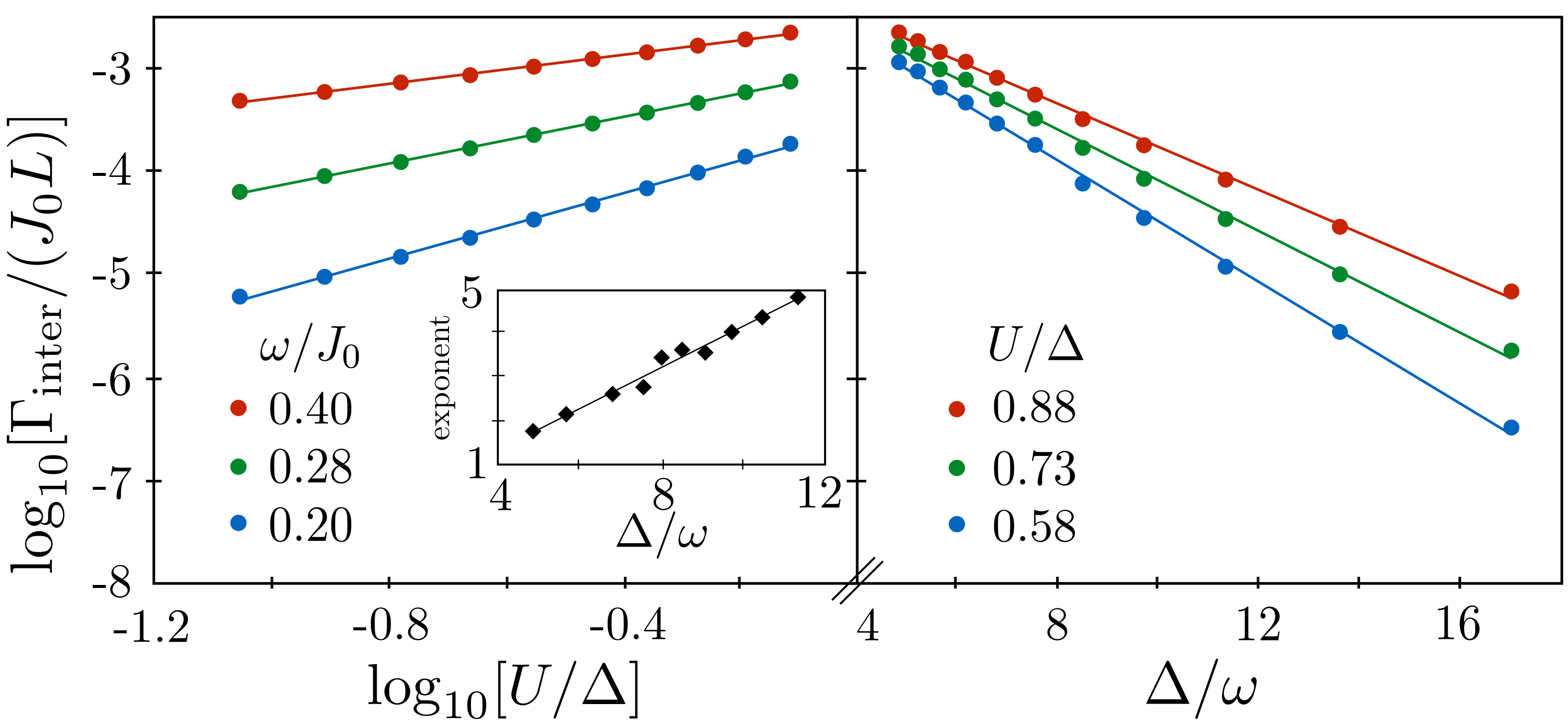
High-order perturbation theory predicts exponential suppression of scattering in $1/\omega$

interaction

minimal order of photon absorption

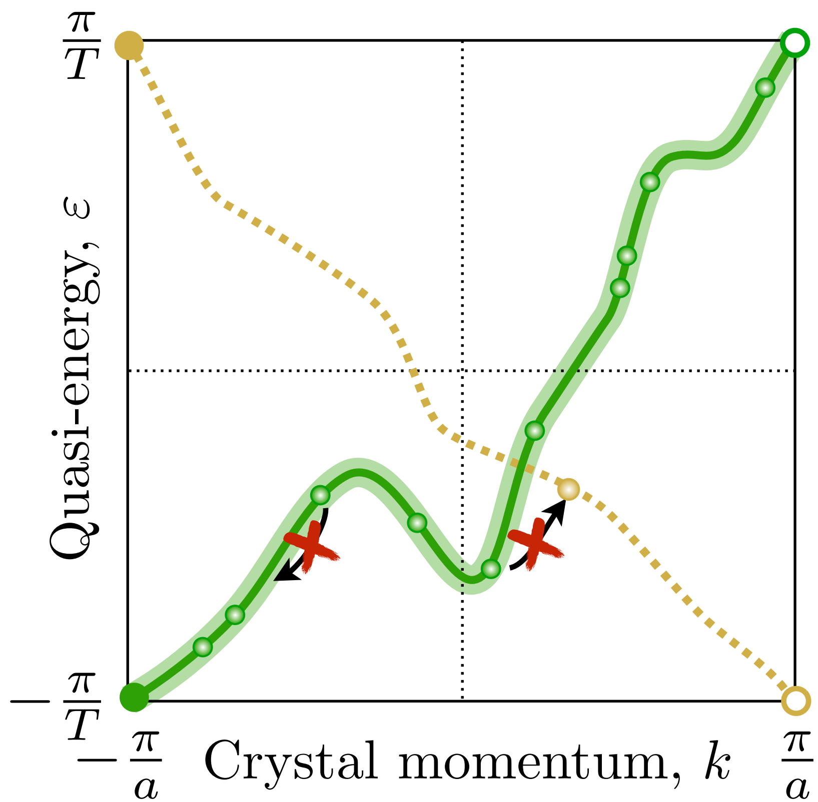
$$\Gamma_{\text{inter}} \sim \left(\frac{\alpha U}{\Delta}\right)^{\frac{\Delta}{\delta m \omega}}$$

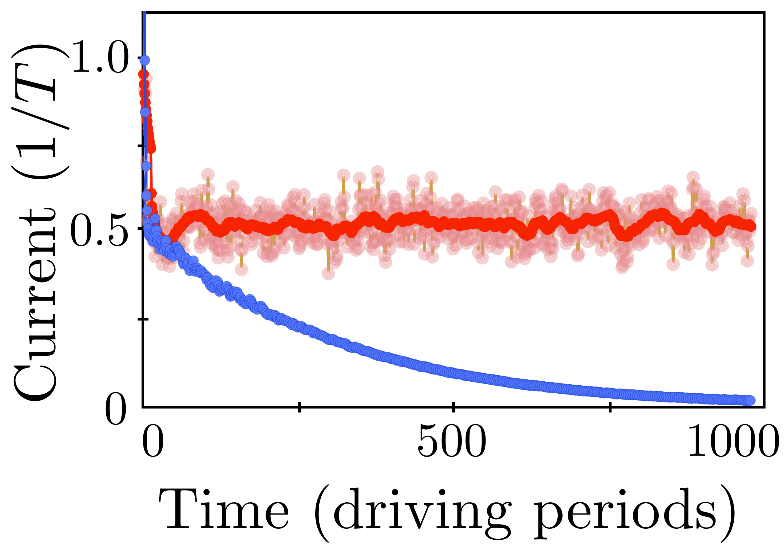
minimal instantaneous band gap



N. Lindner, E. Berg, and MR, arXiv:1603.03053.

Interaction-induced infinite-temperature-like quasi-steady state shows universal behavior for exponentially long time





* New regime of prethermalization and strongly interacting non-equilibrium topological matter exposed

Summary and open questions

Floquet bands in 1D may exhibit non-trivial quasi-energy winding

Chiral quasi-steady states form when interband scattering is suppressed

Universality of quasi-steady behavior apparently persists even when intraband scattering is *fast* compared with driving frequency

Novel prethermalization regime may extend to other dimensions

For details see: N. Lindner, E. Berg, and MR, PRX 7, 011018 (2017)

Contact: rudner@nbi.dk

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Floquet formalism maps time-dependent problem to eigenvalue problem on enlarged space

$$\boxed{H(t) = H_0 + \Delta e^{i\omega t} + \Delta^\dagger e^{-i\omega t} \longrightarrow \mathcal{H}\varphi_n = \varepsilon_n \varphi_n}$$

$$
\begin{array}{ccc}
 & \begin{array}{ccccc} & & z' & & \\ \cdots & 1 & 0 & -1 & \cdots \end{array} & \\
\mathcal{H} = & \left(\begin{array}{ccc} & & \\ H_0+\omega & \Delta & \\ \Delta^\dagger & H_0 & \Delta \\ & \Delta^\dagger & H_0-\omega \\ & & \end{array}\right) & \begin{array}{cc} \vdots & \\ 1 & \\ 0 & z \\ -1 & \\ \vdots & \end{array}
\end{array}
$$

Two-particle state: $\psi_{n\alpha}(\mathbf{k},t) = e^{-i\varepsilon_n(\mathbf{k})t} \sum_{z=-\infty}^{\infty} \underline{\varphi_{n\alpha}^{(z)}(\mathbf{k})} e^{iz\omega t}$

$\mathbf{k} = (k_1, k_2)$

n : band index

α : basis state index

z : harmonic index

Scattering: treat Floquet matrix as many-band Hamiltonian

Evolve in "fake time" with Floquet "extended" Hamiltonian

$$\boxed{i\partial_\tau \varphi = (\mathcal{H}_0 + \mathcal{V})\varphi}$$

Kinetic energy + driving

$$\mathcal{H}_0 = \begin{pmatrix} H_0+\omega & \Delta & \\ \Delta^\dagger & H_0 & \Delta \\ & \Delta^\dagger & H_0-\omega \end{pmatrix}$$

Interactions <u>diagonal</u> in *z*

$$\mathcal{V} = \begin{pmatrix} V & 0 & \\ 0 & V & 0 \\ & 0 & V \end{pmatrix}$$

Express "fake time" evolution of Fourier vector $\varphi(\tau)$ in terms of its Fourier transform $\tilde{\varphi}(\varepsilon)$

$$\varphi(\tau) = \int_{-\infty}^{\infty} d\varepsilon \, e^{i\varepsilon\tau} \tilde{\varphi}(\varepsilon)$$

$$\boxed{\tilde{\varphi}(\varepsilon) = \tilde{\varphi}^{(0)}(\varepsilon) \; + \; \mathcal{G}_0(\varepsilon)\mathcal{T}(\varepsilon)\, \tilde{\varphi}^{(0)}(\varepsilon)}$$

"free" initial state

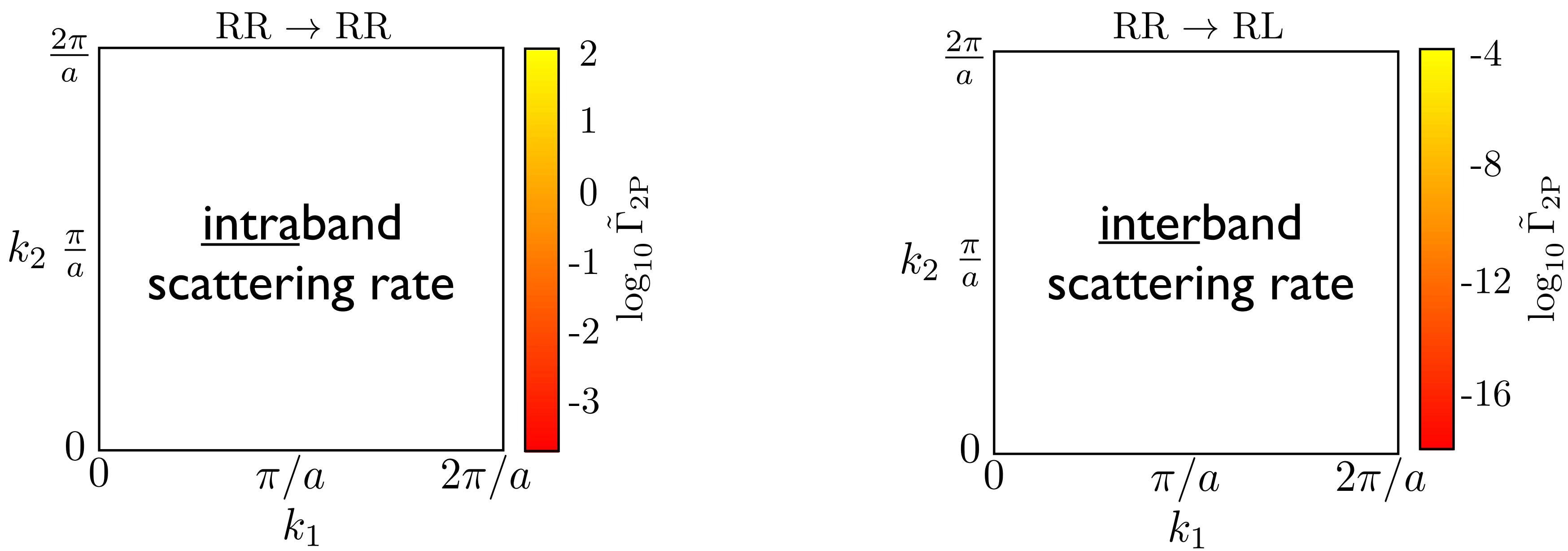
Floquet T-matrix and Green's function:

$$\boxed{\begin{aligned} \mathcal{T}(\varepsilon) &= \mathcal{V} + \mathcal{V}\,\mathcal{G}_0(\varepsilon)\mathcal{T}(\varepsilon) \\ \mathcal{G}_0(\varepsilon) &= (\varepsilon - \mathcal{H}_0 + i\delta)^{-1} \end{aligned}}$$

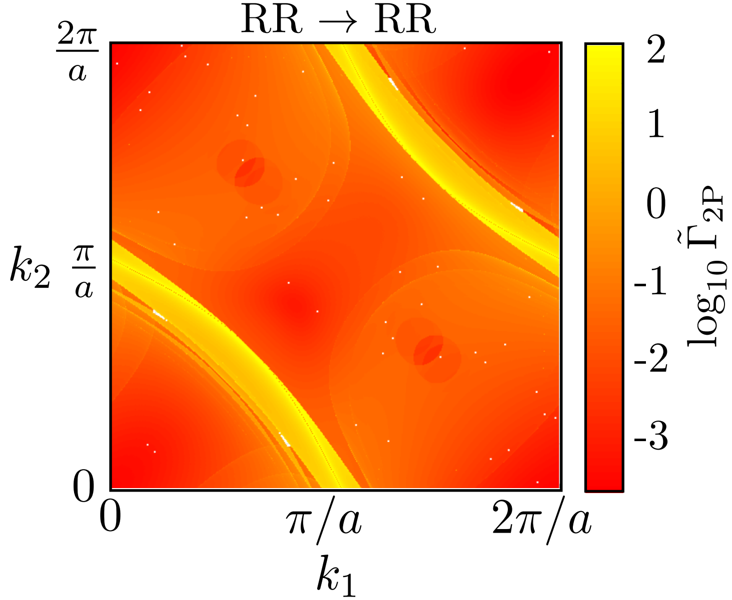
Floquet scattering rates within Born approximation studied in:

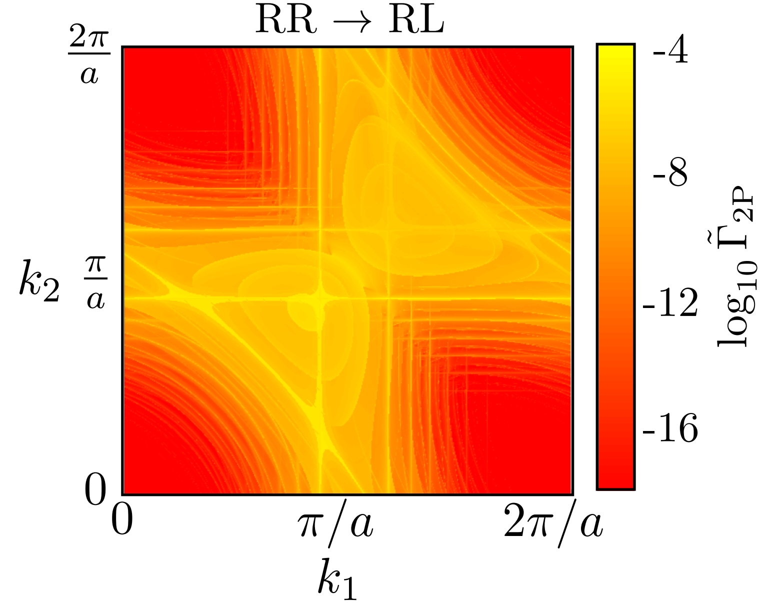
T. Bilitewski and N. R. Cooper, Phys. Rev. A **91**, 033601 (2015).

Interband scattering strongly suppressed at Born level



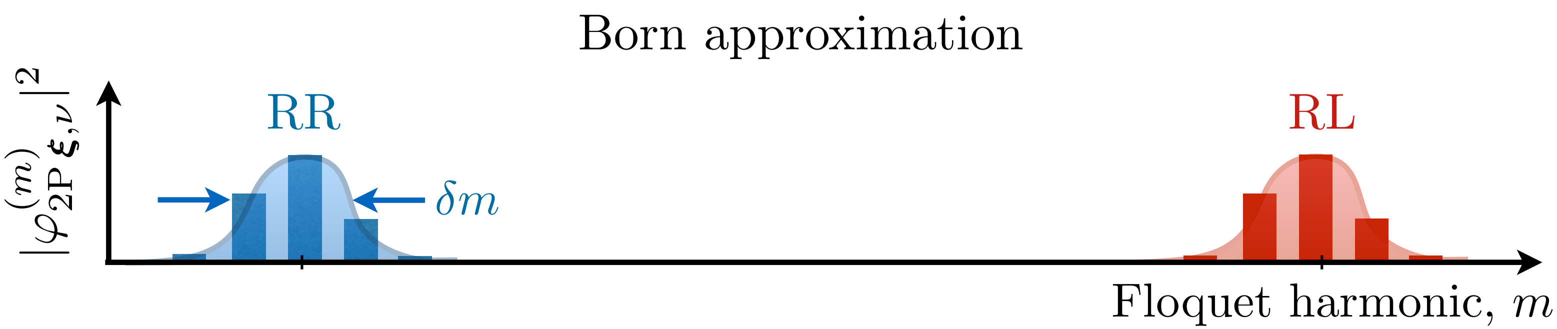
Interband scattering strongly suppressed at Born level



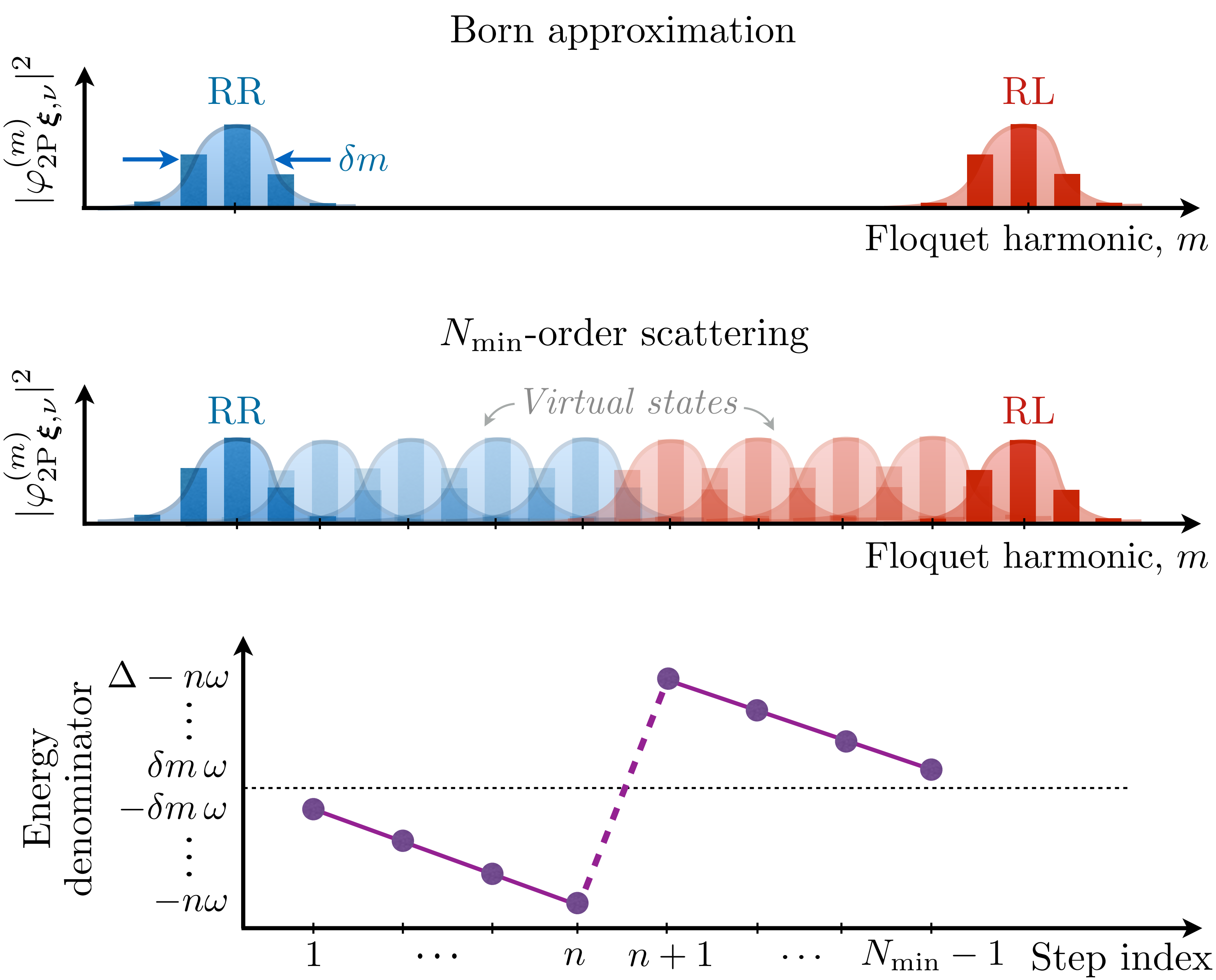


Interbanding scattering strongly suppressed at Born level

High-order perturbation theory predicts exponential suppression of scattering in $1/\omega$



High-order perturbation theory predicts exponential suppression of scattering in $1/\omega$



Finite size effects are weak at L = 16 unit cells, N = 8 particles