Chaos, Random Matrix Theory and Spectral Properties of the SYK Model

Jacobus Verbaarschot

jacobus.verbaarschot@stonybrook.edu

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Stony Brook University

Acknowledgments

Collaborators: Antonio Garcia-Garcia (Cambridge)

References

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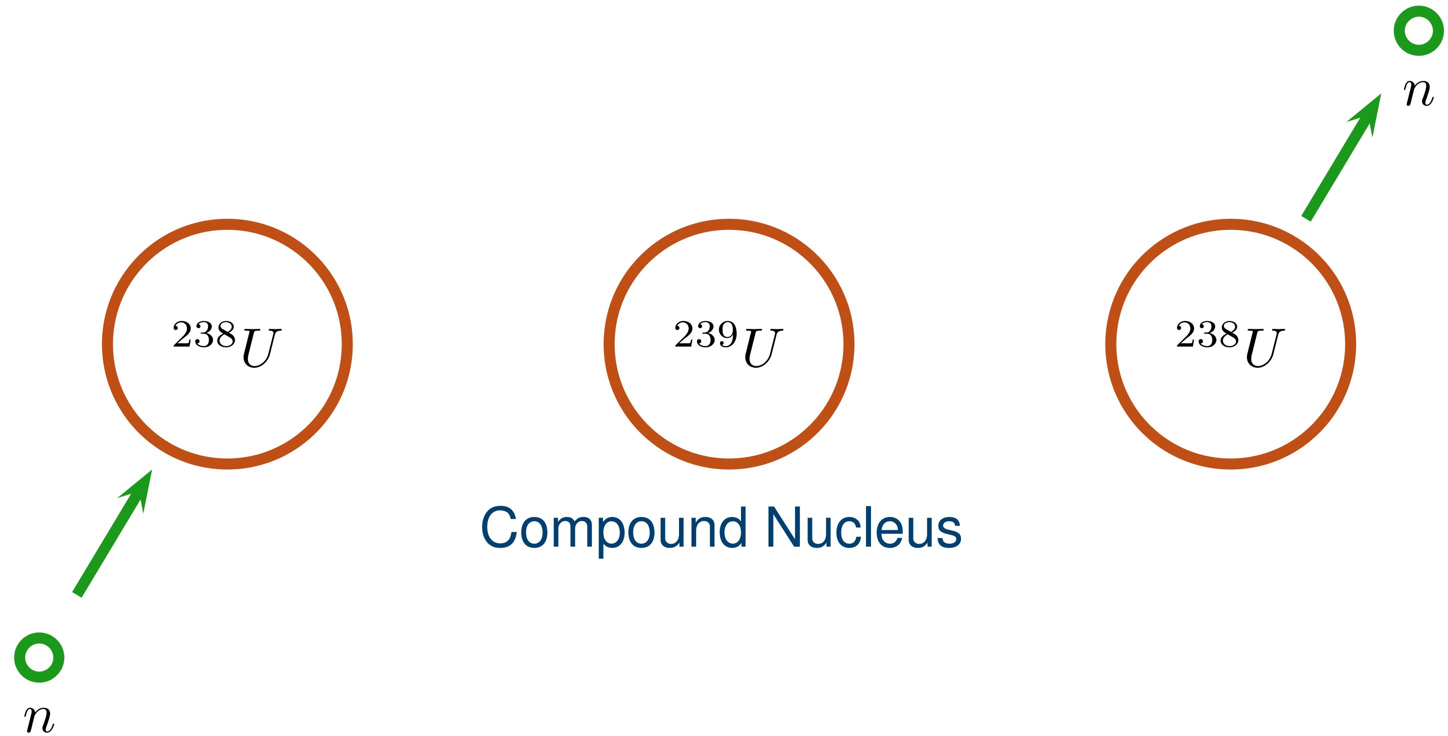
Quantum States of Black Hole

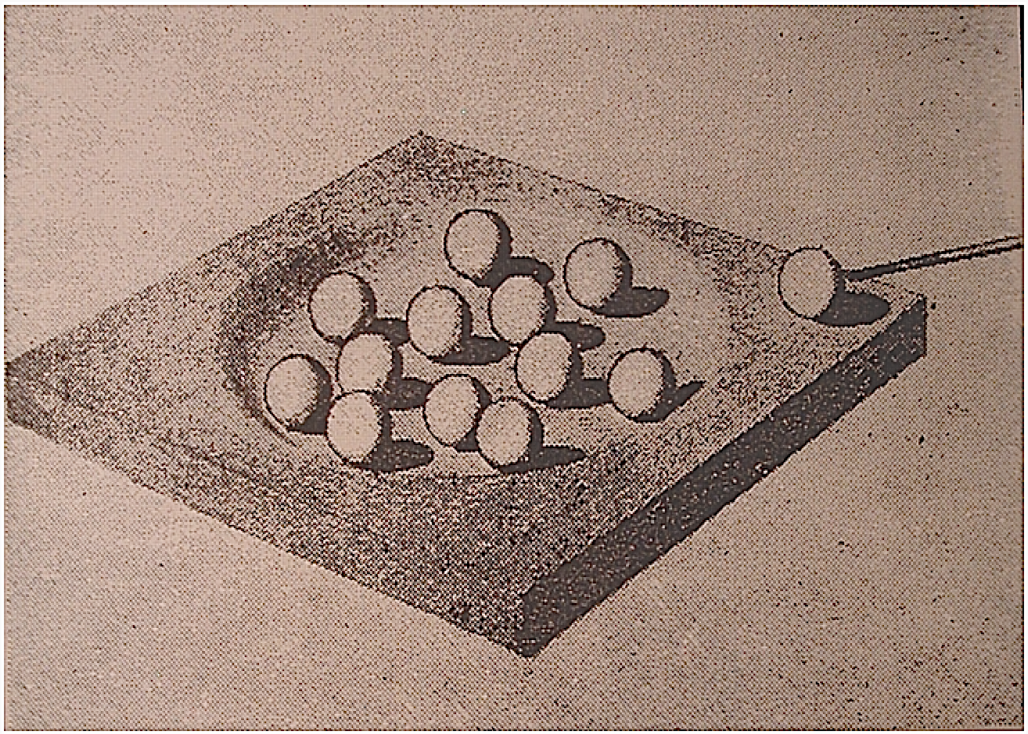
- A black hole is a finite system and therefore has discrete quantum states, in fact resonances because they decay.
- All information that goes into a black hole has been scrambled. Therefore, the information content of these quantum states should be minimized.
- What is the density of states?
- What are the correlations of the eigenvalues?
- Let us have a look at another physical system with these properties.

Compound Nucleus



- Formation and decay of a compound nuclear are independent.
- Because the system is chaotic, all information on its formation got lost.

Bohr's Model of a Compound Nucleus



Bohr, Nature 1934

Guhr-Müller-Groeling-WeidenmÏler-1999

Compound Nucleus is Chaotic

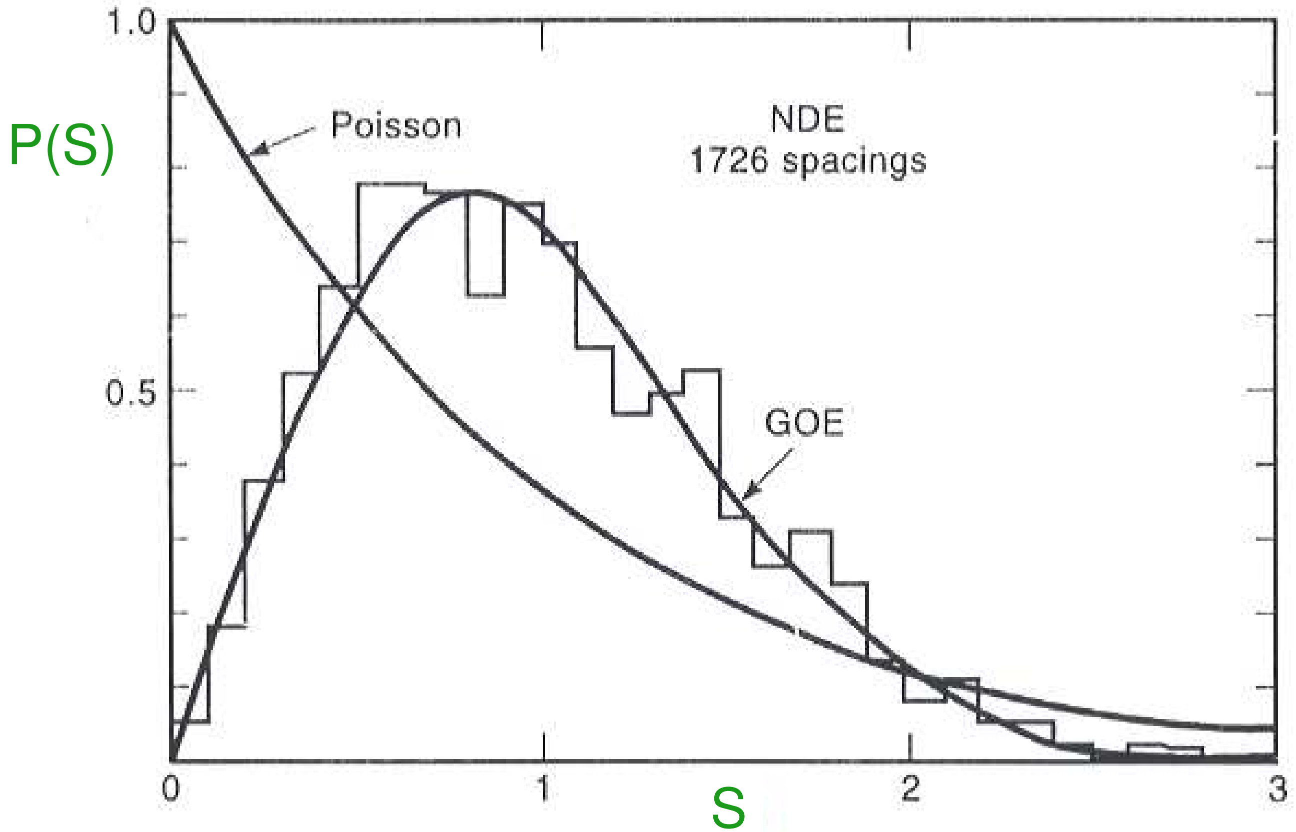
- Most likely a compound nucleus saturates the quantum bound on chaos obtained recently by Maldacena, Shenkar and Stanford. Black holes are believe to saturate this bound as well.
- To some extent, a compound nucleus has no hair, as is the case for a black hole.
- *Bohigas-Giannoni-Schmidt Conjecture*: If a system is classically chaotic, its eigenvalues are correlated according to random matrix theory.

Quantum Hair of a Compound Nucleus

Total cross section versus energy (in eV).

Garg-Rainwater-Petersen-Havens,1964

Nuclear Data Ensemble



Nearest neighbor spacing distribution of an ensemble of different nuclei normalized to the same average level spacing.

Bohigas-Haq-Pandey, 1983

Random Matrix Theory

H $\quad T \quad A \quad P(H)$

Hamiltonian → H

Anti-Unitary Symmetry → T

Anti-Commutator Symmetry → A

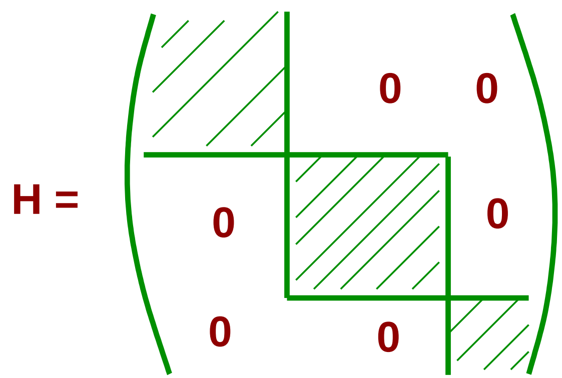
Probability Distribution → $P(H)$

Example: Time reversal invariant system,

$$T\psi = \psi^* \qquad (T^2 = 1)$$

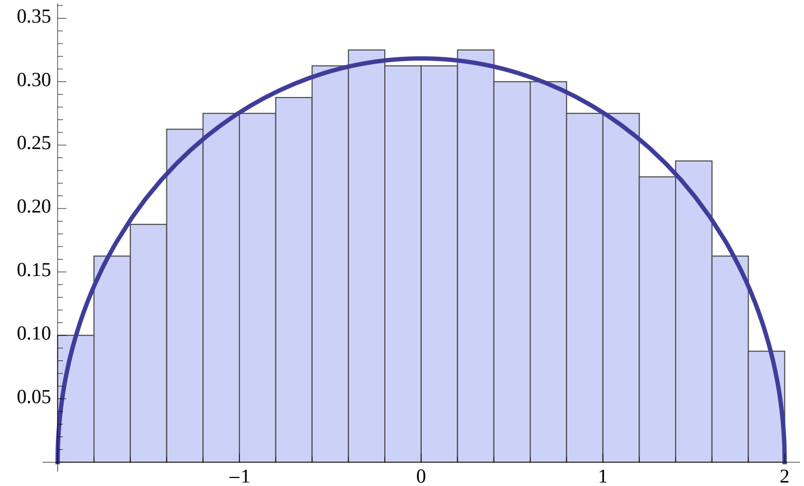
$$H^* = H\ , \qquad P(H) = e^{-N\,\mathrm{Tr}H^\dagger H}$$

$N \times N$ matrix → H



Wigner Semi-Circle

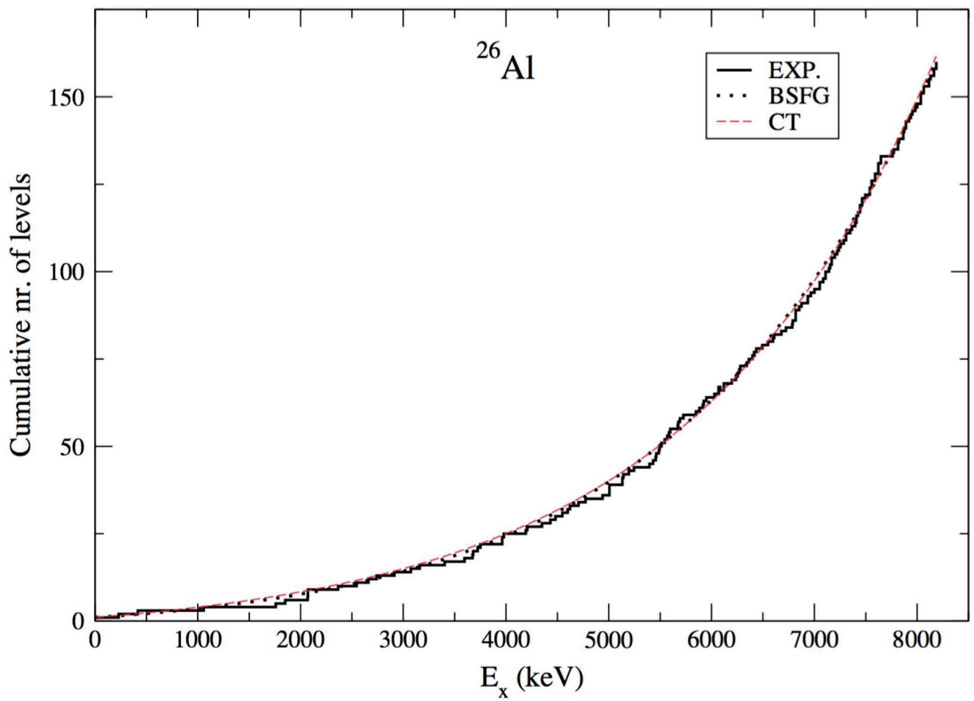
If the matrix elements are independent and have the same distribution, the eigenvalues are distributed according to as semi-circle in the limit of very large matrices



This is the case for a wide range of probability distributions which for convenience is usually taken to be a Gaussian, and a semicircular eigenvalue distribution is found for all 10 classes of random matrices.

Motivation for the Two-Body Random Ensemble

- The nuclear level density behaves as $e^{\alpha\sqrt{E}}$.
- The nuclear interaction is mainly a two-body interaction.
- Random matrix theory describes the level spacings, but it is and N-body interaction with a semicircular level density.



T. von Egidy

Two Body Random Ensemble

$$H = \sum_{\alpha\beta\gamma\delta} W_{\alpha\beta\gamma\delta} a_\alpha^\dagger a_\beta^\dagger a_\gamma a_\delta.$$

French-Wong-1970

Bohigas-Flores-1971

labels of the fermionic creation and annihilation operators run over N single particle states. The Hilbert space is given by all many particle states containing m particles with $m = 0, 1, \cdots, N$.

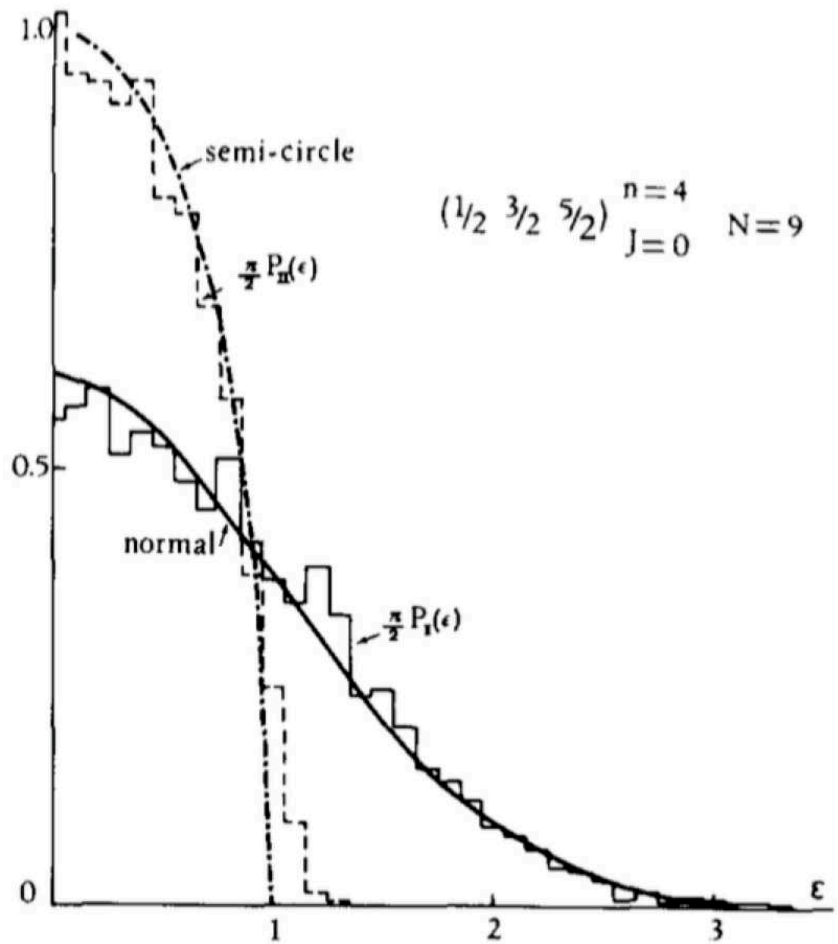
The dimension of the Hilbert space is: $\sum \binom{N}{m} = 2^N$.

- $W_{\alpha\beta\gamma\delta}$ is Gaussian random.
- The Hamiltonian is particle number conserving.
- The matrix elements of the Hamiltonian are strongly correlated.

Brody-et-al-1981, Brown-Zelevinsky-Horoi-Frazier-1997,

Izrailev-1990,Kota-2001,Benet-Weidenmüller-2002,Zelevinsky-Volya-2004

First Numerical Results



Comparison of the spectral density of the GOE and the two-body random ensemble for the sd-shell. Bohigas-Flores-1971

The Sachdev-Ye-Kitaev Model

The SYK Model

Partition Function

The Sachdev-Ye-Kitaev (SYK) Model

The two-body random ensemble from nuclear physics also has become known as the SYK model. However, being familiar with the history, we will only reserve this name for the two-body random ensemble with Majorana fermions Sachdev-Ye-1993,Kitaev-2015

$$H = \sum_{\alpha<\beta<\gamma<\delta} W_{\alpha\beta\gamma\delta}\chi_\alpha\chi_\beta\chi_\gamma\chi_\delta.$$

The fermion operators satisfy the commutation relations

$$\{\chi_\alpha, \chi_\beta\} = \delta_{\alpha\beta}.$$

The two-body matrix elements are taken to be Gaussian distributed with variance

$$\sigma^2 = \frac{6}{N^3}.$$

Hilbert space

Majorana particles are their own anti-particles, and the particle number is not a good quantum number.

The commutation relations are those of the Euclidean γ-matrices, and therefore the fermion operators can be represented as Euclidean gamma matrices with $\{\gamma_\alpha, \gamma_\beta\} = \delta_{\alpha\beta}$.

We can introduce

$$a_k = \gamma_{2k-1} + i\gamma_{2k}, \qquad a_k^* = \gamma_{2k-1} - i\gamma_{2k}$$

This gives $N/2$ creation operators resulting in a Hilbert space of dimension

$$\sum_{k=0}^{N/2} \binom{N/2}{k} = 2^{N/2}.$$

Spectrum and Partition Function

The partition function of N fermions with Hamiltonian H is given by

$$Z(\beta) = \mathrm{Tr} e^{-\beta H} = \int dE \rho(E) e^{-\beta E}.$$

The spectral density is thus given by the Laplace transform of the partition function.

The partition function can be interpreted as the trace of time evolution operator in imaginary time. Feynman told us how to rewrite the time evolution operator as a path integral.

Path Integral Formulation

$$Z(\beta) = \mathrm{Tr} e^{-\beta H} = \int D\chi e^{-\int_0^\beta d\tau \left[\chi \frac{d}{d\tau}\chi + H(\chi)\right]}.$$

where the χ are Grassmann valued functions of τ. See talk by Alex Kamenev.

Generally, we are interested in the free energy. The logarithm of the partition function can be calculated using the replica trick.

This offers an alternative way to study spectral properties which is complementary to the usual way of evaluating the generating function for the resolvent

$$\langle \det(H + z) \rangle.$$

Physical Interpretation

The partition function is that of a system of $N/2$ interacting fermions. The low-temperature expansion is thus given by

$$\begin{aligned}\beta F &= \beta E_0 + \frac{dF}{dT} + \frac{1}{2} T \frac{d^2 F}{dT^2} \\ &= \beta E_0 + S + \frac{1}{2} cT,\end{aligned}$$

where E_0 is the ground state energy, S is the entropy and cT the specific heat.

- E_0 , S and c are extensive.
- The total number of states for N fermions is $2^{N/2}$, so that the noninteracting part of the entropy is $S = \frac{N}{2} \log 2$.

Bethe Formula

The level density is given by the Laplace transform of the spectral density.

$$
\begin{aligned}
\rho(E) &= \int_{r-i\infty}^{r+i\infty} d\beta e^{\beta E} Z(\beta) \\
&= \int_{r-i\infty}^{r+i\infty} d\beta \beta^{-3/2} e^{\beta E} e^{-\beta E_0 + S + \frac{c}{2\beta}}
\end{aligned}
$$

The integral can be done resulting in

$$\rho(E) = \sinh(\sqrt{2cE}).$$

This gives the Bethe formula for the nuclear level density.

Bethe-1936

Spectral Density of the SYK Model

Large N Limit

Leading Corrections

Analytical Result for the Spectral Density

Bethe Formula

Spectral Density

The spectral density can be obtained from the moments

$$\langle \mathrm{Tr} H^{2p} \rangle = \mathrm{Tr} \langle \left(\sum_\alpha W_\alpha \Gamma_\alpha \right)^{2p} \rangle$$

with Γ_α a product of four gamma matrices. The Gaussian integral is equal to the sum over all pair-wise contractions.

When $2p \ll N$, the Γ_α do not have common gamma matrices and they commute. Since

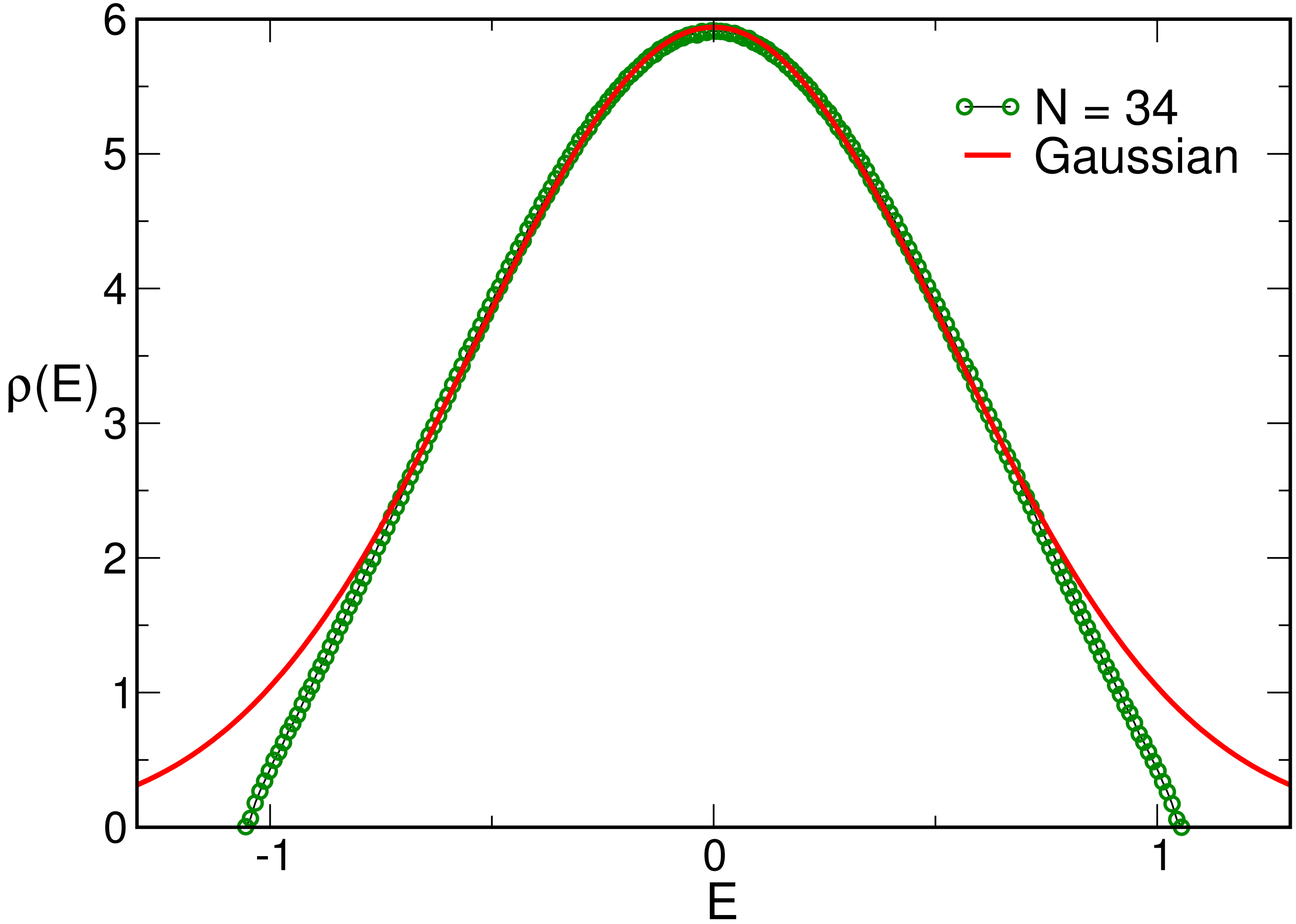
$$\Gamma_\alpha^2 = 1$$

all contractions contribute equally resulting in

$$\langle \mathrm{Tr} H^{2p} \rangle = (2p-1)!! \langle \mathrm{Tr} H^2 \rangle)^p$$

which gives a Gaussian distribution. Mon-French-1975, Garcia-JV-2016

Level Density



The center of the spectum is close to Gaussian but the tail deviates strongly. Garcia-JV-2016

Level Density and Partition Function

- $1/N$ corrections to the level density contribute to the free energy in the thermodynamical limit.

$$\rho(\lambda) = e^{Nf(E/E_0)} = e^{-Na_2(E/E_0)^2 + Na_4(E/E_0)^4 + \cdots} \quad \text{with} \quad E_0 \sim N$$

Partition function

$$Z(\beta) = \int dE e^{-\beta E} e^{-Nf(E/E_0)}$$

Saddle point equation

$$\beta = f'(\bar{E}/E_0) \quad \text{or} \quad \bar{E}/E_0 = f'^{-1}(\beta).$$

Partition function

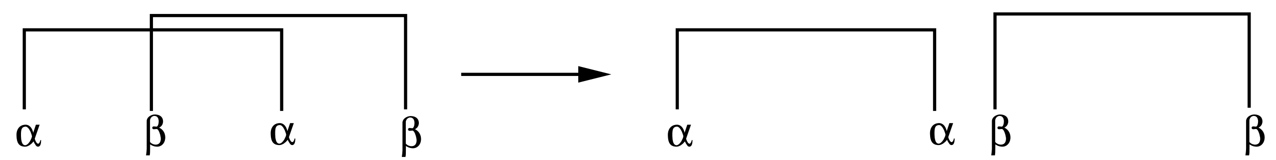
$$Z(\beta) = e^{-\beta E_0 f'^{-1}(\beta) + Nf(f'^{-1}(\beta))}.$$

Moments for Large N

- For large N, the moments can be calculated exactly if we ignore correlations between contractions.
- A product of four Majorana operators satisfies the commutation relations Garcia-Garcia-JV-2016

$$\Gamma_\alpha \Gamma_\beta + (-1)^p \Gamma_\beta \Gamma_\alpha = 0,$$

where p is the number of γ-matrices they have in common.



This results in the suppression factor of intersecting relative to nested contractions

$$\eta_{N,q} = \binom{N}{q}^{-1} \sum_p (-1)^p \binom{q}{p} \binom{N-q}{q-p}.$$

Spectral Density at Finite N

If α is the number of intersections, the moments are given by

$$\frac{M_{2p}}{M_2^p} = \sum_{\text{contractions}} \eta^\alpha = \frac{1}{(1-\eta)^p} \sum_{k=-p}^{p} (-1)^k \eta^{k(k-1)/2} \binom{2p}{p+k},$$

where the sum has been evaluated by the Riordan-Touchard formula.

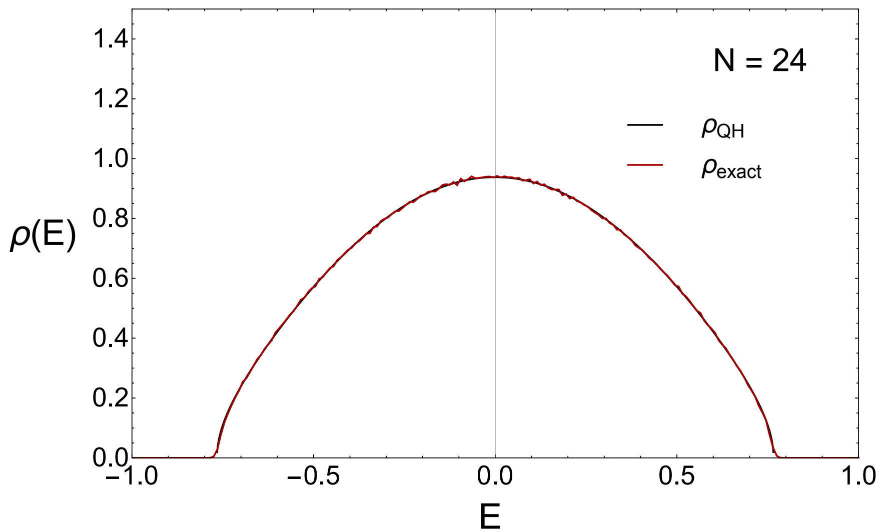
Erdos-2014, Cotler-et-al-2016, Garcia-Garcia-JV-2017

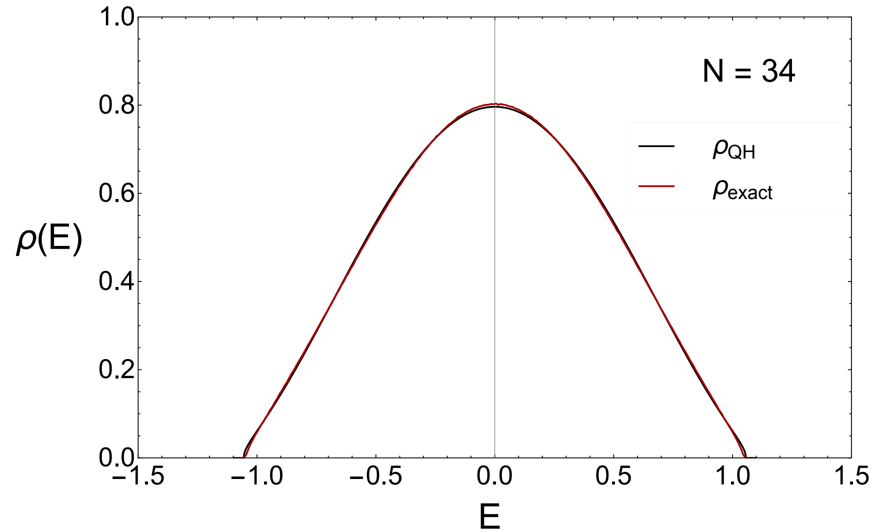
These are the moments corresponding to the weight function of the Q-Hermite Polynomials. This results in the spectral density

$$\rho_{\text{QH}}(E) = c_N \sqrt{1-(E/E_0)^2} \prod_{k=1}^{\infty} \left[1 - 4\frac{E^2}{E_0^2}\left(\frac{1}{2+\eta^k+\eta^{-k}}\right)\right]$$

with $E_0^2 = \frac{4\sigma^2}{1-\eta}$ and σ the variance of the spectral density.

Comparison with Numerical Results

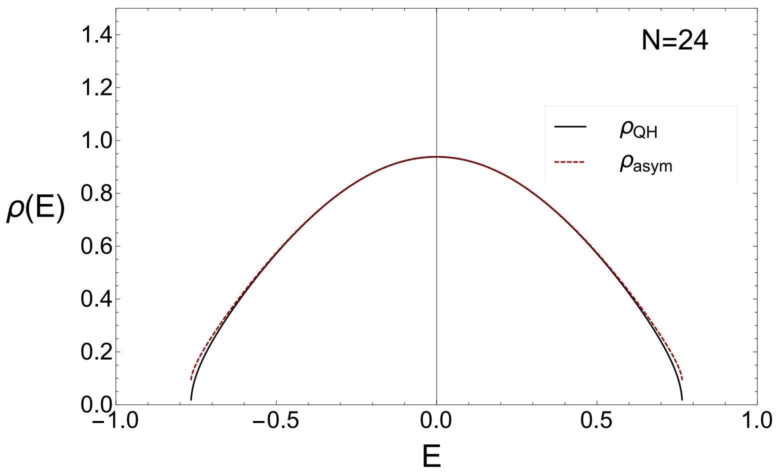


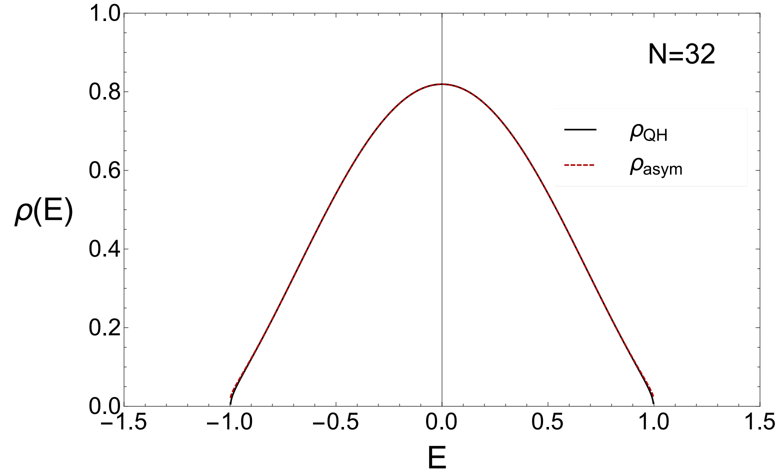


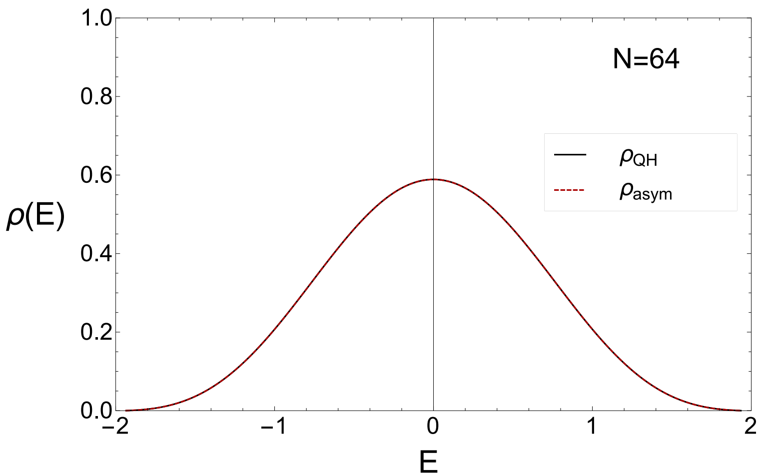
Comparison of the exact spectral density obtained by numerical diagonalization and the Q-Hermite result for the spectral density.

Garcia-Garcia-JV-2017

Simple Formula for Large N







For large N the density $\rho_{\rm QH}(E)$ is given by

$$\rho_{\rm asym}(E) \quad = \quad c_N \exp\left[\frac{2\arcsin^2(E/E_0)}{\log\eta}\right].$$

Garcia-Garcia-JV-2017

Bethe Formula

For large N, the Q-Hermite form is very well appproximated by

$$\rho(E) = c_N \exp\left[\frac{2\arcsin^2(E/E_0)}{\log\eta}\right]\left(1-\exp\left[-\frac{4\pi}{\log\eta}(|\arcsin(E/E_0)|-\frac{\pi}{2})\right]\right).$$

Very close to the ground state, the second term is $\sim\sqrt{1-E/E_0}$.
Because $\log\eta\sim 1/N$, it can be ignored otherwise.

Expansion near the ground state

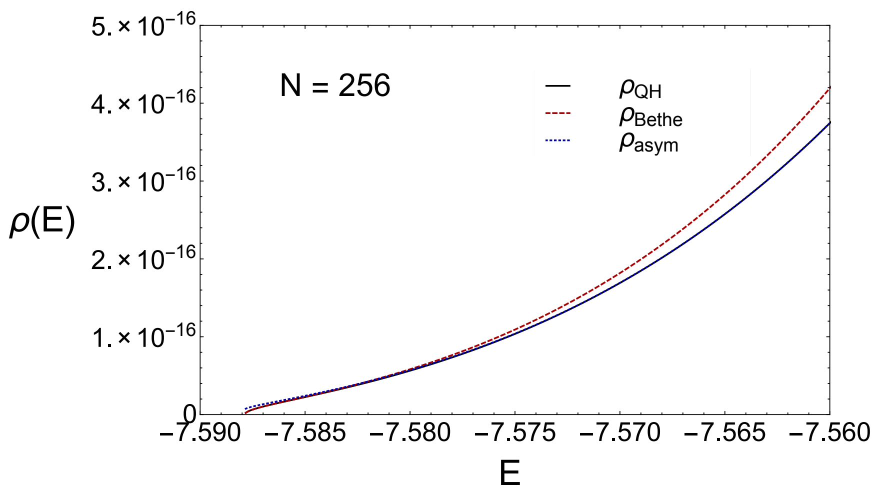
$$\arcsin^2((-E_0+x)/E_0) = \frac{\pi^2}{4} - \pi\sqrt{2x/E_0},$$

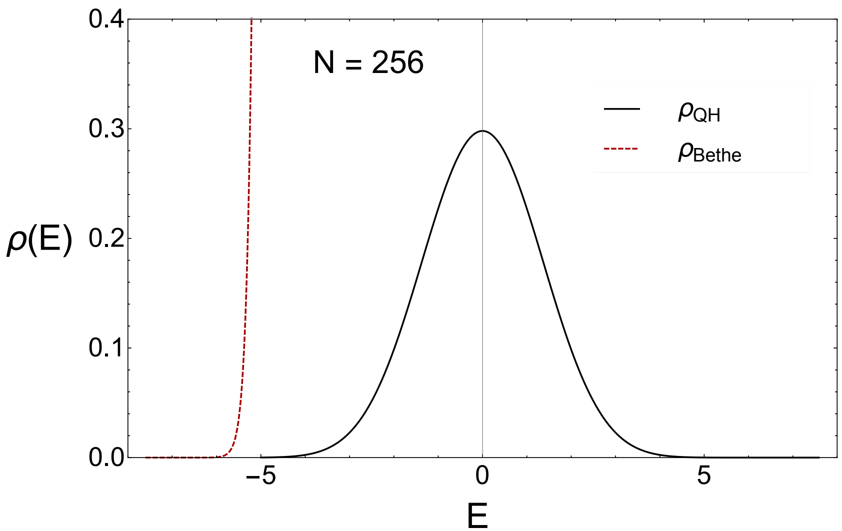
so that

$$\rho(E) = e^{\frac{N}{2}\log 2 - \frac{N}{q^2}\frac{\pi^2}{4}}\sinh\left[\frac{\pi N}{2q^2}\sqrt{2(1-E/E_0)}\right].$$

Cotler-et-al-2016, Garcia-Garcia-JV-2017,Altlang-Bagrets-Kamenev-2017

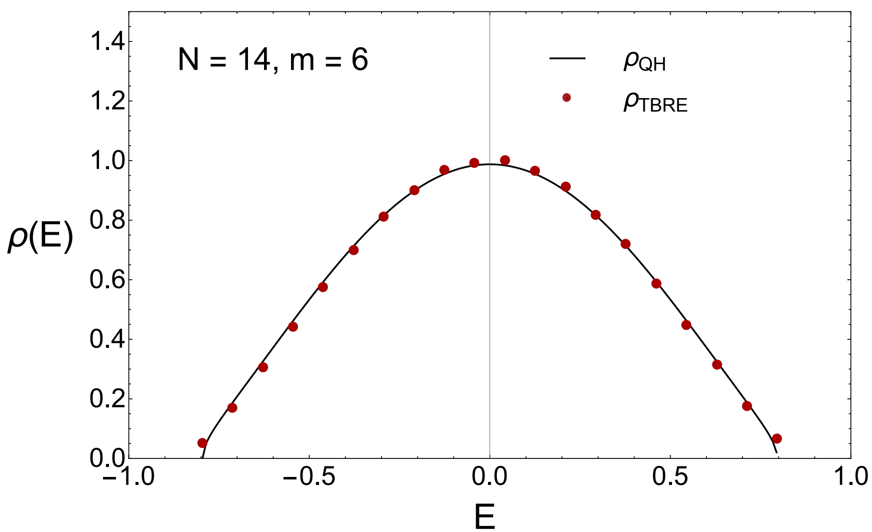
Comparison to the Bethe Formula

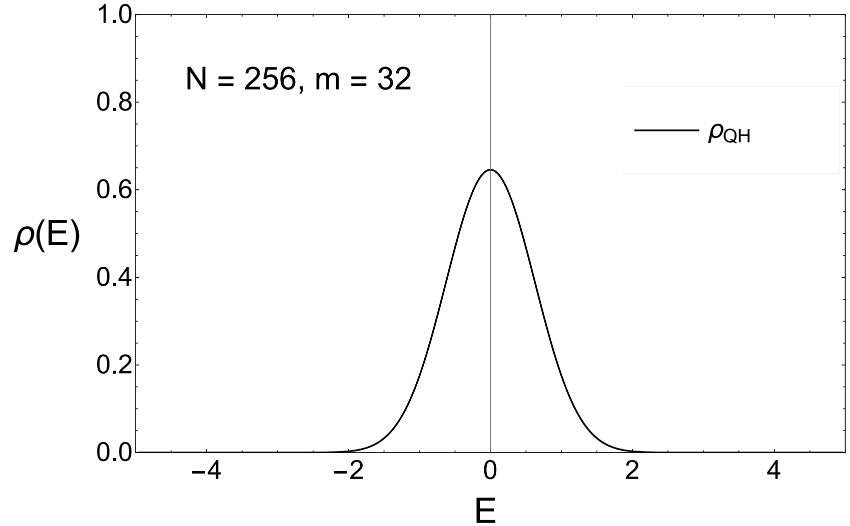




Comparison of the exact Q-Hermite result to the Bethe Formula, $\rho(E) \sim \sinh\sqrt{2c(E - E_0)}$. The Bethe formula is valid in the very tail where the density is non-Gaussian.

Two-Body Random Ensemble





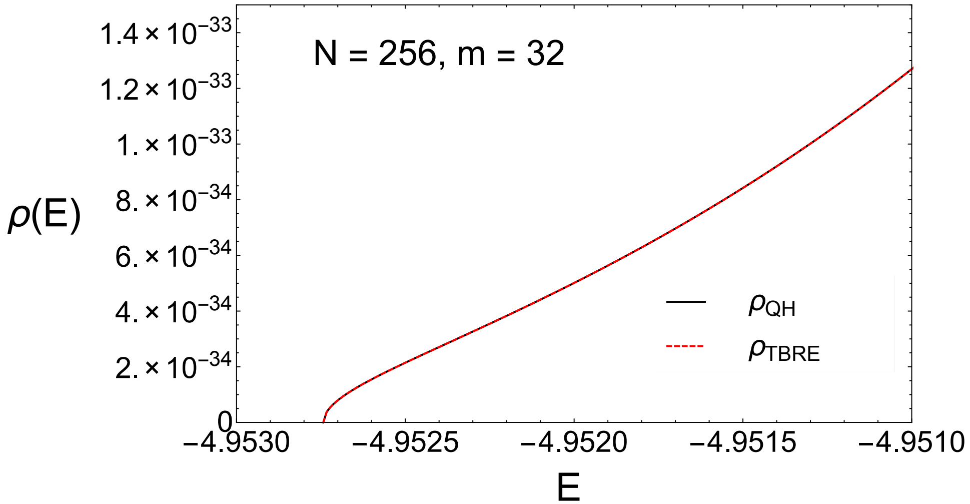
Numerical results for the two body random ensemble obtained by Kolovsky and Shepelyansky (2016) compared to the analytical result based on Q-Hermite polynomials which for large N is given by

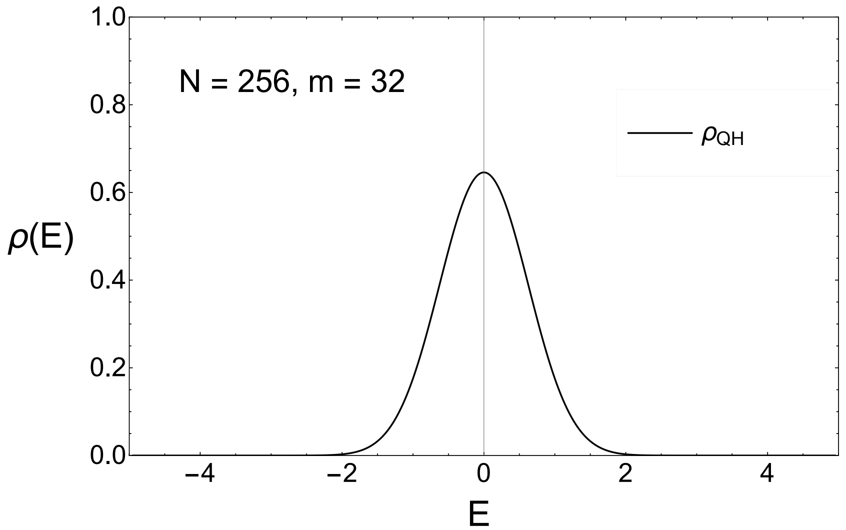
$$\rho_{\rm asym}(E) = c_N \exp\left[\frac{2\arcsin^2(E/E_0)}{\log\eta}\right].$$

with Garcia-Garcia-JV-2017

$$\eta = \left[\binom{N}{k}^{-1}\sum_{p=0}^{k}(-1)^p\binom{k}{p}\binom{N-k}{k-p}\right]^2, \qquad E_0^2 = \frac{2}{(1-\eta)N^3}\binom{N-m+k}{k}\binom{m}{k}.$$

Tail of the Two-Body Random Ensemble





Tail of the level density of the two-body random ensembel. Note that $\binom{256}{32} \approx 5.810^{40}$

Garcia-Garcia-JV-2017

Thermodynamical Properties of the SYK Model

Specific Heat and Entropy

Mean Field Calculation

Mean Field Calculation

The average over the Gaussian distribution is evaluated by a cumulant expansion.

$$\int dW_{\alpha\beta\gamma\delta} e^{-\int d\tau W_{\alpha\beta\gamma\delta}\chi_\alpha\chi_\beta\chi_\gamma\chi_\delta - W^2_{\alpha\beta\gamma\delta}/2\sigma^2} = e^{\frac{\sigma^2}{4!}N^4(\frac{1}{N}\sum_\alpha \chi_\alpha(\tau)\chi_\alpha(\tau'))^4}$$

Inserting the δ function

$$\delta(\frac{1}{N}\sum_\alpha \chi_\alpha(\tau)\chi_\alpha(\tau') - G(\tau,\tau'))$$

and writing the δ function as a Fourier integral results in

$$Z = \int D\Sigma DG e^{-S(\Sigma,G)}$$

with $S(\Sigma, G)$ the effective action.

Maldacena-Stanford-2015, Jevicki-Susuki-Yoon-2016

Saddle Point Approximation

$$S(\Sigma, G) = \frac{N}{2}[\mathrm{Tr}\log(\partial_\tau + \Sigma) + \frac{1}{4}G^4 + \Sigma G]$$

For large N , the integral can be evaluated by a saddle point approximation. The results in the mean field equations

$$-(\partial_\tau + \Sigma)G = 1, \qquad \Sigma = G^3 \quad \mathsf{Maldacena - Stanford - 2015}.$$

When ∂_τ can be neglected, the saddle point equations have a reparameterization invariance, $\tau \to f(\tau)$, and the corresponding soft modes have to be included to evaluate the Green's function.

Altland-Bagrets-Kamenev-2016/2017,Jevicki-Susuki-2016

The mean field equations can be solved in terms of a $1/q$ expansion resulting in the entropy (versus our values of 0.21 and 0.43)

$$\frac{S}{N} = \frac{1}{2}\log 2 - \frac{\pi^2}{64} = 0.19, \qquad c = \frac{\pi^2}{16\sqrt{2}} = 0.44,$$

Analytical Result for the Partition Function

$$Z(\beta) = \int e^{-\beta E} \exp\left[\frac{2\arcsin^2(E/E_0)}{\log\eta}\right].$$

The integral can be calculated by a saddle point approximation

$$Z(\beta) = \int e^{-\beta\bar{E}} \exp\left[\frac{2\arcsin^2(\bar{E}/E_0)}{\log\eta}\right], \quad \text{with} \quad \beta = \frac{4}{E_0\log\eta}\frac{\arcsin(\bar{E}/E_0)}{\sqrt{1-(\bar{E}/E_0)^2}}.$$

For large N , this result is equal to the result of Maldacena and Standord for any β . The low temperature expansion is given by

$$Z(\beta) \sim \frac{1}{\beta^{3/2}} \exp\left[\beta E_0 + \frac{N}{2}\log 2 - \frac{N}{q^2}\frac{\pi^2}{4} + \frac{1}{\beta}\frac{N}{q^2}\frac{\pi^2}{q^2 E_0/N}\right],$$

which is obtained identically from the Bethe formula,
$\rho(E) \sim \sinh(\sqrt{2c(E-E_0)})$.

Spectral Correlations of the SYK Model

Spectral Correlators

Symmetries and Classification

Upper Bound for Lyapunov Exponent

Lyapunov exponent λ

$$\Delta(t) \sim \Delta(0)e^{\lambda t}$$

Energy-time “unertainty relation”

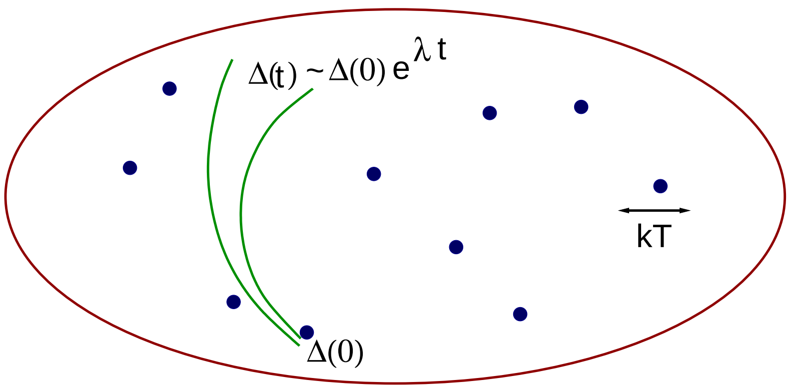
$$\begin{aligned} \Delta t \Delta E &\geq \frac{\hbar}{2} \\ \Delta t &\sim 1/\lambda, \qquad \Delta E \sim \pi k T \end{aligned}$$

So we have the bound

$$\lambda \leq \frac{2\pi k T}{\hbar}$$

Maldacena-Shenker-Stanford-2015

Of the same type at the η/S bound of Son.



Divergence of trajectories in a stadium at temperature T

Spectral Correlations

It has been shown hat the SYK model in maximally chaotic, in the sense that the Lypunov exponent saturates the bound of

$$\lambda_L \leq \frac{2\pi kT}{\hbar}.$$

Kitaev-2015, Maldacena-Shenker-Stanford-2016

- If this is the case its spectrum should behaves as a quantum chaotic system, i.e. the eigenvalue correlations are given by random matrix theory with the corresponding random matrix ensemble determined by the anti-unitary symmetries.
- Black holes also saturate this bound which explains the current interest in this model.
- The quantum properties of black holes are similar to those of compound nuclei.

Classification Summary

N	$(C_1K)^2$	$(C_2K)^2$	C_1KC_2K	RMT	Matrix Elements
2	1	-1	$-i\Gamma_5$	GUE	Complex
4	-1	-1	$-\Gamma_5$	GSE	Quaternion
6	-1	1	$-i\Gamma_5$	GUE	Complex
8	1	1	Γ_5	GOE	Real
10	1	-1	$-i\Gamma_5$	GUE	Complex
12	-1	-1	Γ_5	GSE	Quaternion

Table 1: (Anti-)Unitary symmetries of the SYK Hamiltonian and the corresponding random matrix ensemble. The symmetries are periodic in N modulo 8 (Bott periodicity).

You-Ludwig-Xu-2016, Garcia-Garcia-JV-2016

Spectral Observables

- $P(S)$: the distribution of the spacing of consequetive levels.
- $\Sigma^2(L)$: the variance of the number eigenvalues in an interval that contains L levels on average.
- Spectral form factor

$$g(\beta, t) = \sum_{k,l} e^{-(\beta+it)E_k-(\beta-it)E_l}$$

- These spectral observables are calculating after mappig the spectrum on one with unit average level density. The mapping function is obtained from the average spectral density.
- To increase statistics we can perform a spectral average in addition to andensemble average for the calculation of of $P(S)$ and $\Sigma^2(L)$.

Spectral Correlations

Spectral Density

$$\rho(x) = \left\langle \sum_k \delta(x - E_k) \right\rangle .$$

Two point correlation function

$$\begin{aligned}\rho_2(x,y) &= \left\langle \sum_{kl} \delta(x - E_k)\delta(x - E_l) \right\rangle \\ &= \delta(x-y)\rho(x) + \left\langle \sum_{k \neq l} \delta(x - E_k)\delta(x - E_l) \right\rangle .\end{aligned}$$

The first term is due to self-correlations.

The connected correlator is given by

$$\rho_{2c} = \rho_2(x,y) - \rho(x)\rho(y).$$

Number Variance and Spectral Form Factor

Number variance

$$\Sigma^2(n) = \int_{x_0}^{x_0+n} \int_{x_0}^{x_0+n} dx dy \rho_{2c}(x,y)$$

$$\Sigma^2_{\rm self}(n) = n.$$

Spectral form factor

$$g(\beta, t) = \int dx dy e^{-(\beta+it)x-(\beta-it)y} \rho_2(x,y).$$

Can be split into a connected part, a disconnected part and a part due to the self correlations. The part due to self correlations is given by

$$g_{\rm self}(t) = \int dx \rho(x) e^{-2\beta x} = {\rm constant}.$$

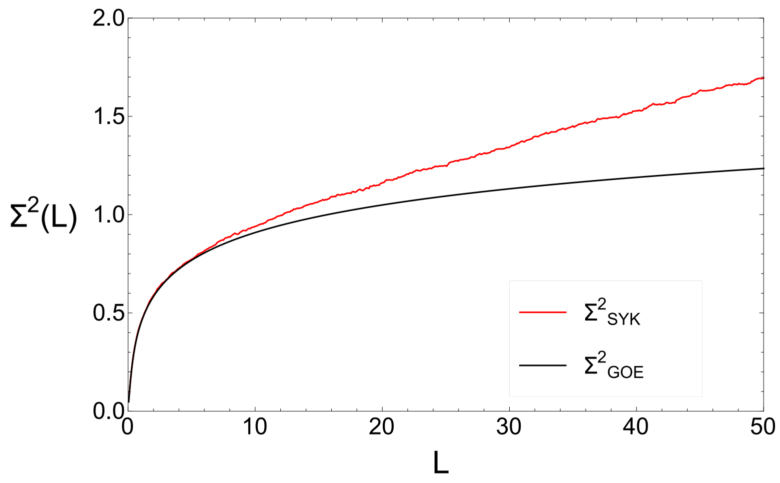
Disconnnected Part of the Spectral Form Factor

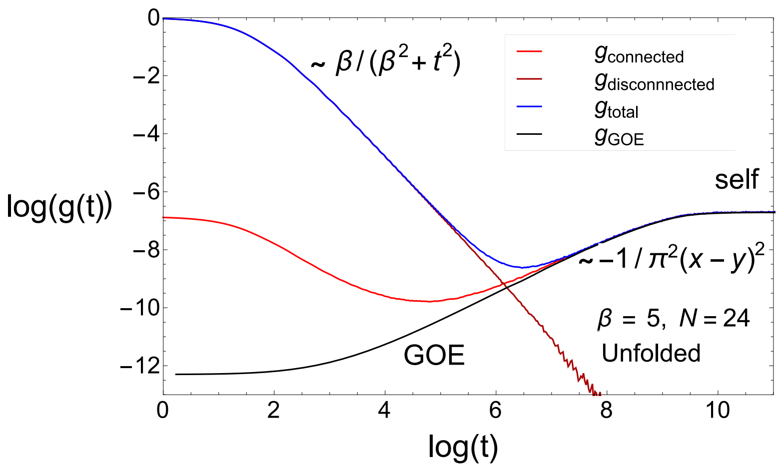
The disconnected contribution can be written as

$$g_{\text{disconnected}} = \left| \int dx e^{-(\beta+it)x} \rho(x) \right|^2 .$$

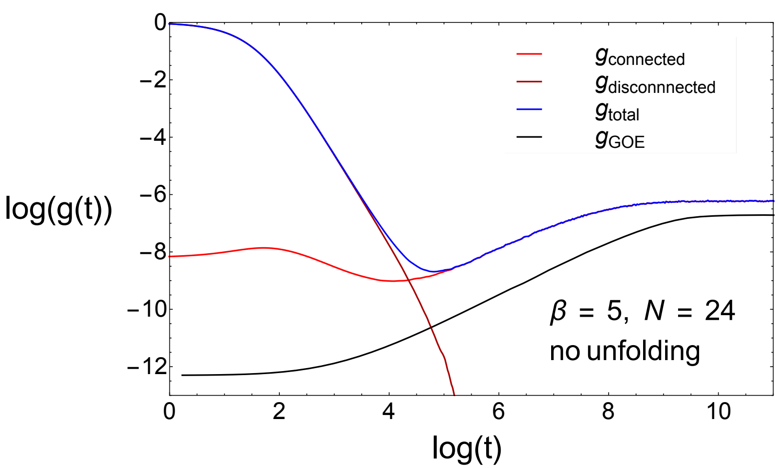
This part for the form factor contains no information on eigenvalue correlations. The chaotic properties of the system are contained in the connnected form factor

Number Variance Versus Spectral Form Factor





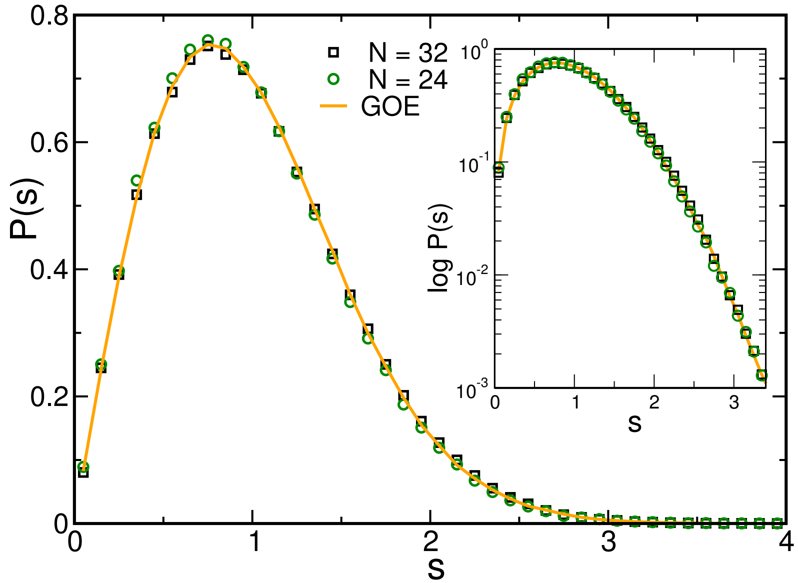
Number variance (left) and spectral form factor (right). $\Sigma^2(L)$ is calculated starting at the 50th eigenvalue above the ground state.

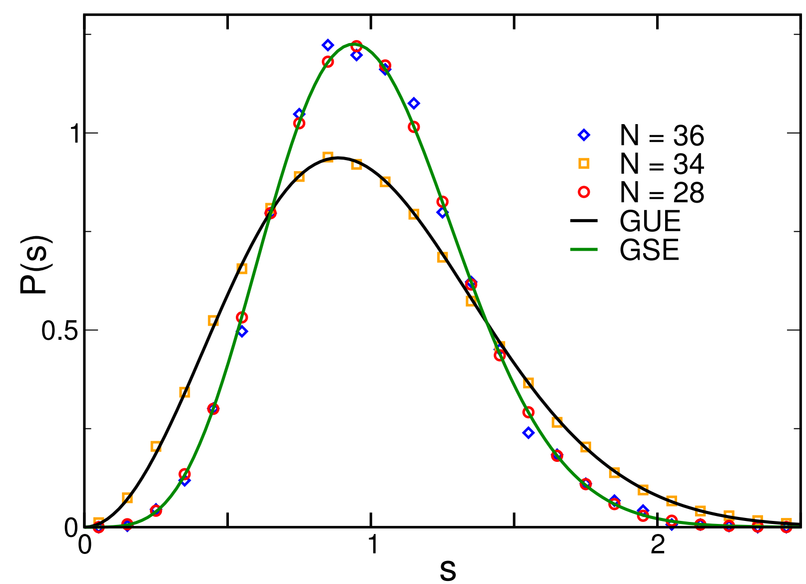


Garcia-Garcia-JV-arXiv:1610.02363

Cotler-et-al-arXiv:1611.04650

Nearest Neigbor Spacing Distribution



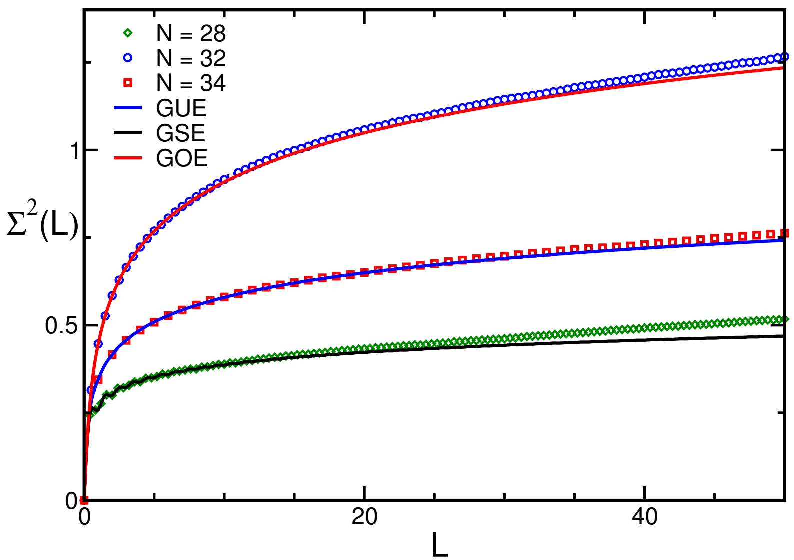


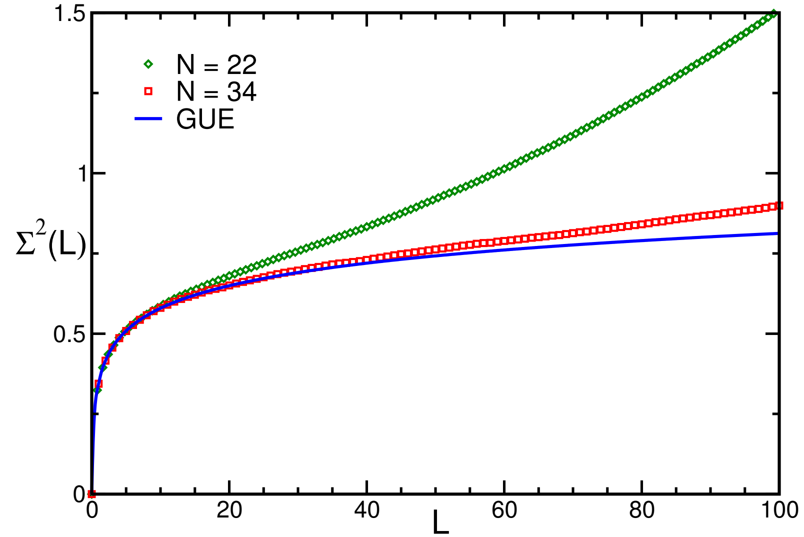
Nearest nieghbor spacing distribution for the bottom (left) and bulk part of the spectrum compared to random matrix theory.

Garcia-Garcia-JV-2016, Garcia-Garcia-JV-2017

This is in agreement with results for the distribution of the ratio of consequetive spacings. You-Ludwig-Xu-2016

Number Variance in the Bulk



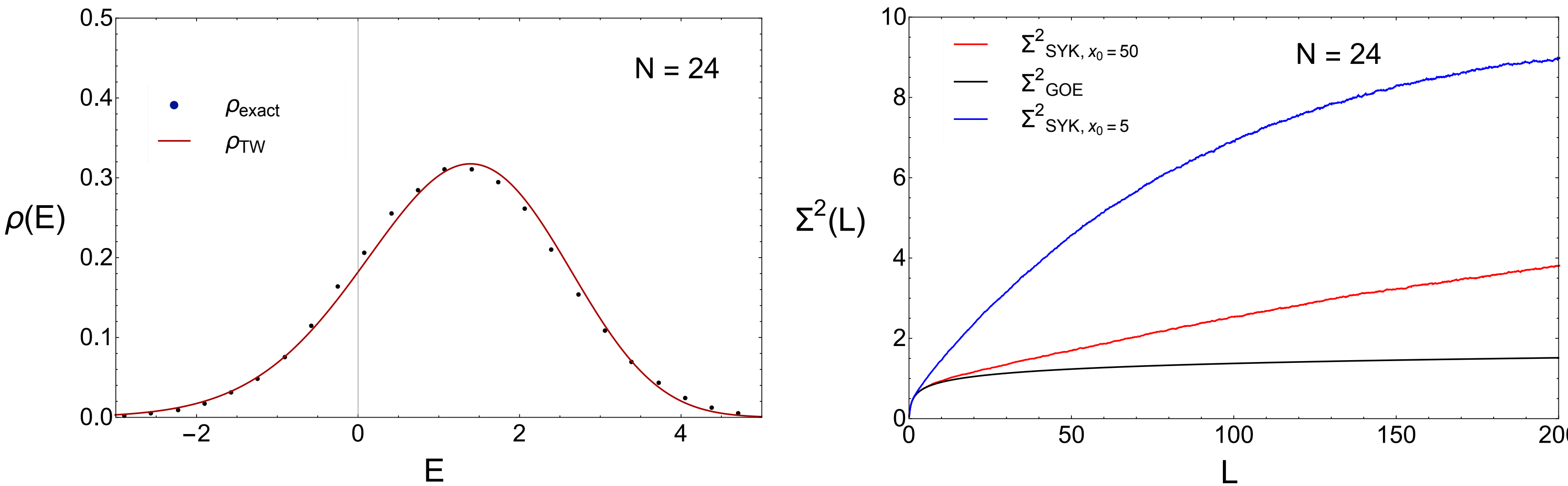


Garcia-Garcia-JV-2016

These results have been confirmed by an independent collaboration who calculated the spectral form factor which is the Fourrier transform of the spectral correlator.

Cotler-Gur-Ari-Hanada-Polchinski-Saad-Shenker-Streicher-Tezuka-2016

Tracy-Widom Distribution



Distribution of the ground state energy compared to the Tracy-Widom distribution of the Gaussian Orhtogonal Ensemble. There is no fitting – the parameter of the Tracy-Widom distribution is fixed by equating its expectation value to the numerical one, at the point $E = 0$, is edge of the spectrum as predicted by the Q-Hermite expresssion.

Garcia-Garica-JV-2017

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