Constant temperature description of the nuclear level densities

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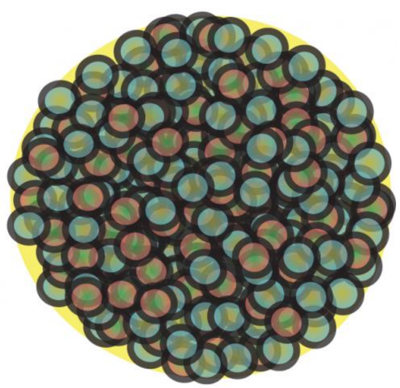
NUCLEI

Overview

- The interactive shell model and effective Hamiltonians
- Spin and parity moments method and nuclear level density
- Applications to reaction rates
- Constant temperature (CT) parameterization
- Features of the CT parameters in the fp shell and beyond
- Summary and Outlook

Microscopic Models of Nuclear Structure

- Take into account the quantum motion of many nucleons
- Some nucleons will be consider active - valence nucleons - , some will be consider to form an “inert” core
- Motion will be considered nonrelativistic: use nonrelativistic many-body Schroedinger equation



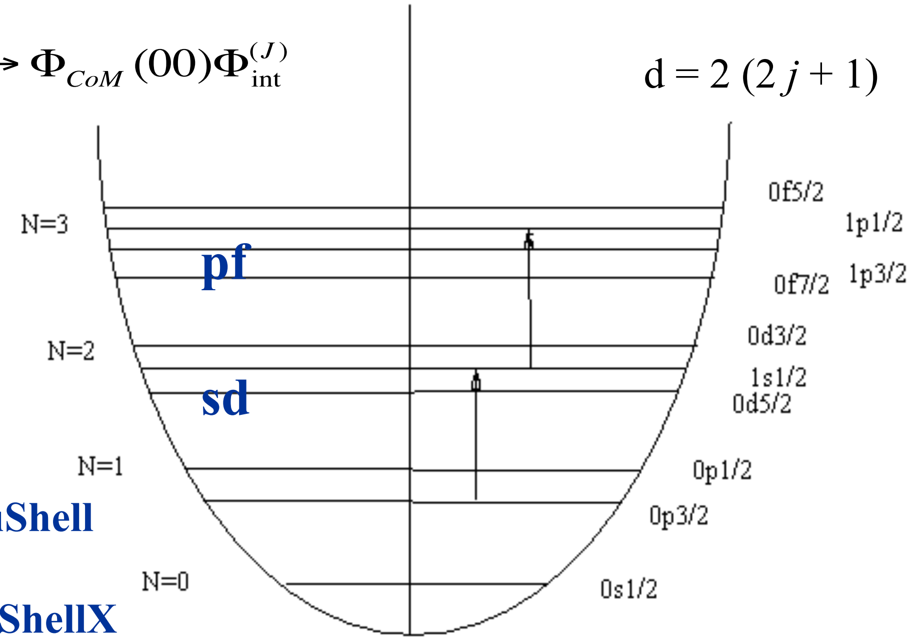
Nuclear Configuration Interaction

$0\hbar\omega$ $$H' = H + \beta(H_{CoM} - 3/2\hbar\omega)$$ **Center-of-mass spurious states**

$1\hbar\omega$ $$\Psi^{(J)} = [\Phi_{CoM}(NL)\Phi_{\text{int}}^{(J')}]^{(J)} \rightarrow \Phi_{CoM}(00)\Phi_{\text{int}}^{(J)}$$

$(0+2)\hbar\omega$

$(1+3)\hbar\omega$ **N_{max}**



$$H = \sum_k \varepsilon_K a_k^+ a_k + \frac{1}{2}\sum_{klmn} V_{kl;mn} a_k^+ a_l^+ a_n a_m + \cdots$$

$$|\alpha> = \sum_i C_i^\alpha \, |i(JT)>$$ **JT-scheme: OXBASH, NuShell**

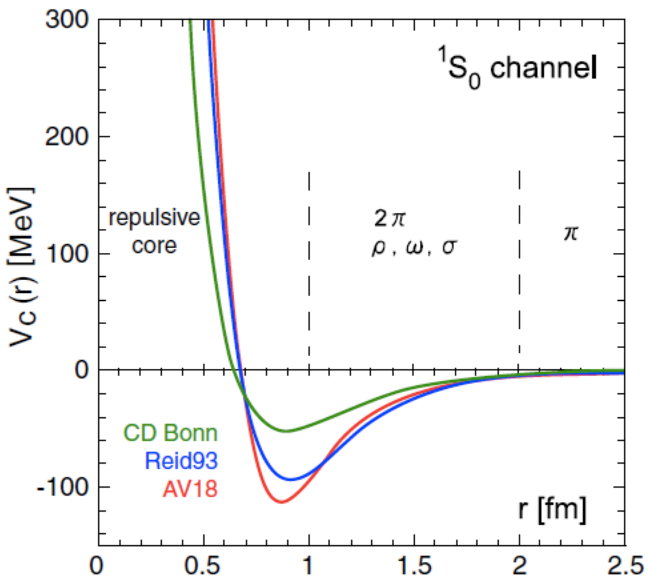
$$|\alpha> = \sum_i C_i^\alpha \, |i(JT_z)>$$ **J-scheme: NATHAN, NuShellX**

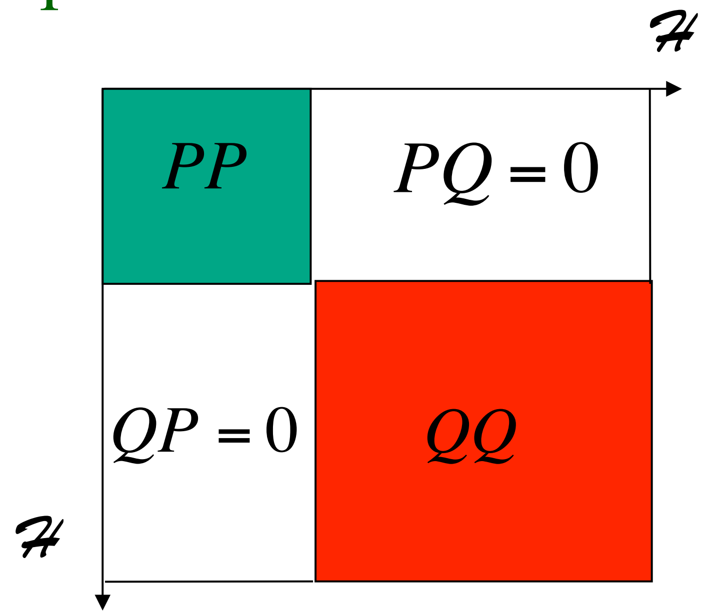
$$|\alpha> = \sum_i C_i^\alpha \, |i(MT_z)>$$ **M-scheme: Oslo-code, Antoine, MFDN, MSHELL, CMichSM, …**

Limits: $10^{10} - 10^{11}$ m-scheme basis states, about 50 s.p. valence states (p+n)

$$\sum_j <i|H|j> C_j^\alpha = E_\alpha C_i^\alpha$$ **Lanczos algorithm: provides few lower energies, especially the M-scheme codes.**

Effective Hamiltonians for Large N $\hbar\omega$ Excitation Model Spaces





"Bare" Nucleon-Nucleon Potentials:

- Argonne V18: PRC 56, 1720 (1997)
- CD-Bonn 2000: PRC 63, 024001 (2000)
- N^3LO: PRC 68, 041001 (2003)
- INOY: PRC 69, 054001 (2004)

$$H = T + \sum_{i<j} V_{ij} + \sum_{i<j<k} V_{ijk} + \cdots$$

$$\Psi_{\mathcal{H}} \rightarrow \Psi_P = P\Psi_{\mathcal{H}}$$

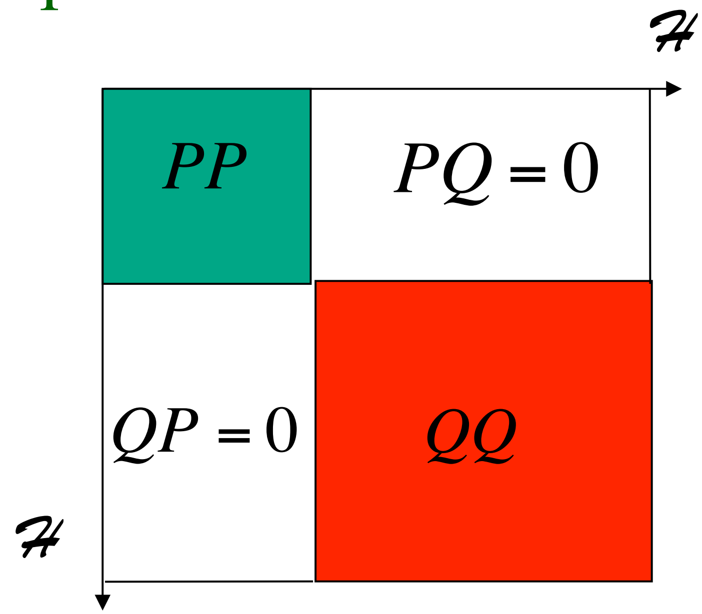
$$\mathcal{H} = U\,H U^{+} = \mathcal{H}_2 + \mathcal{H}_3 + \mathcal{H}_4 + \ldots$$

$$O \rightarrow U\,O\,U^{+}$$

Effective Hamiltonians for Large N $\hbar\omega$ Excitation Model Spaces

Renormalization methods:

- G-matrix: Physics Reports 261, 125 (1995)
- Lee-Suzuki (NCSM): PRC 61, 044001 (2000)
- $V_{\text{low k}}$: PRC 65, 051301(R) (2002)
- Unitary Correlation Operator: PRC 72, 034002 (2004)
- Similarity Renormalization Group (SRG): PRL 103, 082501 (2009)



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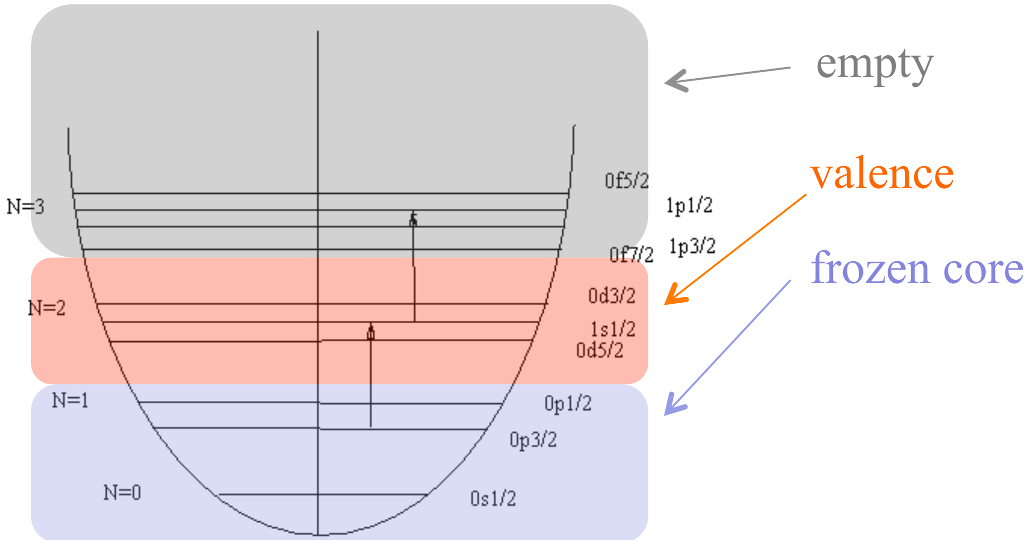
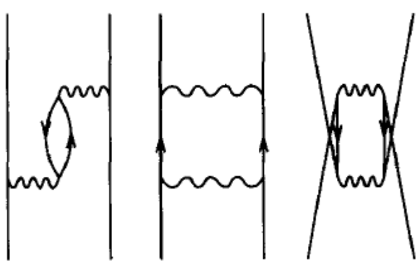
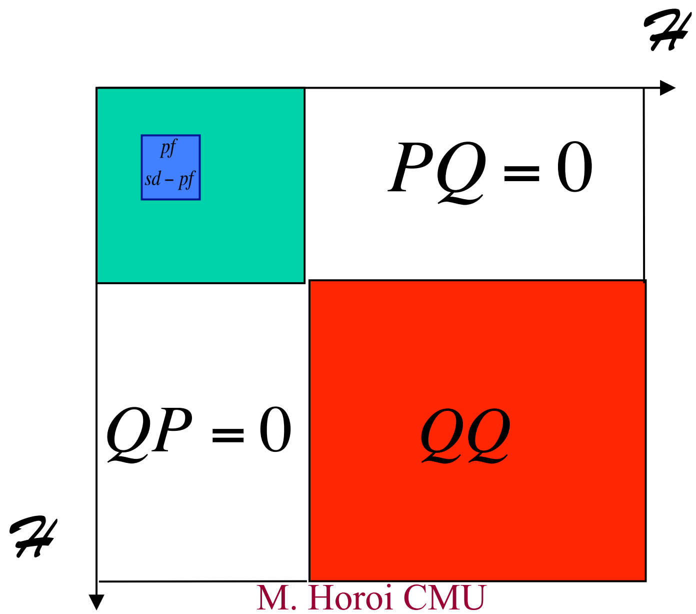
Shell Model Effective Hamiltonians



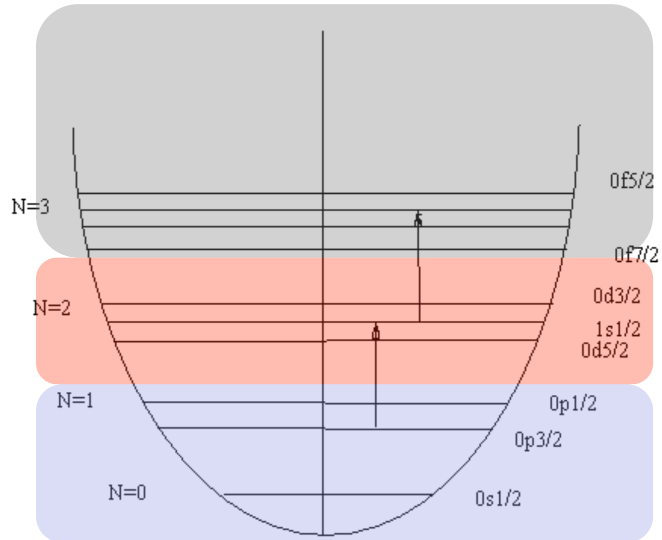
$$H_{valence}\Psi = E_n\Psi$$

core polarization:
Phys.Rep. **261**, 125 (1995)





Shell Model Effective Hamiltonians



$g_A \sigma\tau \xrightarrow{quenched} g_A 0.77 \sigma\tau$

empty

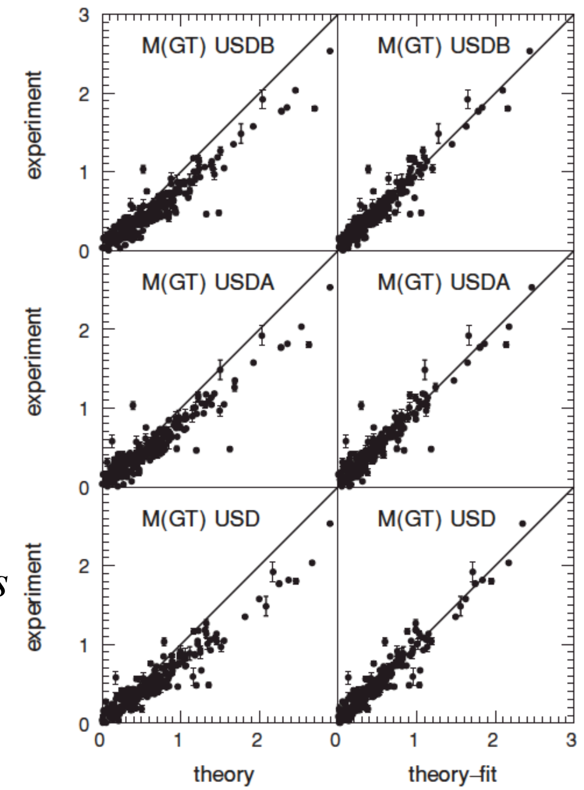
valence

frozen core

$$H_{valence}\Psi = E_n\Psi$$

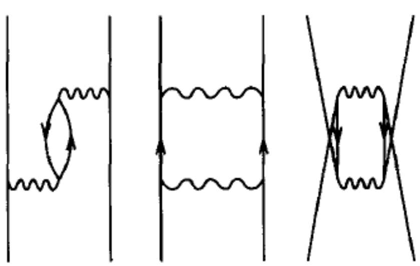
$$H_{valence} = H_{2-body}$$

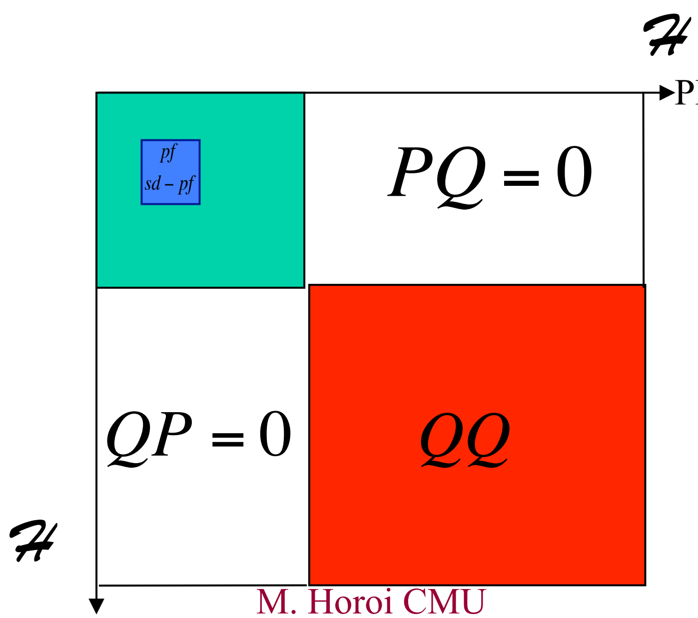
can describe most correlations around the Fermi surface!

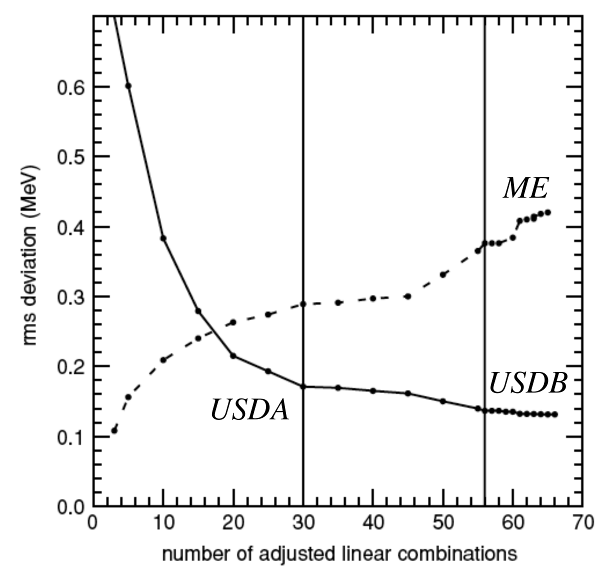


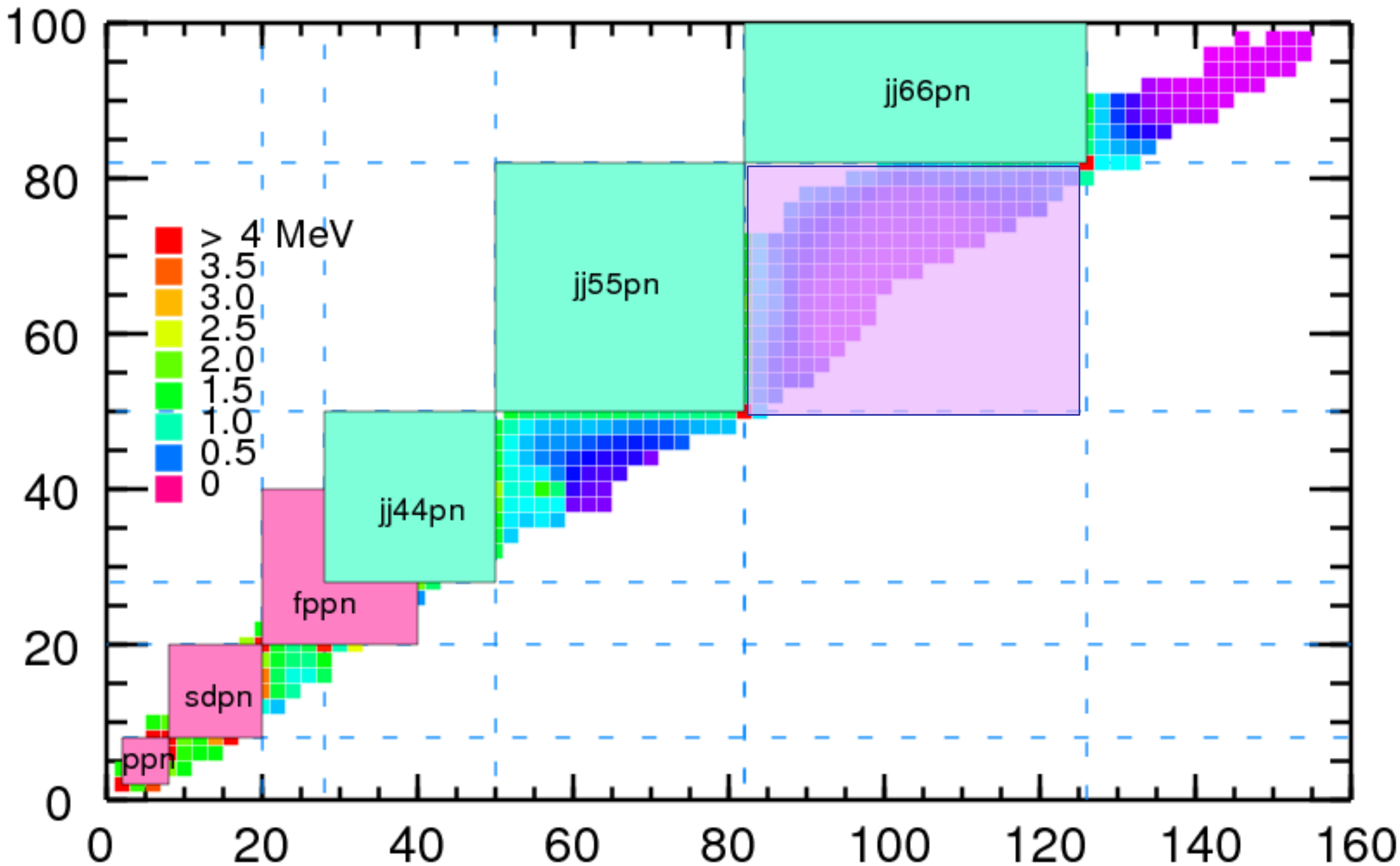
PRC 74, 34315 (2006), 78, 064302 (2008)

core polarization: Phys.Rep. **261**, 125 (1995)







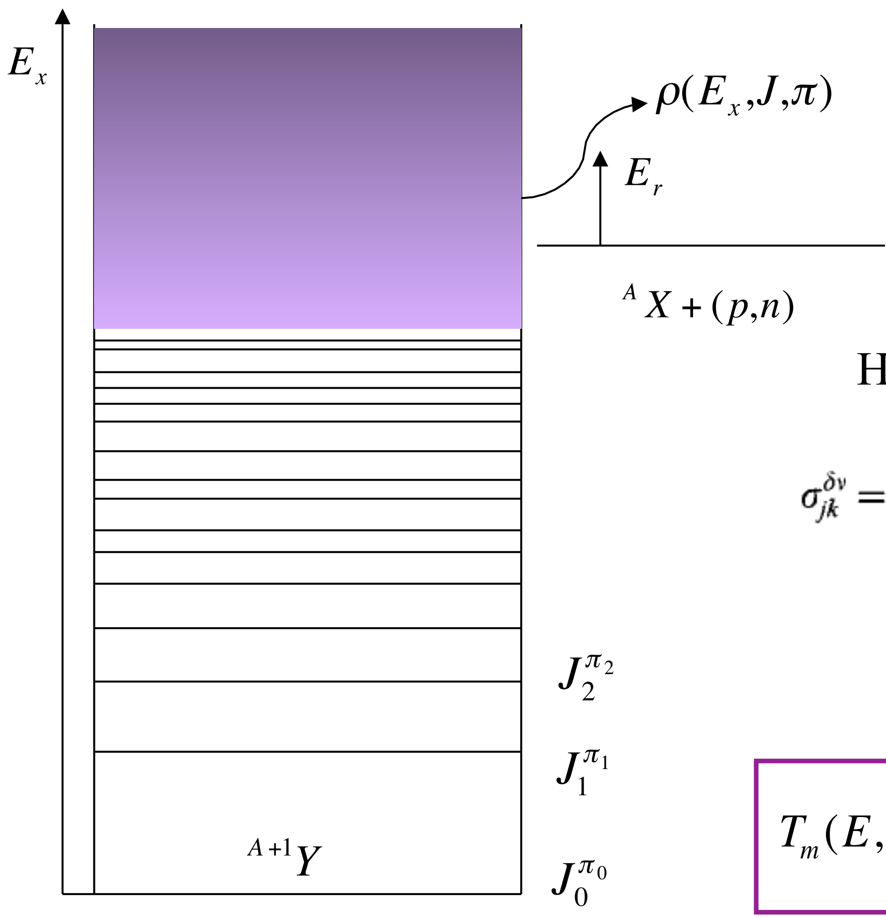


100
80
60
40
20
0
0
20
40
60
80
100
120
140
160
> 4 MeV
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0
jj66pn
jj55pn
jj44pn
fppn
sdpn
ppn

Shell Model: Gold Standard of Nuclear Structure

- ✓ Shell model techniques **describe** and **predict** a large amount of data in light, medium, and heavy nuclei:
  - ✓ Energies and quantum numbers
  - ✓ Electromagnetic transition probabilities
  - ✓ Spectroscopic amplitudes
  - ✓ Nuclear level densities
  - ✓ Beta decay probabilities and charge exchange strength functions
  - ✓ 2ν/0ν Double-beta decay matrix elements

Nuclear Level Densities (NLD)



$(n,\gamma),\ (n,xn),\ (n,n'),\ (n,p),\ (n,f),\ldots$

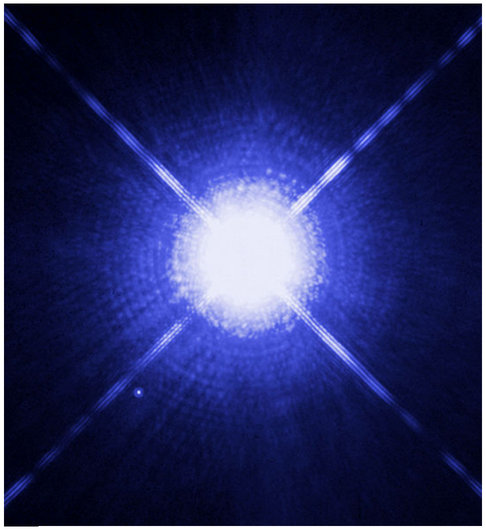
Hauser and Feshbach, Phys. Rev **87**, 366 (1952)

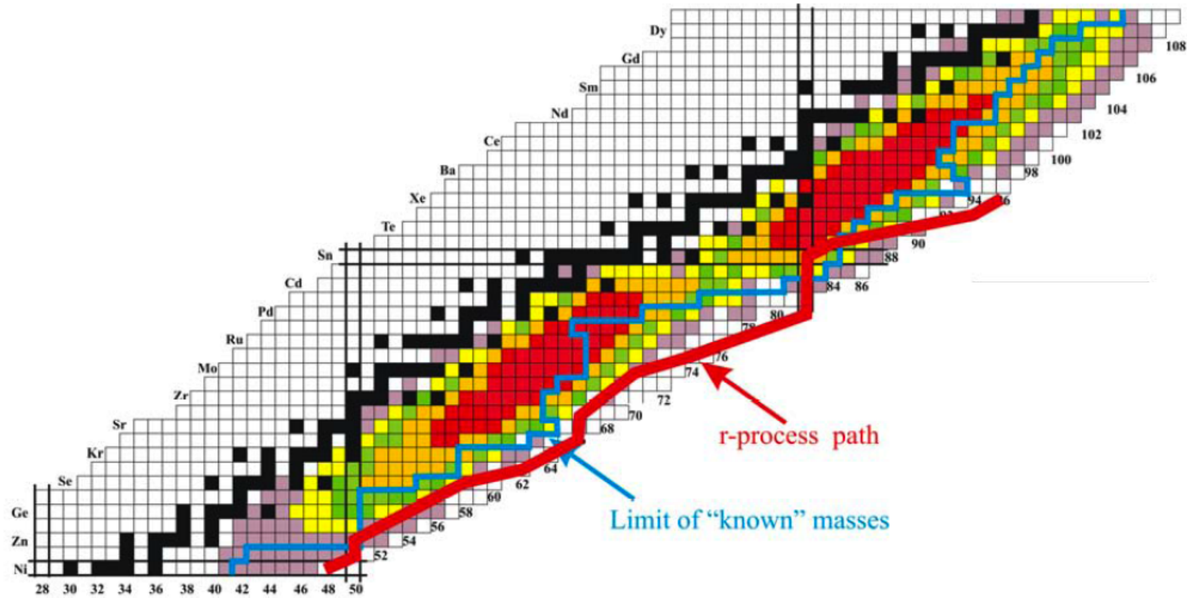
$$\sigma_{jk}^{\delta\nu} = \frac{\pi\hbar^2}{2\mu_{ij}E_{ij}} \frac{1}{(2J_i^{\delta}+1)(2J_j+1)} \times \sum_{J,\pi}(2J+1)\frac{T_j^{\delta}(E,J,\pi,E_j^{\delta},J_j^{\delta},\pi_j^{\delta})\,T_k^{\nu}(E,J,\pi,E_k^{\nu},J_k^{\nu},\pi_k^{\nu})}{\sum_m T_m(E,J,\pi)}$$

$$T_m(E,J,\pi) = \int_{E_{min}}^{E_{max}} T(E,J,\pi;E_x,J_x,\pi_x)\,\rho(E_x,J_x,\pi_x)\,dE_x$$

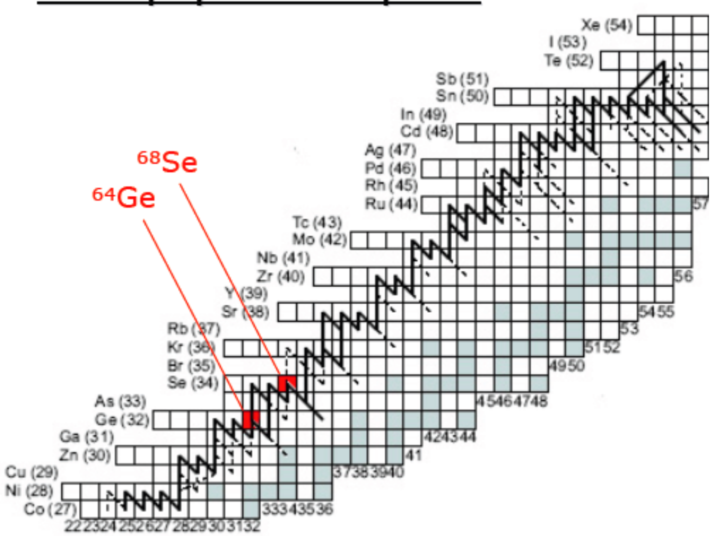
Where are NLD Needed: Nuclear Astrophysics

Binary stars XRB: Sirius





the rp-process path



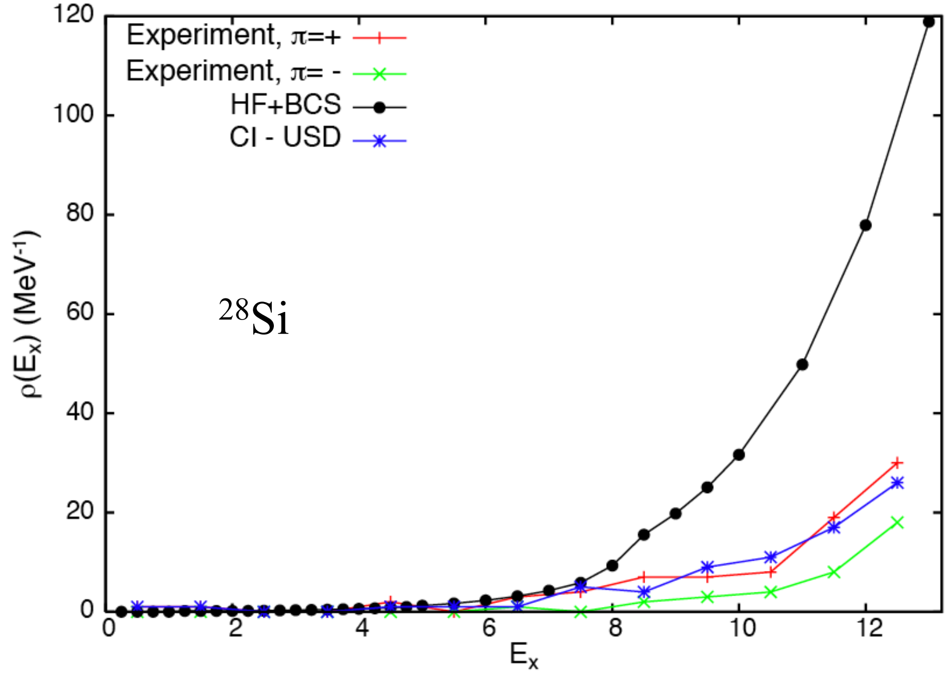
Schatz et al. Phys. Rep 294 (1998) 167-298

SN 1987 A

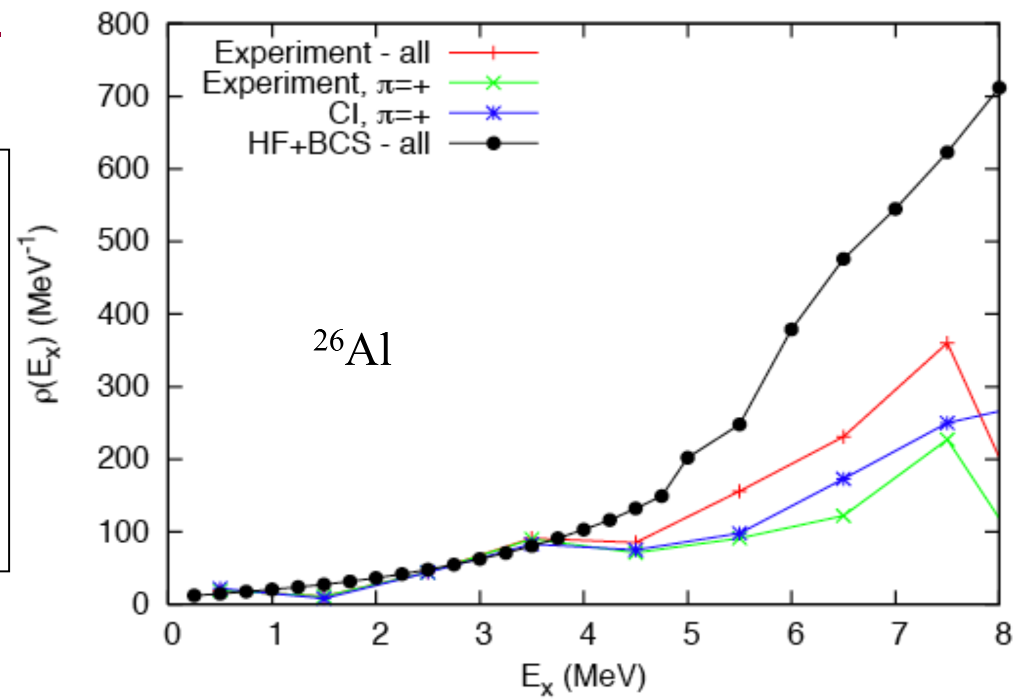
Accurate Nuclear Level Densities

Comparison of:

1. CI
2. HF+BCS
   www-astro.ulb.ac.be/Html/nld.html
3. experimental data



Complete spectroscopy: sd-shell nuclei



Conclusions:

- HF+BCS seems to overestimate the data

- CI seem to accurately describe the data

NMP17
March 7, 2017

NLD and Statistical Spectroscopy

M. Horoi et al. :

PRC **67**, 054309 (2003),

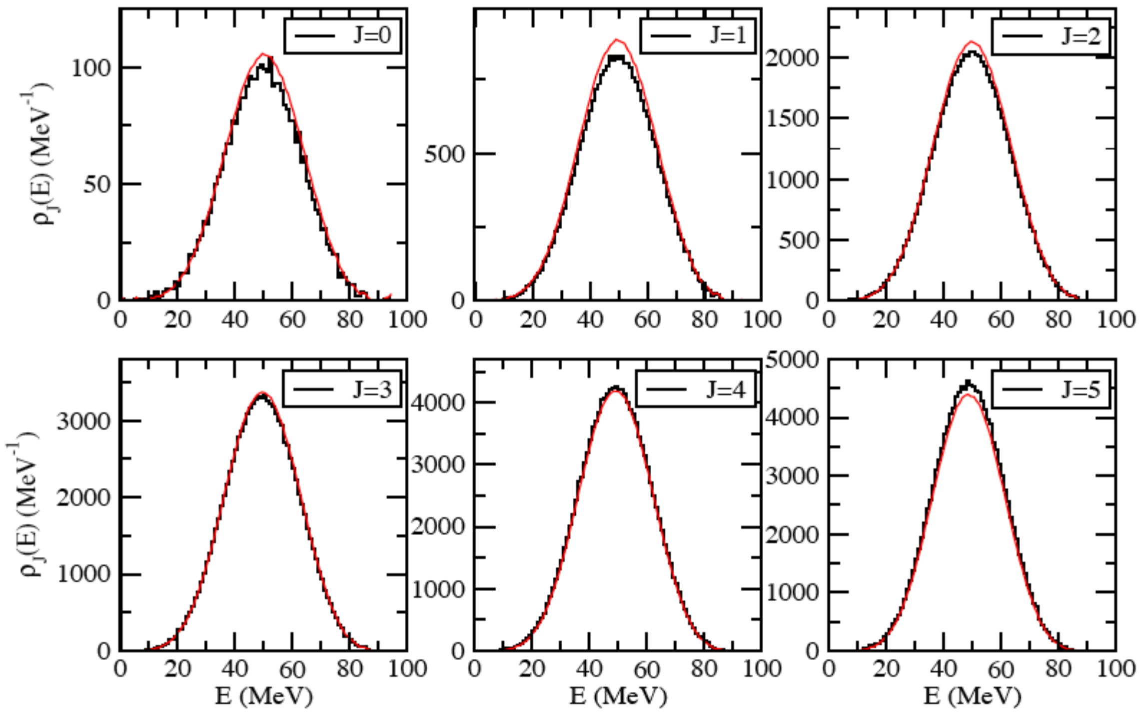
PRC **69**, 041307(R) (2004),

NPA **785**, 142 (2005).

PRL **98**, 265503 (2007)

Configurations: e.g. 4 particles in sd
d3 d5 s1
4 0 0
3 1 0
3 0 1 …
preserve rotational invariance and parity

^{28}Si $\pi = +$ staircase: CI, USD

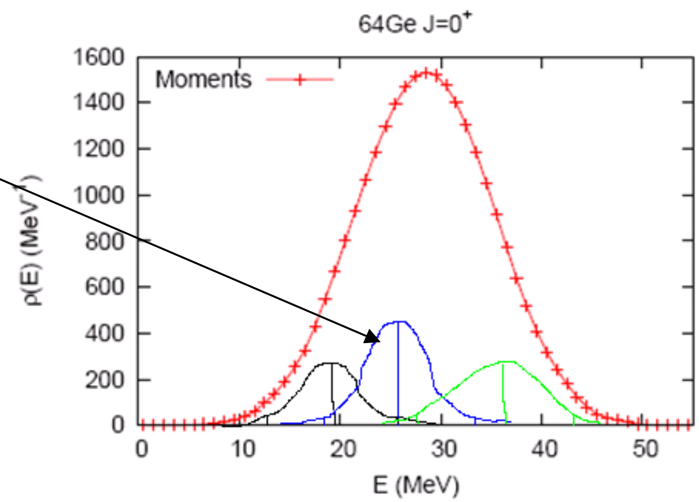


$$\rho(E_x, J, \pi) = \sum_{c \in conf} D_c(J,\pi) G_{FR}\left(E, E_c(J), \sigma_c(J) \right)$$

$$E_c(J), \sigma_c(J) \leftarrow Tr_{SD_c} < M \mid H^q \mid M >_{SD_c}$$

$$E_x = E - E_{g.s.}$$

$E_{g.s.}$ from CI, ECM (PRL **82**, 2064 (1999)), CC, etc.

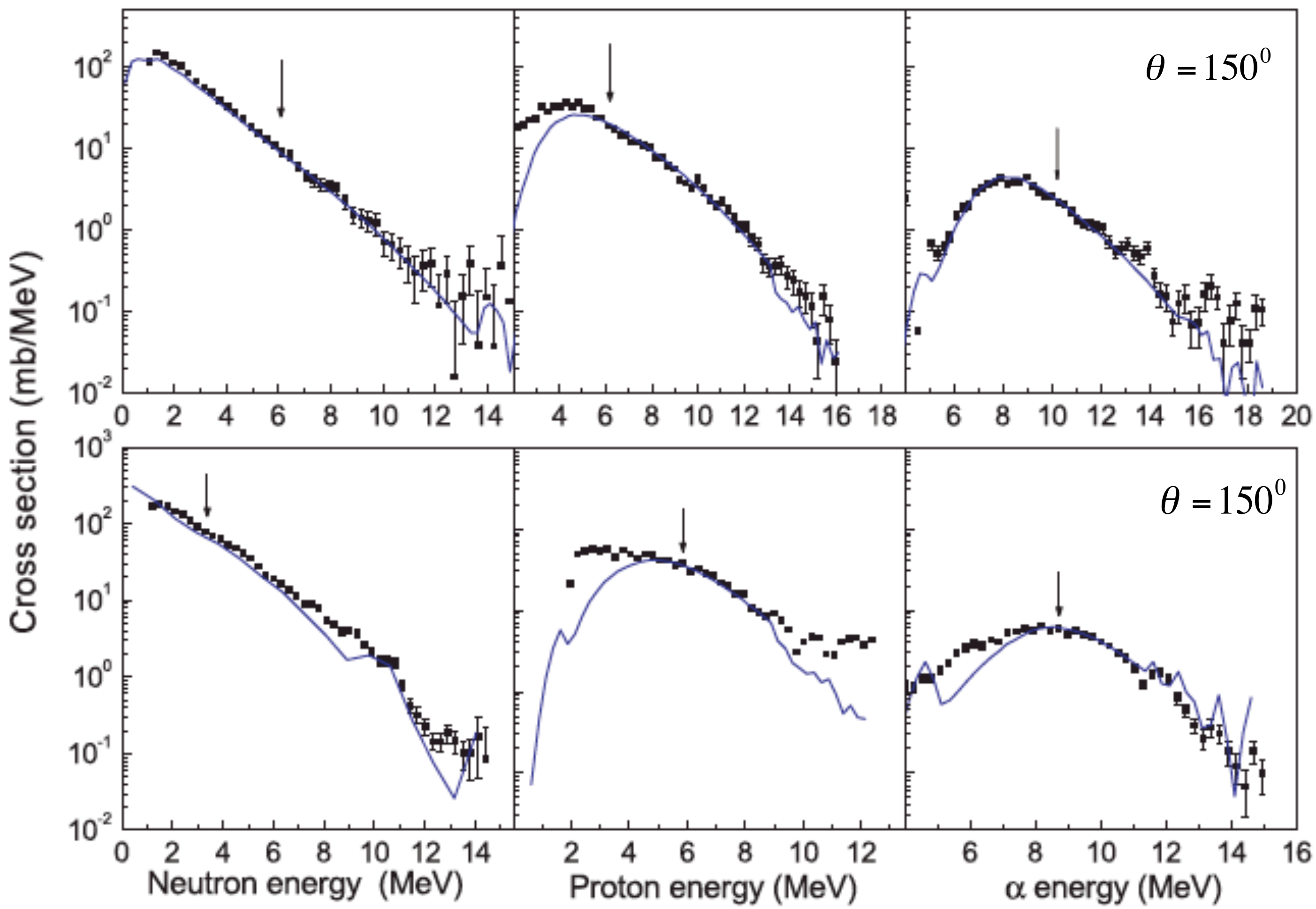


Shell model moments method: pro and con

- Pro
  - Spin and parity dependent centroids take into account the s.p. energy shifting due the monopoles (tensor interaction)
  - Spin and parity dependent widths take into account more realistic spreading, beyond that of pairing
  - No need to consider rotational/vibrational amplifications
- Con
  - Relatively small number of s.p. orbitals in the valence space: natural parity favored (unique)
  - Reliable Hamiltonians hard to obtain
  - Energy of the g.s. could be a problem (but there are some solutions)
  - The configuration distribution could be asymmetric (some solutions here as well)

NLD and Hauser-Feshbach Cross-Sections

From A. Voinov et al., PRC **76**, 044602 (2007)



$\leftarrow\ ^3He + ^{58}Fe$

$E_{^3He} = 10\,MeV$

$^{61}Ni^*$ compund

$\leftarrow\ d + ^{59}Co$

$E_d = 7.5\,MeV$

Comparison with Moments Densities

talys : www.talys.eu

NLD-M1

ldmodel 1: Constant temperature + Fermi gas model

ldmodel 2: Back-shifted Fermi gas model

ldmodel 3: Generalised superfluid model

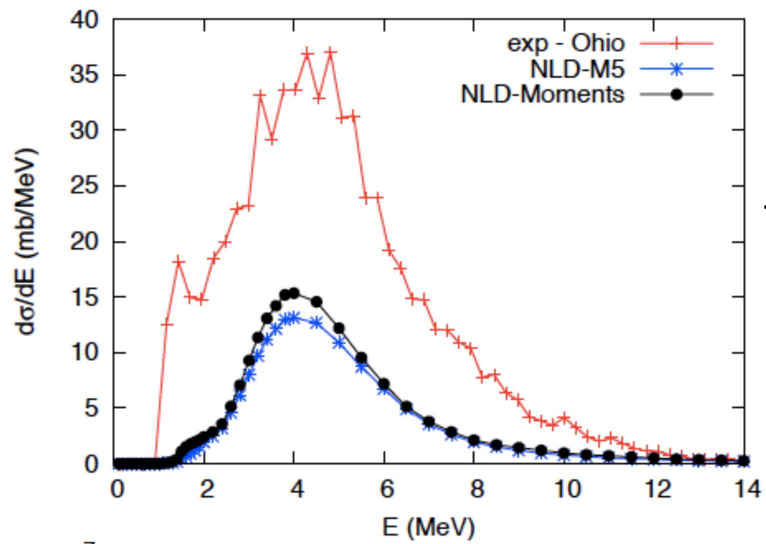
ldmodel 4: Microscopic level densities from Goriely's table

ldmodel 5: Microscopic level densities from Hilaire's table

NLD-M5

Interface:

Moments table -> Hilaire's table

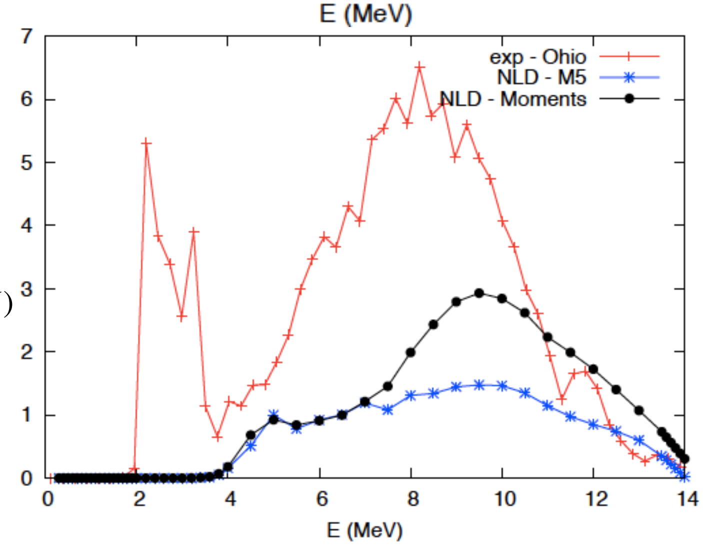


$\leftarrow {}^{58}Fe\left({}^{3}He, p\right){}^{60}Co$

$\theta = 150^0$

$E_{{}^{3}He} = 10.0\ MeV$

$\frac{d\sigma}{dE}$ (mb/MeV)



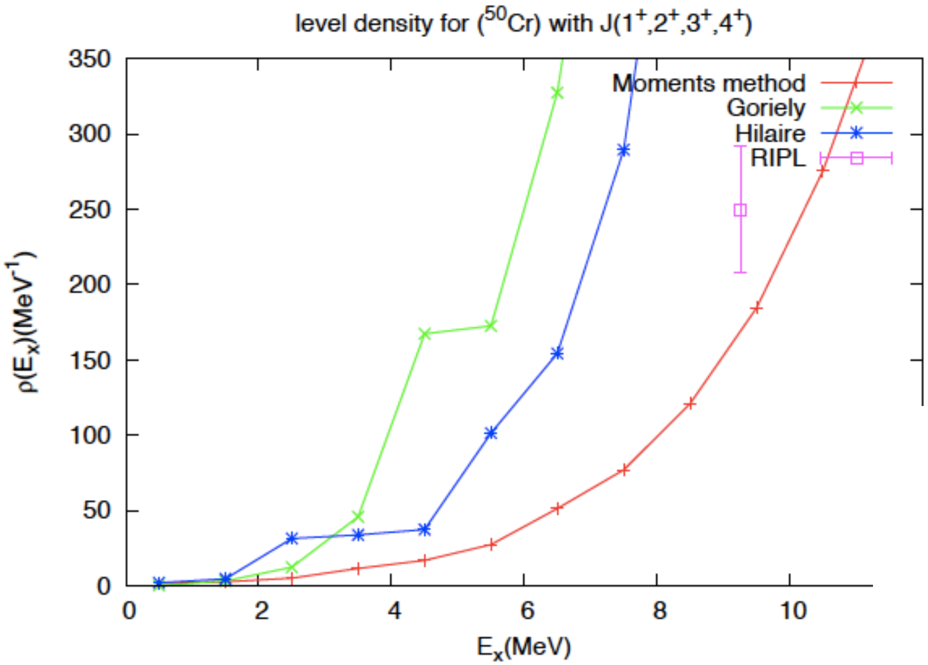
$\leftarrow {}^{59}Co\left(d, \alpha\right){}^{57}Fe$

$\theta = 150^0$

$E_d = 7.5\ MeV$

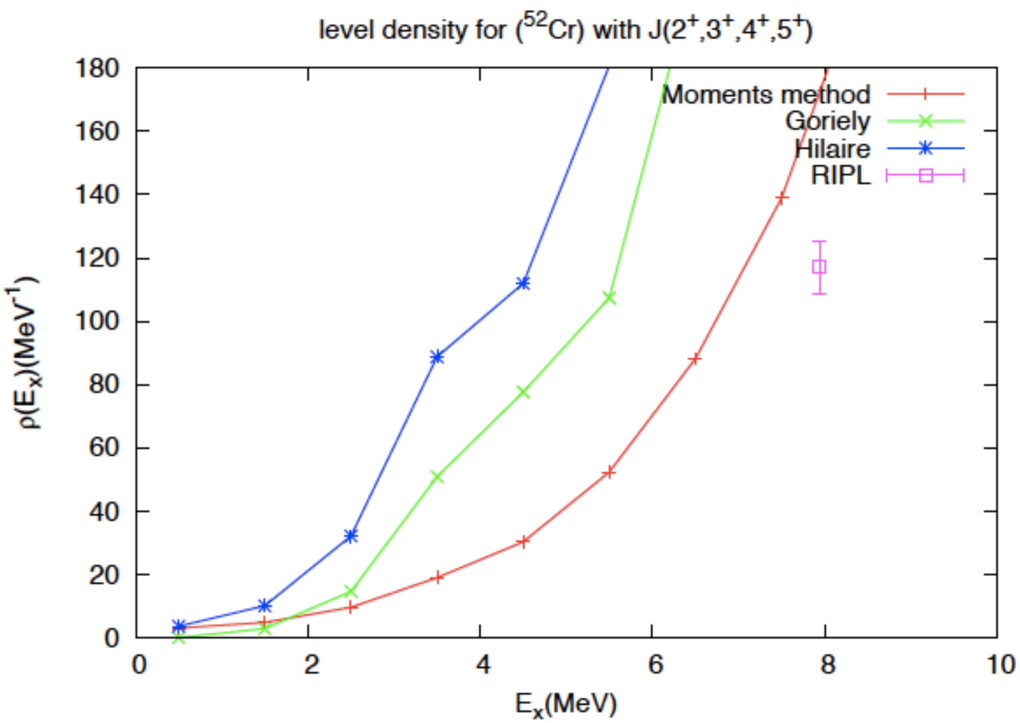
Exp – Ohio: A. Voinov et al., PRC **76**, 044602 (2007)

Comparison with RIPL-2 p-waves neutron resonances data



$^{49}Cr(5/2^{-})+n(l=1,s=1/2)\rightarrow^{50}Cr(J^{\pi}=1^{+},2^{+},3^{+},4^{+})$

$^{51}Cr(7/2^{-})+n(l=1,s=1/2)\rightarrow^{52}Cr(J^{\pi}=2^{+},3^{+},4^{+},5^{+})$



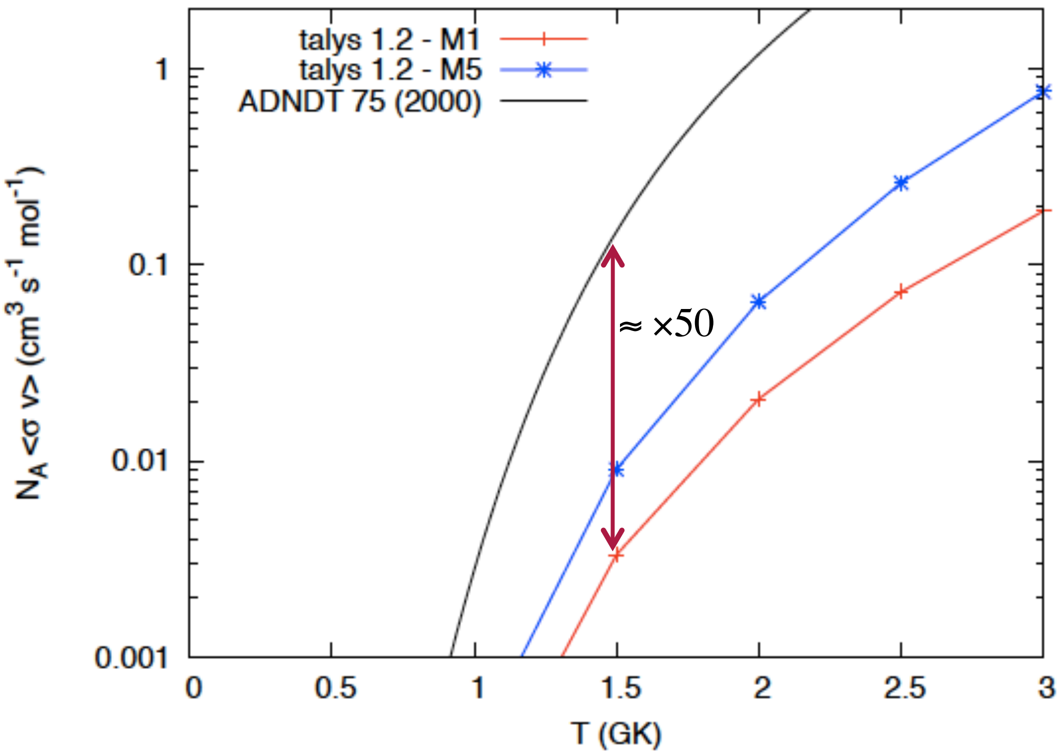
NLD: reaction rates

talys 1.2 : www.talys.eu

Rauscher & Thielemann ADNDT **75**, 1 (2000)

$$N_A\langle\sigma v\rangle^*_{\alpha\alpha'}(T) = \left(\frac{8}{\pi m}\right)^{1/2}\frac{N_A}{(kT)^{3/2}\,G(T)}\int_0^\infty\sum_\mu\frac{(2I^\mu+1)}{(2I^0+1)}\times \sigma^\mu_{\alpha\alpha'}(E)E\exp\left(-\frac{E+E^\mu_x}{kT}\right)dE,$$

$$G(T)=\sum_\mu(2I^\mu+1)/(2I^0+1)e^{-E^\mu_x/kT}\rightarrow\sum_{I,\pi}\int(2I^\pi+1)/(2I^0+1)\rho(E_x,I,\pi)e^{-E_x/kT}dE_x$$



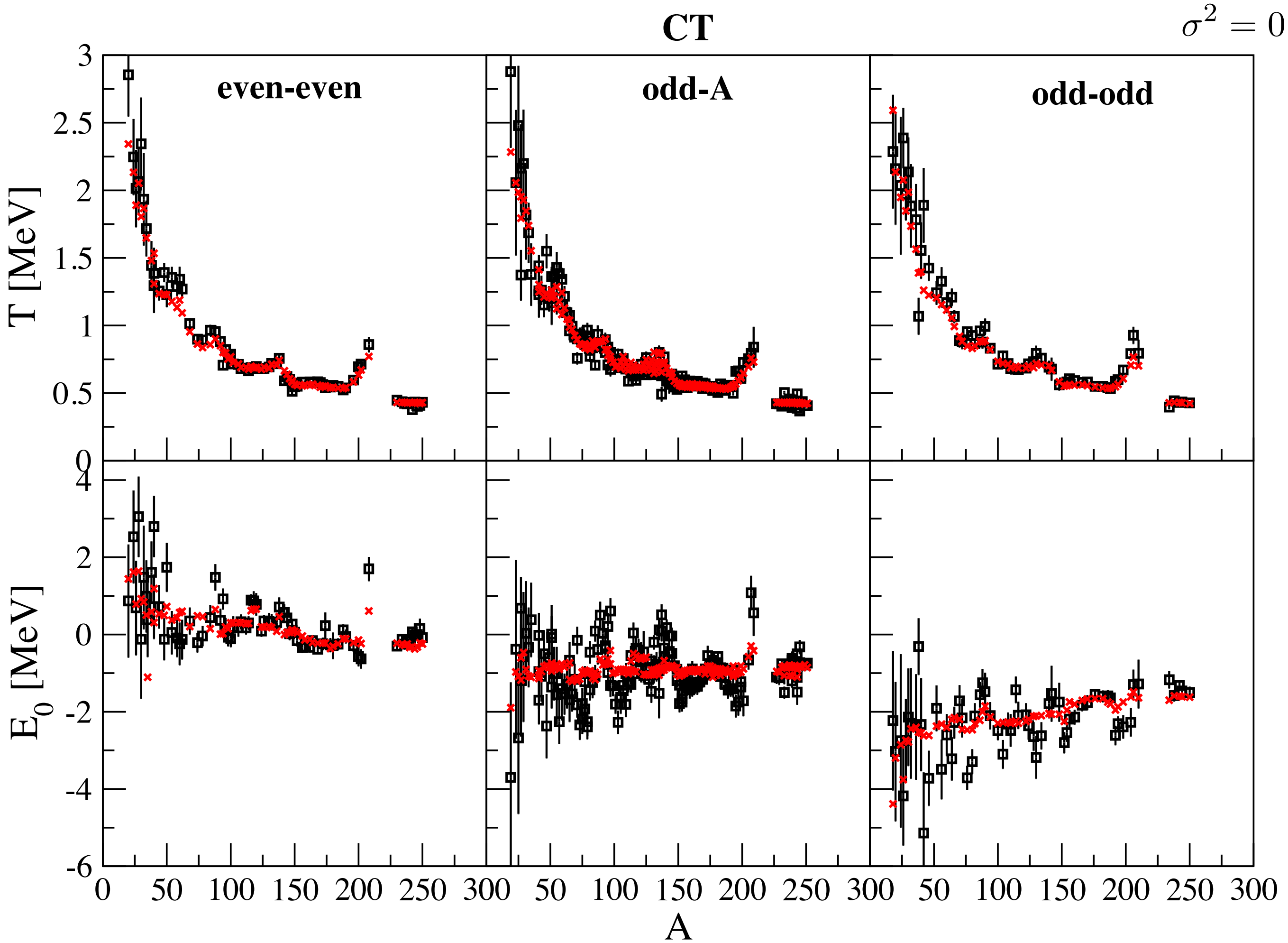
$^{64}Ge(p,\gamma)$

Constant Temperature NLD

$$\rho(E,J) = \frac{1}{2} f(J,\sigma)\rho_{CT}(E) \qquad \rho_{CT}(E) = \frac{1}{T} e^{(E-E_0)/T}$$

$$f(J,\sigma) = e^{-J^2/2\sigma^2} - e^{-(J+1)^2/2\sigma^2} \approx \frac{2J+1}{2\sigma^2} e^{-J(J+1/2)/2\sigma^2}$$

$$\sigma^2 = 0.391 \cdot A^{0.675}(E - 0.5 \cdot Pa')^{0.31}$$



von Egidy & Bucurescu

PRC 72, 044311 (2005), PRC 72, 067304, PRC 80, 054310 (2009), JoP Conf Ser 338, 012028 (2012)

Why constant temperature?

PHYSICAL REVIEW C **75**, 054303 (2007)

Pairing phase transitions in nuclear wave functions

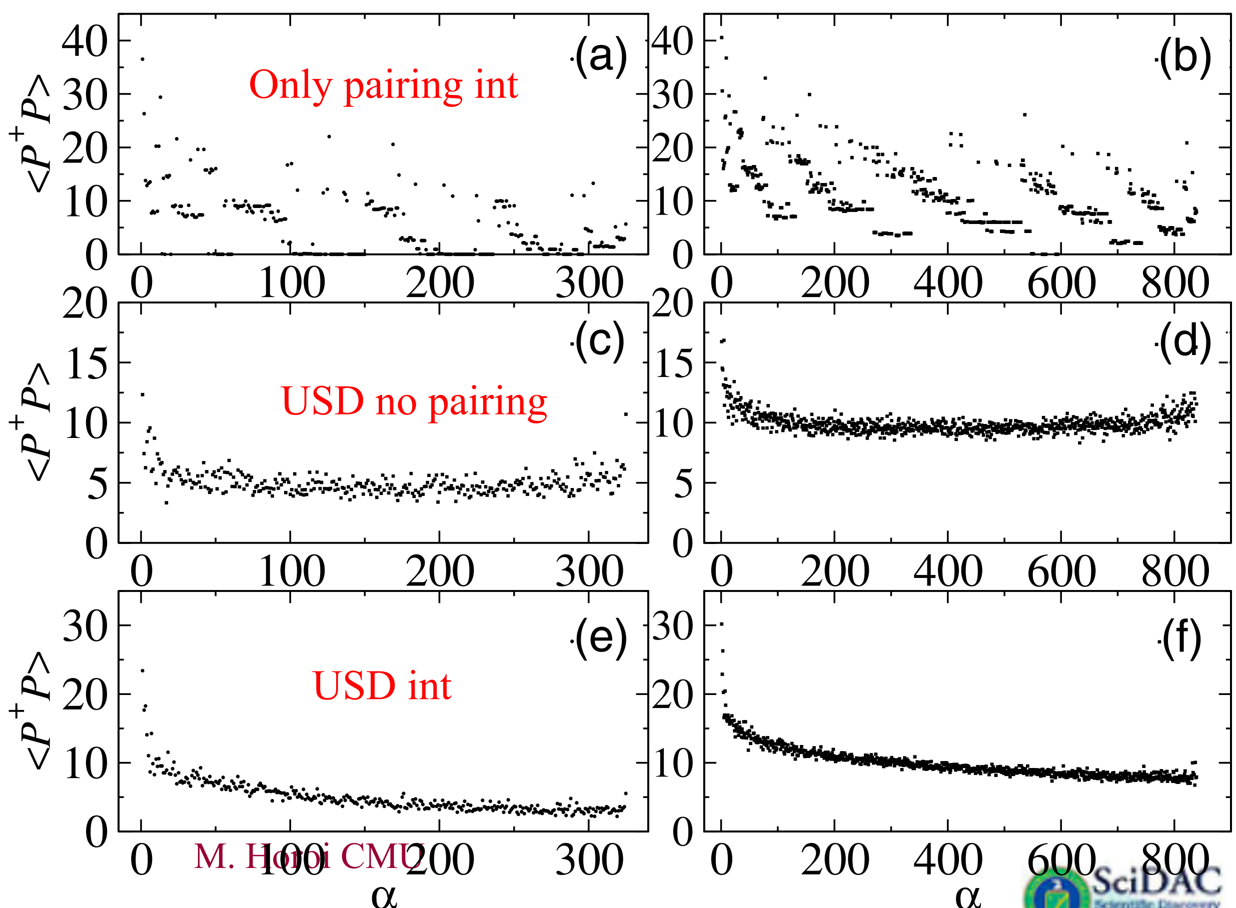
Mihai Horoi[1] and Vladimir Zelevinsky[2]

^{24}Mg $\quad J^{\pi}T = 0^{+}0$ $\quad ^{28}Si$

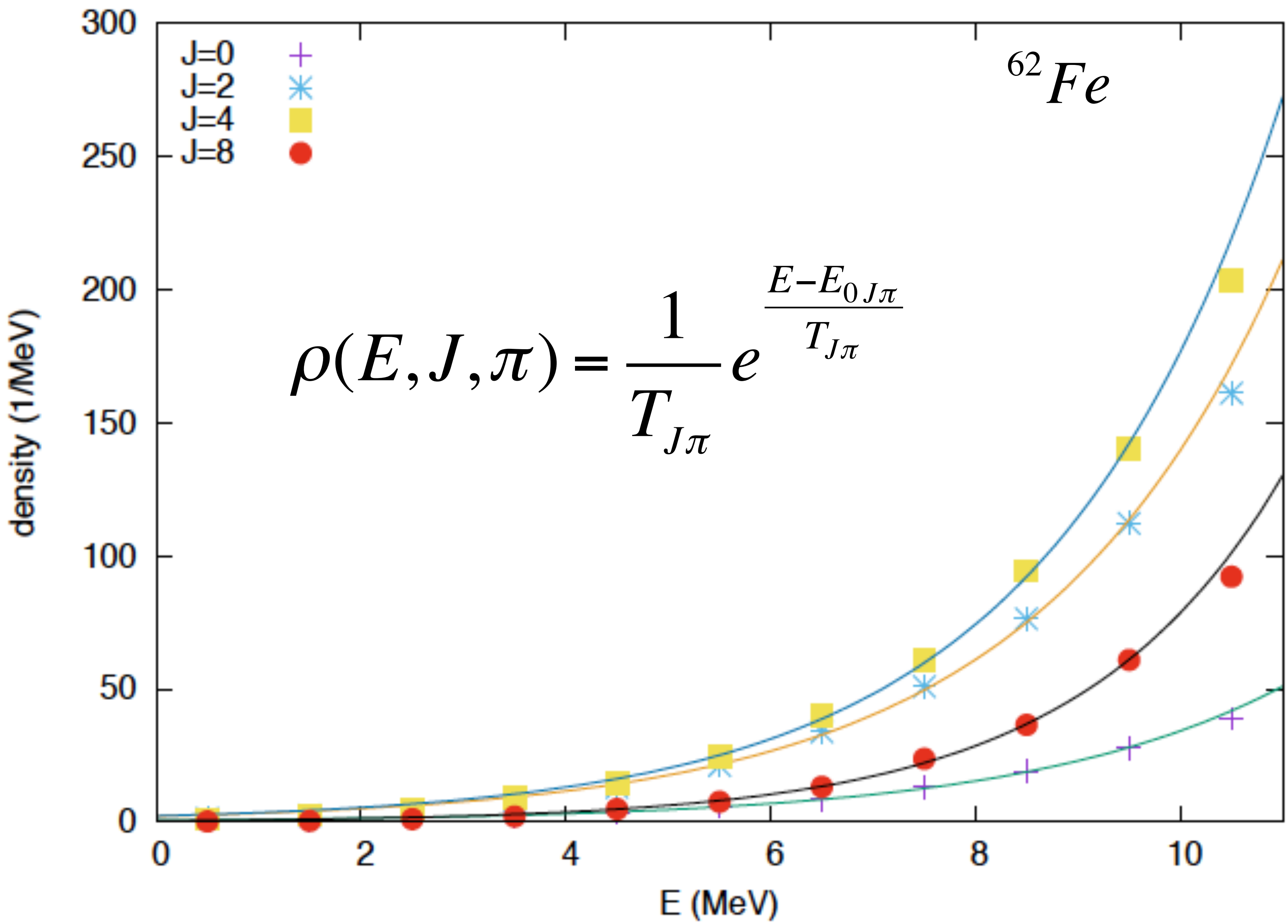
$$\mathcal{H}_P = \sum_{t=0,\pm 1} P_t^{\dagger} P_t,$$

$$P_t = \frac{1}{\sqrt{2}} \sum_j [\tilde{a}_j \tilde{a}_j]_{L=0,T=1,T_3=t},$$

$$P_t^{\dagger} = \frac{1}{\sqrt{2}} \sum_j [a_j^{\dagger} a_j^{\dagger}]_{L=0,T=1,T_3=t}.$$



Constant Temperature vs Moments NLD



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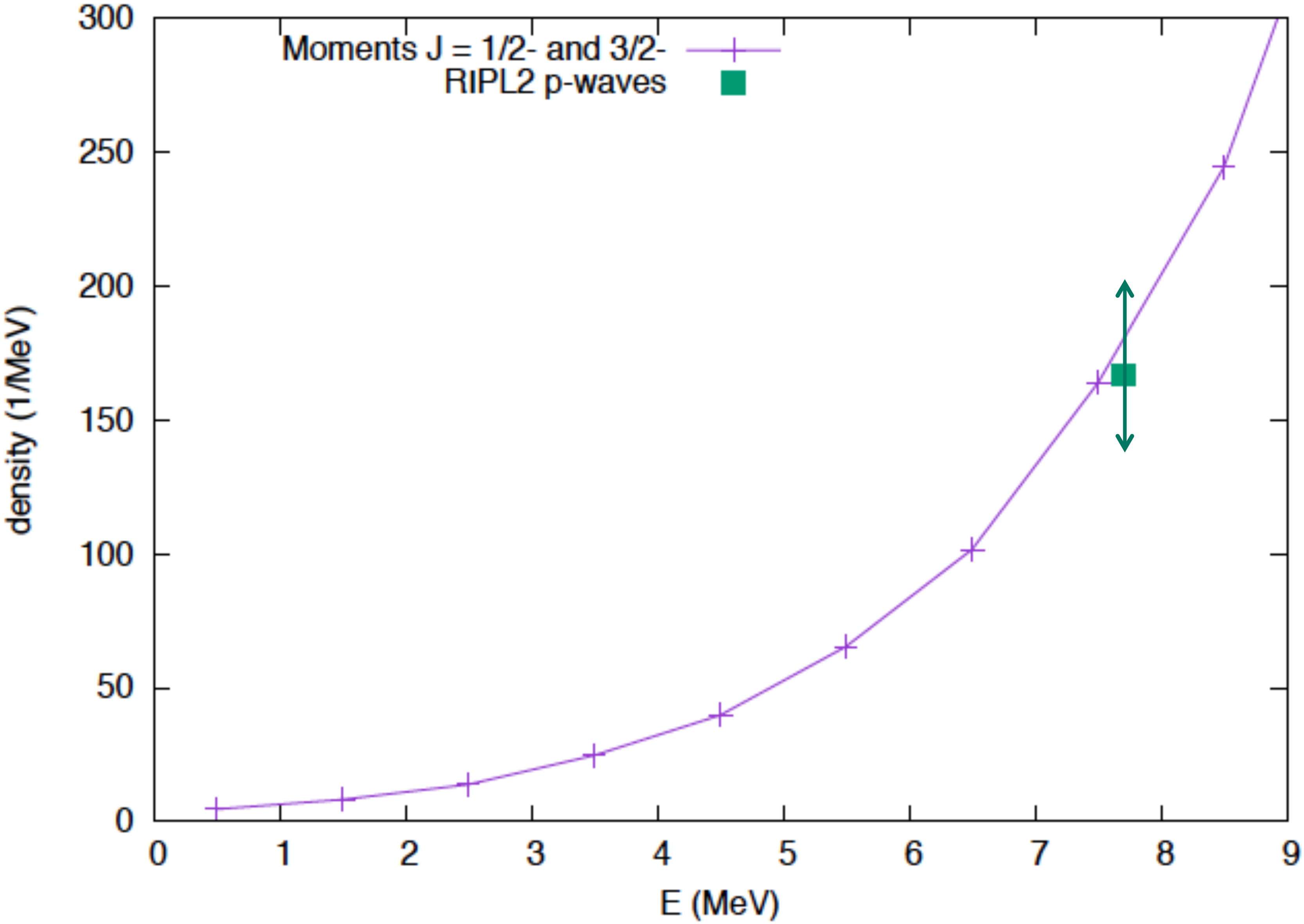

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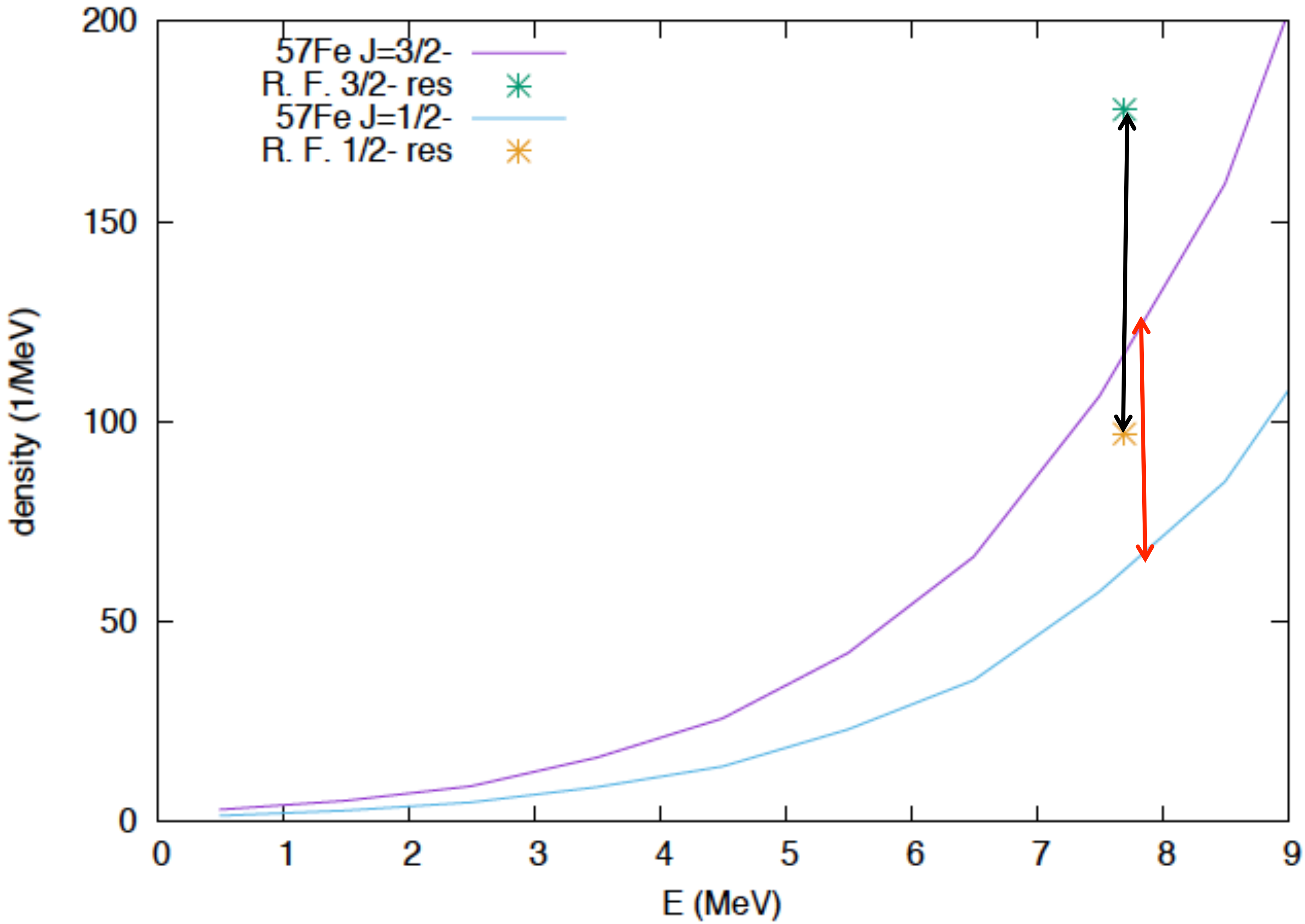

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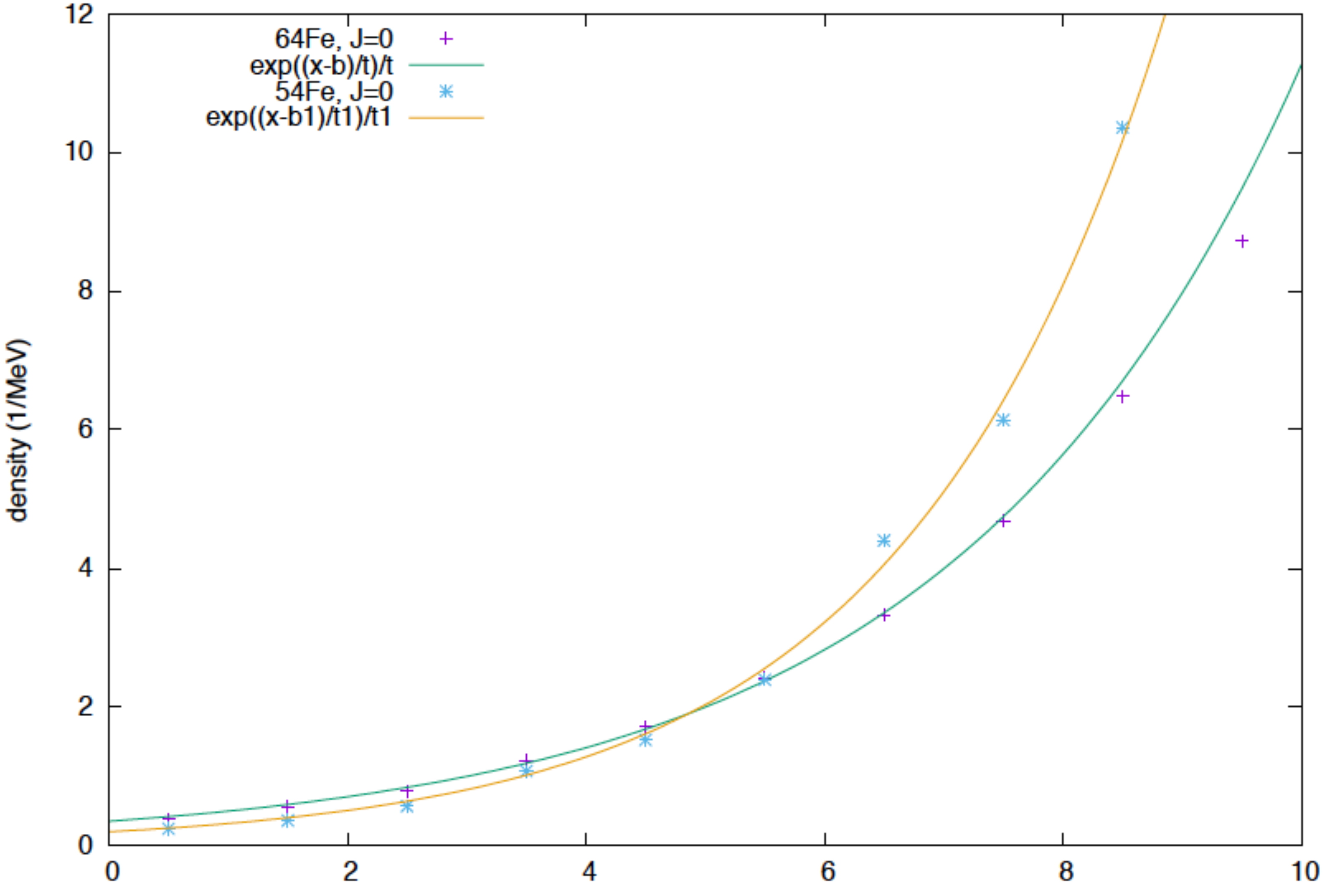

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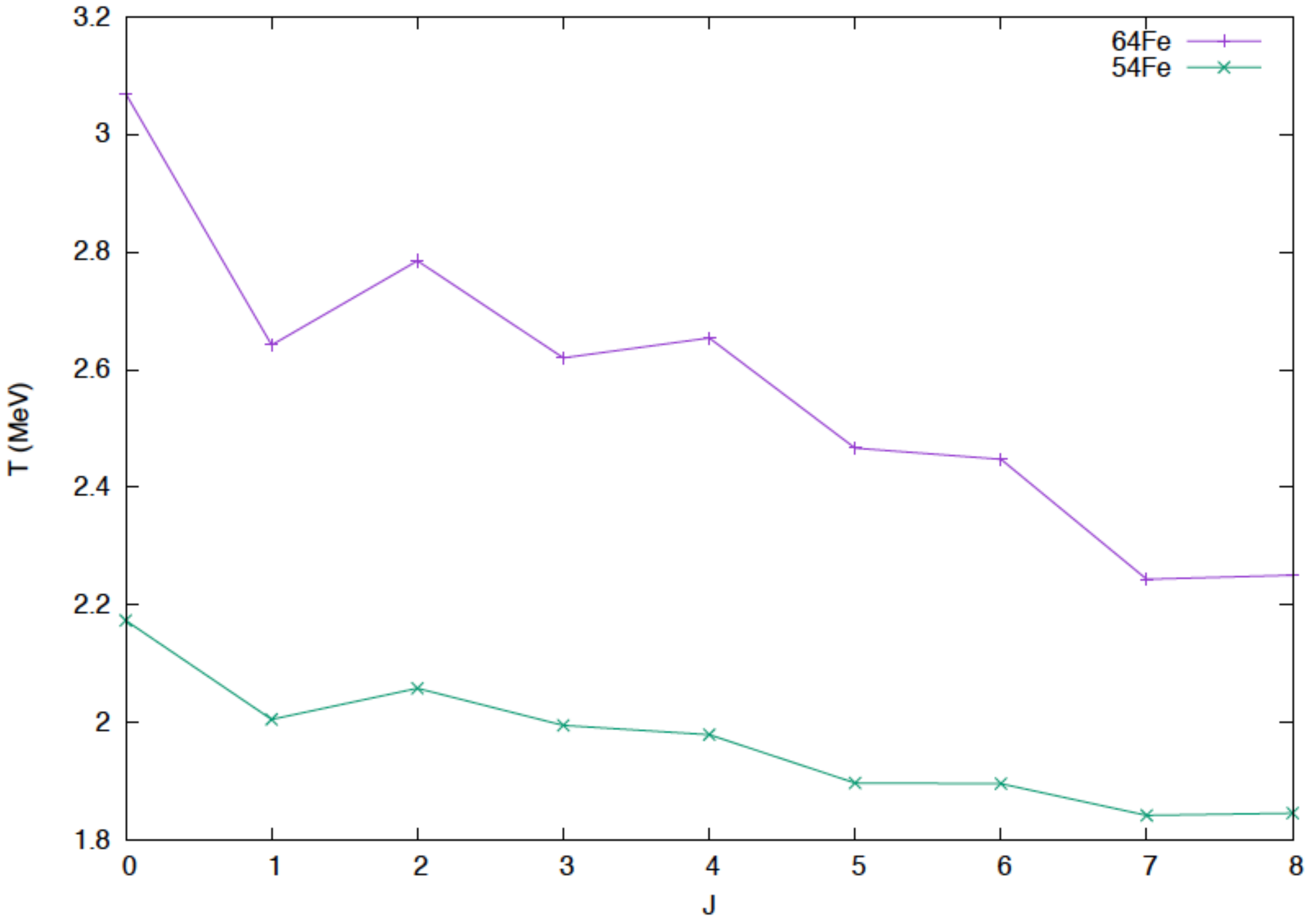

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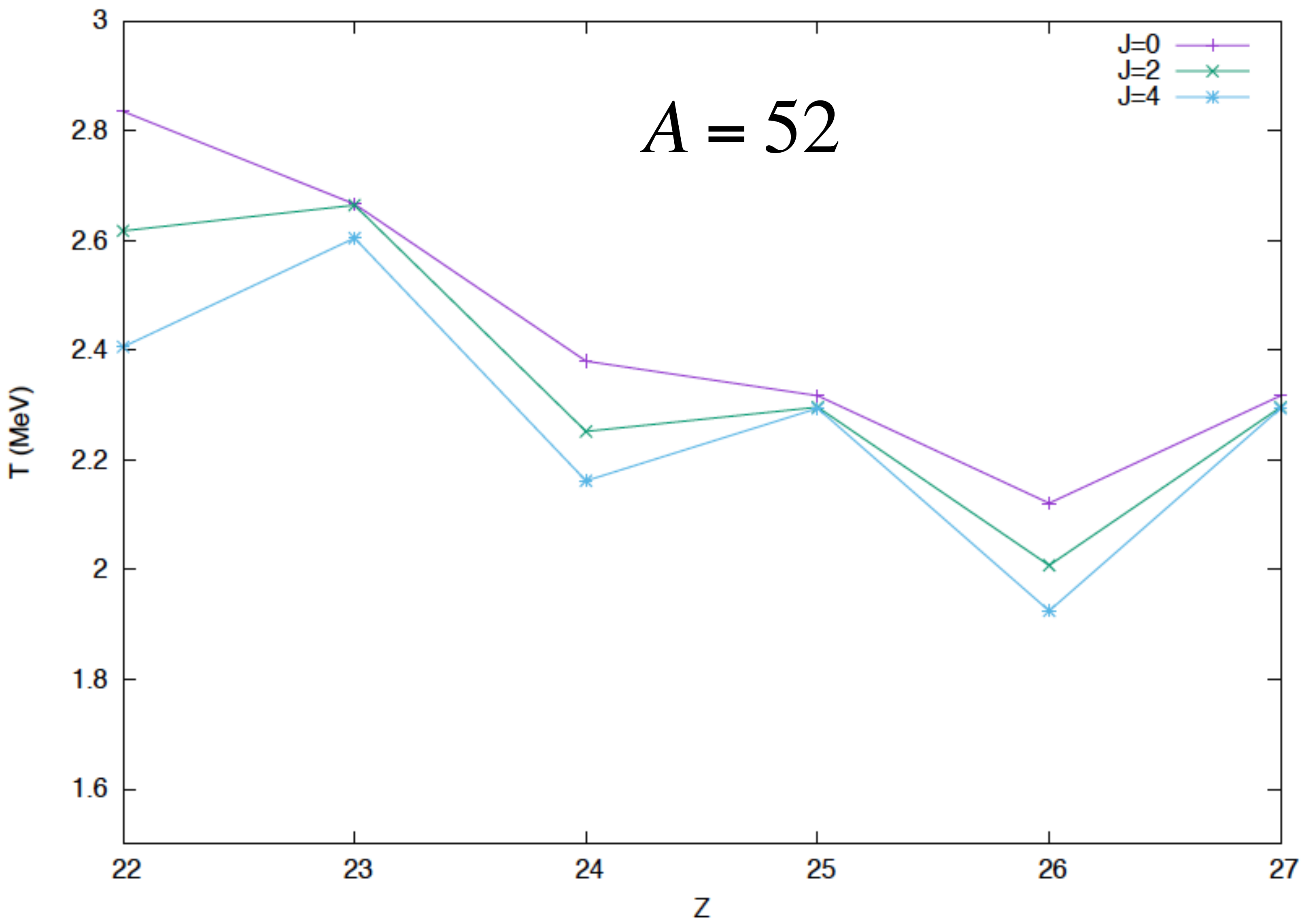

Back to pairing?

PHYSICAL REVIEW C **75**, 054303 (2007)

Pairing phase transitions in nuclear wave functions

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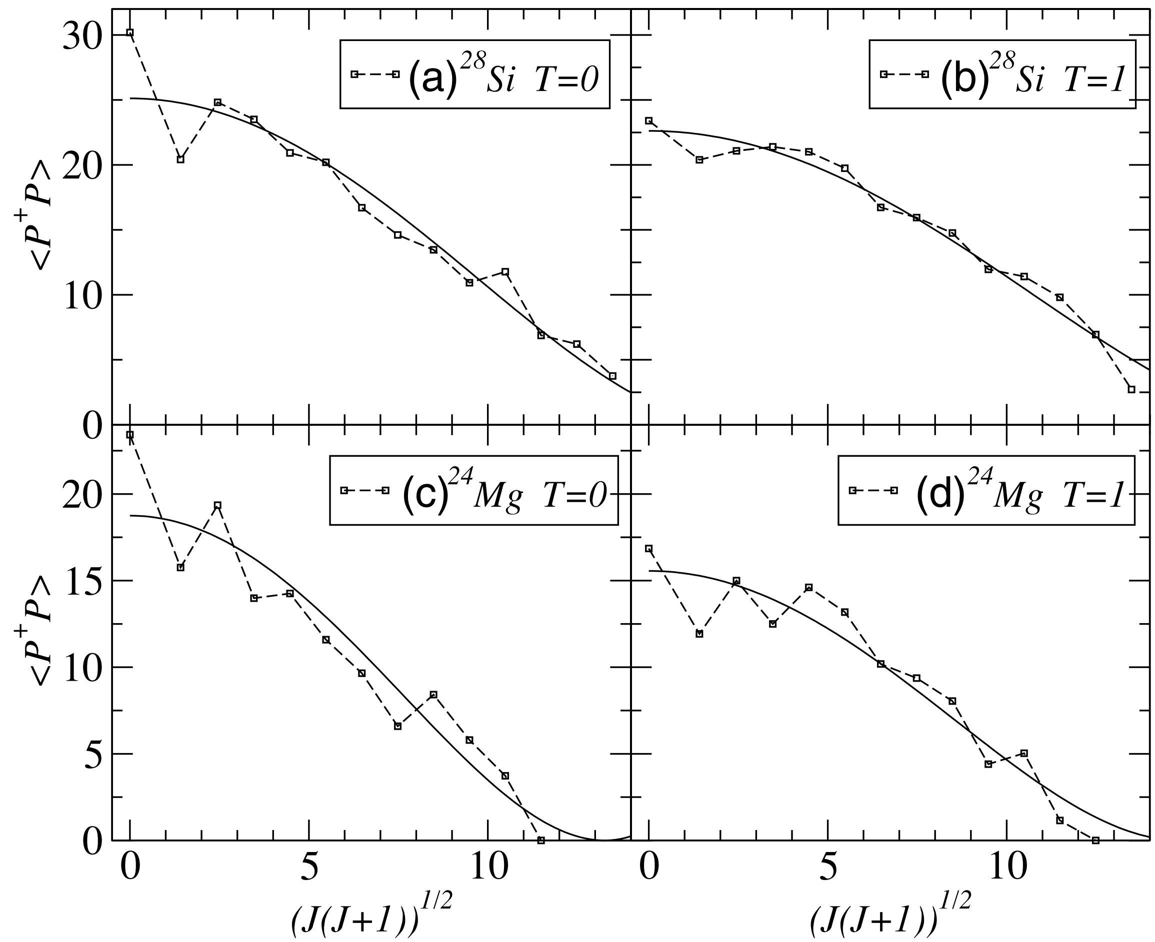

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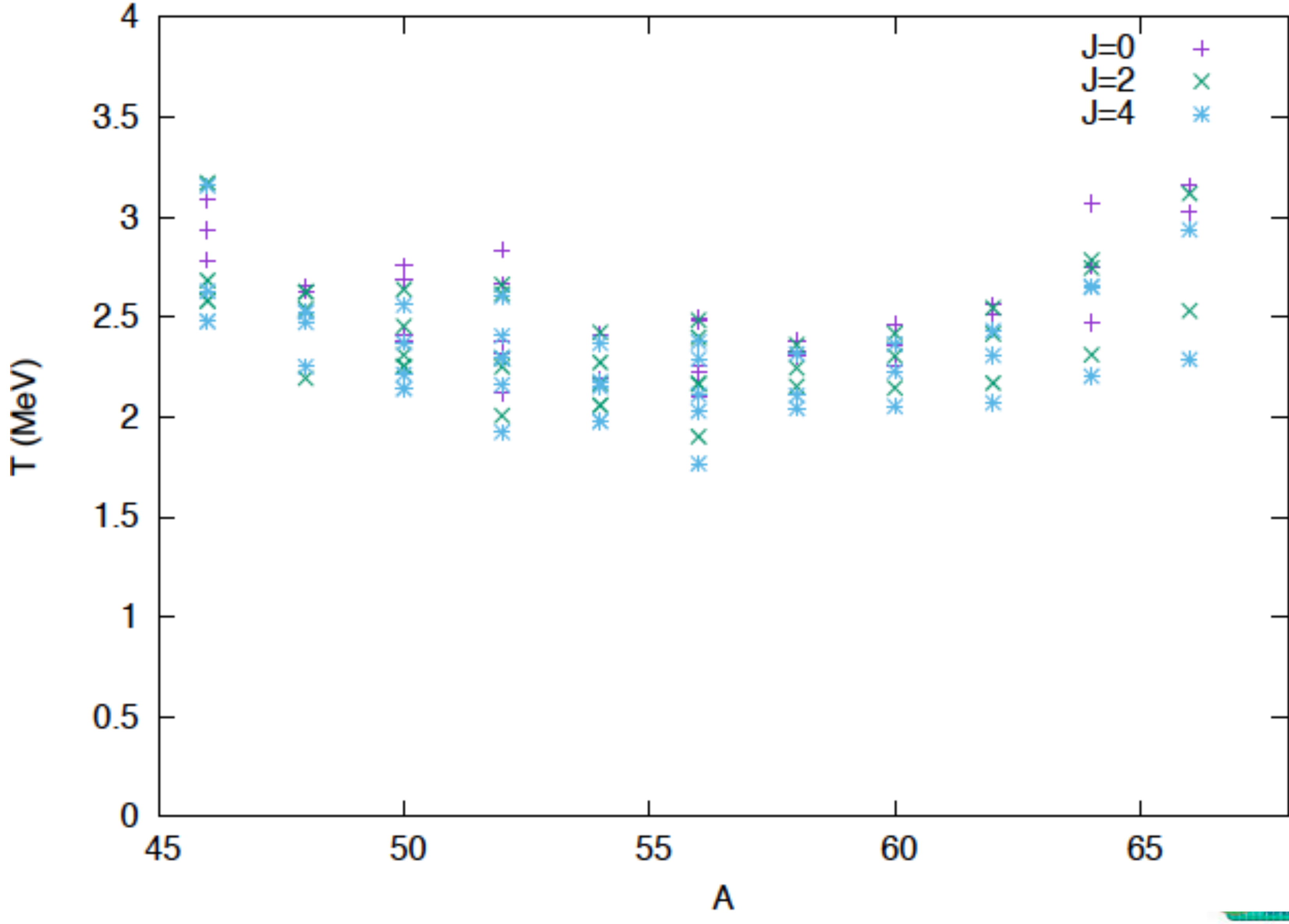

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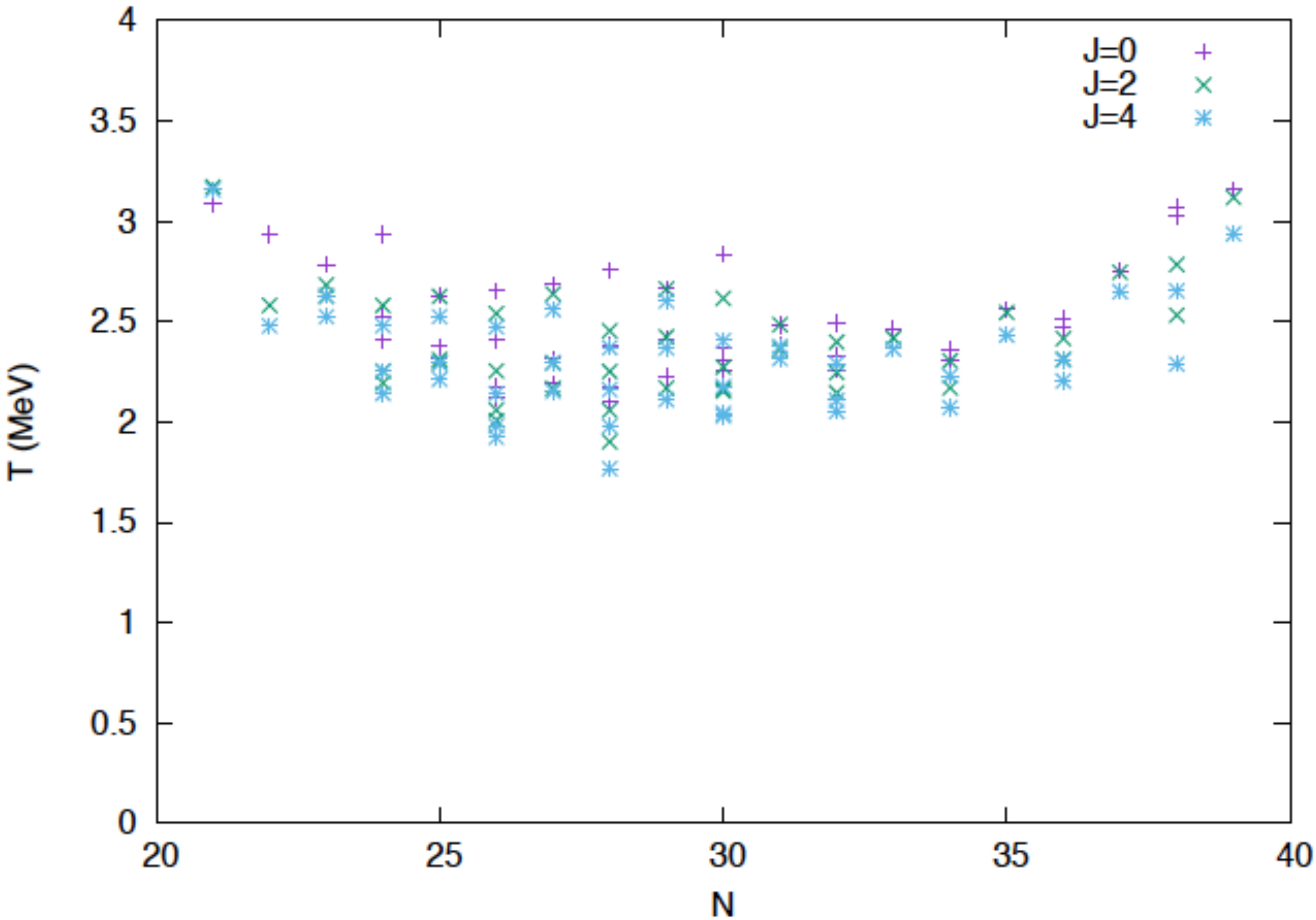

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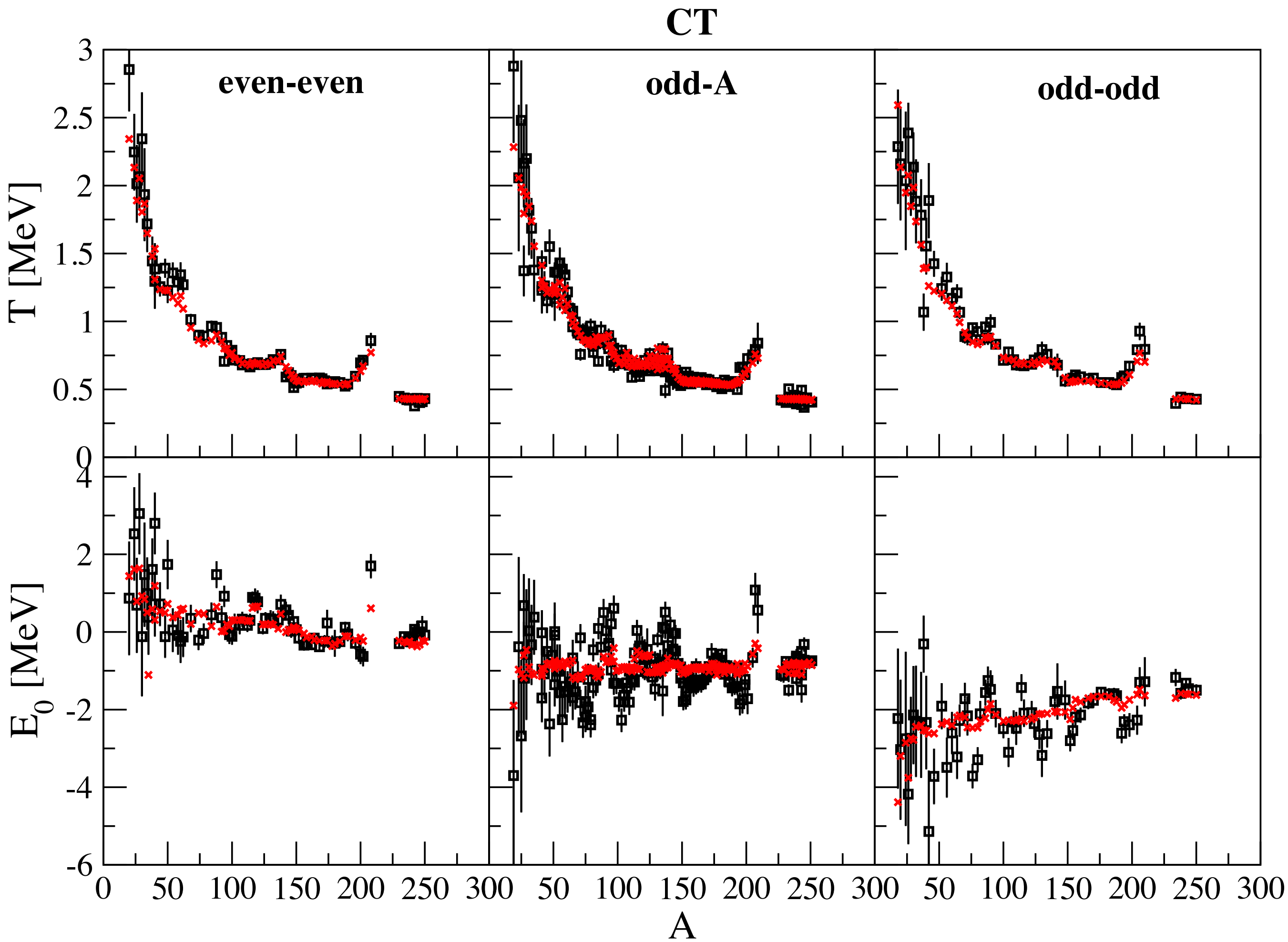

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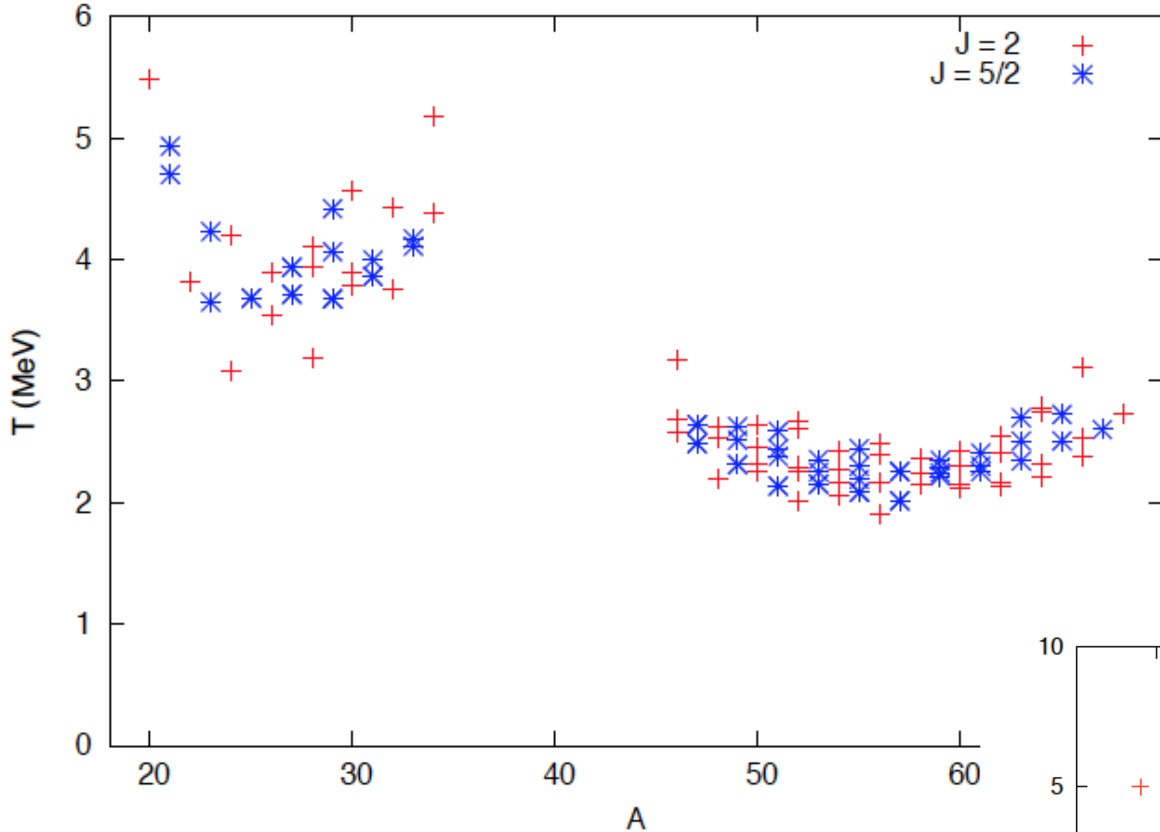
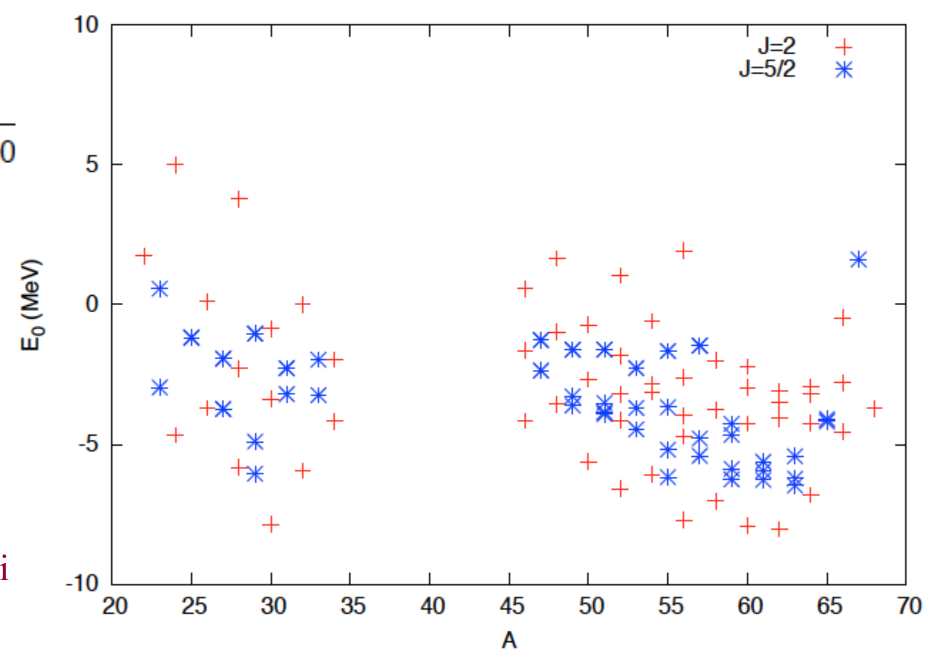
PRC 72, 044311 (2005), PRC 72, 067304, JoP Conf Ser 338, 012028 (2012)

Constant Temperature vs Moments NLD



$$\rho(E,J,\pi)=\frac{1}{T_{J\pi}}e^{\frac{E-E_{0J\pi}}{T_{J\pi}}}$$



Summary and Outlook

- ✓ Shell model techniques **describe** and **predict** a large amount of data in light, medium, and heavy nuclei:
  - ✓ Energies and quantum numbers, Electromagnetic transition probabilities, Spectroscopic amplitudes, Beta decay, charge exchange, 2ν/0ν double-beta decay
  - ✓ Spin and parity dependent nuclear level densities
- ✓ These observables are essential, but:
  - ✓ There is a clear need to obtain accurate effective Hamiltonians for enlarged, but tractable valence spaces.
  - ✓ Effective truncation scheme for configurations (partitions)
- ✓ Constant temperature description of the J-dependent shell model NLD represents a new and powerful technique:
  - ✓ Provides inside into the physics of nuclei as mesoscopic systems
  - ✓ Can provide an efficient interface of the shell model nuclear level densities to reaction codes

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